Necessary ingredients toward a truly model-independent description of light and medium mass nuclei

INT workshop

Chiral EFT: New Perspectives

Chieh-Jen (Jerry) Yang

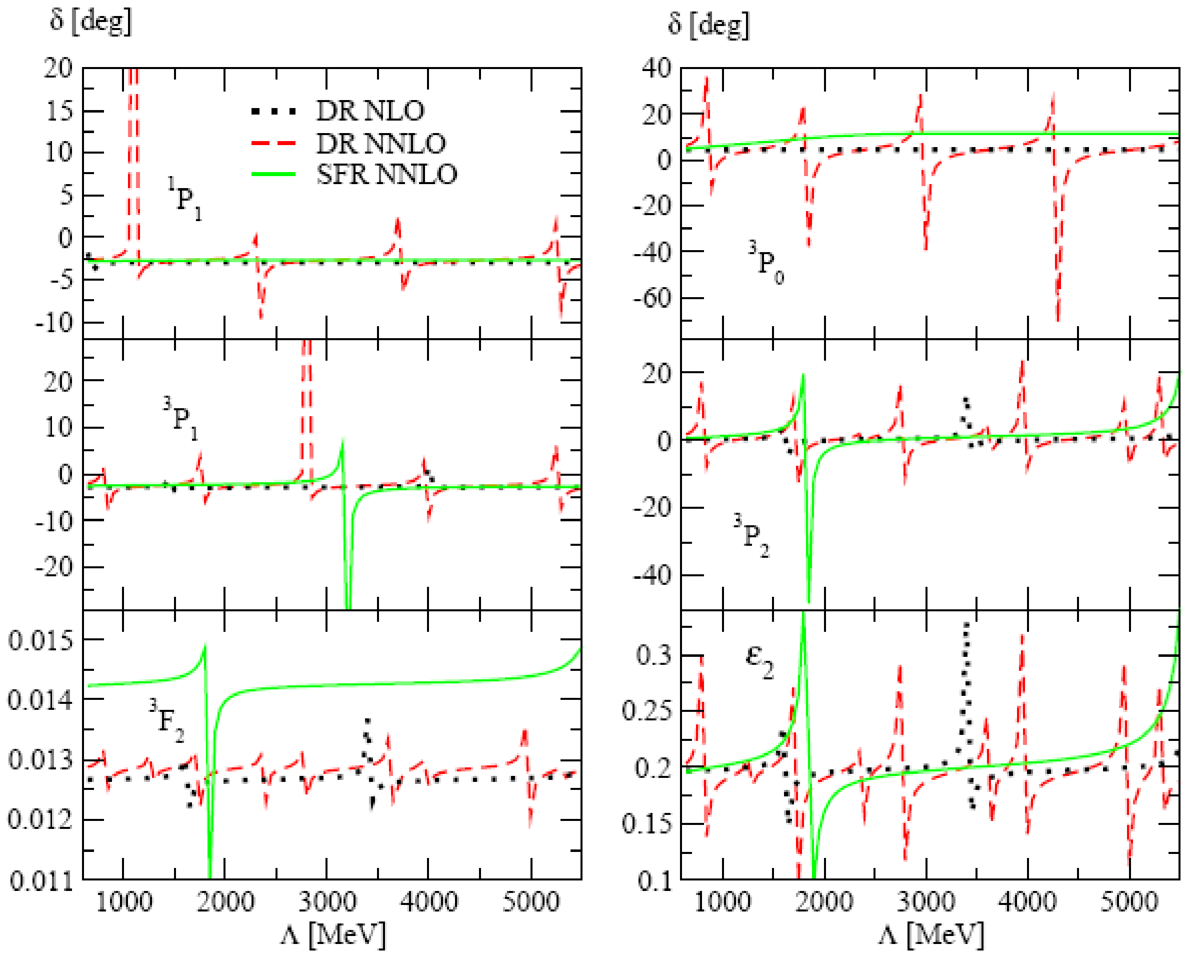
March 19, 2025

nuclear physics

Problems of WPC

WPC is wrong at LO ! (Nogga, Timmermans, van Kolck, *PRC 72 (2005) 054006*)

- Beyond LO: (Yang, Elster, Phillips (2008-2010))



Same story for:
N3LO WPC

Ch. Zeoli, R. Machleidt and D. R. Entem, Few-body syst., 54, 12, 2191 (2012)

In short, WPC might be WPP (pragmatic proposal)

(many in-debate issues, but not the topic today)

More details/opinions could be found in:

Few Body Syst. 62 (2021) 4, 85 and Few Body Syst. 63, no.2, 44 (2022)

Nuclear Effective Field Theories: Reverberations of the early days

U. van Kolck

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July 27, 2021

What Can Possibly Go Wrong?

Harald. W. Grießhammer

Received: date / Accepted: date

Abstract A lot.

The cause of the problem

1. The solution of Schrodinger Eq. with a singular V(~$1/r^n$ with $n \geq 2$) requires suitable boundary conditions (BC) to be meaningful.

2. However, such BC-requirements ***contradict*** with contact terms given by WPC.

3. At each order: BC-requirements ≠ contact terms within WPC.

Root: mathematically they don't match!

This is **not** going to be solved by including higher-order terms in WPC!
(though at NN-level, the problem might be tamed by applying a soft-cutoff $\Lambda \lesssim M_{hi}$)

Keep this in mind

The cause of the problem (of WPC)

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Fully non-perturbative treatment of V

solution

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Keep this in mind

~~**Fully non-perturbative treatment of V**~~

Logical solution: Don't do that, do **DWBA** instead!

By doing that, need to modify NDA also.

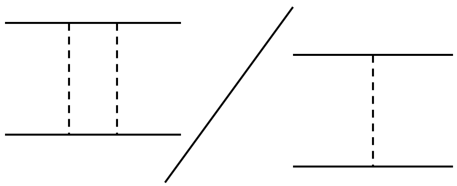
NDA is just an estimate, not always works (c.f. Martin's Monday talk for 1 baryon)

New power counting Decided by RG

Long & Yang, (2010-2012)

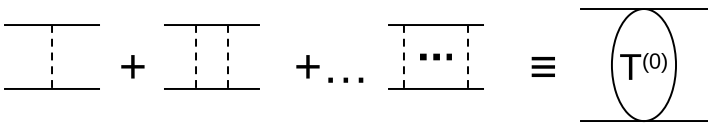
LO: Still iterate to all order (at least for *l*<2).

Reason: van Kolck, Bedaque,… etc.



$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$

Thus, O(Q⁰):

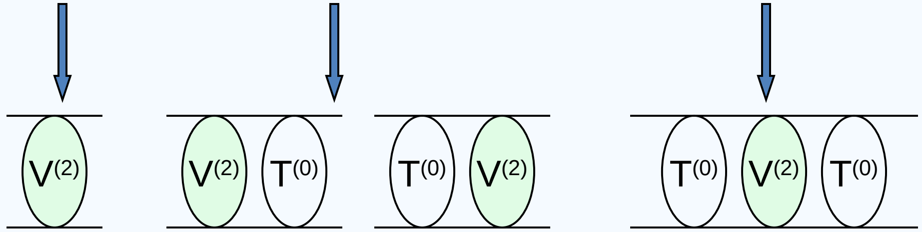


Start at NLO, do perturbation.

$(T = T^{(0)} + T^{(1)} + T^{(2)} + T^{(3)} + \ldots)$

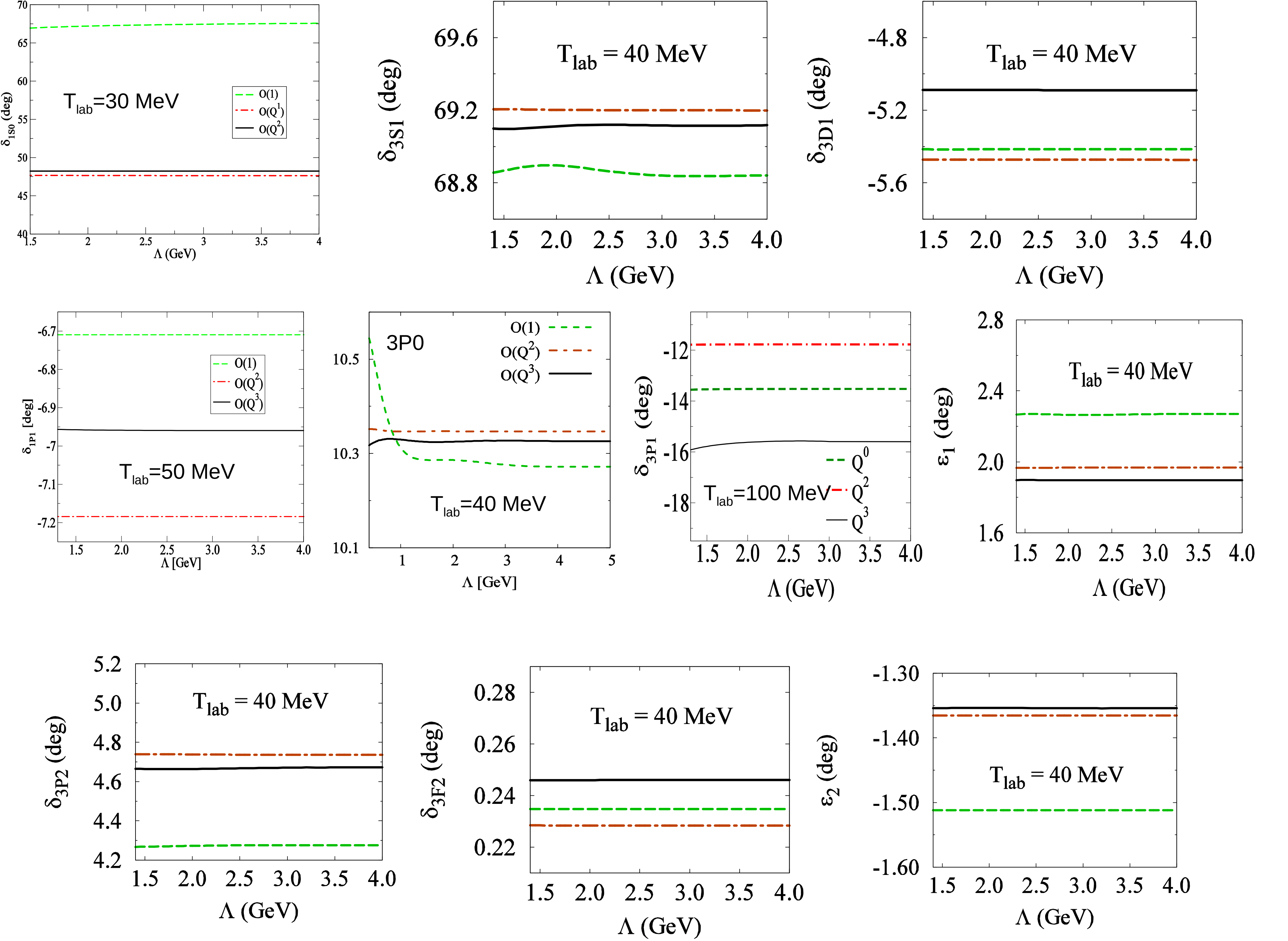
If $V^{(1)}$ is absent:

$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$



$$G \equiv \frac{2M_N}{\pi} \int_0^\Lambda \frac{p^2 dp}{p_0^2 - p^2 + i\varepsilon}$$

$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$



T_lab=30 MeV
O(1)
O(Q^1)
O(Q^2)
δ_1S0 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3S1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3D1 (deg)
Λ (GeV)
O(1)
O(Q^2)
O(Q^3)
T_lab=50 MeV
δ_1P1 [deg]
Λ [GeV]
3P0
O(1)
O(Q^2)
O(Q^3)
T_lab=40 MeV
Λ [GeV]
δ_3P1 (deg)
Q^0
Q^2
Q^3
T_lab=100 MeV
Λ (GeV)
T_lab = 40 MeV
ε_1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3P2 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3F2 (deg)
Λ (GeV)
T_lab = 40 MeV
ε_2 (deg)
Λ (GeV)

A side note: be careful when renormalizing under DWBA

C.-J. Yang, arXiv:2410.08845 [nucl-th]

- |WF>=|WF_LO>+|WF_NLO>+…

- So when renormalizing the perturbative contribution, .e.g, <WF_LO|Contact term|WF_LO>, demanding an exact accuracy to fix LECs at particular E could lead to contradictions.

