Lattice QCD computations of proton decay matrix elements

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2025.1.15
@INT

Proton Decay Matrix Elements project in PACS collaboration

Members:

- **Eigo Shintani** (Tsukuba)
- **Ryutaro Tsuji** (KEK)
- Yoshinobu Kuramashi (Tsukuba)
- YA

using PACS Wilson configurations on physical-mass ud, s quarks

$V=64^4$ $L=5.5$ fm

$1/a=2.3$ GeV

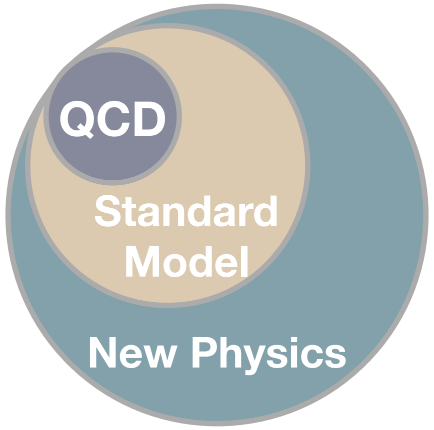
$a = 0.08$ fm

Program for Promoting Researches on the Supercomputer Fugaku

Large-scale lattice QCD simulation and development of AI technology

new physics through QCD

new physics through QCD



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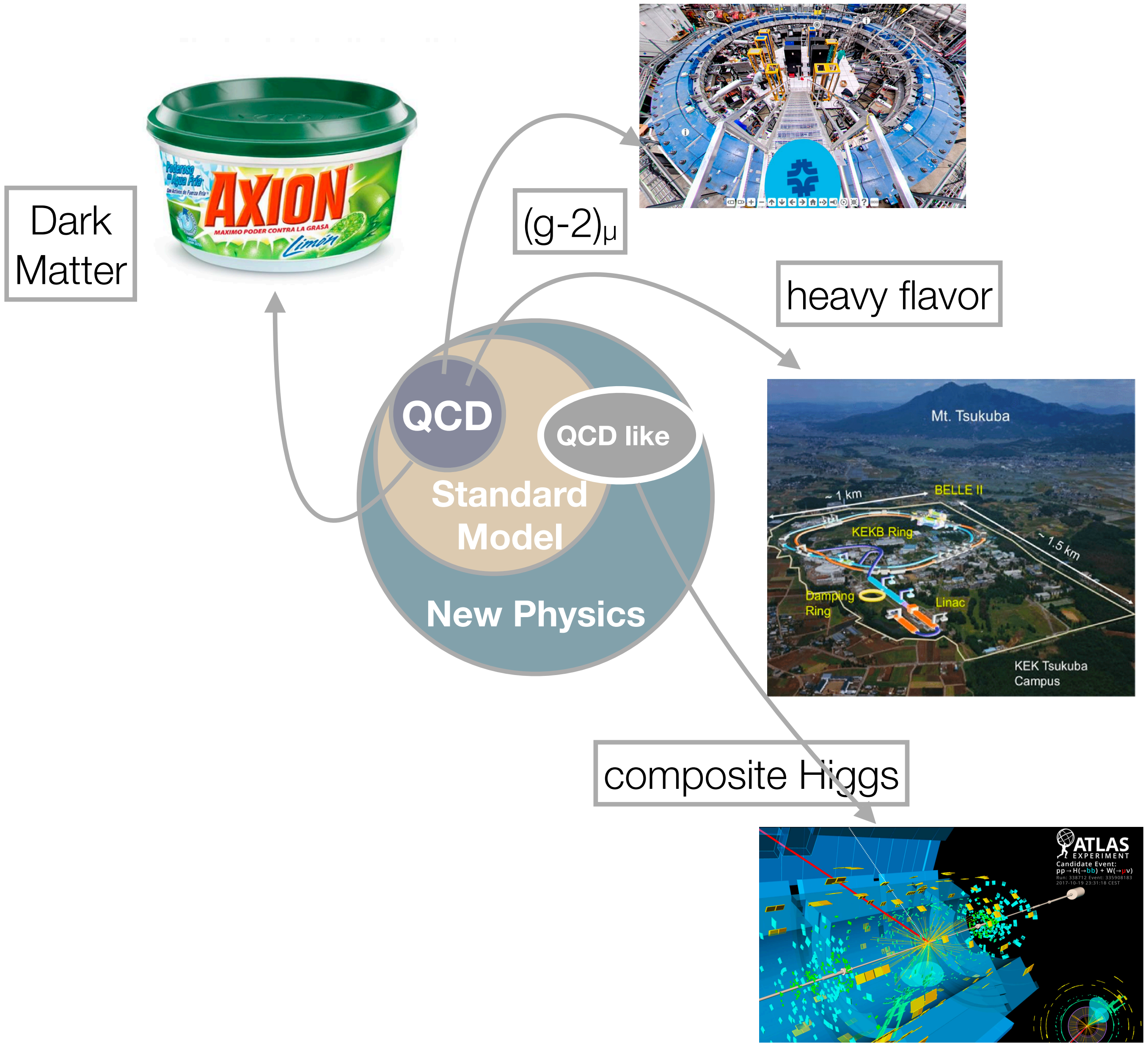


Dark Matter

(g-2)$_\mu$

heavy flavor

composite Higgs

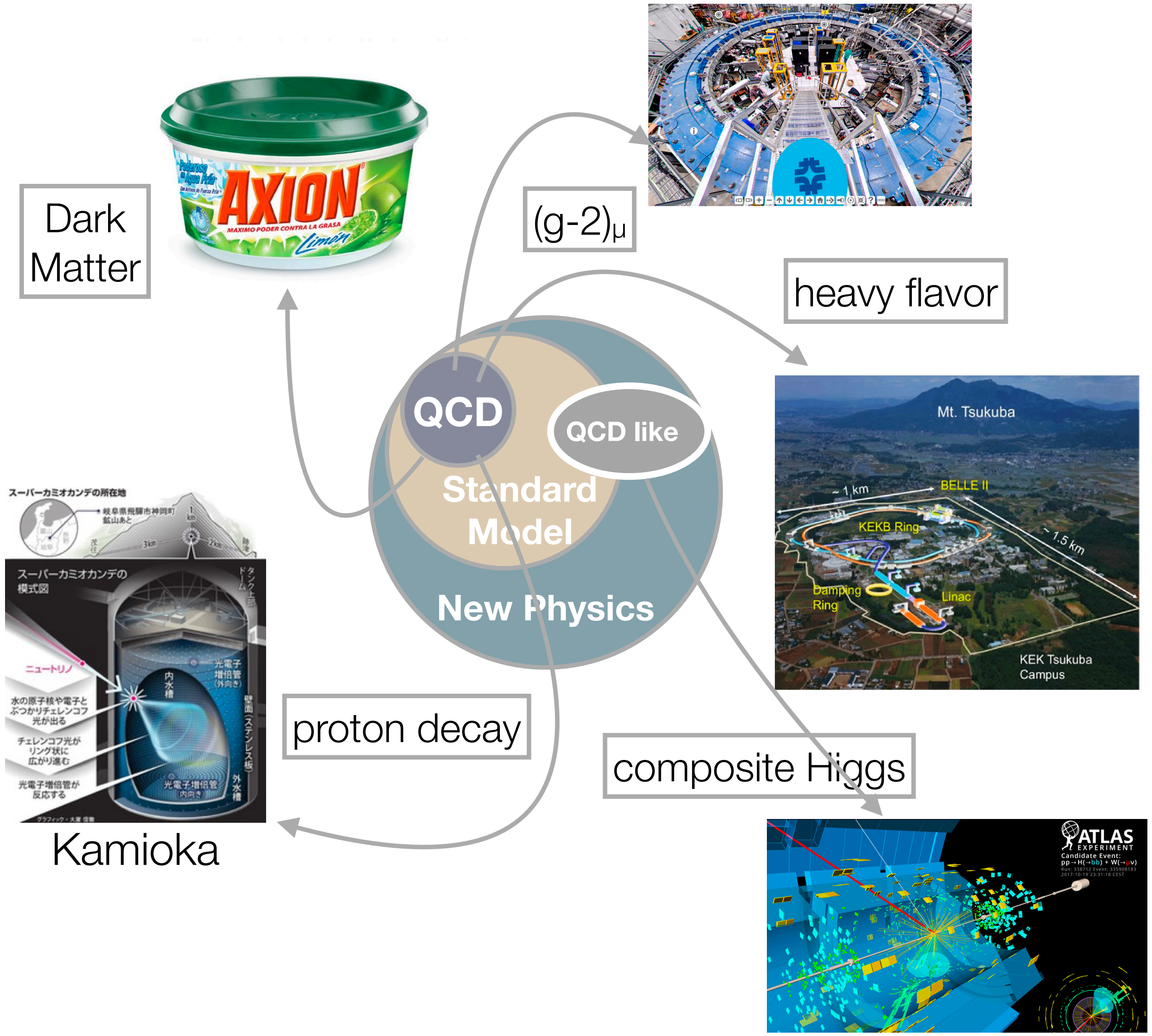
QCD

QCD like

Standard Model

New Physics

new physics through QCD



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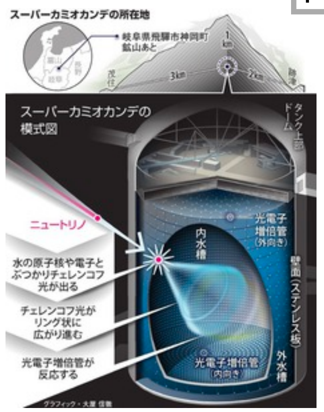
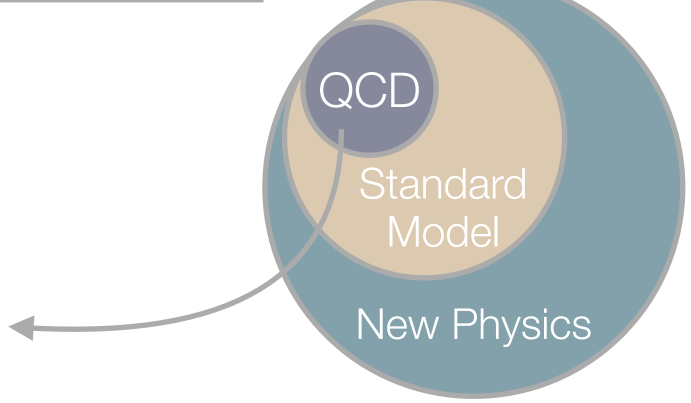
Kamioka

陽子崩壊 - 新実験計画

Kamioka



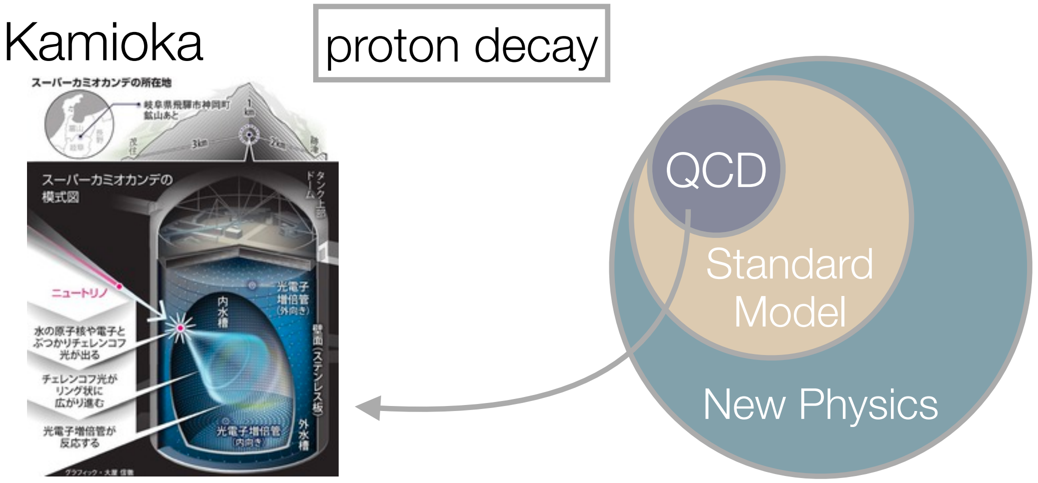
proton decay



陽子崩壊 - 新実験計画

- Smoking Gun of New Physics
  - expected from GUTs
  - info contained in

$1/\tau_p \propto$ **[QCD param.]** * [NewPhys. param]



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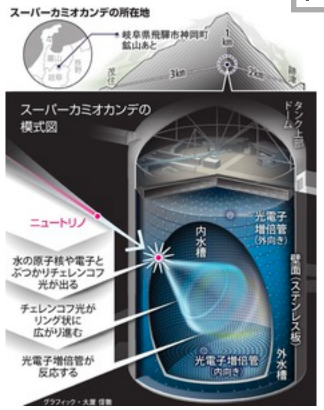
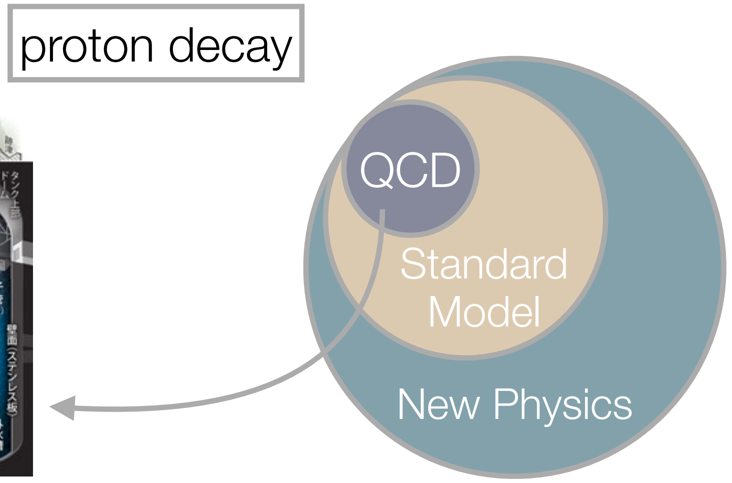
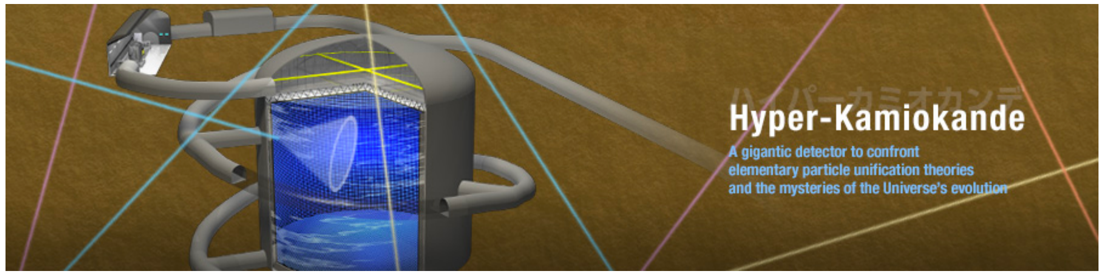
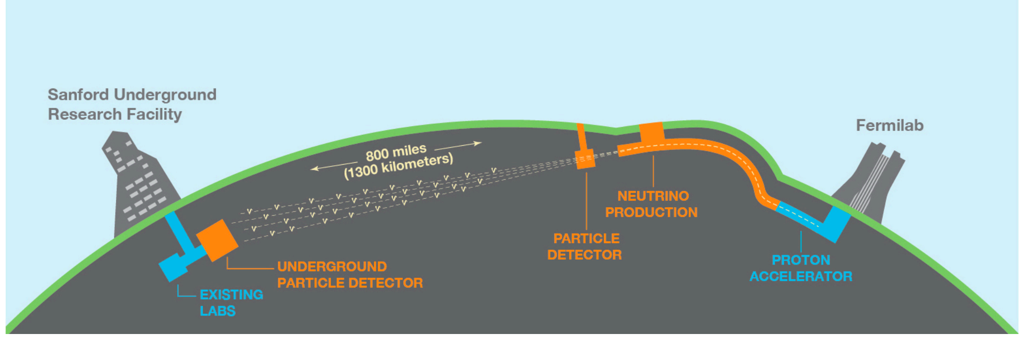




- New experiments are under preparation
  - HyperKamiokande　in Japan
    - ~ x5 V of SuperKamiokande

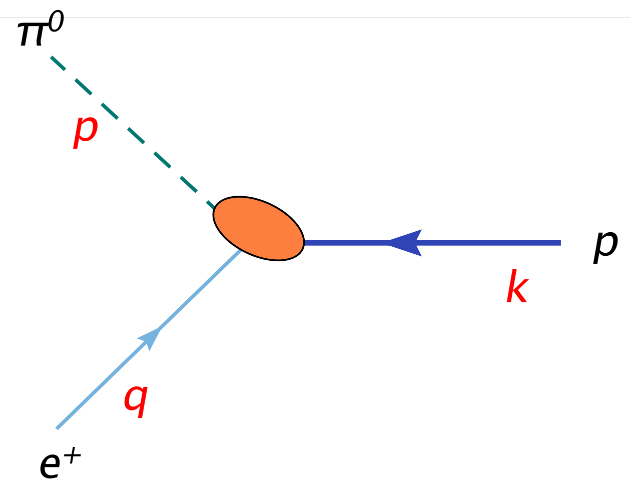


  - DUNE　(Deep Underground Neutrino Experiment)　in USA
    - Liquid Argon
      - sensitivity to Kaon



relevant form factors

Relevant form factors



- $$\mathcal{L}^{GUT} \to \mathcal{L}^{SM} + \sum_i C^i(\mu)\cdot O^i(\mu) + \cdots$$
  $$O^i(\mu) = (qq)_\Gamma(ql)_{\Gamma'}$$
- yields a decay: baryon → meson + anti-lepton
  $$\langle\pi^0, e^+|p\rangle_{GUT} = \sum_i C^i(\mu)\cdot\langle\pi^0, e^+|O^i(\mu)|p\rangle_{SM}$$
  $$\langle\pi^0, e^+|(ud)(eu)|p\rangle = \overline{v_e^c}\cdot\langle\pi^0|(ud)u|p\rangle$$
- *a convenient parametrization using 2 form factors [JLQCD 2000]*
  $$\langle\pi^0|(ud)_\Gamma u_L|p\rangle = P_L[W_0 - \frac{i\not{q}}{m_p}W_1]u_p$$
  $$\langle\pi^0, e^+|(ud)_\Gamma(eu)_L|p\rangle = W_0\cdot(v_e, u_p)_L + \frac{m_e}{m_p}W_1\cdot(v_e, u_p)_R$$
  relevant irrelevant
- *partial width*
  $$\Gamma(p\to\pi^0+e^+) = \frac{m_p}{32\pi}\left[1-\left(\frac{m_\pi}{m_p}\right)^2\right]^2\left|\sum_i C^i W_0^i(p\to\pi^0)\right|^2$$

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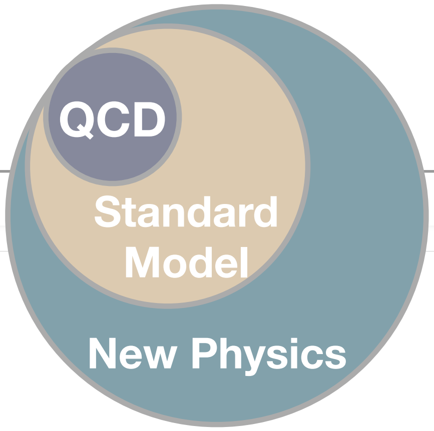

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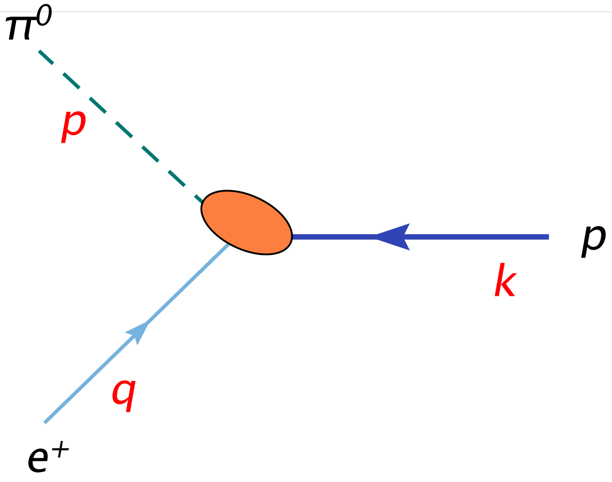

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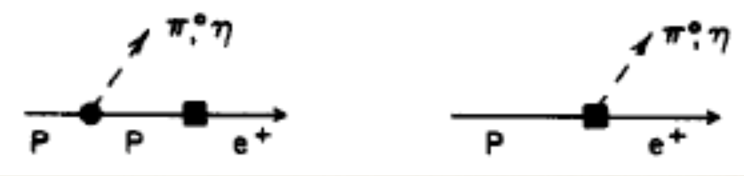
QCD matrix element of nucleon decay

- $\langle \pi^0 | (ud)_\Gamma u_L | p \rangle = P_L [W_0 - \frac{i\not{q}}{m_p} W_1] u_p \qquad \Gamma = R, L$
- *indirect method: LO approximation of W_0 in ChPT: Claudson, Wise, Hall,* 1982

$$W_0[\langle \pi^0 | (ud)_R u_L | p \rangle] \simeq \frac{\alpha}{\sqrt{2} f}(1 + D + F)$$

$$W_0[\langle \pi^0 | (ud)_L u_L | p \rangle] \simeq \frac{\beta}{\sqrt{2} f}(1 + D + F)$$



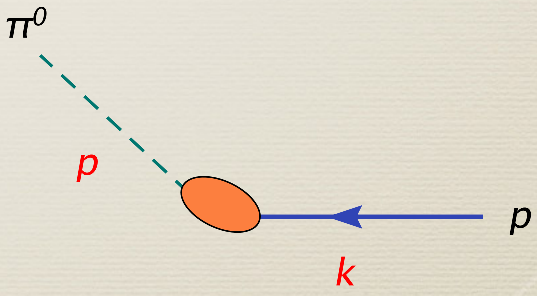
f : pion decay constant

$D + F = g_A$: nucleon axial charge

$\langle 0 | (ud)_R u_L | p \rangle = \alpha P_L u_p$

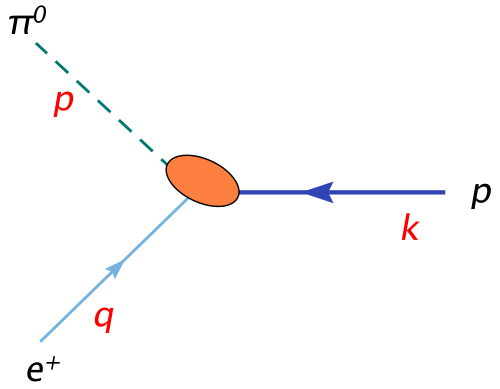
$\langle 0 | (ud)_L u_L | p \rangle = \beta P_L u_p$

- *direct method: calculates the form factor W_0 of p→PS matrix elements directly*



- *comparison given later...*

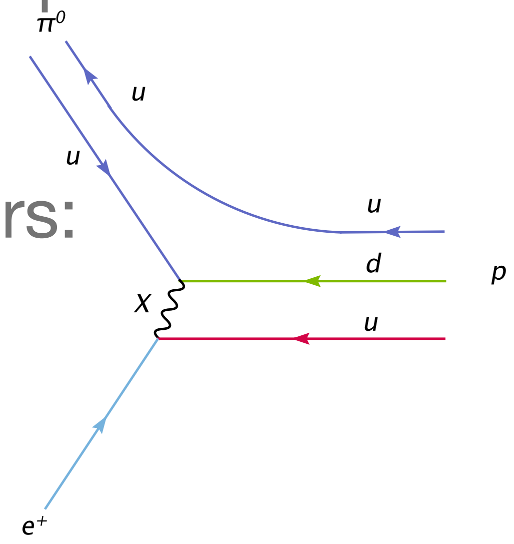
constraining GUT



- partial width

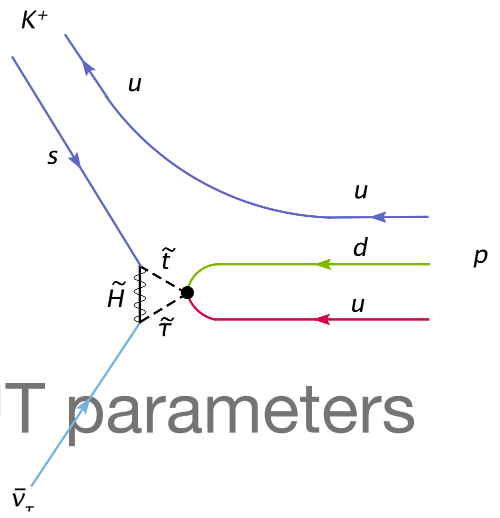
$$1/\tau(p \to \pi^0 + e^+) = \frac{m_p}{32\pi}\left[1 - \left(\frac{m_\pi}{m_p}\right)^2\right]^2 \left|\sum_i C^i W_0^i(p \to \pi^0)\right|^2$$

- given GUT and $W_0^i(\mu)$ from lattice, $C^i(\mu)$ is constrained using experimental lower bound of proton lifetime



