| Equation of State | |
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Which phase transitions are possible in neutron stars?

Jan-Erik Christian



Seattle, August 27, 2024

EOS Measurements with Next-Generation Gravitational-Wave Detectors

| Motivation | Equation of State | Twin Stars | Conclusion |
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Which first order phase transitions to quark matter are possible in neutron stars?

Phys.Rev.D 109

Jan-Erik Christian, Jürgen Schaffner-Bielich, Stephan Rosswog

Universität Hamburg Der Forschung | der lehre | der Bildung

Seattle, August 27, 2024

EOS Measurements with Next-Generation Gravitational-Wave Detectors

Twin Stars

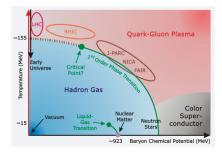
Neutron Stars

- Extremely dense final stage of stellar evolution.
- Used to test GR and emit gravitational waves.
- Masses are well known, radii less so.
- Observables can be calculated with the equation of state (EoS).



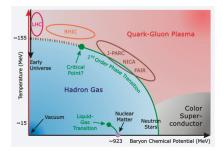
[Artistic render of neutron star merger, LIGO]

| Motivation | Equation of State | |
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| Motivation | Equation of State | Twin Stars | Conclusion |
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| Motivation | | | |

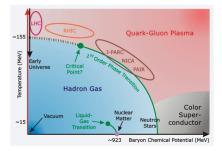
We know:



| Motivation ○● | Equation of State | |
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| Motivation | | |

We know:

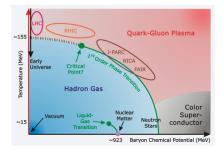
• Low density from terrestrial experiments and theory.



| Motivation ○● | Equation of State | |
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We know:

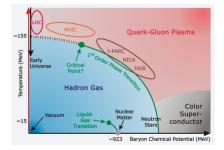
- Low density from terrestrial experiments and theory.
- Astrophysical constraints work at high density.



| Motivation | Equation of State | |
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We know:

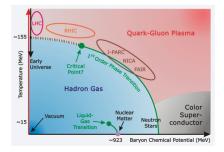
- Low density from terrestrial experiments and theory.
- Astrophysical constraints work at high density.
- A phase transition to QM will take place at some point.



| Motivation ○● | Equation of State | |
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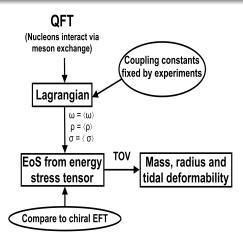
We know:

- Low density from terrestrial experiments and theory.
- Astrophysical constraints work at high density.
- A phase transition to QM will take place at some point.
- Where is the phase transition and how can we tell from mass, radius and tidal deformability constraints?



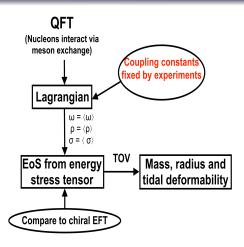
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Relativistic Mean Field Approach



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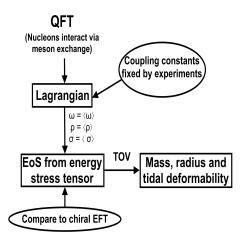
Relativistic Mean Field Approach



Effective mass: $m^*/m = 0.55 - 0.75$ Symmetry energy: J = 30 - 32 MeV Slope parameter: L = 40 - 60 MeV

| Equation of State | |
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Relativistic Mean Field Approach

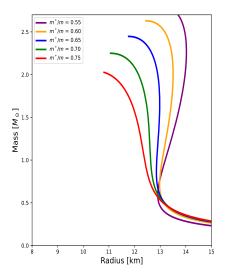


Effective mass: $m^*/m = 0.55 - 0.75$ Symmetry energy: J = 30 - 32 MeV Slope parameter: L = 40 - 60 MeV

J = 32 MeV and L = 60 MeV from chiral EFT.

