Structure of the proton from lattice QCD: 1-D and beyond

Martha Constantinou

Temple University

INT Workshop
Heavy Ion Physics in the EIC Era

August 21, 2024

Outline

★ Approaches to parsonic structure from lattice QCD

★ Recent results on Mellin moments for proton:
- ⇨ Axial form factors
- ⇨ E/M form factors

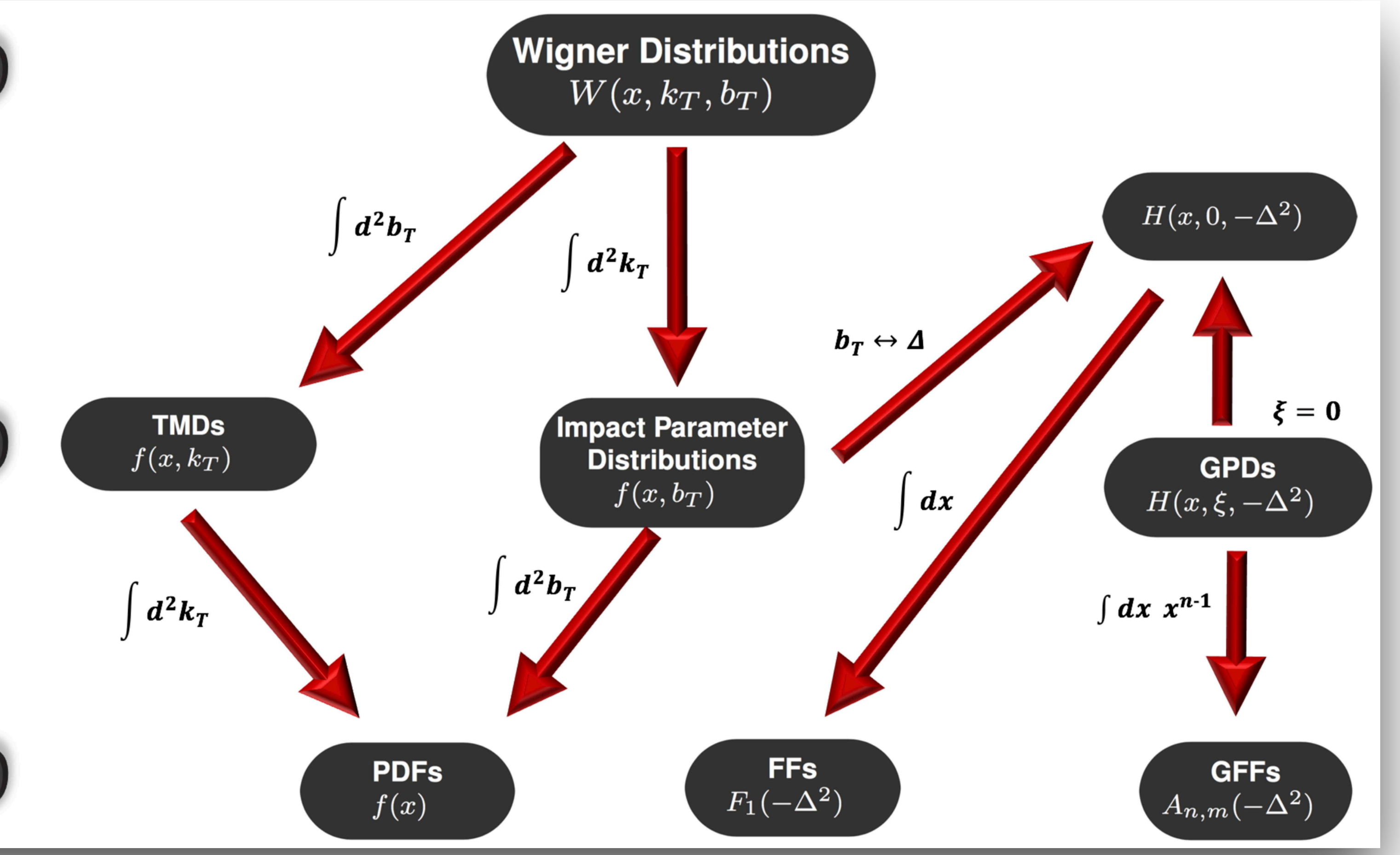
★ x-dependence of GPDs:
- ⇨ leading-twist results
- ⇨ subleasing-twist contributions
- ⇨ new promising method

★ Summary - Outlook

Nucleon Characterization

Wigner distributions

- ★ provide multi-dim images of the parton distributions in phase space
- ★ encode both TMDs and GPDs in a unified picture



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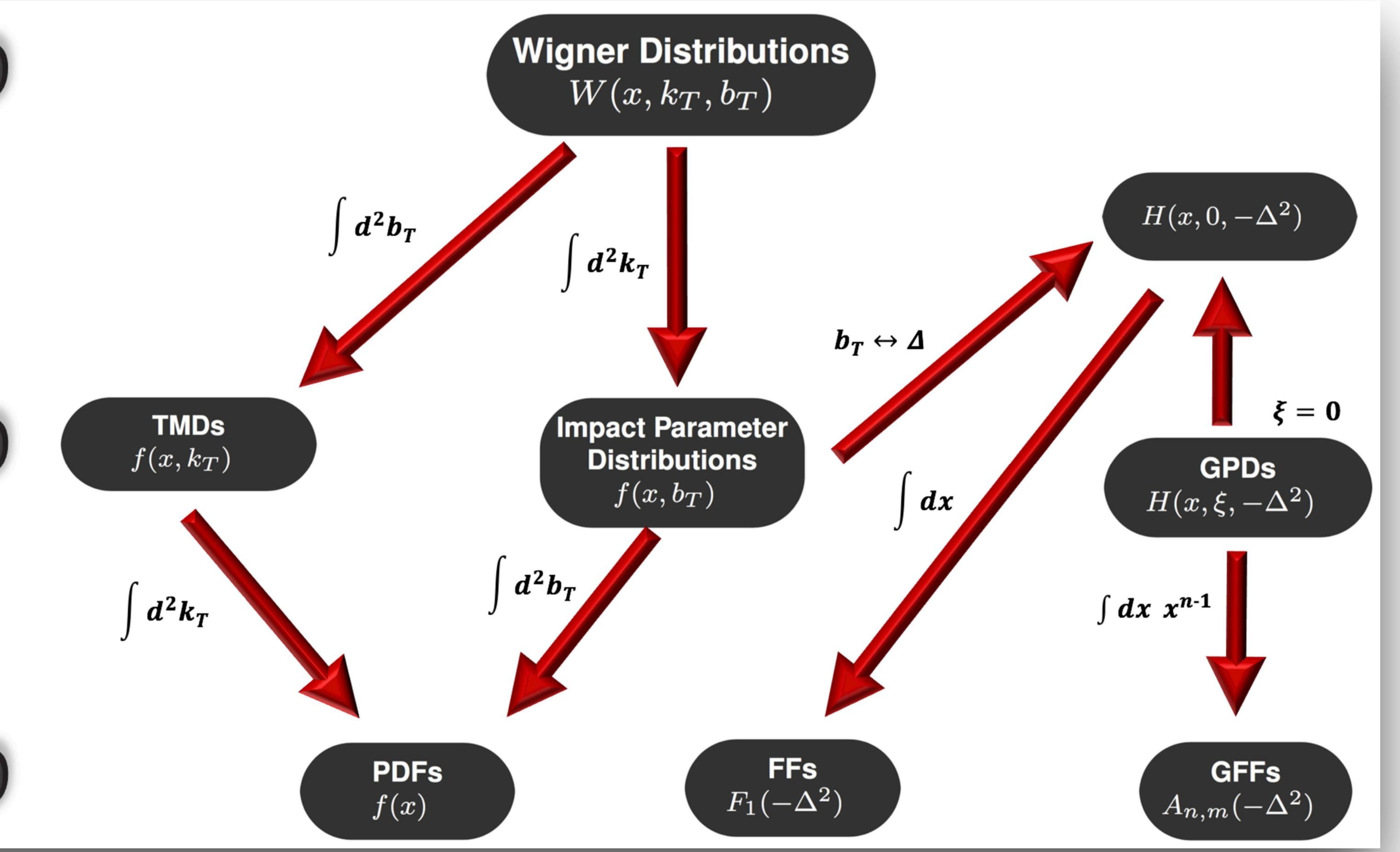
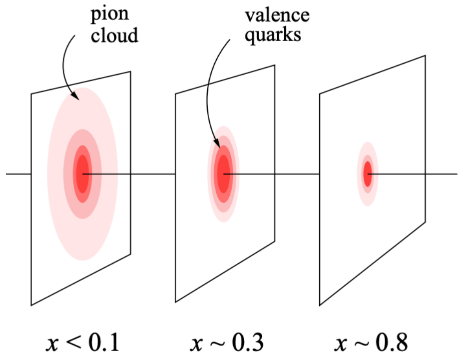


[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]



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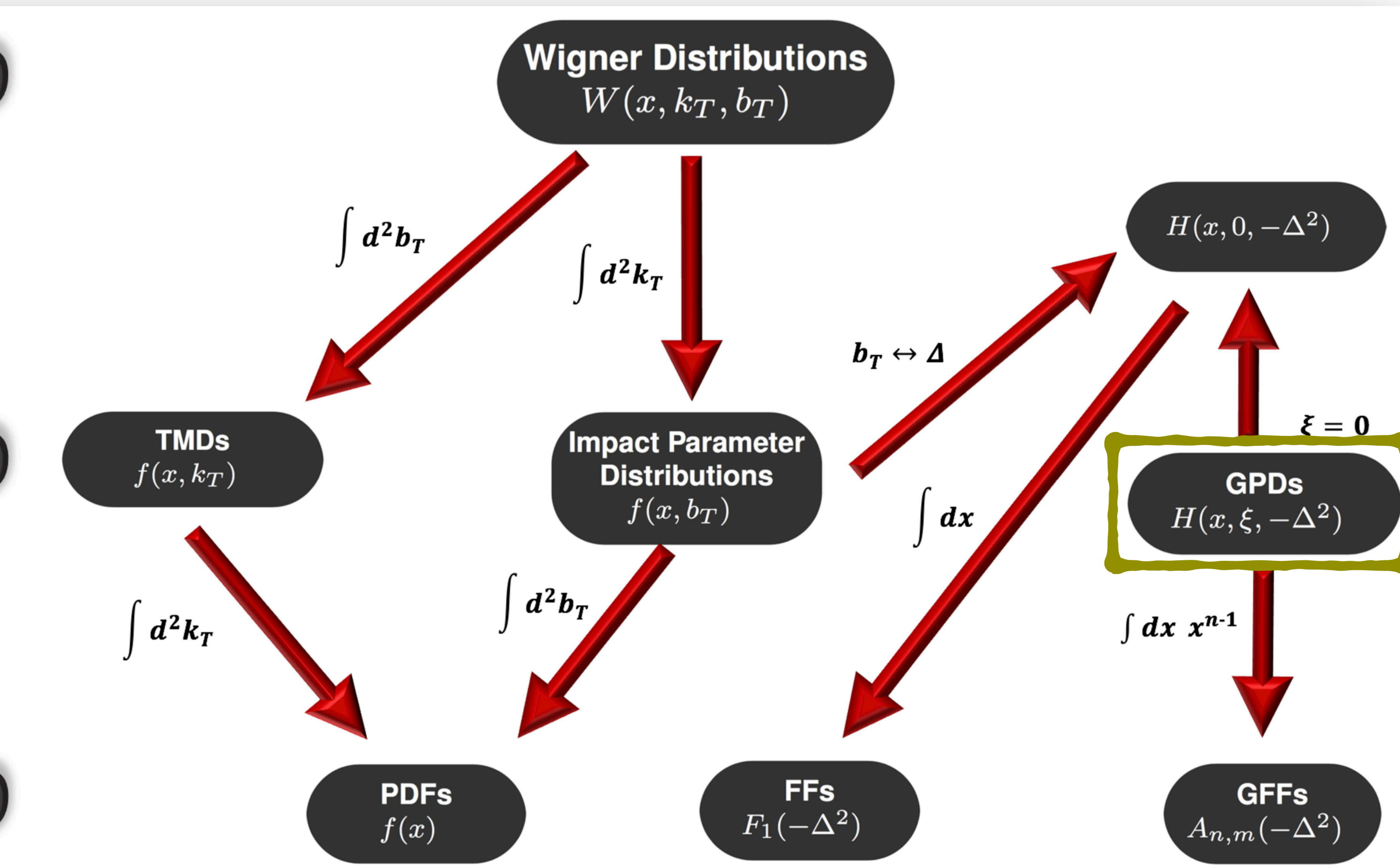

