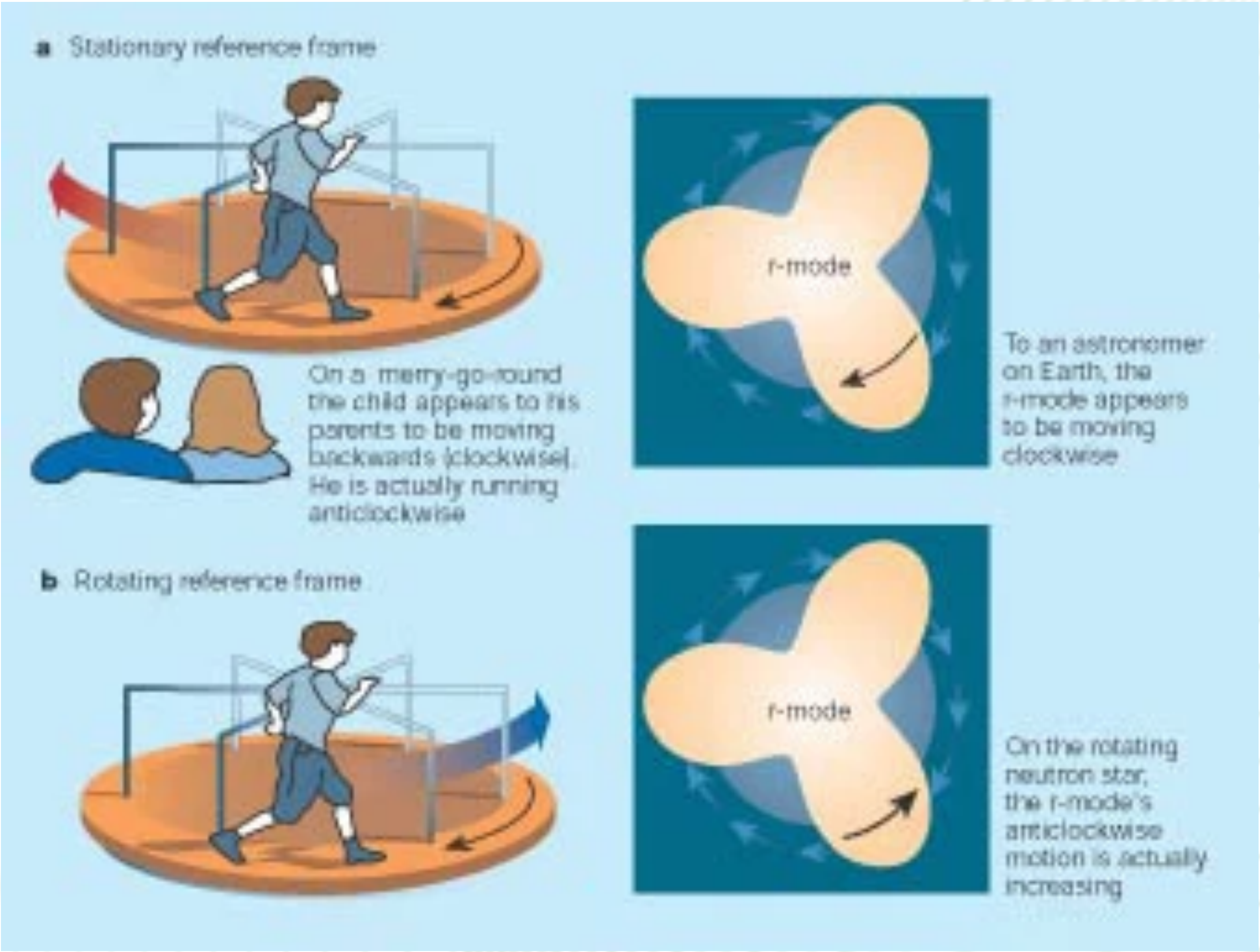
The r-mode puzzle

Cecilia Chirenti



Fryer & Woosley, 2001

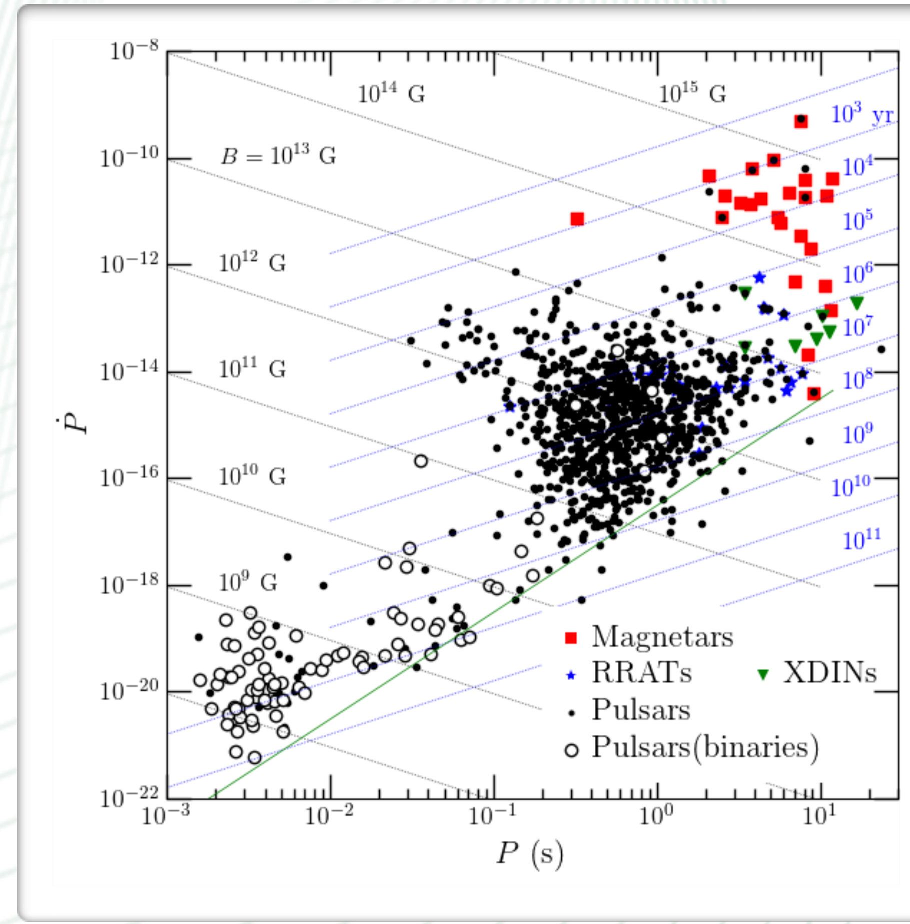
INT Workshop "Discovering Continuous GW with Nuclear, Astro and Particle Physics", November 21 2024, Seattle

Pulsar periods and Keplerian mass-shedding limit

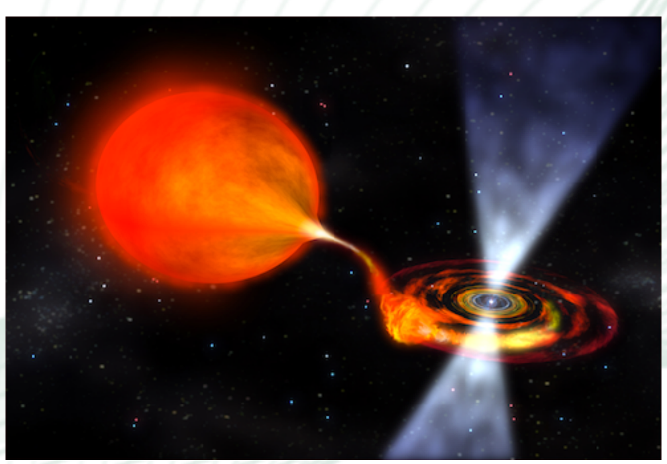
* fastest observed spin to date: 716 Hz for PSR J1748-2446ad

* Accretion can spin up a pulsar, theoretically up to the Keplerian mass-shedding limit:

$$f_K \sim \frac{1}{2\pi}\sqrt{\frac{2GM}{R^3}} \sim 2\,\mathrm{kHz} \text{ or } P_K \sim 500\,\mu\mathrm{s}$$



[Andrew Steiner]



What could be limiting the spins of accreting pulsars?

