Constraining Microscopic Nuclear Dynamics with Multimessenger Astrophysical Data

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Outline

- ⋆ The (evolving) paradigm of nuclear theory
- ⋆ Phenomenological nuclear Hamiltonian
  - nucleon-nucleon (NN) potential
  - irreducible three-nucleon (NNN) interactions
  - relativistic corrections
- ⋆ Impact of NNN interactions on neutron star properties
- ⋆ Constraining NNN potential models with astrophysical data
  - results obtained using available data
  - potential of future gravitational wave observatories
- ⋆ Summary & outlook

The (Evolving) Paradigm of Nuclear Theory

- ⋆ To a remarkable extent, atomic nuclei behave as a collection of point-like protons and neutrons, that can be described in the non-relativistic approximation

- ⋆ Ideally, nuclear theory should be based on a dynamical model capable to describe interactions at all scales relevant to nuclear systems, from deuteron to neutron stars

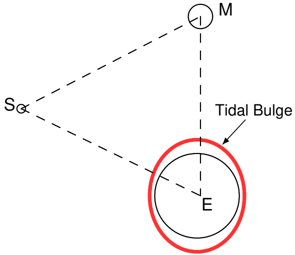
- ⋆ This philosophy has been applied extensively using phenomenological models of the nuclear Hamiltonian, constrained by the observed properties of two- and three-nucleon systems—in both bound and scattering states—and the equilibrium density of isospin-symmetric nuclear matter inferred from nuclear data

The Nuclear Hamiltonian

- The nuclear Hamiltonian consists of a non relativistic kinetic energy term and the potentials v_{ij} and V_{ijk}, accounting for two- and three-nucleon interactions

$$H = \sum_i \frac{\mathbf{p_i}^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

- The inclusion of three-body forces is the price to pay to describe the interactions of composite objects neglecting their internal structure
- The archetypal three-body force appears in the context of gravitational Physics



Phenomenological Models of the NN potential

- Phenomenological potentials describing the full NN interaction can be written in the form

$$v = v_R + \widetilde{v}_\pi$$

where $\widetilde{v}_\pi$ is the OPE potential, stripped of the δ-function contribution

- Phenomenological models, such as the Argonne models of Wiringa *et al*, are written in the form

$$v_{ij} = \sum_p v^p(r_{ij}) O^p_{ij}$$

- The most important contributions are those appearing in the expression of the OPE potential

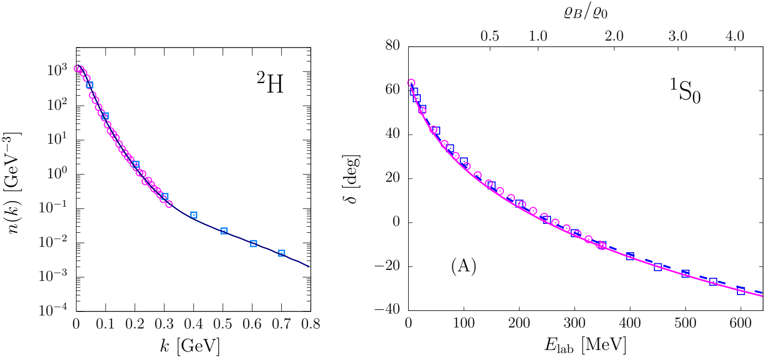
$$O^{p\leq 6}_{ij} = [\mathbb{1}, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}] \otimes [\mathbb{1}, (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)]$$

- More modern models of v_{ij}, such as the Argonne v_{18} (AV18) [PRC **51**, 38 (1995)], include additional terms, taking into account non-static interactions and small violations of charge symmetry.

- At large distances, the AV18 potential reduces to Yukawa's OPE potential

Comparison to Two-Nucleon Data

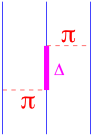
- ⋆ **Left**: momentum distribution in ^{2}H compared to the electron scattering data [M. Bernheim *et al.* NPA **365**, 349 (1981); H. Arenhövel, NPA **384** (1982); C. Ciofi degli Atti *et al.* PRC **36**, 1208 (1987).]
- ⋆ **Right**: nucleon-nucleon scattering phase shifts in the $^1S_0$ channel



PHENOMENOLOGICAL HAMILTONIANS

- ⋆ The full nuclear Hamiltonian is obtained combining phenomenological NN and NNN potentials
- ⋆ Urbana IX NNN potentiall: Fujita-Miyazawa two-pion exchange + phenomenological repulsive term

