First-order nucleon-to-quark phase transition: Thermodynamically-consistent constructions other than Maxwell and Gibbs

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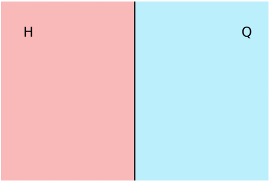
Motivation: Hybrid Stars (?)

- The size of nucleons (uncertain as it may be) implies that deconfined quark matter can exist in the cores of NSs.
- However, such a possibility lacks observational and theoretical support:
  - Measurements of M, R, Λ cannot differentiate normal and hybrid stars.
  - LQCD and PQCD not applicable to NS conditions.
- Possible solution: identify an observable with strong dependence on composition. (Here: g-modes)

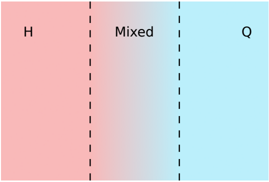
Hybrid Matter: 1st Order Transitions

- Maxwell ("strong", "stiff", ...)
  - Infinite interface tension
  - No phase mixing
  - Local charge neutrality
  - $\varepsilon = \chi(\varepsilon_H + \varepsilon_{eH}) + (1-\chi)(\varepsilon_Q + \varepsilon_{eQ})$



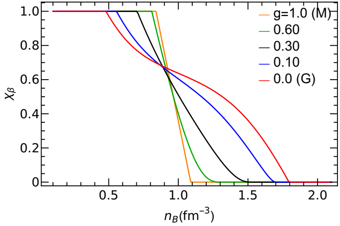
- Gibbs ("weak", "soft", ...)
  - Zero surface tension
  - Complete phase mixing
  - Global charge neutrality
  - $\varepsilon = \chi\,\varepsilon_H + (1-\chi)\,\varepsilon_Q + \varepsilon_{eM}$

- Intermediate case
  - Some phase mixing
  - Charge neutrality is partially local and partially global
  - $\varepsilon = \chi(\varepsilon_H + \eta\,\varepsilon_{eH}) + (1-\chi)(\varepsilon_Q + \eta\,\varepsilon_{eQ}) + (1-\eta)\varepsilon_{eM}$

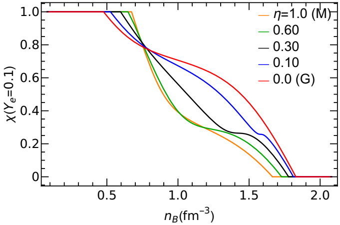


Hybrid Matter: 1st Order Transitions (cont'd)

- Constraints
  - Baryon number conservation
    $1 = \chi(y_n + y_p) + (1-\chi)(y_u + y_d + y_s)/3$
  - Lepton number conservation
    $0 = y_e - \chi\eta y_{eH} - (1-\chi)\eta y_{eQ} - (1-\eta)y_{eM}$
  - Local charge neutrality
    $0 = (y_p - y_{eH}) = (2y_u - y_d - y_s)/3 - y_{eQ}$
  - Global charge neutrality
    $0 = \chi y_p + (1-\chi)(2y_u - y_d - y_s)/3 - y_{eM}$



- Equilibrium (= minimization of ε wrt χ, y_i, η)
  - Mechanical, $P_H + \eta P_{eH} = P_Q + \eta P_{eQ}$
  - Quark weak, $\mu_d = \mu_s$
  - Neutral strong, $\mu_n = \mu_u + 2\mu_d$
  - Charged strong, $\mu_p = 2\mu_u + \mu_d - \eta(\mu_{eH} - \mu_{eQ})$
  - Beta $\mu_d = \mu_u + \eta\mu_{eQ} + (1-\eta)\mu_{eM}$
    -or- $\mu_p = \mu_n - \eta\mu_{eH} - (1-\eta)\mu_{eM}$
  - η optimization, $\varepsilon_{eM} = \chi\varepsilon_{eH} + (1-\chi)\varepsilon_{eQ}$



Equation of State

- Nucleons: Zhao - Lattimer

$$\epsilon_B = \sum_{h=n,p} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{M_B^2 + k^2}\, dk + n_B V(u,x)$$

$$V = 4x(1-x)(a_0 u + b_0 u^{\gamma}) + (1-2x)^2 (a_1 u + b_1 u^{\gamma_1})$$

- Quarks: vMIT

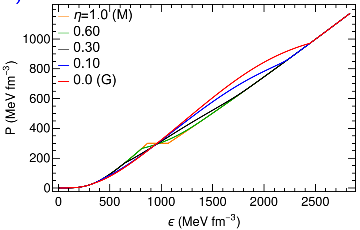
$$\mathcal{L} = \sum_{q=u,d,s} \left[\bar{\psi}_q \left(i\partial\!\!\!/ - m_q - B \right) \psi_i + \mathcal{L}_{\rm int} \right] \Theta$$

