Status update: transport coefficients of strongly coupled plasma from holography

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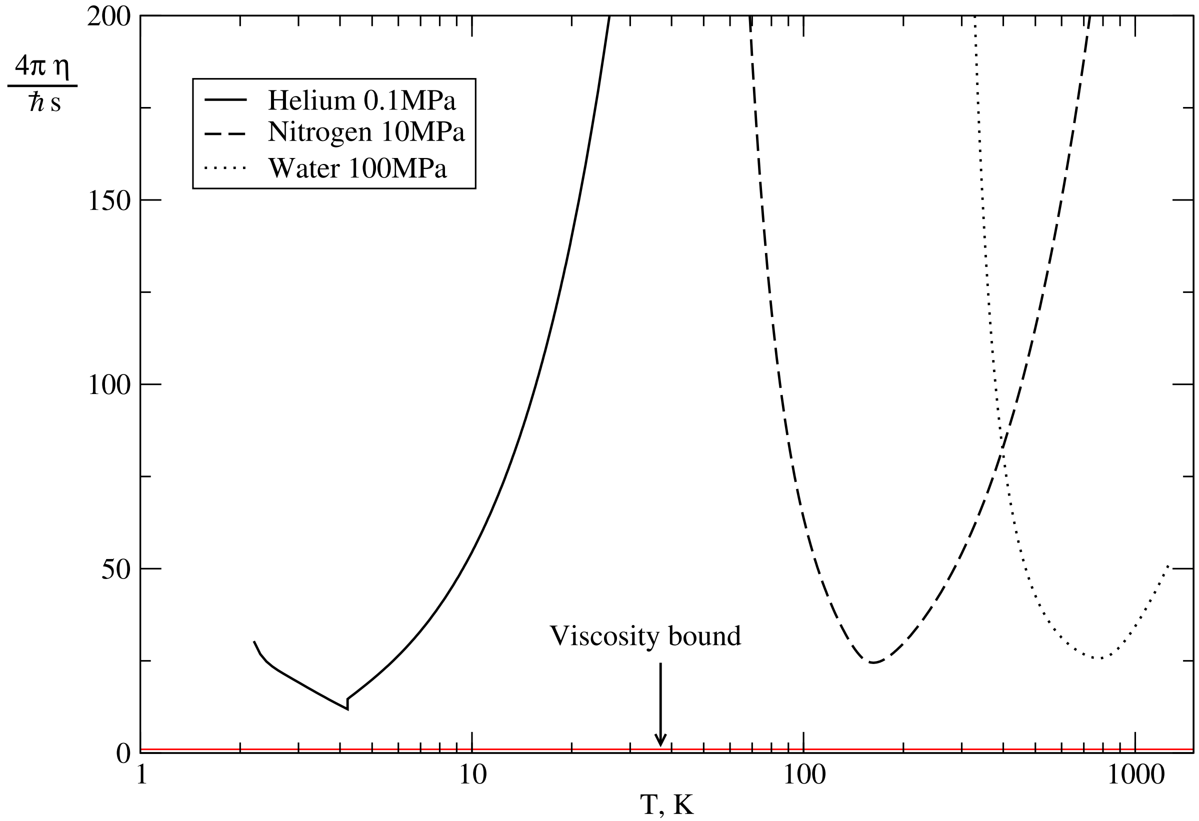
Based on arXiv: $\underbrace{\text{arXiv}:2312.05377}_{\text{(with S.Cremonini and L.Early)}}$ and arXiv:2402.16109

Heavy Ion Physics in the EIC Era, INT, 22 August 2024

$\Longrightarrow$ From Kovtun-Son-Starinets (arXiv:hep-th/0405231):



- universality of $\frac{\eta}{s}$ at strong coupling
- existence the lower bound on $\frac{\eta}{s}$

Outline:

- 3-ways for extract transport coefficients:
  - Kubo formulas
  - dispersion relations
  - near-equilibrium entropy production

- Holography of EOS of QGP:
  - conformal models (*e.g.*, $\mathcal{N} = 4$ SYM)
  - top-down models:
    - explicit breaking of scale inv. (masses, relevant couplings)
    - spontaneous breaking of scale inv. (β-functions)

  - phenomenological (bottom-up) models $\supset$ top-down models

- Beyond 2-derivative gravity:
  - ■ more generic UV fixed points ($c \neq a$)
  - ■ finite 't Hooft coupling corrections
    - – $\frac{\eta}{s}$ is not universal
    - – no lower bound on shear viscosity in gauge theory plasma

- Future directions:
  - ■ DL:
    - – EOS/transport correlations
    - – reconstructing gravitational dual from lattice QCD EOS
    - – string theory vs. phenomenological models

  - ■ Extension of the framework to μ_i (multiple chemical potentials $\Longrightarrow$ transport @ criticality)
  - ■ sensitivity of transport to features of gravitational potentials

$\Longrightarrow$ Consider a (neutral) gauge theory plasma close to equilibrium. **Relativistic hydrodynamics** is an effective theory of the conserved stress-energy tensor $T^{\mu\nu}$, formulated as the series of the local-velocity gradients:

- $\nabla_\mu T^{\mu\nu} = 0$
- $T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \cdots$
- $$T^{\mu\nu} = \underbrace{\mathcal{E}\ u^\mu u^\nu + P\ (g^{\mu\nu} + u^\mu u^\nu)}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[-\eta\, \sigma^{\mu\nu} - \zeta\, (g^{\mu\nu} + u^\mu u^\nu) \nabla \cdot u \right]}_{\mathcal{O}(\partial^1 u):\ \sigma^{\mu\nu} \sim \partial^\mu u^\nu}$$
  $$+ \underbrace{\left[\qquad \cdots \qquad \right]}_{\mathcal{O}(\partial^2 u, (\partial u)^2)} + \cdots$$

  - u^μ — local fluid velocity; $u^\mu u_\mu = -1$
  - η, ζ — shear and bulk viscosities
  - expansion parameter of hydro as EFT: $\frac{1}{T} \cdot |\partial u| \ll 1$

3-ways for extract transport coefficients:

- **Kubo** formulas ($\boldsymbol{x} \equiv \{x_1, x_2, x_3\}$) from equilibrium c.f.:

  - the shear viscosity,

$$\eta = -\lim_{w \to 0} \frac{1}{w} \mathrm{Im}\, G_R(w)$$

$$G_R(w) = -i \int dt d\boldsymbol{x} e^{iwt} \theta(t) \langle [T_{12}(t, \boldsymbol{x}), T_{12}(0, \mathbf{0})] \rangle$$

  - the bulk viscosity,

$$\zeta = -\frac{4}{9} \lim_{w \to 0} \frac{1}{w} \mathrm{Im}\, G_R(w)$$

$$G_R(w) = -i \int dt d\boldsymbol{x} e^{iwt} \theta(t) \langle [\tfrac{1}{2} T_i^i(t, \boldsymbol{x}), \tfrac{1}{2} T_j^j(0, \mathbf{0})] \rangle$$

- **dispersion** spectrum of linearized fluctuations in plasma ($\propto e^{-iwt+i\boldsymbol{q}\cdot\boldsymbol{x}}$):

$$\mathfrak{w} \equiv \frac{w}{2\pi T}\,, \qquad \mathfrak{q} \equiv \frac{|\boldsymbol{q}|}{2\pi T}$$

  - the shear mode,

$$\mathfrak{w} = -i\,\frac{2\pi\eta}{s}\,\mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3)$$

  - the sound,

$$\mathfrak{w} = c_s \cdot \mathfrak{q} - i\,\frac{4\pi\eta}{3s}\left(1 + \frac{3\zeta}{4\eta}\right)\mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3)$$

  where the speed of the sound waves can also be extracted from EOS,

$$c_s^2 = \frac{\partial P}{\partial \mathcal{E}}$$

- **entropy production** in approach to equilibrium:

  - define the non-equilibrium entropy s

$$Ts = P + \mathcal{E}\,, \qquad \partial P = s\partial T$$

and the entropy current S^μ,

$$S^\mu = \underbrace{s\,u^\mu}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[-\frac{u_\nu}{T}\,T^{\mu\nu}_{(1)}\right]}_{\mathcal{O}(\partial^1 u)} + \underbrace{[\cdots]}_{\mathcal{O}(\partial^2 u,(\partial u)^2)} + \cdots$$

  - from hydro EOM,

$$T\partial_\mu S^\mu = \underbrace{\frac{\eta}{2}\,\sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta\,\left(\partial_\lambda u^\lambda\right)^2}_{\mathcal{O}(\partial^2)} + \mathcal{O}(\partial^3)$$

