Thermal evolution of neutron stars in an alternative theory of gravitation

Martin Javier Nava Callejas

IReNA-INT Joint Workshop on Thermal and Magnetic Evolution of Neutron Stars

December 12th, 2024

Main goal:

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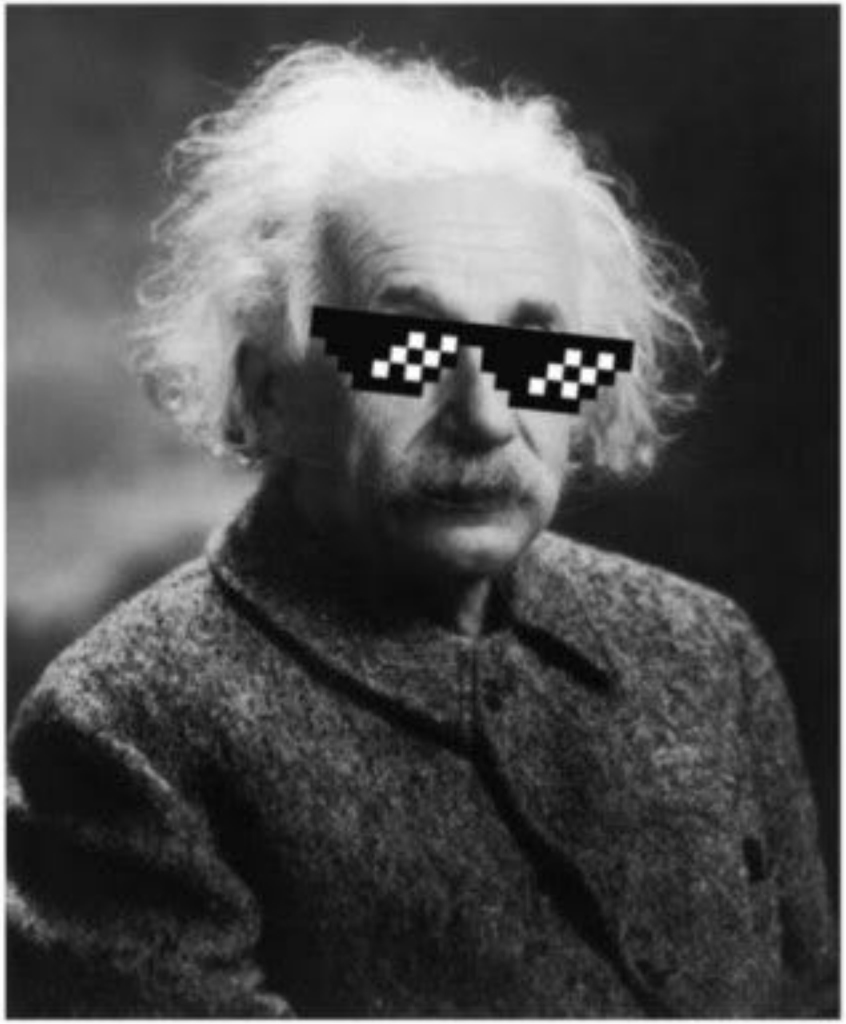

Source: https://medium.com/quantum-outreach/tagged/einstein

Part 1

Why?

Not Wye*

*For “Prelude to Foundation” fans

Again: Why go alternative?

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- Because alternative music is awesome!

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Main reason: scientific curiosity!

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First step: isolated neutron stars!

Modifications: the arrival

- **Stellar evolution: structure & energy transport**

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$g_{\mu\nu}(e^{\Phi}, e^{\Lambda})$$

$\approx$ Time Independent

$$\nabla_\mu (n_{\mathrm{B}} u^\mu) = 0$$

(Baryonic mass conservation)

$$\nabla_\mu (s u^\mu) = \Gamma_{\mathrm{heat}} - \Gamma_{\mathrm{cool}}$$

(Entropy equation)

$$F^\mu = -K\Pi^{\mu\nu}\left[\partial_\nu T + T a_\nu\right]$$

(Flux)

Time Dependent

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(Baryonic mass co

Let's modify structure first!

∇ (

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(Flux)

Time Dependent

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Matter-gravity coupling is another possibility (but requires special care!)

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(Flux)

≈ Time Independent

Time Dependent

Modifications: the arrival

- Stellar ... **ructure** **&** **energy transport**

Matter-gravity coupling is another possibility (but requires special care!)