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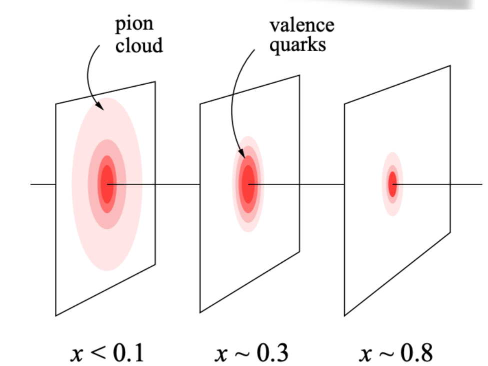

GPDs

★ "Parent" functions for PDFs, FFs, GFFs

★ Multi-dimensional objects

★ Provide correlation between transverse position & longitudinal momentum of the partons in the hadron

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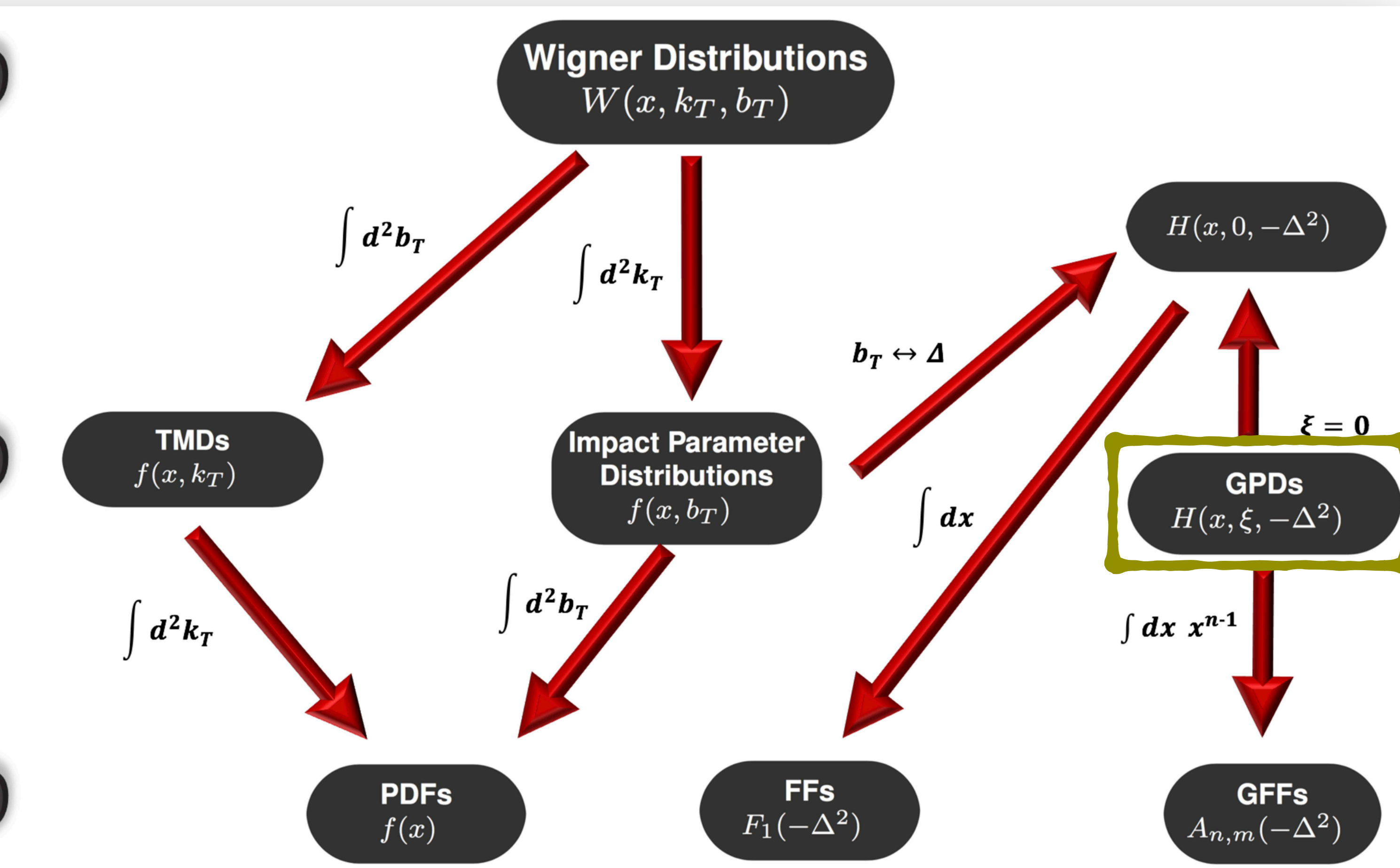

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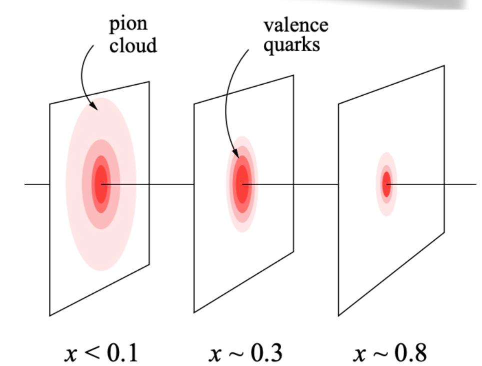

GPDs

★ "Parent" functions for PDFs, FFs, GFFs

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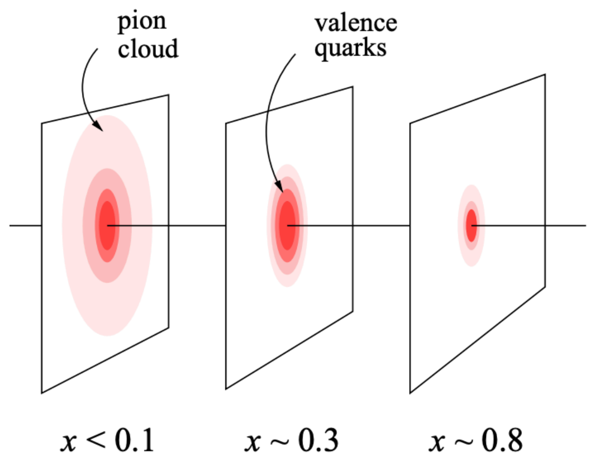
★ Provide correlation between transverse position & longitudinal momentum of the partons in the hadron

★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

Motivation in a nutshell



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1_{mom} + 2_{coord} tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

- ★ Contain physical interpretation on mechanical properties
- ★ Mellin moments connected to e.g., E/M radii, axial mass, spin, mass, …
- ★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathcal{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

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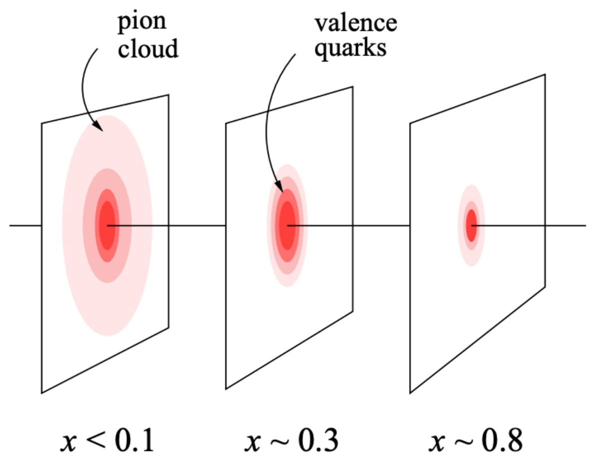

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Essential to complement the knowledge on GPD from lattice QCD

Accessing information on PDFs/GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n} q\right]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\underline{\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)}\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t)+\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P)+\text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t)+\frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P)+\text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t)+\frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t)+\frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t)+\frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P)+\text{ht}$$

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⇨ calculations at physical quark masses

