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Bayesian inference of dense matter EOS: non-relativistic vs relativistic mean field models at zero and finite temperatures

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EOS Measurements with Next-Generation Gravitational-Wave Detectors
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Bayesian inference of mean field EOSs

Introduction

Main part

- Models
- Equations of State of Nuclear Matter
- Constraints
- Markov Chain Monte-Carlo Setup
- Key results

Conclusions

Introduction

In the context of rapidly growing multi-messenger astrophysics it is becoming more and more important to combine together diverse data describing properties of superdense matter. This data comes from theoretical and experimental nuclear physics and from astrophysics.

One of the best ways to achieve this is to employ statistical analysis methods, in particular, Bayesian inference techniques.

The purpose of our work is to investigate how different combinations of constraints and various mean field models affect the inferred properties of dense matter and neutron stars (NSs).

Models

We have considered three mean field models:

- [relativistic] Covariant Density Functionals (CDF a.k.a. RMF) model with simplified Density-Dependent (DD) couplings, proposed by Malik et al. [ApJ **930**, 17 (2022)].
- [non-relativistic] standard Skyrme (Sk) and extended Brussels-Skyrme (BSk) models.

Parameters of the models:

- DD CDF: 6 free parameters.
- Sk [for infinite NM]: 7 free parameters.
- BSk [for infinite NM]: 13 free parameters, out of which we have fixed 2.

Equation of State of Nuclear Matter

A quick reminder about EOSs and Nuclear Empirical Parameters (NEPs)

EOS can be viewed in terms of energy per nucleon (E/A) as a function of baryon number density (n_B) and isospin asymmetry (δ).

NEPs are obtained by expanding E/A around nuclear saturation density ($n_\mathrm{sat} \approx 0.16\ \mathrm{fm}^{-3}$) and isospin symmetry ($\delta = 0$):

$$\frac{E(n_\mathrm{B},\delta)}{A} = E_0(n_\mathrm{B},0) + \delta^2 E_\mathrm{sym}(n_\mathrm{B},0) + \ldots,\ n_\mathrm{B} = n_\mathrm{n} + n_\mathrm{p},\ \delta = \frac{n_\mathrm{n} - n_\mathrm{p}}{n_\mathrm{B}}$$

with

$$E_0(n_\mathrm{B},0) = \sum_{i=0,1,2,\ldots} \frac{1}{i!} X_\mathrm{sat}^{(i)} \mathcal{X}^i = E_\mathrm{sat} + \frac{1}{2} K_\mathrm{sat} \mathcal{X}^2 + \frac{1}{6} Q_\mathrm{sat} \mathcal{X}^3 + \ldots$$

$$E_\mathrm{sym}(n_\mathrm{B},0) = \sum_{j=0,1,2,\ldots} \frac{1}{j!} X_\mathrm{sym}^{(j)} \mathcal{X}^j = J_\mathrm{sym} + L_\mathrm{sym} \mathcal{X} + \frac{1}{2} K_\mathrm{sym} \mathcal{X}^2 + \ldots$$

$$,\ \mathcal{X} = \frac{n_\mathrm{B} - n_\mathrm{sat}}{3 n_\mathrm{sat}}$$

Constraints

We have considered five types of constraints (all at $T = 0$):

1. The best-know NEPs: $n_{\rm sat}, E_{\rm sat}, K_{\rm sat}, J_{\rm sym}$.
2. Energy per nucleon and/or pressure in pure neutron matter (PNM) at various densities based on χEFT calculations.
3. Effective masses: Dirac [relativistic] or Landau [non-relativistic].
4. Maximum neutron star (NS) mass constraint: $M_{\rm G}^{*} > 2.0\ {\rm M}_{\odot}$.
5. "Physical" constraints: NSs' EOSs should be thermodynamically stable & causal; effective masses should be from 0 to 1 [bare nucleon mass]; $\boldsymbol{v}_{\mathbf{F}} < \boldsymbol{c}$ [non-relativistic].

For various reasons, we have not employed mass-radius constraints based on *NICER* results and tidal deformability constraints based on gravitational waves (GWs) data. We have, however, checked our results against those constraints *a posteriori*.

Caveats:

- Inference of NEPs from experiments is model dependent; thus, relativistic and non-relativistic models have different "target" values of NEPs.
- Due to the absence of data, we always treat NEPs as independent constraints; in reality, they are most likely correlated.
- For technical reasons, we have employed different sets of χEFT calculations as constraints for relativistic and non-relativistic models; the difference is minor, though.

Additional considerations: for non-relativistic models, we have considered correlations between the values that energy per nucleon and/or effective masses in PNM have at different densities.

Markov Chain Monte-Carlo Setup

With the abovementioned considerations and given the vector of model parameters $\boldsymbol{\Theta}$ and vector of constraints (*targets)* **D**, the likelihood function for the "run" q can be written as

$$\log \mathcal{L}_q(\boldsymbol{\Theta}\,|\,\mathbf{D}) \propto \underbrace{-\frac{1}{2}\sum_{i=1}^{N_q^{\text{uncorr.}}}\left(\frac{d_i-\xi_i(\boldsymbol{\Theta})}{\sigma_i}\right)^2}_{\text{uncorrelated constraints}} \underbrace{-\frac{1}{2}\sum_{j=1}^{N_q^{\text{corr.}}}\sum_r\sum_s\left(\text{cov}^{-1}\right)_{rs}^{(j)}\delta\xi_r^{(j)}\delta\xi_s^{(j)}}_{\text{correlated constraints (if any)}}$$

where $\xi_i(\boldsymbol{\Theta})$ is the value of quantity i computed from the model defined by parameters $\boldsymbol{\Theta}$.

NSs' maximum mass & "physical" constraints are introduced as "hard walls". Any models $\boldsymbol{\Theta}$ that result in violation of any of those constraints are immediately rejected by setting their likelihood to extremely low value $\approx \exp(-10^{10})$.

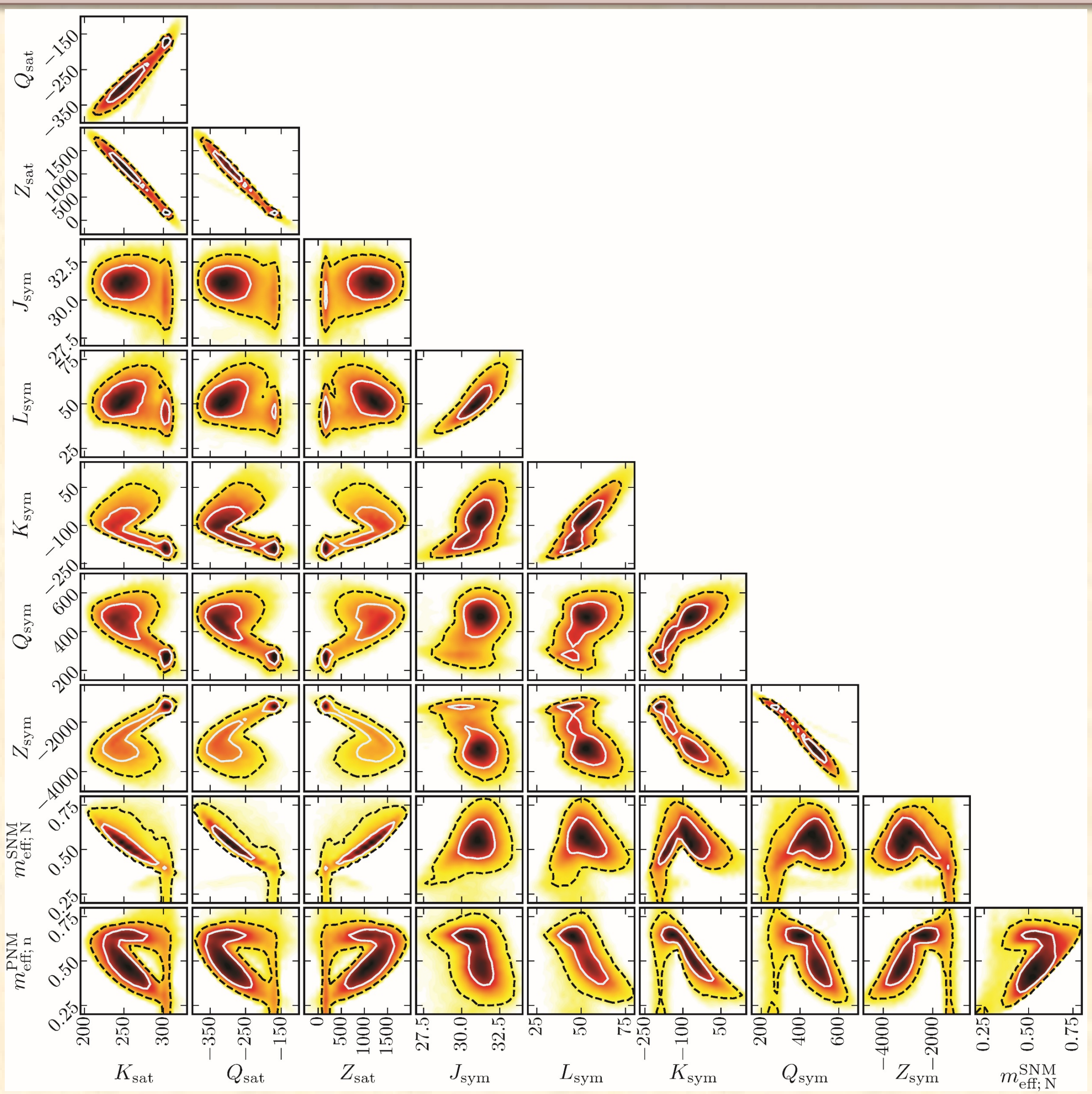
Results

Key message #1 $(T = 0)$: Correlations are model and setup dependent.