(However, these equations will be affected as well by different structure!)

$$\nabla_\mu(n_\mathrm{B}u^\mu) = 0$$

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$$\nabla_\mu(su^\mu) = \Gamma_\mathrm{heat} - \Gamma_\mathrm{cool}$$

(Entropy equation)

$$F^\mu = -K\Pi^{\mu\nu}\left[\partial_\nu T + Ta_\nu\right]$$

(Flux)

Time Dependent

Modifications: a visit to the Lagrangian

(How are alternative theories of gravity constructed?)

Instead of $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, focus on the Lagrangian density $\mathscr{L}_{\text{grav}}$

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$$\mathscr{L}_{\mathrm{grav,\ Hilbert-Einstein}} \propto R$$

$$\mathscr{L}_{\mathrm{grav,new}} \propto f(R) + f(\mathrm{Other\ \ fields})$$

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$$\mathscr{L}_{\text{grav,new}} \propto f(R) + f(\text{Other fields})$$

$$\mathscr{L}_{\text{grav,attemped}} \propto R + \alpha R^2$$

Modifications: learning to fly

(How are alternative theories of gravity constructed?)

$8\pi G$

Recipe:

1. Construct your Lagrangian,
2. Derive the equations for the structure
3. Solve the equations for structure!

$$\mathscr{L}_{\text{grav,attemped}} \propto R + \alpha R^2$$

"New" equations of structure

- Second order ODEs

(remaining 1D!)

- Scalar curvature becomes a "new" independent variable

- "Free" parameter α

$\alpha \sim 10^{14}$ cm^2

to observe differences with GR

$$\frac{d\Phi}{dr} = \frac{1}{A_1}\left\{\frac{4\pi G P r e^{2\Lambda}}{c^4} - \frac{[A_2 e^{2\Lambda} + 2A_3]}{4r}\right\}, \tag{15}$$

$$\frac{d\Lambda}{dr} = \frac{1}{A_1}\left\{\frac{4\pi G \varepsilon r e^{2\Lambda}}{c^4} + \frac{[A_2 e^{2\Lambda} + 2A_3]}{4r} + \alpha r \frac{d^2R}{dr^2}\right\}, \tag{16}$$

$$\frac{d^2R}{dr^2} = \frac{1}{A_6}\left\{\frac{A_1 A_4 e^{2\Lambda}}{6\alpha} - \frac{1}{r}\left[A_5 - \frac{A_2 e^{2\Lambda}}{2}\right]\frac{dR}{dr}\right\}, \tag{17}$$

where, for simplicity of notation, the following functions have been defined:

$$A_1 = 1 + 2\alpha R + \alpha r \frac{dR}{dr}, \tag{18}$$

$$A_2 = \alpha r^2 R^2 - 4\alpha R - 2, \tag{19}$$

$$A_3 = 1 + 2\alpha R + 4\alpha r \frac{dR}{dr}, \tag{20}$$

$$A_4 = \frac{8\pi G}{c^4}(3P - \varepsilon) + R, \tag{21}$$

$$A_5 = \frac{4\pi G}{c^4} r^2 e^{2\Lambda}(P - \varepsilon) + 1 + 2\alpha R - 2\alpha r \frac{dR}{dr}, \tag{22}$$

$$A_6 = 1 + 2\alpha R. \tag{23}$$

From the r-component of the energy-momentum tensor conservation law, Eq. (10), we deduce the differential equation for the pressure,

$$\frac{dP}{dr} = -(P + \varepsilon)\frac{d\Phi}{dr}. \tag{24}$$

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to observe differences with GR

NO “experimental” consensus:

$\alpha \sim 10^{-6}\ \mathrm{cm}^2$ Earth deviations from Newton

$\alpha \sim 10^{15-19}\ \mathrm{cm}^2$ Gravity Probe B & Binary pulsar PSRJ0737-3039

$$\frac{d\Phi}{dr} = \frac{1}{A_1}\left\{\frac{4\pi G P r e^{2\Lambda}}{c^4} - \frac{[A_2 e^{2\Lambda} + 2A_3]}{4r}\right\}, \qquad (15)$$

$$\frac{d\Lambda}{dr} = \frac{1}{A_1}\left\{\frac{4\pi G \varepsilon r e^{2\Lambda}}{c^4} + \frac{[A_2 e^{2\Lambda} + 2A_3]}{4r} + \alpha r \frac{d^2 R}{dr^2}\right\}, \qquad (16)$$

$$\frac{d^2 R}{dr^2} = \frac{1}{A_6}\left\{\frac{A_1 A_4 e^{2\Lambda}}{6\alpha} - \frac{1}{r}\left[A_5 - \frac{A_2 e^{2\Lambda}}{2}\right]\frac{dR}{dr}\right\}, \qquad (17)$$

where, for simplicity of notation, the following functions have been defined:

Structure has been shown as completely different in perturbative vs non-perturbative schemes, so maybe Earth experiments cannot “constraint” this value

Modifications: the revival

- **Stellar evolutio ructure & energy transport**

No change in energy transport = NSCool can be safely used!