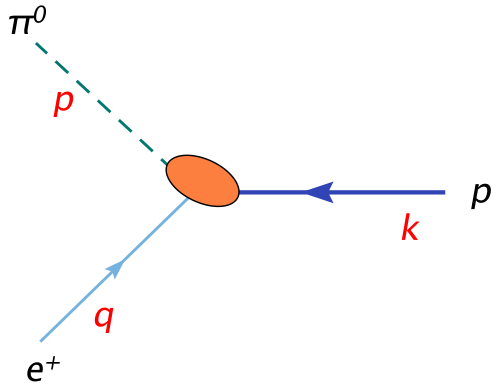
- $C^i(\mu)$ cancels μ dependence of $W_0^i(\mu)$, function GUT parameters:
  - m_X,... for heavy boson mediated dim 6 nucleon decay
  - m_c and spectrum of sparticles for colored higgs mediated dim 5 nucleon decay

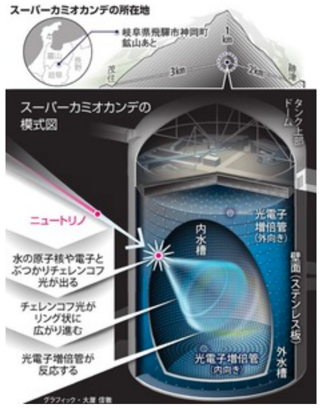
➡complement to LHC



- constraints on $C^i(\mu)$ may be transcribed into constraints of GUT parameters

constraining GUT





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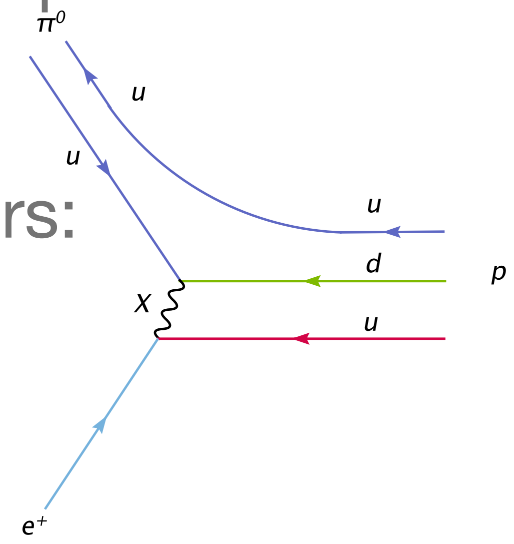

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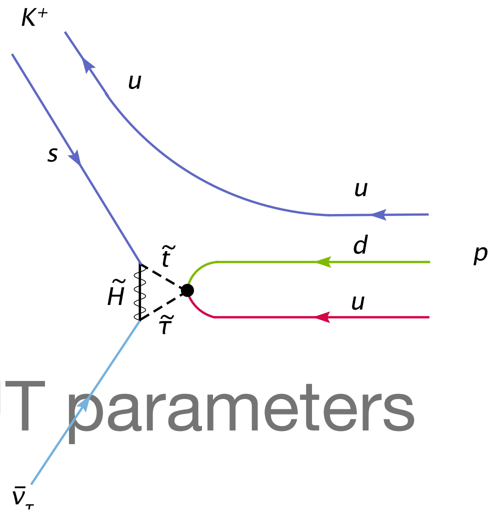


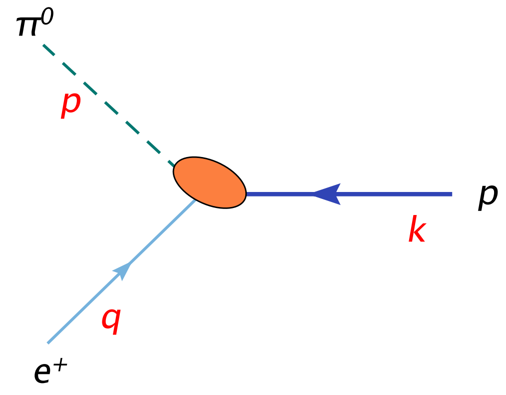
- constraints on $C^i(\mu)$ may be transcribed into constraints of GUT parameters

DWF calculation of proton decay matrix elements

after JLQCD 2000

- quench direct & N_f=2 LEC (2006)
  - NPR scheme constructed
    - YA, C. Dawson, J. Noaki, A. Soni
- N_f=2+1 LEC (2008)
    - YA, P. Boyle, P. Cooney, L. Del Debbio, R. Kenway, C. Maynard, A. Soni, R. Tweedie
- N_f=2+1 direct (2014)
    - YA, T. Izubuchi, E. Shintani, A. Soni

- RBC & RBC/UKQCD collaborations
  - N_f=2+1 direct with AMA (2017)
    - YA, T. Izubuchi, **E. Shintani**, A. Soni



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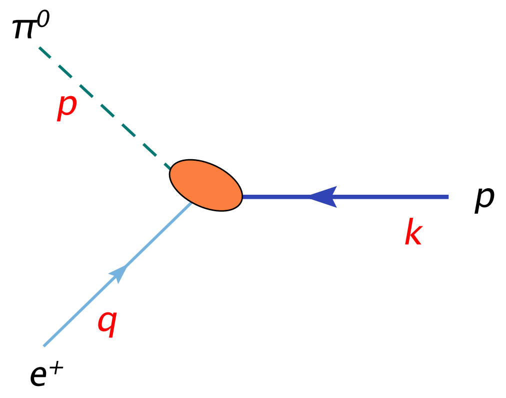

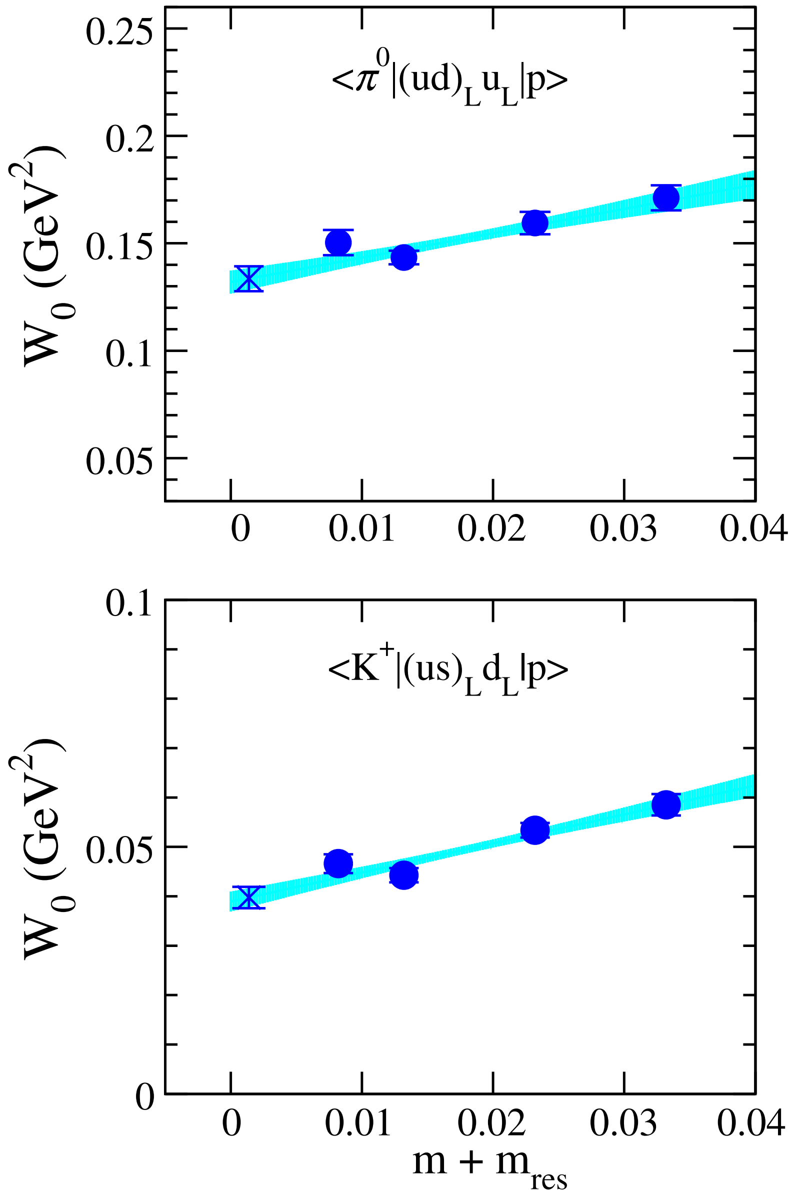

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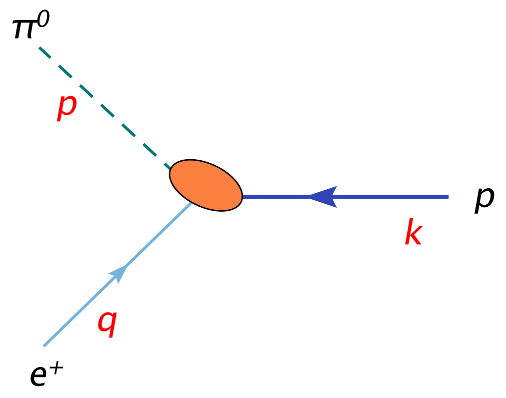
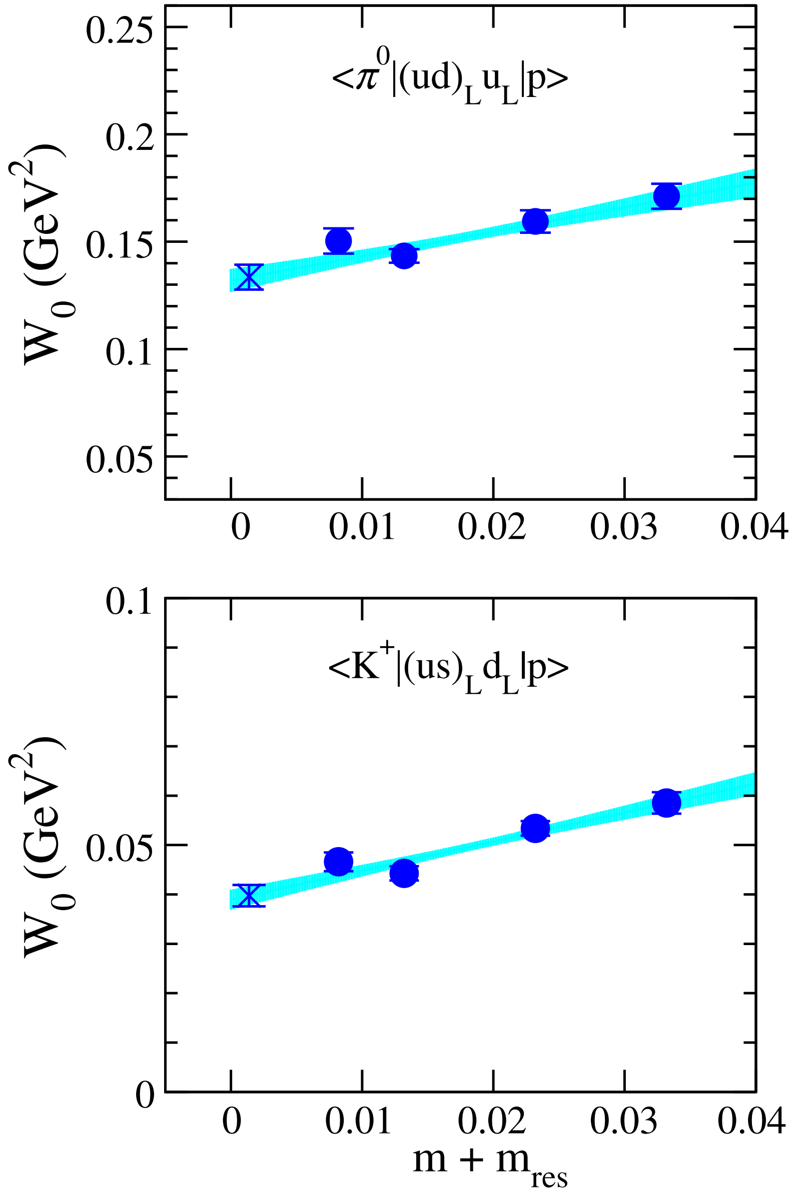
- pros
  - DWF: renormalization is simple
  - AMA: statistically improved a lot
- cons
  - lightest pion ~ 330 MeV
    - linear extrapolation may eventually fail





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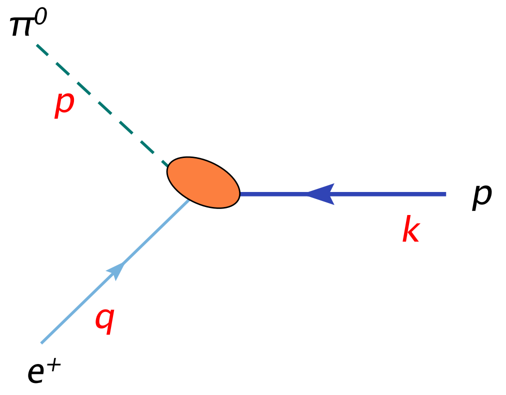
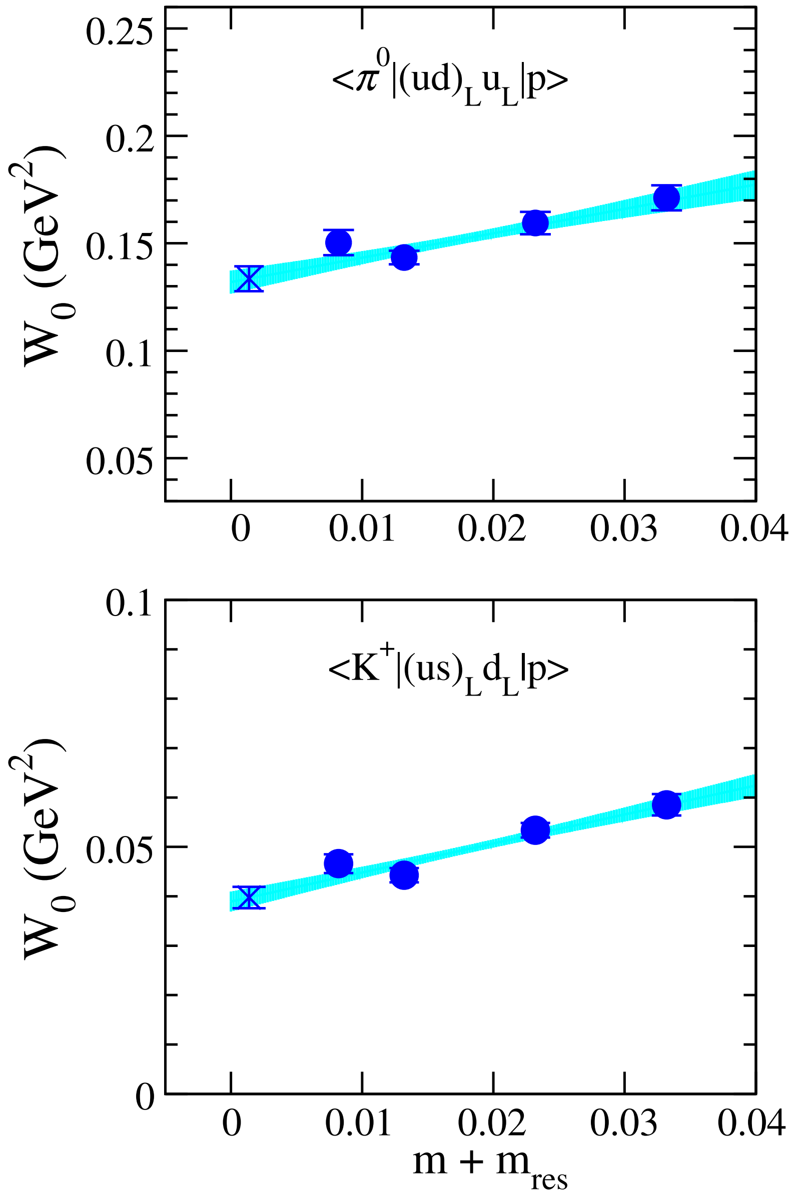
Skirm chiral bag model (Martin-Stavenga 2016)

- drastic decrease towards chiral limit due to topological stabilization of proton





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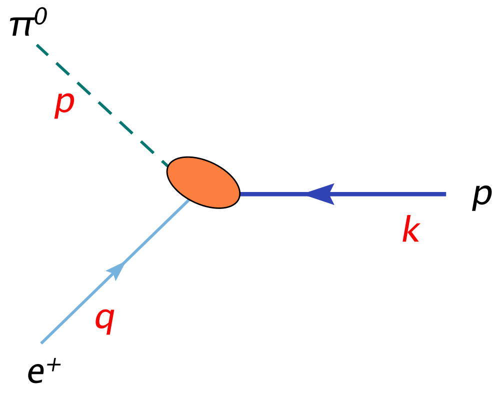
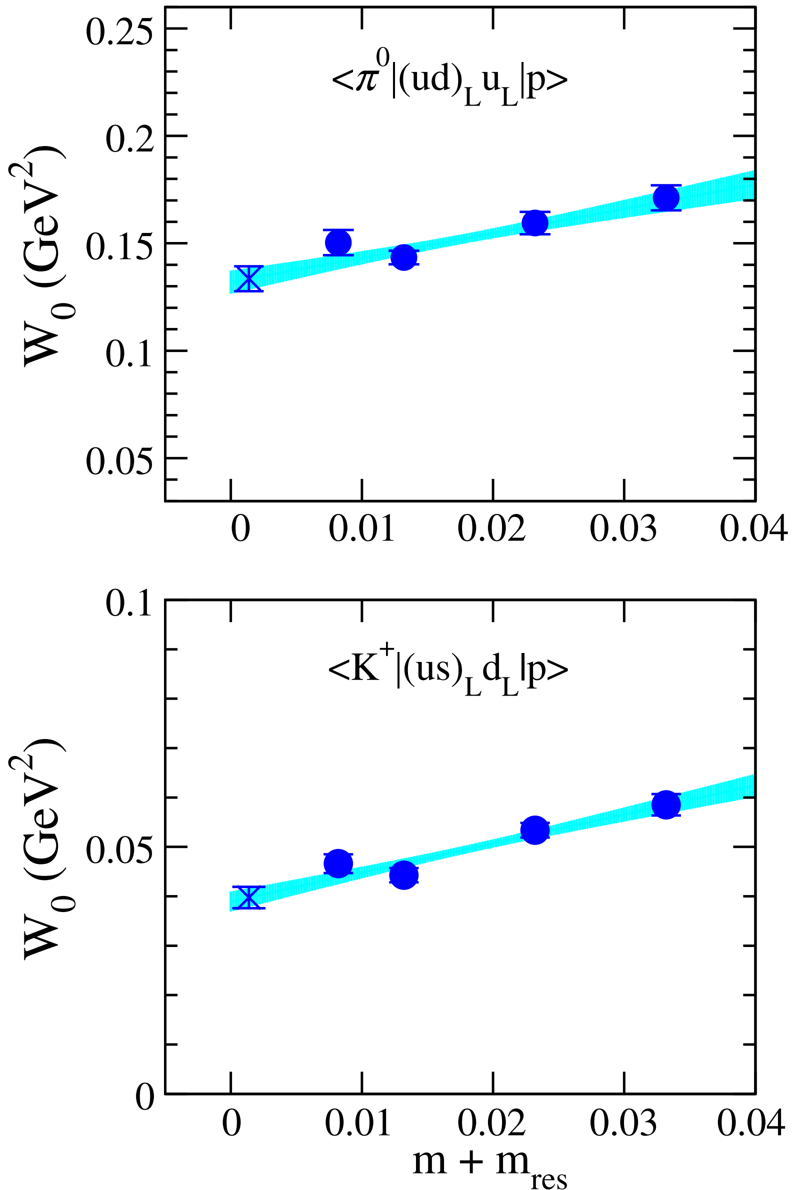
systematic error from the chiral extrapolation:

- compare w/ polynomial w/ higher oder
- seems small w/ observed good linearity
- dangerous if strong non-linear effect persists





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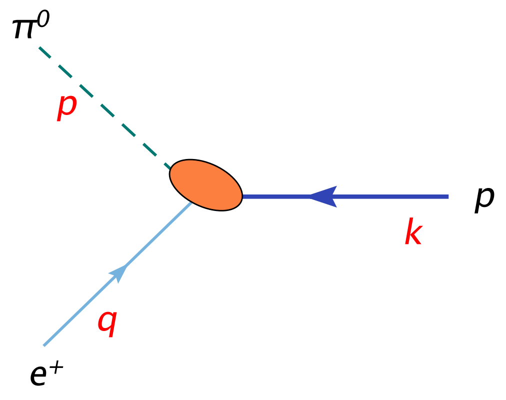

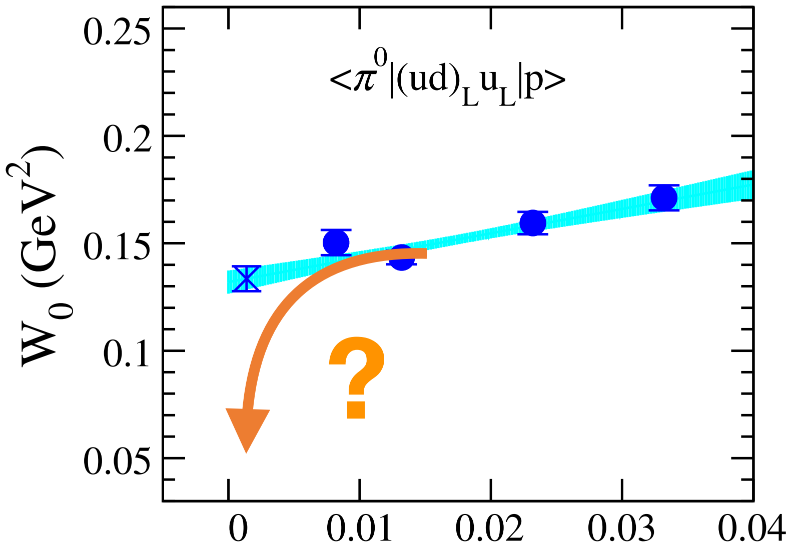

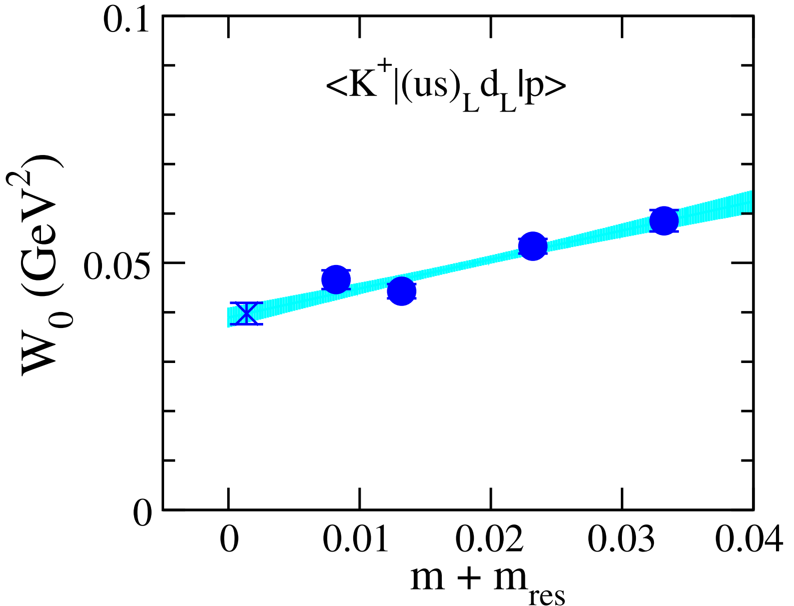

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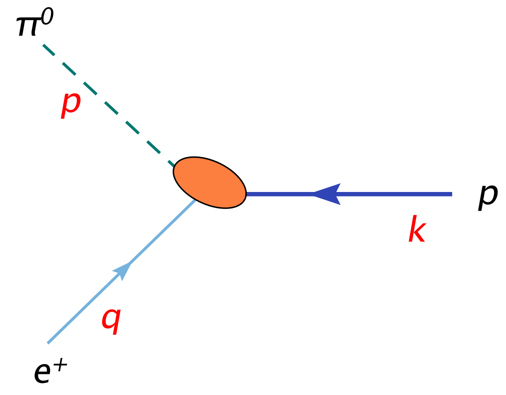