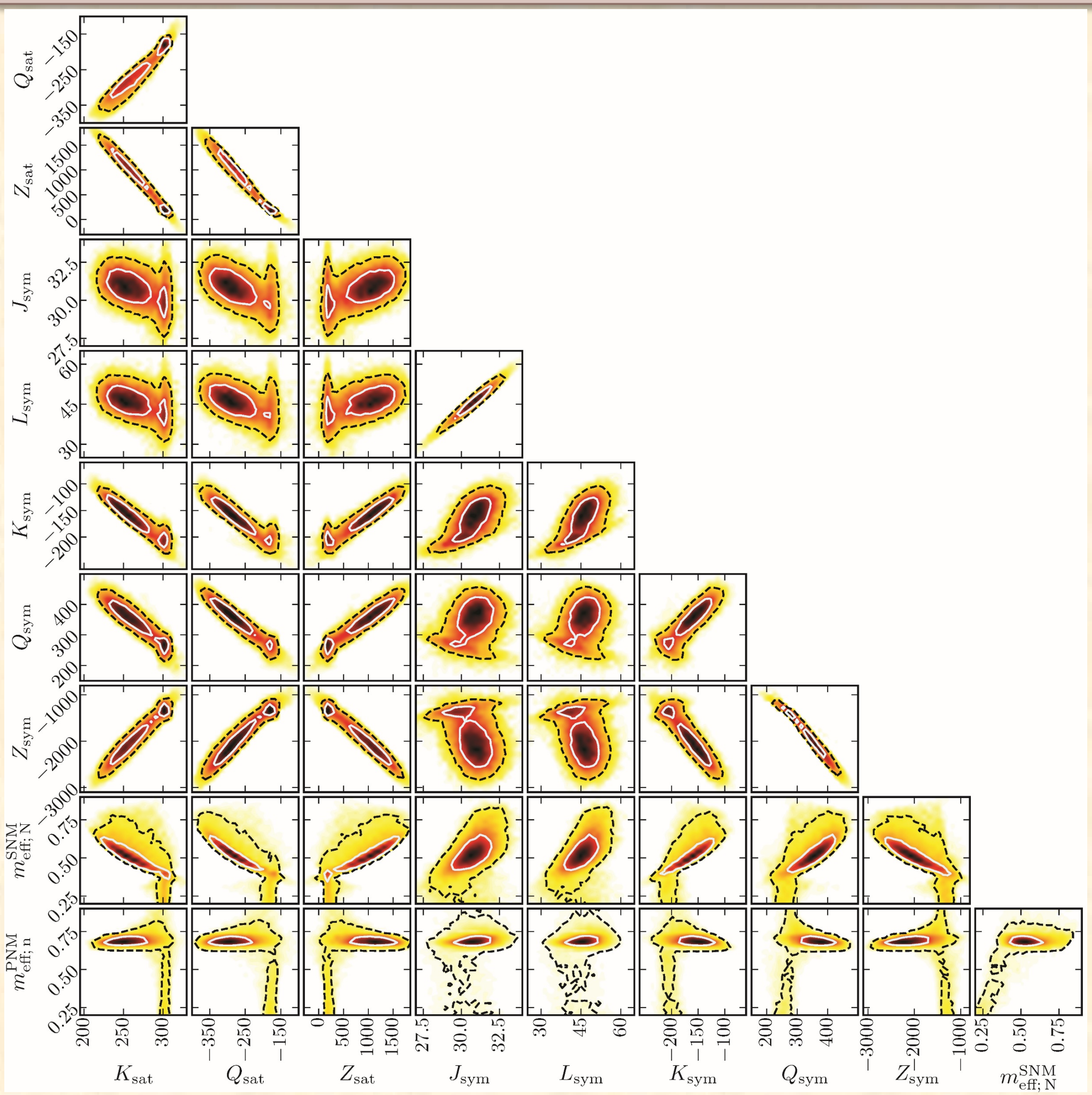
DD CDF model, constraints on E/A in PNM vs P in PNM