Magnetic torques could limit the spin up due to accretion (also r-modes?) [Andersson, Kokkotas & Schutz, 1999, Bildsten, 1998]

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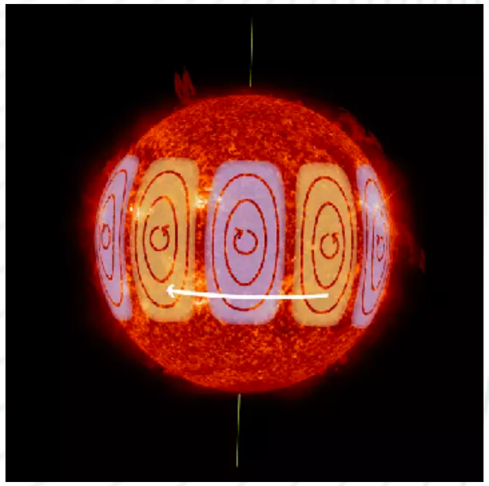
r-modes and the CFS instability

solar rossby waves



[MPS/NASA]

* r-modes: fluid modes of oscillation, restored by the Coriolis force

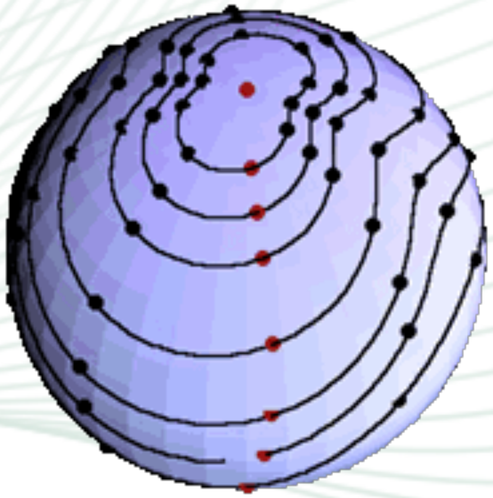
* generically unstable due to the emission of gravitational waves (CFS instability)
  [Chandrasekhar, 1970; Friedman & Schutz, 1978; Andersson 1998; Friedman & Morsink, 1998]

for the dominant $\ell = m = 2$ r-mode

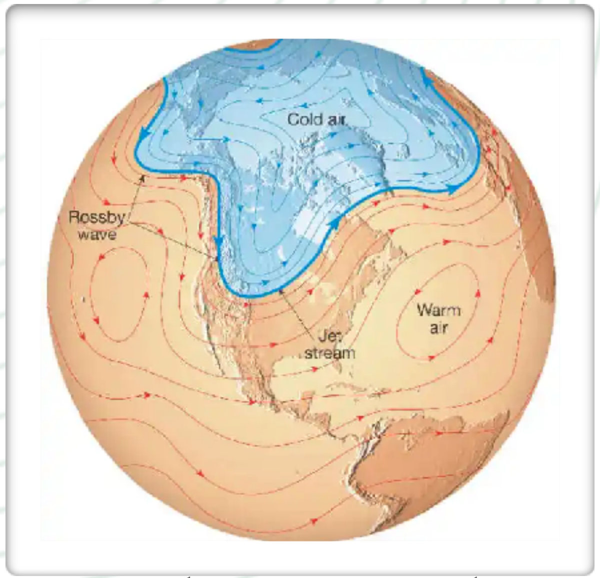
$\sigma_R = (2/3)\Omega$ (corotating frame)

$\sigma = \sigma_R - m\Omega = -(4/3)\Omega$ (inertial frame)

Instability condition: a counter-rotating mode appears as corotating for an inertial observer



[C. Hanna & B. Owen]



rossby waves on Earth

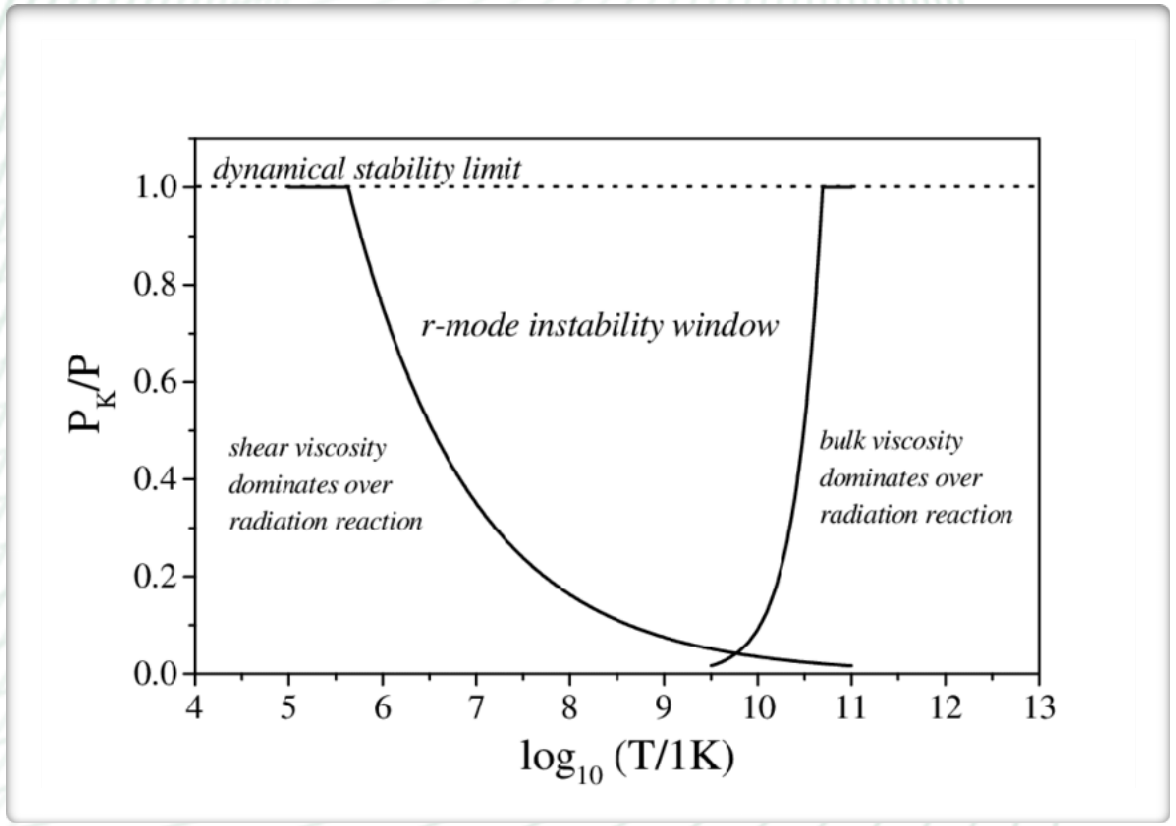
Neutron star r-modes and the instability window

* Linear instability; nonlinear couplings should set the saturation amplitude
  [Arras et al., 2003; Bondarescu, Teukolsky & Wasserman, 2007]

* Shear and bulk viscosity shape the instability window

$$\frac{1}{t} = -\frac{1}{\tau_{gw}}\left(\frac{1\ \mathrm{ms}}{P}\right)^{p_{gw}} + \frac{1}{\tau_{bv}}\left(\frac{1\ \mathrm{ms}}{P}\right)^{p_{bv}}\left(\frac{T}{10^9\ \mathrm{K}}\right)^{6} + \frac{1}{\tau_{sv}}\left(\frac{10^9\ \mathrm{K}}{T}\right)^{2}$$