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R} \quad , \quad V_{ijk}^{2\pi} = A_{2\pi} \times$$

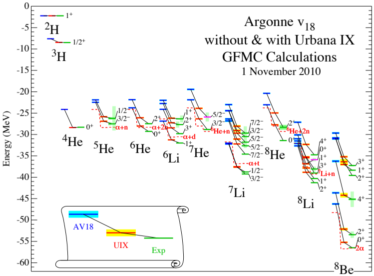


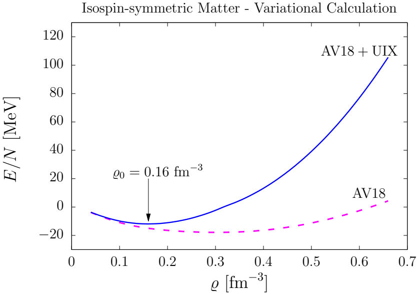
$$V_{ijk}^{R} = U_0 \times \sum_{\text{cycl}} T^2(r_{ij}) T^2(r_{ij}) \quad , \quad T(r) = \left(1 - e^{-cr^2}\right)^2 \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x}$$

- ► The strength of $V^{2\pi}$ ($A_{2\pi}$) is adjusted to reproduce the observed ground state energies of ^{3}He and ^{4}He
- ► the strength of the isoscalar repulsive term V^R (U_0) is adjusted to reproduce the empirical equilibrium density of isospin-symmetric matter (SNM), inferred fom nuclear data

AV18 + UIX Hamiltonian

Spectra of light nuclei [PRC **64**, 014001 (2001)] and binding energy of SNM [PRC **58**, 1804 (1998)] obtained from the AV18 + UIX Hamiltonian





NNN interactions, provide a small negative correction to the binding energies of light nuclei. In SNM their contribution is positive, and becomes large at supranuclear densities

Relativistic Corrections to the NN potential

- The effects of relativistic corrections to the AV18 + UIX Hamiltonian on the properties of the three- and four-nucleon systems have been analysed by Forest *et al.* [PRC **60**, 014002 (1999)] using Monte Carlo techniques.

- The results of these studies show that only the boost correction to the NN potential—needed to take into account the motion of the total momentum of the interacting pair—provides a significant contribution to the energy.

- Leading boost correction to v_{ij}, derived by Friar [PRC **12**, 695 (1975)] and Forest *et al.* [PRC **52**, 568 (1995)]

$$v_{ij}(\mathbf{r}) \to v_{ij}(\mathbf{r}) + \delta v_{\mathbf{ij}}(\boldsymbol{P}, \boldsymbol{r}) \ ,$$

$$\delta v_{ij}(\boldsymbol{P}, \boldsymbol{r}) = -\frac{P^2}{8m^2} v^s_{ij}(\boldsymbol{r}) + \frac{(\boldsymbol{P} \cdot \boldsymbol{r})}{8m^2} \boldsymbol{P} \cdot \boldsymbol{\nabla} v^s_{ij}(\boldsymbol{r}) \ ,$$

where $\mathbf{P} = \mathbf{p}_i + \mathbf{p}_j$, and v^s_{ij} denotes the static part of the NN potential.

Boost Corrections to the Energy

- Ground-state energies are obtained combining the boost-corrected NN potential and a modified NNN potential

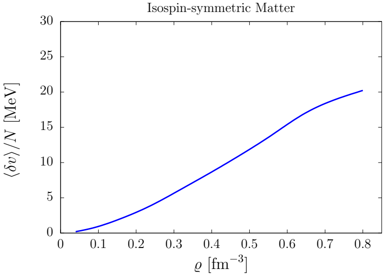
$$H \to H_R = \sum_i \frac{\mathbf{p_i}^2}{2m} + \sum_{j>i} [v_{ij} + \delta v_{ij}] + \sum_{k>j>i} V^*_{ijk} \; .$$

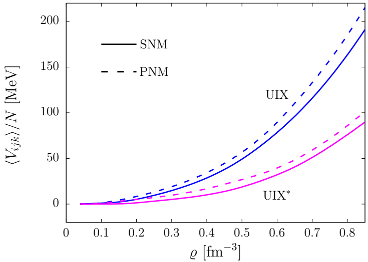
- The boost interaction, δv_{ij} provides a positive contribution of $\sim$0.9 and $\sim$1.9 MeV in ^{3}He and ^{3}He, respectively, which entails a corresponding softening of the repulsive NNN potential V^R. On the other hand, $V^{2\pi}$ is left unchanged.
- The full correction to $\langle H \rangle$ is