No jacket required!*

$\approx$ Ti ndependent

$$\nabla_\mu (n_B u^\mu) = 0$$

(Baryonic mass conservation)

$$\nabla_\mu (s u^\mu) = \Gamma_{\text{heat}} - \Gamma_{\text{cool}}$$

(Entropy equation)

$$F^\mu = -K \Pi^{\mu\nu} \left[\partial_\nu T + T a_\nu \right]$$

(Flux)

Time Dependent

*For Phil Collins fans. (Yet, Genesis was better with Peter!)

Checking old results: maximum mass increases!

PROBING STRONG FIELD $f(R)$ GRAVITY AND … PHYS. REV. D **107,** 104057 (2023)

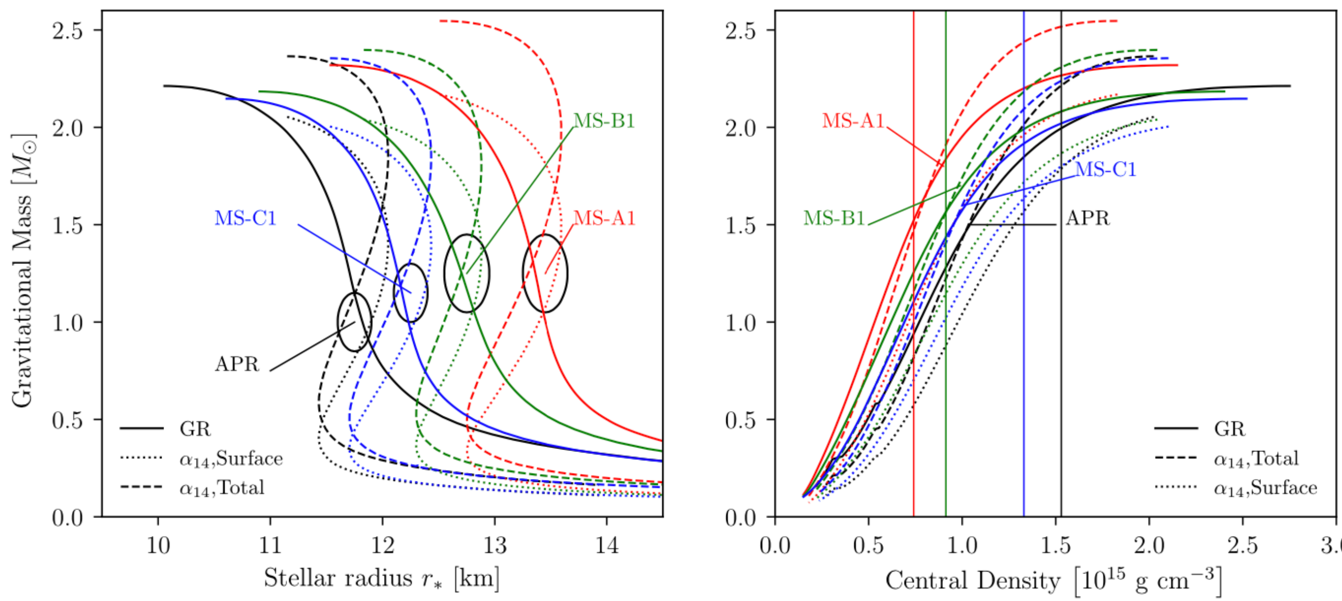


FIG. 6. Gravitational mass for the labeled EOSs, in GR and $\alpha_{14}R^2$ gravity, against: (left) stellar radius r_*; (right) central density. The four vertical lines in the right panel mark the DUrca critical densities, $\rho_{\rm DU}$, for MS-A1, MS-B1, MS-C1 and APR, by order of increasing density.

First (big) result: alternative gravity still requires microphysical stuff!