$\Longrightarrow$ Holographic dictionary:

$$\left.\begin{array}{l}\textbf{Kubo} \Longrightarrow \text{equilibrum c.f. in holographic black hole} \\ \textbf{dispersion} \Longrightarrow \text{holographic black hole QNMs} \\ \textbf{entropy production} \Longrightarrow \text{AH dynamics of black hole}\end{array}\right\} \Longrightarrow \text{same } \{\eta, \zeta\}$$

$\Longrightarrow$ Consistent picture in holography

Holography of EOS of QGP

- **conformal models**: $\mathcal{N}=4$ supersymmetric Yang-Mills theory

■ $SU(N)$ gauge theory A_μ + bosons ϕ_i + fermions ψ_a = maximally supersymmetric and scale invariant $\Longrightarrow \mathcal{L}_{SYM}[A_\mu, \psi_a, \phi_i]$

■

$$Z_{SYM}[\mathcal{M}_4] \equiv \underbrace{\int [dA d\psi d\phi] \epsilon^{i \int_{\mathcal{M}_4} d^4x \mathcal{L}_{SYM}}}_{\text{gauge theory}} = \underbrace{e^{i\, S_5[\partial\mathcal{M}_5 = \mathcal{M}_4]}}_{\text{dual ``gravity'' in 5-dim}}$$

■ classical gravity approximation:

$$\begin{cases} '\text{t Hooft limit}: & N \to \infty\,, g_{YM}^2 \to 0 \text{ with } \lambda \equiv g_{YM}^2 N = \text{const} \\ \text{strong coupling}: & \lambda \to \infty \end{cases}$$

$\Longrightarrow$

$$S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} \mathrm{d}^5x \sqrt{-g}\,(R+12)\ , \qquad G_5 = \frac{\pi}{2N^2}$$

$$\text{SYM thermal states} \qquad \Longleftrightarrow \qquad \text{black holes of } S_5$$

- AdS-Schwarzschild black hole:

$$ds_5^2 = \frac{r_0^2}{u}\left(-(1-u^2)dt^2 + d\boldsymbol{x}^2\right) + \frac{du^2}{4u^2(1-u^2)}$$

  - $u \to 1$ BH horizon
  - $u \to 0 \iff \mathcal{M}_5 \to \partial\mathcal{M}_5 = \mathcal{M}_4 = \mathbb{R}^{3,1}: , \qquad ds^2_{\mathcal{M}_4} = -dt^2 + d\boldsymbol{x}^2$

- BH temperature T and the entropy density s:

$$T = \frac{r_0}{\pi}\,, \qquad s \equiv \frac{\text{horizon area density}}{4G_5} = \frac{r_0^3}{4G_5}$$

$\Longrightarrow$ Thermal properties of BH are interpreted as thermal properties of strongly coupled $\mathcal{N} = 4$ SYM plasma (trade $r_0 \leftrightarrow T$):

- the energy density

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T^4 = \frac{3}{4}\epsilon_{SB}$$

- the pressure

$$P = \frac{1}{8}\pi^2 N^2 T^4$$

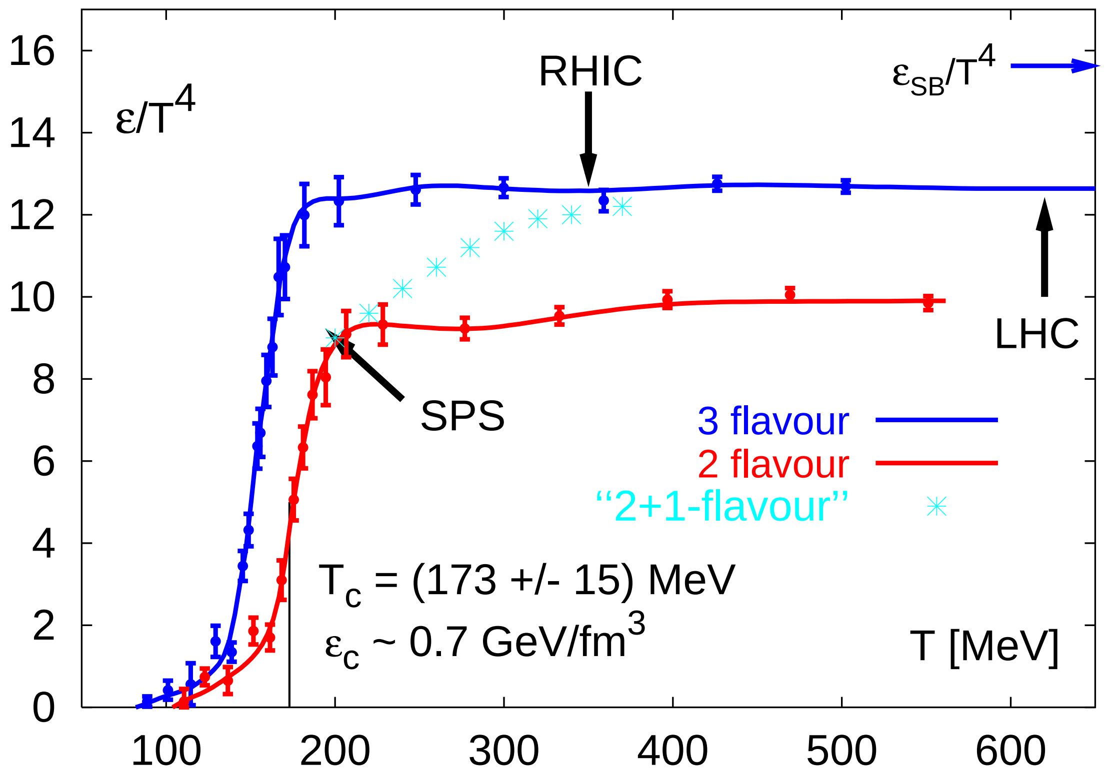
- the entropy density

$$s = \frac{1}{2}\pi^2 N^2 T^3$$

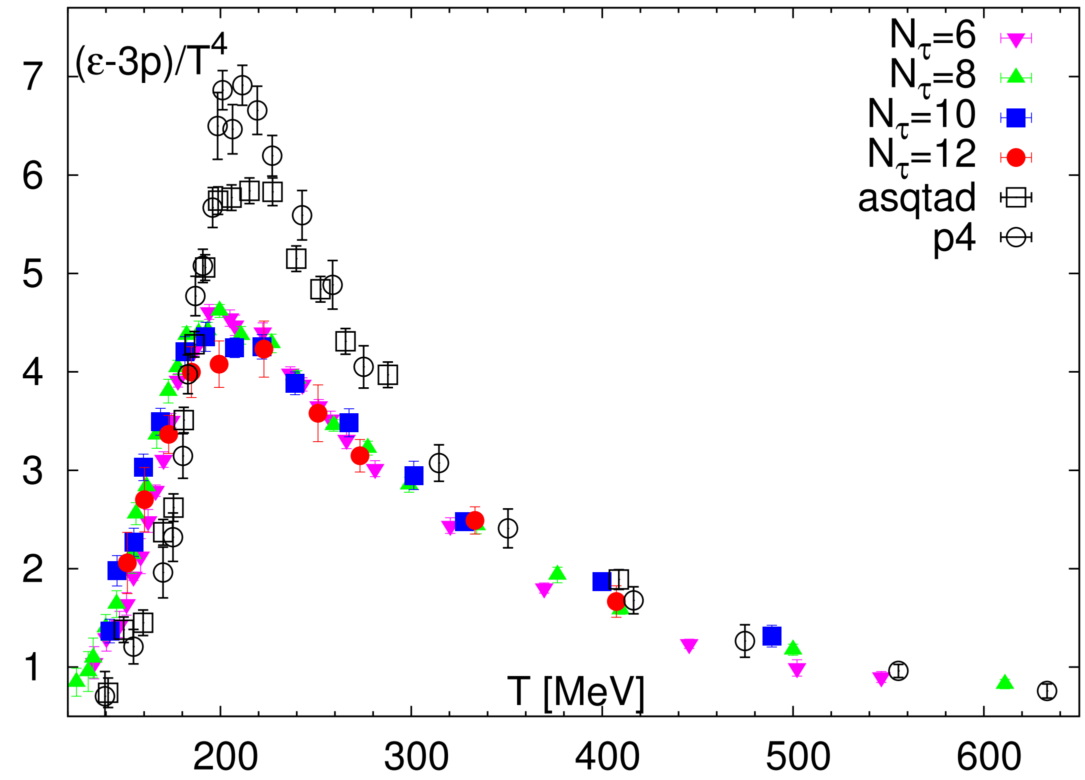
$\Longrightarrow$ Transport in $\mathcal{N} = 4$ (Policastro-Son-Starinets)