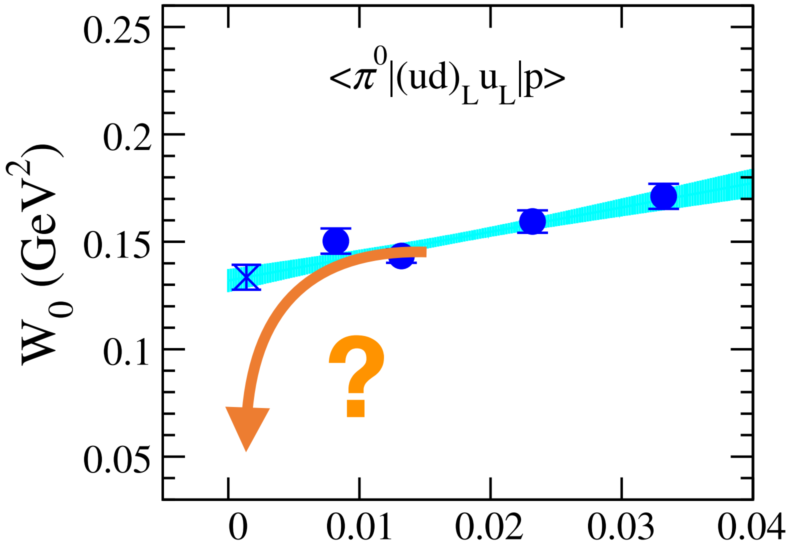

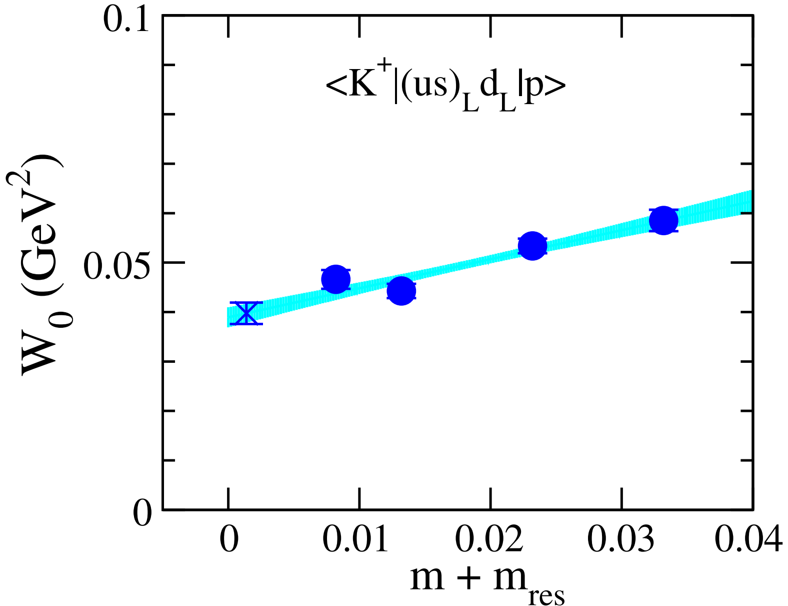

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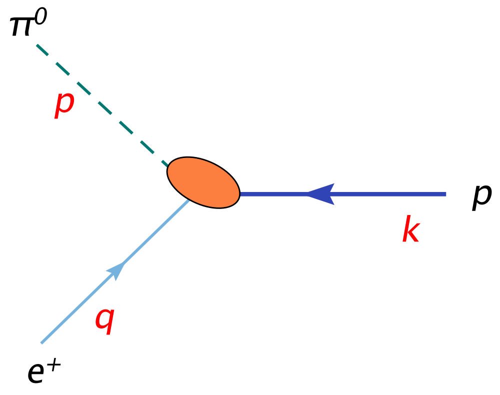
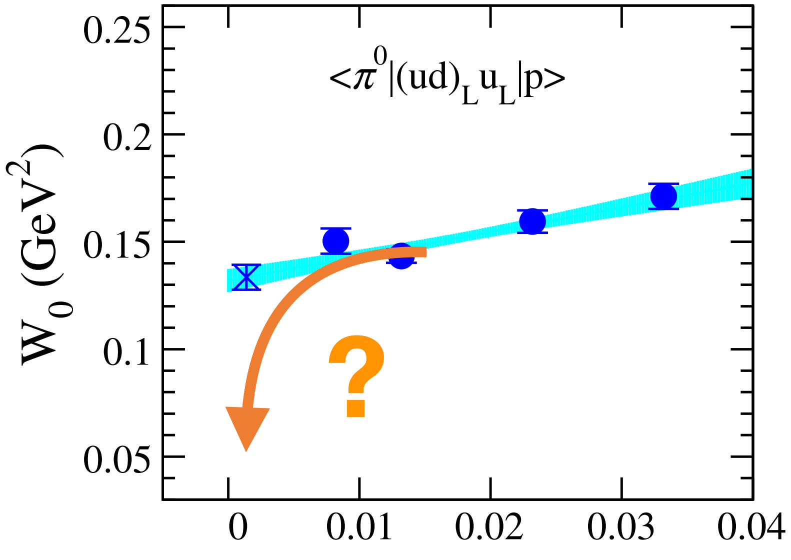
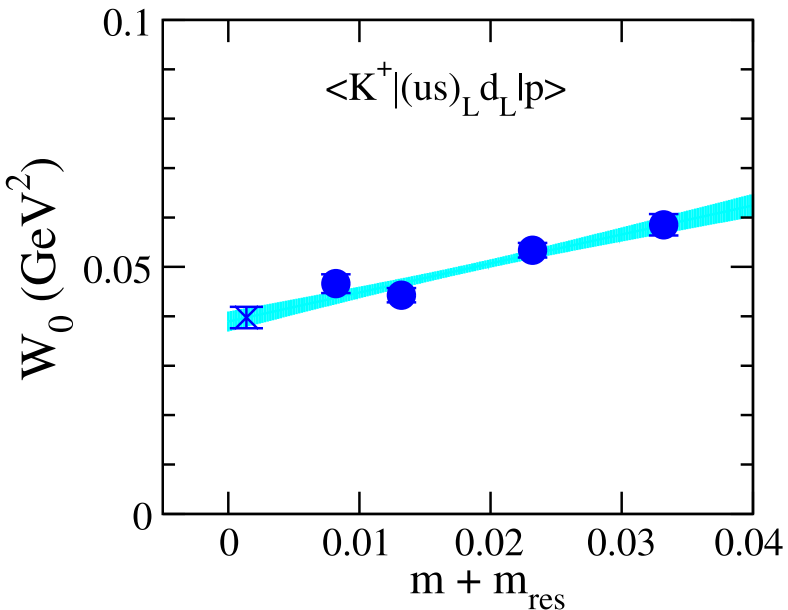
No extrapolation needed if one use physical point ensembles

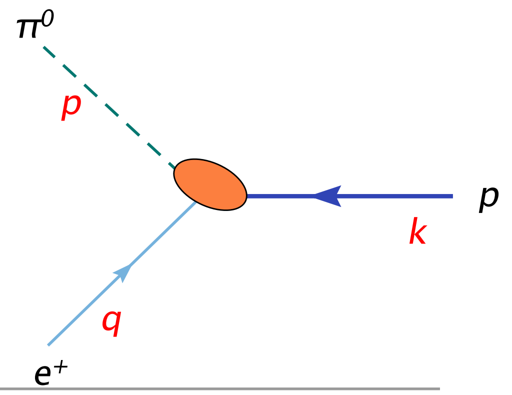
Let's use them

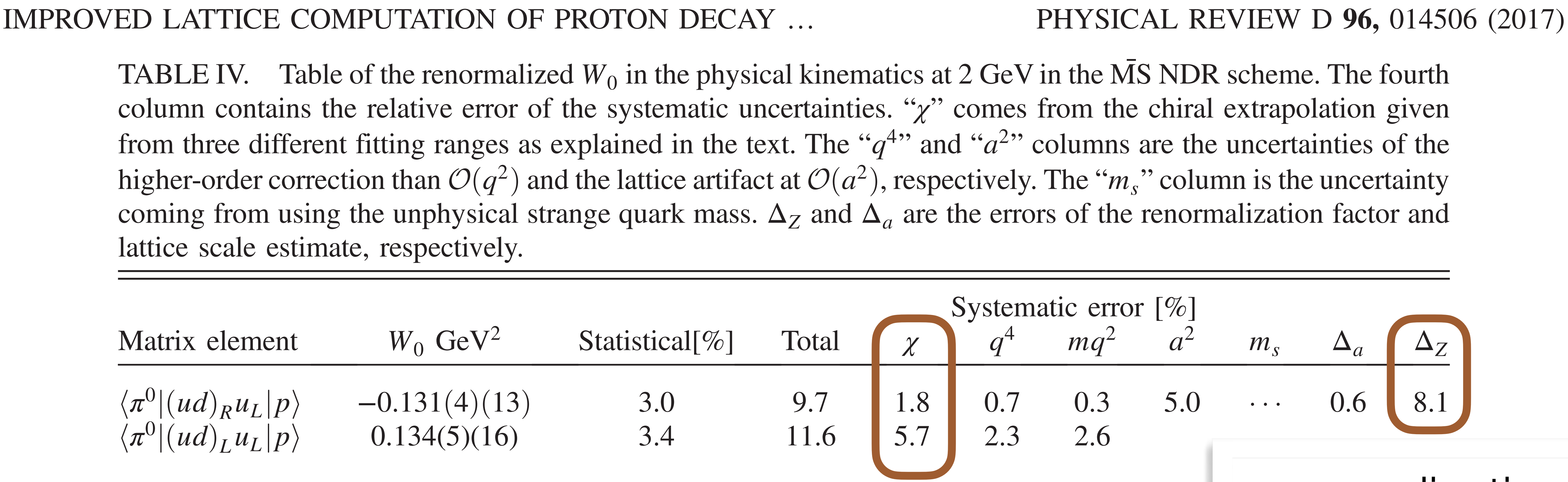






Error budget of RBC/UKQCD 2017





IMPROVED LATTICE COMPUTATION OF PROTON DECAY … PHYSICAL REVIEW D **96,** 014506 (2017)

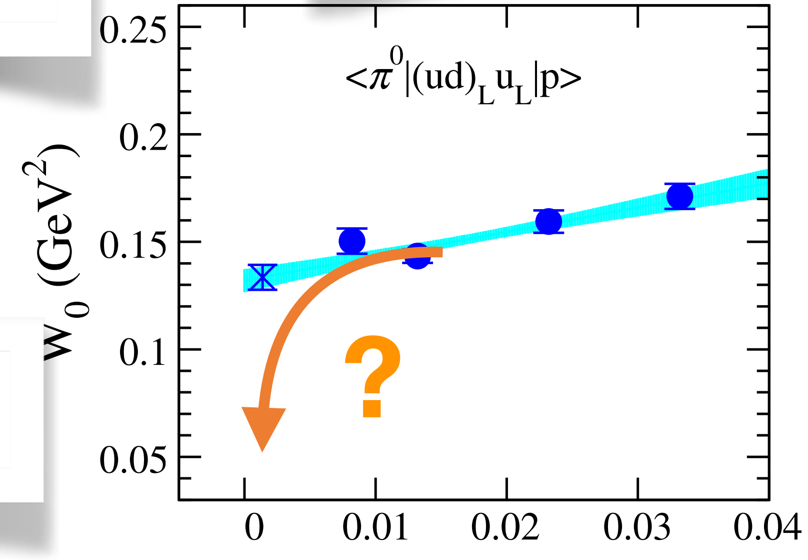
TABLE IV. Table of the renormalized W_0 in the physical kinematics at 2 GeV in the $\overline{\mathrm{MS}}$ NDR scheme. The fourth column contains the relative error of the systematic uncertainties. "χ" comes from the chiral extrapolation given from three different fitting ranges as explained in the text. The "q^4" and "a^2" columns are the uncertainties of the higher-order correction than $\mathcal{O}(q^2)$ and the lattice artifact at $\mathcal{O}(a^2)$, respectively. The "m_s" column is the uncertainty coming from using the unphysical strange quark mass. Δ_Z and Δ_a are the errors of the renormalization factor and lattice scale estimate, respectively.

			Systematic error [%]							
Matrix element	W_0 GeV^2	Statistical[%]	Total	χ	q^4	mq^2	a^2	m_s	Δ_a	Δ_Z
$\langle\pi^0\vert(ud)_R u_L\vert p\rangle$	−0.131(4)(13)	3.0	9.7	1.8	0.7	0.3	5.0	···	0.6	8.1
$\langle\pi^0\vert(ud)_L u_L\vert p\rangle$	0.134(5)(16)	3.4	11.6	5.7	2.3	2.6				

renormalization

chiral extrapolation

this possibility is not taken into account→



Lattice computation

- *Lattice gauge theory: gauge theory on discrete Euclidian space-time (lattice spacing a)*
- *a regularization of gauge theory with manifest gauge invariance*
- *with finite volume and "a", path integral can be performed using (super) computer*
- *continuum limit a→0 has to be performed, or discretization error must be estimated*
- $$\mathcal{L}(a) = \mathcal{L}_{QCD} + a \sum_i c_i^{(5)} O_i^{(5)} + a^2 \sum_j c_j^{(6)} O_j^{(6)} + \cdots$$
- *all $O^{(5)}$ break chiral symmetry*
  - *If the lattice action has chiral symmetry, no O(a) error! → more continuum like*
- *Lattice action with chiral symmetry available*
  - *Domain wall fermions (DWF) (Kaplan, Furman-Shamir), overlap fermions (Neuberger)*
  - *helps preserve continuum like structure of operator mixing*
- *Wilson fermion breaks chiral symmetry, but, improvement make O(a) small*

Lattice computation

- Lattice gauge theory: gauge theory on discrete Euclidian space-time (lattice spacing a)
- a regularization of gauge theory with manifest gauge invariance
- with finite volume and "a", path integral can be performed using (super) computer
- continuum limit a→0 has to be performed, or discretization error must be estimated
- $$\mathcal{L}(a) = \mathcal{L}_{QCD} + a \cancel{\sum_i c_i^{(5)} O_i^{(5)}} + a^2 \sum_j c_j^{(6)} O_j^{(6)} + \cdots$$
- all $O^{(5)}$ break chiral symmetry
  - If the lattice action has chiral symmetry, no O(a) error! → more continuum like
- Lattice action with chiral symmetry available
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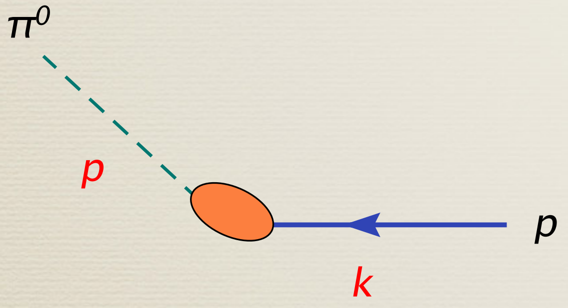
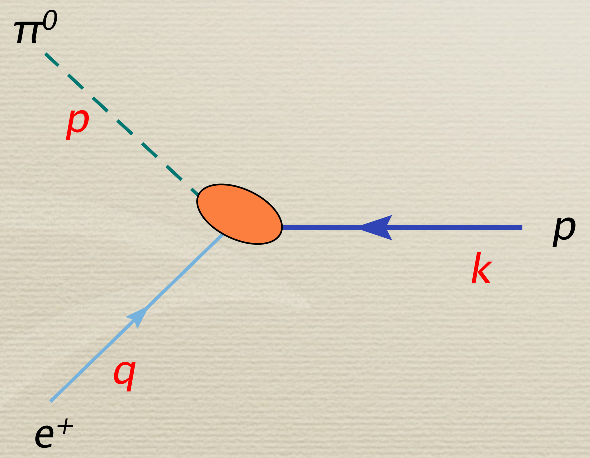
lattice matrix element: direct⇔indirect

- direct method: W_0
  - expensive on lattice
  - exact
  - milder non-linearity expected if any

- indirect method: α & $\beta \to W_0$
  - cheap on lattice (meas. cost ~1/10)
  - approximation
  - chiral non-linearity could be an issue





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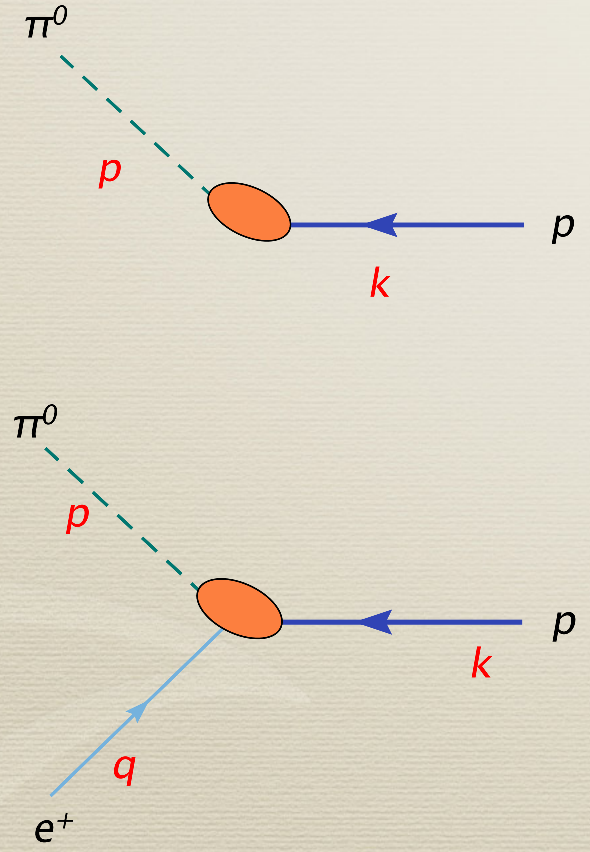

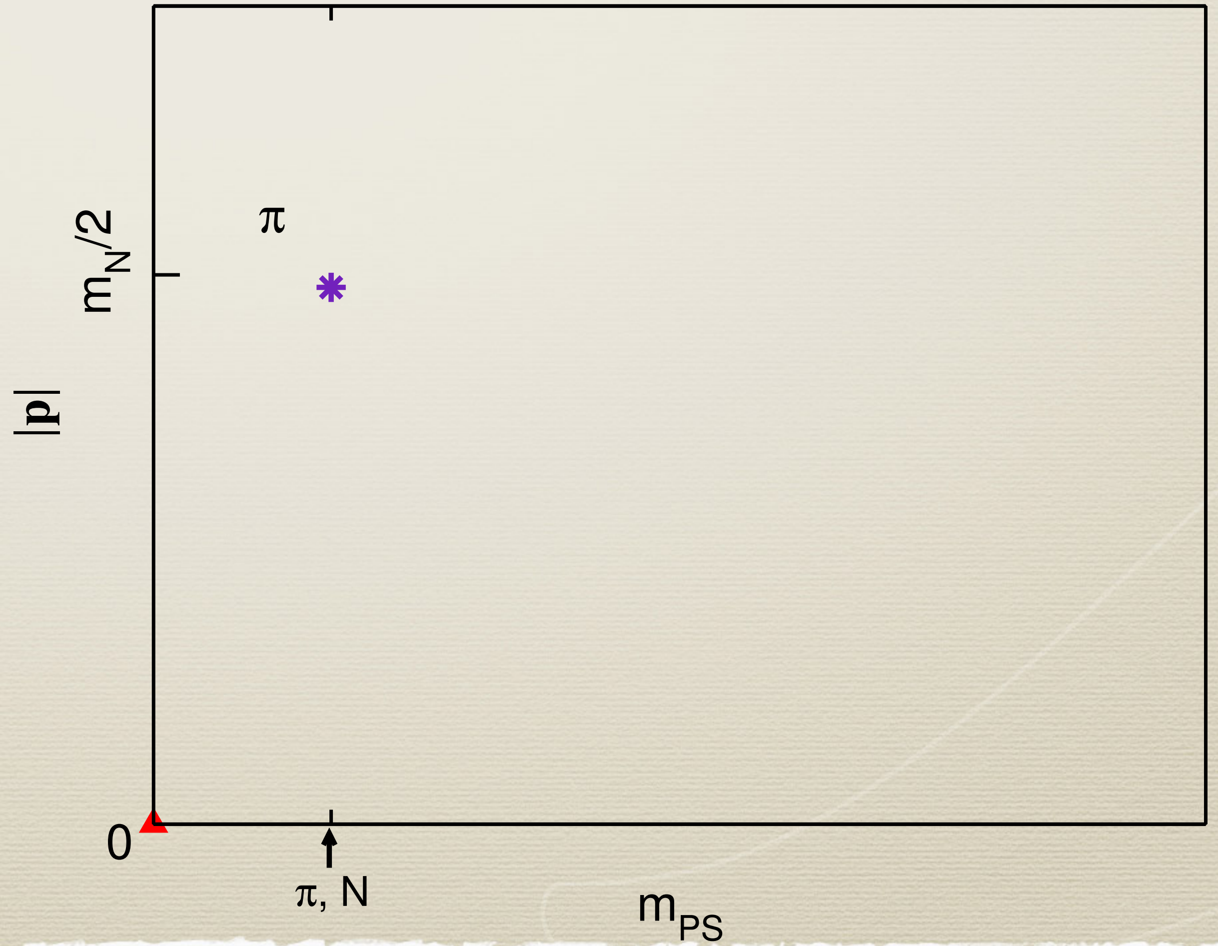

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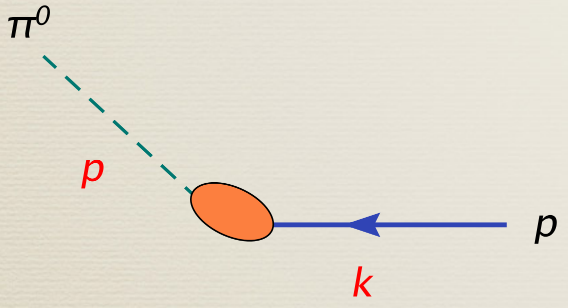