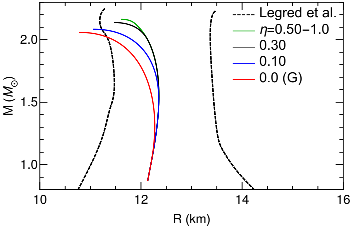
$$\mathcal{L}_{\rm int} = -G_v \sum_q \bar{\psi} \gamma_\mu V^\mu \psi + \left(m_V^2/2 \right) V_\mu V^\mu$$

$$\epsilon_Q = \sum_q \epsilon_{\rm FG,q} + \frac{1}{2} \left(\frac{G_v}{m_V} \right)^2 n_Q^2 + B$$

- Leptons: noninteracting, relativistic fermions

$$\epsilon_L = \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{m_L^2 + k^2}\, dk$$





Finite Temperature

- $\sum_i \frac{\gamma}{2\pi^2} \int_0^{k_{Fi}} k^2 \epsilon_{ki}\ dk \longrightarrow \sum_i \frac{\gamma}{2\pi^2} \int_0^{\infty} k^2 \frac{\epsilon_{ki}}{\exp\left[\frac{(\epsilon_{ki}-\mu_i)}{T}\right]+1}\ dk$

  where $\epsilon_{ki} = (k_i^2 + m_i^2)^{1/2} \frac{\delta\varepsilon}{\delta\tau_i} + \frac{1}{n_B} \frac{\delta\varepsilon}{\delta y_i}$

  - For ZL and vMIT, $\epsilon_{ki} = (k_i^2 + m_i^2)^{1/2} + U(n_B, \{y_i\})$

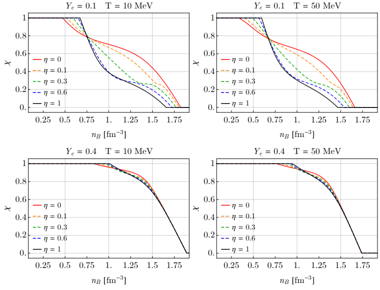
    $\Rightarrow$ Thermal effects of an ideal gas with $m_i^* = (k_{Fi}^2 + m_i^2)^{1/2}$

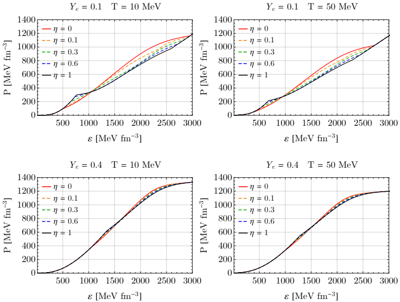
- $\epsilon \longrightarrow \mathcal{F}$ (inclusive of antiparticle contributions)

  Minimization leads to the same equilibrium conditions as in the $T = 0$ case (except for η optimization where $\epsilon \rightarrow \mathcal{F}$).

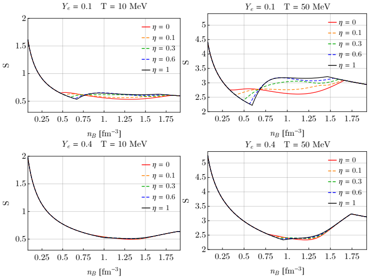
  Additionally, $\mu_i = -\mu_{\bar{i}}$.

Finite Temperature: Some Results



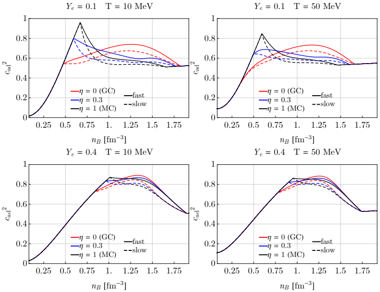


- Increasing T shifts the phase coexistence region to lower densities; width decreases.
- Increasing Y_e leads to weaker sensitivity on η.
- Nonmonotonic behavior of S as a function of n_B in the mixed phase. Similar traits exhibited by C_V and C_P.