Problematic Λ_*, where $E^* \in p_{tye}$

(Extremely narrow window)

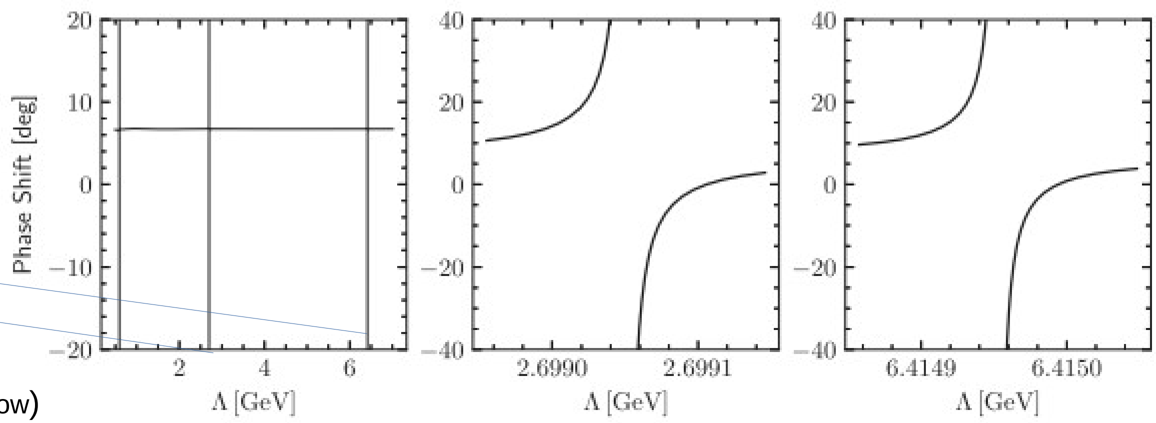


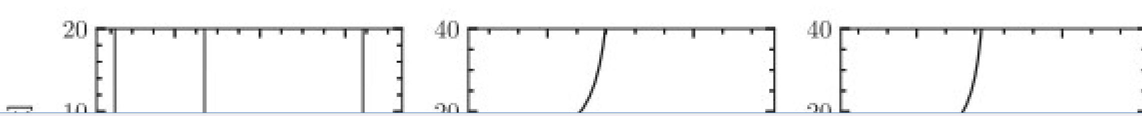
FIG. 5. Cutoff dependence of $^3P_0$ phase shift calculated at the fixed laboratory energy of $T_{lab} = 130$ MeV using the approach of Ref. [30] at NLO. The middle and right panels show zoomed regions in the vicinity of two exceptional cutoffs.

A.M. Gasparyan, E. Epelbaum, PRC 107, 034001 (2023)

One needs to be careful when renormalizing under DWBA

- |WF>=|WF_LO>+|WF_NLO>+…

- So when renormalizing the perturbative contribution, .e.g, <WF_LO|Contact term|WF_LO>, demanding an exact accuracy to fix LECs at particular E could lead to contradictions.



Problematic Λ_*,
where $E^* \in p_{tye}$

(Extremely narrow

at

Root of the problem (**nothing to do with PC**, but a general feature of perturbative corrections)

This happens because you force the corrections under DWBA
(or EFT results truncated at finite-order)
to have an accuracy exceeding what they should be.

C.-J. Yang, arXiv:2410.08845 [nucl-th]

For PC of Long & Yang

- Adopting $xf_a(\Lambda)+(1-x)f_b(\Lambda+\Lambda/1000)$ (or: f_a sharp cutoff, f_b as a super-gaussian) solves the issue.

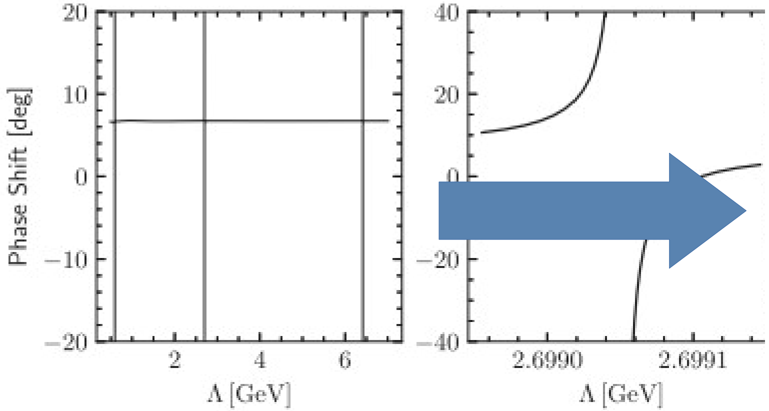
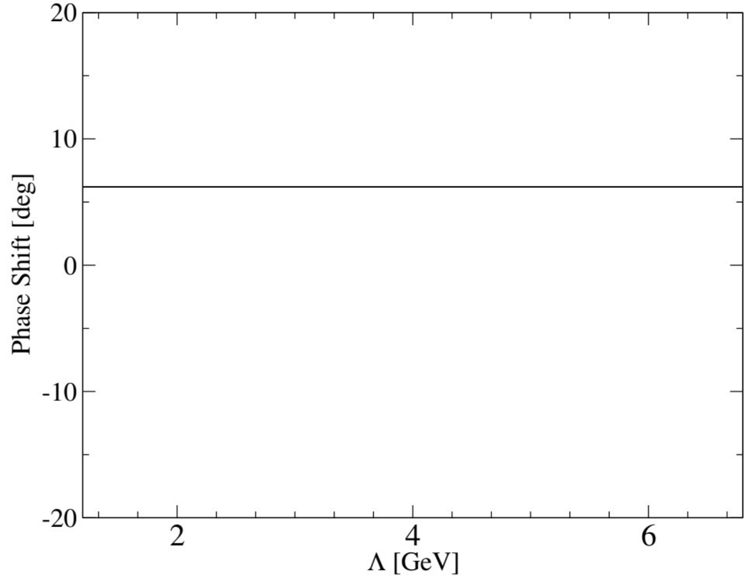


FIG. 5. Cutoff dependence of $^3P_0$ phase shift calculated at the fixed laboratory energy at NLO. The middle and right panels show zoomed regions in the vicinity of two exceptior



C.-J. Yang, arXiv:2410.08845 [nucl-th]

So RG is satisfied at NN-level now

Question: Is this behavior extendable from interaction-level to all nuclei?

Answer: In principle yes, but...

One example (Pionless EFT)

NN-only → fine for deutron
→ Thomas collapsing for triton

Heuristic reason: (U. van Kolck, Les Houches Lectures on Effective Field Theories for Nuclear and (some) Atomic Physics 2019):

Number of pairs of two-body interactions in an A-body system is A(A-1)/2, while the corresponding appearance of the kinetic term is A-1 (one of the kinetic terms goes into the total c.m. of the system).

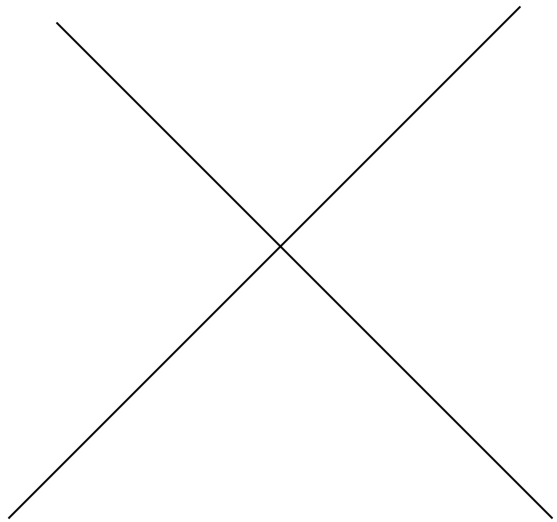
For A=3, the interaction pairs consist of v12, v23 and v13, but are only accompanied by two kinetic terms. The extra pair of the purely attractive interaction (becomes sharp δ at $\Lambda \to \infty$) causes the system to collapse.

Solution without destroying PC at NN → Promote **3NF** to LO!

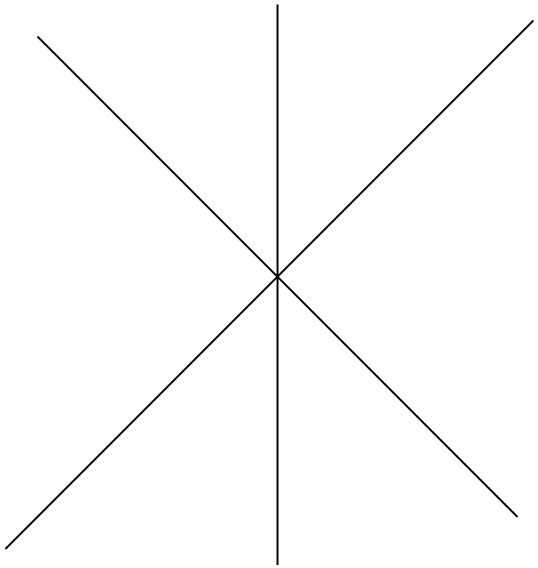
↓

RG-based promotion

Naïve dimensional analysis (NDA)



2 nucleon force



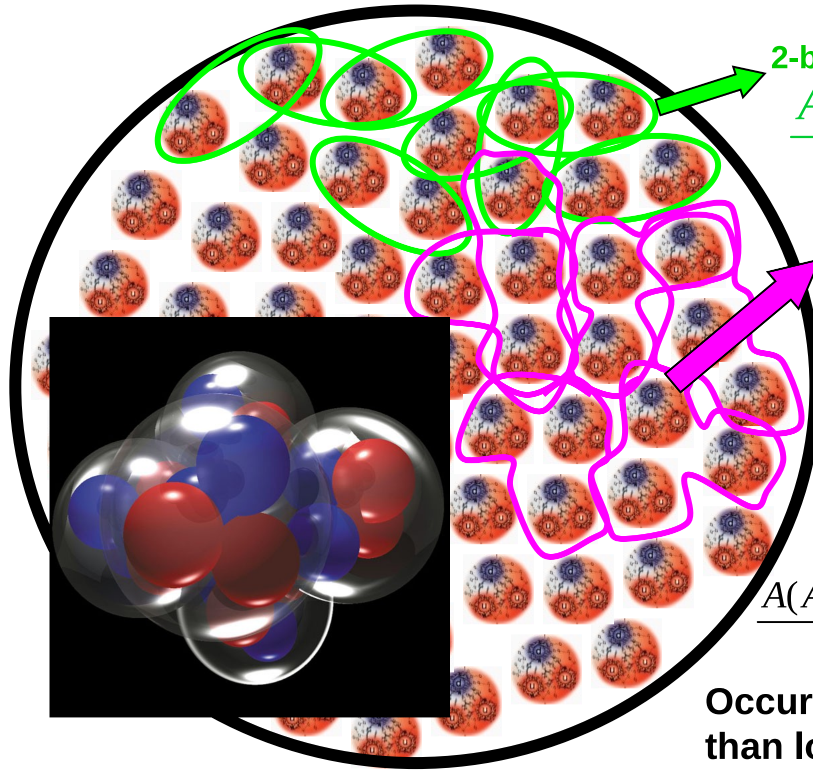
3 nucleon force

$$\text{3NFs/2NFs} \approx \frac{N^{+}N}{f_{\pi}^{2}M_{hi}} \approx \frac{\rho_0}{93^2 \cdot 500} \approx 0.28$$

However, NDA doesn’t take A into account!

Many-body forces in complex systems

- Some of many-body couplings are genuine and unknown, i.e., cannot be derived from NN couplings.
- They are estimated to be weaker by naïve dimension analysis (NDA).
- However, their importance can grow in a large system.



2-body pairs

$$\frac{A(A-1)}{2!}$$

3-body subsets

$$\frac{A(A-1)(A-2)}{3!}$$

:

:

***n*-body subsets**

$$\frac{A(A-1)\cdots(A-n+1)}{n!} \equiv C_n^A$$

Occurs more frequently than lower-body forces !

A	number of doublets $\frac{A(A-1)}{2}$	number of triplets $\frac{A(A-1)(A-2)}{6}$
3	3	1
4	6	4
5	10	10
6	15	20

C.-J. Yang, Eur.Phys.J.A 56 (2020) 3, 96

“A choose n” enhancements

$$C_n^A = \frac{A(A-1)(A-2)\ldots(A-n+1)}{n!}$$

- In a self-bound system, the above enhancement won’t be fully counted. For example, an n-body subset will have nearly zero contribution if its constituents span a distance much larger than the range of the n-body forces.→ density saturates, not $\rightarrow \infty$.