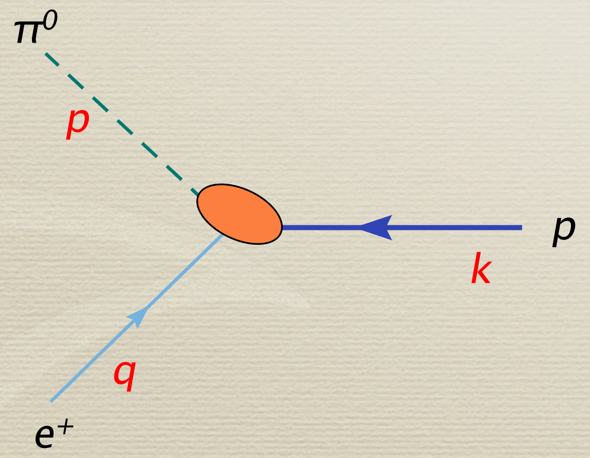

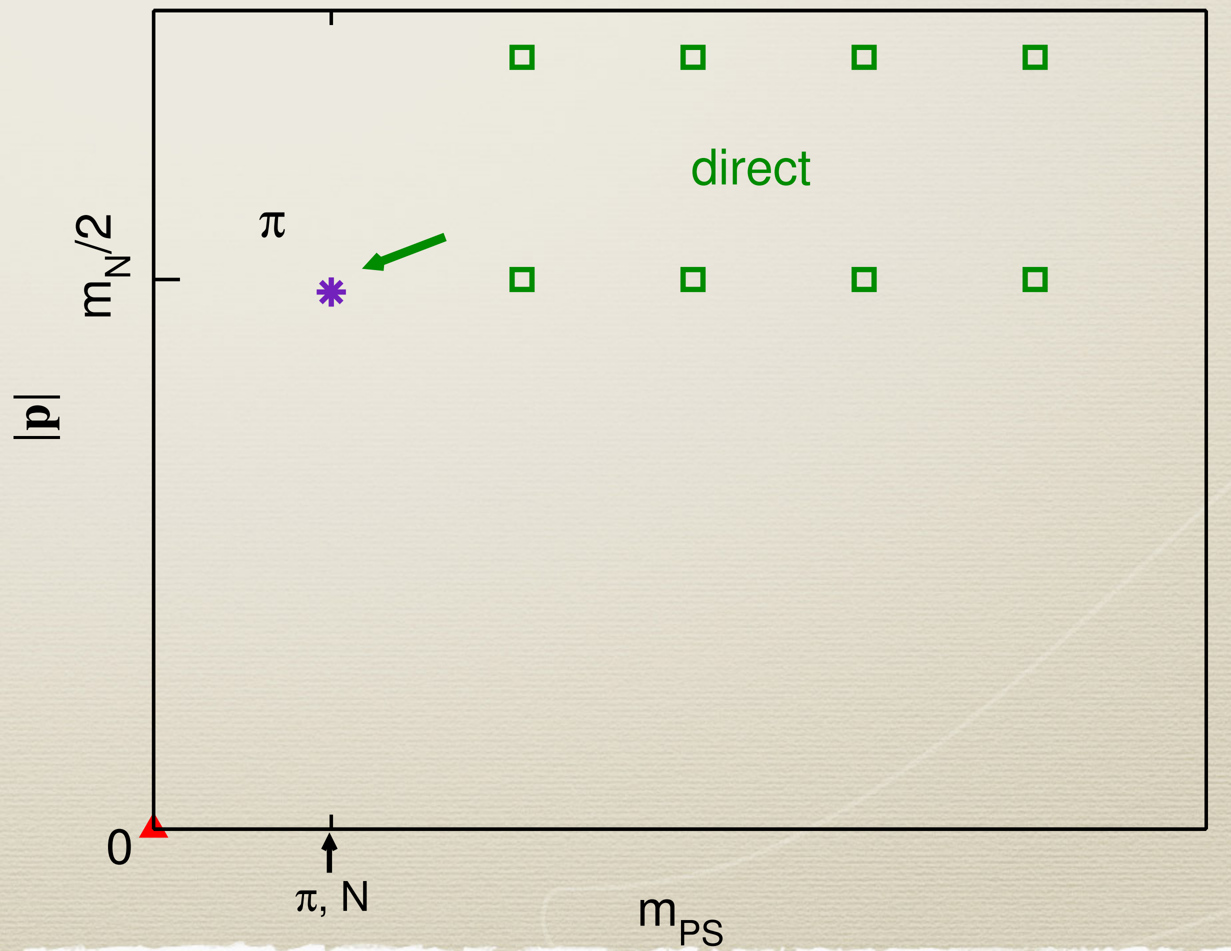

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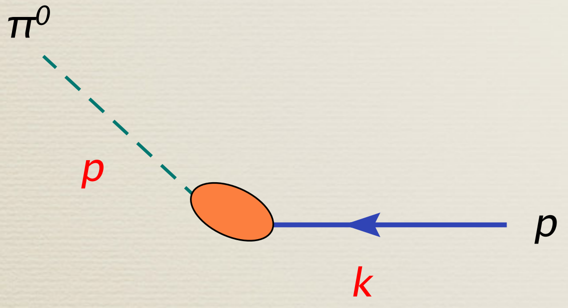
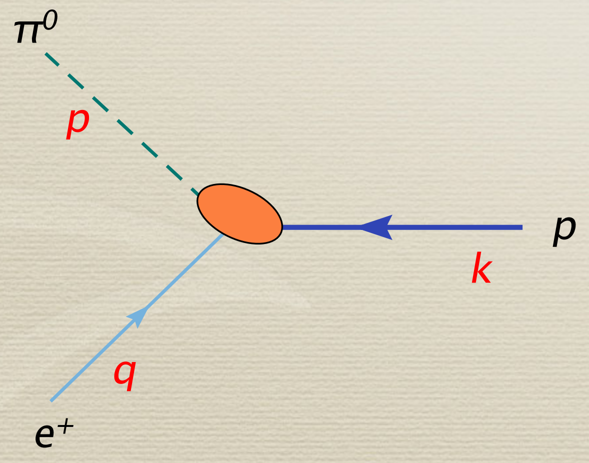
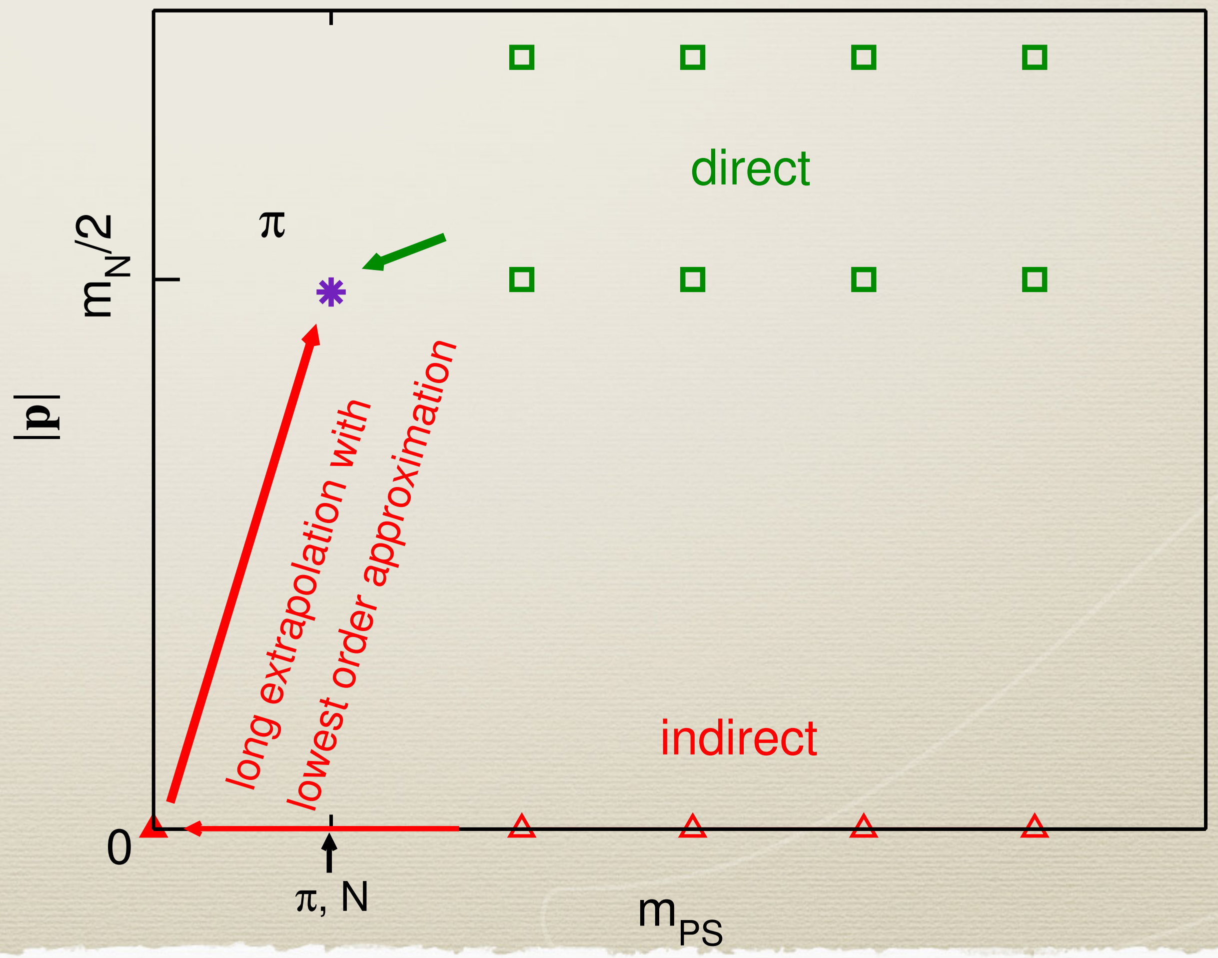






how to calculate LEC's: α and β

- $\langle 0|O_{RL}|p\rangle = \alpha u_p$

$$\begin{aligned} O_{RL} &= (\overline{u^c} P_R d)\cdot P_L u \\ J_p &= (\overline{u^c}\gamma_5 d)\cdot u \end{aligned}$$

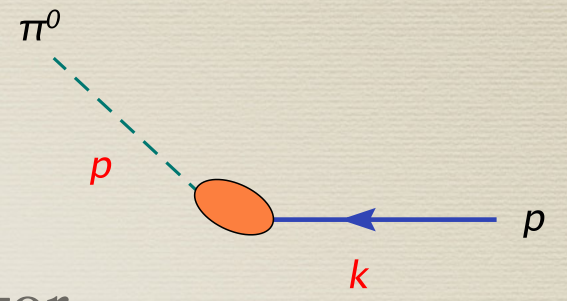
(color indices contracted with ε^{ijk} to make singlet)

:proton interpolation operator

- $$\begin{aligned} \sum_{\vec{x}} \langle 0|O_{RL}(\vec{x},t)\cdot \overline{J}_p(0)|0\rangle &= \sum_{\vec{x}} \langle 0|O_{RL}(\vec{x},t)\cdot \sum_i |i\rangle \frac{1}{2E_i}\langle i|\cdot \overline{J}_p(0)|0\rangle \\ &= \sum_i e^{-E_i t}\frac{1}{2E_i}\langle 0|O_{RL}(0)|i\rangle\langle i|\overline{J}_p(0)|0\rangle \\ (\text{large } t) &\rightarrow e^{-m_p t}\frac{1}{2m_p}\langle 0|O_{RL}(0)|p\rangle\langle p|\overline{J}_p(0)|0\rangle \end{aligned}$$

- $$\sum_{\vec{x}} \langle 0|J_p(\vec{x},t)\cdot \overline{J}_p(0)|0\rangle \rightarrow e^{-m_p t}\frac{1}{2m_p}\langle 0|J_p(0)|p\rangle\langle p|\overline{J}_p(0)|0\rangle$$
  - linear combination of products of 3 quark propagators
  - quark propagators: inverse of domain-wall fermion Dirac operator
  - engineering the interpolation operator necessary to have good S/N

Direct method to calculate W₀



- $$\langle \pi^0 | O_{RL} | p \rangle = P_L[W_0 - \frac{i\not{q}}{m_p} W_1] u_p$$

  $$O_{RL} = (\overline{u^c} P_R d) \cdot P_L u$$

  $$J_p = (\overline{u^c} \gamma_5 d) \cdot u$$:proton interpolation operator

  $$J_\pi^0 = \frac{1}{\sqrt{2}} (\bar{u} \gamma_5 u - \bar{d} \gamma_5 d)$$:pion interpolation operator

- *three point function with momentum injection to pion in proton's rest frame*

  $$\sum_{\vec{y}} \sum_{\vec{x}} e^{i\vec{p}\cdot(\vec{y}-\vec{x})} \langle 0 | J_{\pi^0}(\vec{y}, t') \cdot O_{RL}(\vec{x}, t) \cdot \overline{J}_p(0) | 0 \rangle$$

  $$(t' \gg t \gg 0) \ \rightarrow e^{-E_\pi (t'-t)} e^{-m_p t} \frac{1}{2E_\pi} \frac{1}{2m_p} \langle 0 | J_{\pi^0} | \pi^0(\vec{p}) \rangle \langle \pi^0(\vec{p}) | O_{RL}(0) | p \rangle \langle p | \overline{J}_p(0) | 0 \rangle$$

- $$\sum_{\vec{x}} \langle 0 | J_p(\vec{x}, t) \cdot \overline{J}_p(0) | 0 \rangle \quad \rightarrow \quad e^{-m_p t} \frac{1}{2m_p} \langle 0 | J_p(0) | p \rangle \langle p | \overline{J}_p(0) | 0 \rangle$$

- $$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | J_{\pi^0}(\vec{x}, t) \cdot J_{\pi^0}(0) | 0 \rangle \quad \rightarrow \quad e^{-E_\pi t} \frac{1}{2E_\pi} \langle 0 | J_{\pi^0}(0) | \pi^0(\vec{p}) \rangle \langle \pi^0(\vec{p}) | J_{\pi^0}(0) | 0 \rangle$$

- *through some projection/subtraction, W_0 is obtained.*

Operator Property

- Operator $qqql$:
  - Lorentz symmetry: $(\overline{q^c}\Gamma q)(\overline{l^c}\Gamma' q)$
  - $SU(3)_c$ singlet: $\epsilon_{ijk}q^iq^jq^kl$
  - $SU(2)\times U(1)$ symmetry determines the relative coefficients of the operators in low energy Lagrangian.
  - relevant for nucleon decay: $q=u,d,s$ $(m_c>m_N)$.
  - at QCD scale, lepton is treated trivially, so we are left with

$$\mathcal{O}=\epsilon_{ijk}(u^{iT}C\Gamma d^j)\Gamma' s^k$$

Lorentz spinor

(u,d,s) simply labeling different flavors, not necessarily mean real flavors

- $\Gamma\Gamma'$ Lorentz structure variation with fixed flavor ordering:
  - notation: $S=1$, $P=\gamma_5$, $V=\gamma_\mu$, $A=\gamma_\mu\gamma_5$, $T=\sigma_{\mu\nu}$:
  - $\mathcal{P}^-$: SS, PP, AA, VV, TT.
  - $\mathcal{P}^+$: SP, PS, AV, VA, $T\tilde{T}$.

Operator

$$(\Gamma\Gamma')_{uds} = \epsilon_{ijk}(u^{iT}C\Gamma d^j)\Gamma' s^k$$

- $\mathcal{P}^-$ operators
  - $(SS)_{uds}$, $(SS)_{dsu}$, $(SS)_{sud}$
  - $(PP)_{uds}$, $(PP)_{dsu}$, $(PP)_{sud}$
  - $(AA)_{uds}$
  - $(VV)_{uds}$
  - $(TT)_{uds}$
- $\mathcal{S}$: switching ($u \leftrightarrow d$): $(\Gamma\Gamma')_{dus} = \pm(\Gamma\Gamma')_{uds}$
- Any vector, tensor indices can be eliminated.
  - $\Gamma(\Gamma') = S, P$ or L, R do everything: common form in the low energy effective Lagrangian → Weinberg PRL 43 (1979) 1566.
- $(SS)_{uds} + (SS)_{dsu} + (SS)_{sud} + (PP)_{uds} + (PP)_{dsu} + (PP)_{sud} = 0$
- $(\Gamma\Gamma)_{uds}$ with (SS, PP, AA, VV, TT) can be used as a complete set: We use them for the operator renormalization.
- Similar property for $\mathcal{P}^-$.

Renormalization: mixing

$\Gamma\ \Gamma'$

	$\mathcal{S}^-$	$\mathcal{S}^+$
$\mathcal{P}^-$	SS, PP, AA	VV, TT
$\mathcal{P}^+$	SP, PS, AV	VA, $T\tilde{T}$

$$\begin{pmatrix} SS \\ PP \\ AA \end{pmatrix}_{ren} = \left(\quad {Z'_{ND}}^{ij} \quad \right) \begin{pmatrix} SS \\ PP \\ AA \end{pmatrix}_{latt} , \quad \begin{pmatrix} SP \\ PS \\ -AV \end{pmatrix}_{ren} = \left(\quad {Z'_{ND}}^{ij} \quad \right) \begin{pmatrix} SP \\ PS \\ -AV \end{pmatrix}_{l}$$

$$\begin{pmatrix} R \cdot L \\ L \cdot L \\ A \cdot LV \end{pmatrix}_{ren} = \left(\quad {Z_{ND}}^{ij} \quad \right) \begin{pmatrix} R \cdot L \\ L \cdot L \\ A \cdot LV \end{pmatrix}_{latt} , \quad \begin{pmatrix} R \cdot R \\ L \cdot R \\ A \cdot RV \end{pmatrix}$$: similar.

- $(\Gamma\Gamma')_{udu} \equiv \epsilon_{ijk}(\bar{u}^{ci}\Gamma d^j)\Gamma' u^k$ is renormalized in the same way.
- Wilson fermion (Richards, Sachrajda, Scott):
  $(RL)_{ren} = Z(RL)_{latt} + \frac{\alpha_s}{4\pi} Z_{mix}(LL)_{latt} + \frac{\alpha_s}{4\pi} Z'_{mix}(A \cdot LV)_{latt}$
  no other terms appear at any order.

RI/MOM scheme renormalization

$$G^a(x_0, x_1, x_2, x_3) = \langle \mathcal{O}^a_{uds}(x_0)\bar{u}(x_1)\bar{d}(x_2)\bar{s}(x_3)\rangle.$$

- a labels chiral structure type $\Gamma\Gamma'$.

$$\mathcal{O}^a_{uds\ ren} = Z^{ab}_{ND} O^b_{uds\ latt}$$

- Momentum p for all three external quarks, amputate it with three quark propagators:

$$\Lambda^a(p^2) = \text{F.T. } G^a(0, x_1, x_2, x_3)|_{Amp}.$$

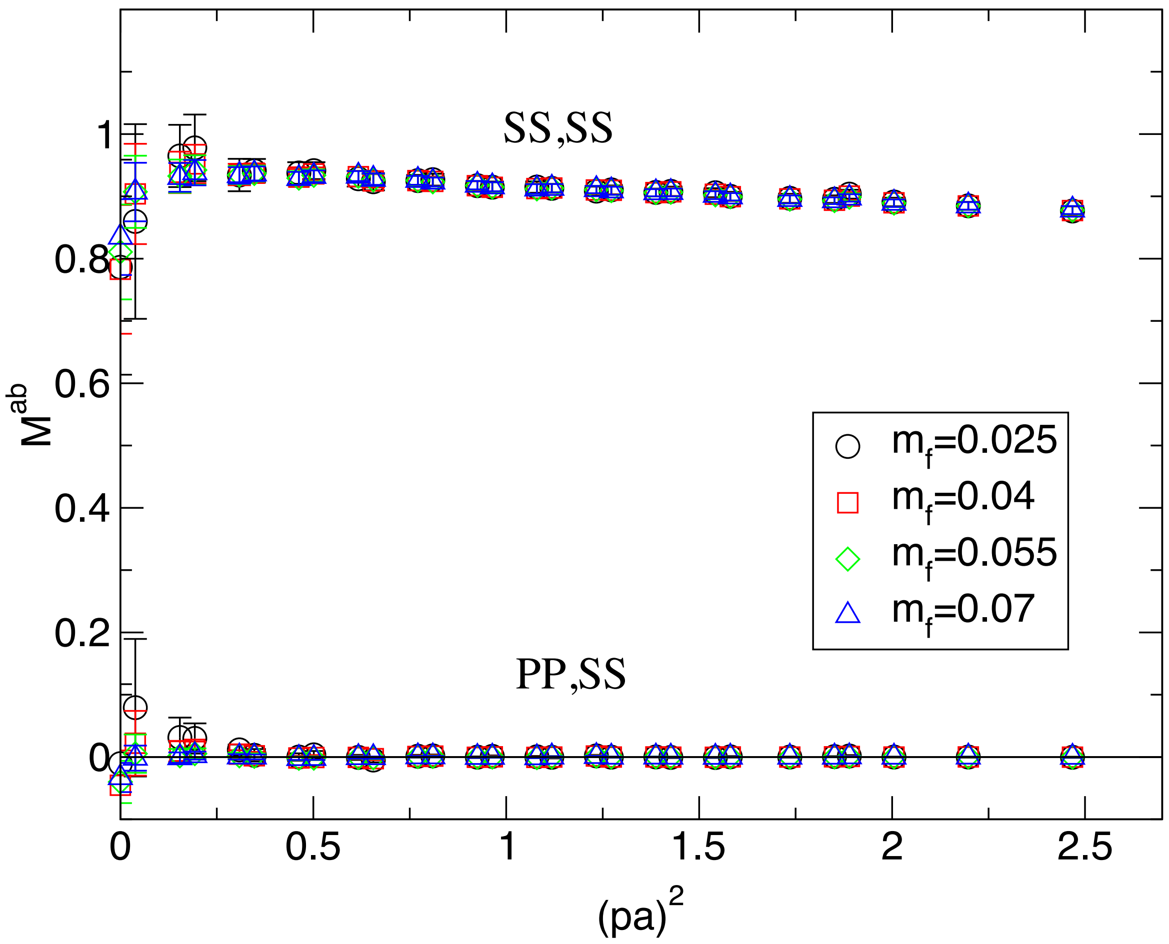
- Renormalization condition reads at scale p,

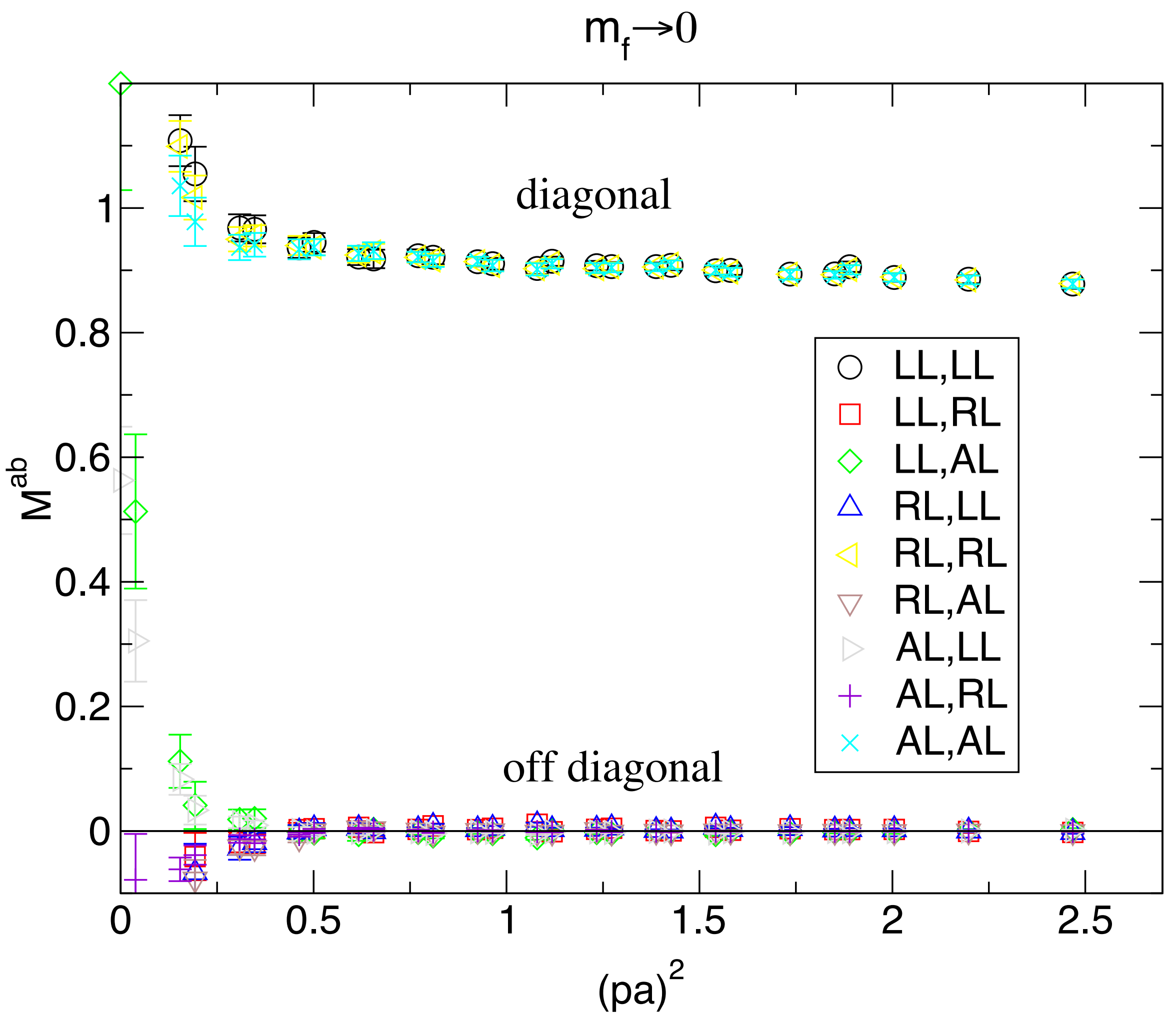
$$P^a_{ijk\ \beta\alpha\ \delta\gamma} \cdot Z_q^{-3/2} Z^{bc}_{ND} \Lambda^c_{ijk\ \alpha\beta\ \gamma\delta} = \delta^{ab},$$

$$M^{ab} = P^a_{ijk\ \beta\alpha\ \delta\gamma} \cdot \Lambda^b_{ijk\ \alpha\beta\ \gamma\delta} \rightarrow Z_q^{3/2} (Z^{-1}_{ND})^{ab}.$$

Operator Mixing ?

$\beta = 0.87$ DBW2 glue ($a^{-1} = 1.3$ GeV), and DWF with $L_s = 12$ and $M_5 = 1.8$: $m_{\text{res}} \simeq 1.6 MeV$.





- Small mass dependence.

- Off-diagonal elements negligible.
  → No mixing.