[Lindblom, Owen & Morsink, 1998;
Andersson, Kokkotas & Schutz, 1999]



[Kokkotas, 2003]

typical values: $\tau_{gw} \sim 20$ s, $\tau_{bv} \sim 10^{10}$ s, $\tau_{sv} \sim 10^{8}$ s,
$p_{gw} \sim 6, p_{bv} \sim 2, p_{sv} \sim 0$

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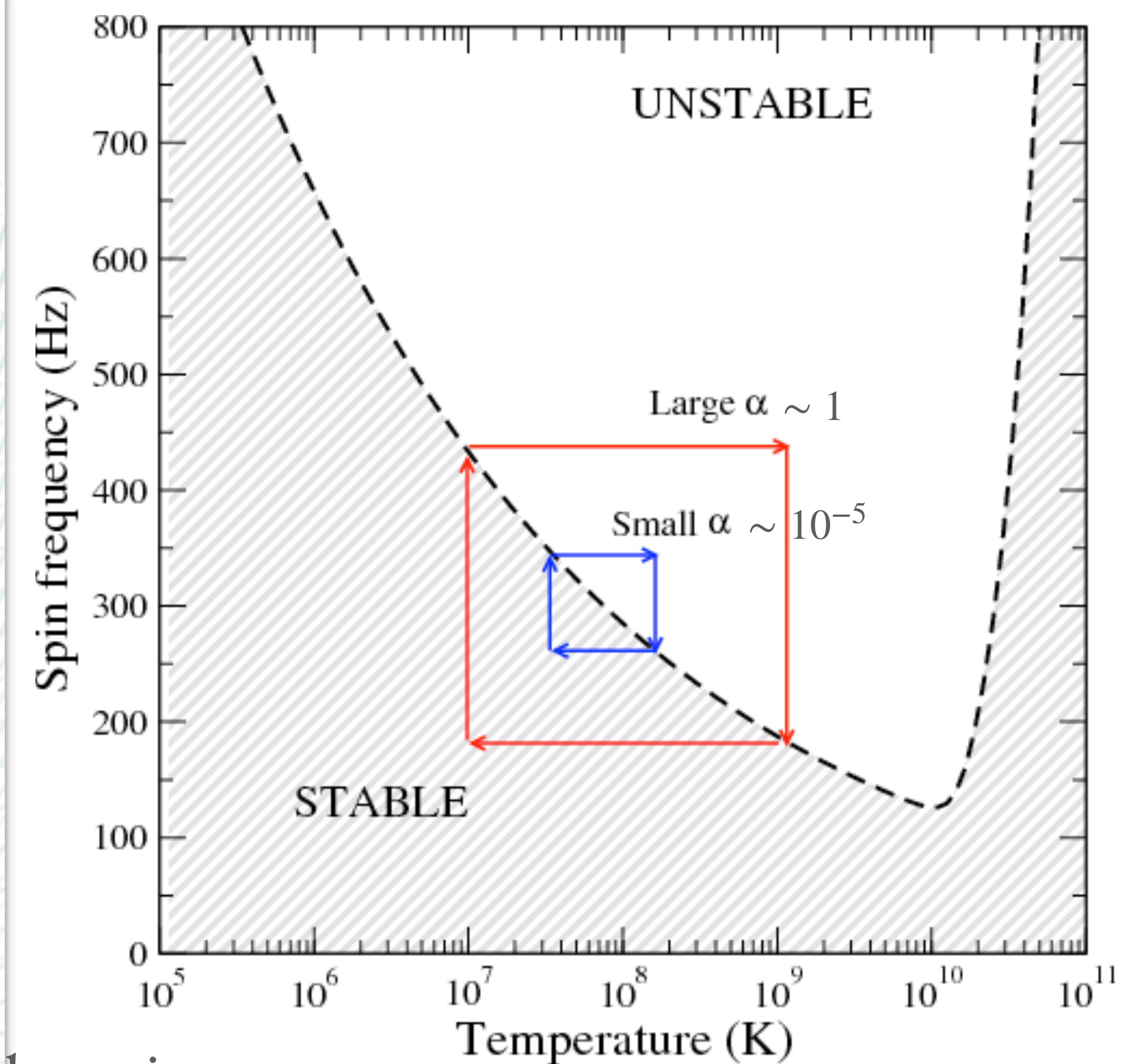
What happens in the r-mode instability?

* the r-mode grows exponentially until it reaches a saturation amplitude α
* the star heats up due to the unstable mode
* the r-mode spins down the star
* the star cools down
* accretion spins up the star again



[Haskell, Degenaar and Ho, 2012]

Equilibrium between spin up and spin down is unstable: neutron star follows a limit cycle

[Levin, 1999, Andersson et al. 2000]

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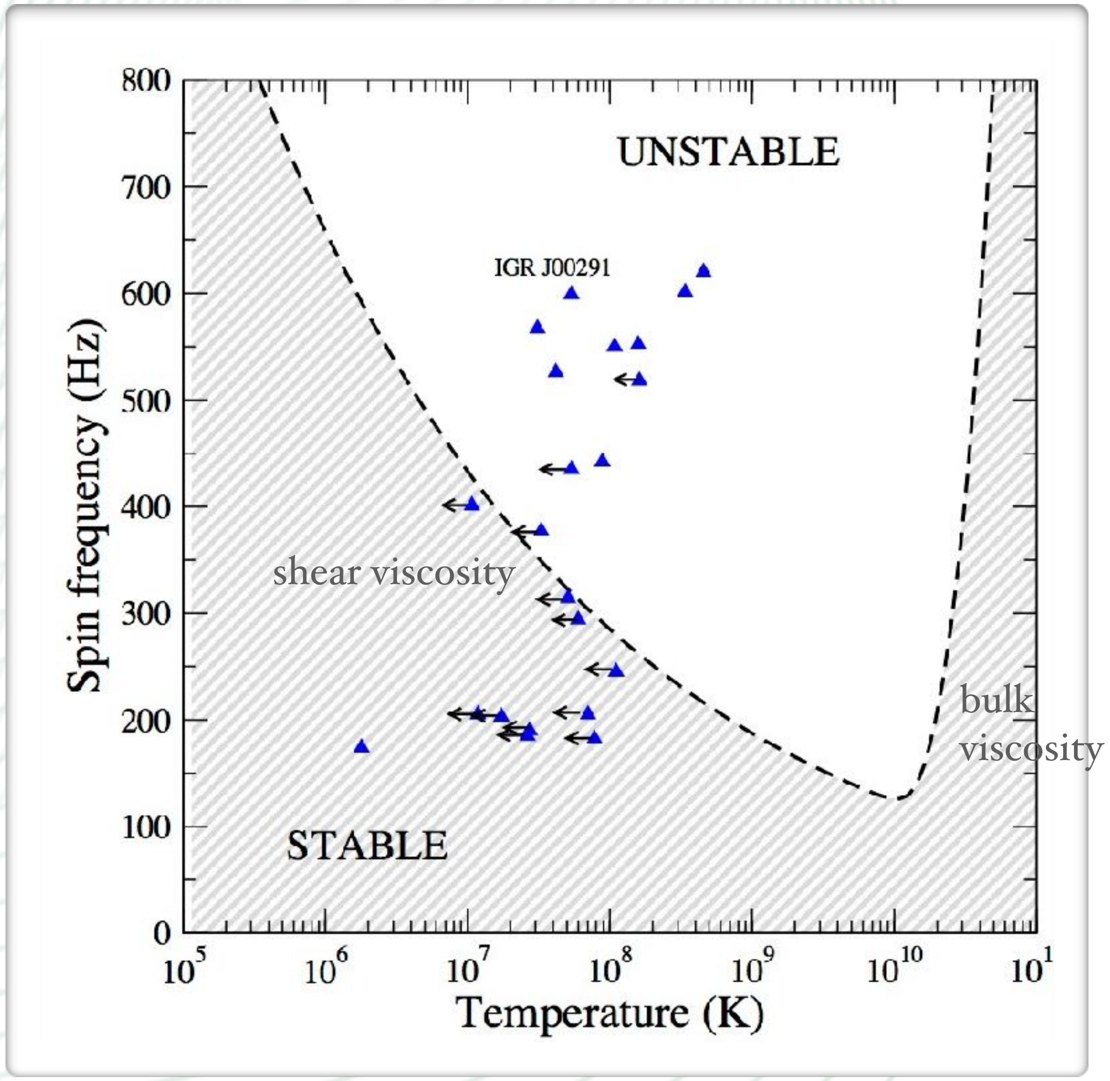
Neutron stars observed in the instability window

