Equation of State	

Which phase transitions are possible in neutron stars?

Jan-Erik Christian



Seattle, August 27, 2024

EOS Measurements with Next-Generation Gravitational-Wave Detectors

Motivation	Equation of State	Twin Stars	Conclusion

Which first order phase transitions to quark matter are possible in neutron stars?

Phys.Rev.D 109

Jan-Erik Christian, Jürgen Schaffner-Bielich, Stephan Rosswog

Universität Hamburg Der Forschung | der lehre | der Bildung

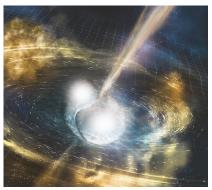
Seattle, August 27, 2024

EOS Measurements with Next-Generation Gravitational-Wave Detectors

Twin Stars

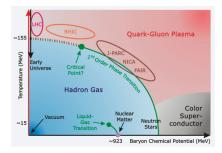
Neutron Stars

- Extremely dense final stage of stellar evolution.
- Used to test GR and emit gravitational waves.
- Masses are well known, radii less so.
- Observables can be calculated with the equation of state (EoS).



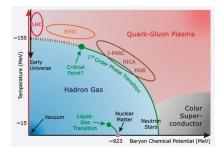
[Artistic render of neutron star merger, LIGO]

Motivation	Equation of State	
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Motivation	Equation of State	Twin Stars	Conclusion
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Motivation			

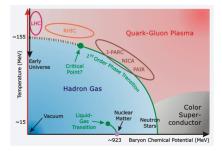
We know:



Motivation ○●	Equation of State	
Motivation		

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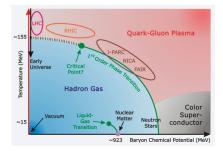
• Low density from terrestrial experiments and theory.



Motivation ○●	Equation of State	

We know:

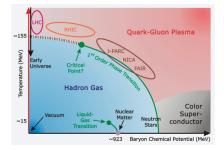
- Low density from terrestrial experiments and theory.
- Astrophysical constraints work at high density.



Motivation	Equation of State	
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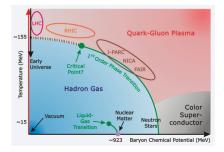
- Low density from terrestrial experiments and theory.
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Motivation ○●	Equation of State	

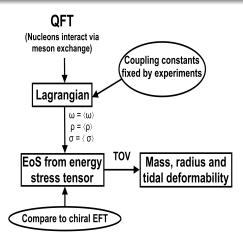
We know:

- Low density from terrestrial experiments and theory.
- Astrophysical constraints work at high density.
- A phase transition to QM will take place at some point.
- Where is the phase transition and how can we tell from mass, radius and tidal deformability constraints?



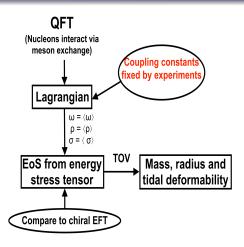
Equation of State	
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Relativistic Mean Field Approach



Equation of State	
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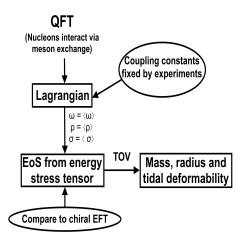
Relativistic Mean Field Approach



Effective mass: $m^*/m = 0.55 - 0.75$ Symmetry energy: J = 30 - 32 MeV Slope parameter: L = 40 - 60 MeV

Equation of State	
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Relativistic Mean Field Approach

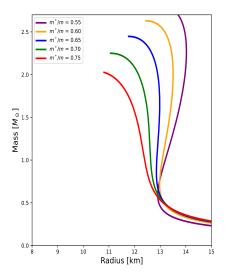


Effective mass: $m^*/m = 0.55 - 0.75$ Symmetry energy: J = 30 - 32 MeV Slope parameter: L = 40 - 60 MeV

J = 32 MeV and L = 60 MeV from chiral EFT.