	$\Gamma_{\sigma,0}$	$\Gamma_{\omega,0}$	$\Gamma_{\rho,0}$	a_σ	a_ω	a_ρ	$n_{\rm sat}$	$E_{\rm sat}$	$K_{\rm sat}$	$Q_{\rm sat}$	$Z_{\rm sat}$	$J_{\rm sym}$	$L_{\rm sym}$	$K_{\rm sym}$	$Q_{\rm sym}$	$Z_{\rm sym}$	$m_{\rm eff}$	$\widetilde{E}_1$	$\widetilde{E}_2$	$\widetilde{E}_3$	P_1	P_2	P_3	$M^*_{\rm G}$	$M^*_{\rm B}$	$n^*_{\rm c}$	$\rho^*_{\rm c}$	$P^*_{\rm c}$	$R_{1.4}$	$\Lambda_{1.4}$	$R_{2.0}$
$\Gamma_{\sigma,0}$	-0.00	0.00	-0.22	-0.01	-0.00	-0.11	0.01	-0.02	0.03	0.03	-0.02	-0.14	0.06	0.14	-0.18	0.06	-0.01	-0.13	-0.17	-0.10	0.04	0.05	0.03	0.03	0.03	-0.04	-0.04	-0.01	0.04	0.04	0.04
$\Gamma_{\omega,0}$	0.00	0.00	-0.22	-0.01	-0.00	-0.11	0.01	-0.02	0.03	0.03	-0.02	-0.14	0.06	0.14	-0.18	0.06	-0.01	-0.12	-0.17	-0.10	0.04	0.05	0.03	0.03	0.03	-0.04	-0.04	-0.01	0.04	0.04	0.04
$\Gamma_{\rho,0}$	-0.22	-0.22	0.00	-0.09	-0.15	-0.67	0.10	-0.04	-0.17	-0.16	0.20	-0.14	0.59	-0.00	-0.68	0.82	0.21	-0.61	-0.11	-0.13	0.31	0.59	0.51	-0.08	-0.07	0.16	0.18	0.22	0.07	-0.05	-0.08
a_σ	-0.01	-0.01	-0.09	0.00	-0.01	-0.04	0.01	-0.01	0.02	0.02	0.01	-0.05	0.00	-0.01	-0.04	0.01	0.01	-0.02	-0.04	-0.03	0.00	0.00	-0.00	0.02	0.02	-0.02	-0.02	0.03	0.02	0.02	0.03
a_ω	-0.00	-0.00	-0.15	-0.01	0.00	-0.06	0.01	-0.01	0.03	0.03	-0.00	-0.07	0.01	0.03	-0.08	0.02	0.00	-0.05	-0.09	-0.05	0.01	0.01	0.00	0.03	0.03	-0.04	-0.04	0.01	0.03	0.03	0.04
a_ρ	-0.11	-0.11	-0.67	-0.04	-0.06	0.00	0.13	-0.07	-0.05	-0.06	0.07	-0.82	-0.07	0.23	0.03	-0.07	0.08	-0.15	-0.78	-0.86	-0.34	-0.07	-0.09	-0.02	0.00	0.06	0.06	0.09	-0.09	-0.08	-0.05
$n_{\rm sat}$	0.01	0.01	0.10	0.01	0.01	0.13	0.00	0.02	-0.01	-0.01	0.01	0.19	-0.15	-0.08	0.16	-0.15	0.00	0.16	0.10	0.08	-0.04	-0.13	-0.12	0.00	0.00	-0.01	-0.01	-0.01	-0.04	-0.02	-0.01
$E_{\rm sat}$	-0.02	-0.02	-0.04	-0.01	-0.01	-0.07	0.02	0.00	-0.01	-0.01	0.01	-0.05	0.06	0.03	-0.08	0.08	0.01	-0.06	-0.03	-0.02	0.02	0.05	0.05	-0.01	-0.01	0.01	0.01	0.01	0.01	0.00	-0.00
$K_{\rm sat}$	0.03	0.03	-0.17	0.02	0.03	-0.05	-0.01	-0.01	0.00	0.00	-0.02	-0.08	0.01	0.15	-0.14	0.01	-0.03	-0.08	-0.14	-0.07	0.04	0.01	-0.01	0.01	0.01	-0.02	-0.03	-0.04	0.00	0.02	0.02
$Q_{\rm sat}$	0.03	0.03	-0.16	0.02	0.03	-0.06	-0.01	-0.01	0.00	0.00	-0.03	-0.08	0.02	0.16	-0.14	0.02	-0.03	-0.08	-0.14	-0.06	0.04	0.02	0.00	0.01	0.01	-0.02	-0.02	-0.05	0.01	0.02	0.02
$Z_{\rm sat}$	-0.02	-0.02	0.20	0.01	-0.00	0.07	0.01	0.01	-0.02	-0.03	0.00	0.09	-0.01	-0.11	0.14	-0.02	0.01	0.08	0.14	0.07	-0.03	-0.01	0.01	-0.03	-0.03	0.04	0.04	0.02	-0.02	-0.03	-0.04
$J_{\rm sym}$	-0.14	-0.14	-0.14	-0.05	-0.07	-0.82	0.19	-0.05	-0.08	-0.08	0.09	0.00	0.78	0.06	-0.88	0.92	0.10	-0.72	-0.16	-0.12	0.35	0.73	0.64	-0.08	-0.08	0.13	0.15	0.14	0.14	0.00	-0.04
$L_{\rm sym}$	0.06	0.06	0.59	0.00	0.01	-0.07	-0.15	0.06	0.01	0.02	-0.01	0.78	0.00	-0.23	-0.09	0.02	-0.02	0.15	0.71	0.82	0.30	0.04	0.03	0.03	0.02	-0.05	-0.05	-0.04	0.06	0.05	0.05
$K_{\rm sym}$	0.14	0.14	-0.00	-0.01	0.03	0.23	-0.08	0.03	0.15	0.16	-0.11	0.06	-0.23	0.00	0.27	-0.40	-0.12	0.26	-0.02	0.09	0.07	-0.19	-0.21	0.12	0.12	-0.17	-0.19	-0.13	0.01	0.10	0.12
$Q_{\rm sym}$	-0.18	-0.18	-0.68	-0.04	-0.08	0.03	0.16	-0.08	-0.14	-0.14	0.14	-0.88	-0.09	0.27	0.00	0.11	0.15	-0.29	-0.83	-0.95	-0.37	-0.09	-0.12	-0.07	-0.05	0.14	0.14	0.14	-0.15	-0.15	-0.13
$Z_{\rm sym}$	0.06	0.06	0.82	0.01	0.02	-0.07	-0.15	0.08	0.01	0.02	-0.02	0.92	0.02	-0.40	0.11	0.00	-0.03	0.28	0.91	0.97	0.50	0.10	0.05	0.01	-0.00	-0.04	-0.03	-0.03	0.05	0.04	0.03
$m_{\rm eff}$	-0.01	-0.01	0.21	0.01	0.00	0.08	0.00	0.01	-0.03	-0.03	0.01	0.10	-0.02	-0.12	0.15	-0.03	0.00	0.09	0.15	0.08	-0.04	-0.02	-0.00	-0.04	-0.04	0.04	0.04	0.01	-0.03	-0.04	-0.04
$\widetilde{E}_1$	-0.13	-0.12	-0.61	-0.02	-0.05	-0.15	0.16	-0.06	-0.08	-0.08	0.08	-0.72	0.15	0.26	-0.29	0.28	0.09	0.00	-0.65	-0.76	-0.24	0.09	0.09	-0.06	-0.04	0.10	0.11	0.09	-0.05	-0.08	-0.08
$\widetilde{E}_2$	-0.17	-0.17	-0.11	-0.04	-0.09	-0.78	0.10	-0.03	-0.14	-0.14	0.14	-0.16	0.71	-0.02	-0.83	0.91	0.15	-0.65	0.00	-0.13	0.39	0.73	0.63	-0.10	-0.10	0.15	0.18	0.16	0.12	-0.03	-0.07
$\widetilde{E}_3$	-0.10	-0.10	-0.13	-0.03	-0.05	-0.86	0.08	-0.02	-0.07	-0.06	0.07	-0.12	0.82	0.09	-0.95	0.97	0.08	-0.76	-0.13	0.00	0.41	0.85	0.76	-0.06	-0.06	0.09	0.10	0.10	0.21	0.06	0.00
P_1	0.04	0.04	0.31	0.00	0.01	-0.34	-0.04	0.02	0.04	0.04	-0.03	0.35	0.30	0.07	-0.37	0.50	-0.04	-0.24	0.39	0.41	0.00	0.28	0.29	0.01	-0.01	-0.03	-0.03	-0.04	0.16	0.11	0.06
P_2	0.05	0.05	0.59	0.00	0.01	-0.07	-0.13	0.05	0.01	0.02	-0.01	0.73	0.04	-0.19	-0.09	0.10	-0.02	0.09	0.73	0.85	0.28	0.00	0.05	0.02	0.01	-0.05	-0.04	-0.03	0.06	0.05	0.04
P_3	0.03	0.03	0.51	-0.00	0.00	-0.09	-0.12	0.05	-0.01	0.00	0.01	0.64	0.03	-0.21	-0.12	0.05	-0.00	0.09	0.63	0.76	0.29	0.05	0.00	0.02	0.01	-0.03	-0.02	-0.01	0.03	0.02	0.02
$M^*_{\rm G}$	0.03	0.03	-0.08	0.02	0.03	-0.02	0.00	-0.01	0.01	0.01	-0.03	-0.08	0.03	0.12	-0.07	0.01	-0.04	-0.06	-0.10	-0.06	0.01	0.02	0.02	0.00	-0.00	0.01	0.00	-0.05	-0.00	0.00	-0.01
$M^*_{\rm B}$	0.03	0.03	-0.07	0.02	0.03	0.00	0.00	-0.01	0.01	0.01	-0.03	-0.08	0.02	0.12	-0.05	-0.00	-0.04	-0.04	-0.10	-0.06	-0.01	0.01	0.01	-0.00	0.00	0.01	0.00	-0.05	-0.01	-0.00	-0.01
$n^*_{\rm c}$	-0.04	-0.04	0.16	-0.02	-0.04	0.06	-0.01	0.01	-0.02	-0.02	0.04	0.13	-0.05	-0.17	0.14	-0.04	0.04	0.10	0.15	0.09	-0.03	-0.05	-0.03	0.01	0.01	0.00	0.00	0.05	-0.01	-0.01	-0.01
$\rho^*_{\rm c}$	-0.04	-0.04	0.18	-0.02	-0.04	0.06	-0.01	0.01	-0.03	-0.02	0.04	0.15	-0.05	-0.19	0.14	-0.03	0.04	0.11	0.18	0.10	-0.03	-0.04	-0.02	0.00	0.00	0.00	0.00	0.05	0.01	-0.00	0.00
$P^*_{\rm c}$	-0.01	-0.01	0.22	0.03	0.01	0.09	-0.01	0.01	-0.04	-0.05	0.02	0.14	-0.04	-0.13	0.14	-0.03	0.01	0.09	0.16	0.10	-0.04	-0.03	-0.01	-0.05	-0.05	0.05	0.05	0.00	-0.03	-0.04	-0.05
$R_{1.4}$	0.04	0.04	0.07	0.02	0.03	-0.09	-0.04	0.01	0.00	0.01	-0.02	0.14	0.06	0.01	-0.15	0.05	-0.03	-0.05	0.12	0.21	0.16	0.06	0.03	-0.00	-0.01	-0.01	0.01	-0.03	0.00	-0.00	0.00
$\Lambda_{1.4}$	0.04	0.04	-0.05	0.02	0.03	-0.08	-0.02	0.00	0.02	0.02	-0.03	0.00	0.05	0.10	-0.15	0.04	-0.04	-0.08	-0.03	0.06	0.11	0.05	0.02	0.00	-0.00	-0.01	-0.00	-0.04	-0.00	0.00	0.01
$R_{2.0}$	0.04	0.04	-0.08	0.03	0.04	-0.05	-0.01	-0.00	0.02	0.02	-0.04	-0.04	0.05	0.12	-0.13	0.03	-0.04	-0.08	-0.07	0.00	0.06	0.04	0.02	-0.01	-0.01	-0.01	0.00	-0.05	0.00	0.01	0.00