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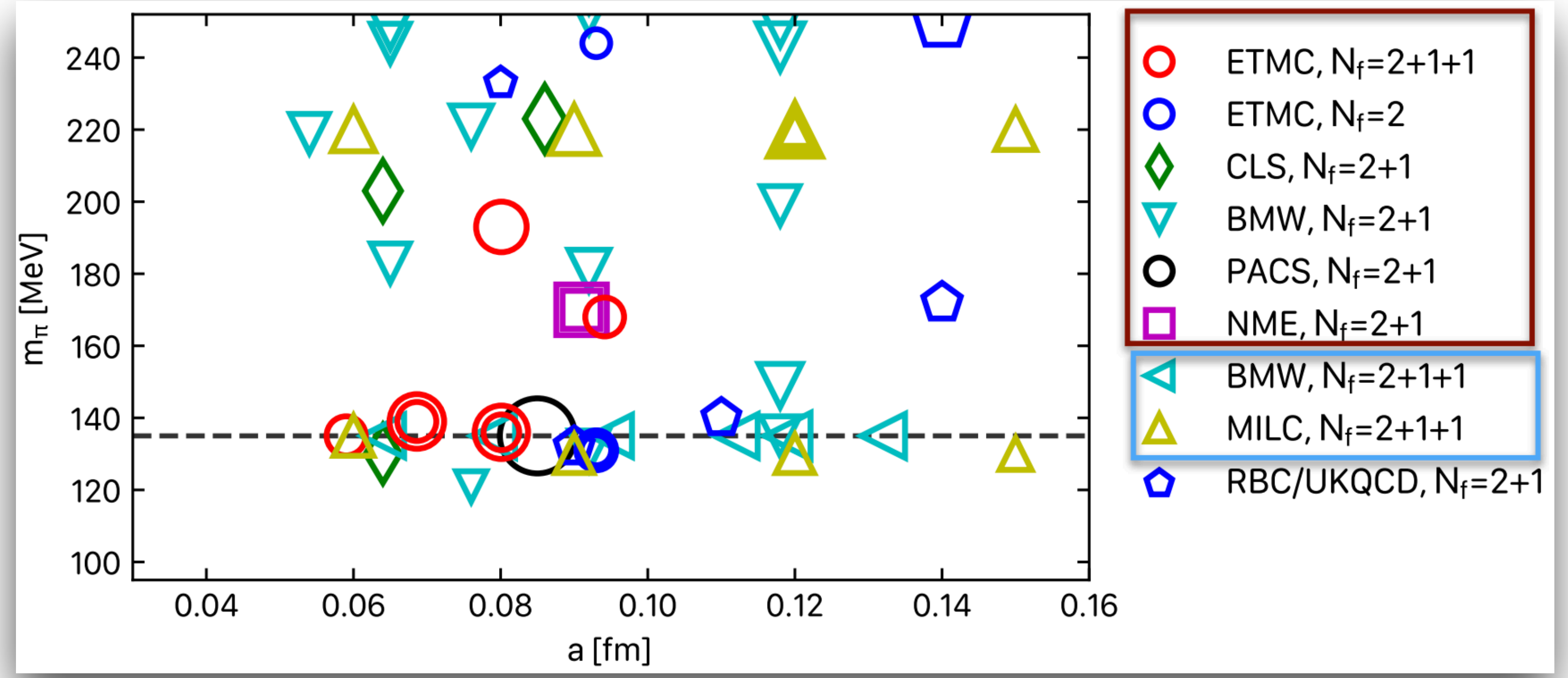

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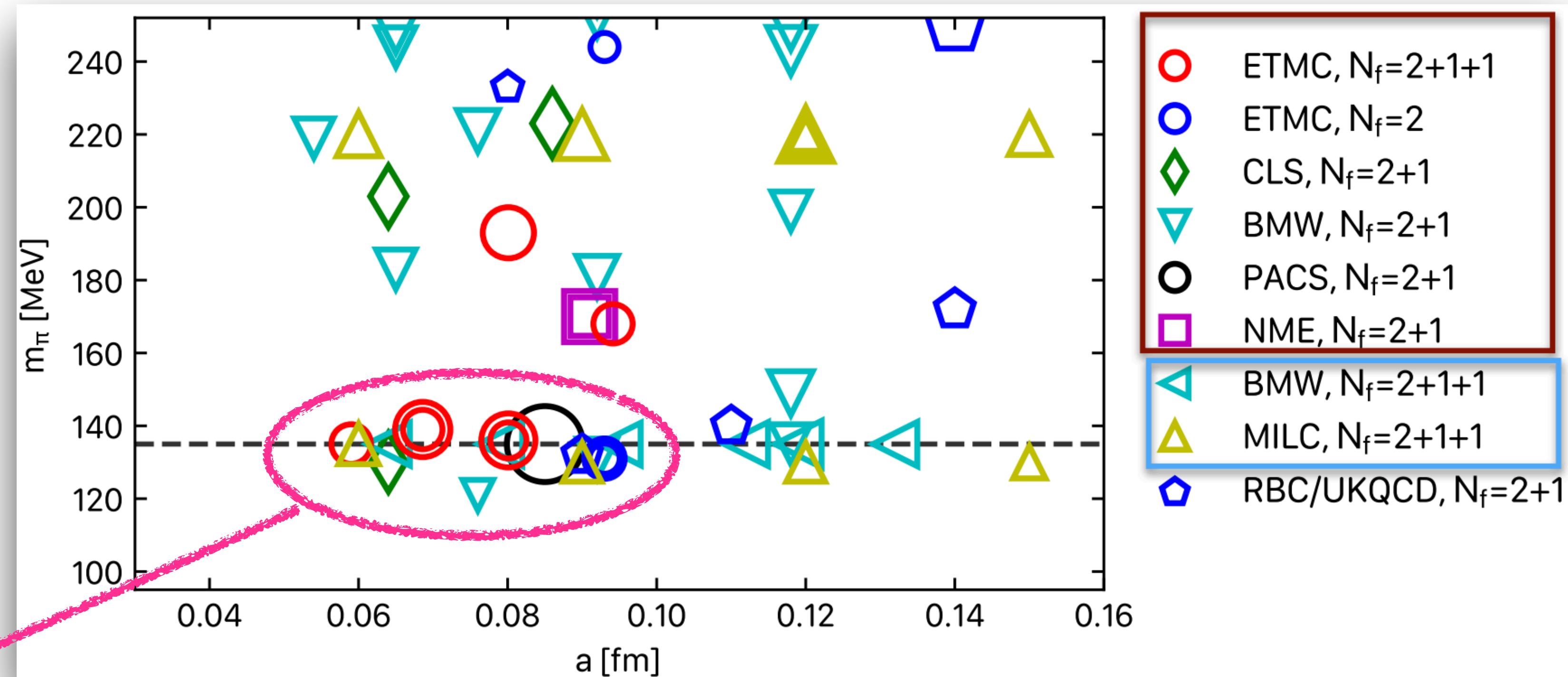

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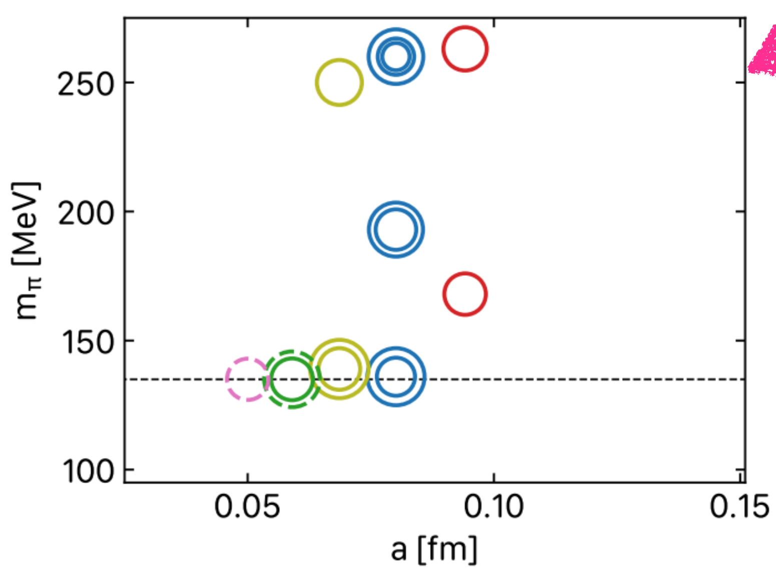
Extended Twisted Mass Collaboration

ETMC update



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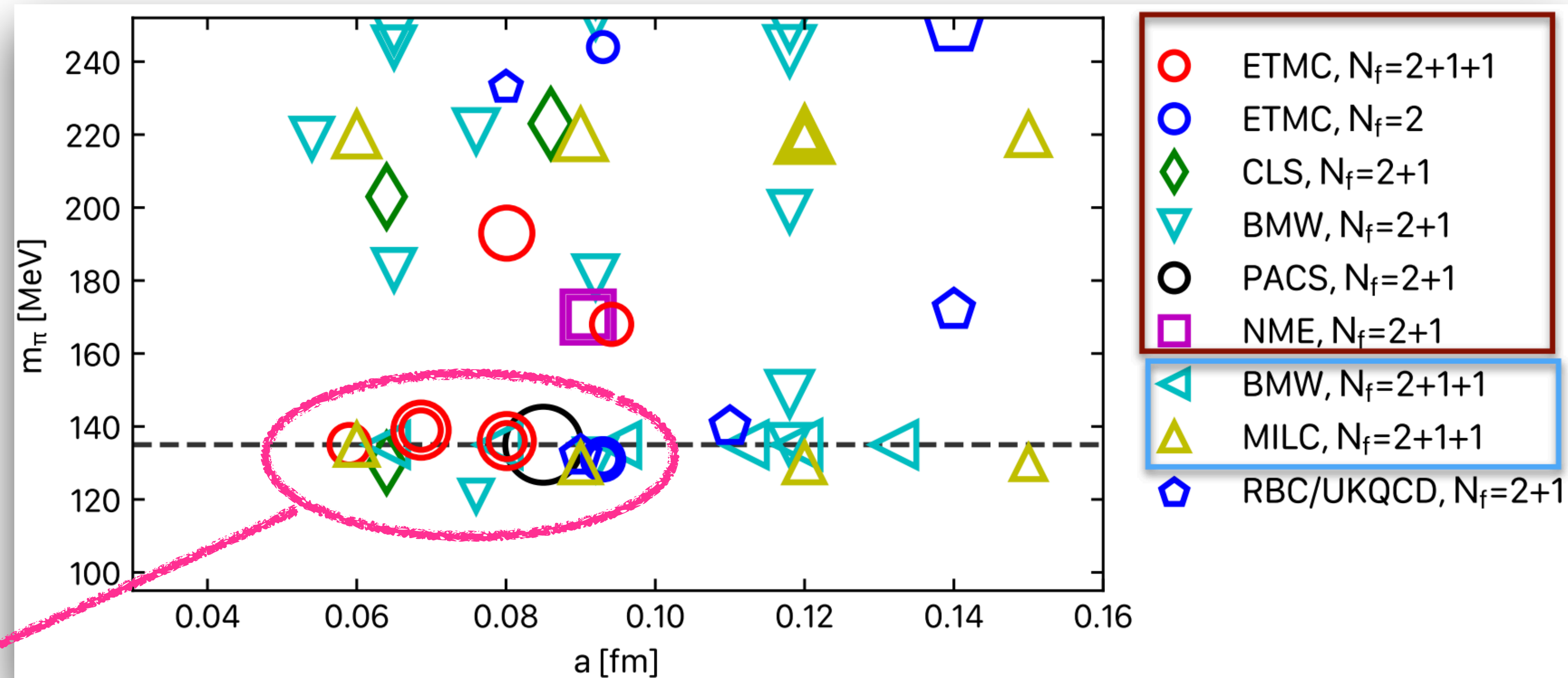