- Neutron stars in LMXBs and MSRPs are found in the r-mode instability window
- Should they be observed there? Some stars are so deep into the window that they should have large α ($\alpha > 10^{-3}$)
- For large (small) values of α, the star should spend less than 1 % (30 %) of the time inside the window
- Number of sources that should be on:

$$N_{\rm on} = \frac{\tau_{\rm on}}{\tau_{\rm cycle}} \sim 5 \times 10^{-8} \alpha^{-1.6} N_{\rm sample}$$

[Heyl, 2002]



[Haskell, Degenaar and Ho, 2012]

temperature information is model-dependent and uncertain…

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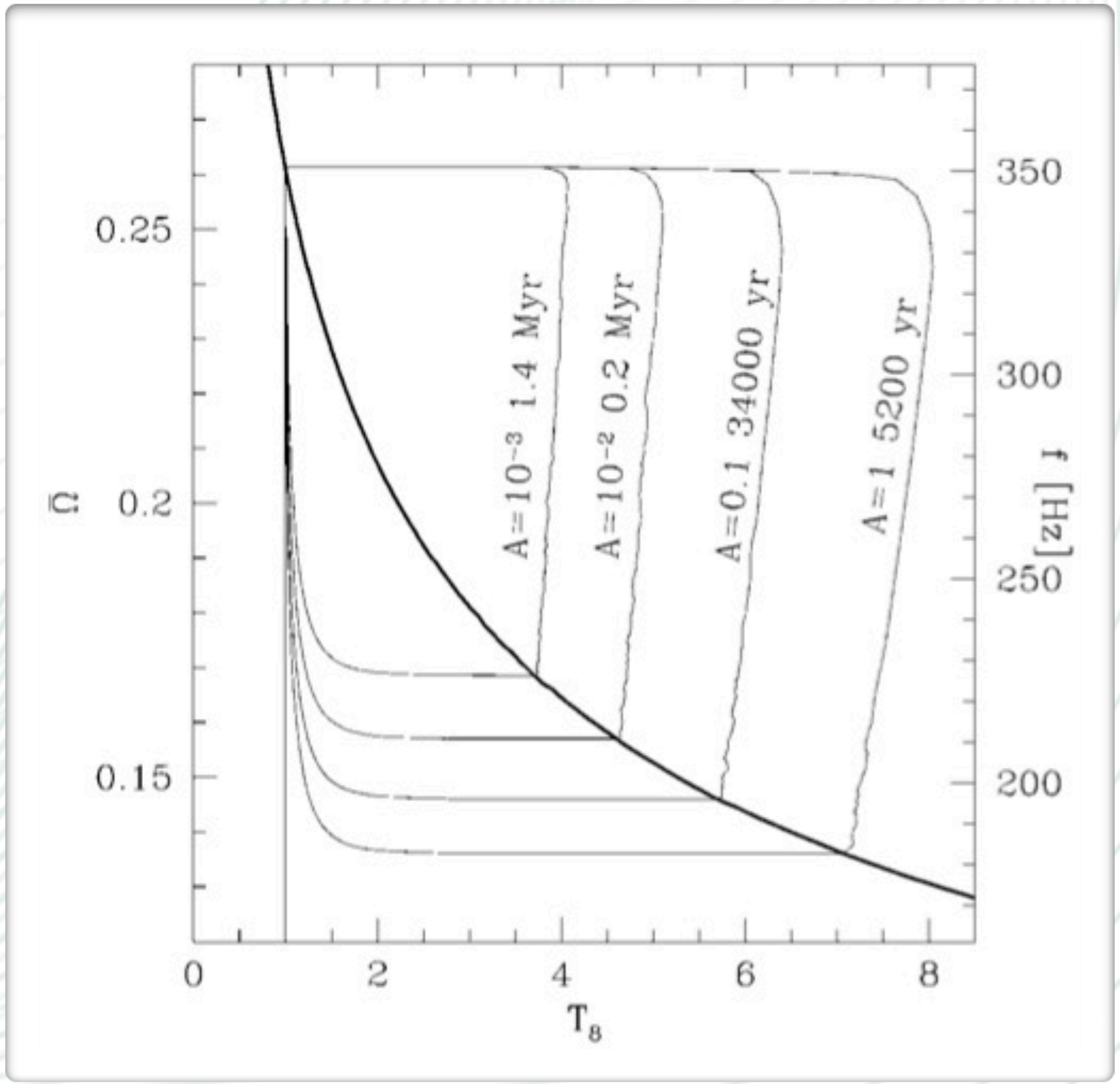
Duration of the GW emission in the r-mode cycle

The time for the system to complete the circuit depends only weakly on A (saturation amplitude), here 3.7 – 4.8 Myr.

Curves are labelled with the duration of the GW emission.

The duty cycle ranges from ~ 30 % to ~ 0.0001 % .



[Heyl, 2002]

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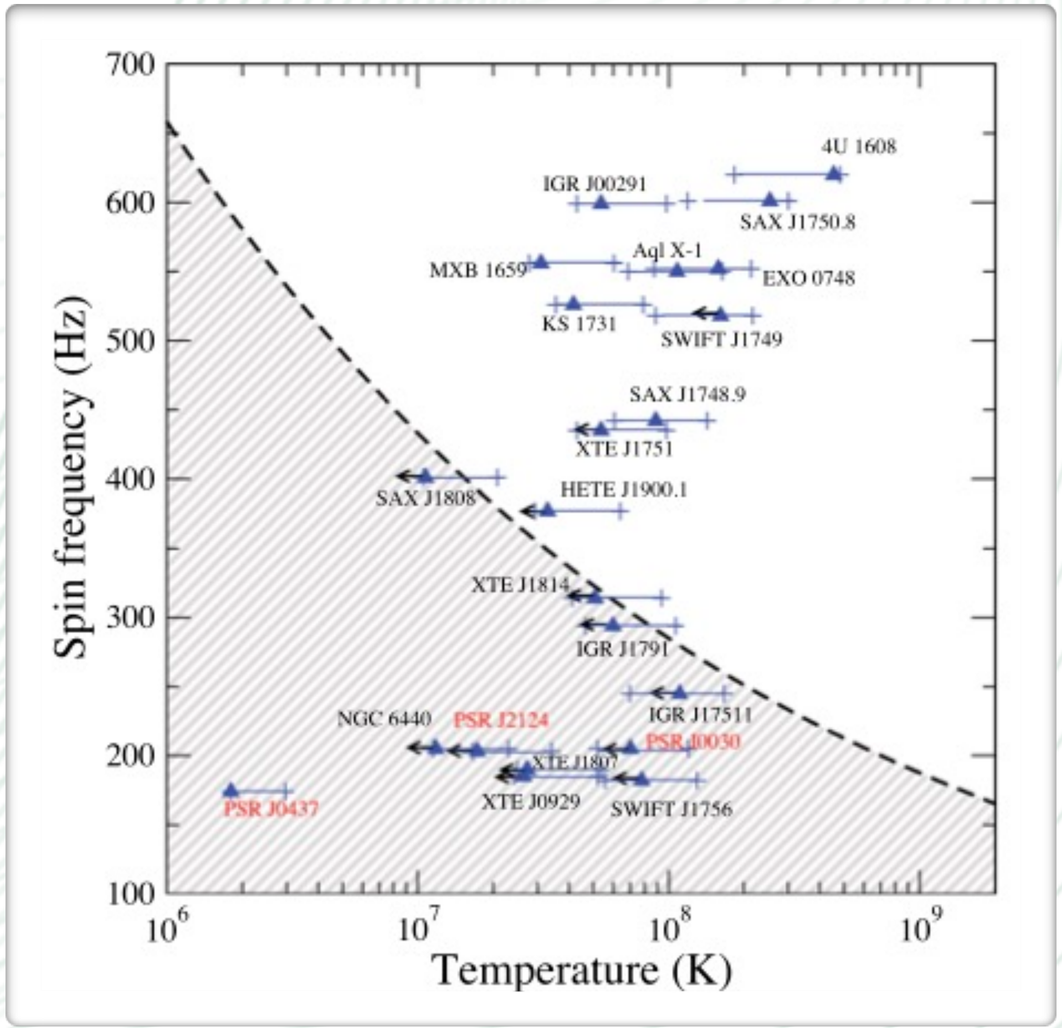
Uncertainties in the temperature estimates