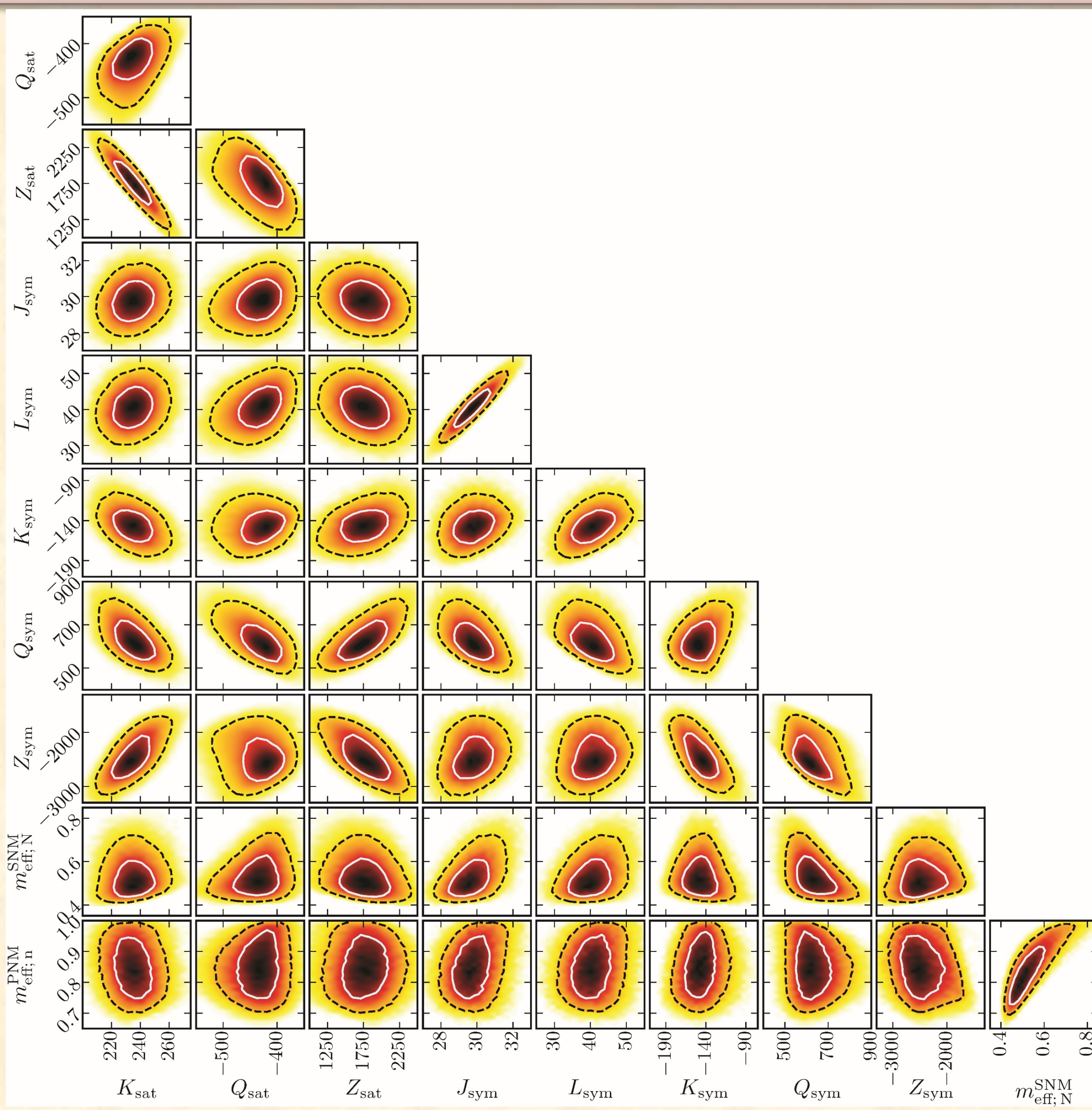
Correlations between NEPs, Sk, uncorrelated constraints on E/A in PNM, no v_{F} constraint.



Correlations between NEPs, Sk, correlated constraints on E/A in PNM, no v_F constraint.



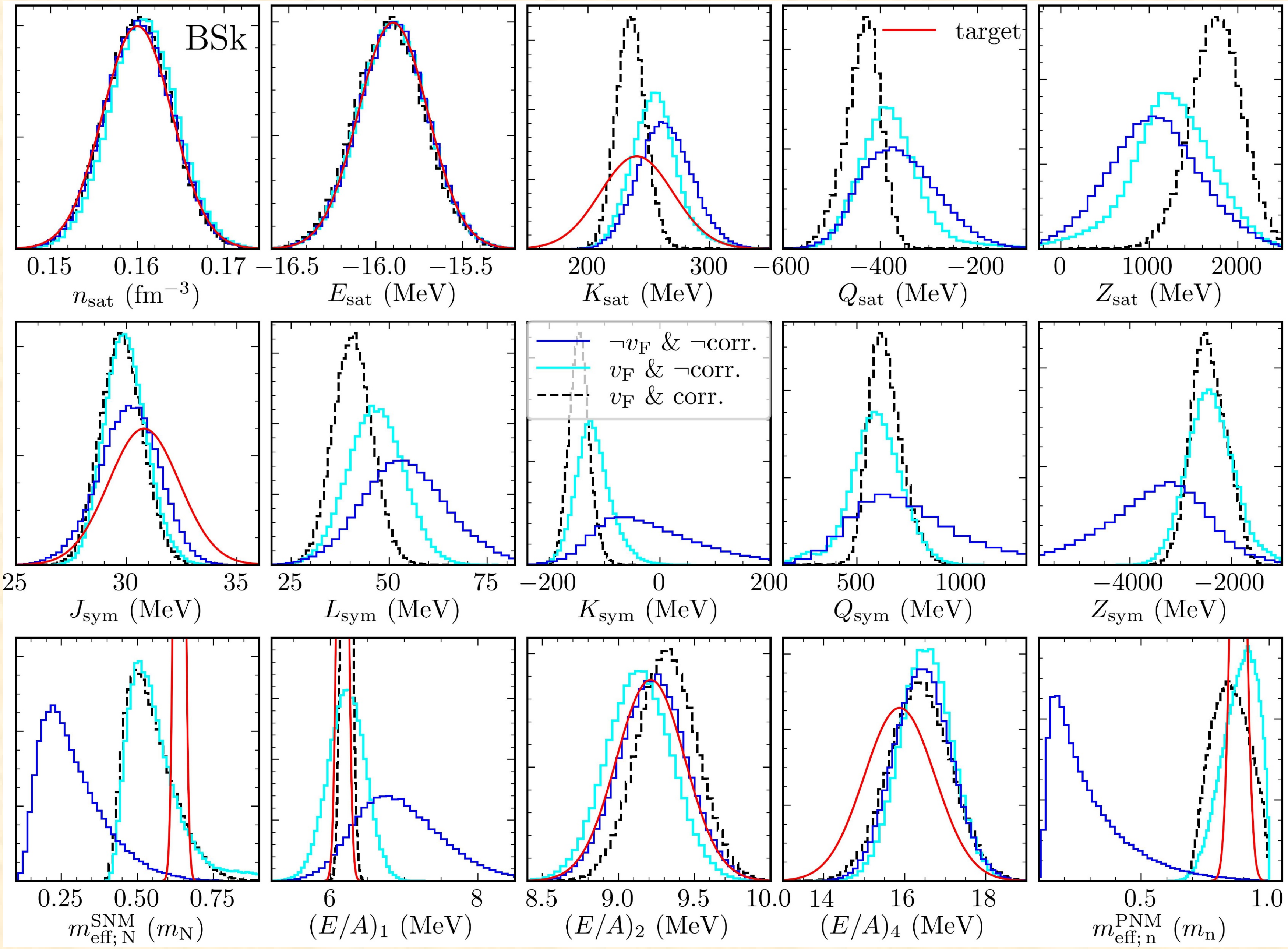
Correlations between NEPs, BSk, correlated constraints on E/A in PNM, v_{F} is constrained.