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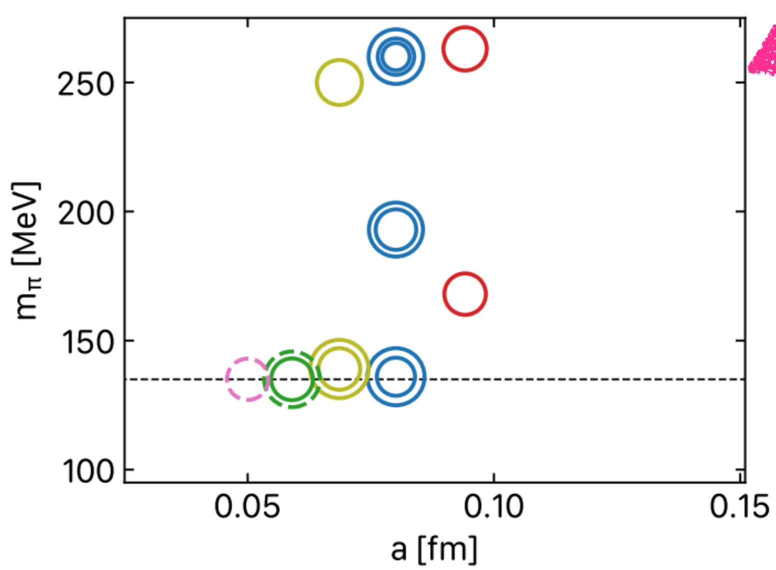
ETMC update



Simulations for hadron structure and beyond

Ensemble	V/a^4	β	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64	$64^3 \times 128$	1.778	0.07957(13)	140.2(2)	3.62
cC211.060.80	$80^3 \times 160$	1.836	0.06821(13)	136.7(2)	3.78
cD211.054.96	$96^3 \times 192$	1.900	0.05692(12)	140.8(2)	3.90

Nucleon Form Factors

The case of the EM form factors

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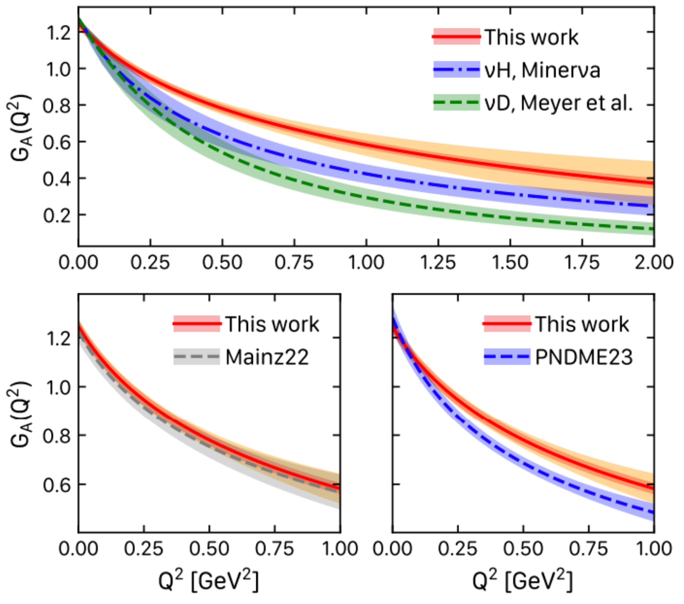
(Axial form factors: backup slides)



E/M form factors

[(ETMC) Alexandrou et al., PRD 100 (2019) 1, 014509]

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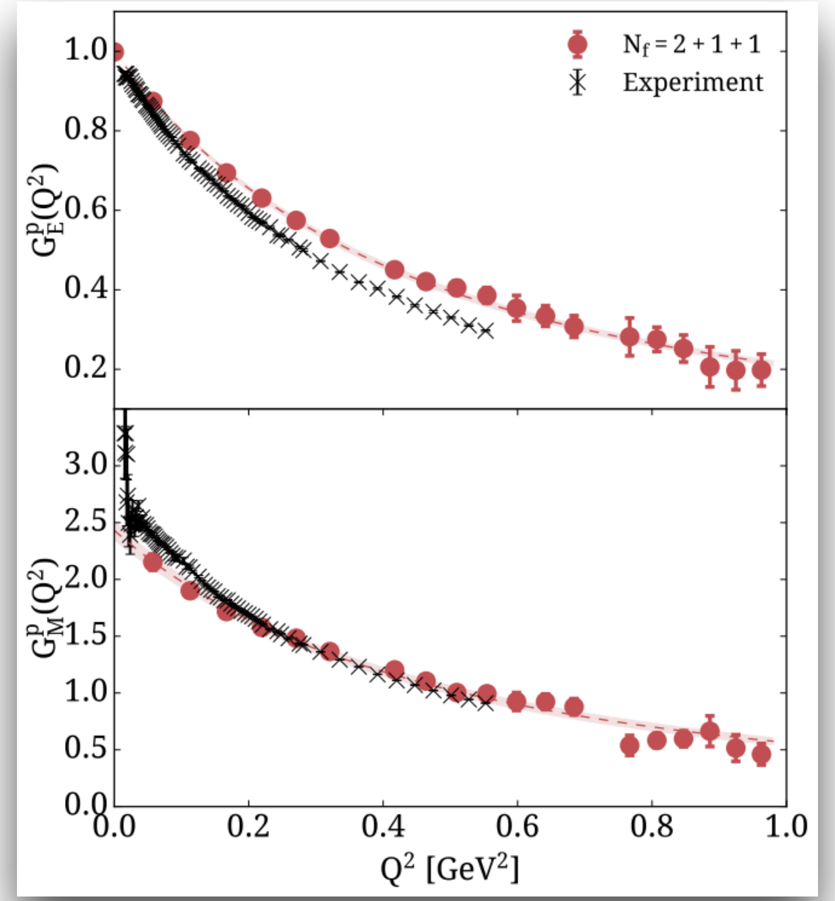

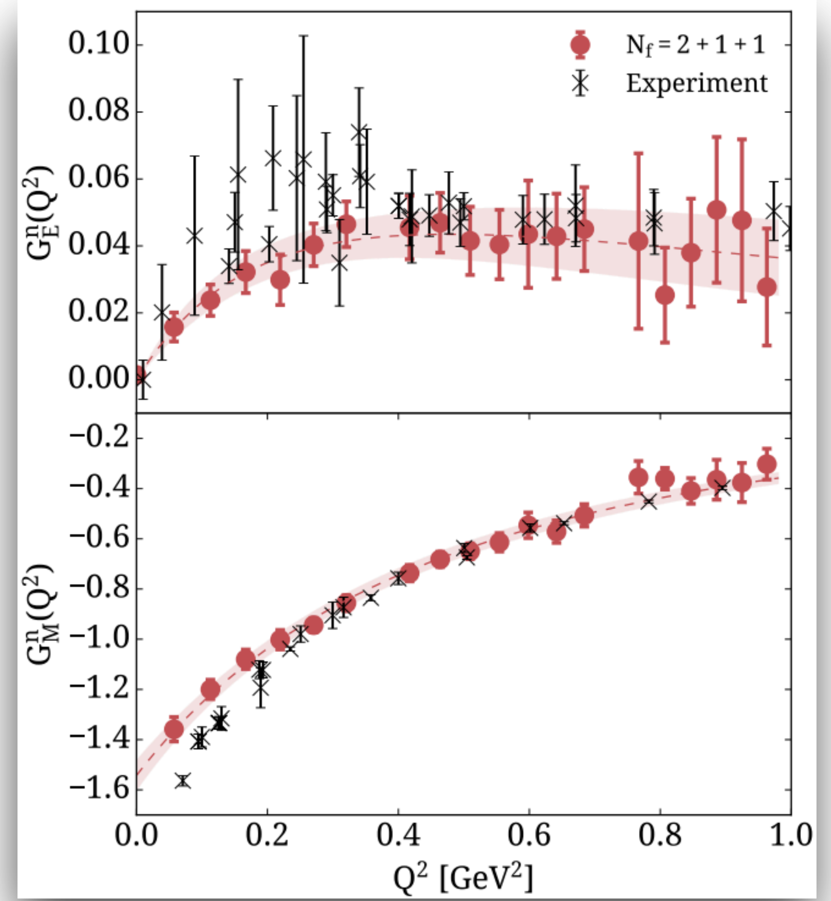

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★ High accuracy results may be valuable for experimental data

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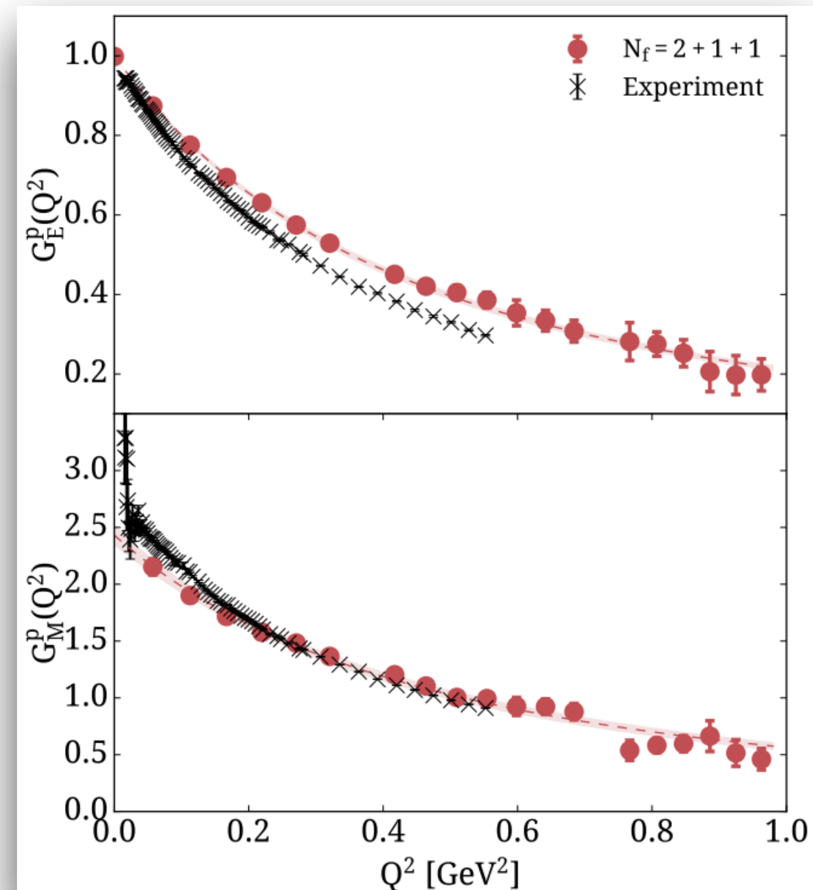