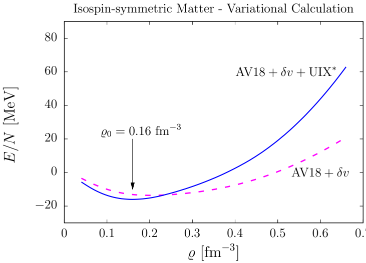
$$\delta E_R = \langle \delta v \rangle - \gamma \langle V^R \rangle \quad , \quad \gamma = 0.37 \; .$$

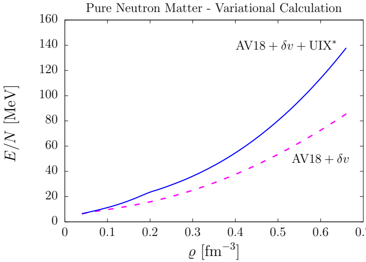
- The above relativistic corrections are included in the energies of pure neutron matter (PNM) and isospin-symmetric matter SNM computed by Akmal Pandharipande & Ravenhall [PRC **58**, 1804 (1988)].

Boost Corrections in Nuclear Matter



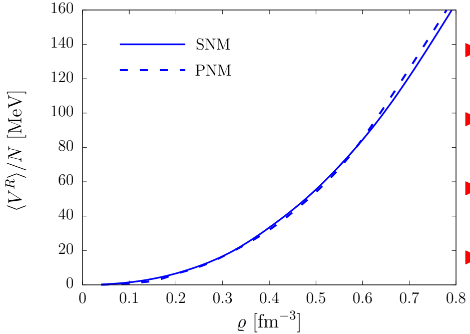






NNN Repulsion in Nuclear Matter

- Contribution of repulsive NNN interactions to the energy of SNM and PNM, obtained using the AV18 + δv + UIX* Hamiltonian



- Repulsive NNN interactions are isoscalar
- Provide large contributions to nuclear matter energy
- Largely determine the stiffness of the EOS of neutron star matter
- Are totally unconstrained at supranuclear density

- Can astrophysical data constrain the strength of NNN interactions in dense matter?

Impact of V^R on Neutron Star Properties

- ⋆ We have generated a set of EOS using the parametrisation of the EOS of Akmal *et al.* [PRC **58**, 1804 (1998)]

$$\varrho \frac{E}{N} = \epsilon(\varrho, x_p) = \epsilon_K(\varrho, x_p) + \epsilon_I(\varrho, x_p)$$

and replacing

$$\langle V^R \rangle \to \alpha \langle V^R \rangle \Longrightarrow \epsilon_I(\varrho, x_p, \alpha) \to \epsilon_I(\varrho, x_p) + (\alpha - 1) \frac{\varrho}{N} \langle V^R \rangle$$

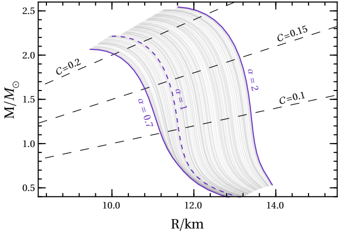
- ⋆ The case $\alpha = 1$ corresponds to the EOS of Akmal *et al.*, providing the baseline for our analysis. The range of α has been chosen in such a way as to limit to $\sim 15\%$ the displacement of the equilibrium density of SNM from its empirical value
- ⋆ Using the above parametrisation, we have obtained the EOSs of β-stable matter needed to perform calculations of neutron star properties for any given value of α

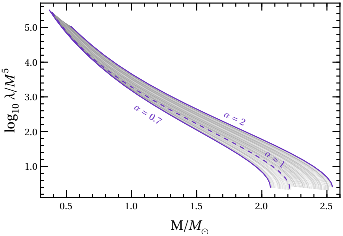
Constraining α through Bayesian Inference

- ⋆ We have considered a family of neutron star configurations specified by the value of α, employed to obtain the EOS, and the central pressure

$$\{\alpha, p_c\} \rightarrow \{M, R, \Lambda\}$$

- ⋆ Mass-radius and mass-tidal deformability for $0.7 \leq \alpha \leq 2.0$





Bayesian Inference Framework

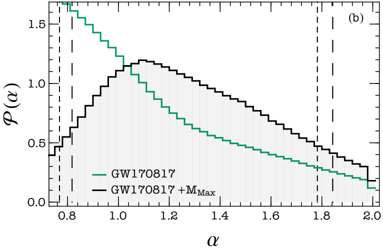
- Given a set of observations O^i of m neutron stars, Bayes' theorem can be used to infer the distribution of $\{\alpha, \vec{p}_c\} = \{\alpha, p_c^1, \ldots, p_c^m\}$
- We have sampled the posterior distribution