• Setup following: [Hornick et al. 2018, Phys. Rev. C]

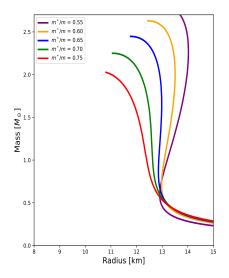
	Equation of State		
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Mass-Rad	ius Relations		



Equation of State ○●○○○	

Mass-Radius Relations

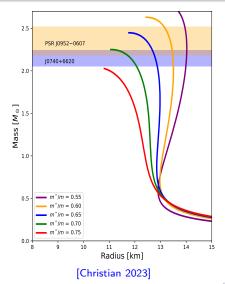
- Increasing the central pressure increases the mass.
- *m**/*m* is directly linked to an EoS's stiffness.
- Stiffer EoSs feature higher maximal masses and larger radii, they are less compact.



Equation of State ○○●○○	

Mass-Radius Constraints

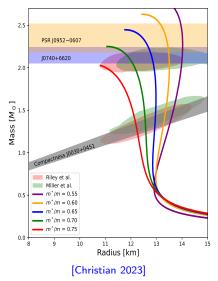
• Neutron stars with $2 M_{\odot}$ are known



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Mass-Radius Constraints

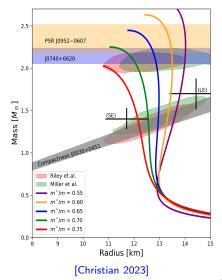
- Neutron stars with 2 M_{\odot} are known
- NICER measured radii between 11 - 16 km



Equation of State ○○●○○	

Mass-Radius Constraints

- Neutron stars with 2 M_{\odot} are known
- NICER measured radii between 11 - 16 km
- Potential candidates after NICER reanalysis (Vinciguerra et al. 2023)



Equation of State	

 In a binary system the companions tidal field induce a quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

Equation of State ○○○●○	

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Equation of State	
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• Upper limit for combined value:

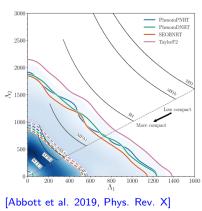
$$\tilde{\Lambda} = \tilde{\Lambda} \left(\Lambda_1, m_1, \Lambda_2, m_2 \right) \le 720$$

Equation of State	
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$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

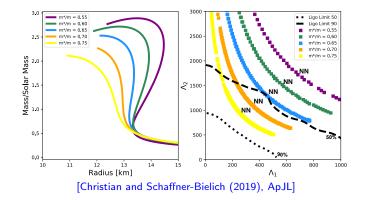
- Obtain dimensionless form: $\Lambda = \frac{\lambda}{m^5}$
- Upper limit for combined value:



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ight) \leq 720$$

Equation of State	
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Closer Look: Tidal Deformability Constraint

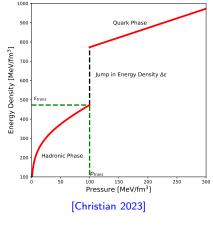


• Only EoSs with $m^*/m \ge 0.65$ are soft enough to fit the data.

Equation of State	Twin Stars ●○○○○○	

Constant Speed of Sound Quark Matter

- First order phase transition at critical pressure *p*_{trans}.
- Parameterization is well known. [Alford et. al. 2013, Phys. Rev. D]
- We use $c_{QM} = 1$.

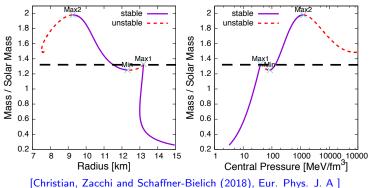


$$\epsilon(p) = egin{cases} \epsilon_{HM}(p) \ \epsilon_{HM}(p_{trans}) + \Delta \epsilon + c_{QM}^{-2}(p-p_{trans}) \end{cases}$$

	Equation of State	Twin Stars ○●○○○○	
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Twin Star Solutions

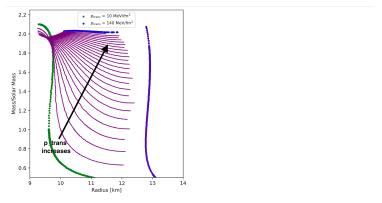
 Phase transition can lead to twin star solutions, where two stars have the same mass, but different radii.