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\,, \qquad \frac{\zeta}{s} = 0$$

- QCD thermodynamics from lattice; (Karsch, Laermann, hep-lat/0305025). The plateau is $\sim 80\%$ of the SB result — close to 3/4 in SYM thermodynamics



$\Longrightarrow$ From A.Bazarov et.al (HotQCD Collaboration), arXiv:1407.6387:



$\Longrightarrow$ The violation of the conformality,

$$\frac{\epsilon - 3p}{\epsilon} \sim 50\%$$

at the maximum

QCD $\neq$ CFT! — part I

- quark masses $\Longrightarrow$ explicit breaking of the scale invariance
  - holographic model ($\mathcal{N}=2^*$ model):

$$\mathcal{L}_{SYM} \longrightarrow \mathcal{L}_{SYM} + \delta\mathcal{L}\,, \qquad \delta\mathcal{L} = -2\int d^4x\,\left[\lambda_2\,\mathcal{O}_2 + \lambda_3\,\mathcal{O}_3\right]$$

where $\dim[\mathcal{O}_\Delta]=\Delta$; $\lambda_\Delta \neq 0$ is its coupling constant; *e.g.*, :

$$\underbrace{\mathcal{O}_2 \sim \mathrm{Tr}\,|\phi|^2}_{\text{boson mass-term operator}} \qquad , \qquad \underbrace{\mathcal{O}_3 \sim \mathrm{Tr}\,\overline{\psi}\psi}_{\text{fermion mass-term operator}}$$

- QCD has a strong coupling scale $\Longrightarrow$ "spontaneous" breaking of scale invariance ($\beta_{QCD}<0$); confinement/deconfinement+χSB
  - holographic model (Klebanov-Strassler model):

$$\mathcal{N}=1, SU(N)\ \text{SYM}\ +\ \text{irrelevant in the IR operators}$$

$\Longrightarrow$ Holographic implementation of non-conformal models:

$$S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[\underbrace{R + 12}_{\text{CFT sector}} \qquad \underbrace{- \frac{1}{2} \sum_i (\partial \phi_i)^2 - V\{\phi_i\}}_{\text{deformation}} \right]$$

$$\phi_i \leftrightarrow \mathcal{O}_{\Delta_i} \,, \qquad m_i^2 \equiv \left. \frac{\partial^2 V}{\partial \phi_i^2} \right|_{\phi_i = 0} = \Delta_i (\Delta_i - 4)$$

- $\mathcal{N} = 2^*$ model:

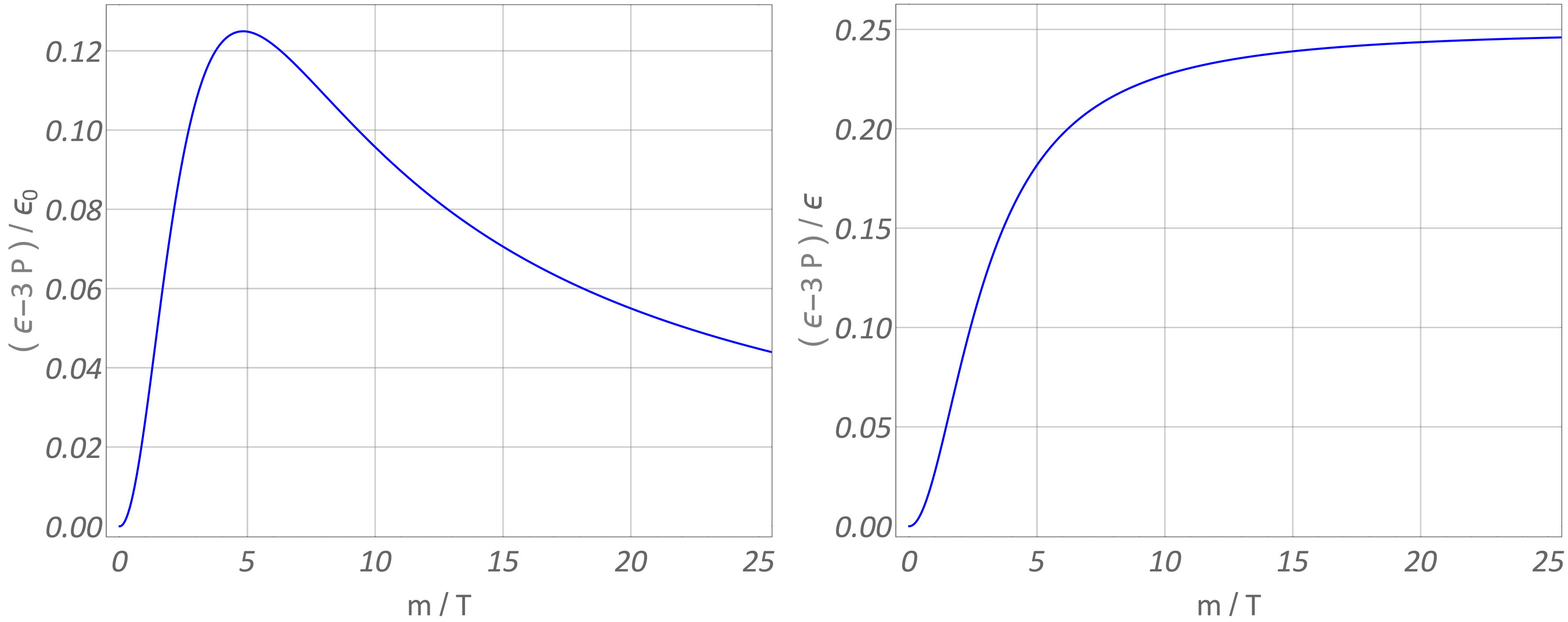
$$V\{\phi_2, \phi_3\} = -4 e^{-\frac{2}{\sqrt{6}}\phi_2} - 8 e^{\frac{1}{\sqrt{6}}\phi_2} \cosh\left[\frac{1}{\sqrt{2}}\phi_3\right] + e^{\frac{4}{\sqrt{6}}\phi_2} \sinh^2\left[\frac{1}{\sqrt{2}}\phi_3\right]$$

- Klebanov-Strassler model:

$$V = V\{\phi_3^{(1)}, \phi_3^{(2)}, \phi_4, \phi_6, \phi_7, \phi_8\}$$

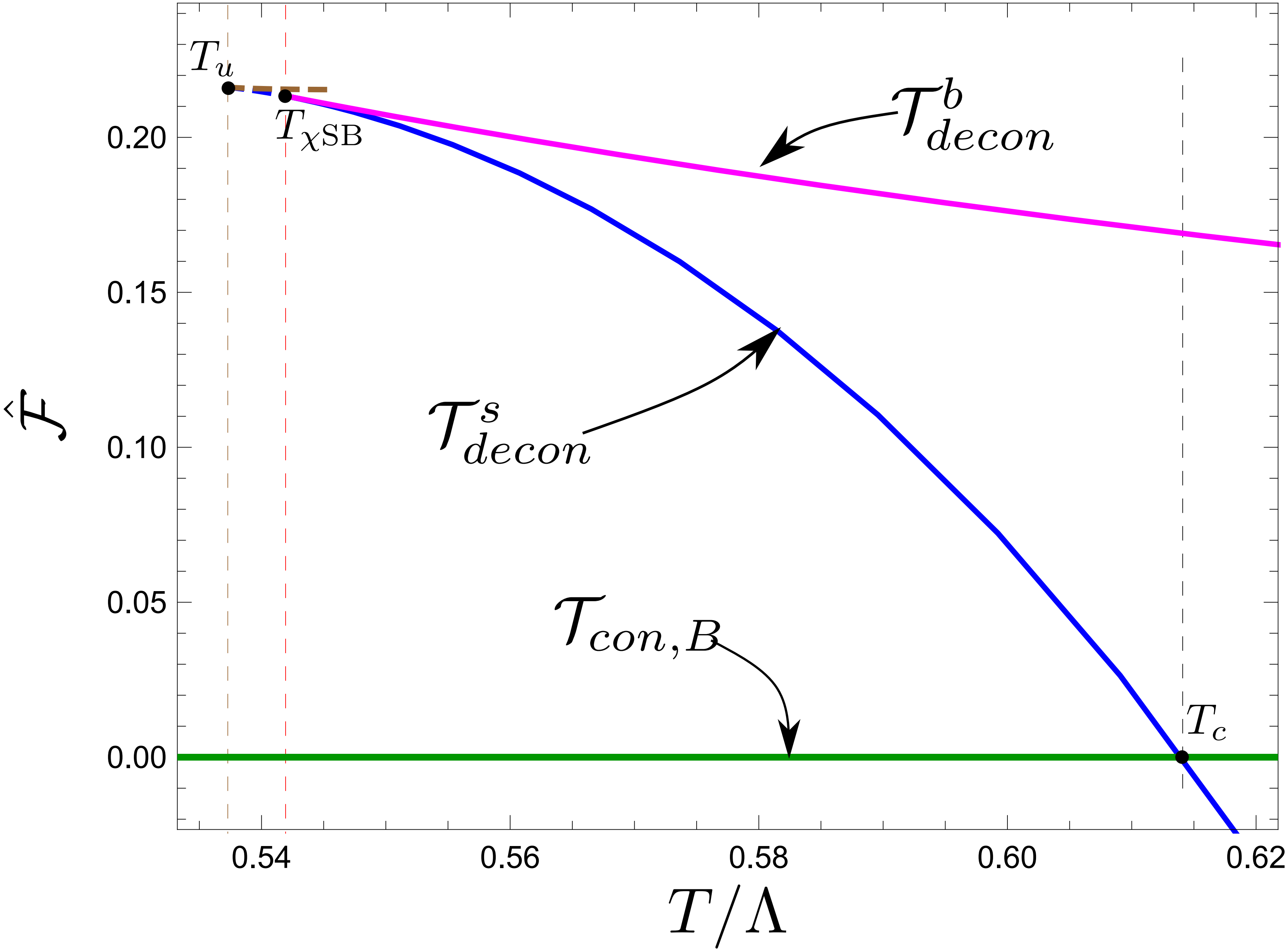
- holographic phenomenology: **anything goes!**