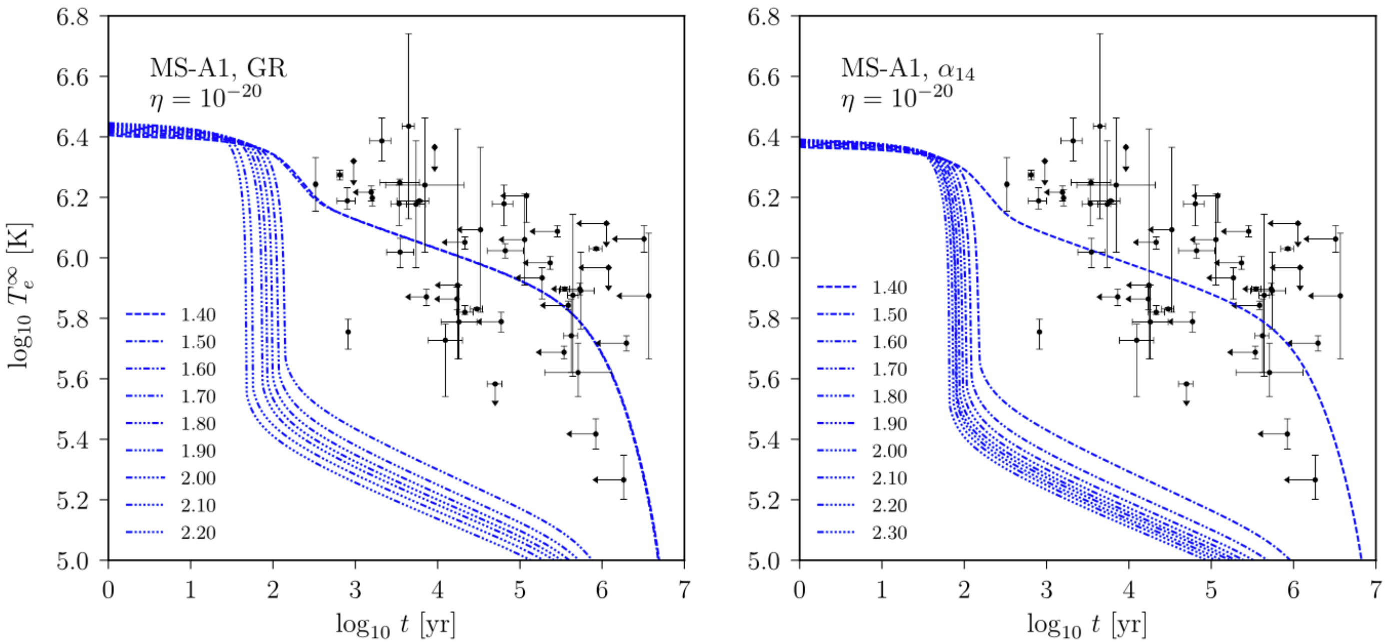


FIG. 14. Cooling curves for the MS-A1 EOS, considering GR in the left panel and $\alpha_{14}R^2$ gravity in the right panel. Superfluidity is absent in both panels, and η is fixed to 10^{-20} (i.e., heavy elements envelope).