• Setup following: [Hornick et al. 2018, Phys. Rev. C]

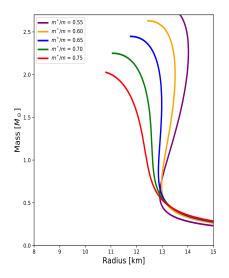
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| Mass-Rad | ius Relations | | |



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Mass-Radius Relations

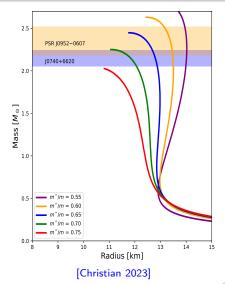
- Increasing the central pressure increases the mass.
- *m**/*m* is directly linked to an EoS's stiffness.
- Stiffer EoSs feature higher maximal masses and larger radii, they are less compact.



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Mass-Radius Constraints

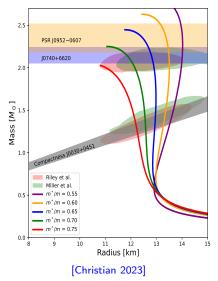
• Neutron stars with $2 M_{\odot}$ are known



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Mass-Radius Constraints

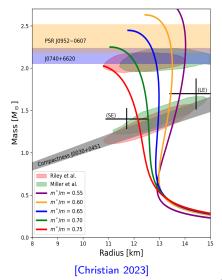
- Neutron stars with 2 M_{\odot} are known
- NICER measured radii between 11 - 16 km



| Equation of State ○○●○○ | |
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Mass-Radius Constraints

- Neutron stars with 2 M_{\odot} are known
- NICER measured radii between 11 - 16 km
- Potential candidates after NICER reanalysis (Vinciguerra et al. 2023)



| Equation of State | |
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 In a binary system the companions tidal field induce a quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

| Equation of State ○○○●○ | |
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 In a binary system the companions tidal field induce a quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

• Obtain dimensionless form:

$$\Lambda = \frac{\lambda}{m^5}$$

| Equation of State | |
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 In a binary system the companions tidal field induce a quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

• Obtain dimensionless form:

$$\Lambda = \frac{\lambda}{m^5}$$

• Upper limit for combined value:

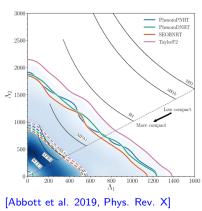
$$\tilde{\Lambda} = \tilde{\Lambda} \left(\Lambda_1, m_1, \Lambda_2, m_2 \right) \le 720$$

| Equation of State | |
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 In a binary system the companions tidal field induce a quadrupole moment:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

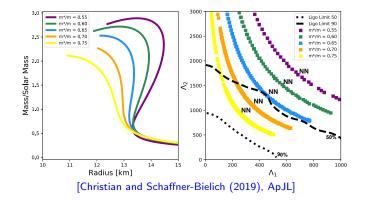
- Obtain dimensionless form: $\Lambda = \frac{\lambda}{m^5}$
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$$ilde{\Lambda} = ilde{\Lambda} \left(\Lambda_1, m_1, \Lambda_2, m_2
ight) \leq 720$$

| Equation of State | |
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Closer Look: Tidal Deformability Constraint

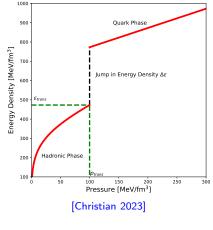


• Only EoSs with $m^*/m \ge 0.65$ are soft enough to fit the data.

| Equation of State | Twin Stars ●○○○○○ | |
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Constant Speed of Sound Quark Matter

- First order phase transition at critical pressure *p*_{trans}.
- Parameterization is well known. [Alford et. al. 2013, Phys. Rev. D]
- We use $c_{QM} = 1$.