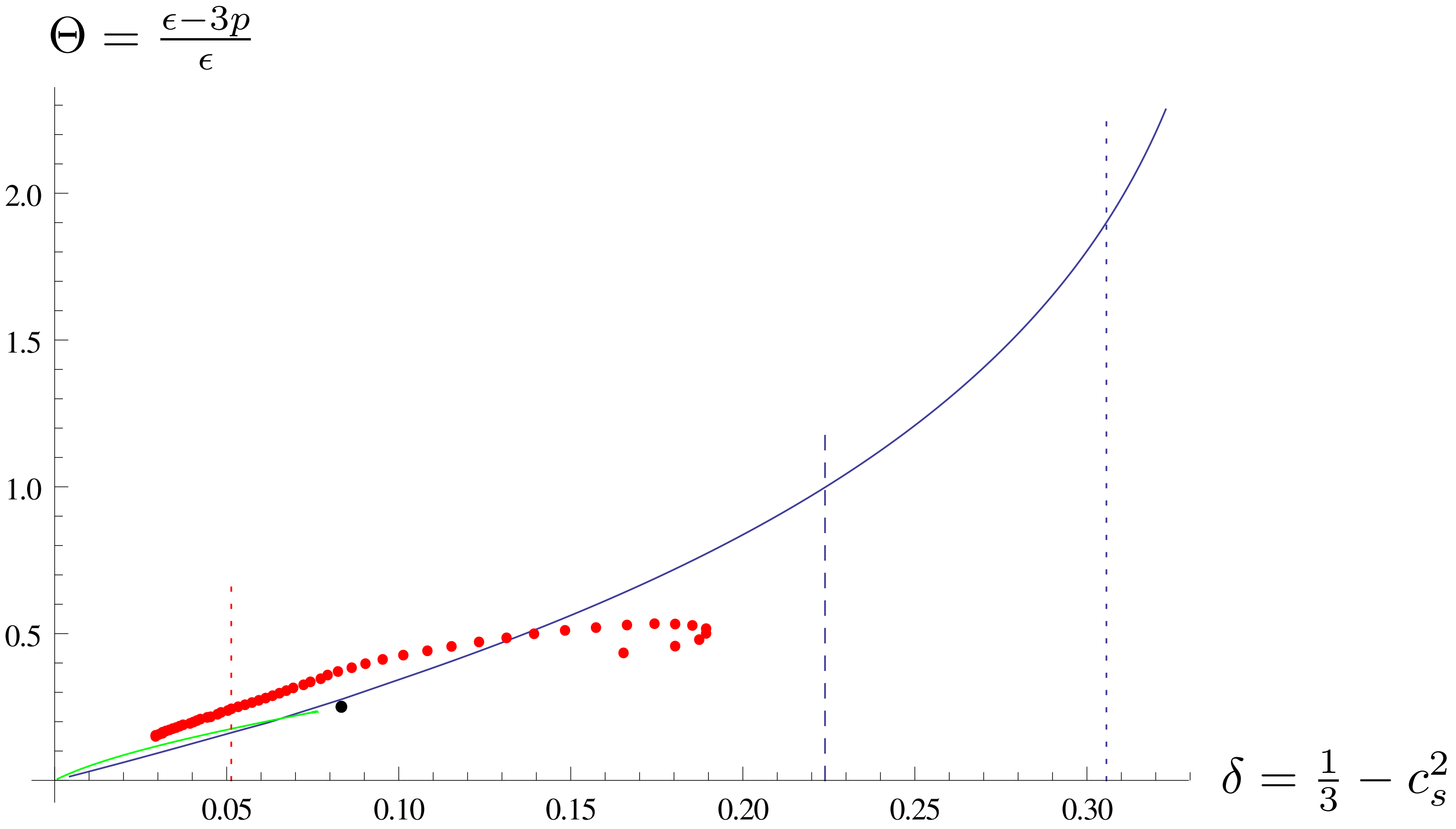
$\Longrightarrow$ From $\mathcal{N} = 2^*$ BH thermodynamics:



- (L) Trace of the energy-momentum tensor normalized to the energy density of $\mathcal{N} = 4$ SYM ($\epsilon_0 = \frac{3}{8}\pi^2 N_c^2 T^4$ with N_c denoting the number of colors) as a function of m/T. The results indicate that, thermodynamically, the effects of the conformal symmetry breaking are the strongest at $m/T \approx 4.8$.
- (R) Trace anomaly in deep IR — approach to a CFT_5

$\Longrightarrow$ From Klebanov-Strassler BH thermodynamics:





- lattice QCD (the red dots)
- $\mathcal{N} = 2^*$ (the solid green line)
- KS gauge theory (the solid blue line)
- vertical lines: $T = 0.3\text{GeV}$ (red), phase transitions in KS (blue)

$\Longrightarrow$ **Theorem:** for any model,

$$S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R + 12 - \frac{1}{2} \sum_i (\partial\phi_i)^2 - V\{\phi_i\} \right]$$

the shear viscosity is universal:

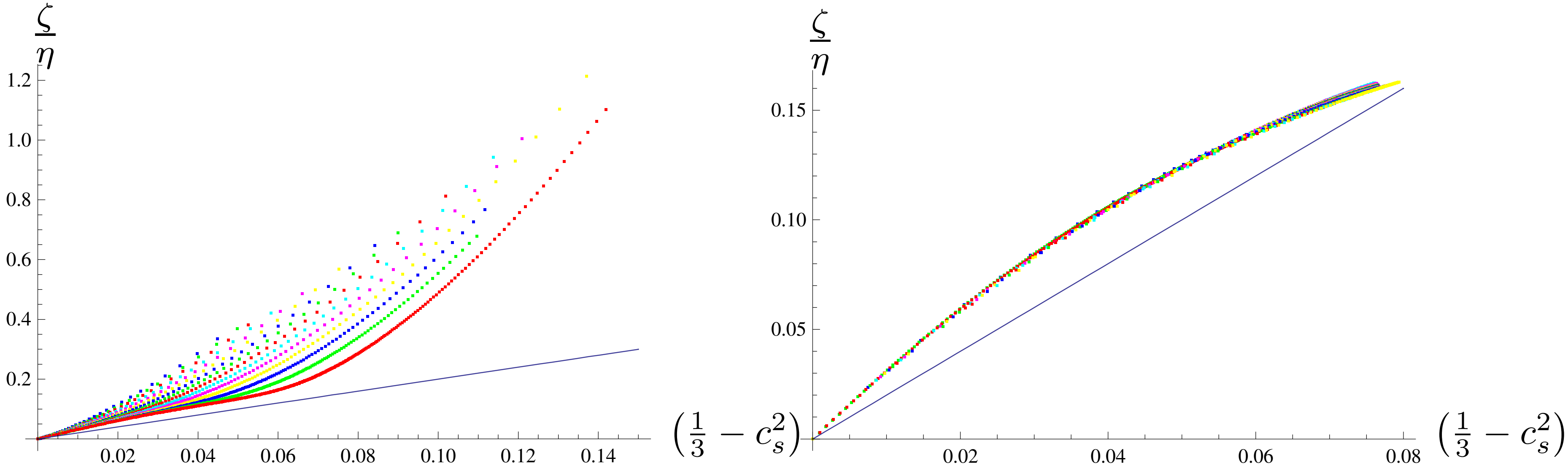
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \approx 0.08$$