Treatment of Z_q

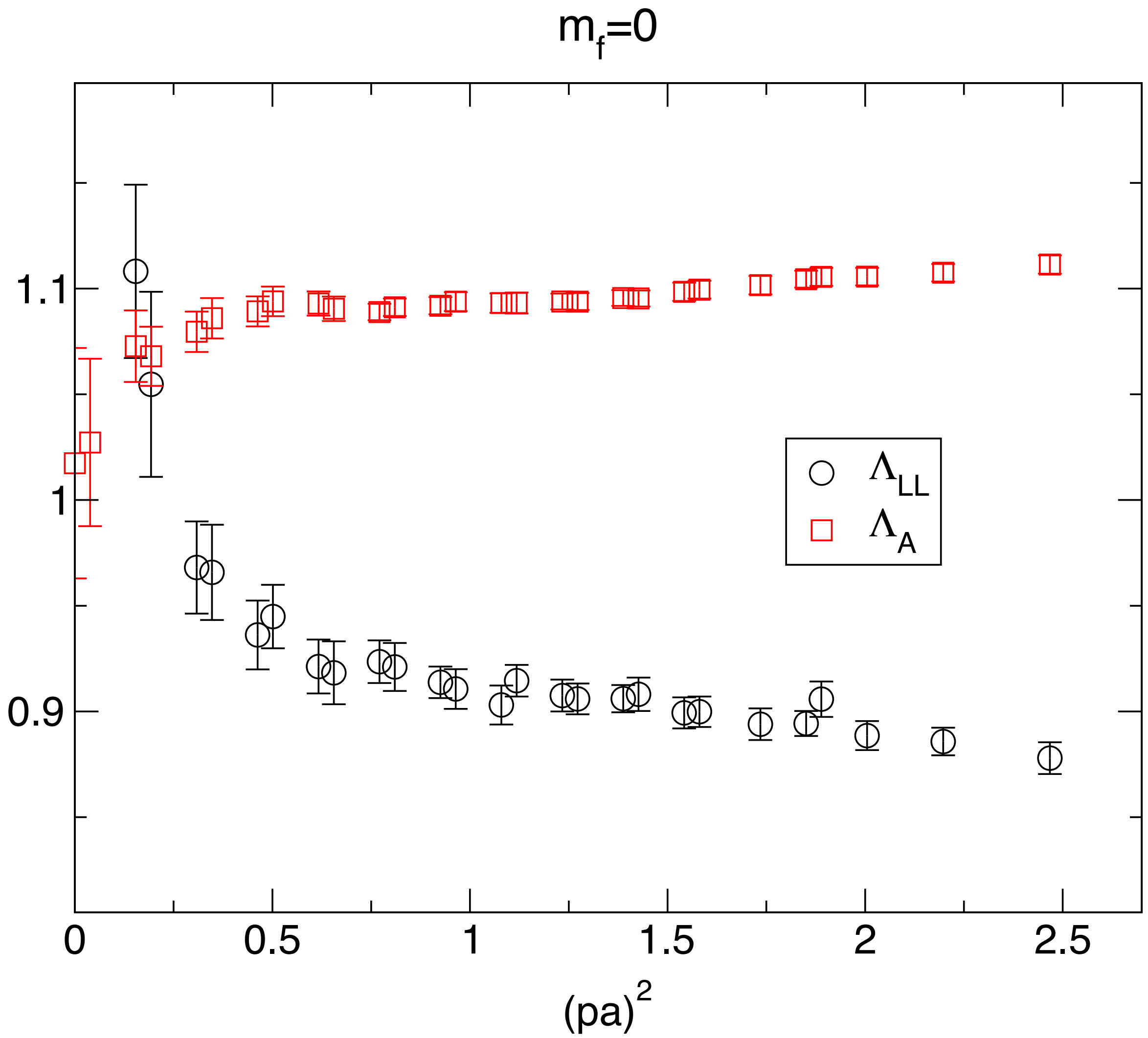
$$M^{LL,LL} \rightarrow Z_q^{3/2}/Z_{ND}^{LL,LL}.$$

$$P_A\Lambda_A \rightarrow Z_q Z_A^{-1}.$$

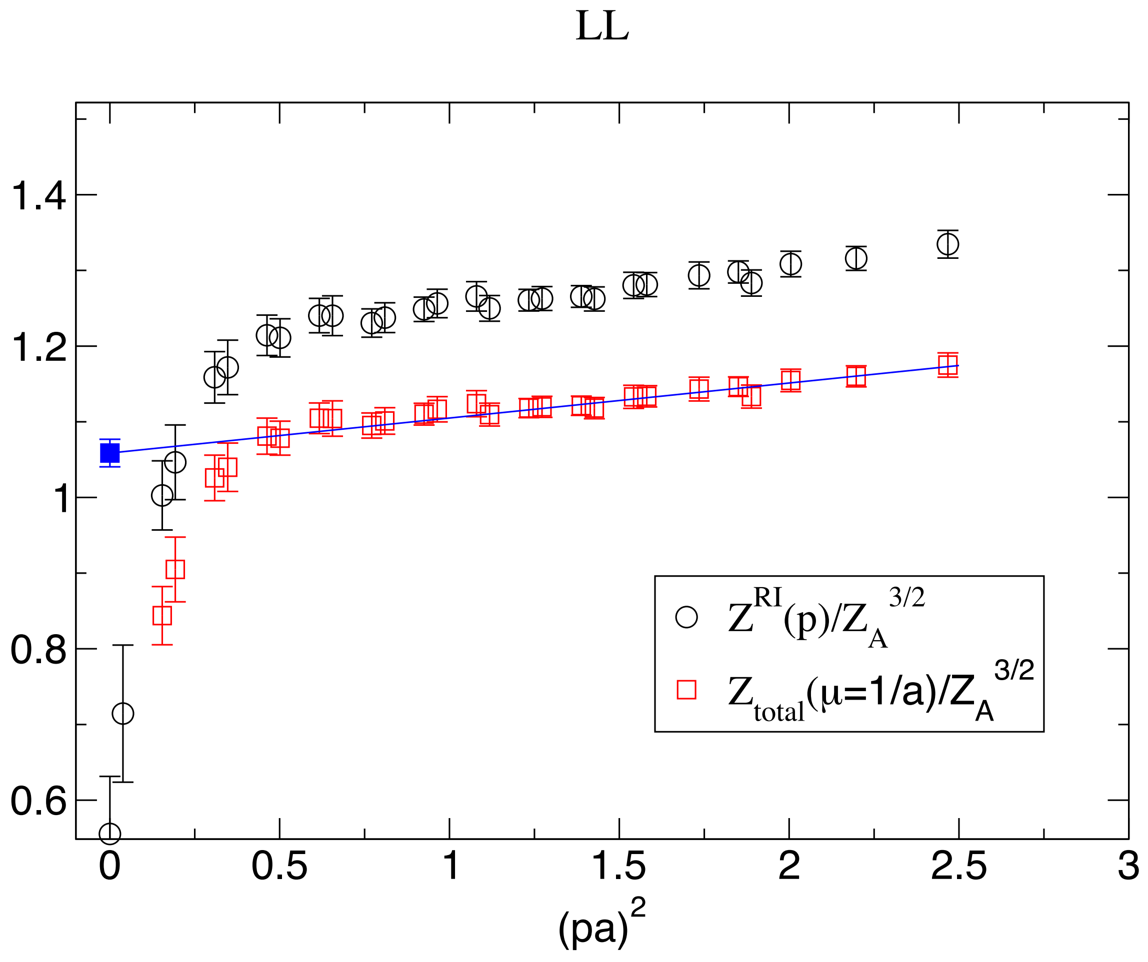
$$(P_A\Lambda_A)^{3/2}/M^{LL,LL} \rightarrow Z_{ND}^{LL,LL}/Z_A^{3/2}.$$

$$Z_A = 0.7798(5) \leftarrow \frac{\langle \mathcal{A}_\mu P\rangle}{\langle A_\mu P\rangle}$$

- Z_A has no scale dependence (Null anomalous dimension for A_μ).
- $(P_A\Lambda_A)^{3/2}/M^{LL,LL}$ has same scale dependence as Z_{ND} and possible $O(a^2)$ discretization error.



Matching to $\overline{MS}$ (NLO)



$$\mathcal{O}^{\overline{MS}}(\mu) = \underbrace{\overbrace{U^{\overline{MS}}(\mu;p)\frac{Z^{\overline{MS}}(p)}{Z^{RI}(p)}}^{perturbative} Z^{RI}(p)}_{Z_{total}(\mu)} \mathcal{O}^{latt}$$

- 2-loop anomalous dimension: Nihei, Arafune (94)
- 1-loop matching (finite part): This work
- NPR: This work
- The product is independent of p. Let's set $\mu = 1/a$.

$$Z_A = 0.7798(5) \quad \rightarrow \quad Z_{total}^{LL,LL}(\mu = 1/a) = 0.73(1).$$

$\beta = 0.87$ DBW2 glue ($a^{-1} = 1.3$ GeV), and DWF with $L_s = 12$ and $M_5 = 1.8$:

Recent improvement in RI/MOM renormalization

- issues
  - p must be large enough, to control perturbation theory
  - p must be small enough, to control lattice artifact (pa)^n
  - $\Lambda_{QCD} \ll p \ll 1/a$ Window problem
- solution
  - super fine lattice
  - perturbation theory: higher order
  - scheme: less contamination of low energy physics

NLO→NNLO

RI/SMOM

Proton Decay Matrix Elements projects

Collaboration in Japan

- **Eigo Shintani** (Tsukuba)
- **Ryutaro Tsuji** (KEK)
- Yoshinobu Kuramashi (Tsukuba)
- YA

using PACS Wilson configurations

$V=64^4$ $L=5.5$ fm

$1/a=2.3$ GeV

$a = 0.08$ fm

Program for Promoting Researches on the Supercomputer Fugaku

Large-scale lattice QCD simulation and development of AI technology

Collaboration in USA

- **Jun-Sik Yoo** (KEK/Stony Brook)
- Taku Izubuchi (BNL/RBRC)
- Amarjit Soni (BNL)
- Sergey Syritsyn (Stony Brook/RBRC)
- YA

using RBC/UKQCD DWF configurations

$V=24^3$x64, 32^3x64 $L=4.7$ fm

$1/a=1$ and 1.4 GeV

$a = 0.2, 0.14 \text{ fm} \rightarrow 0$

[PRD 105, 0074501 (2022)]

use of **PACS** ensemble @ physical pion mass

N_f=2+1 PACS ensemble

- Iwasaki gauge β=1.82
- **stout smeared Wilson fermion**: ρ=0.1, N=6
- ud and s quarks are on **physical point**
- $1/a$ = 2.333(18) GeV
- 64^4 is mostly used in this study: $m_\pi L$=3.8

statistical note

- ~100 configurations
- for each config
  - matrix elements: **AMA**
    - one exact and
    - 256 sloppy solves
  - NPR:
    - single point source

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Renormalization:

Richards et al 1987 lattice perturbation theory

(Notation JLQCD2000)

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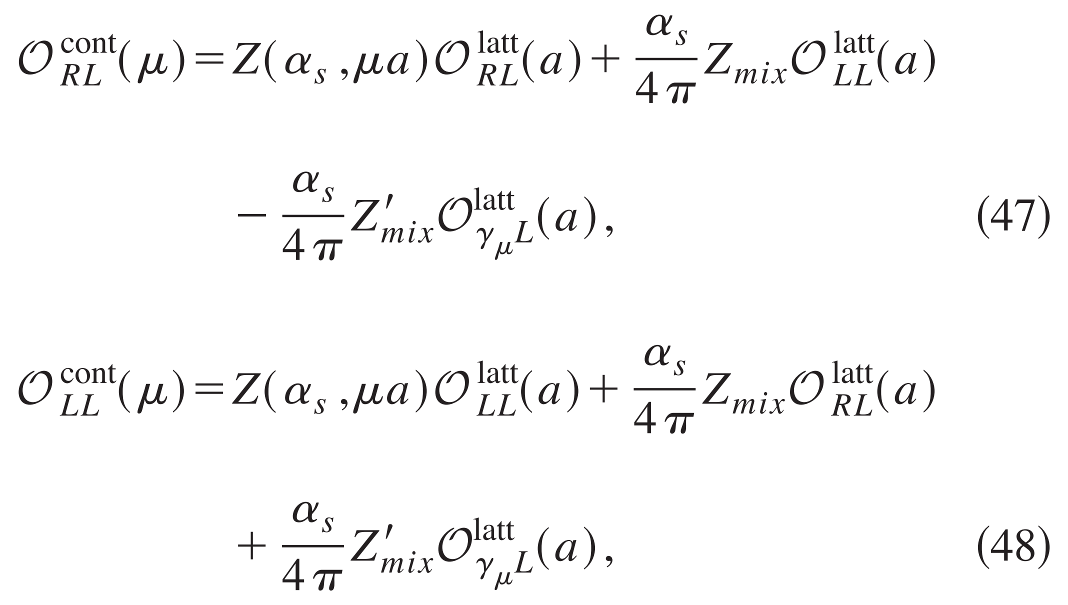


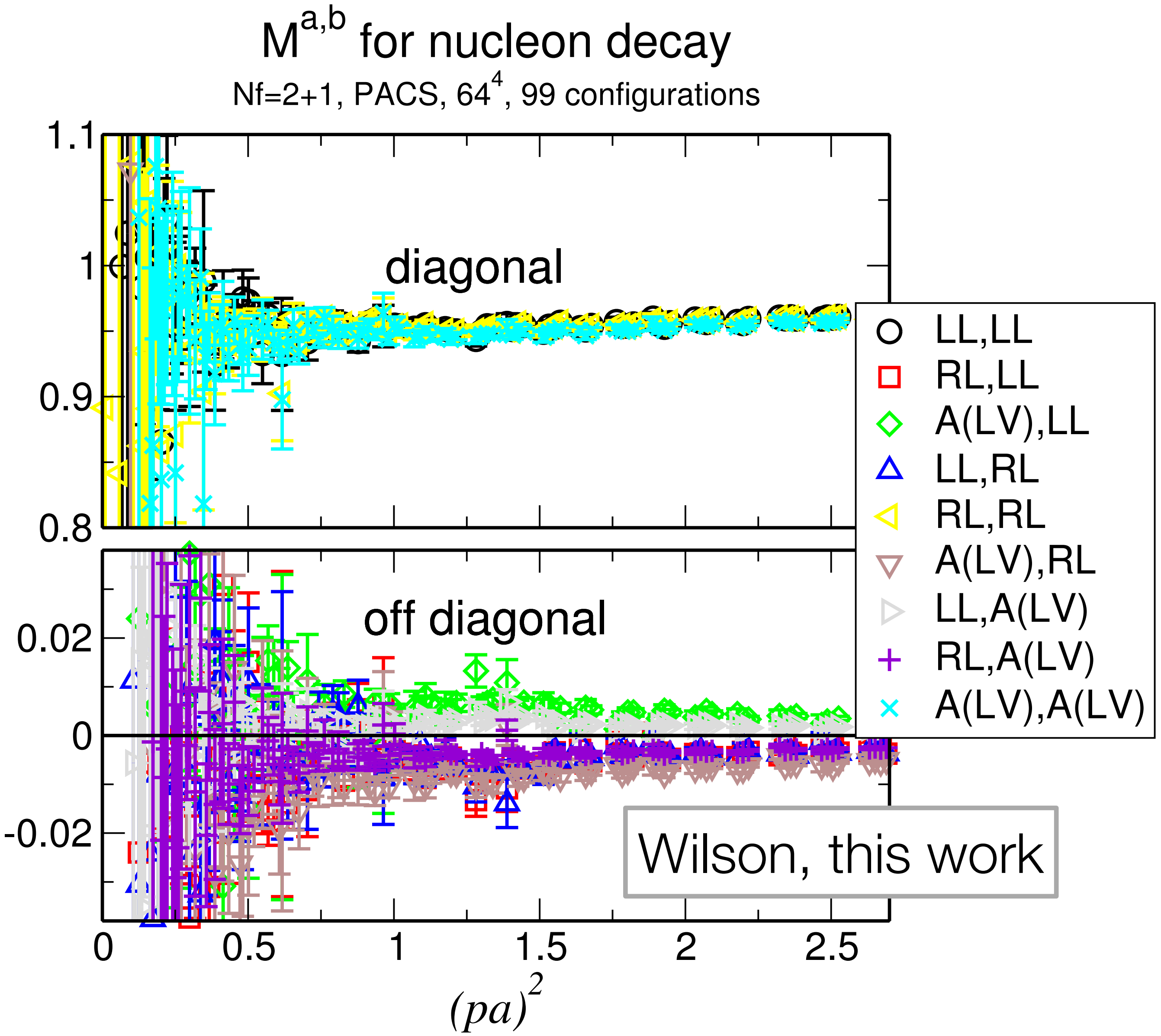
$$\mathcal{O}^{\rm cont}_{RL}(\mu)=Z(\alpha_s,\mu a)\mathcal{O}^{\rm latt}_{RL}(a)+\frac{\alpha_s}{4\pi}Z_{mix}\mathcal{O}^{\rm latt}_{LL}(a)$$

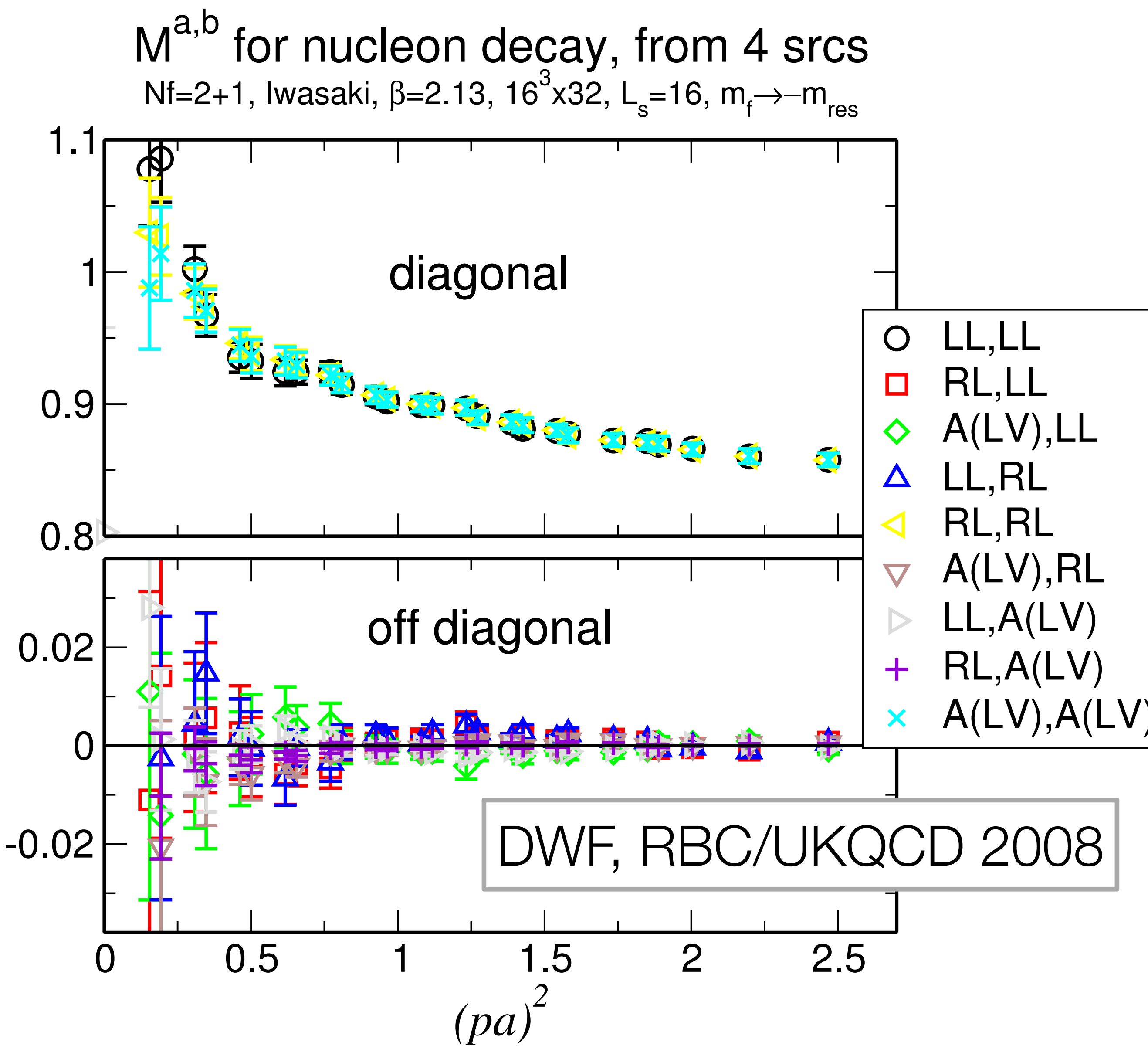
$$-\frac{\alpha_s}{4\pi}Z'_{mix}\mathcal{O}^{\rm latt}_{\gamma_\mu L}(a), \qquad (47)$$

$$\mathcal{O}^{\rm cont}_{LL}(\mu)=Z(\alpha_s,\mu a)\mathcal{O}^{\rm latt}_{LL}(a)+\frac{\alpha_s}{4\pi}Z_{mix}\mathcal{O}^{\rm latt}_{RL}(a)$$

$$+\frac{\alpha_s}{4\pi}Z'_{mix}\mathcal{O}^{\rm latt}_{\gamma_\mu L}(a), \qquad (48)$$

RI/MOM 3q vertex matrix: comparison with DWF





$$\left(\begin{array}{rcl} O_{RL} & = & (\overline{u^c} P_R d) \cdot P_L s \\ O_{LL} & = & (\overline{u^c} P_L d) \cdot P_L s \\ O_{A(LV)} & = & (\overline{u^c} \gamma_\mu \gamma_5 d) \cdot P_L \gamma_\mu s \end{array}\right.$$

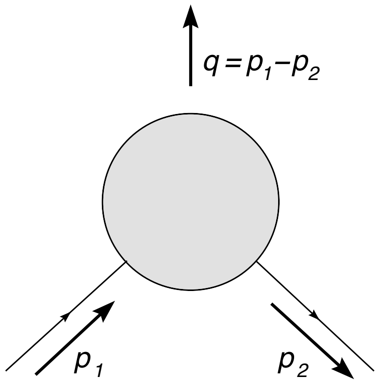
- $(pa)^2$ dep bit different → lattice artifact dominant
- Wilson:

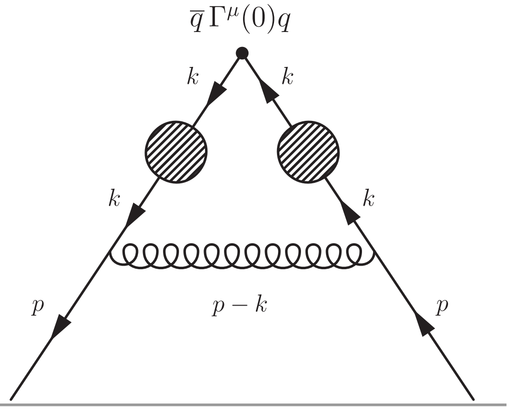
  off-diagonal larger than DWF, but, ≲ 1% → treated as negligible below

renormalization

- ratio of 3q, 2q vertex function → $Z_{ND}/Z_A^{3/2}, Z_{ND}/Z_V^{3/2}$
    - input Z_V or Z_A from SF scheme →Z_{PD}

renormalization





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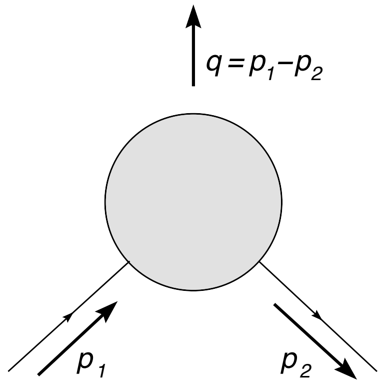
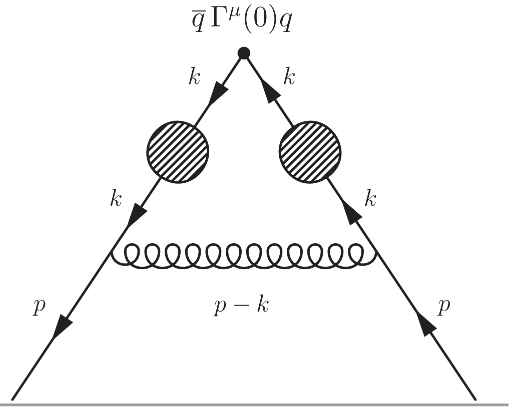
    - input Z_V or Z_A from SF scheme →Z_{PD}
- DWF (RBC/UKQCD) [2006〜2017]
    - wave function renormalization (bilinear) uses exceptional momenta (q_μ=0)
        - sensitive to the change of m_f and SSB → source of systematic error

renormalization





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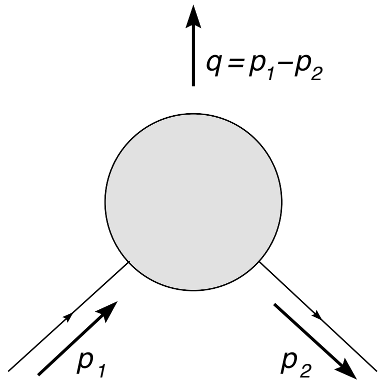
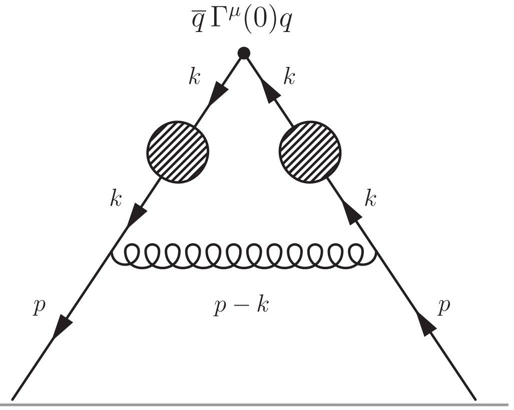
renormalization





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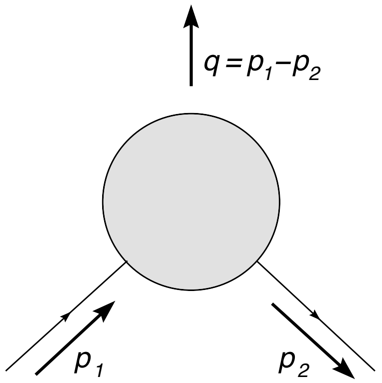
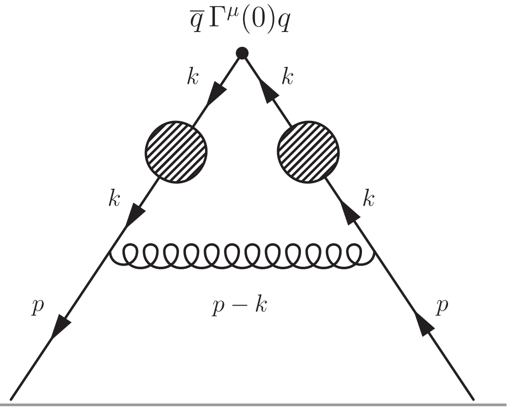
- utilize RI/**S**MOM wave function renormalization: A_μ, V_μ

    ➡proton decay **S**MOM schemes: SMOM, SMOM_{γ_μ}

    - had expected effect on bilinear and 4q operators
    - $\overline{\text{MS}}$ matching w/ NLO perturbation theory
        - totally: 2x2 = 4 schemes

renormalization





- ratio of 3q, 2q vertex function → $Z_{ND}/Z_A^{3/2}, Z_{ND}/Z_V^{3/2}$
    - input Z_V or Z_A from SF scheme →Z_{PD}
- DWF (RBC/UKQCD) [2006〜2017]
    - wave function renormalization (bilinear) uses exceptional momenta (q_μ=0)
        - sensitive to the change of m_f and SSB → source of systematic error
    - 3 quark op uses non-exceptional momentum
- utilize RI/**S**MOM wave function renormalization: A_μ, V_μ