- On the other hand, those small contributions could still add up to become sizable, due to the fact that there are many of them.

- Thus, the growth of n-body forces in large systems depends on multiple factors such as the **range** and the **form** of interactions, the mass of particles, etc., ➔ **Require actual ab-initio calculations to check the PC.**

Another ingredient of promoting 3NF and 4NF

- Combine NDA and "A choose n":

Combine both:

Approx. with nuclear saturation density

$$\frac{C_n^A F_n}{C_m^A F_m} = \frac{A-m}{n}\left(\frac{\rho_0}{f_\pi^2 M_{\rm hi}}\right)^{n-m} = \frac{A-m}{n}\left(\frac{142(\rm MeV)}{M_{\rm hi}}\right)^{n-m}$$

~1

NN and NNN becomes the same important starting from **A=13-26** (M_{hi}=500-1000 MeV)

*NNN and NNNN becomes the same important starting from A=17-34.

*5^+-body force is more suppressed (s≥1), only equal to NNNN after A>500.

As nuclear forces are short-range, the enhancement can be weaker.

Opposite opinions (from various resources)

1. Double count the combinatorial factor?

P. Navrátil, G. P. Kamuntavičius, and B. R. Barrett, Phys. Rev. C 61, 044001(2000).

(A2) $$\left\langle \sum_{i<j=1}^{A} V_{ij} \right\rangle = \frac{1}{2}A(A-1)\langle V(\sqrt{2}\vec{\eta}_{A-1})\rangle$$ Total from NN=(combinatorial factor)*(V_{NN})

One should arrange (power count) on this!

Arranged by NDA

(A4) $$\left\langle \sum_{i<j<k=1}^{A} V_{ijk} \right\rangle = \frac{1}{6}A(A-1)(A-2)\langle V(\vec{\vartheta}_{A-2},\vec{\eta}_{A-1})\rangle$$ Total from NNN=(combinatorial factor)*(V_{NNN})

So, No double counting!

2. Nucleons only interact with nearby nucleons (i.e., the factor is there, but is weaken to a negligible degree)

=> Model space to converge ab-initio ≉ Hartree-Fock → The impact of not nearby interaction in nuclear binding will be ≳ the size of |(converged result) – (HF)|.

=> Compare **the same weakening** in NN to NNN (i.e., weakening also applies to NN).

=> The growing of NNN does stop at saturation (A≈56), with the exception of extreme conditions (e.g., the core of a neutron star).

3. Not enough evidence (e.g., **Bayesian** analysis on WPC does not see such a need).

=> So far it also says **WPC is o.k.** on almost everything (if Λ is restricted).

=> The wrong pole at LO without NNN **only shows up when Λ>500 MeV**.

Heuristic opposite opinions (regarding the sign of 3NF)

1. Normally the contribution of 3NF is repulsive (to prevent Thomas collapsing for triton).
But for Λ>600 MeV, you need overall attractive 3NF. Maybe a too high Λ causes the LO 2NF (i.e., OPE) to lose "the range".

2. Interactions within EDF have repulsive 3-body/density-dep term.

=> Thus the problem is mainly 2NF instead of 3NF.

Answer to opposite opinions:

1. EDF stops at HF, i.e., finite and low-Λ. For low-Λ, 3NF indeed provides repulsion (which is the case for our Λ=450 MeV), but it needs not to keep the sign always.

2. In EFT everything runs with cutoff. LECs could run from -∞ to ∞ (limit-cycle) across Λ. Since LECs in both 2NF and 3NF run with Λ, there will be a "good" cutoff/regulator that the effect of cD, cE cancels with each other and minimizes 3NF → traditionally accounted that "*cutoff/regulator/things providing the range*" into V_{nn}.

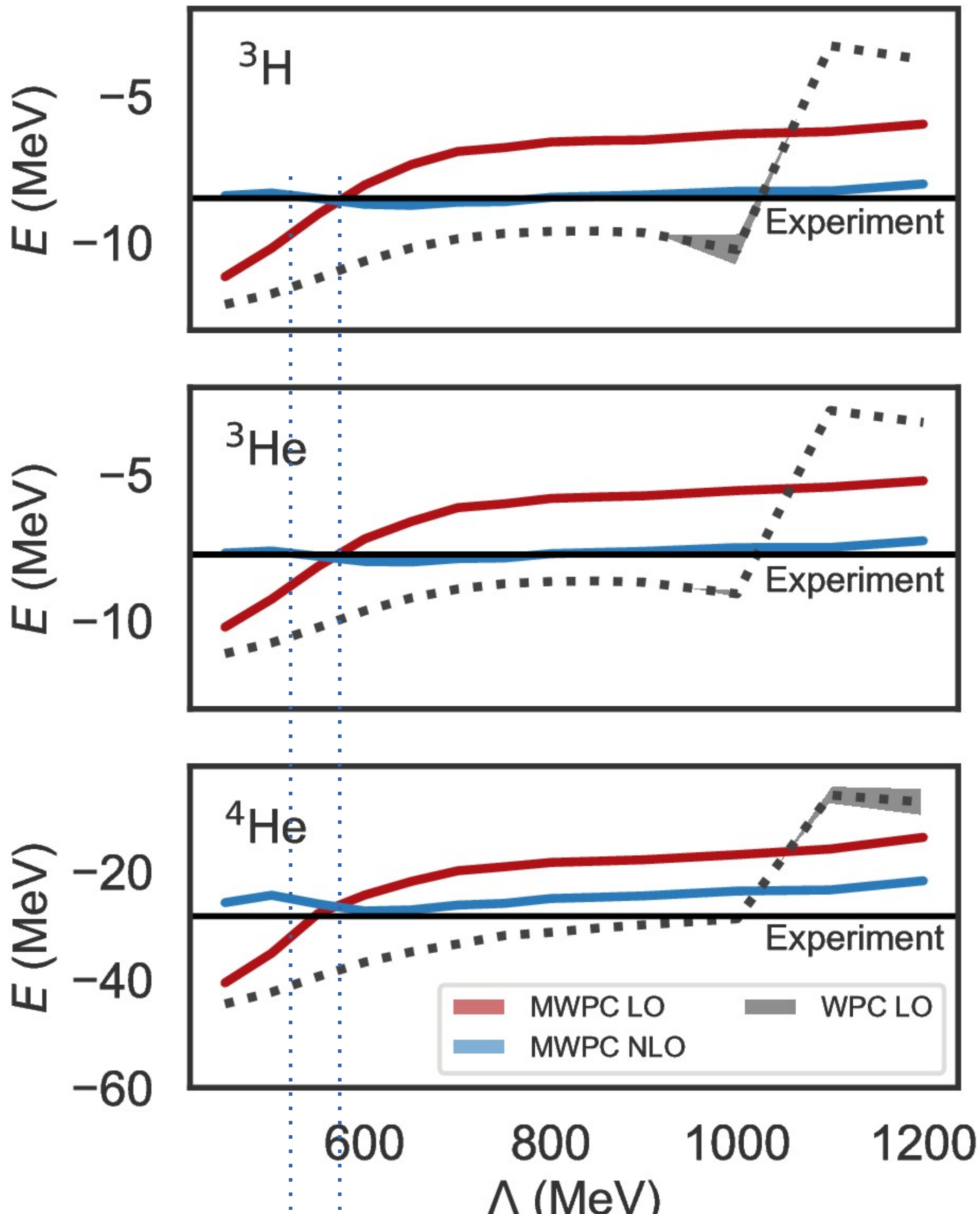
3. So the "imperfect feature" in the 2NF (including "the range") are interchangeable with 3NF, at least to certain degree (c.f. density-dep t3-term in Skyrme).

Trust, but verify

Test NN-only for A=2,3,4. See if they converged reasonably to exp. data.

Let’s start from light systems: where 3NFs are small

Use only 2NF up to next-to leading order, do ^{3}H, ^{3}He, ^{4}He



“Good” cutoff: ≈450-500 MeV

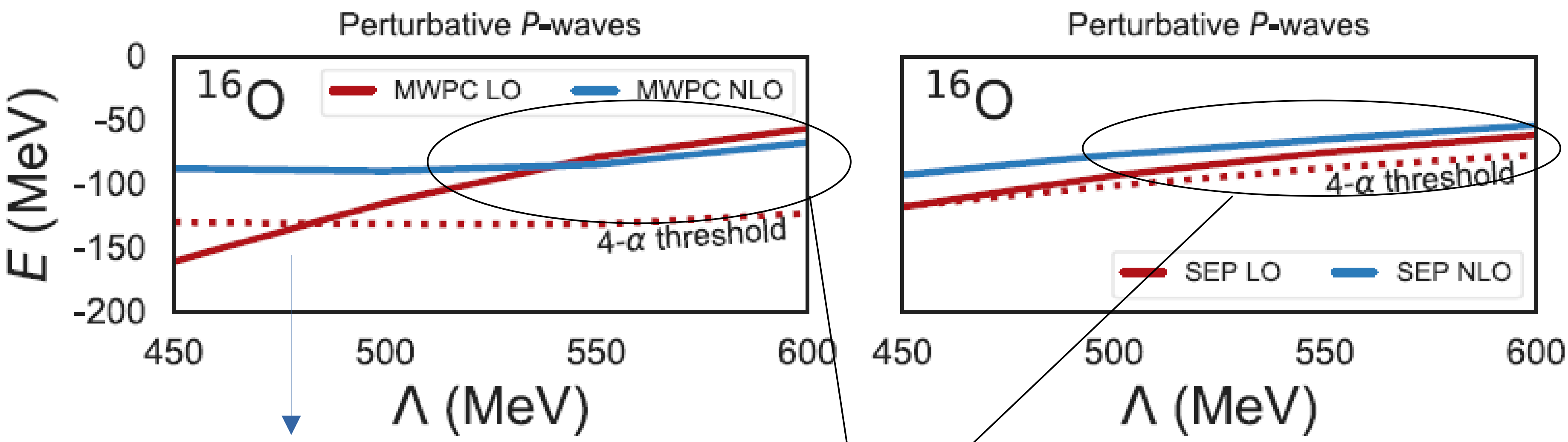
Conclusion:
2NFs up to NLO works
for A≤4 systems.

C.-J. Yang, A. Ekstrom, C. Forssen, G. Hagen,
PRC 103 (2021) 5, 054304.

For A up to 3 see also:
Nogga et al, PRC 72 (2005), 054006
Song et al, PRC 96 (2017), 024002.

So far so good, let’s increase to A=16

^{16}O results (LO, NN only)



"Good" cutoff: ≈450-500 MeV

^{16}O non-physical !

MWPC:

At LO, Nogga, Timmerman, van Kolck PC
(*Phys.Rev.C 72 (2005) 054006)*

NLO, plus Long & Yang PC
(*Phys.Rev.C* 86 (2012) 024001)

SEP: NN 1s0 adopts dibaryon field
(Phys.Rev.C 97 (2018) 2, 024001)

Perturbative P-waves: PC by S. Wu & B. Long (*Phys.Rev.C* 99 (2019) 2, 024003)

Wrong ^{16}O pole

The same NN interaction generates ^{16}O with the *wrong* pole structure (not stable w.r.t. 4α decay) at LO. Also, deformed state becomes deeper than spherical state.

Same thing for WPC, PC improved with auxiliary dibaryon fields, and pionless EFT.

M. S. Sánchez, C.-J. Yang, Bingwei Long, U. van Kolck, Phys.Rev. C97 (2018) no.2, 024001.

In fact, nobody got ^{16}O right at LO yet!

- We have exhausted all possibilities (dibaryon, perturbative P-waves, different fitting of LECs) we could think of in the NN sector.

What to do then (to restore the correct pole)?

- "Improved action" applied to LO.

L. Contessi, M. Schäfer, U. van Kolck, Phys.Rev.A 109 (2024) 2, 022814
L. Contessi, M. Pavon Valderrama, and U. van Kolck, arXiv:2403.16596 [nucl-th]

- "Combinatorial factor" should kick in, and promote 3NF to LO.