- surprising, because by dimensional analysis, for a generic non-conformal model is expected to be a functional of ratios

$$\frac{\lambda_{\Delta_i}}{T^{4-\Delta_i}}$$

for every nonzero coupling $\lambda_{\Delta_i} \neq 0$ of an operator $\mathcal{O}_i$

$\Longrightarrow$ bulk viscosity in $\mathcal{N} = 2^*$:

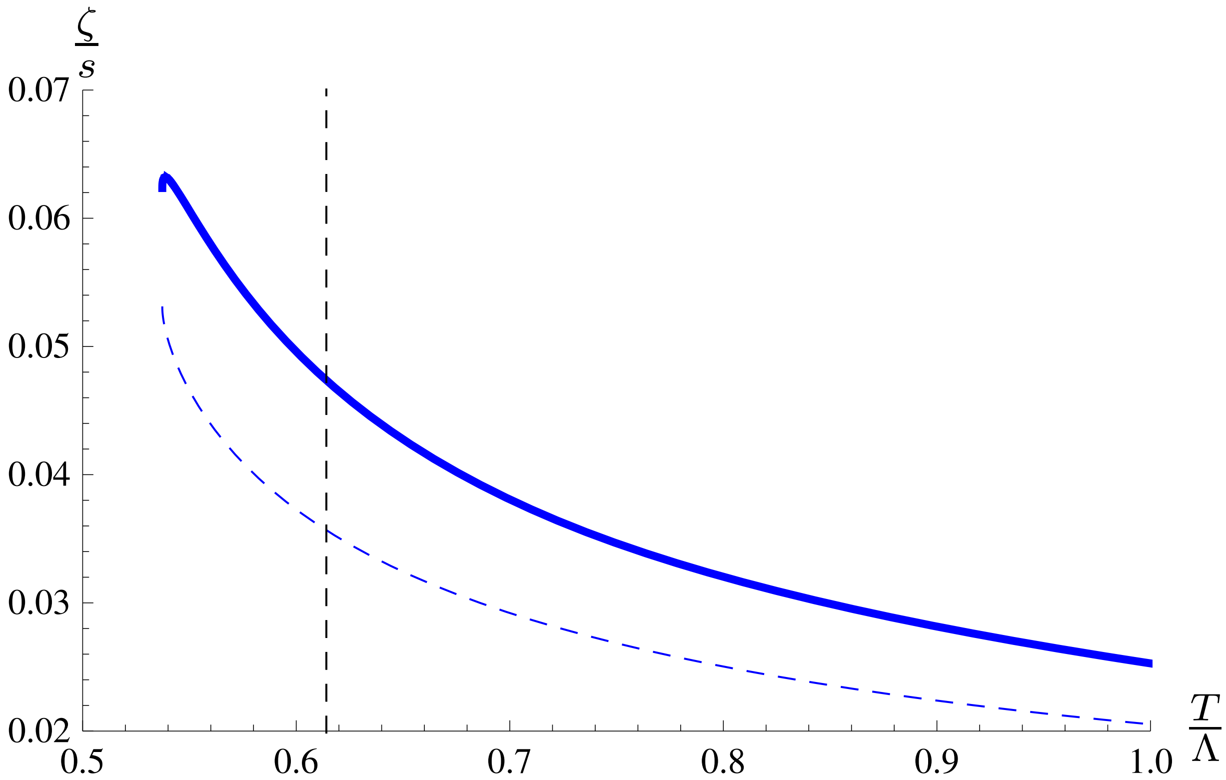


- Left: $m_f < m_b\,,\qquad \frac{m_f^2}{m_b^2} \in [0.2, 0.9]$
- Right: $m_f \ge m_b\,,\qquad \frac{m_f^2}{m_b^2} \in [1, 6]$

- The solid line represents the bulk viscosity "bound"

$$\frac{\zeta}{\eta} \ge 2\left(\frac{1}{3} - c_s^2\right)$$

$\Longrightarrow$ bulk viscosity in Klebanov-Strassler model:



- dashed blue: the bulk viscosity bound
- dashed vertical: T_c for the deconfinement transition in KS

Holographic lessons at $N = \infty$ and $\lambda = \infty$:

- $\frac{\eta}{s}$ is universal
- bulk viscosity is typically $\zeta \lesssim \eta$, becoming larger in models with stronger breaking of supersymmetry
- $$\frac{\zeta}{\eta} \sim \mathcal{O}(1) \;\times \left(\frac{1}{3} - c_s^2\right)$$

  side note: at weak coupling $\frac{\zeta}{s} \sim \left(\frac{1}{3} - c_s^2\right)^2$

QCD $\neq$ CFT! — part II

- QCD number of colors

$$\frac{1}{N} = \frac{1}{3} \neq 0$$

- QCD *'t Hooft coupling*

$$\frac{1}{\lambda_{QCD}} \neq 0$$

$\Longrightarrow$ In the past, holographic analysis in this directions were limited to conformal theories (few explicit results); technically very difficult

Beyond 2-derivative gravity-I:

$$S_5 = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R+12-\frac{1}{2}\sum_i\left(\partial\phi_i\right)^2 - V\{\phi_i\} + \underbrace{\beta\cdot\delta L}_{\rm higher-der} + \mathcal{O}(\beta^2)\right]$$

- eight-derivative curvature corrections described by:

$$\delta L_4 \equiv C^{hmnk}C_{pmnq}C_h{}^{rsp}C^q{}_{rsk} + \frac{1}{2}C^{hkmn}C_{pqmn}C_h{}^{rsp}C^q{}_{rsk}\,,$$

where C is the Weyl tensor.

▪ if the UV fixed point is $\mathcal{N}=4$ $SU(N)$ supersymmetric Yang-Mills theory with a gauge coupling g^2_{YM}, then

$$\beta \equiv \frac{1}{8}\zeta(3)\ \left(g^2_{YM}N_c\right)^{-3/2}$$

- using Kubo formulas (+ many technical tricks):
  - shear viscosity:

$$\left.\frac{\eta}{s}\right|_{\delta L_4} = \frac{1}{4\pi}\left(1 - \beta\cdot\frac{1}{72}(V-12)\left[3\sum_i(\partial_i V)^2 + 5(V-12)^2\right]\right),$$

    where $\partial_i V \equiv \frac{\partial V}{\partial \phi_i}$

  - bulk viscosity:

$$9\pi\left.\frac{\zeta}{s}\right|_{\delta L_4} = \left(1 + \frac{5}{144}\beta(V-12)^3\right)\sum_i z_{i,0}^2 - \beta\cdot\frac{5}{24}(V-12)\ \sum_i(z_{i,0}\cdot\partial_i V)^2\,.$$

    where $z_{i,0}$ are the values of the gauge invariant scalar fluctuations, at zero frequency, evaluated at the black hole horizon

$\Longrightarrow$ **anything goes** for the scalar sector of the theory

$\Longrightarrow$ old results for $\mathcal{N}=4$ SYM ($V\equiv 0$):

- shear viscosity (Buchel-Liu-Starinets):

$$\frac{\eta}{s}\bigg|_{\delta L_4}^{CFT}=\frac{1}{4\pi}\left(1+\beta\cdot 120\right)$$