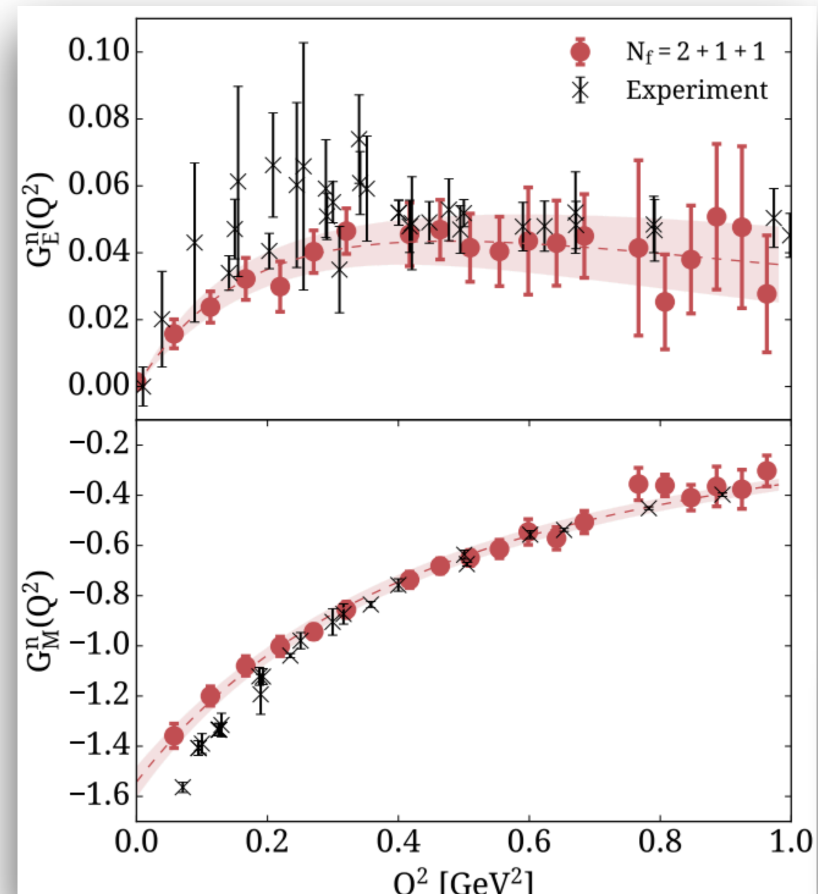

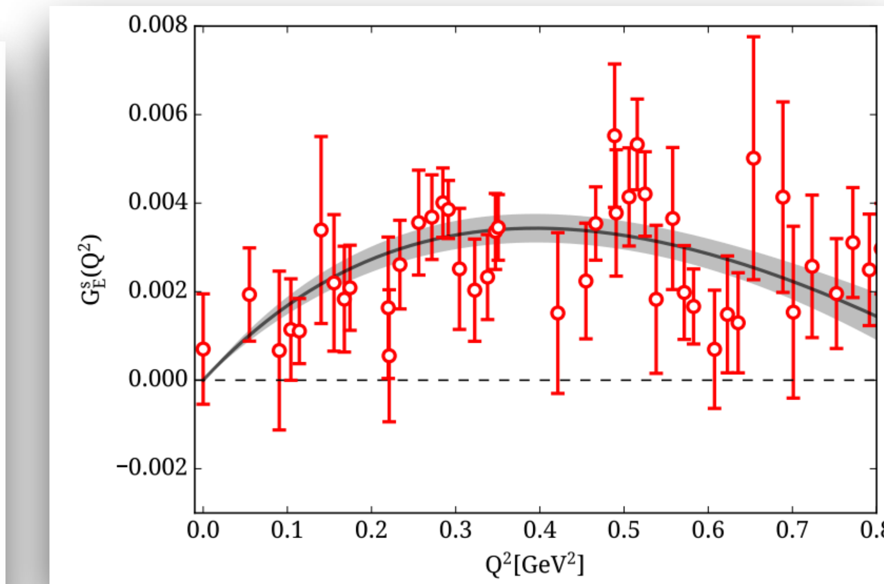

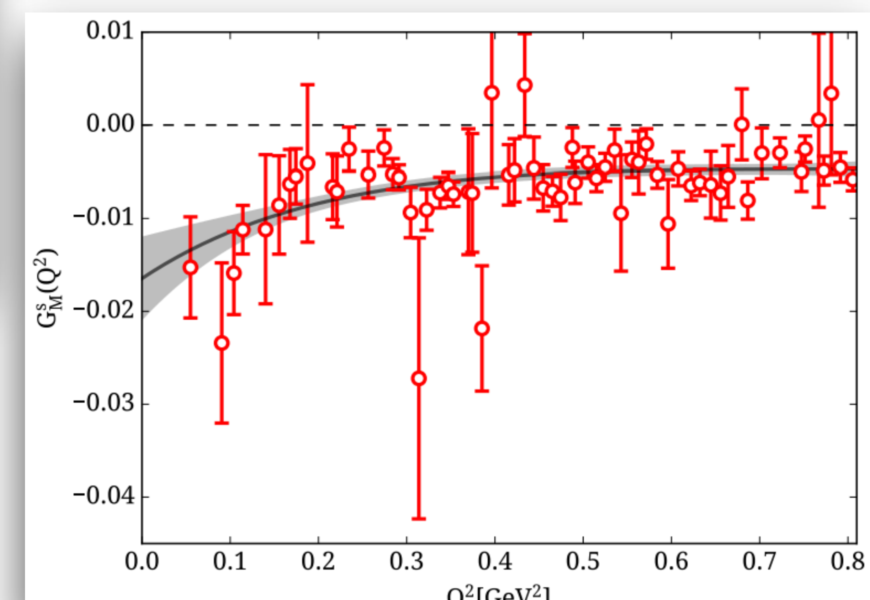

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E/M form factors

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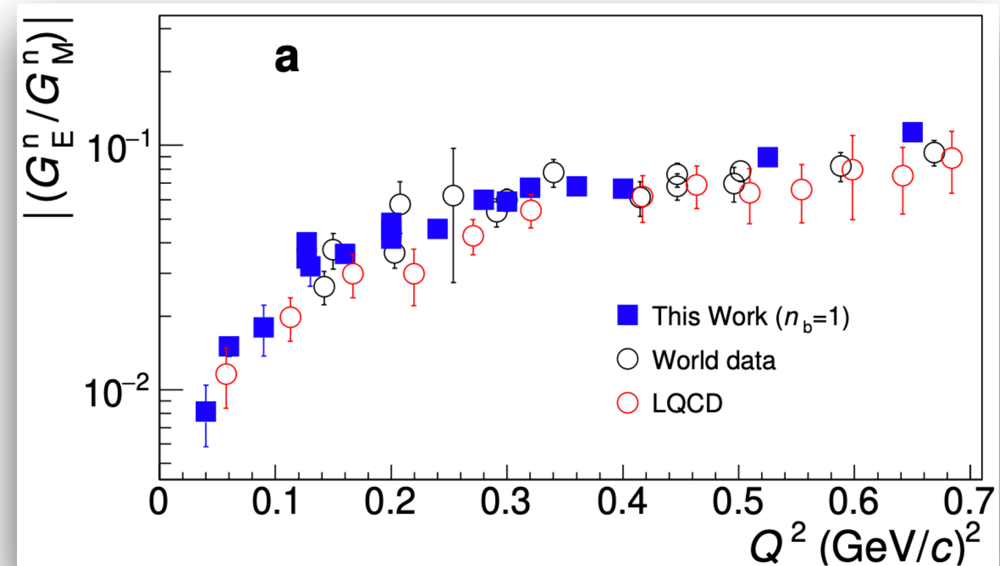
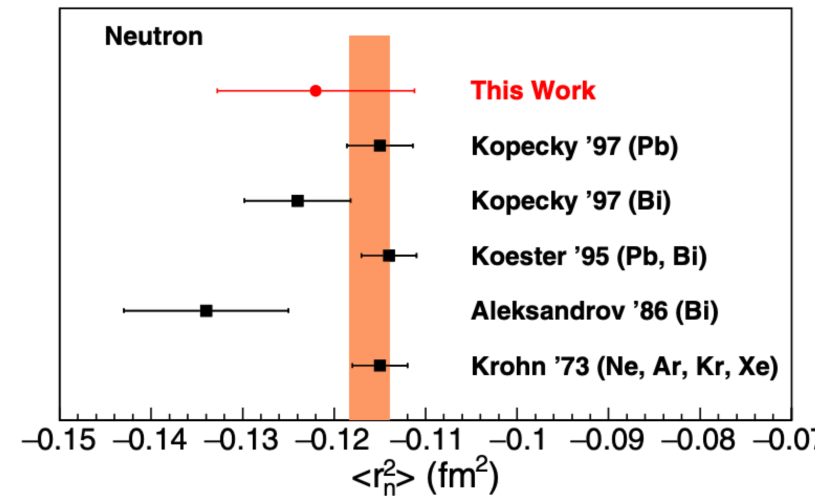
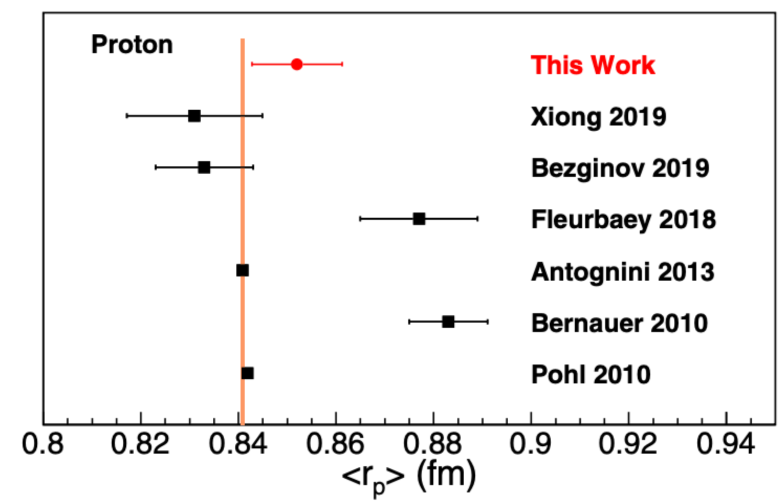
Synergy with experimental data



[Atac et al., Nature Comm. 12, 1759 (2021)]

★ Coverage in regions where data is sparse





[H. Atac et al., Eur. Phys. J. A 57, 65 (2021)]

Recent updates from various collaborations:

ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

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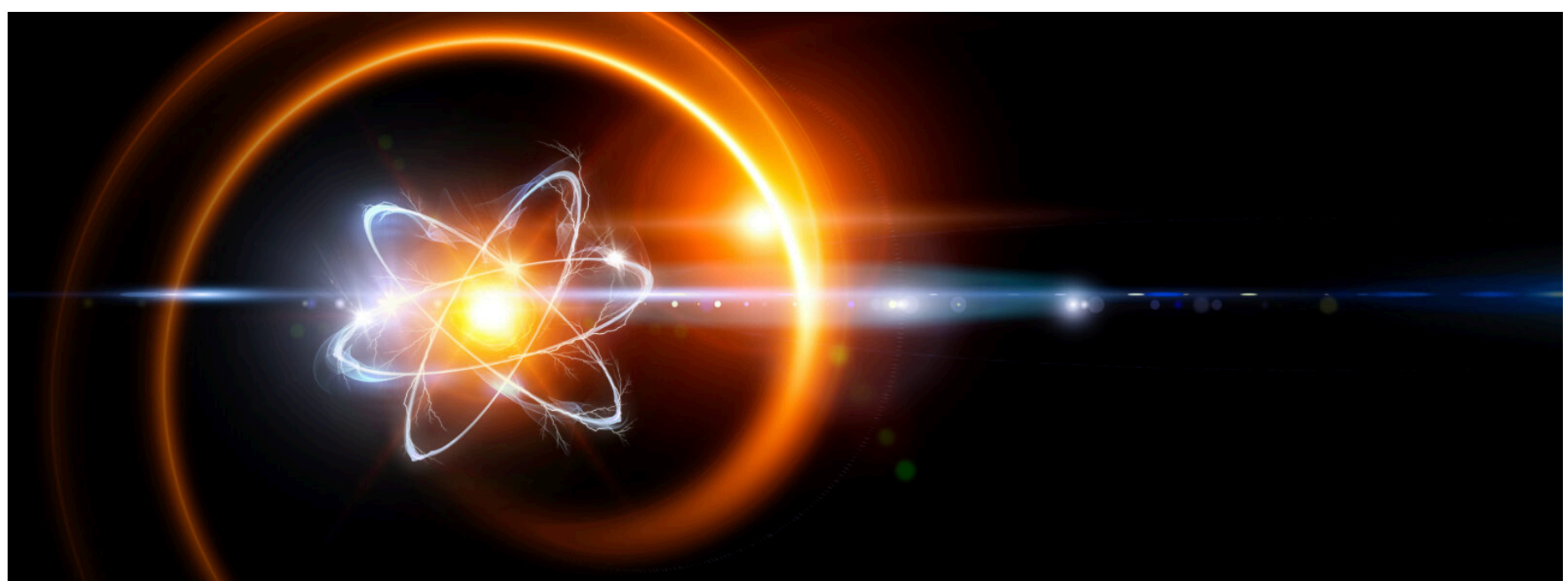
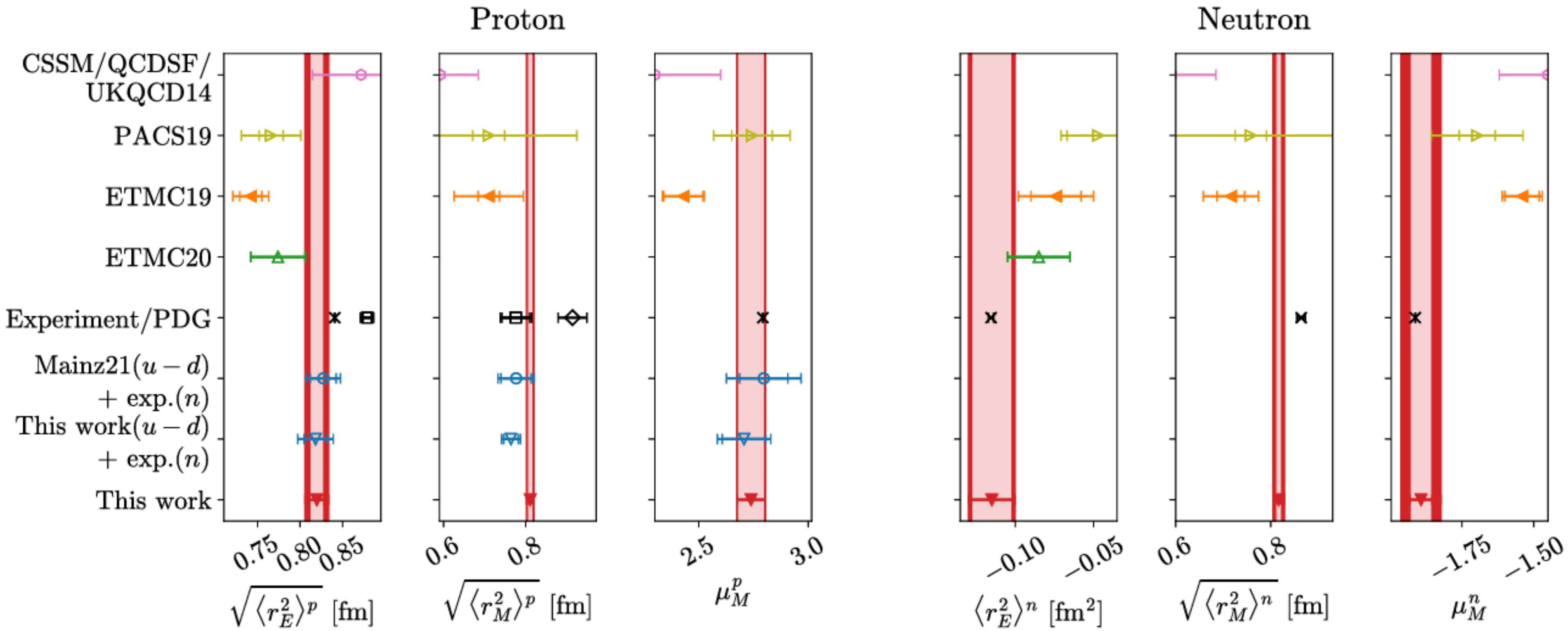
August 5 - 9, 2024

TOWARDS IMPROVED HADRON TOMOGRAPHY WITH HARD EXCLUSIVE REACTIONS



See talk by D. Djukanovic

Comparison to others em FF



$$\langle r_E^2\rangle^p = (0.672 \pm 0.014 \text{ (stat)} \pm 0.018 \text{ (syst)}) \text{ fm}^2,$$

$$\langle r_M^2\rangle^p = (0.658 \pm 0.012 \text{ (stat)} \pm 0.008 \text{ (syst)}) \text{ fm}^2,$$

$$\mu_M^p = 2.739 \pm 0.063 \text{ (stat)} \pm 0.018 \text{ (syst)},$$

$$\langle r_E^2\rangle^n = (-0.115 \pm 0.013 \text{ (stat)} \pm 0.007 \text{ (syst)}) \text{ fm}^2$$

$$\langle r_M^2\rangle^n = (0.667 \pm 0.011 \text{ (stat)} \pm 0.016 \text{ (syst)}) \text{ fm}^2,$$

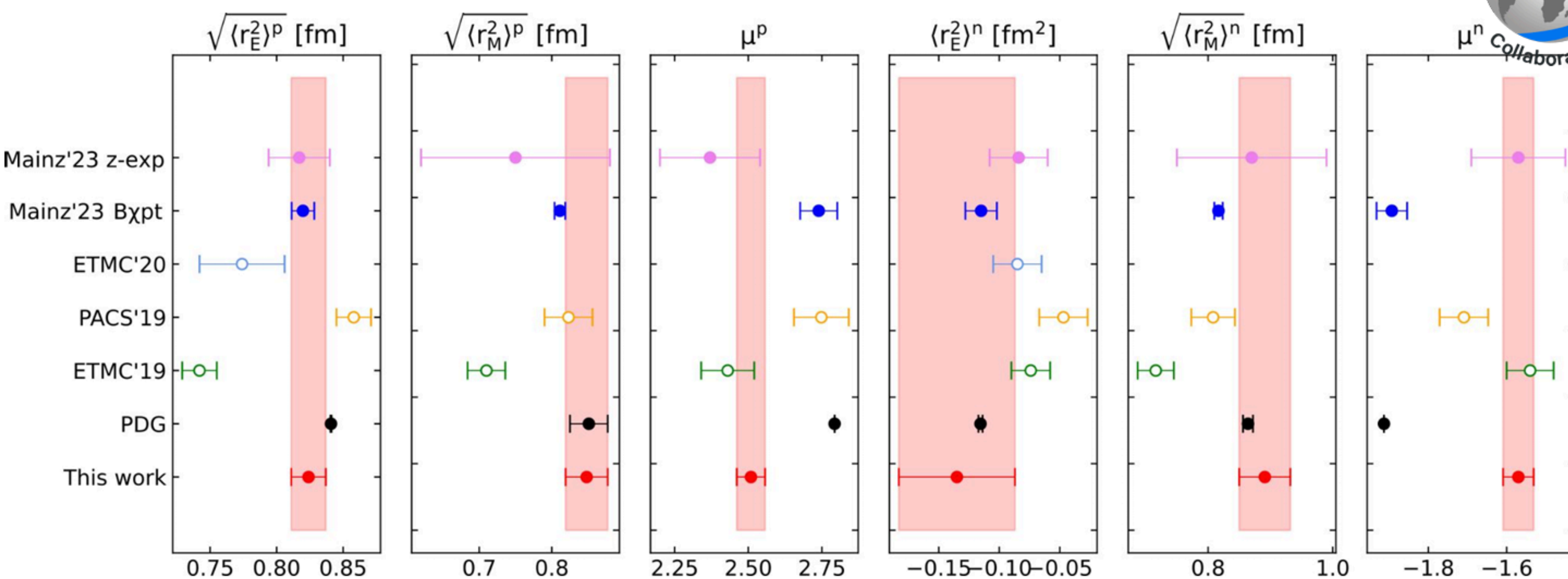
$$\mu_M^n = -1.893 \pm 0.039 \text{ (stat)} \pm 0.058 \text{ (syst)}.$$