C.J. Yang, A. Ekström, C. Forssén, G. Hagen, G. Rupak, U. van Kolck, Eur.Phys.J.A 59 (2023) 10, 233

Estimations

- Combine NDA and "A choose n":

Combine both:

Approx. with nuclear saturation density

$$\frac{C_n^A F_n}{C_m^A F_m} = \frac{A-m}{n}\left(\frac{\rho_0}{f_\pi^2 M_{\rm hi}}\right)^{n-m} = \frac{A-m}{n}\left(\frac{142({\rm MeV})}{M_{\rm hi}}\right)^{n-m}$$

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*5^+-body force is more suppressed (s≥1), only equal to NNNN after A>500.

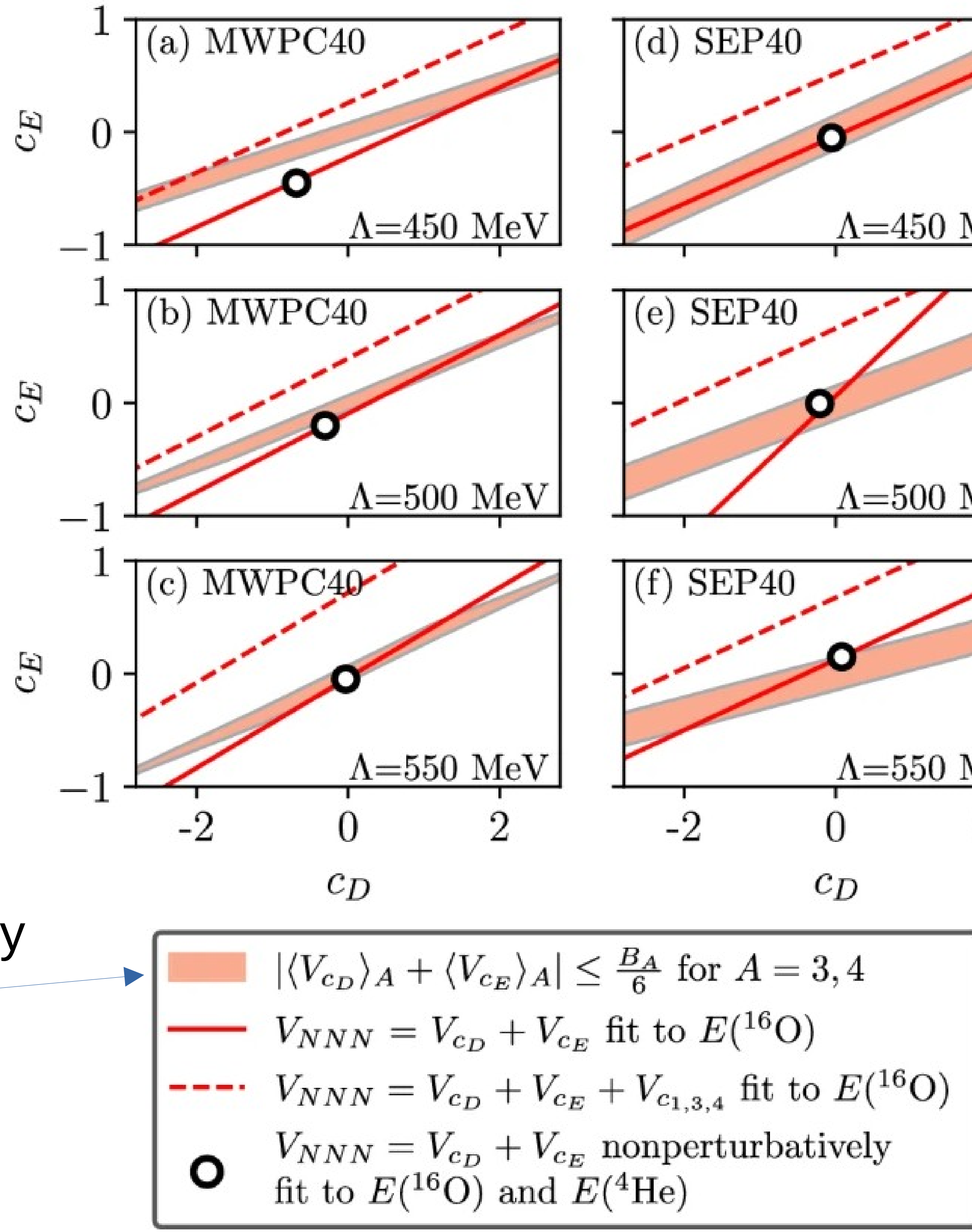
As nuclear forces are short-range, the enhancement can be weaker.

NNN will be LO for A>13

^{16}O has A=16!

=> Already need NNN at LO

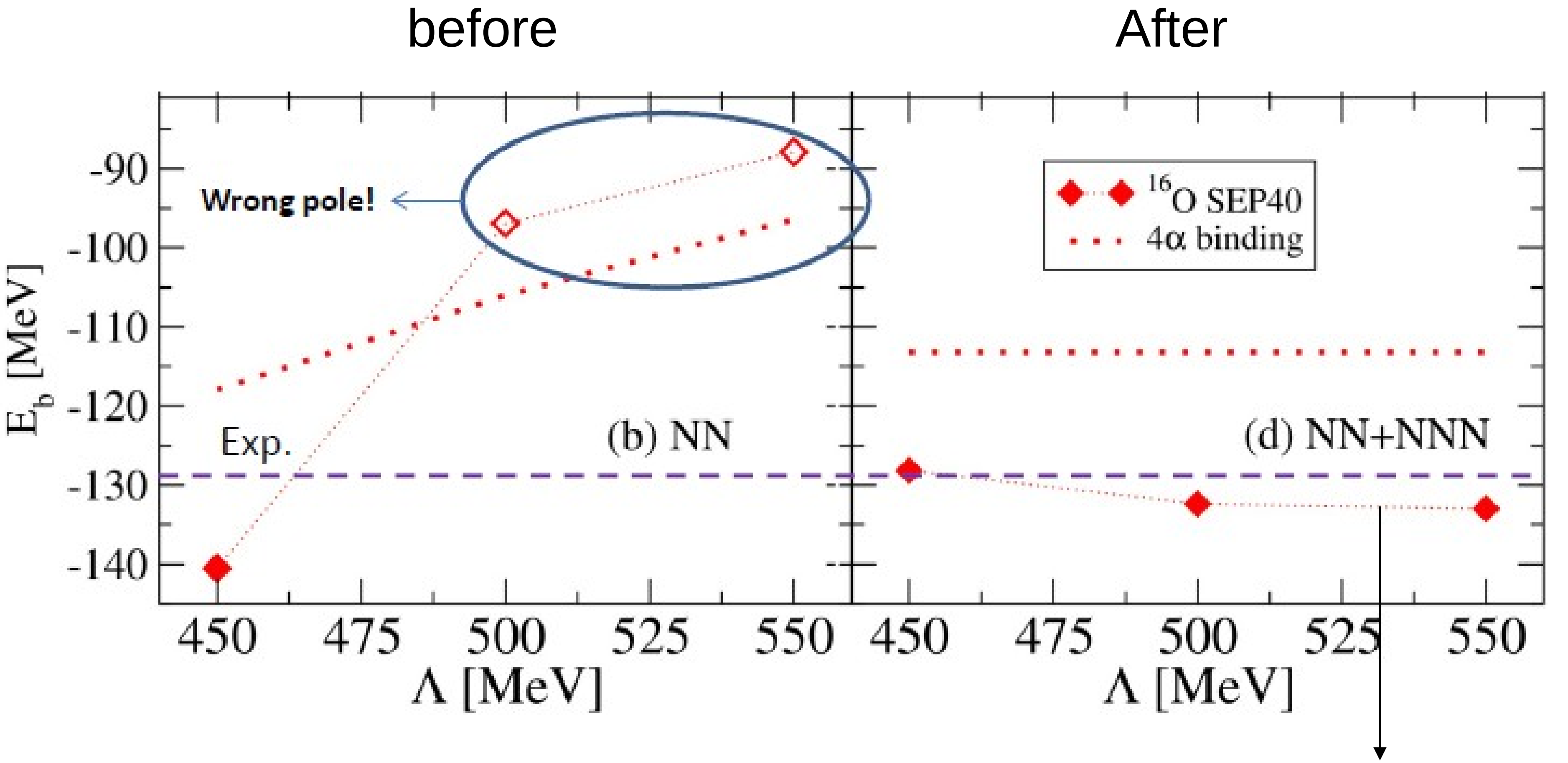
First, need to fit/renormalize cD, cE, which do not appear at LO for A=2-4, but are there for 16O.



c_D, c_E determined by a restricted fit.

With 3NFs' size limited to be NNLO on A≤4 systems

C.J. Yang, A. Ekström, C. Forssén, G. Hagen, G. Rupak, U. van Kolck, Eur.Phys.J.A 59 (2023) 10, 233

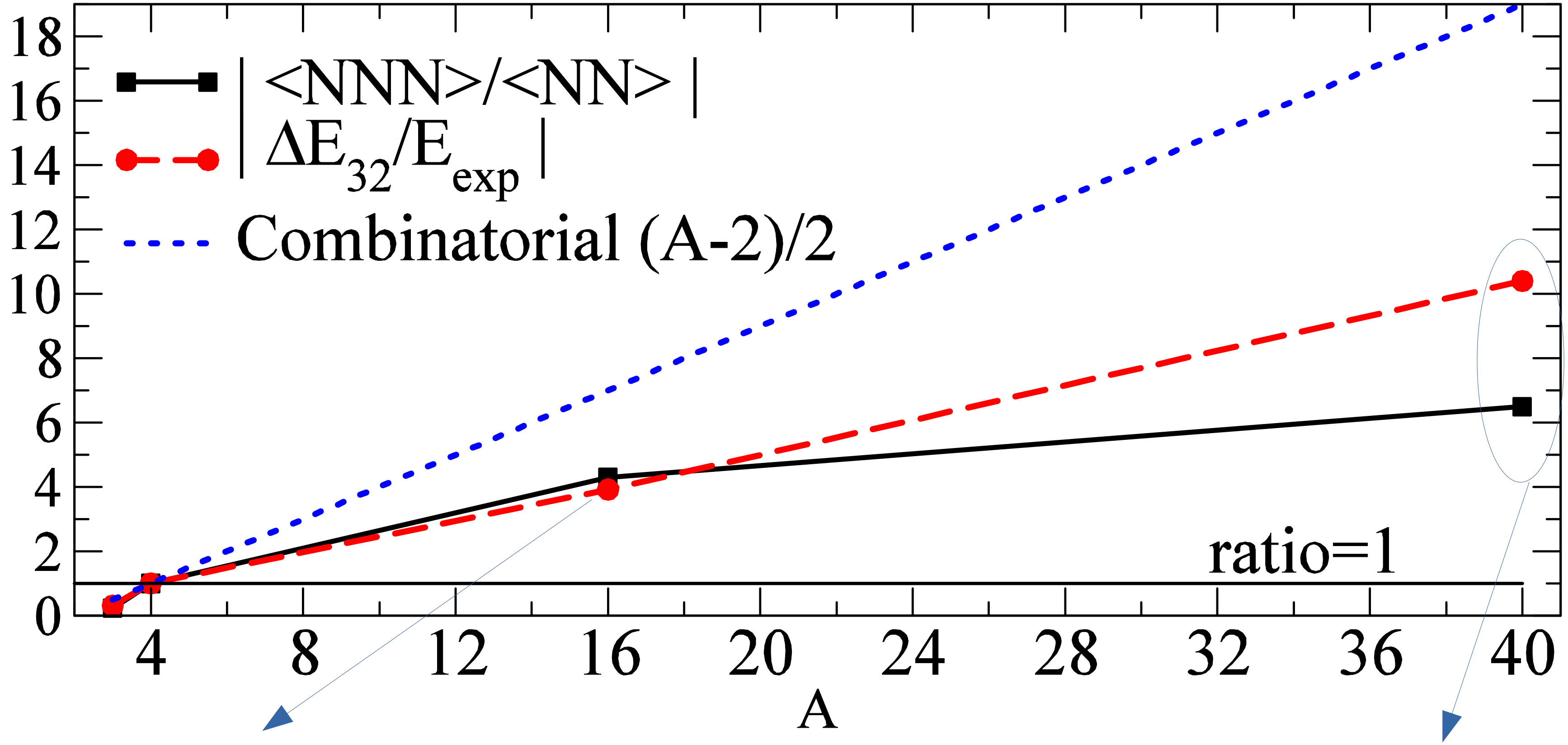


Problem solved! ^{16}O great already at LO!

Moreover...