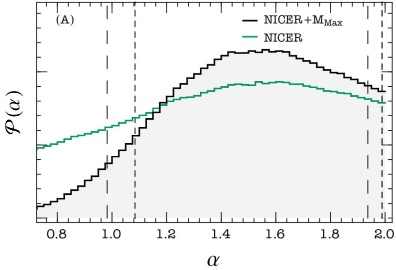
$$\mathcal{P}(\alpha, \vec{p}_c|\vec{O}) \propto \mathcal{P}_0(\alpha, \vec{p}_c) \prod_{i=1}^{m} \mathcal{L}(O^i|\alpha, p_c^i)$$

  - $\mathcal{P}_0(\alpha, \vec{p}_c)$ prior distribution
  - $\mathcal{L}(O^i|\alpha, p_c^i)$ likelihood of the i-th observation

  using the Markov Chain Monte Carlo technique
- The distribution $\mathcal{P}(\alpha)$ has been then obtained marginalising over $\vec{p}_c$
- Data set
  - GW observation of the binary system GW170817, made by the LIGO/Virgo Collaboration (masses and tidal deformabilities)
  - Observation of the millisecond pulsars PSR J0030+0451 made by the NICER satellite (mass and radius)
  - Precise determination of the maximum neutron star mass observed so far, $M = 2.14^{+0.1}_{-0.09}\ M_\odot$ [ApJ Lett. **918**, L29 (2021)]

GW170817 & NICER + M_{max}

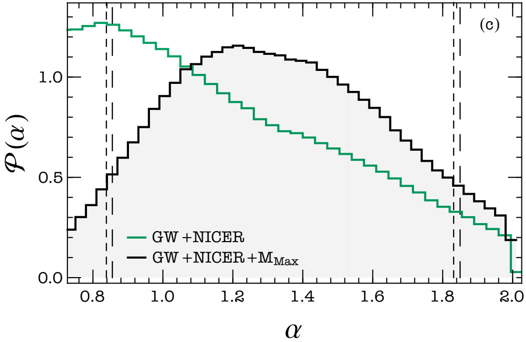




- GW170817 data alone not very constraining
- NICER looks somewhat more informative
- The maximum mass turns out to be the strongest constraint
- The inferred values of α are

$$\alpha_{GW} = 1.25^{+0.48}_{-0.53} \quad , \quad \alpha_{EM} = 1.52^{+0.43}_{-0.47}$$

GW170817 + NICER + M_{max}



- GW170817 dominates if taken alone with NICER
- Full dataset still mainly affected by the maximum mass
- The analysis, yielding

$$\alpha_{GW} = 1.32^{+0.48}_{-0.51}$$

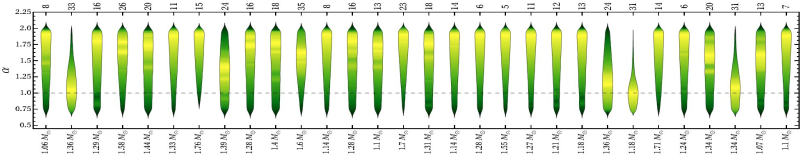
indicates that observations are sensitive to the strength of repulsive NNN interations

POTENTIAL OF FUTURE GW OBSERVATIONS

- ⋆ The study based on the available data has been extended using a set of *simulated GW observations* that will be feasible in the future using both upgraded and new interferometers
- ⋆ The analysis includes observations of 30 binary neutron star events made by
  - the LIGO Hanford, LIGO Livingston, and Virgo interferometers at design sensitivity
  - The future third-generation interferometer Einstein Telescope
- ⋆ For each observatory, two sets of events have been generated using EOSs corresponding to different α
  - the strength of NNN interactions was set to $\alpha = 1$ and $\alpha = 1.3$
  - the sky location and inclination were assumed to be uniformly distributed over the sky
  - the chirp mass of each event, $\mathcal{M} = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}$, was assumed to be known with infinitesimal precision