www.hi-mainz.de

HIM HELMHOLTZ Helmholtz-Institut Mainz

See talk by S. Bacchio

Comparison with other studies



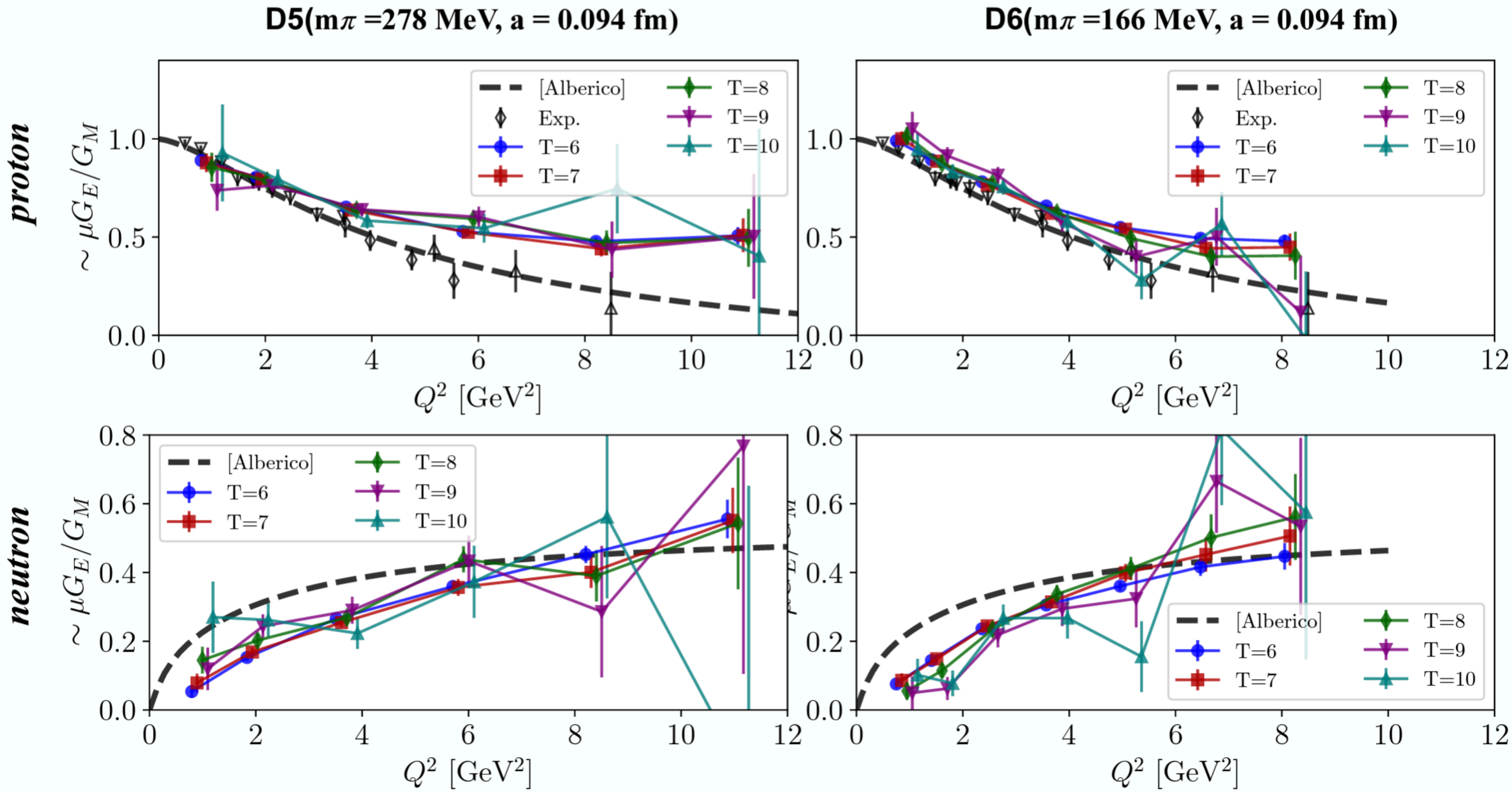
- Good agreement for radii with PDG
- Tension of our results in magnetic moments

PRELIMINARY

See talk by S. Syritsyn

Proton&Neutron G_E/G_M : Connected+Disconnected

$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}\overset{T\to\infty}{=}G_E/G_M$$ where $\begin{pmatrix} p^{(\prime)} & = m_N\sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N\cosh\lambda^{(\prime)}\end{pmatrix}$



Sergey Syritsyn | Nucleon Form Factors at High Q^2 from LQCD | ECT* Workshop, Trento, Aug 5–9, 2024

Access to higher Mellin moments not trivial

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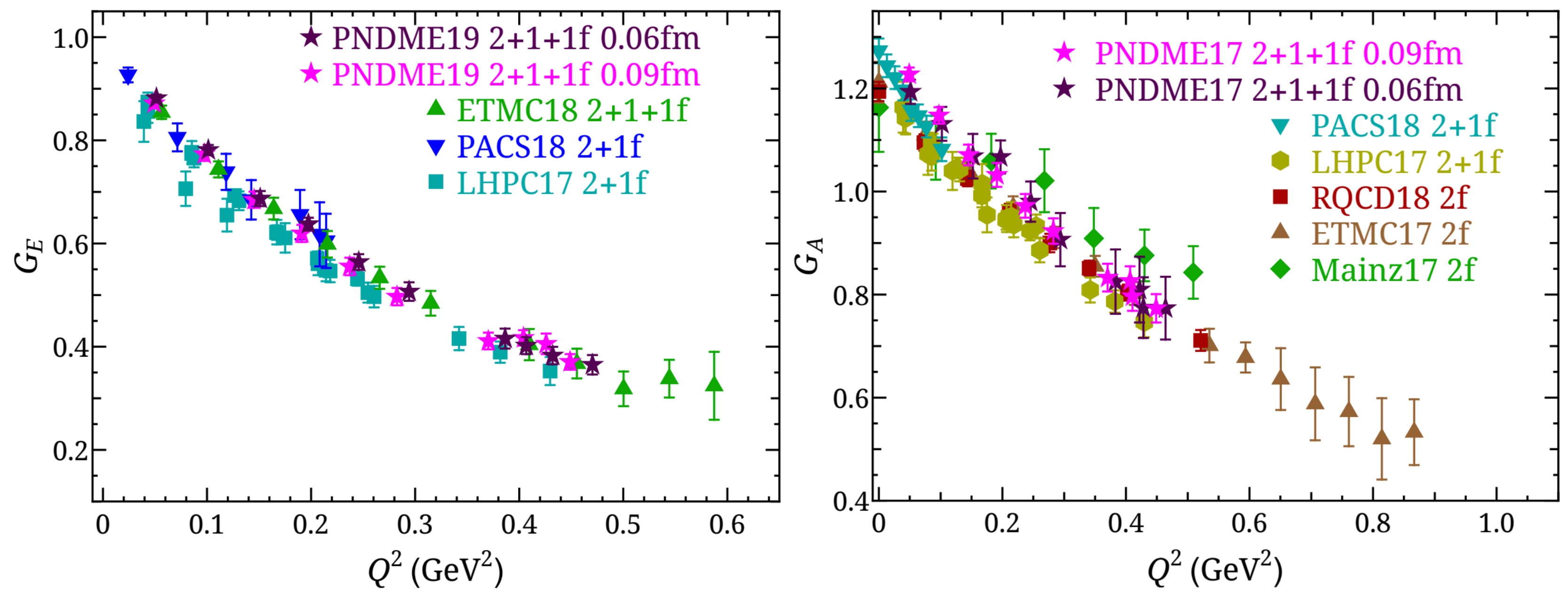
★ Form factors



- Simulations at physical point available by multiple groups
- Precision data era (control of systematic uncertainties)

Access to higher Mellin moments not trivial

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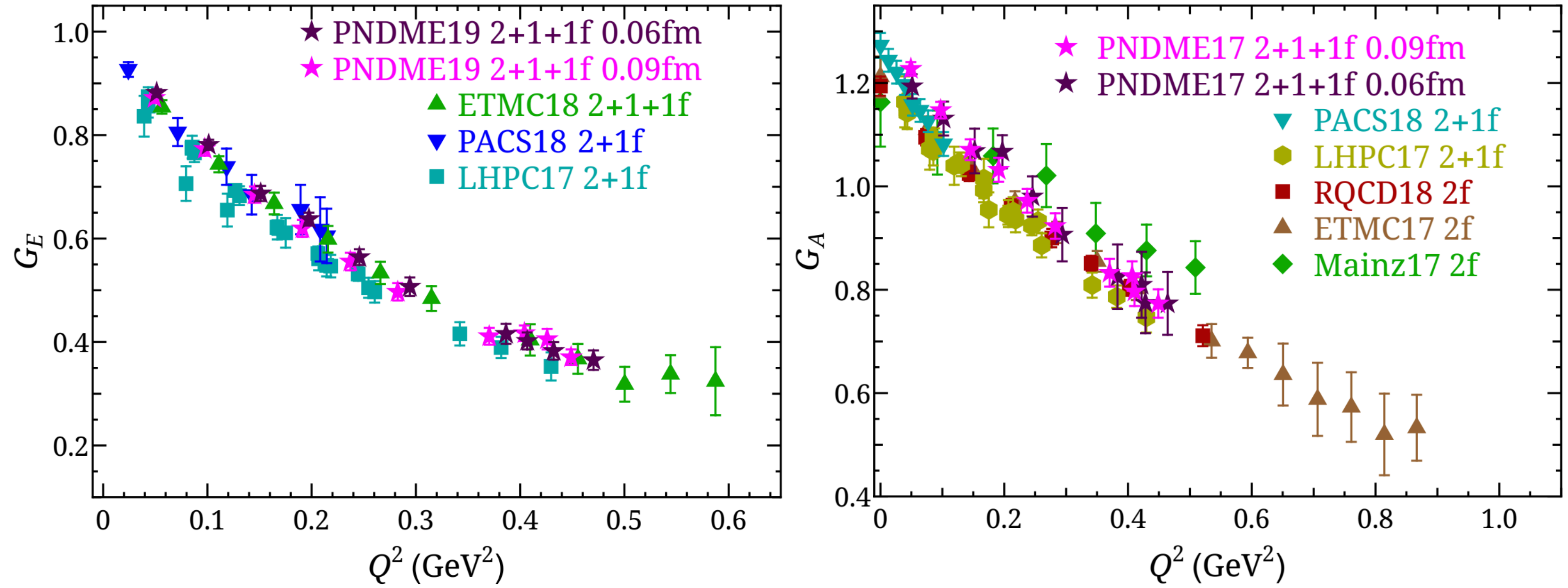

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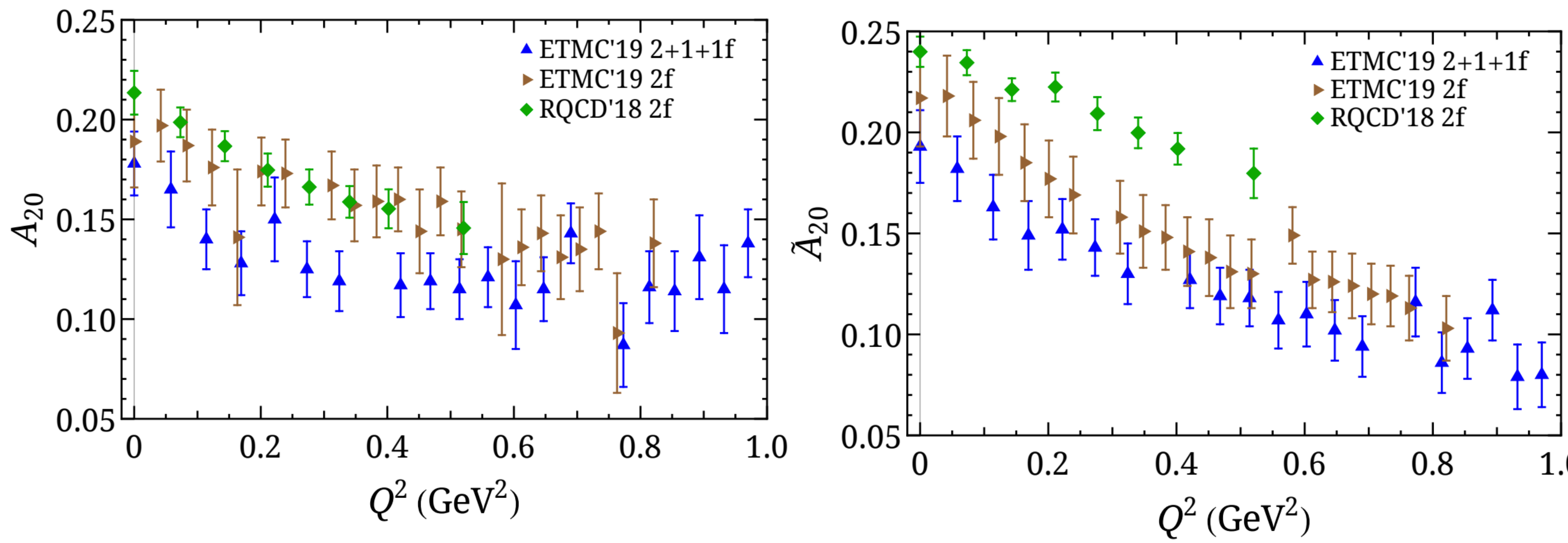
★ Generalized form factors