Real Growth (accounting all effects) of 3NF/2NF with A

Λ=450 MeV, i.e., NNN provides repulsion

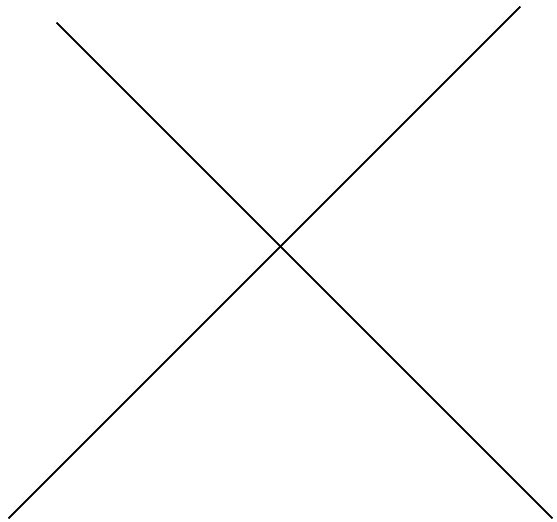


(Number of particles in the nuclei)

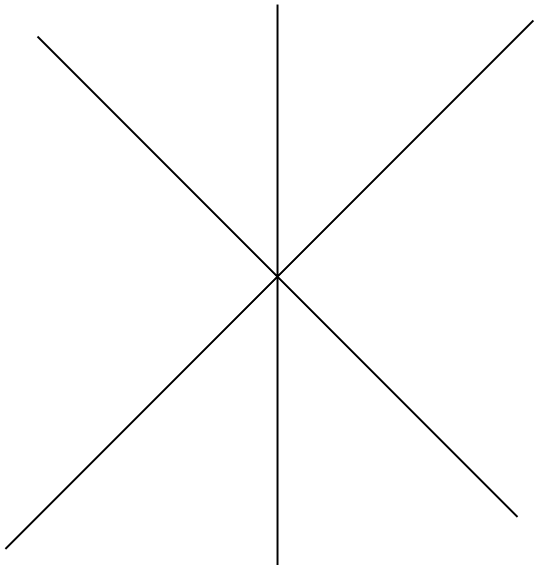
~5 times (promoted at least 1 order ~1/3)

6~10 times

Naïve dimensional analysis (NDA)



2 nucleon force



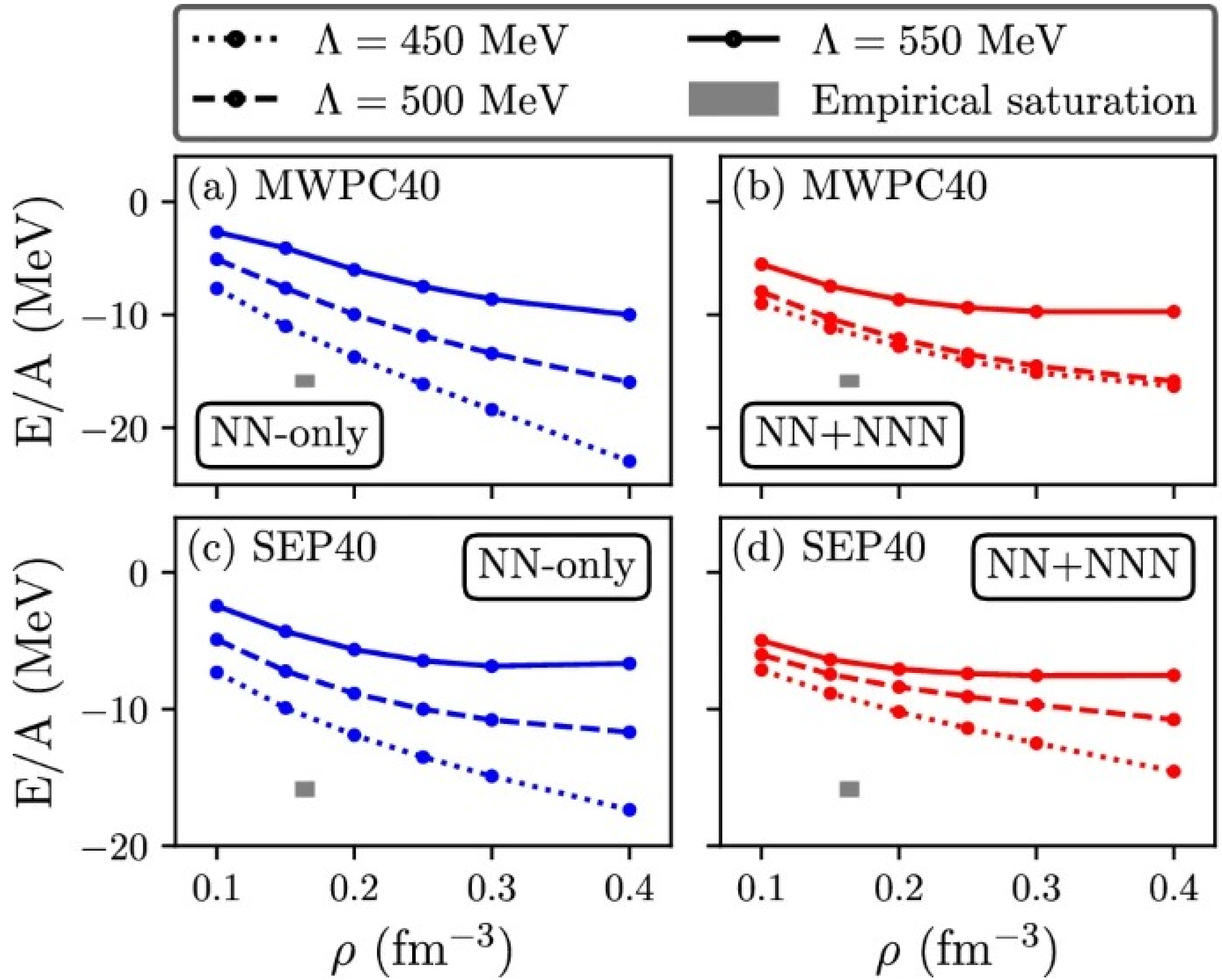
3 nucleon force

$$\text{3NFs/2NFs} \approx \frac{N^{+}N}{f_{\pi}^{2} M_{hi}} \approx \frac{\rho_0}{93^2 \cdot 500} \approx 0.28$$

This suggests:
3NFs are LO at least for $A \geq 16$

NNNN at LO for larger A?

No-go test by nuclear matter (EoS)

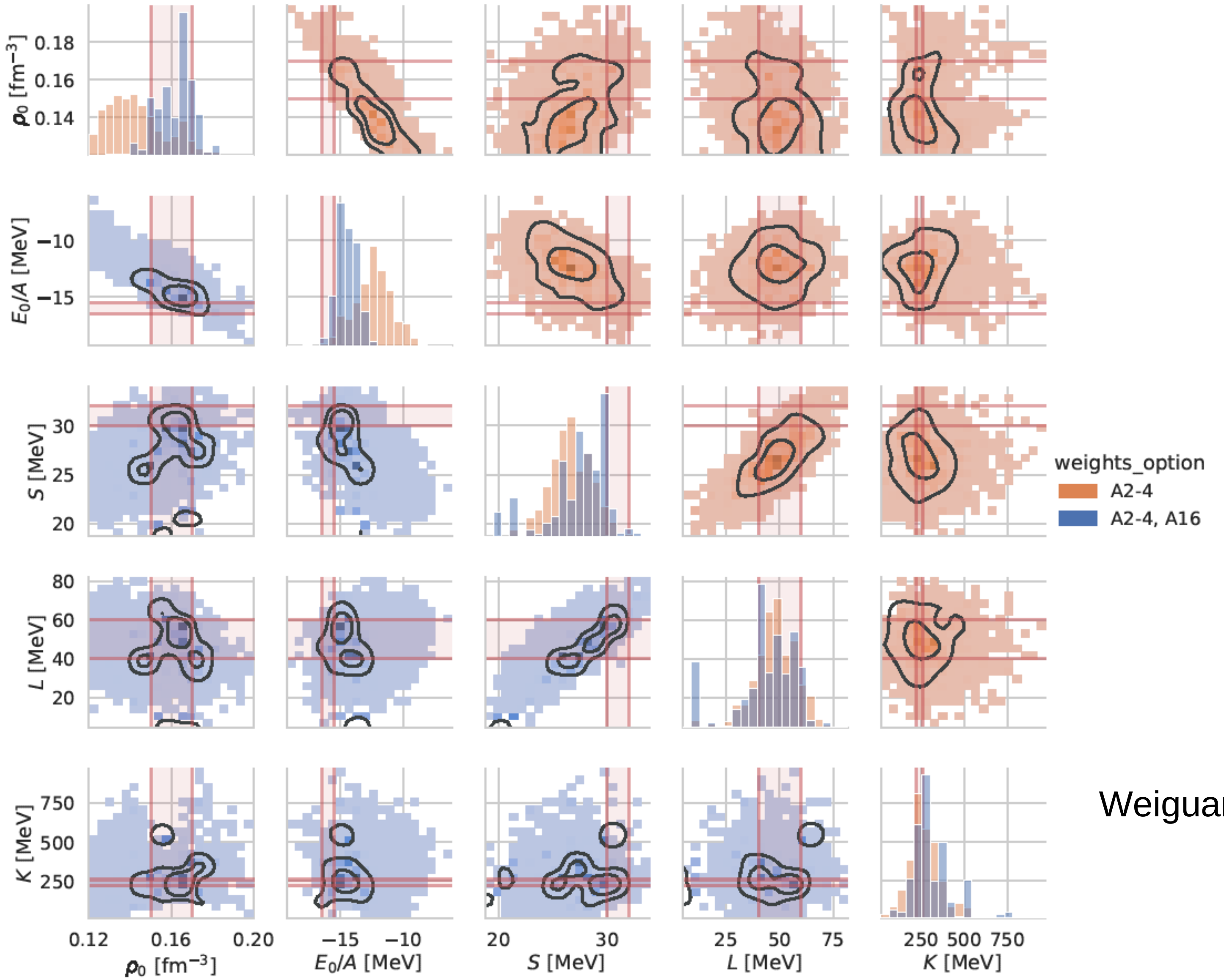


Conclusion: NN+NNN seems **no enough** !

Review of current status: nuclear structure

- Most ab-initio calculations adopt chiral EFT potential organized under Weinberg power counting (WPC).
- Good results (w.r.t. exp. data) for light systems, if low-energy constants (LECs) are renormalized at NN/NNN-level.
- But not quite the same for ^{16}O (or heavier)➔ **need to refit** (optimize) the potential and **sacrifice** NN.→ what's ab-inito debate.

Optimizing NNLO with Δ(1232)



Weiguang Jiang, et al.

Necessary ingredients of model-independent EFT

1. A-dependence

This means:

(1a) Need to promote 3- and 4-body forces at A>4

(1b) V and LECs need to change with A (or if you know the ad hoc density)

↓

c.f. $NNLO_{opt}$ $NNLO_{sat}$, $NNLO_{go}$, EM_{magic}, etc.

2. RG-invariance

This means:

(2a) Need more contact terms than NDA

(2b) Cannot adopt an entirely non-perturbative treatment

↓

Why? How about using low-cutoffs?

Let's see the pros and cons of perturbative correction first

Non-perturbative v.s. perturbative treatment

Non-per. is necessary for bound-states (pole in the S-matrix), but it:

- Often (if not always) destroys PC arranged on the potential-level.
- Gives rise to m_π-dep problem (if OPE is iterated).
- Avoided level-crossing.

Good if you want to shift poles across threshold or create a new one.
Repels/forbids states to be close to each other (or make it very hard).

Non-per + perturbative on subleading orders:

- ~~Often (if not always) destroys PC arranged on the potential-level.~~
- Gives rise to m_π-dep problem (if OPE is iterated).
- Allow level-crossing.