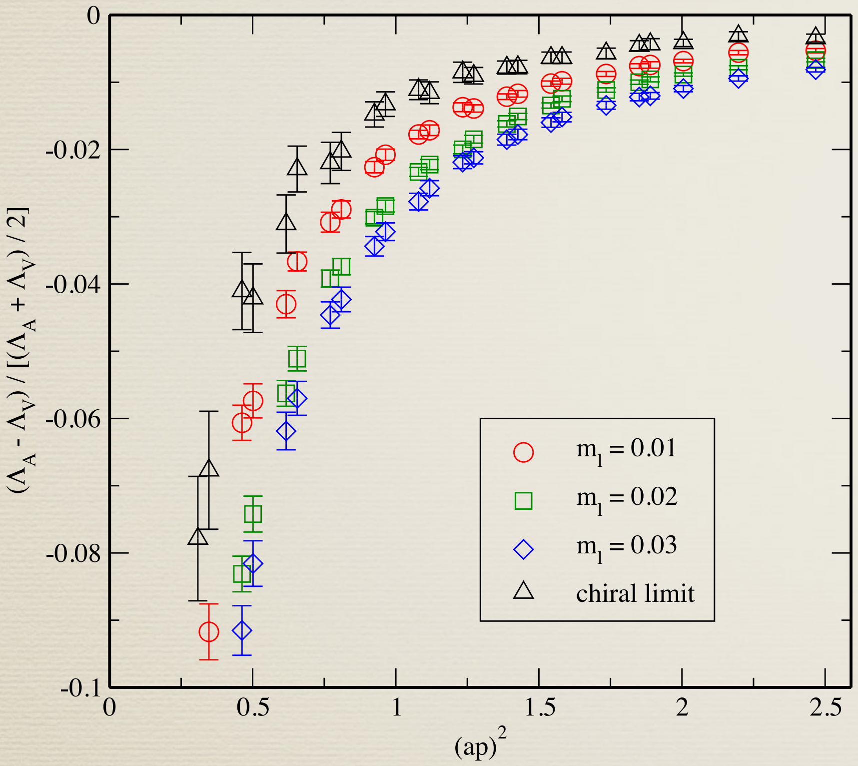
  ➡proton decay **S**MOM schemes: SMOM, $\mathrm{SMOM}_{\gamma_\mu}$
    - had expected effect on bilinear and 4q operators
    - $\overline{\mathrm{MS}}$ matching w/ NLO perturbation theory
        - totally: 2x2 = 4 schemes

SMOM schemes:
fully utilize
non-exceptional momenta

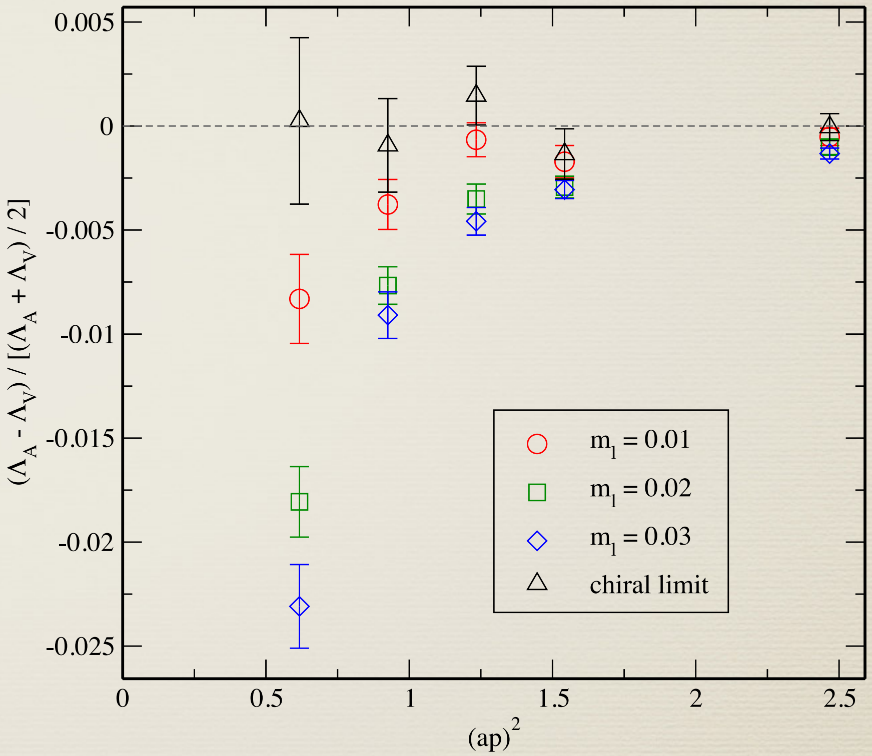
A test of non-exceptional mom

$\Lambda_A - \Lambda_V$: RBC/UKQCD [PRD 2008]

exceptional

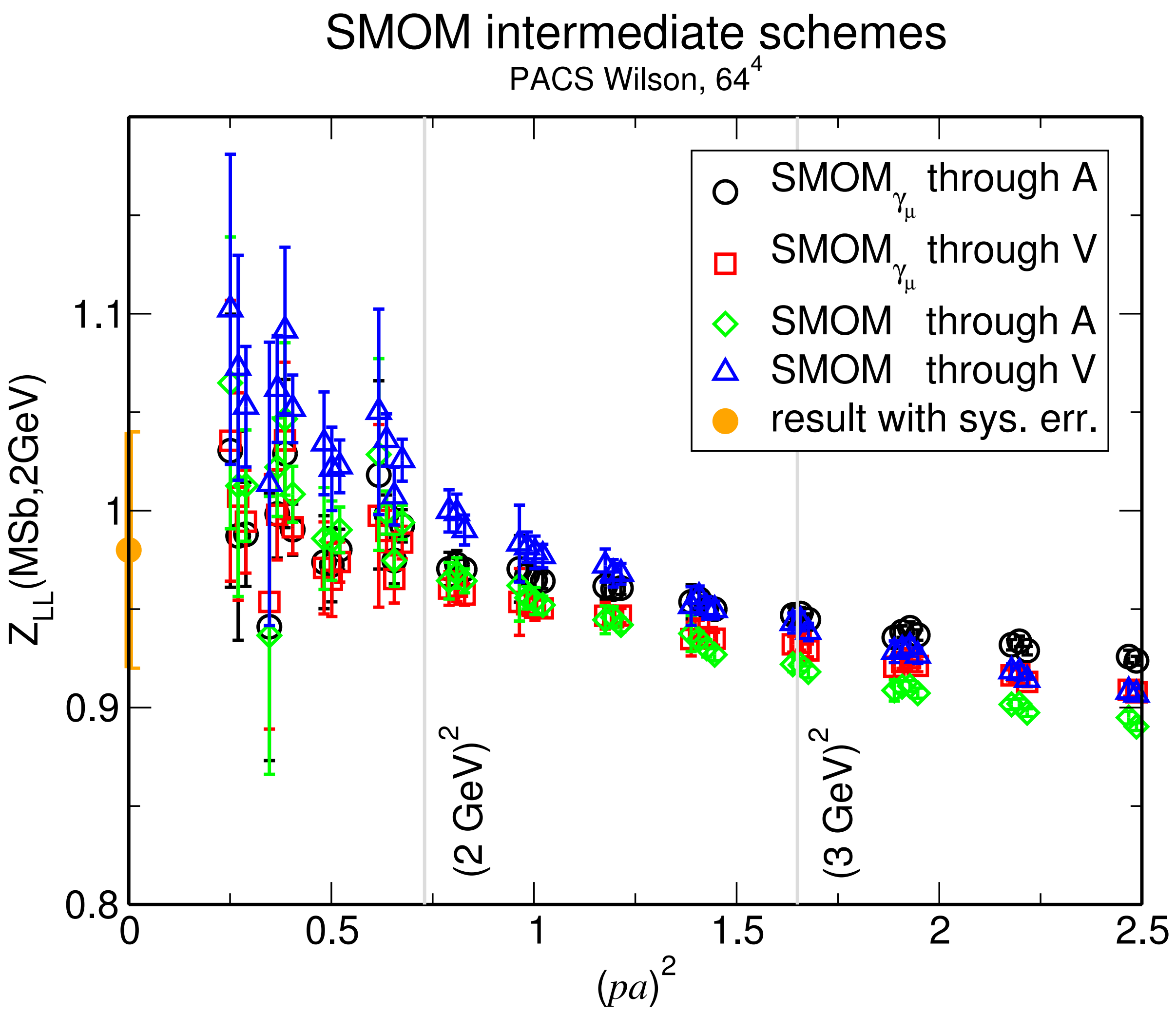


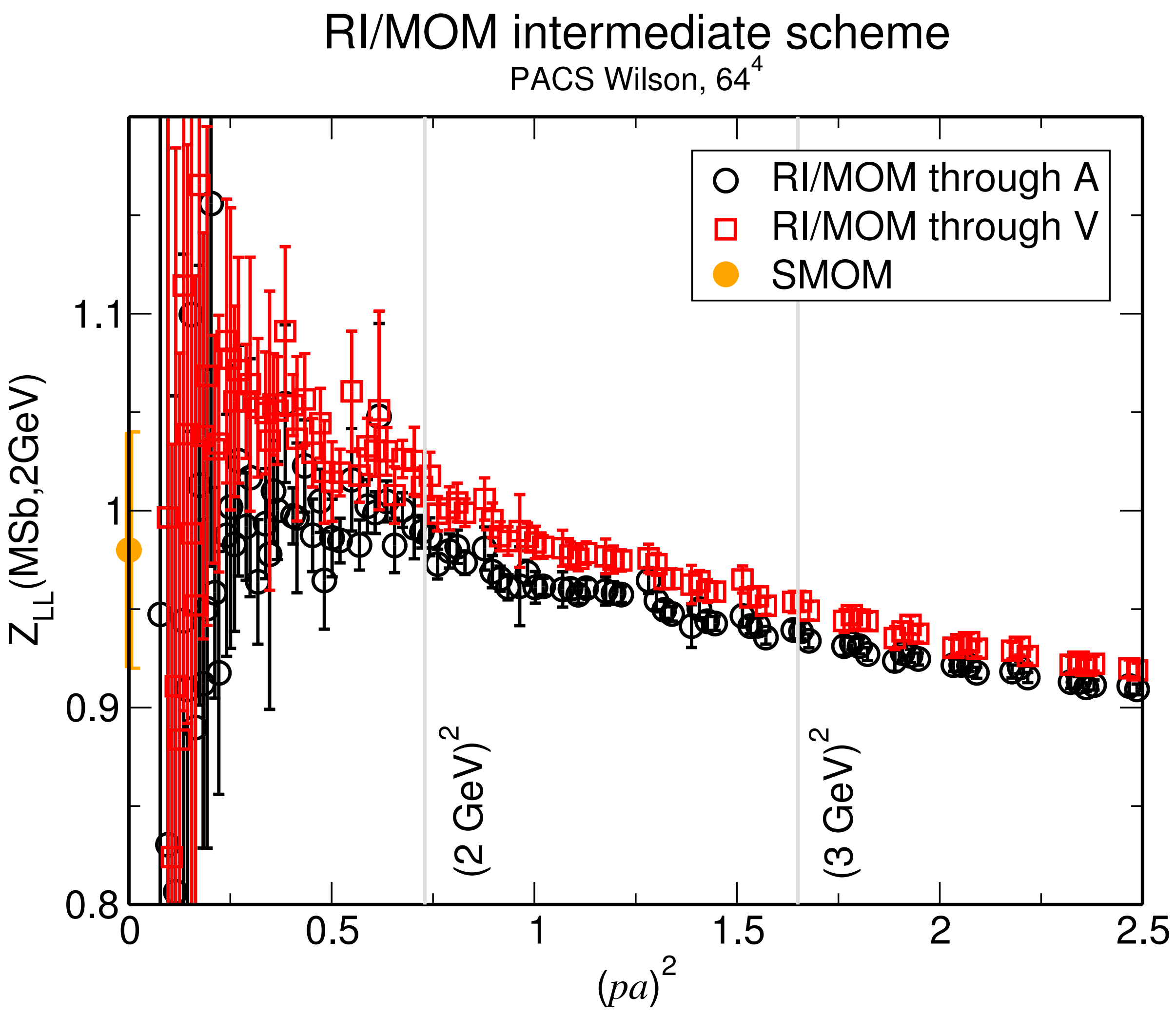
non-exceptional



- The success created a very good motivation to invest in non-exceptional momenta

$\overline{\text{MS}}$ Z(2GeV) from RI/SMOM schemes ver.1

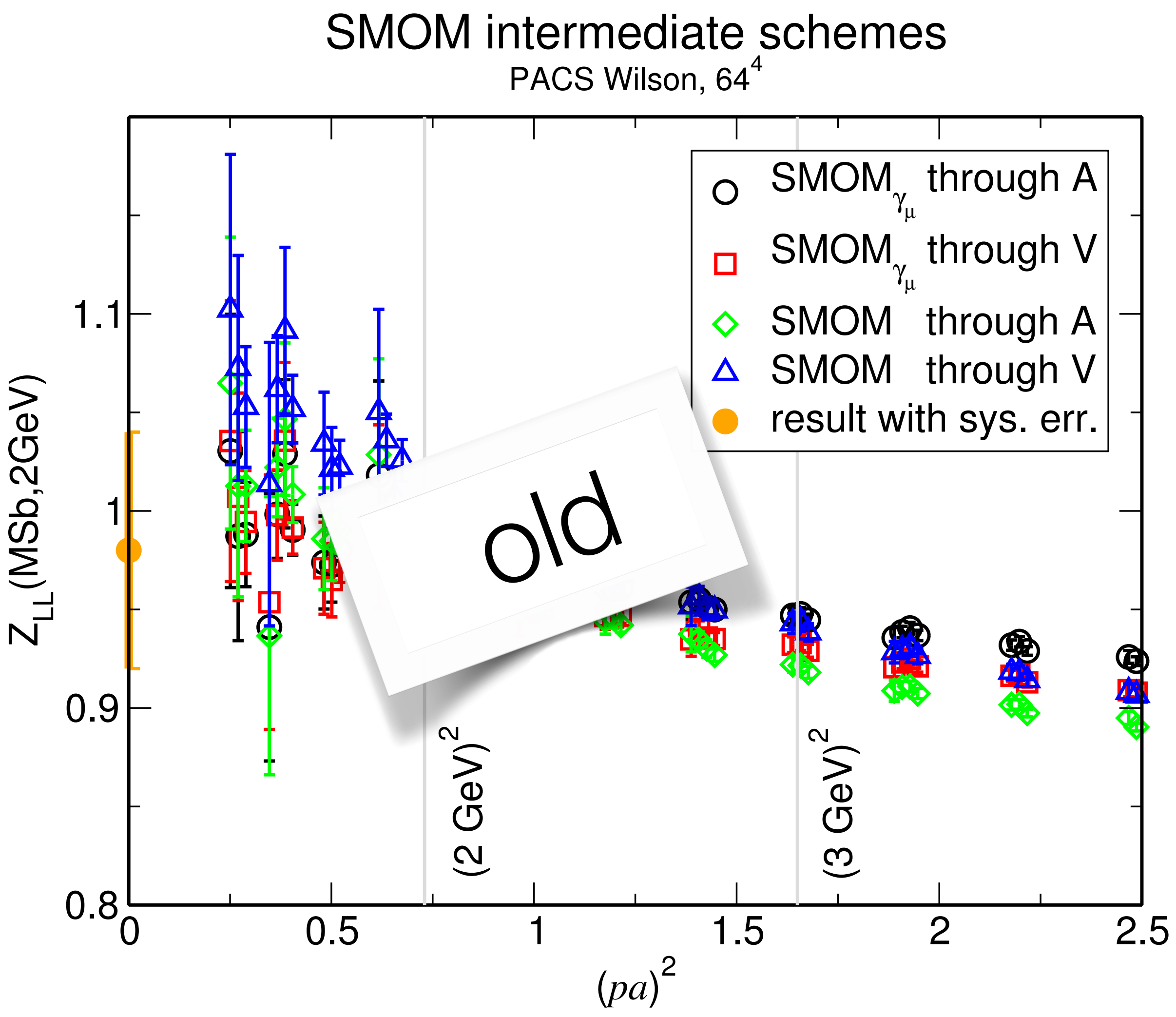

SMOM intermediate schemes
PACS Wilson, 64^4


RI/MOM intermediate scheme
PACS Wilson, 64^4

- Z_{LL}(MSb, 2GeV) = 0.98 (6)
- Z_{RL}(MSb, 2GeV) = 0.98 (7)

@ Lattice 2019

$\overline{\text{MS}}$ Z(2GeV) from RI/SMOM schemes improved


SMOM intermediate schemes

PACS Wilson, 64^4

Improvement:

[Tsuji et al Lattice 2024]

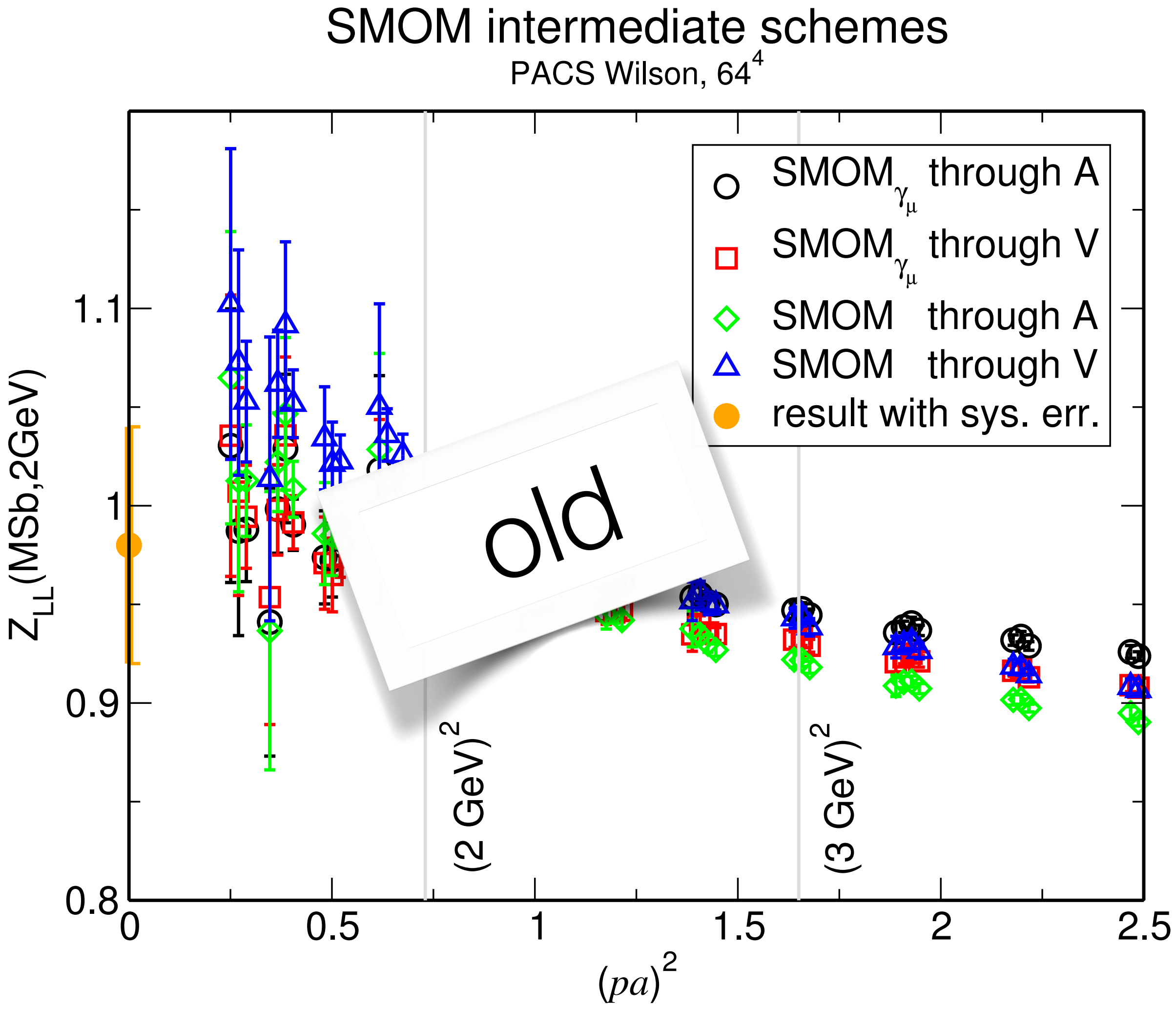
- (improved stat.)
- use of SYM3q scheme
  - ➡ NNLO available
- remove *(pa)*^{2} and higher
- remove non-pert. eff.
  - ➡ fit variation
- estimate PT truncation
  - ➡ several interm. scheme

- Z$_{\text{LL}}$(MSb, 2GeV) = 0.98 (6)
- Z$_{\text{RL}}$(MSb, 2GeV) = 0.98 (7)

- $Z^{LL}_{\overline{\text{MS}}} = 1.018(6)_{\text{stat}}(37)_{\text{sys}}$
- $Z^{RL}_{\overline{\text{MS}}} = 1.016(5)_{\text{stat}}(41)_{\text{sys}}$

- note: Z$_{\text{LL}}$(MSb, 2GeV) = Z$_{\text{RL}}$(MSb, 2GeV) ≃ 1 → bare ME ≃ ren. ME

$\overline{\mathrm{MS}}$ Z(2GeV) from RI/SMOM schemes improved



Improvement:

[Tsuji et al Lattice 2024]

- (improved stat.)
- use of SYM me
  - ➡ N
- $a)^2$ and higher
- remove non-pert. eff.
  - ➡ fit variation
- estimate PT truncation
  - ➡ several interm. scheme

preliminary

- Z_{LL}(MSb, 2GeV) = 0.98 (6)
- Z_{RL}(MSb, 2GeV) = 0.98 (7)

➡

- $Z^{LL}_{\overline{\mathrm{MS}}} = 1.018(6)_{\mathrm{stat}}(37)_{\mathrm{sys}}$
- $Z^{RL}_{\overline{\mathrm{MS}}} = 1.016(5)_{\mathrm{stat}}(41)_{\mathrm{sys}}$

- note: Z_{LL}(MSb, 2GeV) = Z_{RL}(MSb, 2GeV) ≃ 1 → bare ME ≃ ren. ME

NPR schemes and matching - utilizing RI/MOM

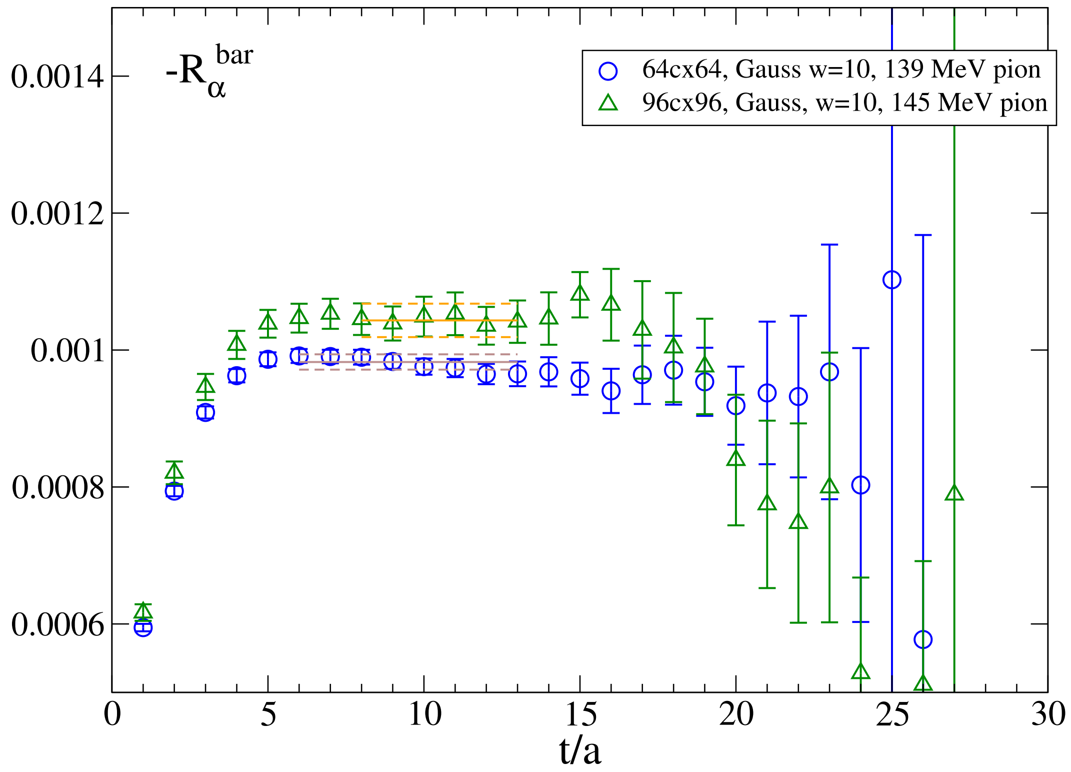
NPR schemes and matching - utilizing RI/MOM

- three quark vertex
  - original:
    - $p_1 = p_2 = p_3 = p,\ q = 3p, \mu^2 = p^2$ (non-exceptional) [RBC 2006] NLO
  - symmetric 3q:
    - $p_1 + p_2 + p_3 = q = 0, p_1^2 = p_2^2 = p_3^2 = p^2 = \mu^2$ (exceptional) [Gracey 2013] NNLO
  - SMOM
    - $p_1^2 = p_2^2 = p_3^2 = q^2 = \mu^2$ NEW! (yet) (non-exceptional) [Kniehl, Veretin 2023] NNLO
- bilinear (q-qbar) vertex for wave function renormalization
  - RI/MOM [Martinelli et al 1993, 1995]
    - $p_1 = p_2, q = 0$ (exceptional)
  - RI/SMOM (symmetric momentum configuration)
    - $p_1^2 = p_2^2 = q^2$ (non-exceptional) [RBC/UKQCD 2007]
    - two schemes: $SMOM$ and $SMOM_{\gamma_\mu}$[Sturm et al 2009] NLO, [Almeida,Sturm 10] NNLO