Factors:

* uncertain envelope composition (considered in this plot)
* some systems may still be thermally relaxing to a steady state after an outburst and may have sizeable temperature gradients in the crust [Brown & Cumming, 2009]
* resulting factor of a few uncertainty seems to have no qualitative impact



[Haskell, Degenaar and Ho, 2012]

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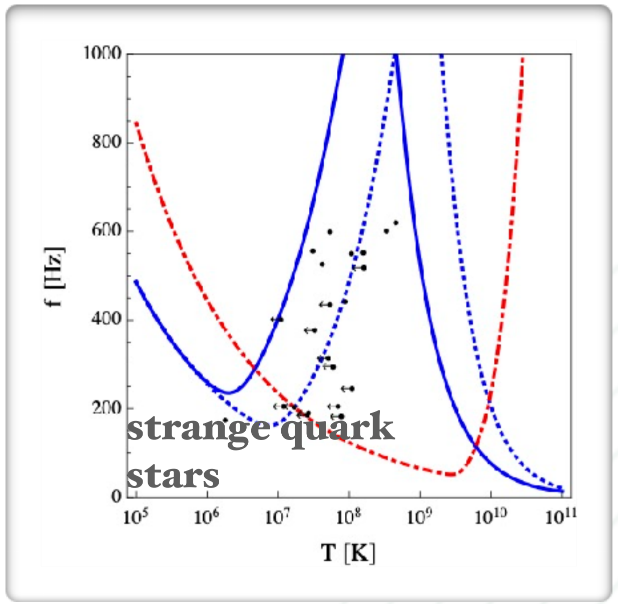
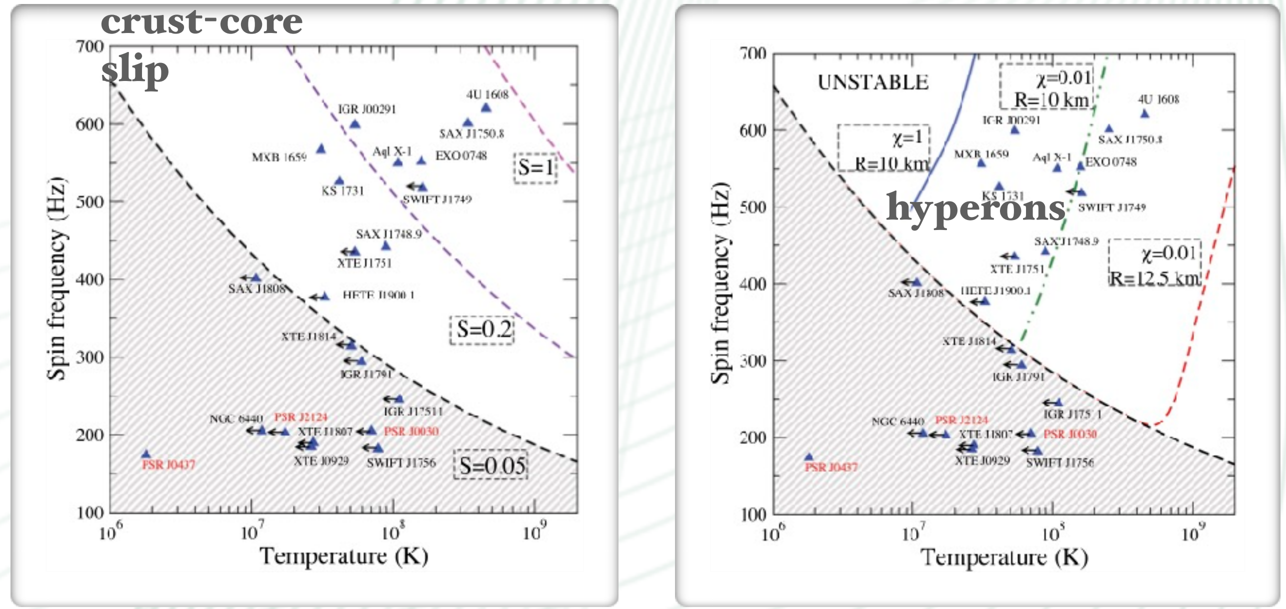
r-mode windows? Alternative models and uncertainties

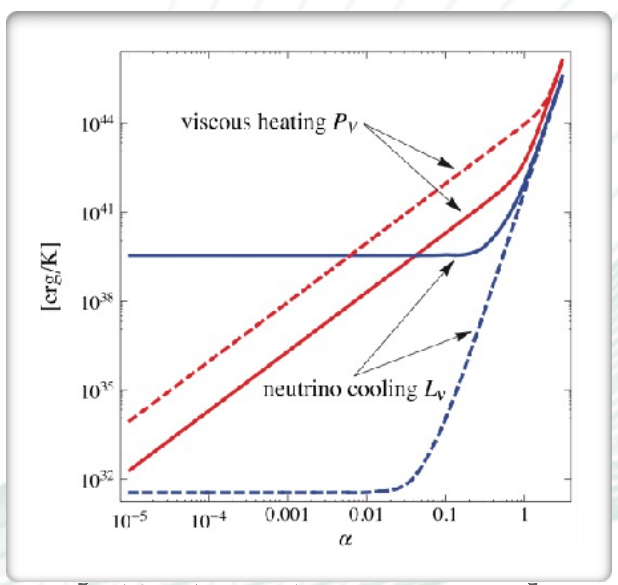


[Schwenzer, 2012]