- bulk viscosity (Benincasa-Buchel):

$$\frac{\zeta}{s}\bigg|_{\delta L_4}^{CFT}=\beta\cdot 0$$

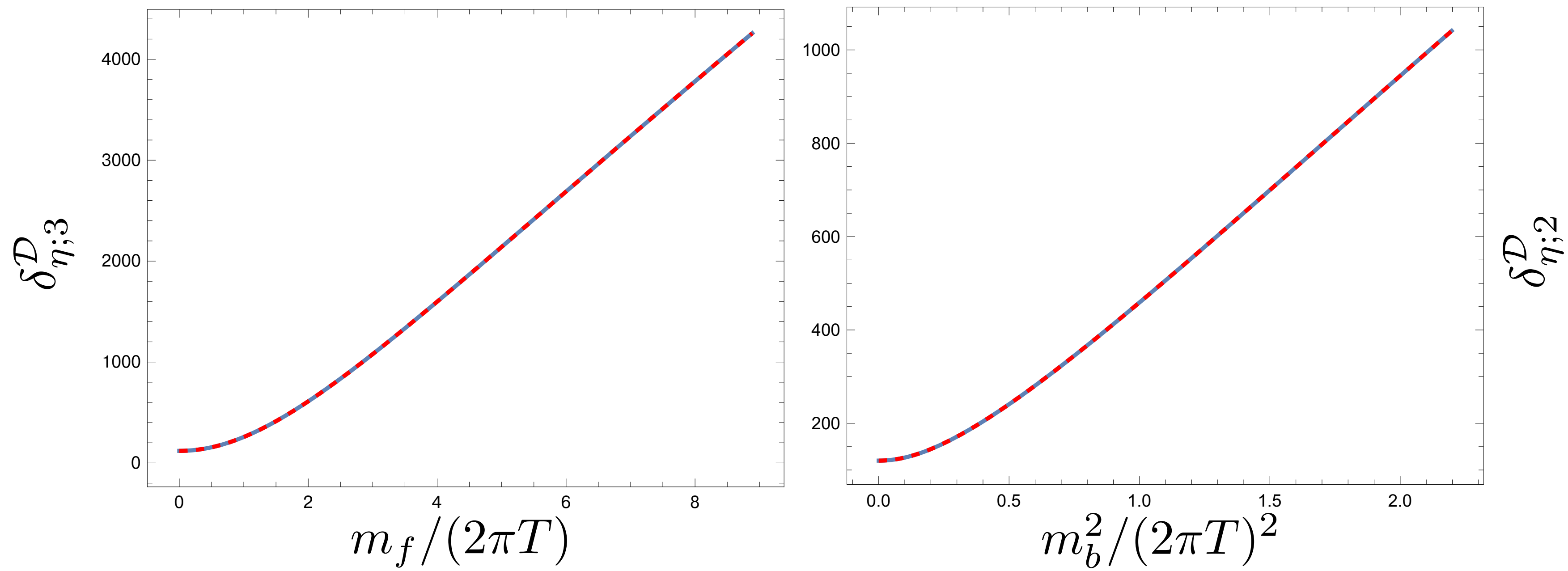
$\Longrightarrow$ no T dependence because for a conformal model no dimensional parameters

$\Longrightarrow$ $(\mathcal{D}_{4,\Delta})$: δL_4 model with

$$V = \frac{m^2}{2}\,\phi^2\,, \qquad m^2 = \Delta(\Delta-4)\,, \qquad \Delta = \{2,3\}$$

- shear viscosity:

$$\frac{\eta}{s}\bigg|_{\mathcal{D}} = \frac{1}{4\pi}\left(1+\beta\cdot\delta^{\mathcal{D}}_{\eta;\Delta}\right)$$

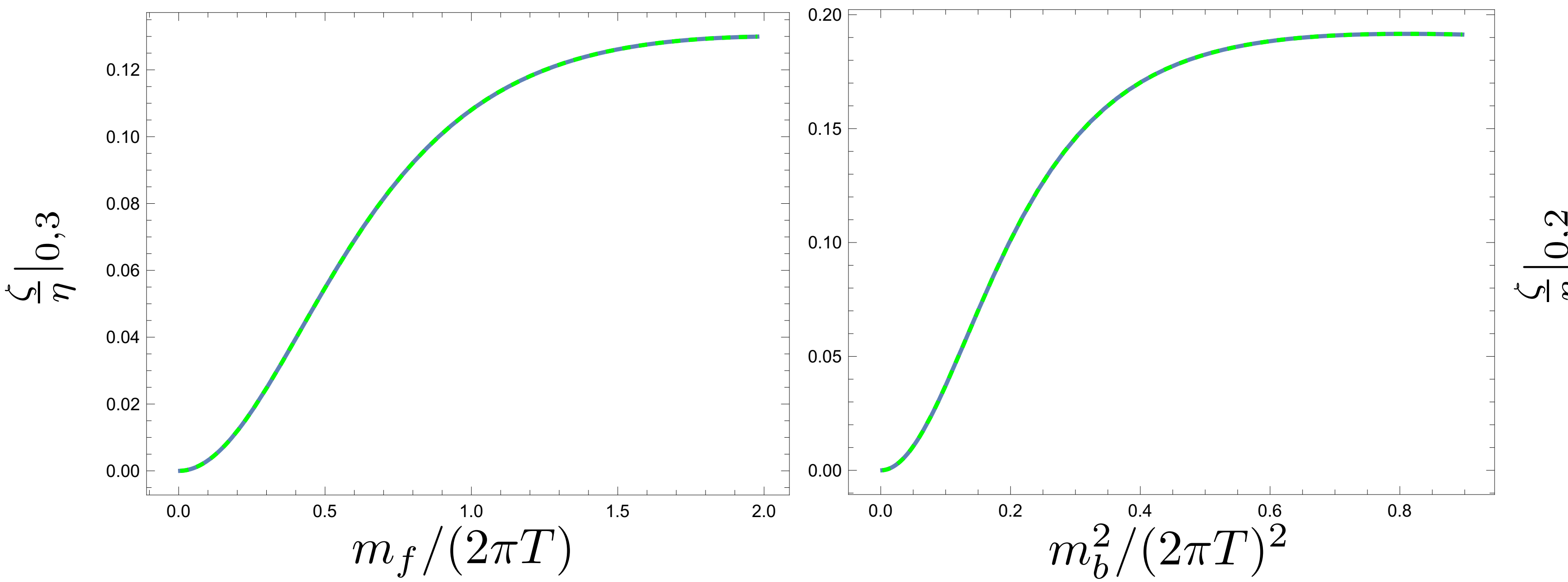


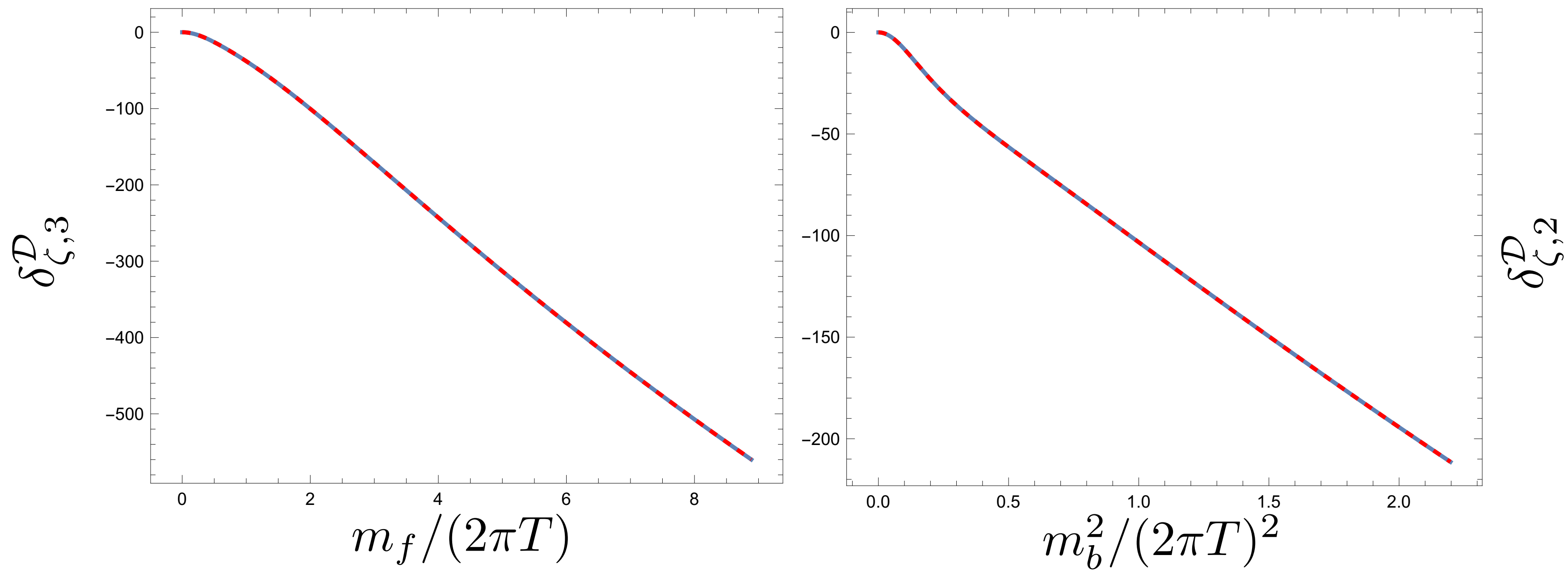
— huge sensitivity to finite coupling corrections