Equation of State	Twin Stars	
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Parameter Effects on MR Relation; Hybrid vs Twin

• *p*_{trans} determines the first branch's maximum and the shape of the second branch.

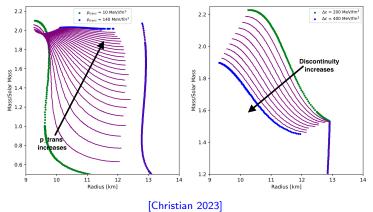


[Christian 2023]

Equation of State	Twin Stars	
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Parameter Effects on MR Relation; Hybrid vs Twin

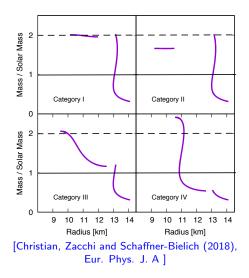
- *p*_{trans} determines the first branch's maximum and the shape of the second branch.
- $\Delta \epsilon$ strongly influences the second's maximum by determining the position of the second branch.



Equation of State	Twin Stars	
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Categories of Twin Stars

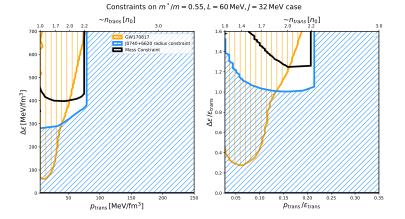
- Category I: Both maxima meet mass constraint M_{data}.
- Category II: Only the hadronic maximum exceeds *M*_{data}.
- **Category III**: Only the hybrid maximum exceeds M_{data} .
- Category IV: Only hybrid stars can be observed.



Equation of State	Twin Stars	
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Constraints on Stiff Equation of State

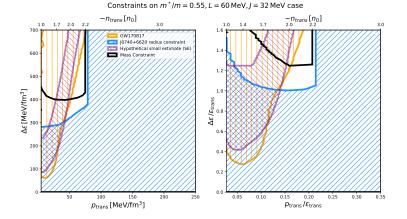
• The GW170817 constraint can be met with a phase transition.



Equation of State	Twin Stars	
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Constraints on Stiff Equation of State

• The GW170817 constraint can be met with a phase transition.

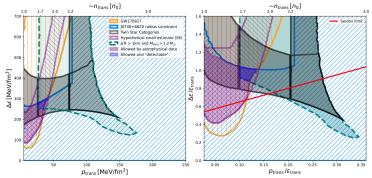


Jan-Erik Christian Phase Transitions in Neutron Stars

Equation of State	Twin Stars ○○○○●○	

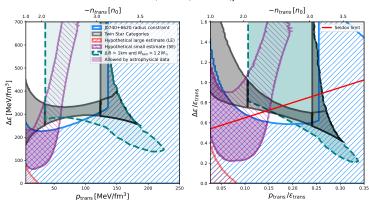
Constraints on Stiff Equation of State

- The GW170817 constraint can be met with a phase transition.
- A hypothetical well determined "small" star does not constrain a stiff EoS further.



Equation of State	Twin Stars	
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Constraints on Softer Equation of state

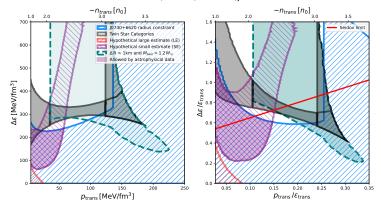


Constraints on $m^*/m = 0.65$, L = 60 MeV, J = 32 MeV case

Equation of State	Twin Stars	
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Constraints on Softer Equation of state

• Large parameter space allowed by constraints.