[Haskell, Degenaar and Ho, 2012]

dashed: 10^8 K
solid: $10^9 K$

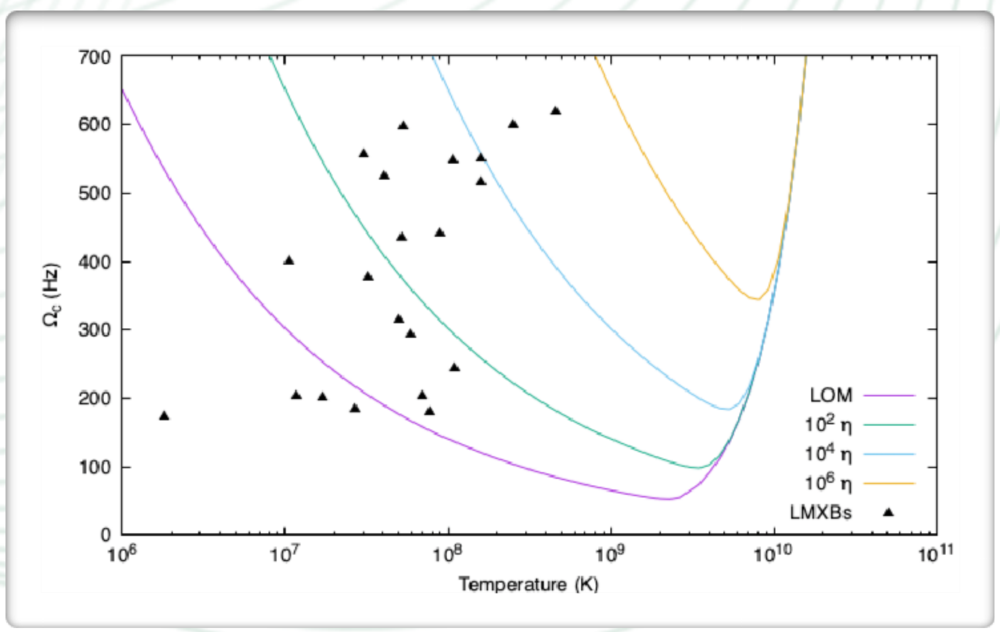


[Alford & Schwenzer, 2014]

shear viscosity:

$\eta = 347\rho^{9/4}T^{-2}$

[Cutler & Lindblom, 1987]



Corrections to the r-mode frequency

* For the $\ell = m = 2$ r-mode, the observed gravitational wave frequency σ is related to the rotating frame frequency σ_R by
$$\sigma = \sigma_R - 2\Omega = (\kappa - 2)\Omega, \quad \kappa \equiv \frac{\sigma_R}{\Omega}$$
* For a slowly rotating Newtonian star, $\sigma_R = \frac{2m\Omega}{\ell(\ell+1)} = (2/3)\Omega$
* Relativistic effects are quantified by the compactness *M/R*.
* The effect of fast rotation can be expressed as $\kappa = \kappa_0 + \kappa_2 \frac{\Omega^2}{\pi G \bar{\rho}_0}$
* If we know κ_0 and κ_2 as functions of the compactness and observe an r-mode, we can solve for *M/R*!

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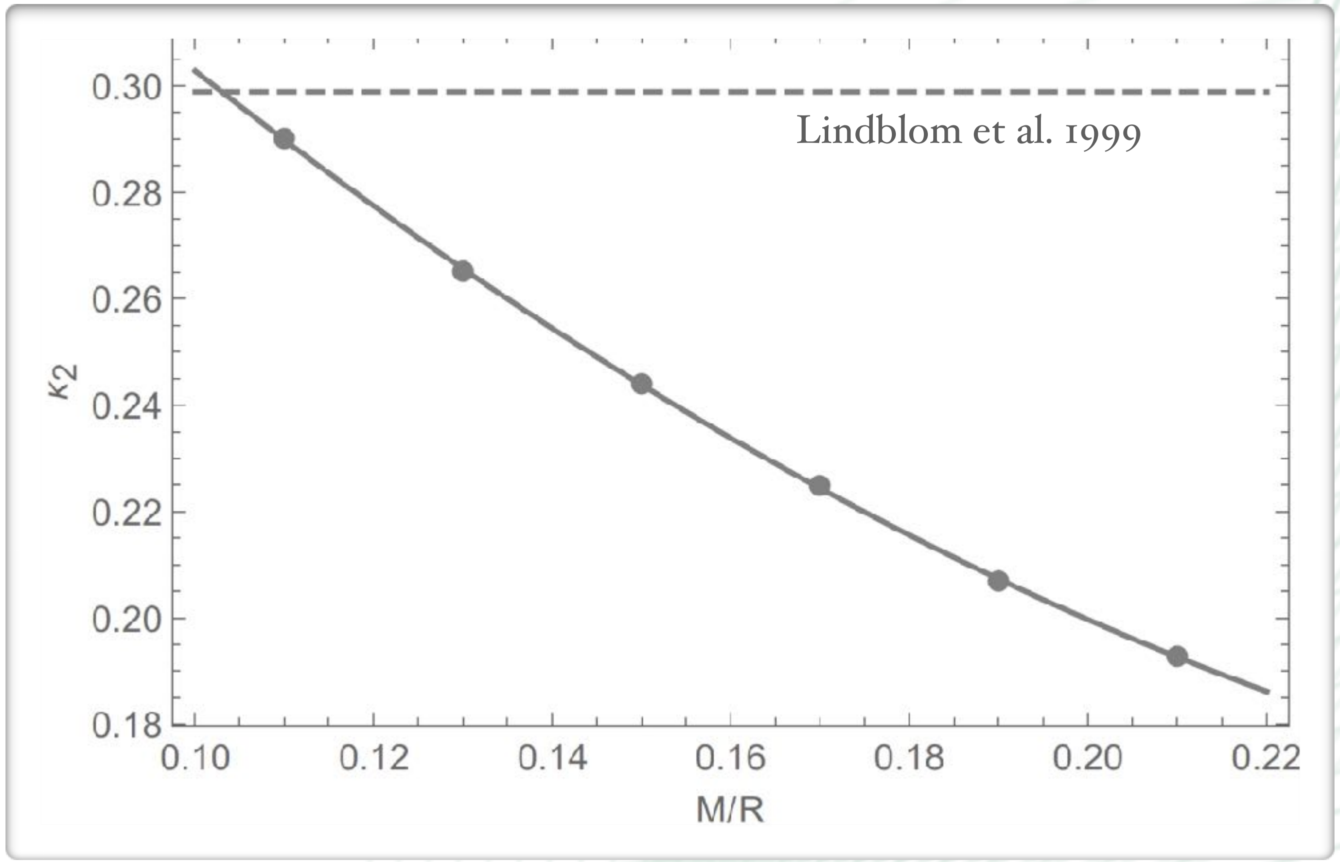
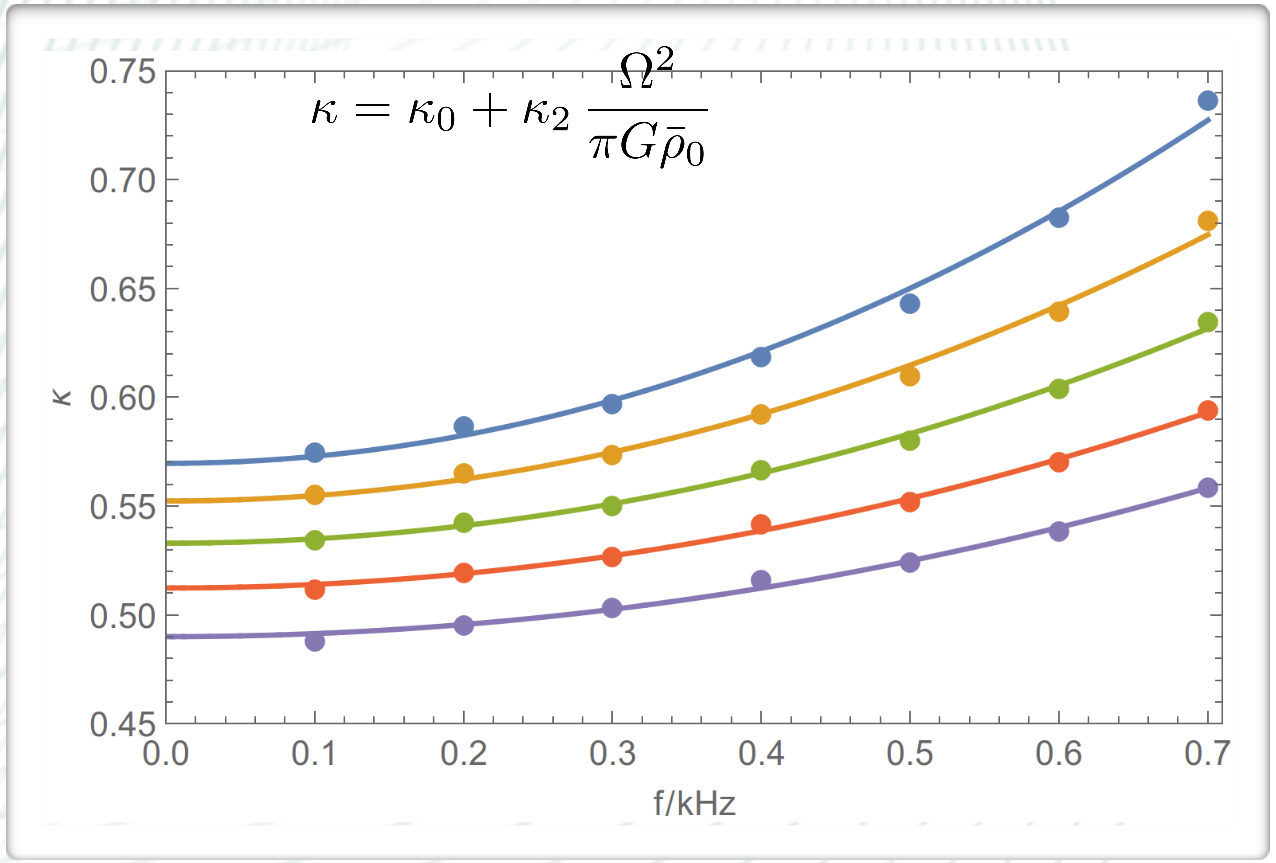
Corrections to the r-mode frequency