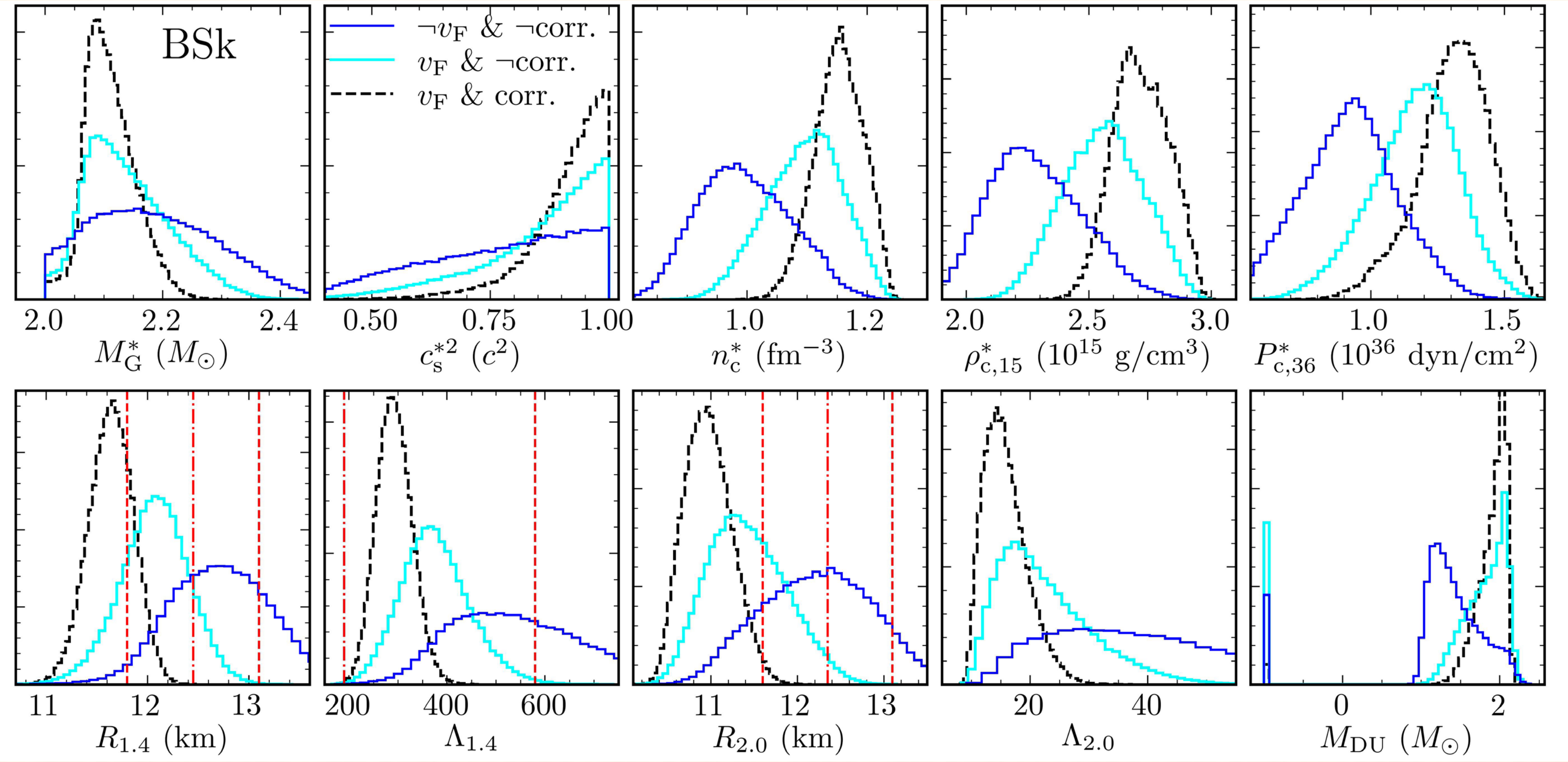
Results #2

Key message #2 ($T = 0$): Non-relativistic models very easily become unphysical and keeping them physical strongly impacts the results.

BSk, with (cyan solid) and without (blue solid) v_F constraint



BSk, with (cyan solid) and without (blue solid) v_F constraint

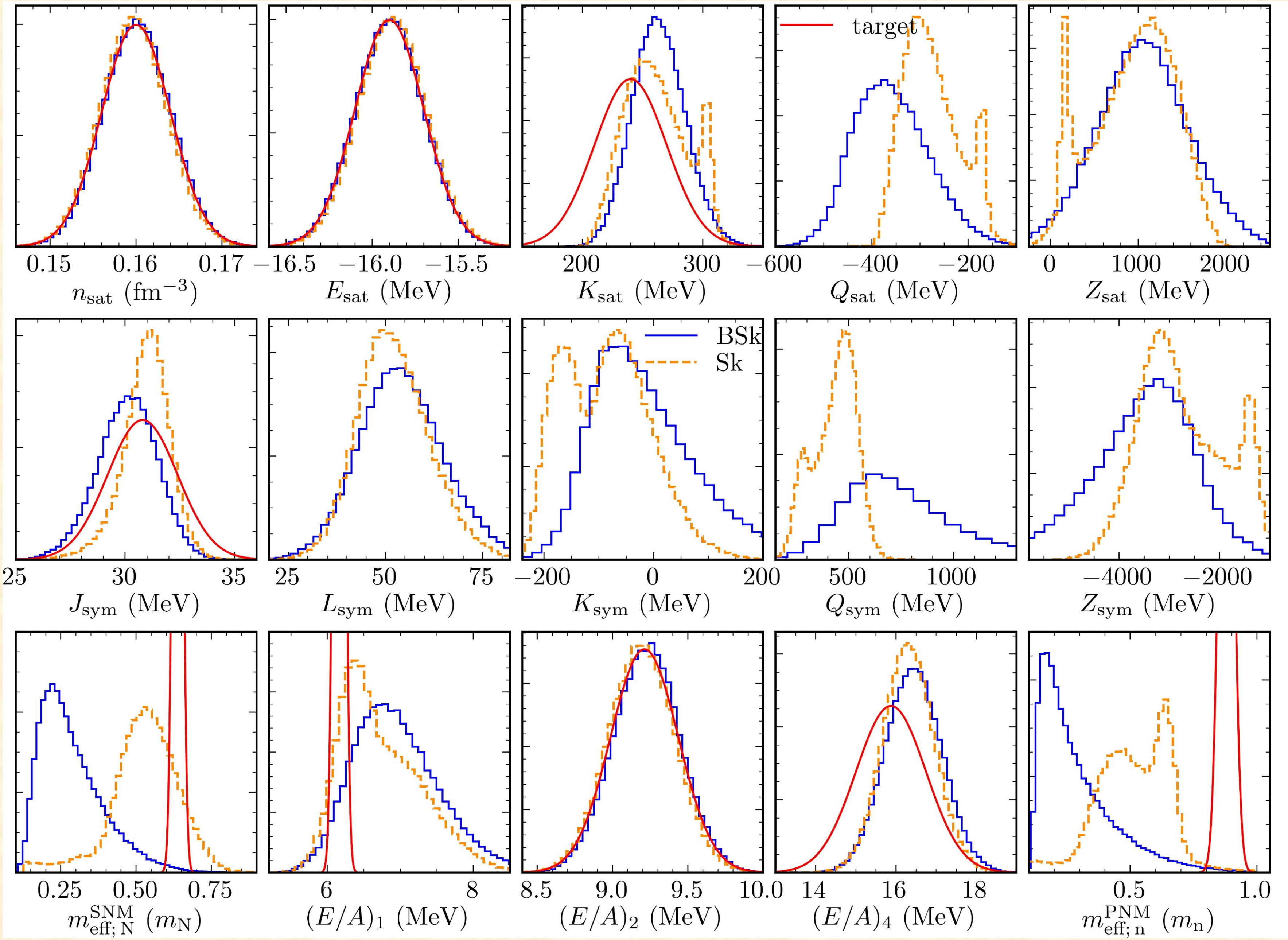


Data on radii are from M. C. Miller *et al.*, ApJL **918**, L28 (2021); 68% CI.