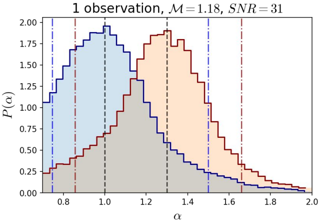
Mock Data: LIGO/Virgo

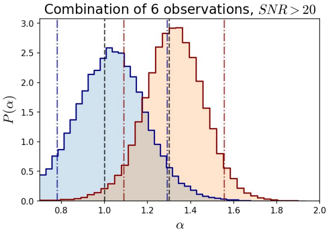
- Posterior densities inferred from simulated GW data, assuming $\alpha = 1$. Top and bottom axes give SNR and chirp mass



Only few, low-mass and high-SNR, events provide a meaningful constraint on α

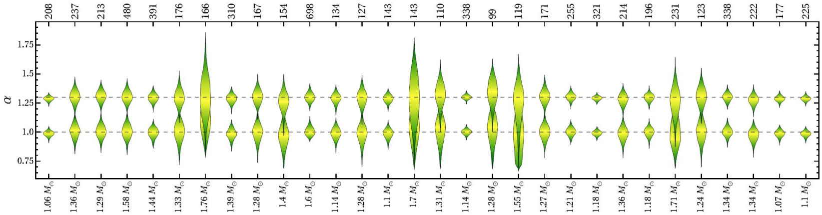
- Probability distributions of α





Mock Data: Einstein Telescope

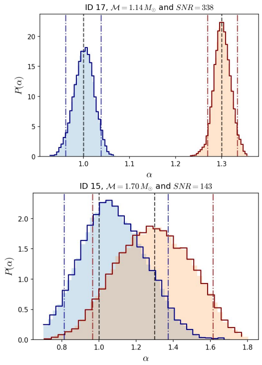
- Posterior densities inferred from simulated GW data, assuming $\alpha = 1$ and $\alpha = 1.3$ Top and bottom axes give SNR and chirp mass

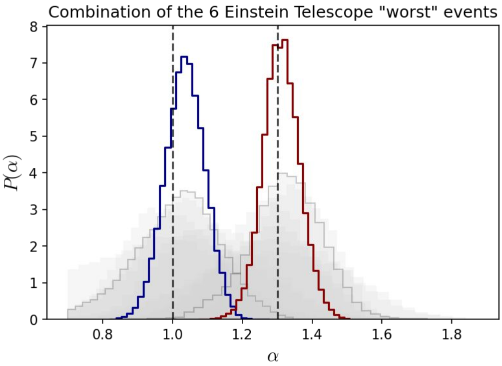


- In most of cases, the large SNRs allow the posteriors corresponding to the injected values of α to be clearly separated
- It appears that even a single observation made by the Einstein Telescope may allow to constrain the strength of NNN interactions

Mock data: Einstein Telescope

- In the few cases in which posterior distributions overlap, stacking of few observations still allows to clearly resolve the peaks corresponding to $\alpha = 1$ and 1.3





Summary & Outlook

- ⋆ The long anticipated advent of multimessenger astrophysics is providing unprecedented access to neutron star properties
- ⋆ The available data are being extensively employed to constrain the EOS of dense nuclear matter. We have explored the possibility to push these studies to a deeper level, in which observations are used to infer information on the underlying microscopic model of nuclear dynamics
- ⋆ Stronger constraints on repulsive NNN interactions will allow to improve the understanding of the stiffness of the nuclear EOS at high densities, and clarify the importance of relativistic boost interactions
- ⋆ Early results of our analysis indicate that the data collected by existing facilities, while showing a sensitivity to the strength of the repulsive NNN potential, α, lack the accuracy needed for its precise determination
- ⋆ More recent studies, performed using simulated data, suggest that the advent of third-generation GW interferometers, notably the Einstein Telescope, may dramatically improve the picture, allowing to pin down the value of α with the observation of even a single neutron star event

Credits & References

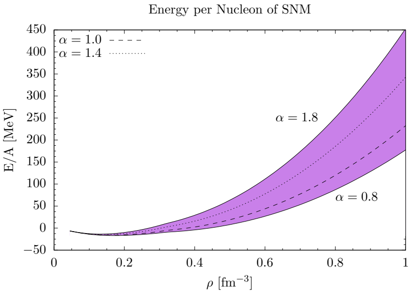
- ⋆ The analysis discussed in this talk is the result of the work of my collaborators
  - Andrea Sabatucci (INFN Pisa)
  - Andrea Maselli (GSSI)
  - Costantino Pacilio (Milano Bicocca)
  - Alessandro Lovato (ANL)
- ⋆ References