The rotational correction κ_2 is also a function of the compactness M/R!



[Chirenti & Jasiulek, 2018]

From top to bottom: M/R = 0.12, 0.14, 0.16, 0.18 and 0.20



[Chirenti & Jasiulek, 2017]

Observations of r-modes in LMXBs? XTE J1751-305 and 4U 1636-536

* Strohmayer & Mahmoodifar reported in 2014 oscillations consistent with r-modes in RXTE data:
    * XTE J1751-305: κ = 0.5727597 and f = 435 Hz (found in the 2002 discovery outburst)
    * 4U 1636-536: κ = 0.5645388 and f = 582 Hz (found in the 2001 thermonuclear superburst)
* More observations? NICER should be able to see these oscillations, if they are there! (But see also "Where are the r-modes?" by Mahmoodifar and Strohmayer, 2017).

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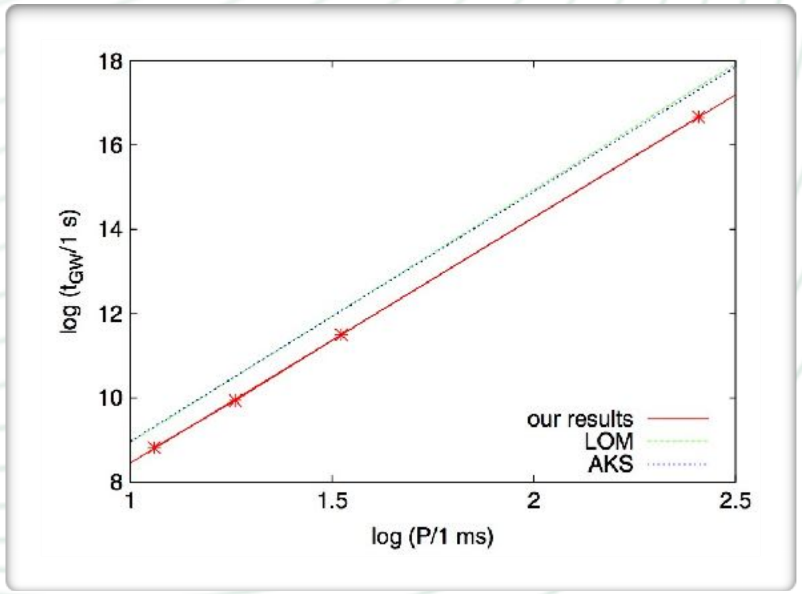
Caveats: constraints from measured spin-down and alternatives

* **Problem:** the r-mode amplitude derived from the observed modulation amplitude leads to a spin down rate too large to be consistent with the observations
  [Strohmayer & Mahmoodifar 2014]
* **Alternatives:** rotationally modified g-modes, r-modes modified by a solid crust, surface g-modes or r-modes, unstable toroidal crustal modes or amplification at the surface of a core r-mode with lower amplitude
  [Andersson, Jones & Ho, 2014; Lee, 2014]
* **Questions:**

  shape of the instability window?

  r-mode amplitude × signal amplitude?



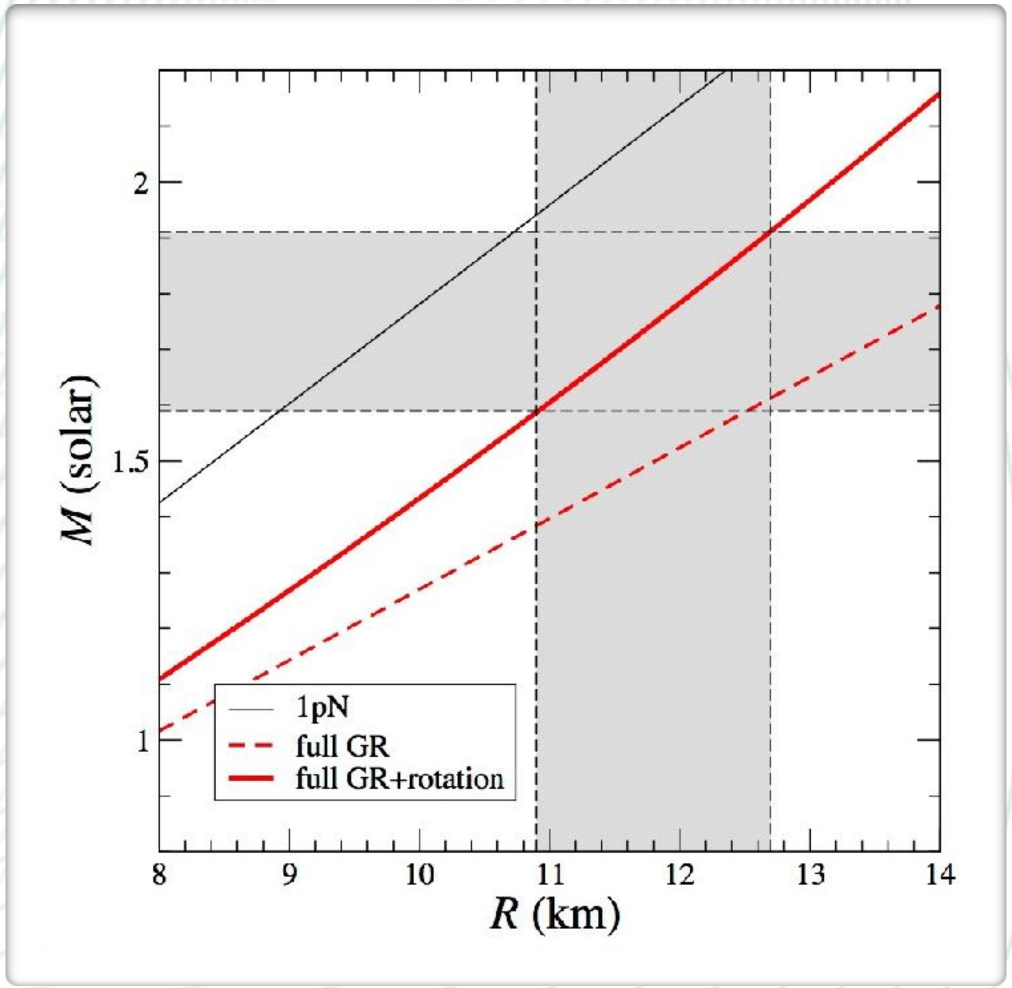
[Chirenti & Skákala, 2013]