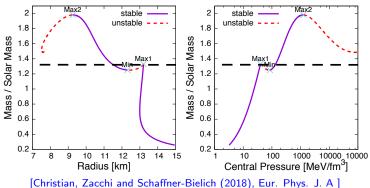


$$\epsilon(p) = egin{cases} \epsilon_{HM}(p) \ \epsilon_{HM}(p_{trans}) + \Delta \epsilon + c_{QM}^{-2}(p-p_{trans}) \end{cases}$$

| | Equation of State | Twin Stars ○●○○○○ | |
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Twin Star Solutions

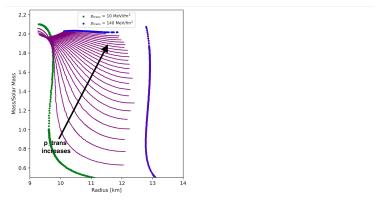
 Phase transition can lead to twin star solutions, where two stars have the same mass, but different radii.



| Equation of State | Twin Stars | |
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Parameter Effects on MR Relation; Hybrid vs Twin

• *p*_{trans} determines the first branch's maximum and the shape of the second branch.

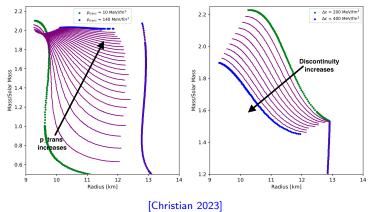


[Christian 2023]

| Equation of State | Twin Stars | |
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Parameter Effects on MR Relation; Hybrid vs Twin

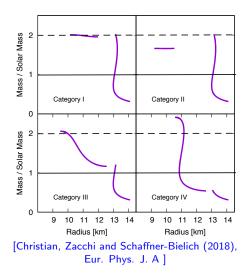
- *p*_{trans} determines the first branch's maximum and the shape of the second branch.
- $\Delta \epsilon$ strongly influences the second's maximum by determining the position of the second branch.



| Equation of State | Twin Stars | |
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Categories of Twin Stars

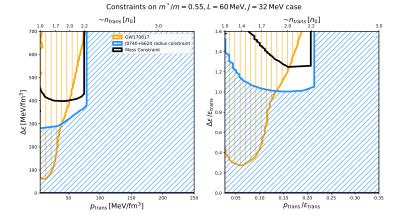
- Category I: Both maxima meet mass constraint M_{data}.
- Category II: Only the hadronic maximum exceeds *M*_{data}.
- **Category III**: Only the hybrid maximum exceeds M_{data} .
- Category IV: Only hybrid stars can be observed.



| Equation of State | Twin Stars | |
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Constraints on Stiff Equation of State

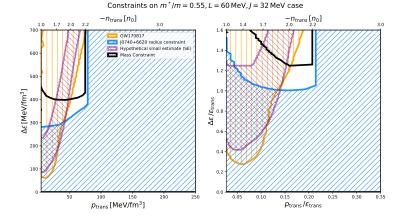
• The GW170817 constraint can be met with a phase transition.



| Equation of State | Twin Stars | |
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Constraints on Stiff Equation of State

• The GW170817 constraint can be met with a phase transition.

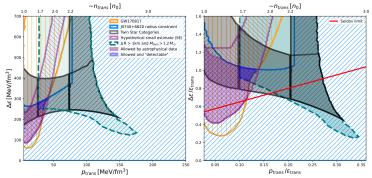


Jan-Erik Christian Phase Transitions in Neutron Stars

| Equation of State | Twin Stars ○○○○●○ | |
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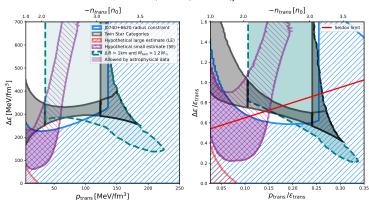
Constraints on Stiff Equation of State

- The GW170817 constraint can be met with a phase transition.
- A hypothetical well determined "small" star does not constrain a stiff EoS further.



| Equation of State | Twin Stars | |
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Constraints on Softer Equation of state

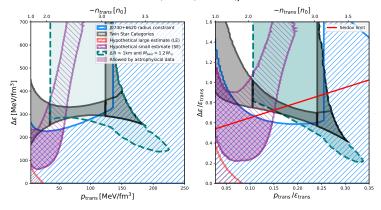


Constraints on $m^*/m = 0.65$, L = 60 MeV, J = 32 MeV case

| Equation of State | Twin Stars | |
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Constraints on Softer Equation of state

• Large parameter space allowed by constraints.