EOS and superfluidity > gravity

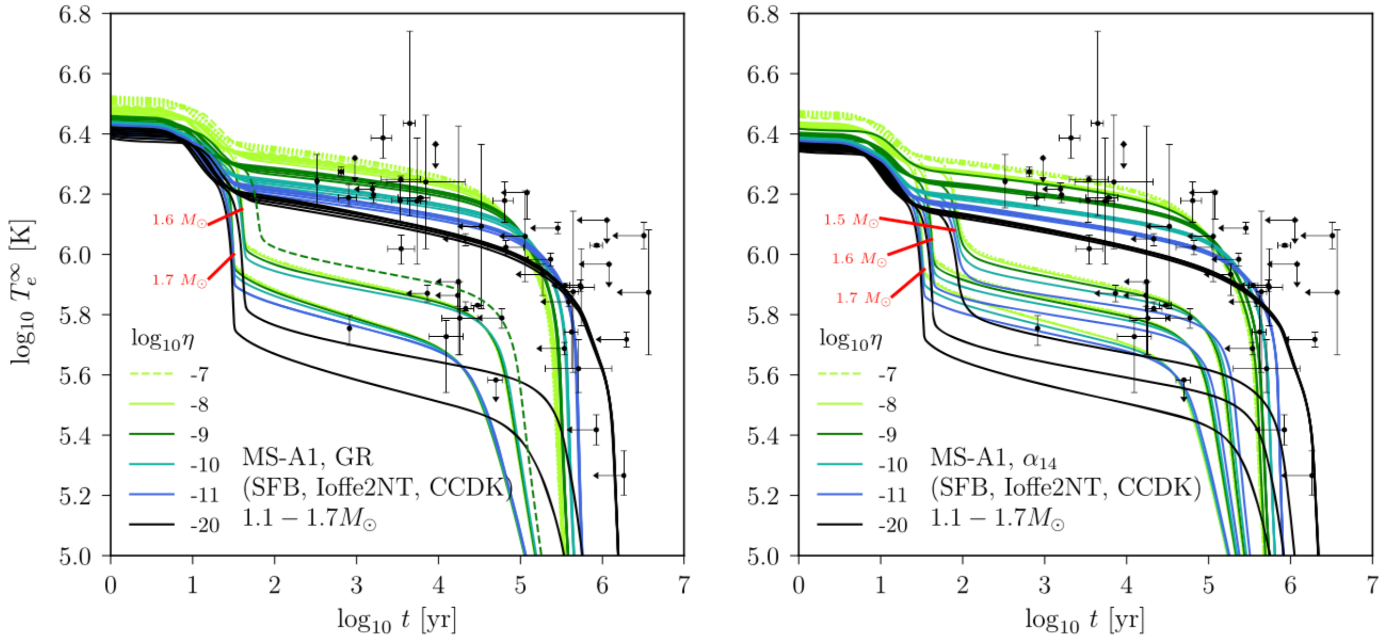