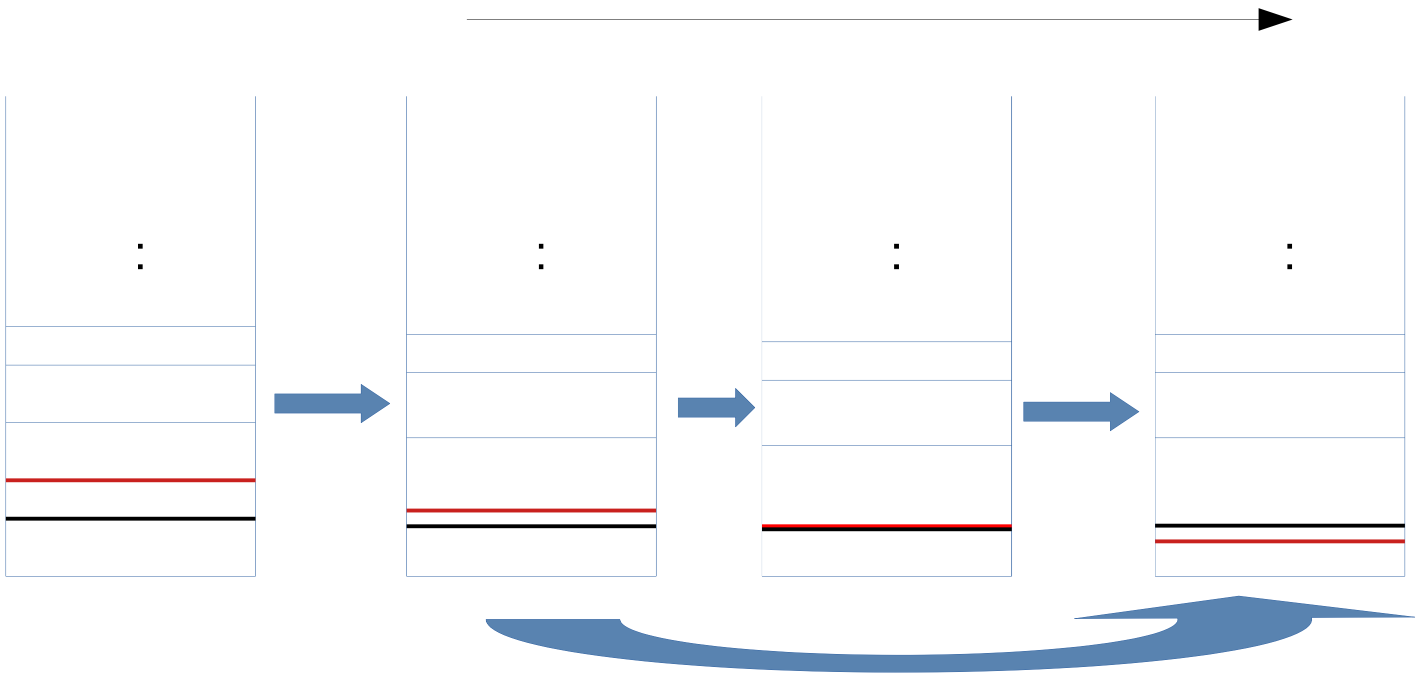
~~Good if you want to shift poles across threshold or create a new one.~~
Allows states to be close to each other ~~(or make it very hard)~~.

Perturbative treatment of NLO

LO: V_{LO}

NLO: $E_{LO}+C_{nlo}\langle WF_{LO}|V_{NLO}|WF_{LO}\rangle$

With increasing C_{nlo}



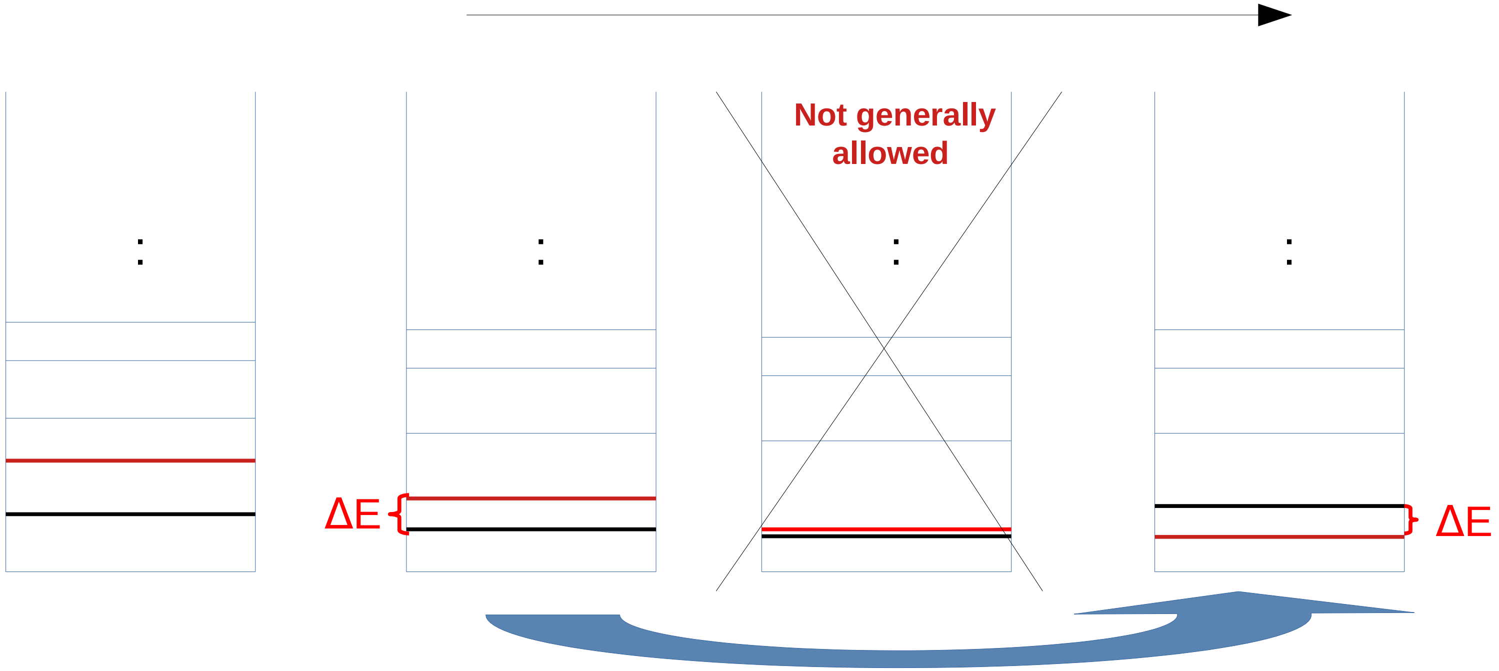
Level-crossing happened!

Gap between states can be zero

Non-perturbative treatment of NLO

LO: $[V_{LO}]_{diagonalize}$

NLO: $[V_{LO}+C_{nlo}V_{NLO}]_{diagonalize}$
With increasing C_{nlo}



Avoided level-crossing !
There's a minimum and finite separation between levels!

A crucial difference w.r.t. per-treatments

(Lower the cutoff in the iteration won't change this characteristic!)

In terms of reproducing **ground state** properties, traditional way (non-per.) is o.k.

starts from A>3

But for excited states (maybe for resonances too), depend on the actual data, **you might want to have the flexibility to reproduce smaller gaps**.

(Adjusting/fine-tuning $V_{non\text{-}per}$ is likely to re-shuffle all eigen-energies)

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starting from A>3

So, (low-cutoff + non-per.) ≉ per. correction

But for excited states (maybe for resonances too), depend on the actual data, **you might want to have the flexibility to reproduce smaller gaps**.

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Summary

Why modified PC?

- Because it provides solutions/improvements of conceptual problem of WPC (allow RG to be o.k., or aka, a systematical control of the uncertainty).

Why A-dep PC?

- The combinatorial enhancement becomes important for A>10. This makes the promotion of many-body forces (NNN and NNNN) ***necessary***!

I don't like it either, but sometimes the correct way happens to be the painful way.

Advertisement: An extreme scenario

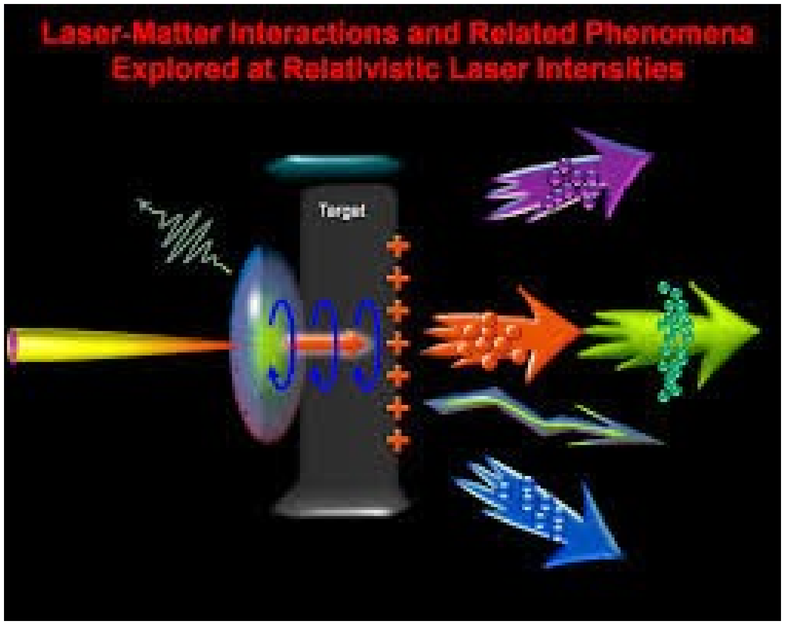
What if we irradiate nuclei with very intensive neutron/proton/gamma/ion beams?
Combinactorial enhancement will be enlarged further, with
3- and higher-body force not just promoted, but dominated!

<u>Nuclear/particle physics at intense limit</u>

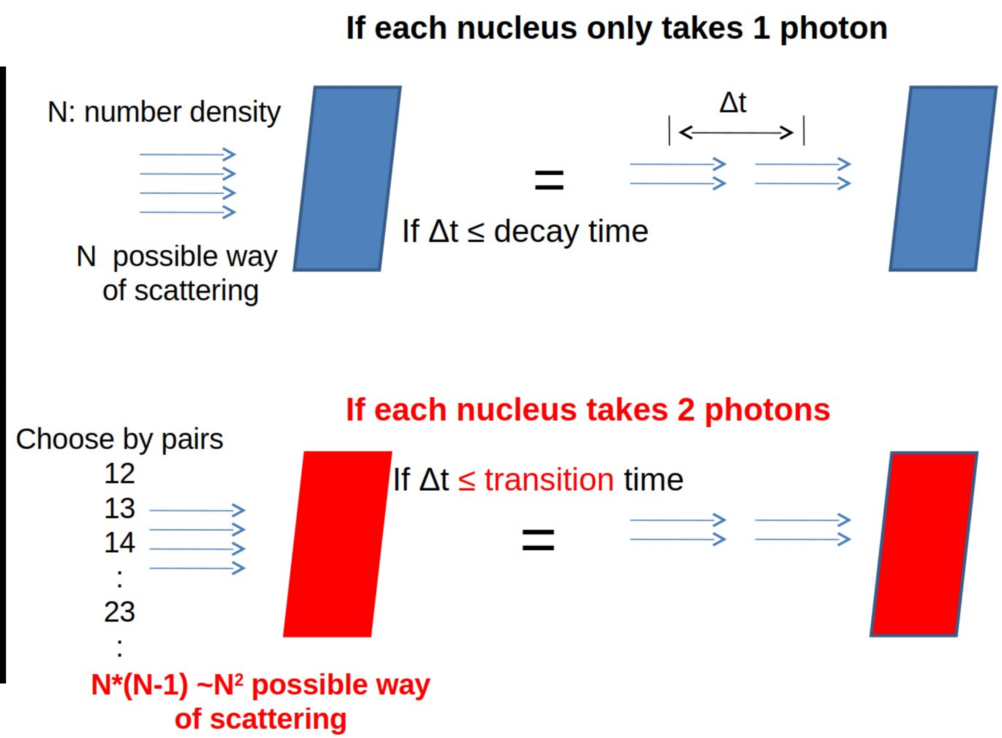
This is not a science fiction!

ELI-NP, Romania
10 PW lasers

A very simple explanation
(beams from laser-plasma interaction)



https://www.icuil.org/



- Key: total # maybe not great, but the intensity is,

∵ compress in time + space.

10~50 fs   3~100 μm

Result in some new phenomena/applications:

Isomer pumping: arXiv: 2404.07909 [nucl-th]

Graser: 2404.10025 [physics.optics]

And more to be explored!

Thank you!