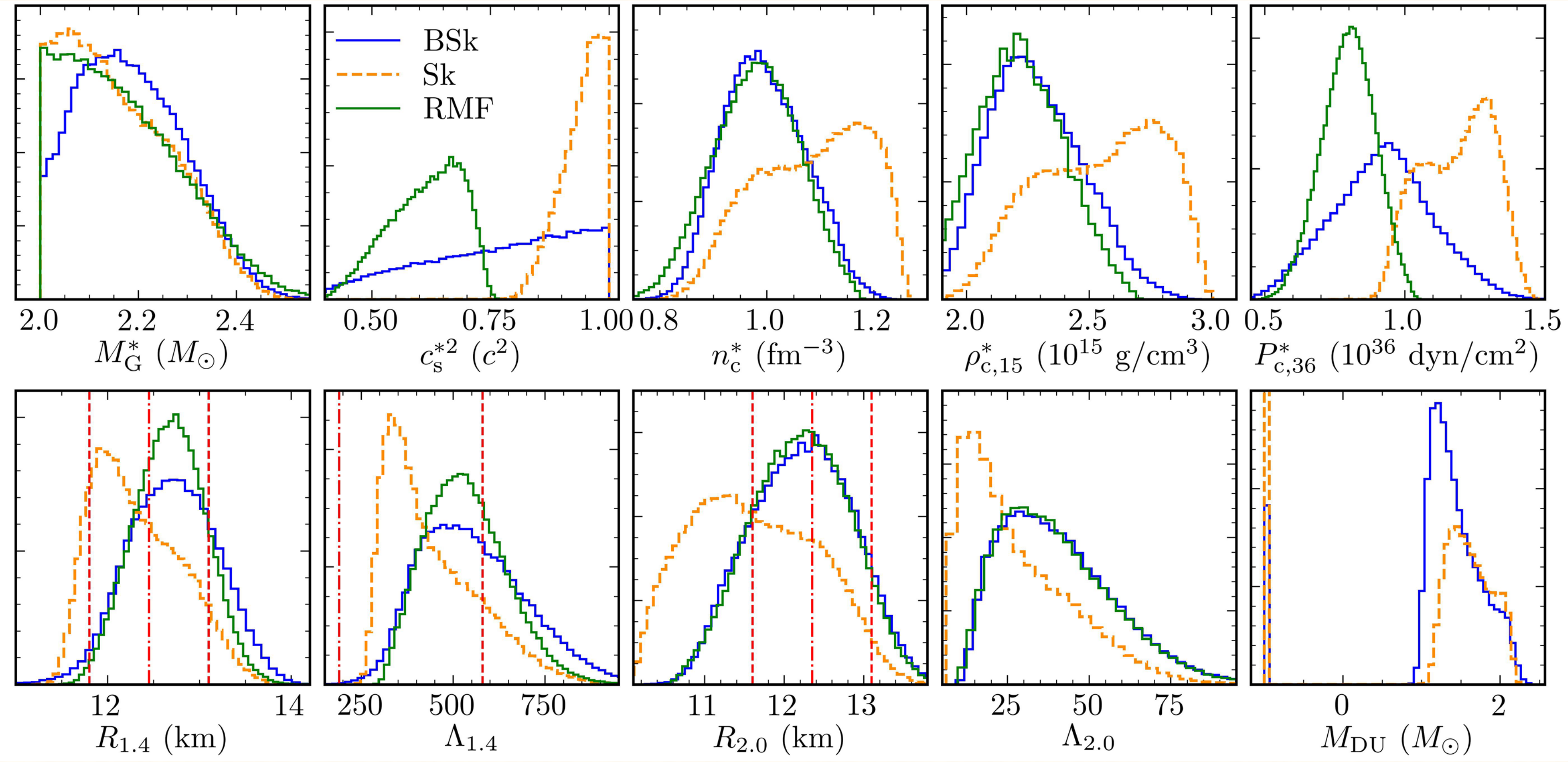
Data on tidal deformability are from B. P. Abbott *et al.*, PRL **121**, 161101 (2018); 90% CI.

Key message #3 ($T = 0$): Results depends on models and constraints.

BSk (blue solid) vs Sk (orange dashed), identical constraints:



BSk vs Sk vs DD CDF [RMF], very similar constraints



Data on radii are from M. C. Miller *et al.*, ApJL **918**, L28 (2021); 68% CI.

Data on tidal deformability are from B. P. Abbott *et al.*, PRL **121**, 161101 (2018); 90% CI.

Key message #4 (finite-*T*): BSk energy-density functional allows for U-shape behavior of the Landau effective mass and, thus, for negative thermal pressure.

$$P_{\rm th} = P\left(n_{\rm B}, T, Y_{\rm p}\right) - P\left(n_{\rm B}, T=0, Y_{\rm p}\right) = \frac{2}{3}\sum_{i={\rm n,p}} Q_i \left(\frac{\hbar^2}{2m_{{\rm L,eff},i}}\tau_i - \frac{3}{5}\tau_{{\rm F},i} n_i\right),$$

$$Q_i = 1 - \frac{3}{2}\frac{n_{\rm B}}{m_{{\rm L,eff},i}}\frac{\partial m_{{\rm L,eff},i}}{\partial n_{\rm B}}, \quad \tau_{{\rm F},i} = \frac{p_{{\rm F},i}^2}{2m_{{\rm L,eff},i}}$$

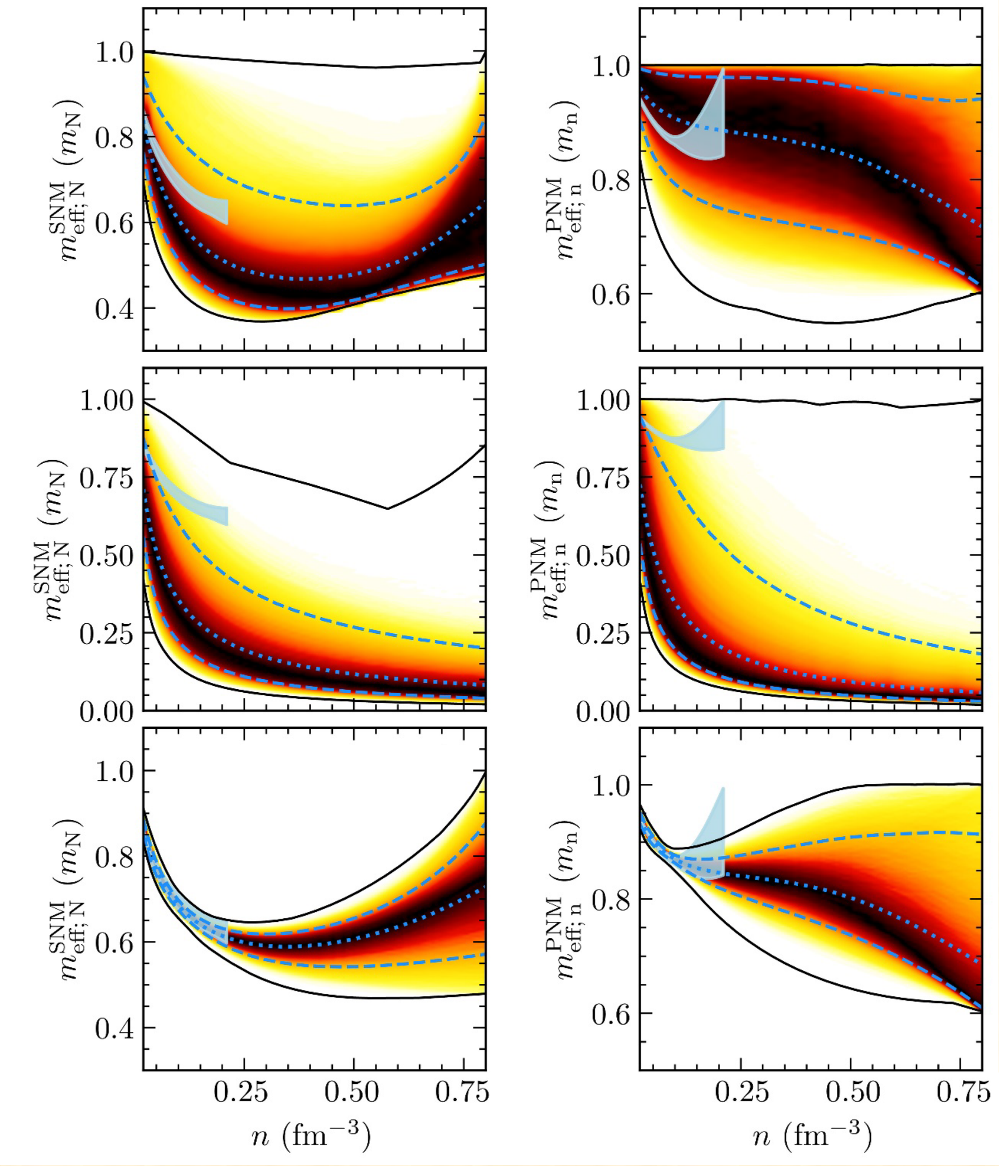
Expression from C. Constantinou *et al.*, PRC **89**, 065802 (2014).