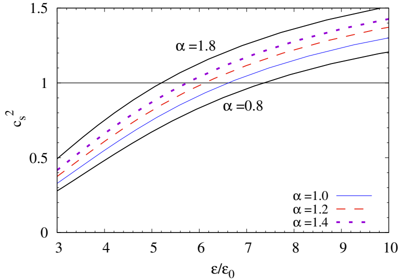
  - A. Sabatucci & OB, Phys. Rev. C **101**, 045807 (2020)
  - A. Maselli, A. Sabatucci, & OB, Phys. Rev. C **103**, 065804 (2021)
  - A. Sabatucci, OB, A. Maselli, & C. Pacilio, Phys. Rev. D **106**, 083010 (2022)
  - A. Sabatucci, OB, & A. Lovato, arXiv:2406.05732 [nucl-th], submitted to PRC (2024)

Backup slides

Impact of V^R on Nuclear Matter Properties

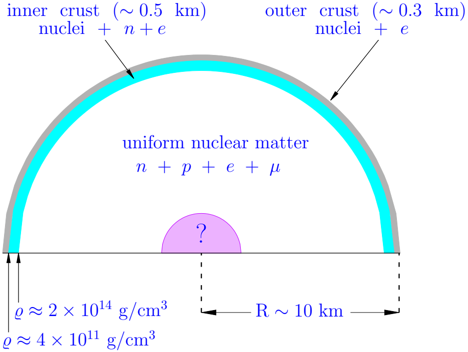
Density depependence of the binding energy per nucleon of SNM (left) and the squared speed of sound in β-stable matter (right) corresponding to different values of α





One-slide Introduction to Neutron Stars

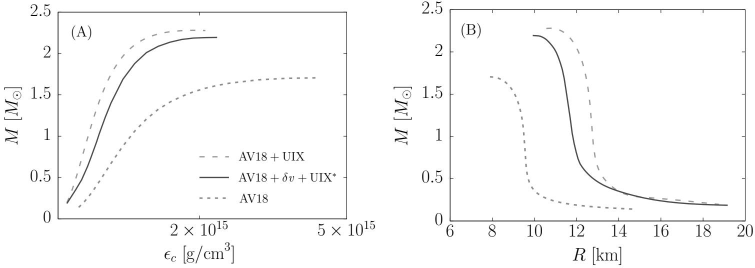
- Overview of NS structure (Recall: $T \sim 10^9$ K $\ll T_F \sim 10^{12}$ K)

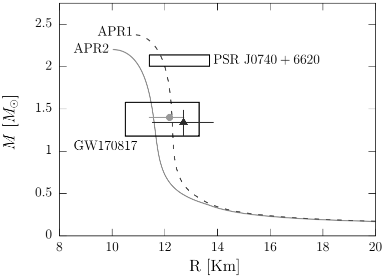


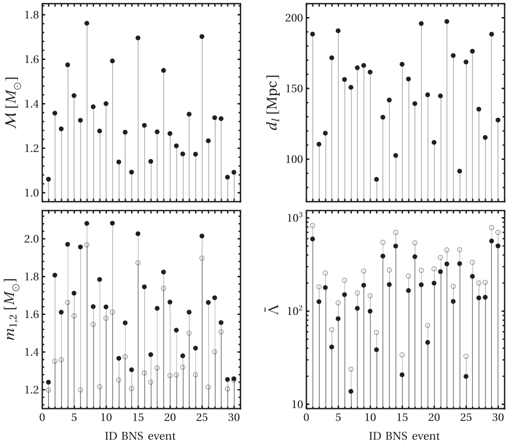
- NS properties such as mass, radius and tidal deformability are largely determined by the equation of state (EOS) of matter in its interior,

$$\left\{ \begin{array}{l} \text{energy density} : \epsilon(\varrho) = (E(\varrho) + Nm)/V \\ \text{pressure} : P(\varrho) = -\partial E(\varrho)/\partial V \end{array} \right. \quad \Rightarrow P(\epsilon)$$

Impact of Boost Corrections on NS properties







⋆ Component masses, luminosity distance, chirp mass, and tidal parameter for the catalogue of NS binaries

Comparison Between Present and Future Constraints

- ⋆ Neutron star mass-radius relations, obtained from EOSs corresponding to the distributions $\mathcal{P}(\alpha)$ resulting from our analysis
  - Left panel: available observations
  - Right two panel: simulated observations with the Einstein Telescope