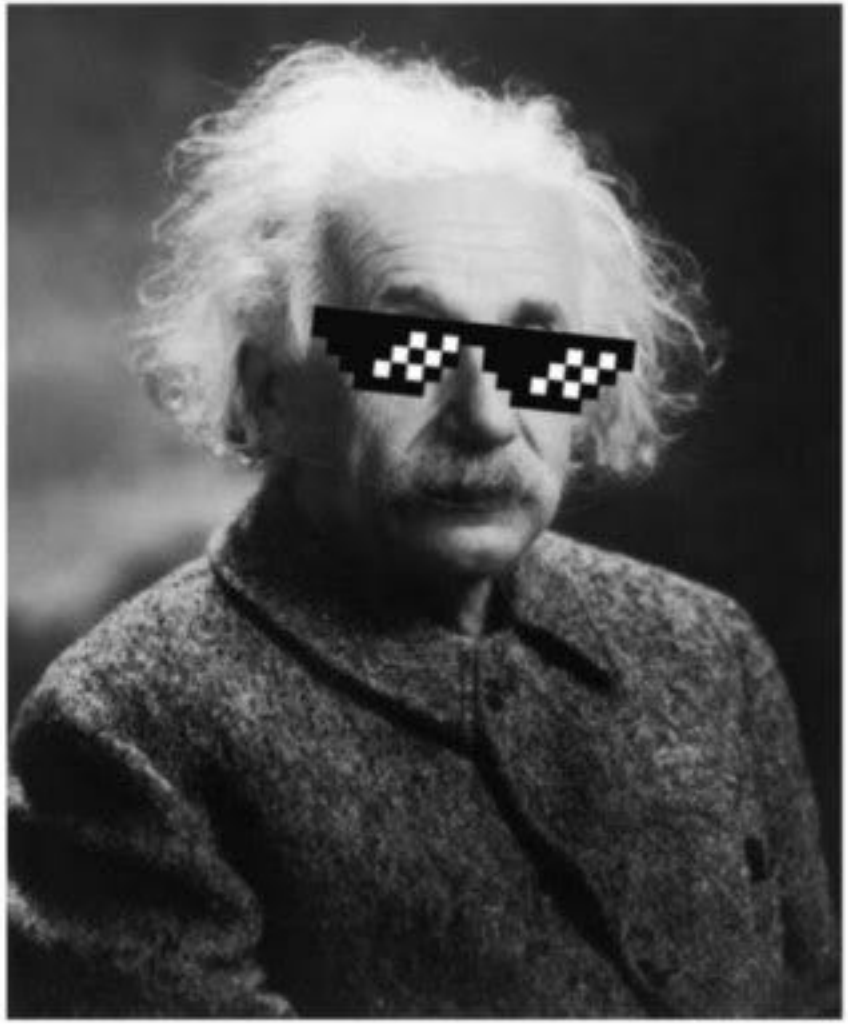
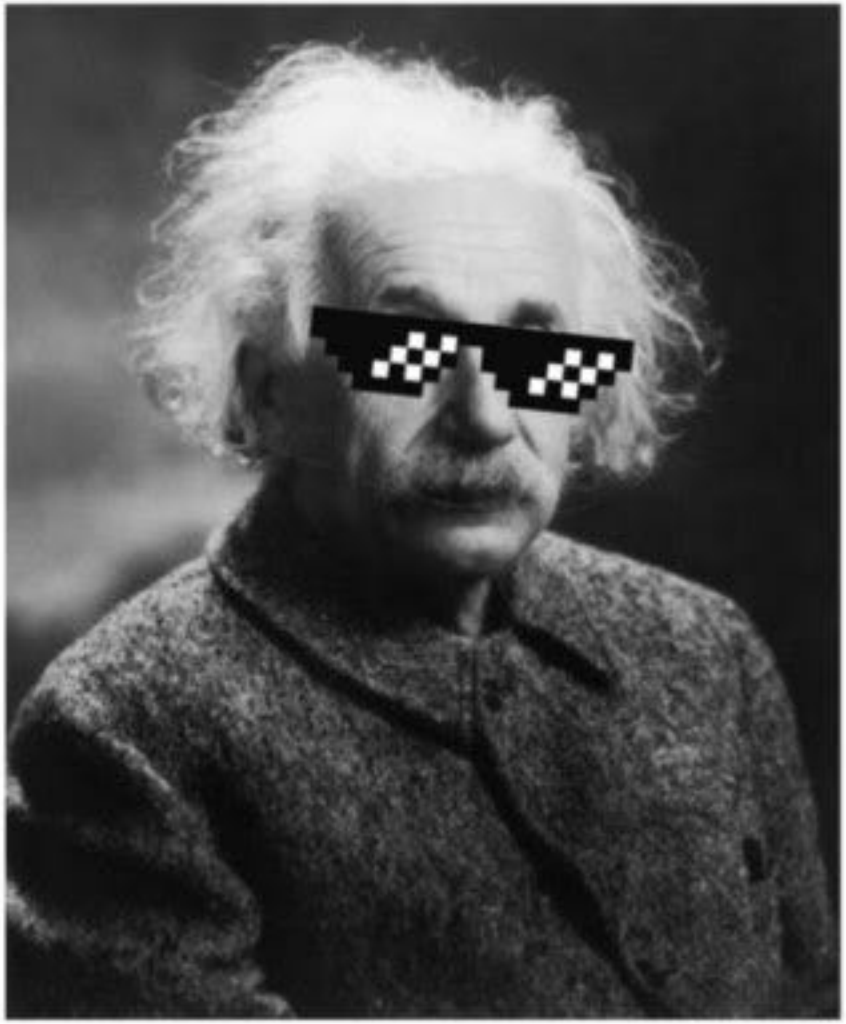