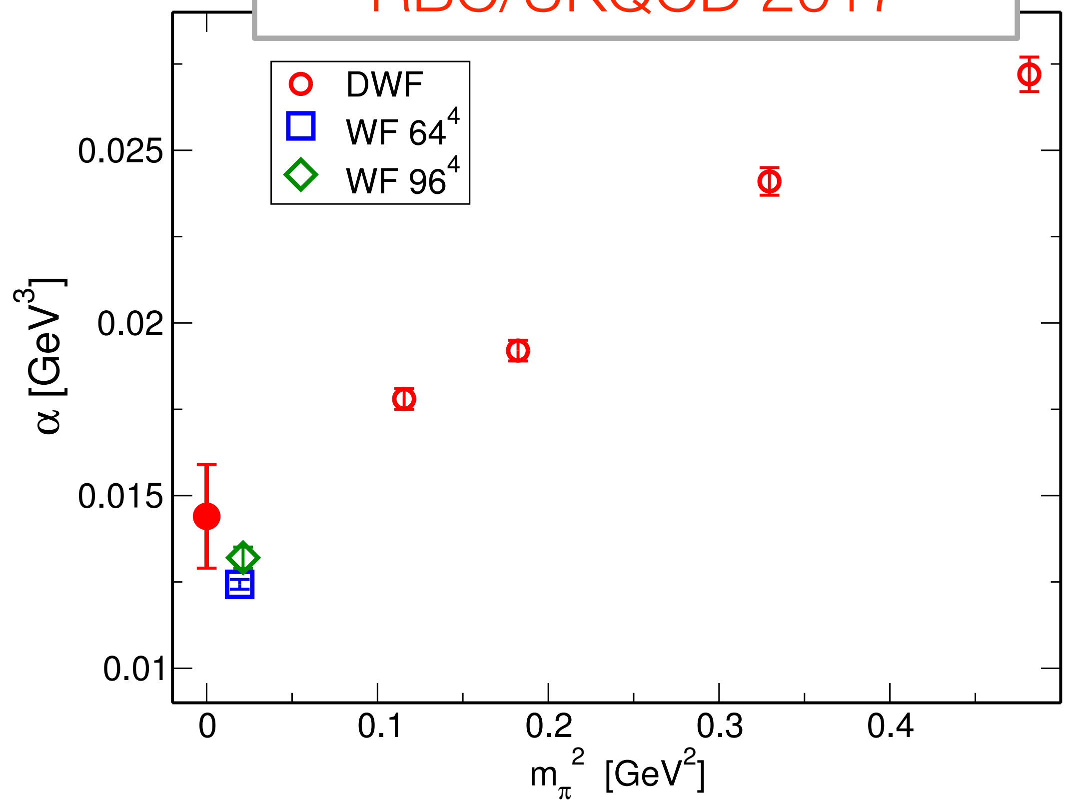
proton decay LEC: α, β

Low energy constants of proton decay

- LO ChPT $W_0[\langle\pi^0|(ud)_R u_L|p\rangle] \simeq \frac{\alpha}{\sqrt{2}f}(1+D+F)$
- ratio of 2pt fuctions



comparing **this work** and
RBC/UKQCD 2017

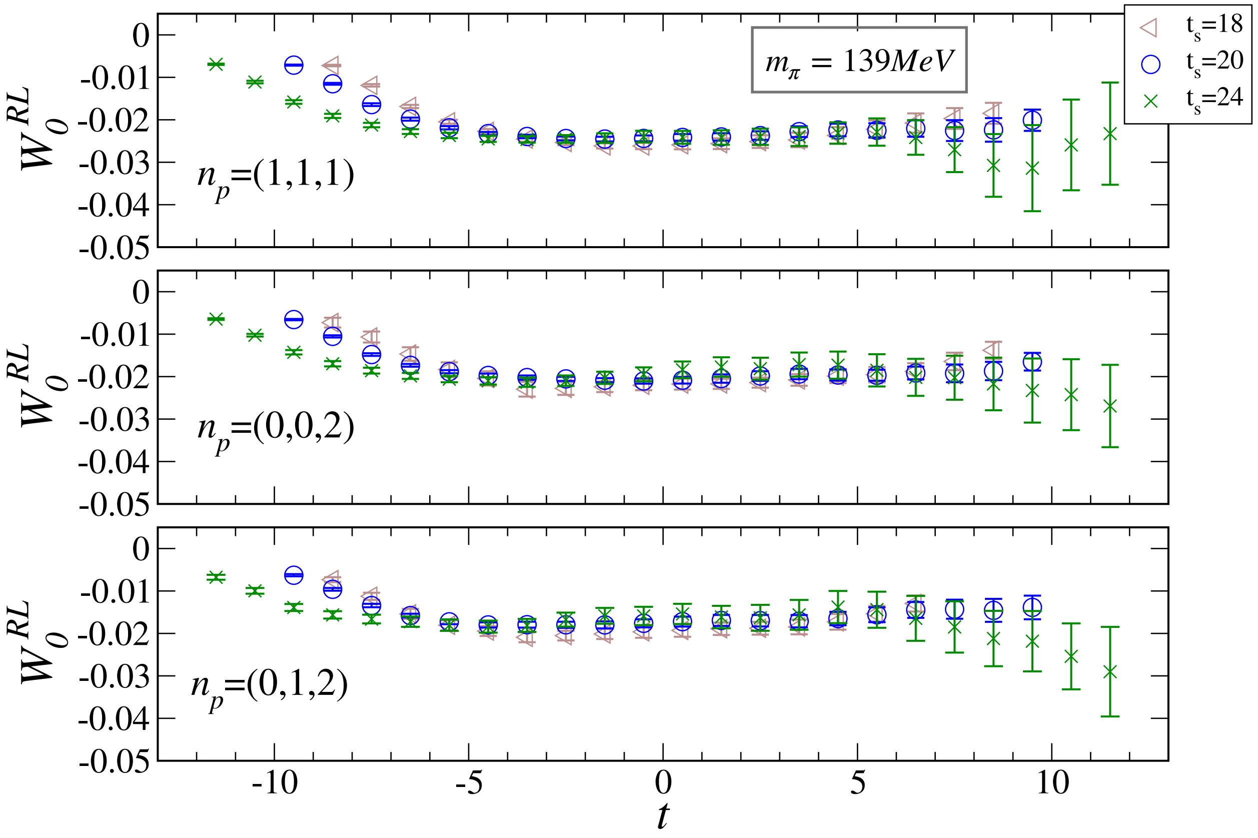


- α consistent with earlier DWF computation w/ long chiral extrapol.
- β as well
- no big surprise happening when going down to physical ud mass

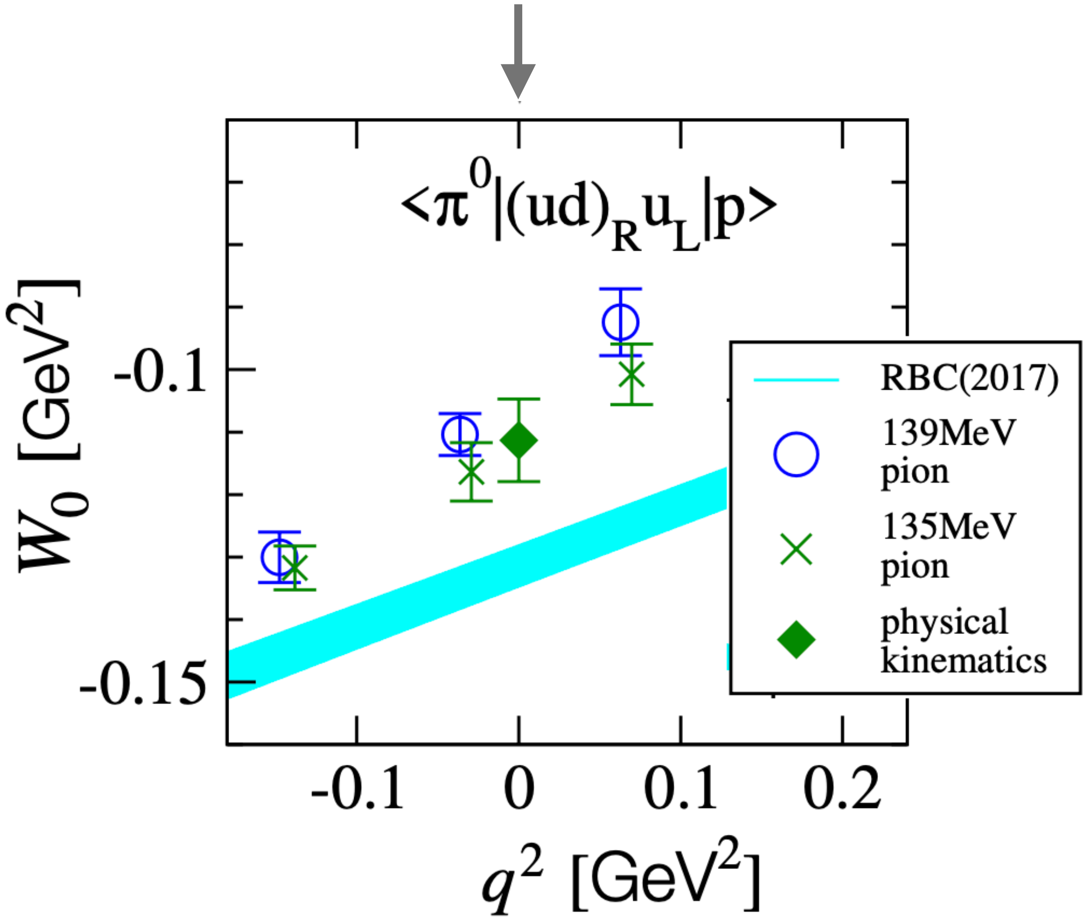
proton decay form factor W_0 for pion final state

Relevant form factor W_0 $\langle\pi^0|(ud)_R u_L|p\rangle$ as an example

- from ratio of 3 and 2 point functions
- meson momentum $\vec{p} = \frac{2\pi}{L}\vec{n}_p$



on-shell lepton: $-q^2=m_l^2=0$



- $|W_0|$ ~20% smaller than DWF (with a long chiral extrapolation) at q^2=0
- consistent with sys. error ! no big surprise found for $m_f \rightarrow m_{ud}$
- 10% total error is not a dream…

RBC/UKQCD study

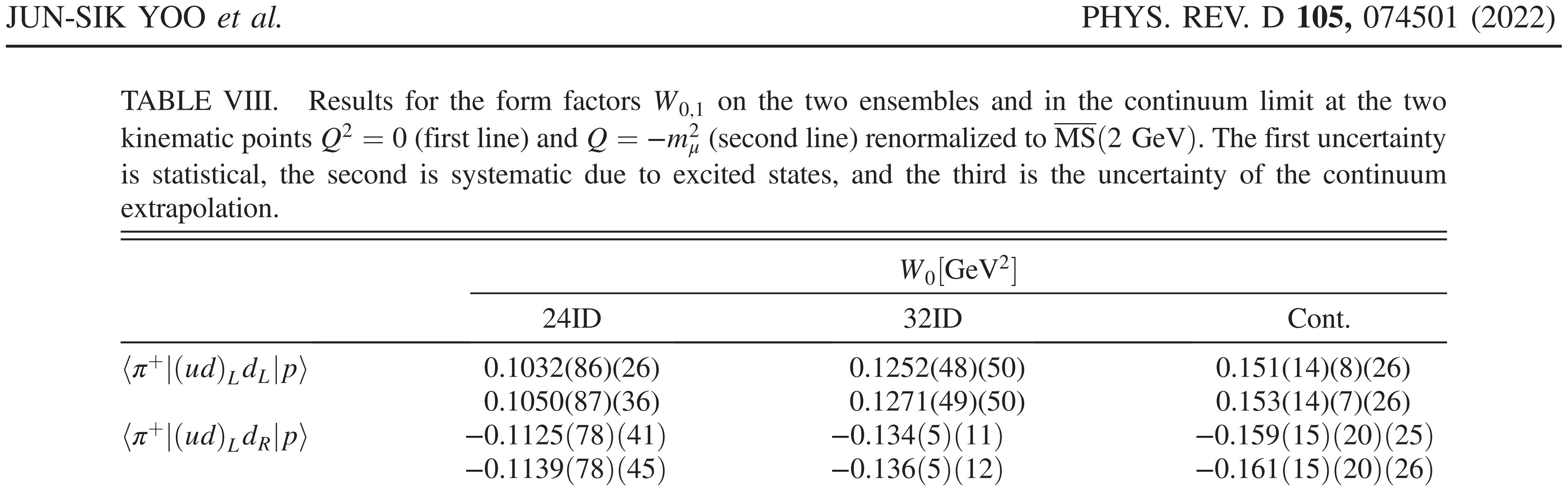
Phys. Rev. D 105, 074501 (2022)

Advantage

- New renormalization scheme (subtraction point)
  - Matching available one order higher (NNLO) : Gracey (2012)
  - → reduced systematic error ~1%
- Two lattice spacings → continuum limit
- Chiral symmetry

Disadvantage

- Coarse lattice a=0.2,0.14fm → large error after continuum extrapolation ~20%



JUN-SIK YOO *et al.* PHYS. REV. D **105,** 074501 (2022)

TABLE VIII. Results for the form factors $W_{0,1}$ on the two ensembles and in the continuum limit at the two kinematic points $Q^2 = 0$ (first line) and $Q = -m_\mu^2$ (second line) renormalized to $\overline{\text{MS}}$(2 GeV). The first uncertainty is statistical, the second is systematic due to excited states, and the third is the uncertainty of the continuum extrapolation.

	$W_0[\text{GeV}^2]$		
	24ID	32ID	Cont.
$\langle\pi^+\|(ud)_L d_L\|p\rangle$	0.1032(86)(26)	0.1252(48)(50)	0.151(14)(8)(26)
	0.1050(87)(36)	0.1271(49)(50)	0.153(14)(7)(26)
$\langle\pi^+\|(ud)_L d_R\|p\rangle$	−0.1125(78)(41)	−0.134(5)(11)	−0.159(15)(20)(25)
	−0.1139(78)(45)	−0.136(5)(12)	−0.161(15)(20)(26)

RBC/UKQCD results

Phys. Rev. D 105, 074501 (2022)

Blue solid symbols are the current best estimate

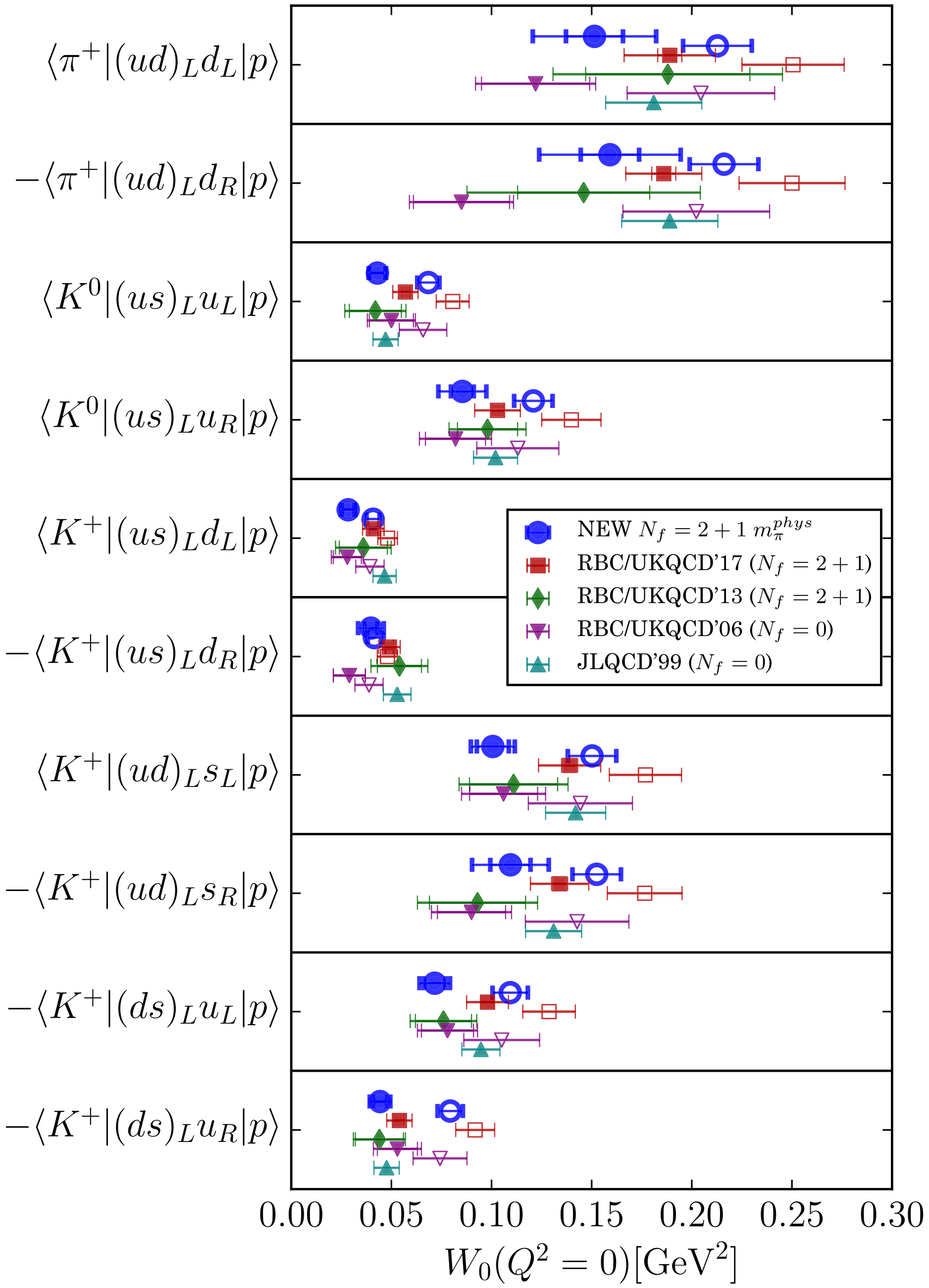
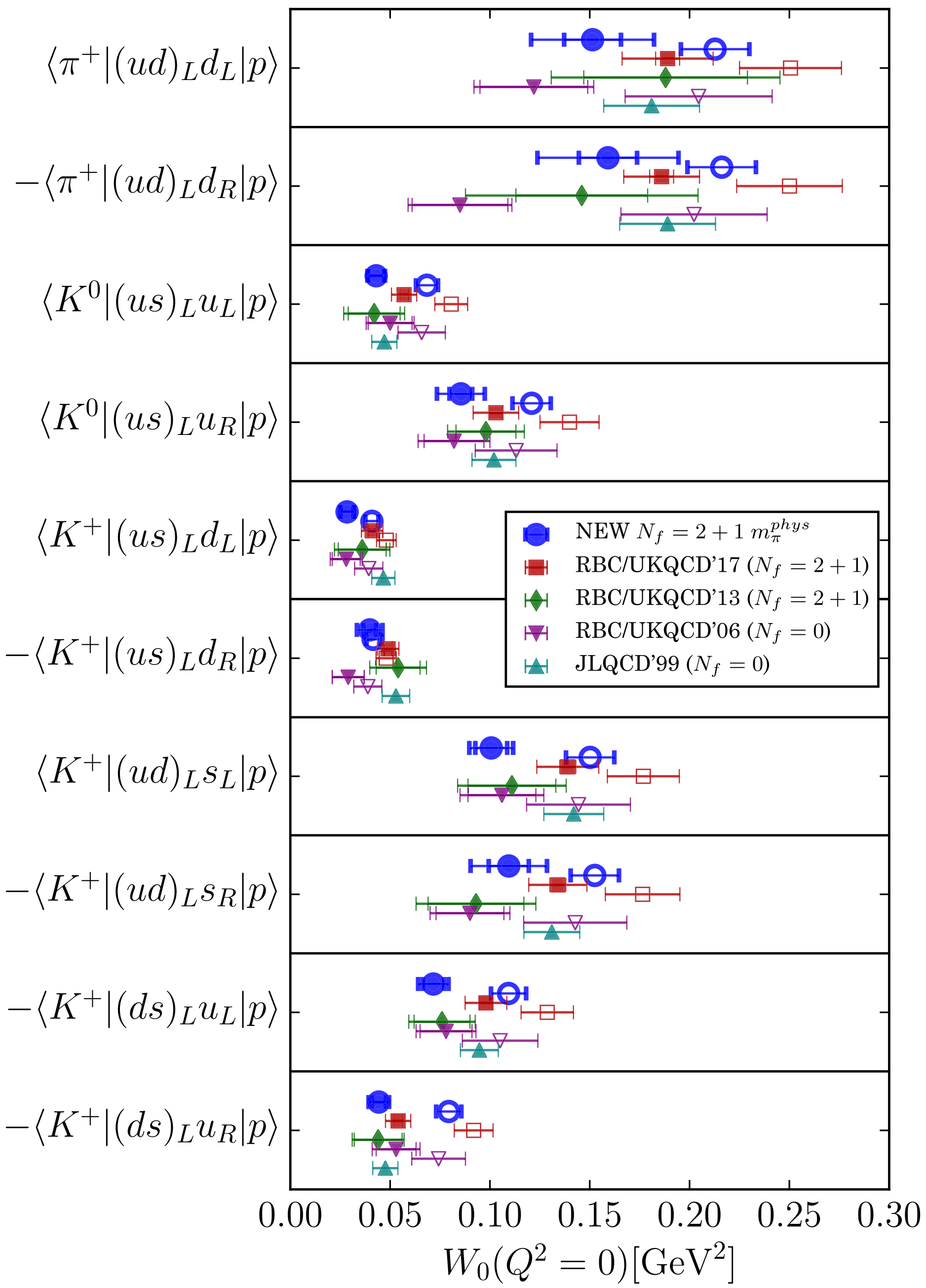


FIG. 14. Comparison of our results ("NEW") for the proton decay amplitudes $W_0(0)$ computed directly (filled symbols) and indirectly (open symbols) to previous determinations [38,40,42]. All results are renormalized to the $\overline{\mathrm{MS}}$(2 GeV) scheme.

RBC/UKQCD results

Phys. Rev. D 105, 074501 (2022)



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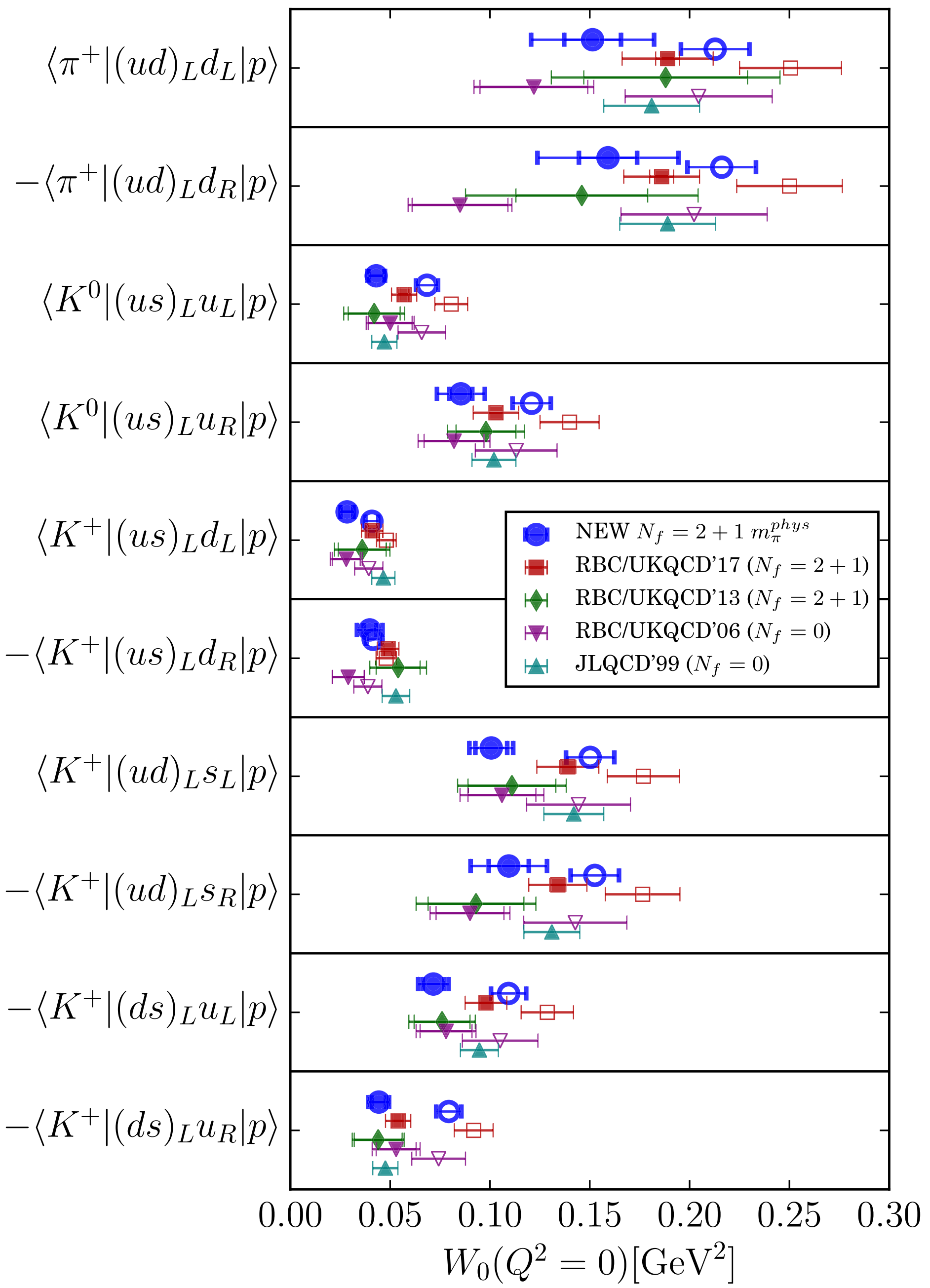
Blue solid symbols are the current best estimate

LEC ?