A few thought-provoking questions

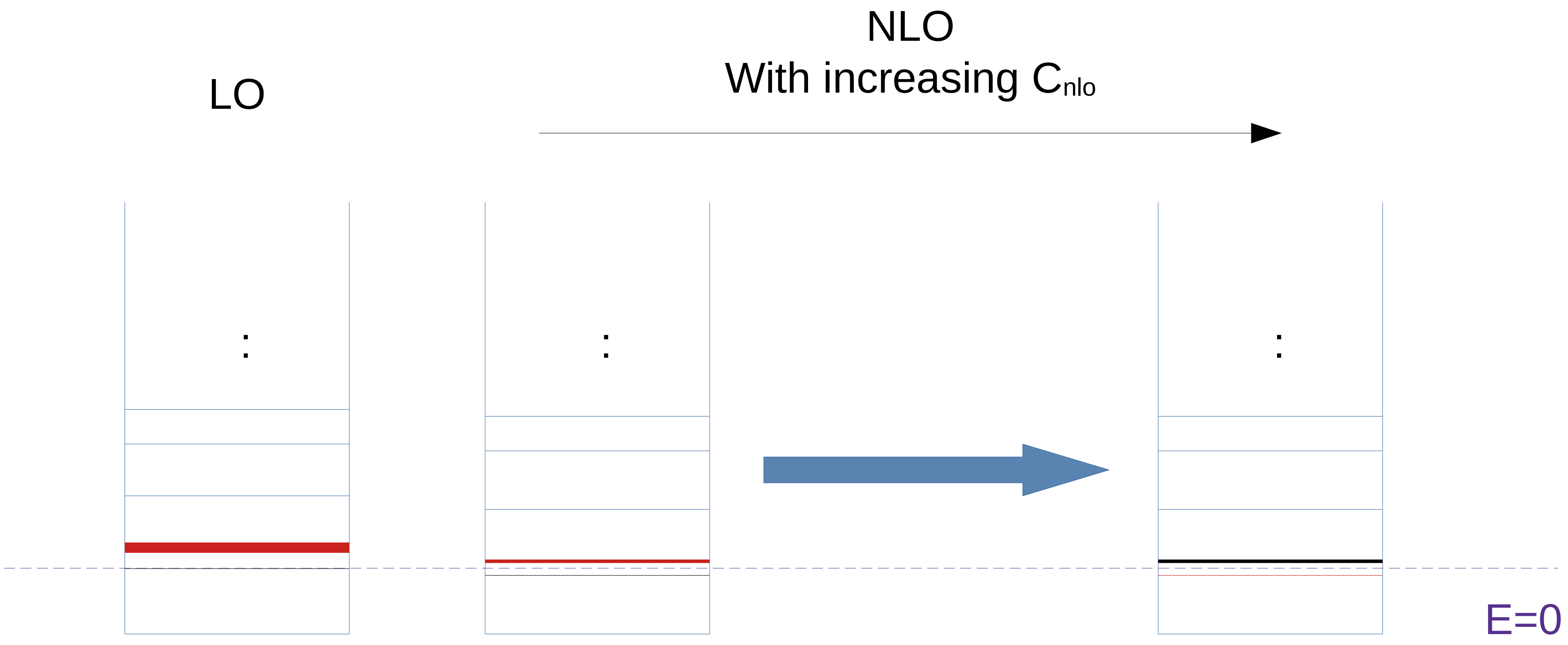
1. Are we going back to (EFT-inspired) models→i.e., build whatever describes data? → The error might be controlled (and even reduced at higher-orders to some degree) by a carefully chosen Λ + fitting procedure + Bayesian analysis?

Or, we insist to do the truly EFT-based approach (there might be more things to learn with try & error)?

2. Can WPC (and it's rel. version) solve Ay puzzle?

3. Any doubt on 'the importance of many-body forces' and it's dependence on the number of nucleons?

Level-crossing v.s. avoided level-crossing

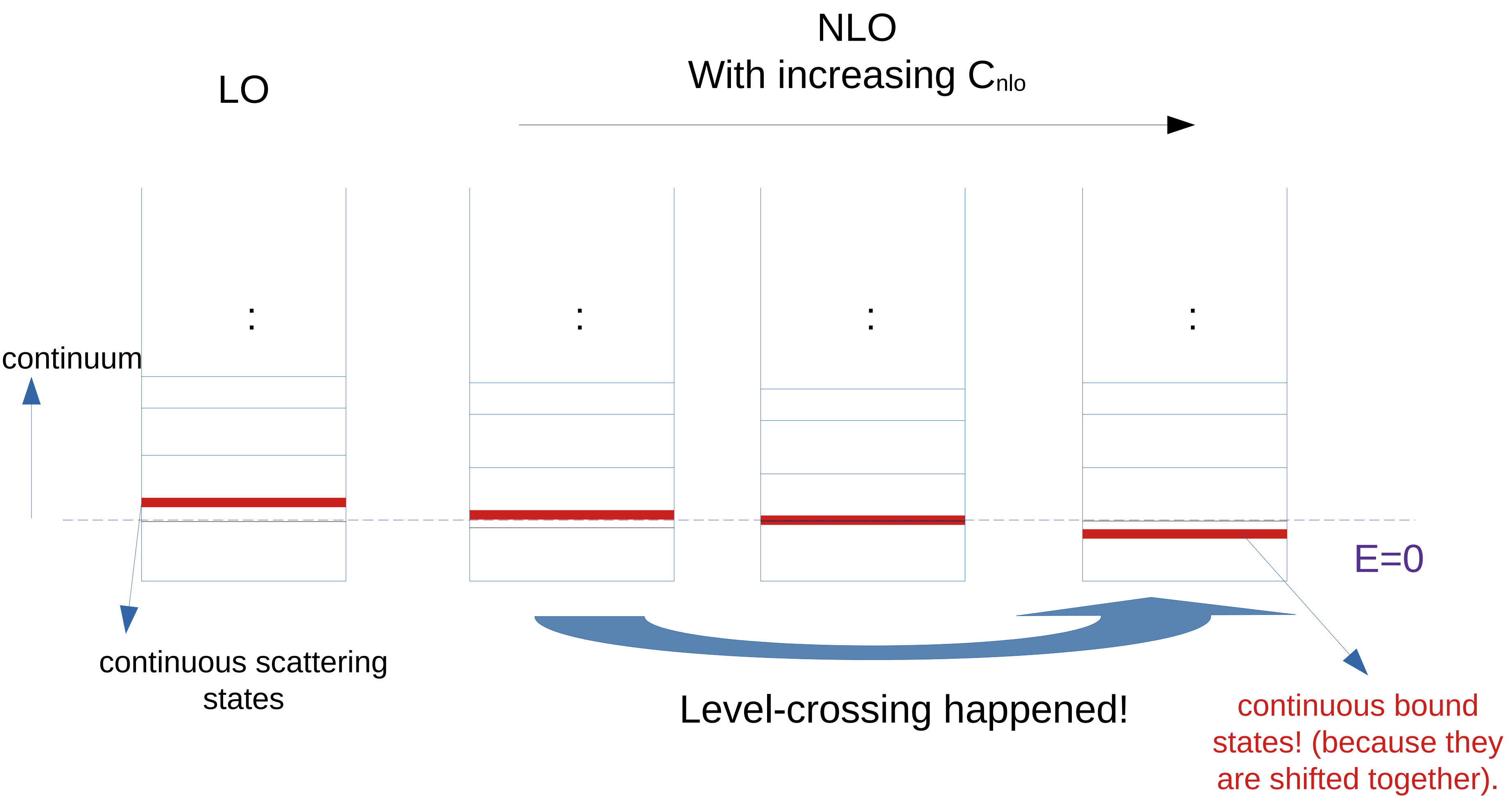


Avoided level-crossing !

There’s a minimum and finite separation between levels!

Non-perturbative treatment allows stand-alone bound-states.

Level-crossing v.s. avoided level-crossing

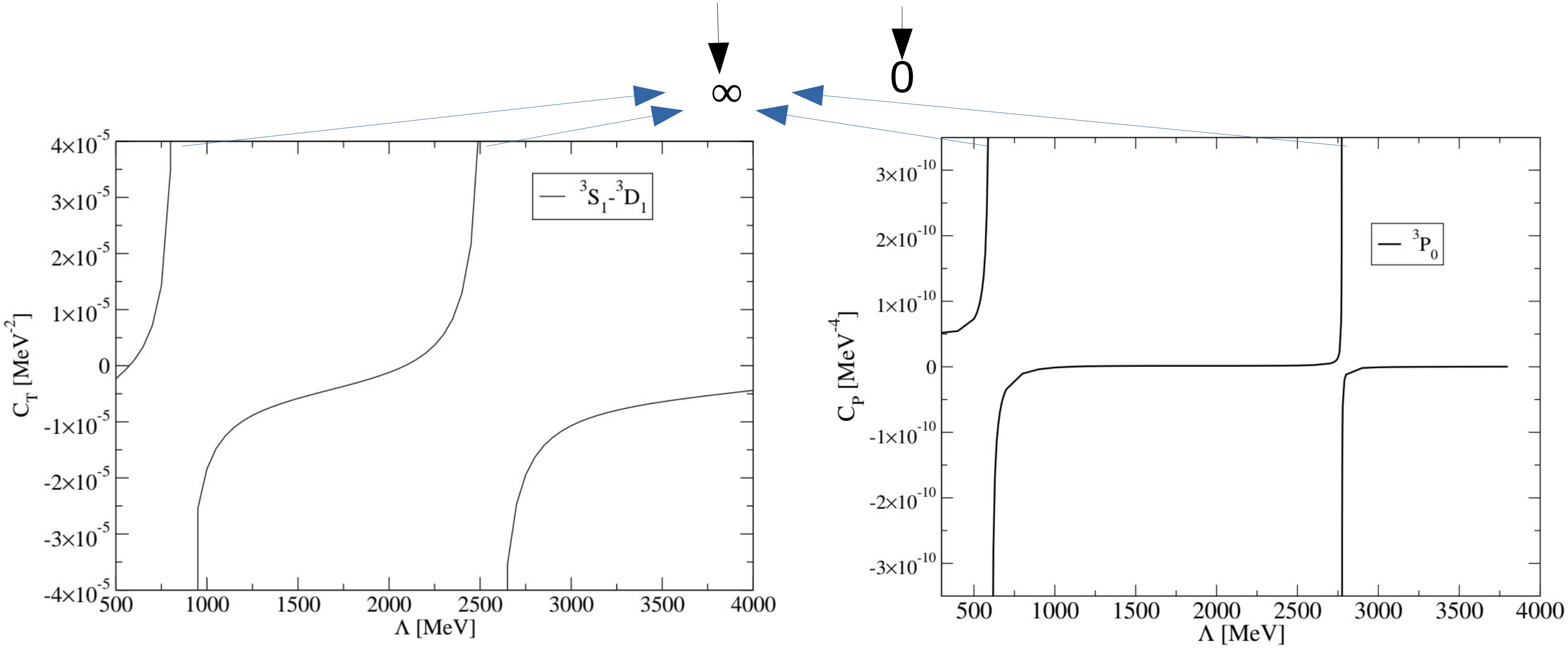


Perturbative treatment of NLO

Origin of the issue

- LECs at LO (non-per. treatment) could have limit-cycle running.
- At LO, this is ok, even exactly at Λ_e where $c(\Lambda_e)=\infty$. Because: (non-per) = (matrix diagonalization), which guarantee that **each eigenvalue $<\Phi_{LO,i}|H_{LO}|\Phi_{LO,i}>=E_i$ is finite**.

∵ <KE> and $<V_{LO}>$ are finite, => $c(\Lambda_e)<\Phi_{LO,i}|\hat{O}_{ct}|\Phi_{LO,i}>$=finite for all i.



However, the same **won't hold** for NLO or higher-orders, if **DWBA** is adopted.

Origin of the issue

- At NLO (or higher), additional CT enters, but unlike LO, where $c(\Lambda_e)\langle\Phi_{LO,i}|\hat{O}_{LO,ct}|\Phi_{LO,i}\rangle$=finite for all i, the DWBA correction $d(\Lambda_*)\langle\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}\rangle\neq$finite for all i (as we are not protected by the eigenvalue feature).

  => At a certain i* (correspond to E*), $\langle\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}\rangle=0$, but for other i it's not!

- This means, if one **choose to renormalize** at E=E*, one faces the choice of using $d\to\infty$, in order to have a non-zero NLO correction. But then observable at other E blow up. On the other hand, using $d\neq\infty$ will make this CT have zero contribution (not good either).