— solid curves on top $\Longrightarrow$ from dispersion relations

- bulk viscosity:

$$\left.\frac{\zeta}{\eta}\right|_{\mathcal{D}} = \left.\frac{\zeta}{\eta}\right|_{0,\Delta} + \beta \cdot \delta^{\mathcal{D}}_{\zeta;\Delta}$$





— huge sensitivity to finite coupling corrections

— solid curves represent corrections extracted from the sound wave channel quasinormal mode of the background black hole

Beyond 2-derivative gravity-II:

$$S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5x\sqrt{-g} \left[R + 12 - \frac{1}{2}\sum_i (\partial\phi_i)^2 - V\{\phi_i\} + \underbrace{\beta \cdot \delta L}_{\text{higher-der}} + \mathcal{O}(\beta^2)\right]$$

- four-derivative curvature corrections described by:

$$\delta L_2 \equiv \alpha_1\ R^2 + \alpha_2\ R_{\mu\nu}R^{\mu\nu} + \alpha_3\ R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}\ ;$$

▪ If c and a are the two central charges of a gauge theory UV fixed point, *i.e.*,

$$\langle T^{\mu}{}_{\mu}\rangle_{CFT} = \frac{c}{16\pi^2}\ I_4 - \frac{a}{16\pi^2} E_4$$

where $\{E_4, I_4\}$ are the Euler density and the square of Weyl curvature of $\mathcal{M}_4$,

$$\beta \cdot \alpha_3 = \frac{c-a}{8c}\ \propto\ \frac{1}{N}$$

- using Kubo formulas (+ many technical tricks):
  - shear viscosity:

$$\left.\frac{\eta}{s}\right|_{\delta L_2} = \frac{1}{4\pi}\left(1 + \beta \cdot \frac{2}{3}\alpha_3 \left(V - 12\right)\right)$$

  - bulk viscosity:

$$9\pi \left.\frac{\zeta}{s}\right|_{\delta L_2} = \left(1 - \frac{2}{3}(V - 12)(5\alpha_1 + \alpha_2 - \alpha_3)\beta\right) \sum_i z_{i,0}^2 + \beta \cdot \frac{4(5\alpha_1 + \alpha_2 - \alpha_3)}{3(V - 12)} \sum_i (z_{i,0} \cdot \partial_i V)^2$$

where $z_{i,0}$ are the values of the gauge invariant scalar fluctuations, at zero frequency, evaluated at the black hole horizon

$\Longrightarrow$ **anything goes** for the scalar sector of the theory

$\Longrightarrow$ old results for $\mathcal{N} = 2$ $Sp(N)$ SYM + matter ($V \equiv 0$):

- shear viscosity (Katz-Petrov):

$$\left.\frac{\eta}{s}\right|_{\delta L_2}^{CFT} = \frac{1}{4\pi}\left(1 - \beta \cdot 8\alpha_3\right)$$

- bulk viscosity:

$$\left.\frac{\zeta}{s}\right|_{\delta L_2}^{CFT} = \beta \cdot 0$$

$\Longrightarrow$ no T dependence because for a conformal model no dimensional parameters

$\Longrightarrow$ $(\mathcal{A}_{2,\Delta}, \mathcal{B}_{2,\Delta}, \mathcal{C}_{2,\Delta})$: δL_2 models with

$$V = \frac{m^2}{2}\,\phi^2\,, \qquad m^2 = \Delta(\Delta-4)\,, \qquad \Delta = \{2,3\}$$

- $\mathcal{A}_{2,\Delta}$:

$$\{\alpha_1, \alpha_2, \alpha_3\} = \{0, 0, 1\}$$

  - higher-derivative in the bulk, and at the horizon: $s \neq \frac{\text{area density}}{4G_5}$

- $\mathcal{B}_{2,\Delta}$:

$$\{\alpha_1, \alpha_2, \alpha_3\} = \{1, -4, 1\}$$

  - Gauss-Bonnet model $\Longrightarrow$ 2-derivative gravity; $s = \frac{\text{area density}}{4G_5}$

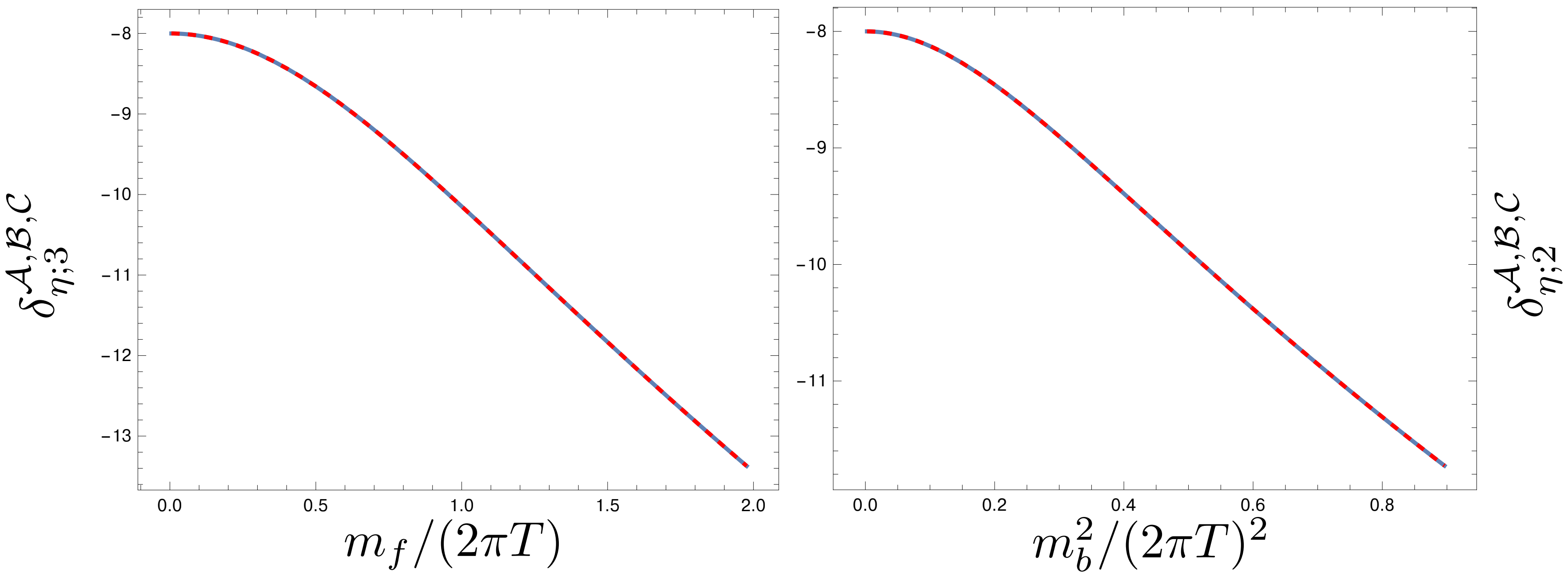
- $\mathcal{C}_{2,\Delta}$:

$$\{\alpha_1, \alpha_2, \alpha_3\} = \{0, 1, 1\}$$

  - higher-derivative in the bulk; effectively 2-derivative at the horizon, $s = \frac{\text{area density}}{4G_5}$

- shear viscosity:

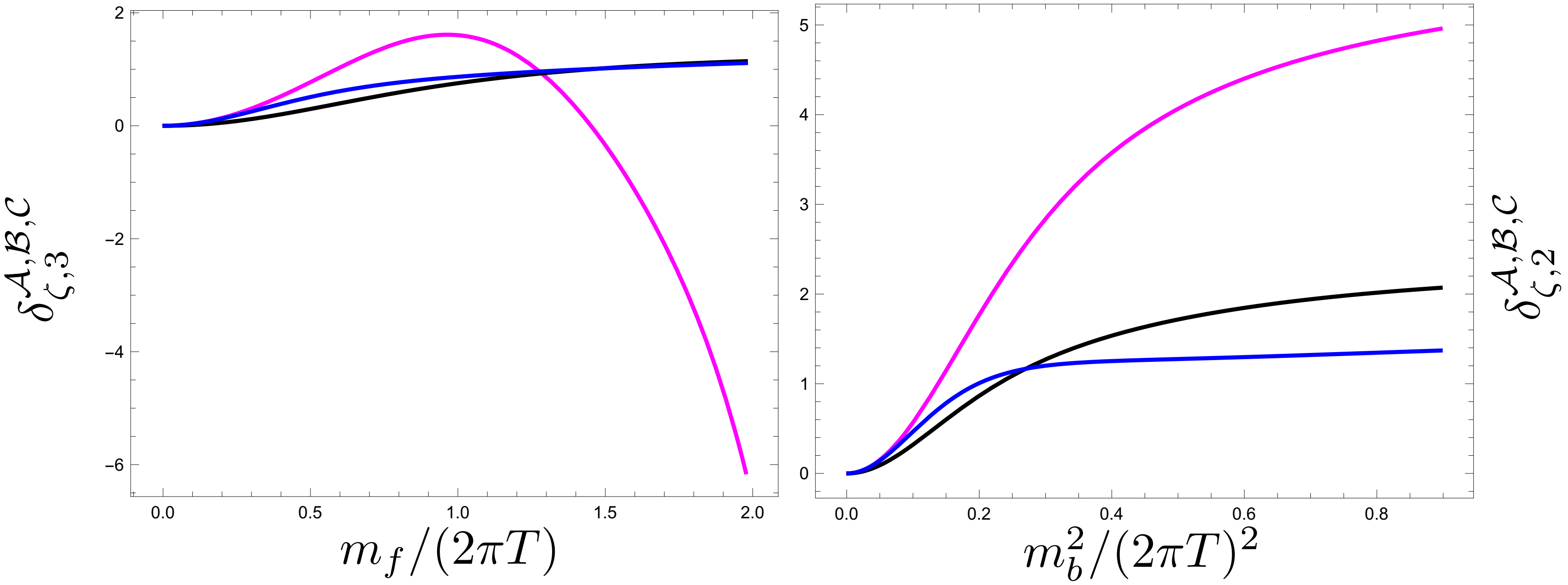
$$\left.\frac{\eta}{s}\right|_{\mathcal{A},\mathcal{B},\mathcal{C}} = \frac{1}{4\pi}\left(1+\beta\cdot\delta_{\eta;\Delta}^{\mathcal{A},\mathcal{B},\mathcal{C}}\right)$$



— all models have the same α_3; $\frac{\eta}{s}$ is insensitive to $\{\alpha_1,\alpha_2\}$
but the entropy density is sensitive!

- bulk viscosity:

$$\frac{\zeta}{\eta}\bigg|_{\mathcal{A},\mathcal{B},\mathcal{C}} = \frac{\zeta}{\eta}\bigg|_{0,\Delta} + \beta \cdot \delta_{\zeta;\Delta}^{\mathcal{A},\mathcal{B},\mathcal{C}}$$



— black curves represent $\mathcal{A}_{2,\Delta}$ models

— blue curves represent $\mathcal{B}_{2,\Delta}$ models

— magenta curves represent $\mathcal{C}_{2,\Delta}$ models

Beyond 2-derivative gravity-III:

While in general:

$$S_5 = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R+12-\frac{1}{2}\sum_i\left(\partial\phi_i\right)^2-V\{\phi_i\}+\underbrace{\beta\cdot\delta L_2}_{\rm higher-der}+\mathcal{O}(\beta^2)\right]$$

for $\mathcal{B}_{2,\Delta}$ models:

$$S_5 = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R+12-\frac{1}{2}\sum_i\left(\partial\phi_i\right)^2-V\{\phi_i\}+\underbrace{\beta\cdot\delta L_2^{(1,-4,1)}}_{\rm Gauss-Bonnet}\right.$$
$$\left.+\cancel{\mathcal{O}(\beta^2)}\right]$$

$\Longrightarrow$ In the past, for holographic CFTS, *i.e.*, $V\equiv 0$, causality of the theory constrains (Maldacena-Hofman & Buchel-Myers):

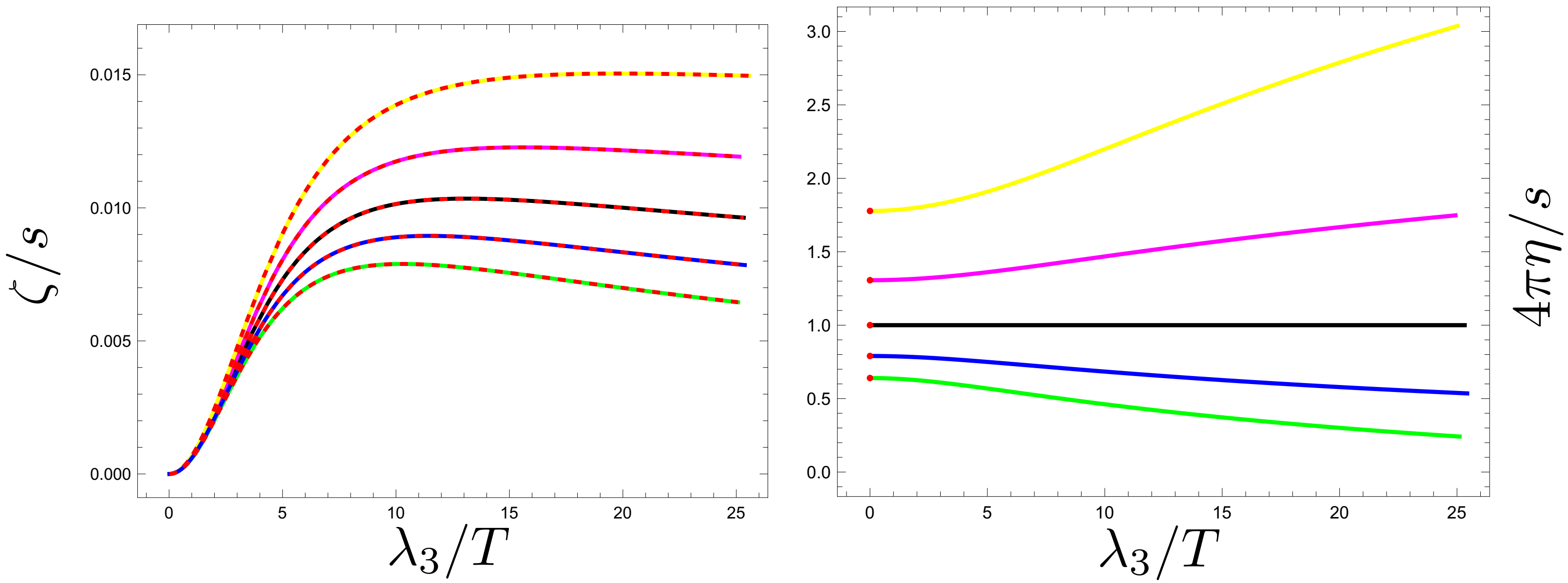
$$-\frac{7}{72}\le\beta\le\frac{9}{200}\qquad\Longleftrightarrow\qquad -\frac{1}{2}\le\frac{c-a}{c}\le\frac{1}{2}$$

$\Longrightarrow$ (Brigante et.al):

$$\frac{4\pi\eta}{s}=1+\beta\cdot\frac{2}{3}\underbrace{\alpha_3}_{=1}\Big(\underbrace{V}_{\equiv 0}-12\Big)=1-8\beta\ \le\ \frac{16}{25}$$

$\Longrightarrow$ Consider a single scalar field model with a potential

$$V = \frac{m^2}{2}\phi^2 \,, \qquad \Delta = 3$$



The color coding of the solid curves is as in

$$c - a/c \;=\; \{-1/2\,,\, -1/4\,,\, 0\,,\, 1/4\,,\, 1/2\}\,.$$

Note that $\frac{4\pi\eta}{s}$ ratio (the green curve) is always below the minimal conformal value

$$\eta/s \;\le\; 1/(4\pi)\cdot 16/25$$

Holographic lessons at $\frac{1}{N} \neq 0$ and $\frac{1}{\lambda} \neq 0$:

- $\frac{\eta}{s}$ is no universal, and does depend on T
- there is no lower bound on $\frac{\eta}{s}$
- there is a huge sensitivity to finite-N, finite-λ corrections
- sensitivity increases as ever higher-derivative corrections are included in the effective dual gravitational action

$\Longrightarrow$ Straightforward to extend:

- more general higher-derivative corrections
- charged gauge theory plasma

Future directions:

- Holography is a "black box" that produces EOS + transport
  $\Longrightarrow$
  produces **training data** for DL NN

- utilize this data to study:
  - EOS/transport correlations
  - reconstructing gravitational dual from lattice QCD EOS
  - string theory vs. phenomenological models

Help wanted: experts in PINNs