RBC/UKQCD results

Phys. Rev. D 105, 074501 (2022)



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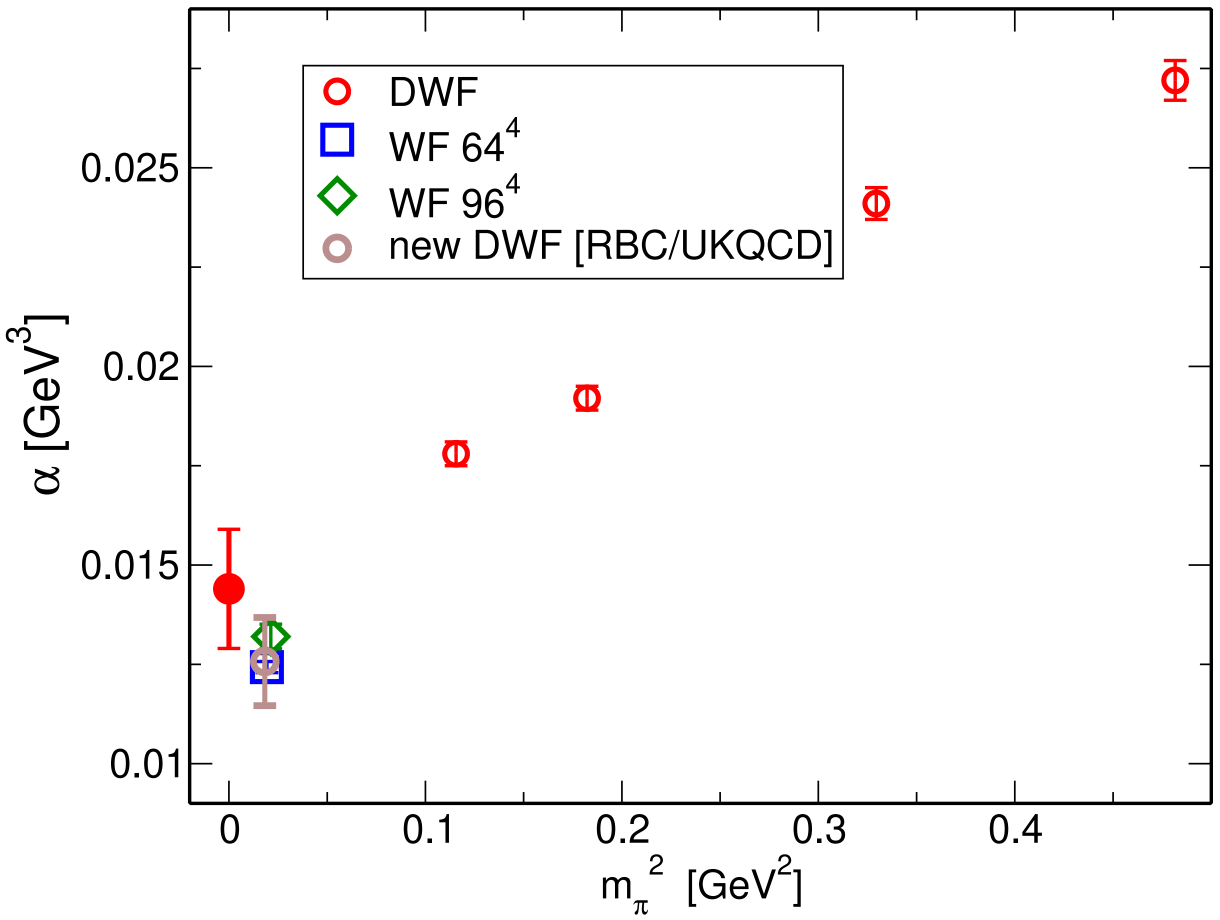
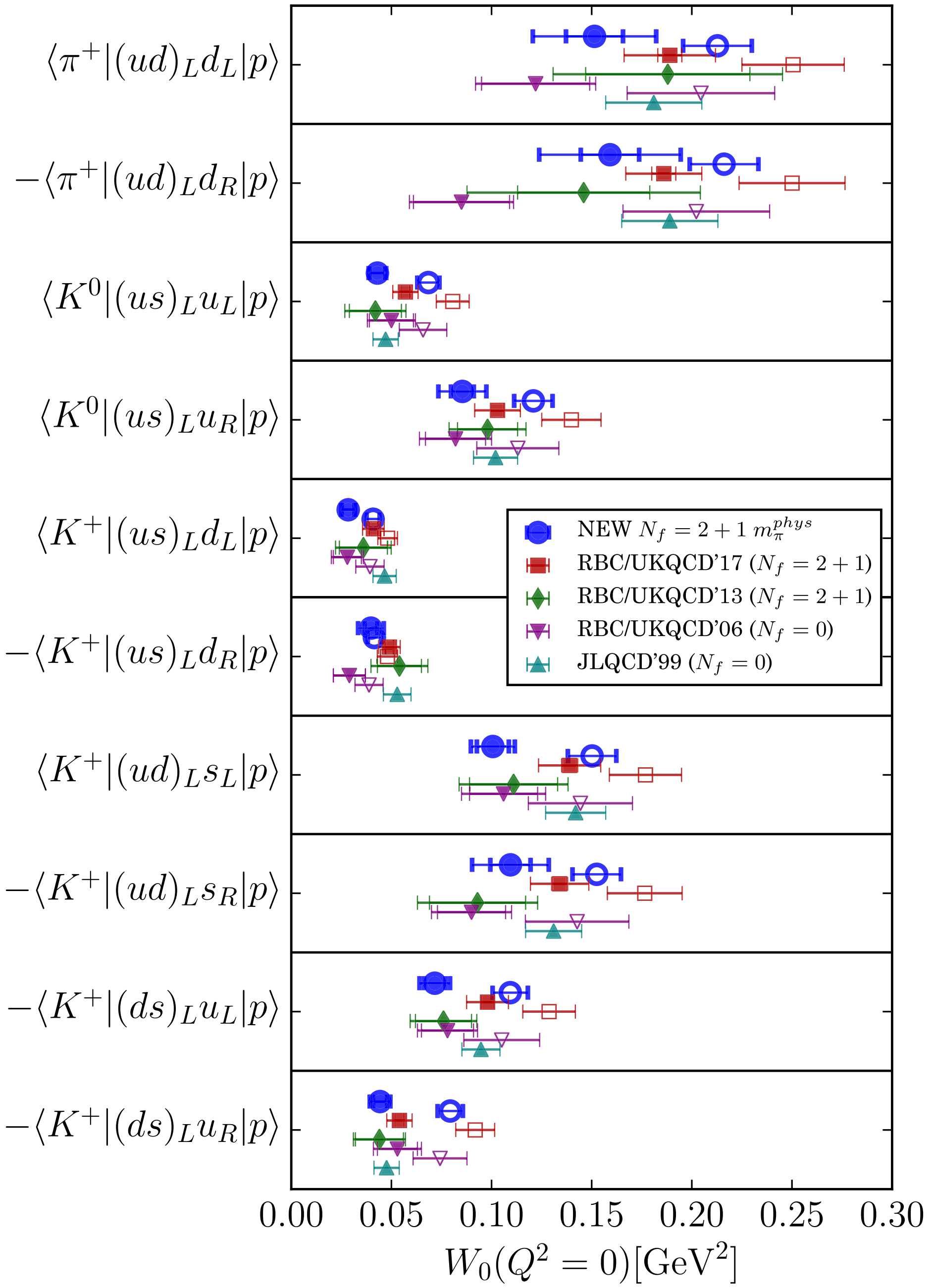


RBC/UKQCD results

Phys. Rev. D 105, 074501 (2022)



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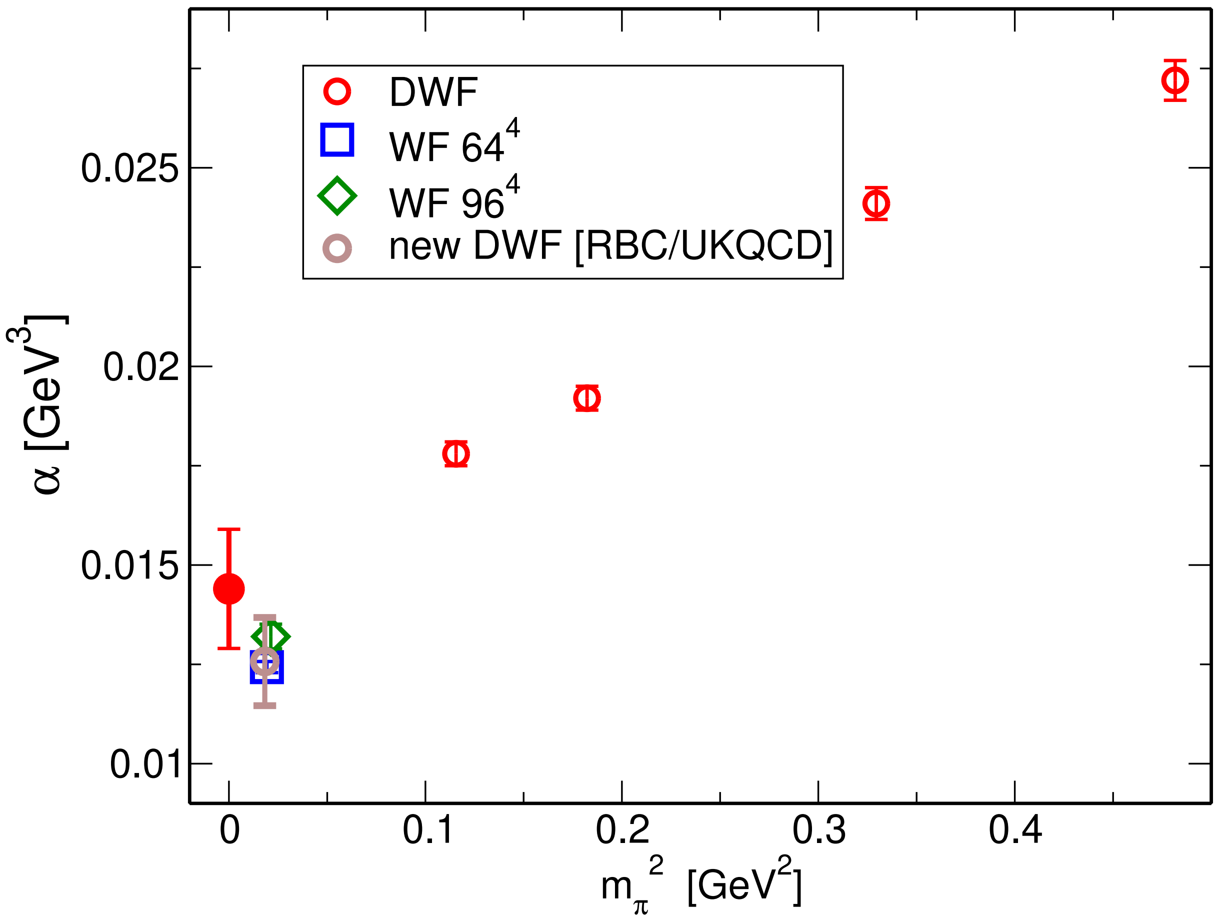
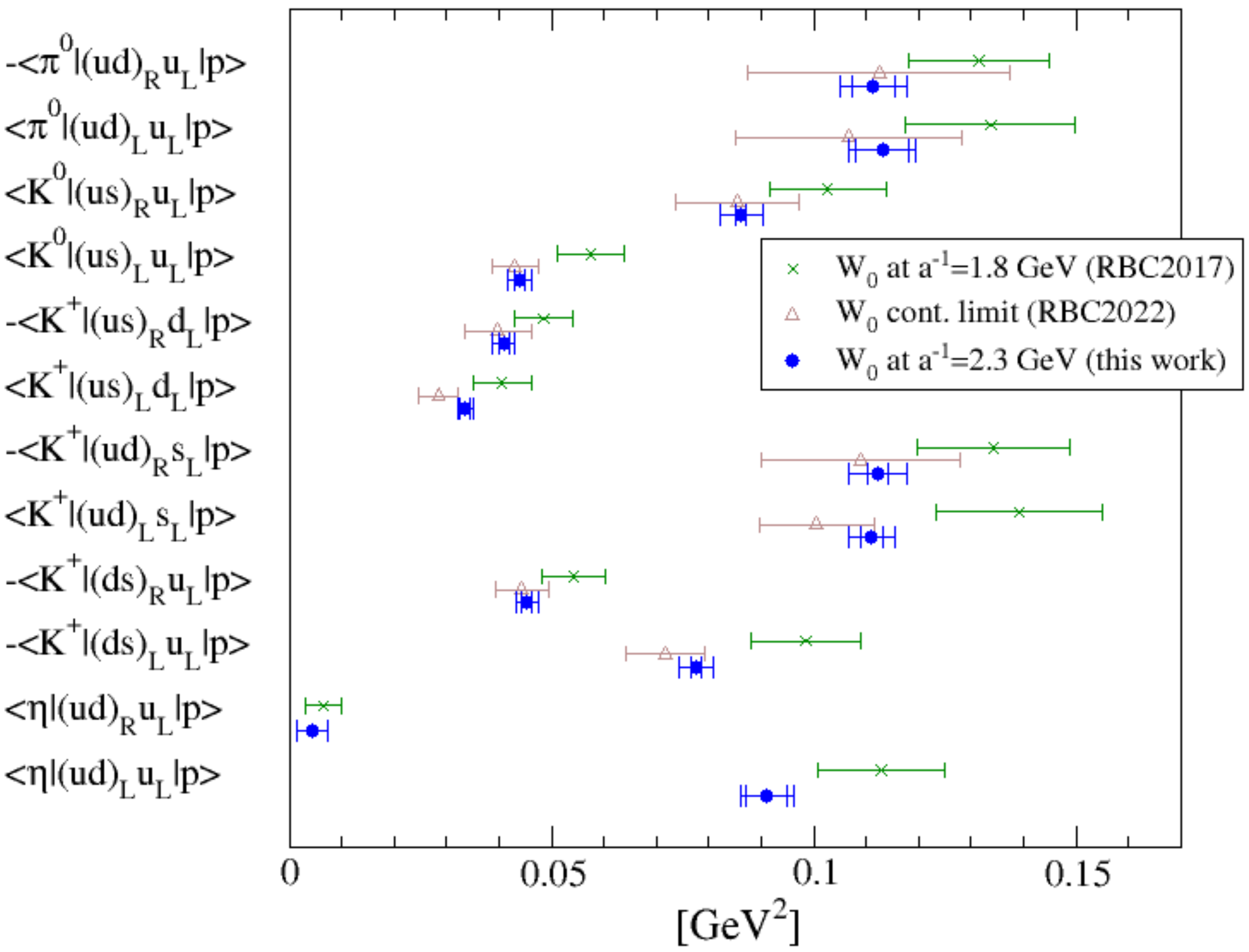


No surprise happened at physical point !

Preliminary PACS results of proton decay FF

$W_0\ (\overline{MS},2GeV)$

- First Wilson fermion physical point (preliminary) results
- compared with DWF (RBC 2017, RBC 2022(physical point, continuum))



- (L)QCD parity invariance:
  - $W_0^{LR} = W_0^{RL}$
  - $W_0^{RR} = W_0^{LL}$
- PACS yet to do:
  - discretization error est.
    - no large error expected
- Consistent with DWF

Summary and outlook

Summary and outlook

- proton decay FFs till 2017 may suffer from chiral extrapolation error
- Now we can do computations on physical mass (no extrapolation)
  - aiming to remove final loose end
- Using PACS Wilson ensemble
  - RI/SMOM non-perturbative renormalization schemes applied
    - robust against SSB and mass effect
  - W_0 ($p \rightarrow \pi^0$), LEC α and β consistent with DWF(2017) [preliminary]
- RBC/UKQCD with domain wall fermion
  - New renormalization scheme help reduce the systematic error
  - W_0 ($p \rightarrow \pi^0$), LEC α and β consistent with DWF(2017)
  - → No chiral limit surprise! ←→ Martin & Stavenga (skirm chiral bag)

PACS analysis to be finalized, envisioning comparable /better accuracy

plan to use PACS10c configurations w/ continuum scaling study

acknowledgements

- Thanks for various help from other PACS members esp
    - Takeshi Yamazaki, Shoichi Sasaki, Natsuki Tsukamoto
- Used Computers
    - K-computer,
    - Oakforest PACS,
    - Cygnus,
    - Supercomputer Fugaku,
    - Wisteria/BDEC-01
- Programs
    - Multidisciplinary Cooperative Research Program in CCS Tsukuba
    - HPCI: hp200062, hp200167, hp210112, hp220079, hp230199
    - 

      Program for Promoting Researches on the Supercomputer Fugaku

      Large-scale lattice QCD simulation and development of AI technology

      JPMXP1020230409

Thank you very much for your attention

appendix

use of RI/SMOM schemes [for PACS analysis]

- original NPR for proton decay
  - wave function renormalization from (axial) vector currents with exceptional momentum configuration
    - which is sensitive to low energy parameters like mass
    - RI/MOM defined $m \rightarrow 0$; we have only physical point $0 \simeq m_{ud} \ll m_s$
- new attempt using SMOM schemes for wave function
  - 3q vertex has not been using exceptional momenta
  - now no exceptional momenta at all
  - safer to use with physical point ensemble
- matching from SMOM schemes to MSb to NLO available
  - (wave function renormalization known to NNLO, 3q unchanged)

RI/SMOM procedure for proton decay operators

- calculate 3q vertex: same momentum p injected from all quark legs
- cancel the wave function renormalization by dividing with local vector or axialvector vertex in SMOM schemes with proper power
- this will provide: $Z_{ND}/Z_A^{3/2}$, $Z_{ND}/Z_V^{3/2}$
- take Z_A, Z_V factor away by inputting those calculated in SF scheme
- Z_{ND} (SMOM) matched to MSb and ran to 2 GeV by NLO PT
- final value and error estimate:
        - error from truncation of PT, $(pa)^2$, contamination of SSB
    - center = (max+min)/2, error = (max-min)/2
    - max, min determined in
        - $(pa)^2$ linear extrapolation: 4 vales: (A,V) x (SMOM, SMOMγμ)
        - scatter of all data in the window 2-3 GeV

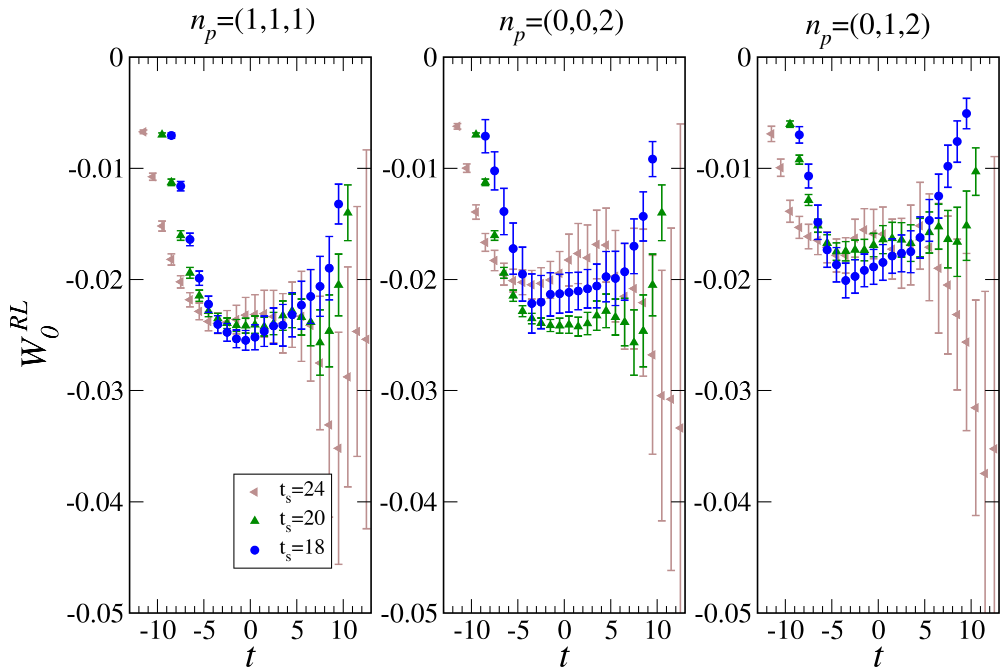
Lattice setup: input and output

- 2+1 flavor lattice QCD computation parameters:
  - gauge coupling, m_{ud}(degenerate), m_s
  - Monte Carlo simulations are done at $m_s^{(sim)} \approx m_s^{(phys)}$, $m_{ud}^{(sim)} > m_s/5$
    - small mass is demanding: cost $\sim 1/m^x$: $x>1$
  - at a fixed gauge coupling, tune m's so that it reproduces ratios of π, K, Ω^- mass.
    - π, K: quark mass dependence best known: NNLO ChPT / analytic also tested
    - Ω^-(sss): no pion chiral logs at NLO: safe to apply linear chiral extrapolation
- all other quantities are predictions, ex: [RBC/UKQCD PRD78(08)114509]
  - f_π=124.1(6.9)MeV ↔ 130.7(0.1)(0.36)[exp], f_K/f_π=1.205(18) ↔ 1.223(12)[exp]
    - one lattice spacing, estimate of $O(a^2)$ systematic error was added.
  - quark masses, B_K...
  - ➡ continuum limit results available: [RBC/UKQCD PRD83(11)074508, PRD84(11)014503]

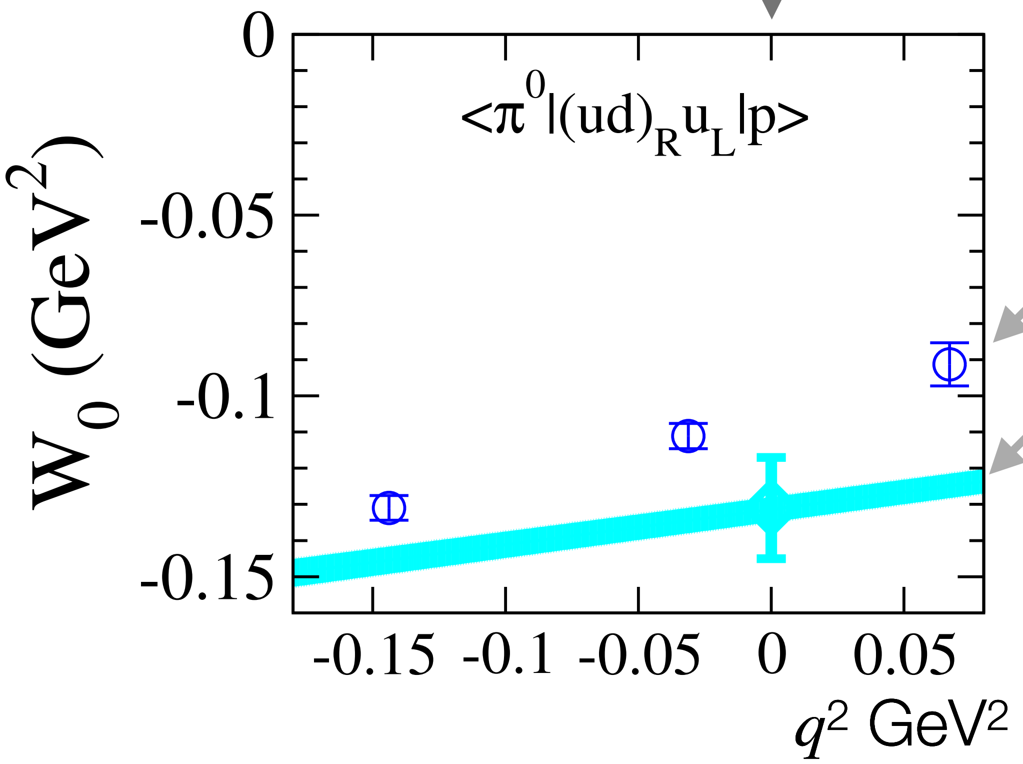
proton decay form factor W_0 for pion final state

Relevant form factor W_0 $<\pi^0|(ud)_R u_L|p>$ as an example

- from ratio of 3 and 2 point functions
- meson momentum $\vec{p} = \frac{2\pi}{L}\vec{n}_p$



on-shell lepton: $-q^2=m_l^2=0$



new

DWF
(RBC/UKQCD
2017)

- $|W_0|$ ~20% smaller than DWF (with a long chiral extrapolation) at q^2=0
- consistent with sys. error ! no big surprise found for $m_f \rightarrow m_{ud}$
- 10% total error is not a dream…