An application: results for XTE J1751-305

Using

κ_0 for fully relativistic uniform density stars [Lokitch, Friedman & Andersson, 2003]

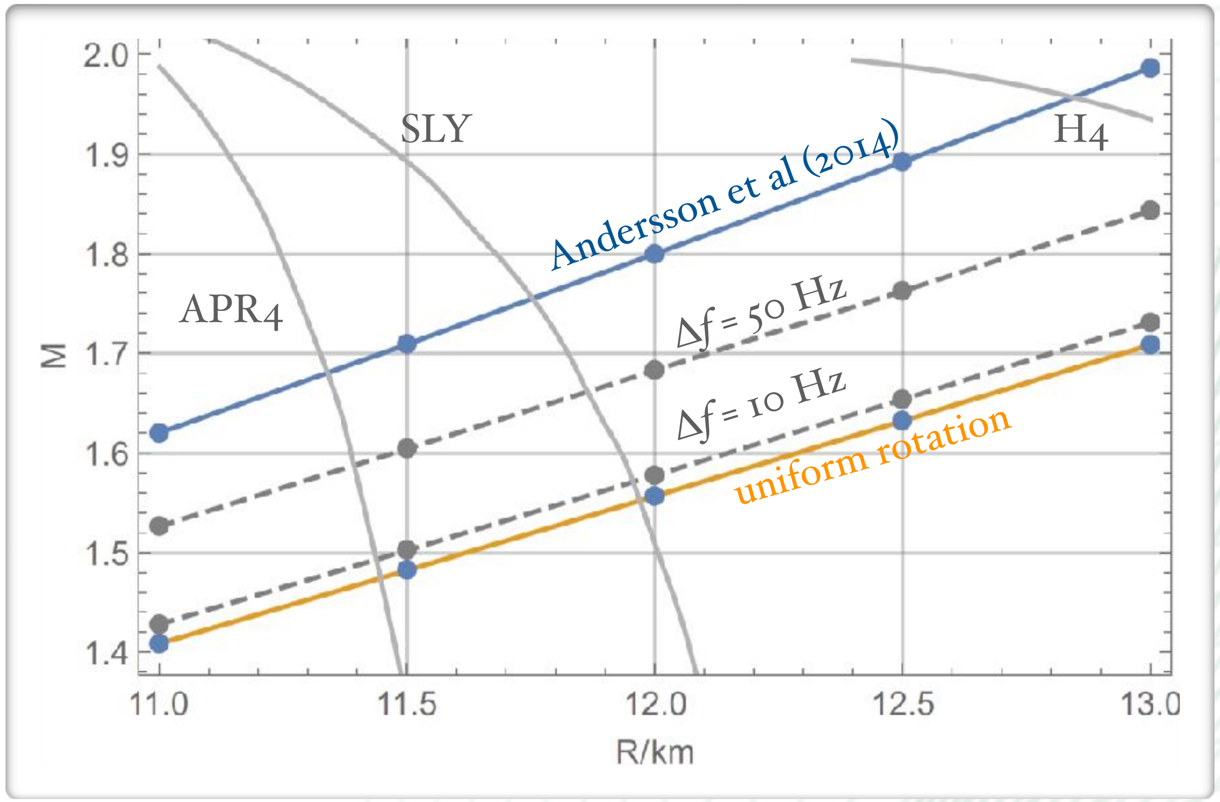
+ κ_2 for fast rotating Newtonian polytropes [Lindblom, Mendell & Owen, 1999]



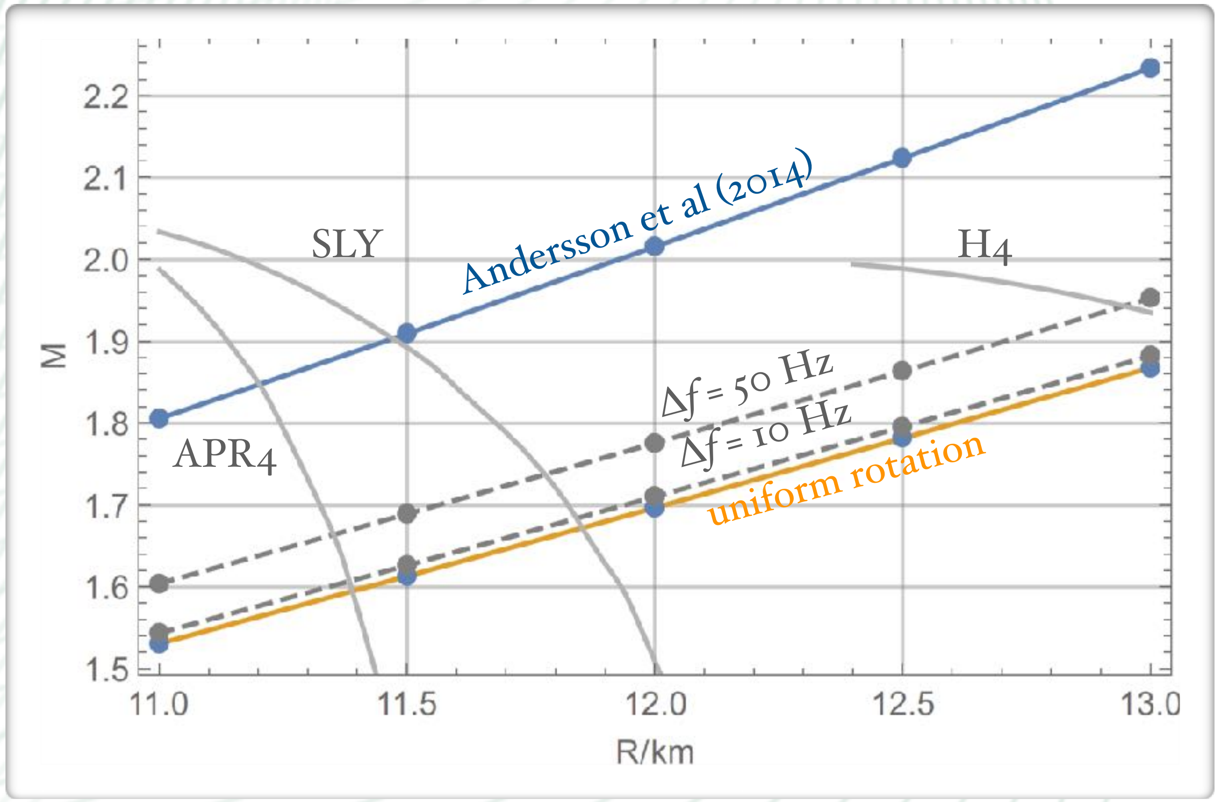
[Anderson, Jones & Ho, 2014]

More *M-R* constraints from the (possible) r-mode frequencies

XTE J1751-305



4U 1636-536



[Chirenti & Jasiulek, 2018]

For a radius range of 11 – 13 km, with the assumed r-mode detections and APR4 + SLY we find the mass estimates:

XTE J1751-305: 1.48 – 1.56 $M_\odot$ 4U 1636-536: 1.60 – 1.68 $M_\odot$

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~~Conclusions~~ Puzzles

Puzzle 1: What limits the spin of accreting neutron stars?

Effect of magnetic torques in accretion process? r-modes?

Puzzle 2: Why are there (so many) neutron stars in LMXBs and MSRPs that seem to be in the r-mode instability window?

T measurements? Modeling of the window? Physical assumptions?

Puzzle 3: r-modes in XTE J1751-305 and 4U 1636-536?

Modified r-modes? g-modes? mode amplitude × amplitude of X-ray modulation?

Puzzle 4: What is the r-mode saturation amplitude?

Will r-modes be detectable with LIGO? With 3G detectors?

Recommendation for observers: Look for r-modes in stars inside the window!

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