A quick reminder of Landau effective mass behavior in BSk

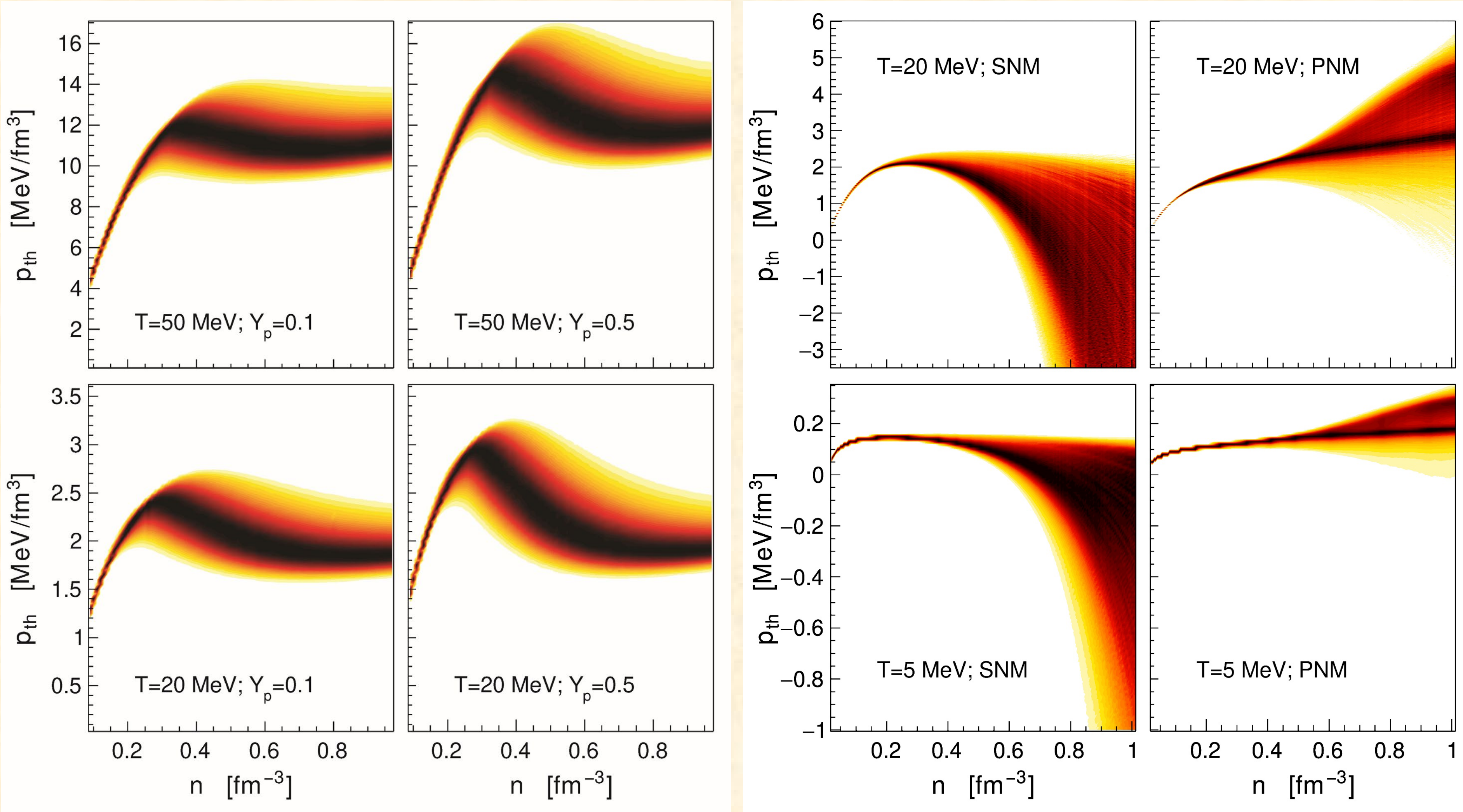
v_F, no m_eff

No v_F, no m_eff

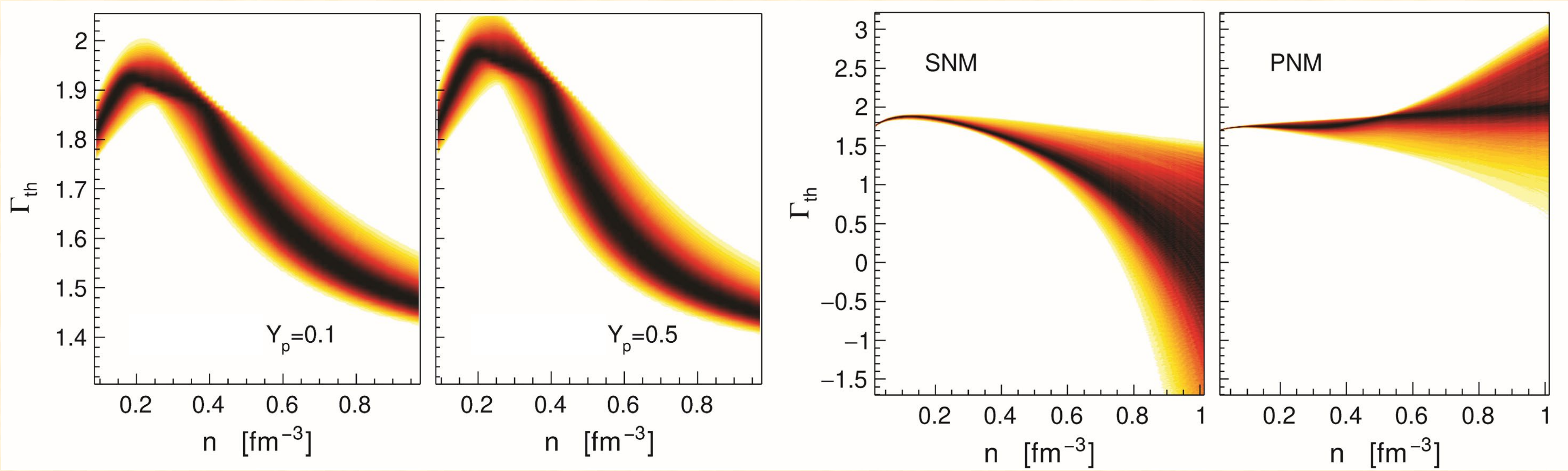
v_F and m_eff



DD CDF (left) vs BSk (right), $P_{\rm th}$



DD CDF (left) vs BSk (right), $\Gamma_{\text{th}} = 1 + p_{\text{th}}/e_{\text{th}}$



Results #4

- Evolution of cataclysmic astrophysical phenomena (CAP) such as CCSN, BNS mergers, and stellar BH formation depend on the properties of finite-T EOS.
- For non-relativistic models microscopic calculations predict U-shape behavior of $m_{\mathrm{L;eff}}$ and, thus, $P_{\mathrm{th}} < 0$. **Is this physical?!**
- A lot of studies of such phenomena rely on standard Skyrme interactions (*have to be kept physical!)* that cannot provide U-shaped $m_{\mathrm{L;eff}}(n)$. This studies, however, demonstrate that $m_{\mathrm{L;eff}}$ is one of the most important “ingredients” of a finite-T EOS.
- Thus, more realistic simulations should employ EOSs that can give U-shaped $m_{\mathrm{L;eff}}(n)$.
- Effects of $m_{\mathrm{D;eff}}$ in RMF models on CAP deserve a systematic investigation. It was demonstrate by different authors that many finite-T quantities depend on $m_{\mathrm{D;eff}}$ and $\partial_n m_{\mathrm{D;eff}}$.
- Microphysics community: calculate & constrain $m_{\mathrm{eff}}(T)$.

Conclusions

- We have employed Bayesian analysis technique in order to investigate how different mean field models of NM and various combinations of nuclear physics and astrophysics constraints affect the inferred properties of NM and NSs.
- Our main conclusions were formulated in four "key messages":
  1. ($T = 0$): Correlations are model and setup dependent.
  2. ($T = 0$): Non-relativistic models very easily become un-physical and keeping them physical strongly impacts the results.
  3. ($T = 0$): Results depends on models and constraints.
  4. (finite-T): BSk energy-density functional allows for negative thermal pressure.

Figures presented here are either taken from our papers, PRC **107**, 045803 (2023); ApJ **966**, 216 (2024); PLB **853**, 138696 (2024); and ArXiv: 2403.19325; or prepared specifically for this presentation.