Fast vs Slow conversion

- The conversion rate between hadronic and quark matter in the mixed phase is unknown.
- If it is sufficiently fast such that no type of reaction can reach equilibrium, then all Y_i are free variables and must be held constant when taking derivatives (equilibrium conditions imposed afterwards).
- If it is slow enough then only β-reactions can be out of equilibrium and thus the only independent particle fraction to be held constant is Y_e (equilibrium conditions imposed before taking the derivative).
- 1st derivatives (e.g. P, S, ...) are not affected by this distinction. This is not the case for 2nd derivatives (c_{ad}, C_V, C_P, ...) and higher.
- Example: c_{ad}



Application: g-modes

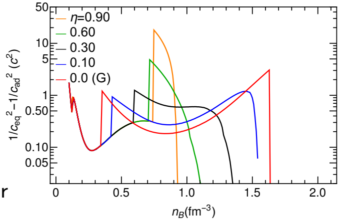
- Global, long-lived, nonradial fluid oscillations resulting from fluid-element perturbations in a stratified environment.
- Slow chemical equilibration generates buoyancy forces to oppose dispacement.
- In stably-stratified systems the opposing force sets up oscillations with a characteristic frequency [Brunt-Väisälä, $N^2 = g^2 \Delta(c^{-2}) e^{\nu-\lambda}$] which depends on both the equilibrium and the adiabatic sound speeds [$\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$].
  - $c_{\rm eq}^2(n_B) = \frac{dP}{d\varepsilon} = \frac{dP_\beta}{dn_{\rm B}} \left(\frac{d\varepsilon_\beta}{dn_{\rm B}} \right)^{-1}$
    mechanical equilibrium restored instantaneously.
  - $c_{\rm ad}^2(n_{\rm B}, x) = \left(\frac{\partial P}{\partial \varepsilon} \right)_x = \left. \frac{\partial P}{\partial n_{\rm B}} \right|_x \left(\left. \frac{\partial \varepsilon}{\partial n_{\rm B}} \right|_x \right)^{-1}$
    $c_{{\rm ad},\beta}^2(n_{\rm B}) = c_{\rm ad}^2[n_{\rm B}, x_\beta(n_{\rm B})]$
    slow restoration of chemical equilibrium because $\tau_\beta \gg \tau_{\rm oscillation}$.
- g-mode oscillations couple to tidal forces; they can be excited in a NS merger and provide information on the interior composition.
- Detection remains a challenge; but within sensitivity of 3rd generation detectors.

g-mode Signals

- $\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$ drives the restoring force for g-mode oscillations. In *npe* matter,

$$c_{\rm ad}^2 = c_{\rm eq}^2 + \left[n_B \left(\frac{\partial\tilde{\mu}}{\partial n_B}\right)_x\right]^2 \left[\mu_n \left(\frac{\partial\tilde{\mu}}{\partial x}\right)_{n_B}\right]^{-1}$$

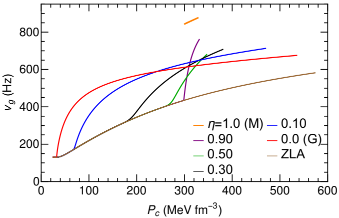
$$\tilde{\mu} = \mu_e + \mu_p - \mu_n \stackrel{\beta-\rm eq.}{\longrightarrow} 0$$



- g-modes in 1st-order hybrid matter have larger frequency range compared to pure nucleon matter $\widehat{=}$ the behavior of $\Delta(c^{-2})$ in the mixed phase.

- Dramatic changes in ν_g require new DOFs not just a smooth change in composition.

- Discontinuity g-modes
  - Generated by the flatness of $P(n_B)$ in a Maxwell mixed phase that leads to a density jump in the core of a hybrid star.
  - Characterized by discontinuous g-mode frequencies.
  - A special case of a compositional g-mode in the limit $\eta \to 1$.



Summary

- Construction of a thermodynamically-consistent framework for the treatment of 1st-order phase transitions intermediate to Maxwell and Gibbs at zero and finite temperature.
- Beware of the Maxwell construction!
- The assumption of slow or fast conversion between hadronic and quark matter in the mixed phase, affects second derivatives of the free energy (e.g. c_{ad}, C_V, C_P) at the $\sim$ 10 % level.
- g-modes can detect nonnucleonic matter in the cores of NS; assuming quark matter (by some other means), g-modes can distinguish between a first-order phase transition and a crossover.
- Discontinuity g-modes as a special case of compositional g-modes in the Maxwell limit.
- (Near) Future:
  - Applications to protoneutron stars (cooling, superfluidity) and binary mergers.
  - Construct EOS that uses the same underlying description for quarks and hadrons.