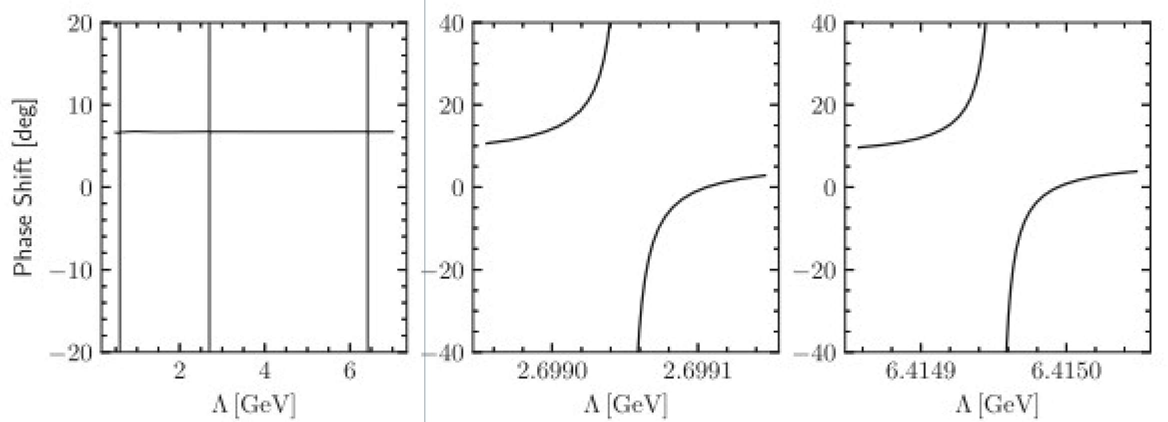


FIG. 5. Cutoff dependence of $^3P_0$ phase shift calculated at the fixed laboratory energy of T_{lab} = 130 MeV using the approach of Ref. [30] at NLO. The middle and right panels show zoomed regions in the vicinity of two exceptional cutoffs.

Allowed to choose anywhere below M_{hi}

In practice (on Long & Yang)

- At NLO (or higher), additional CT enters, but unlike LO, where $c(\Lambda_e)<\Phi_{LO,i}|\hat{O}_{LO,ct}|\Phi_{LO,i}>$=finite for all i, the DWBA correction $d(\Lambda_*)<\Phi_{LO,i}|\hat{O}_{NLO,ct}|\Phi_{LO,i}>\neq$finite for all i (as we are not protected by the eigenvalue feature).

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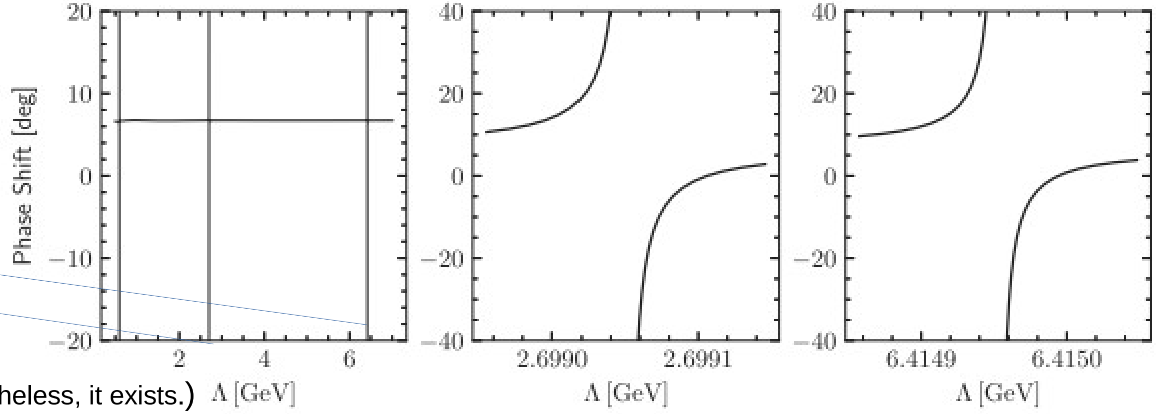
Problematic Λ_*, where $E^*\in p_{tye}$

(Extremely narrow, nevertheless, it exists.)



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Conditions of the breakdown (for the above case$_{Long\&Yang}$):

1. $\hat{O}_{NLO,ct}\neq\hat{O}_{LO,ct}$
2. Adopt Λ very close (>4 significant digits the same) to those problematic Λ_*.
3. Choose to renormalize **exactly** at E* (or exactly on a set of particular E_i, if number of LECs≥2).

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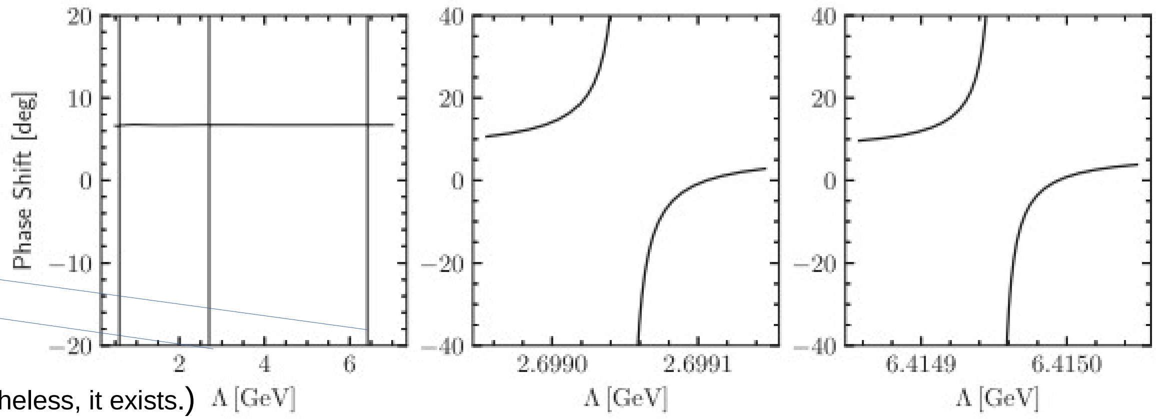

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Key word!

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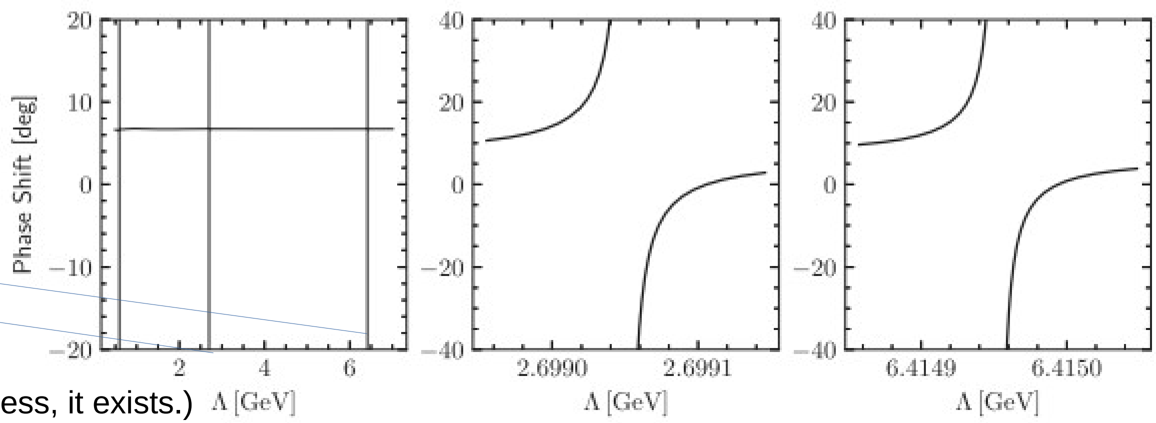

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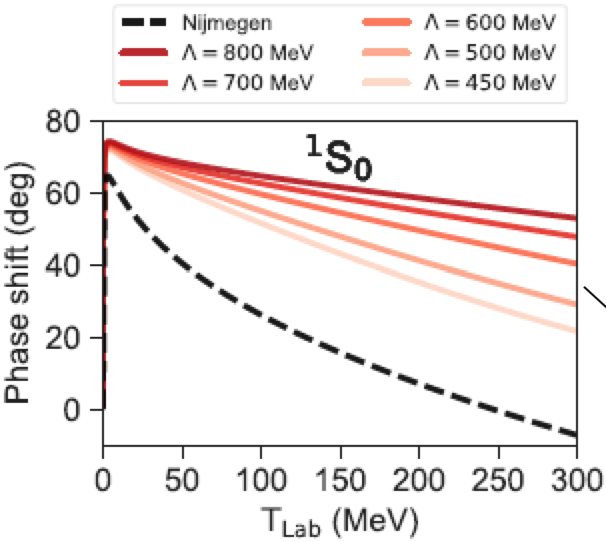
However, the issue occurs only when one treats those incomplete, truncated amplitudes exactly or beyond the degree to which they should be trusted.

Root of the problem (**nothing to do with PC**, but a general feature of perturbative corrections)

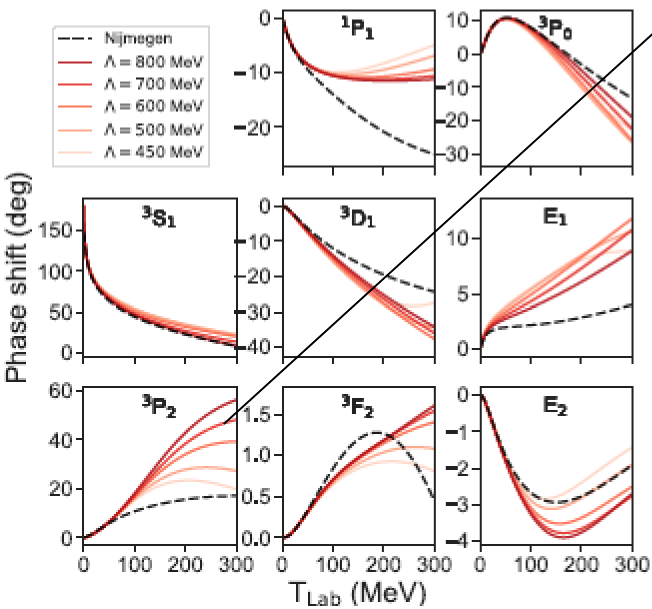
The above has taken $\langle\Phi_{LO,i}|$ (and therefore the **NLO matrix element**) too **exact.**

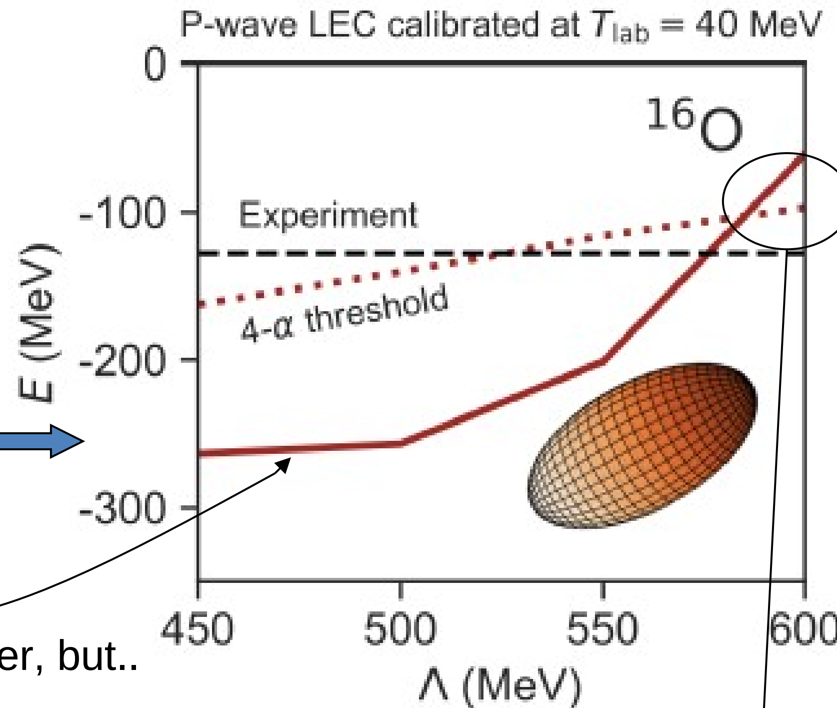
Under EFT, it should always be accompanied by an uncertainty $\sim O(p/M_{hi})^n$.

Further fine-tune (LO, NN only)



over-attractive





Make ^{16}O deeper, but..

^{16}O is still non-physical
(deformed and unbound) !