Thank you!
Peles castle, Sinaia, Romania

Simplified DD CDF model has six parameters: 3 nucleon-meson coupling strengths at saturation density (n_{sat}) and 3 parameters describing the density dependence of those coupling strengths.

$$\Gamma_M(n) = \Gamma_{M,0} h_M(x), \quad x = \frac{n}{n_{\mathrm{sat}}}, \quad M = \sigma, \omega, \rho$$

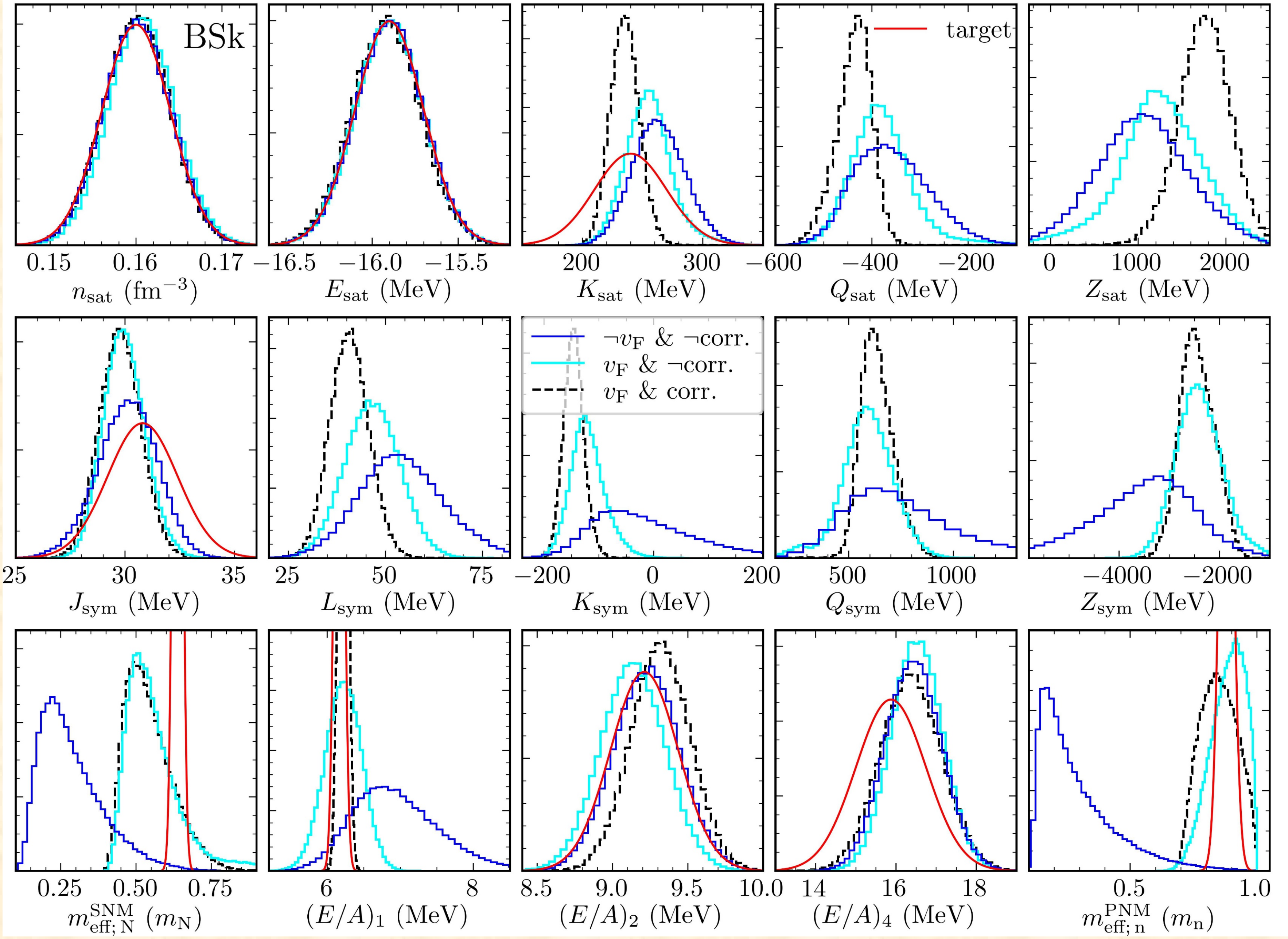
$$h_\sigma(n) = \exp\left[-\left(x^{a_\sigma} - 1\right)\right]$$
$$h_\omega(n) = \exp\left[-\left(x^{a_\omega} - 1\right)\right]$$
$$h_\rho(n) = \exp\left[-a_\rho (x - 1)\right]$$

Six parameters $\{\Gamma_{\sigma,0}, \Gamma_{\omega,0}, \Gamma_{\rho,0}, a_\sigma, a_\omega, a_\rho\}$ completely determine the model.

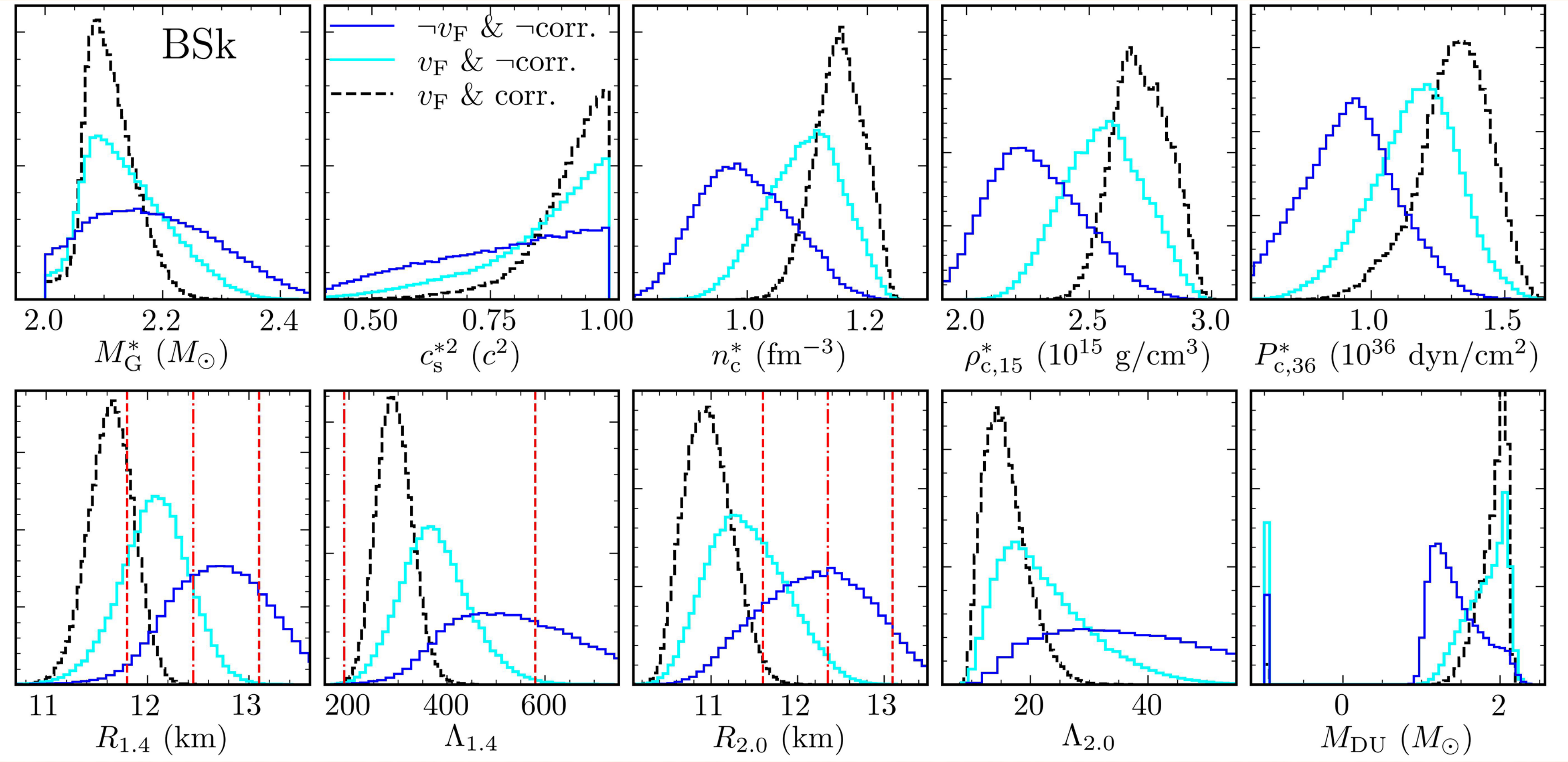
Note: n_{sat} has to be determined self-consistently with the above equations using the fact that the pressure of SNM at saturation is zero.

Key message #3a ($T = 0$): A comment about correlations in the constraints

BSk, with (black dashed) and without (cyan solid) correlations



BSk, with (black dashed) and without (cyan solid) correlations



Data on radii are from M. C. Miller *et al.*, ApJL **918**, L28 (2021);

Data on tidal deformability are from B. P. Abbott *et al.*, PRL **121**, 161101 (2018).