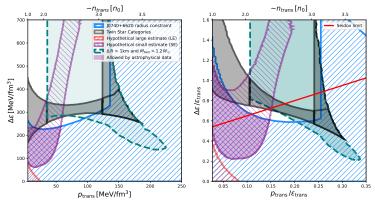


Constraints on $m^*/m = 0.65$, L = 60 MeV, l = 32 MeV case

| Equation of State | Twin Stars | |
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Constraints on Softer Equation of state

- Large parameter space allowed by constraints.
- No significant ΔR in allowed parameter space.

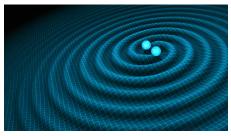


Constraints on $m^*/m = 0.65$, L = 60 MeV, J = 32 MeV case

[Christian et al. 2023]

| Equation of State | Conclusion |
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Summary and Outlook

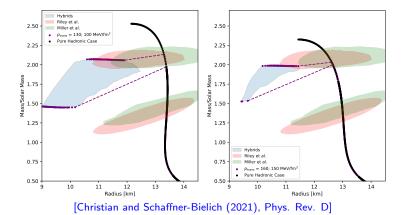


[LIGO]

- Phase transitions in neutron stars create unique mass radius relations and tidal deformability.
- The overlap between easily detectable and possible solution is shrinking rapidly.
- Gravitational wave measurements should be able to probe the area inaccessible by mass and radius constrains.

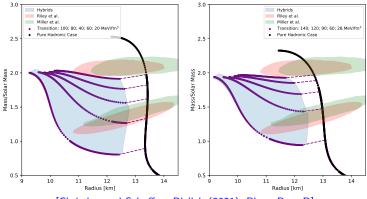
| Equation of State | Conclusion |
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Category I and II NICER constraints



| Equation of State | Conclusion |
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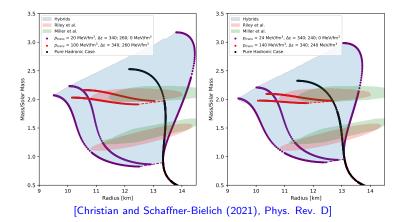
Category III NICER constraints



[Christian and Schaffner-Bielich (2021), Phys. Rev. D]

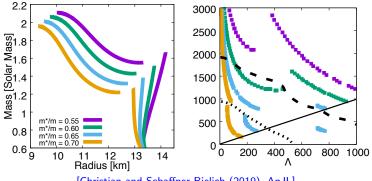
| Equation of State | Conclusion |
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Hybrid stars NICER constraints



| Equation of State | Conclusion |
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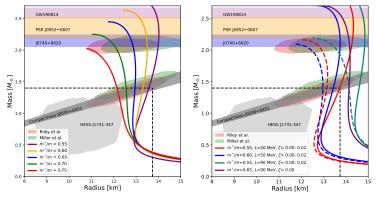
Tidal deformability changes GW170817



[Christian and Schaffner-Bielich (2019), ApJL]

| Equation of State | Conclusion |
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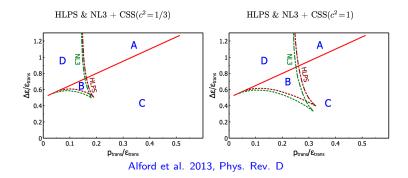
MR constraints for more RMF models



[Christian 2023]

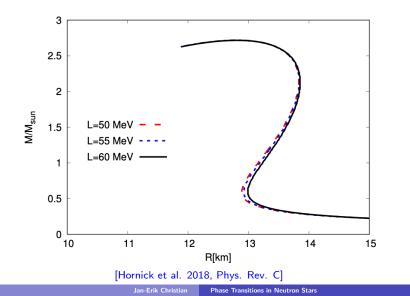
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Influence of c_{QM} and hadronic EoS on parameter space



| Equation of State | Conclusion |
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Backup Slide



| | Equation of State | | Conclusion |
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Parameter Variation