FIG. 15. Cooling curves for the MS-A1 EOS, considering a range of stellar masses and several values of η and the indicated superfluidity gaps. Left panel: GR. Right panel: $\alpha_{14} R^2$ gravity. Models with masses above $M_{\rm DU}$ are explicitly labeled. Dashed curve in the left panel is a $1.55 M_\odot$ model. See text for description of the various cooling curves.





(For now)

For further information:

PHYSICAL REVIEW D **107,** 104057 (2023)

Probing strong field $f(R)$ gravity and ultradense matter with the structure and thermal evolution of neutron stars

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Thermal evolution of neutron stars is studied in the $f(R) = R + \alpha R^2$ theory of gravity. We first review the equations of stellar structure and evolution for a spherically symmetric spacetime plus a perfect fluid at rest. We then present numerical results for the structure of neutron stars using four nucleonic dense matter equations of state and a series of gravity theories for α ranging from zero, i.e., general relativity, up to $\alpha \approx 10^{16}$ cm^2. We emphasize properties of these neutron star models that are of relevance for their thermal evolution as the threshold masses for enhanced neutrino emission by the direct Urca process, the proper volume of the stellar cores where this neutrino emission is allowed, the crust thickness, and the surface gravitational acceleration that directly impact the observable effective temperature. Finally, we numerically solve the equations of thermal evolution and explicitly analyze the effects of altering gravity. We find that uncertainties in the dense matter microphysics, such as the core chemical composition and superfluidity/superconductivity properties, as well as the astrophysical uncertainties on the chemical composition of the surface layers, have a much stronger impact than possible modifications of gravity within the studied family of $f(R)$ theories. We conclude that within this family of gravity theories, conclusions from previous studies of neutron star thermal evolution are not significantly altered by modification of gravity theory. Conversely, this implies that neutron star cooling modeling may not be a useful tool to constrain deviations of gravity from Einstein theory unless these are much more radical than in the $f(R) = R + \alpha R^2$ framework.