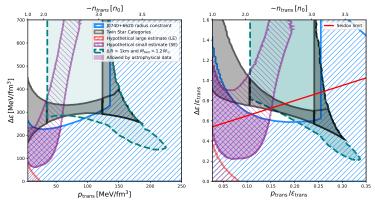


Constraints on $m^*/m = 0.65$, L = 60 MeV, l = 32 MeV case

Equation of State	Twin Stars	

Constraints on Softer Equation of state

- Large parameter space allowed by constraints.
- No significant ΔR in allowed parameter space.

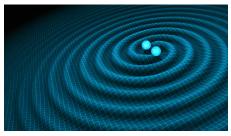


Constraints on $m^*/m = 0.65$, L = 60 MeV, J = 32 MeV case

[Christian et al. 2023]

Equation of State	Conclusion
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Summary and Outlook

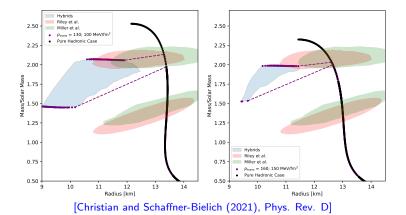


[LIGO]

- Phase transitions in neutron stars create unique mass radius relations and tidal deformability.
- The overlap between easily detectable and possible solution is shrinking rapidly.
- Gravitational wave measurements should be able to probe the area inaccessible by mass and radius constrains.

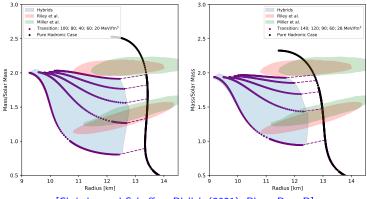
Equation of State	Conclusion

Category I and II NICER constraints



Equation of State	Conclusion

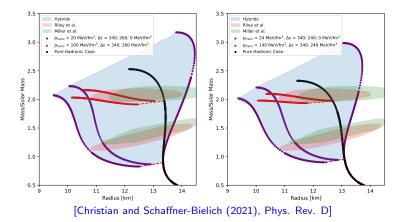
Category III NICER constraints



[Christian and Schaffner-Bielich (2021), Phys. Rev. D]

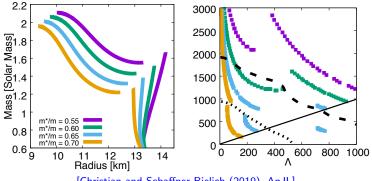
Equation of State	Conclusion

Hybrid stars NICER constraints



Equation of State	Conclusion

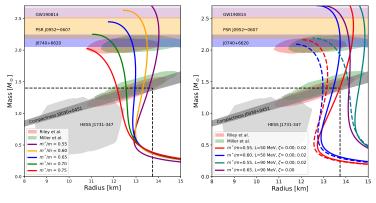
Tidal deformability changes GW170817



[Christian and Schaffner-Bielich (2019), ApJL]

Equation of State	Conclusion

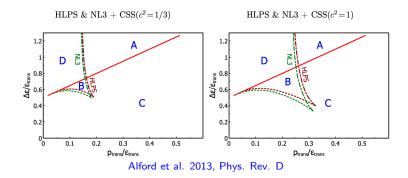
MR constraints for more RMF models



[Christian 2023]

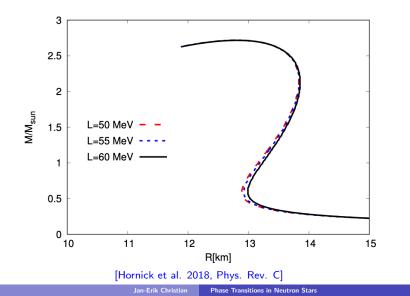
Equation of State	Conclusion

Influence of c_{QM} and hadronic EoS on parameter space



Equation of State	Conclusion
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Backup Slide



	Equation of State		Conclusion
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Parameter Variation