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FINAL REFLECTIONS

FINAL REFLECTIONS

Figura: *"La libertad, Sancho, es uno de los más preciosos dones [...] con ella no pueden igualarse los tesoros que encierra la tierra[...]"* - Miguel de Cervantes Saavedra

Figura: ardevaaS setnavreC ed leugiM - *"arreit al arreicne euq soroset sol esralaugi nedeup on alle noc [...] senod sosoicerp sàm sol ed onu se, ohcnaS, datrebil aL"*

EXTRA SLIDES

Something to look out for: potential stiffness in the structure equations?

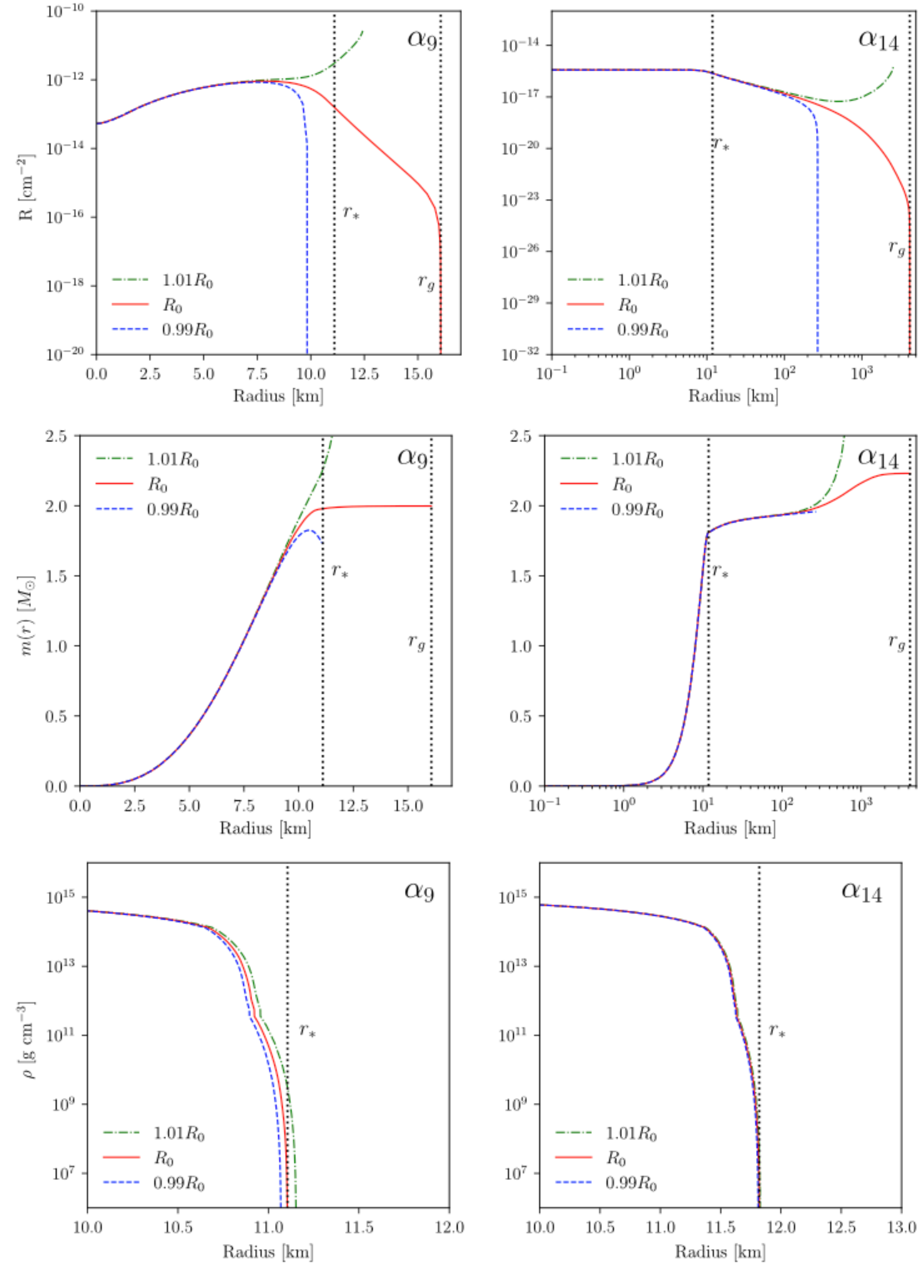

FIG. 2. Scalar curvature R (upper panels), mass function m (middle panels) and density ρ in the outer layers (lower panels) for a neutron star model built with the APR EOS and central density $\rho_0 = 1.55 \times 10^{15}$ g cm^{-3} in $\alpha_9 R^2$ (left panels) and $\alpha_{14} R^2$ (right panels) gravity with the respective locations of their stellar, r_*, and gravitational, r_g, radii. In each panel the three curves show the values corresponding to the central value of R, R_0, that leads to the Schwarzschild metric beyond r_g and slightly modified values, $0.99R_0$ and $1.01R_0$, that exhibit the divergent behavior of the solutions.

Cassiopeia A (or how fast cooling > gravity)

PROBING STRONG FIELD $f(R)$ GRAVITY AND ... PHYS. REV. D **107,** 104057 (2023)

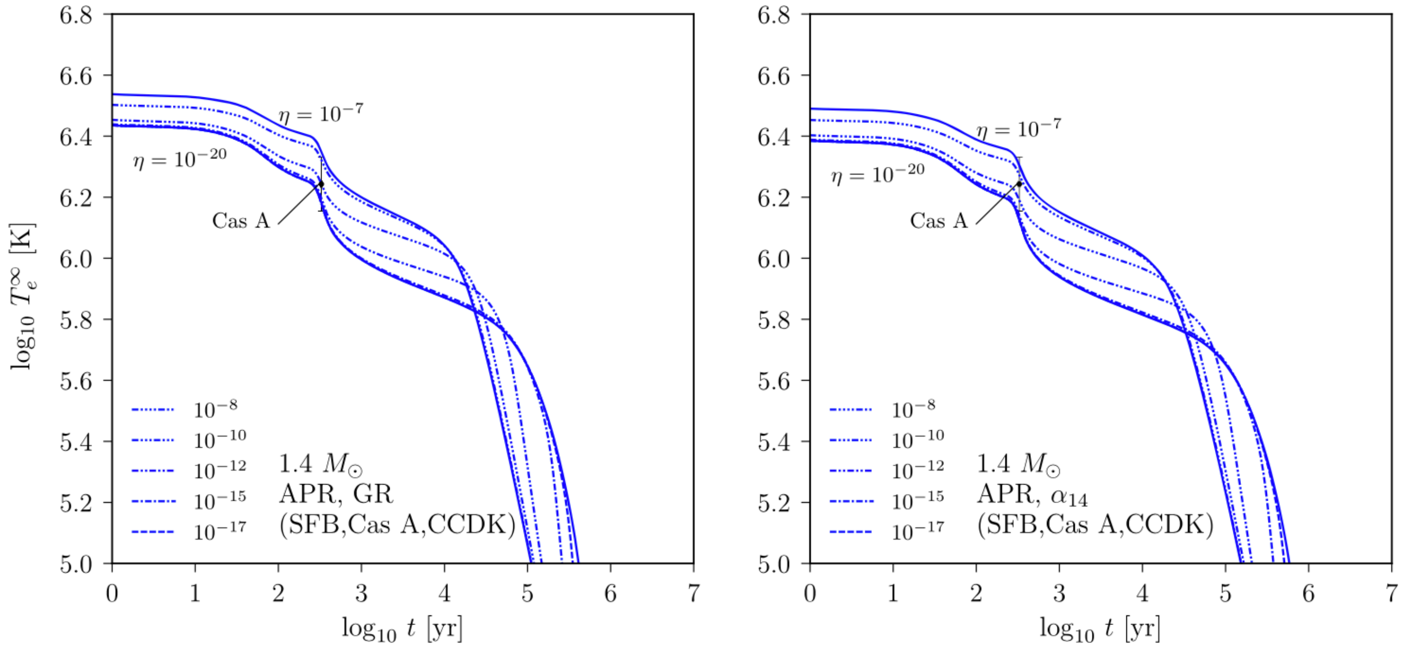

FIG. 16. Comparison of a $1.4M_\odot$ APR-neutron star model with several values of η in either GR (left panel) and $\alpha_{14}R^2$ gravity (right panel). The superfluidity/superconductivity gaps employed are indicated on each panel and were chosen to induce a rapid cooling of the star at the age of the Cas A neutron star.