- Lesser studied compared to FFs at physical point
- Decay of signal-to-noise ratio

Mellin moments of GPDs (form factors and generalized form factors) give incomplete information on GPDs

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GPDs

Through non-local matrix elements of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

Light-Cone:

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

Euclidean lattice:

- ★ Matrix elements of mom.-boosted states and nonlocal operators
- ★ Connection to light-cone GPDs through LaMET [X. Ji, PRL 110 (2013) 262002], SDF [A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}^{\rm GPD}_{\Gamma}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

Access of PDFs/GPDs on a Euclidean Lattice

Light-Cone:

$$F^{[\Gamma]}(x,\Delta;P^3)=\frac{1}{2}\int\frac{dz^3}{2\pi}e^{ik\cdot z}\langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

Euclidean lattice:

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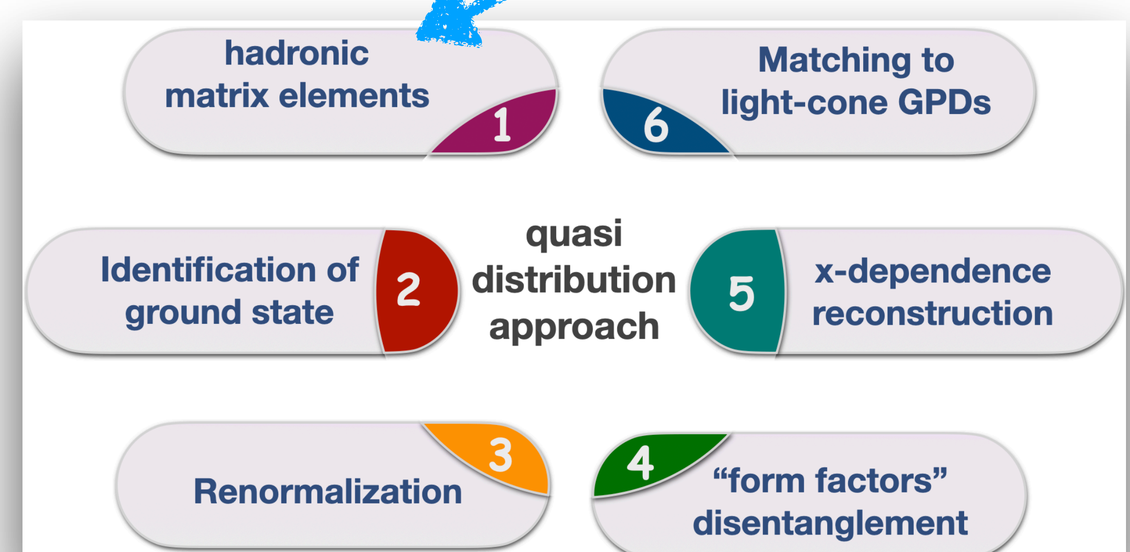
$$\tilde{q}_\Gamma^{\rm GPD}(x,t,\xi,P_3,\mu)=\int\frac{dz}{4\pi}e^{-ixP_3z}\quad\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Computationally intensive

$$\Delta=P_f-P_i$$

$$t=\Delta^2=-Q^2$$

$$\xi=\frac{Q_3}{2P_3}$$



New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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★ Lorentz-invariant parametrization

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	Δ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
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asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Zero-skewness calculation

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Symmetric frame: each momentum requires <u>separate</u> computational resources

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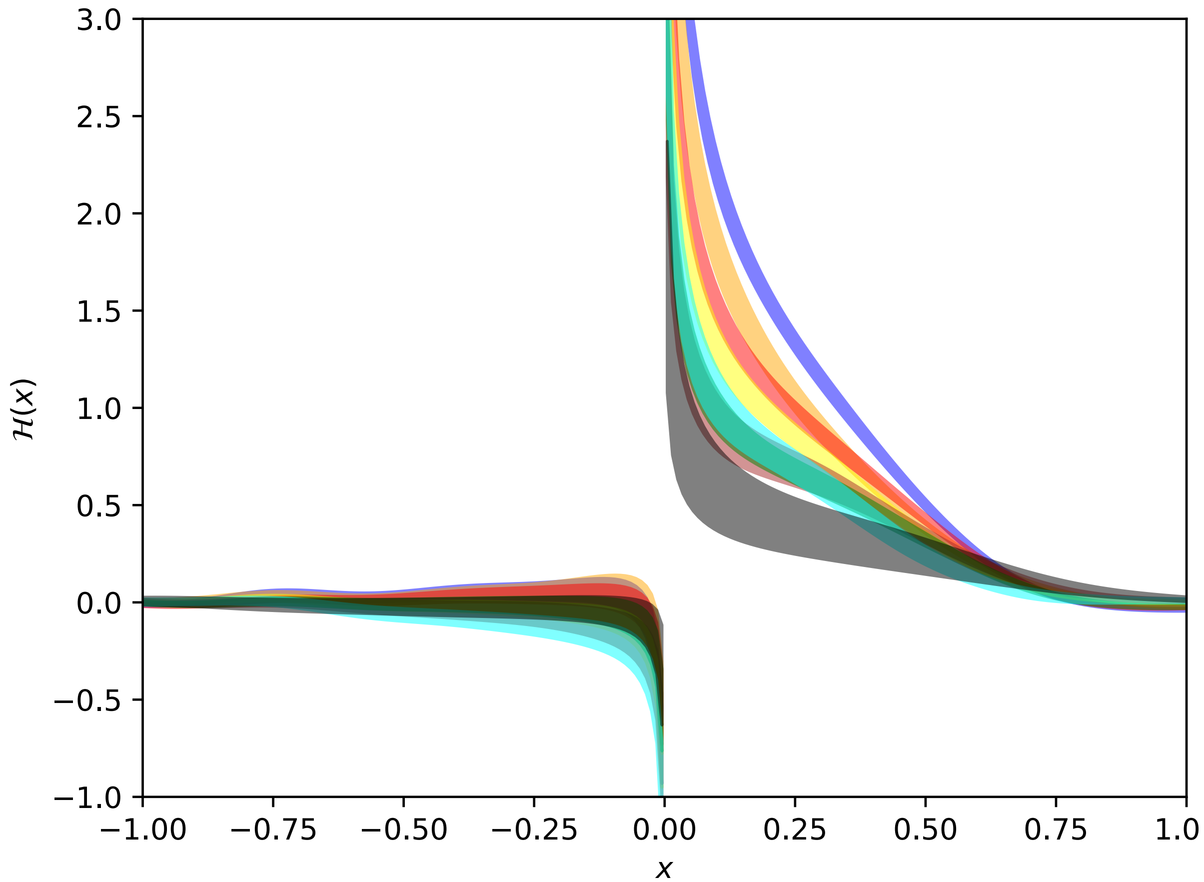
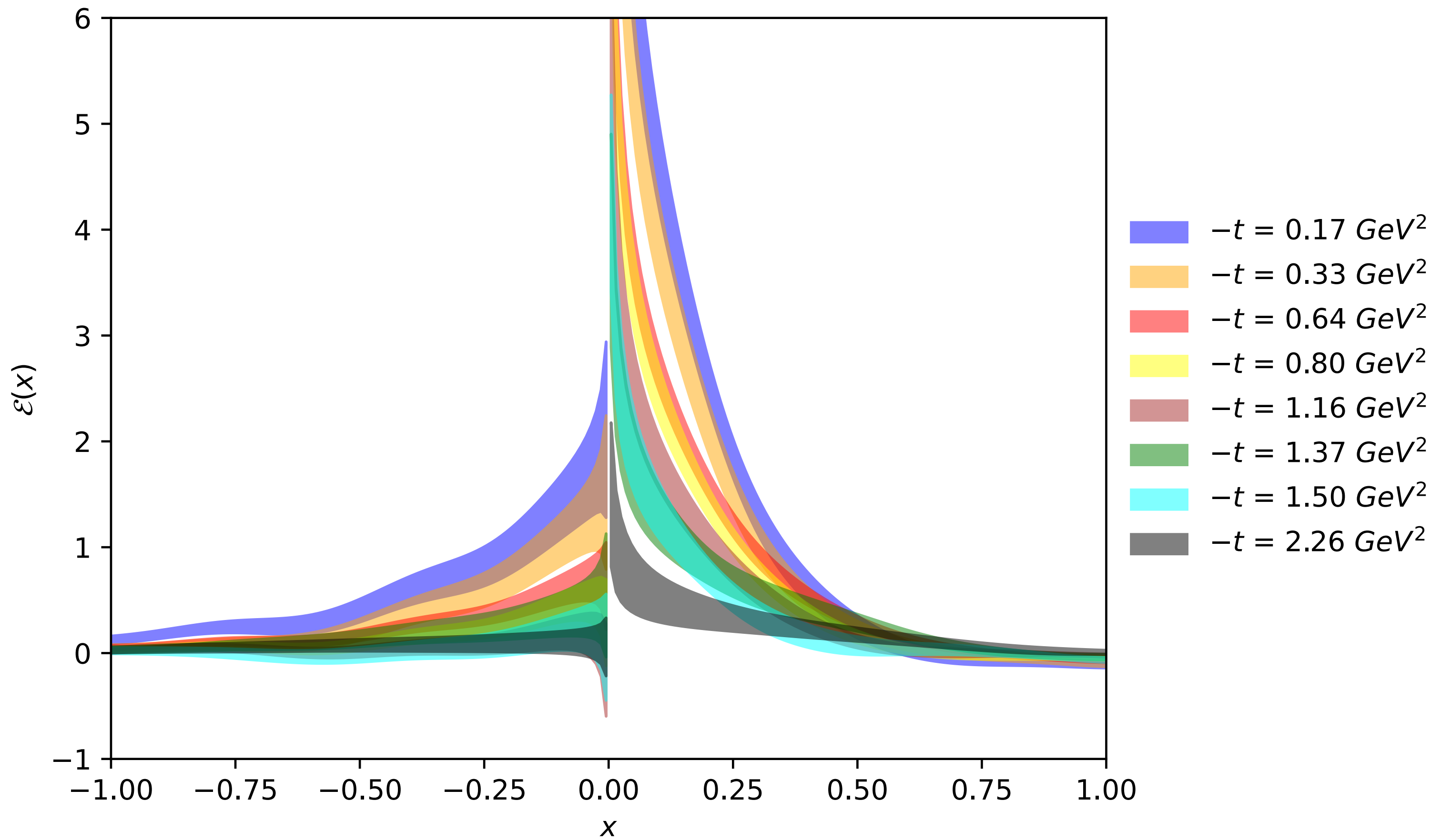
Zero-skewness calculation

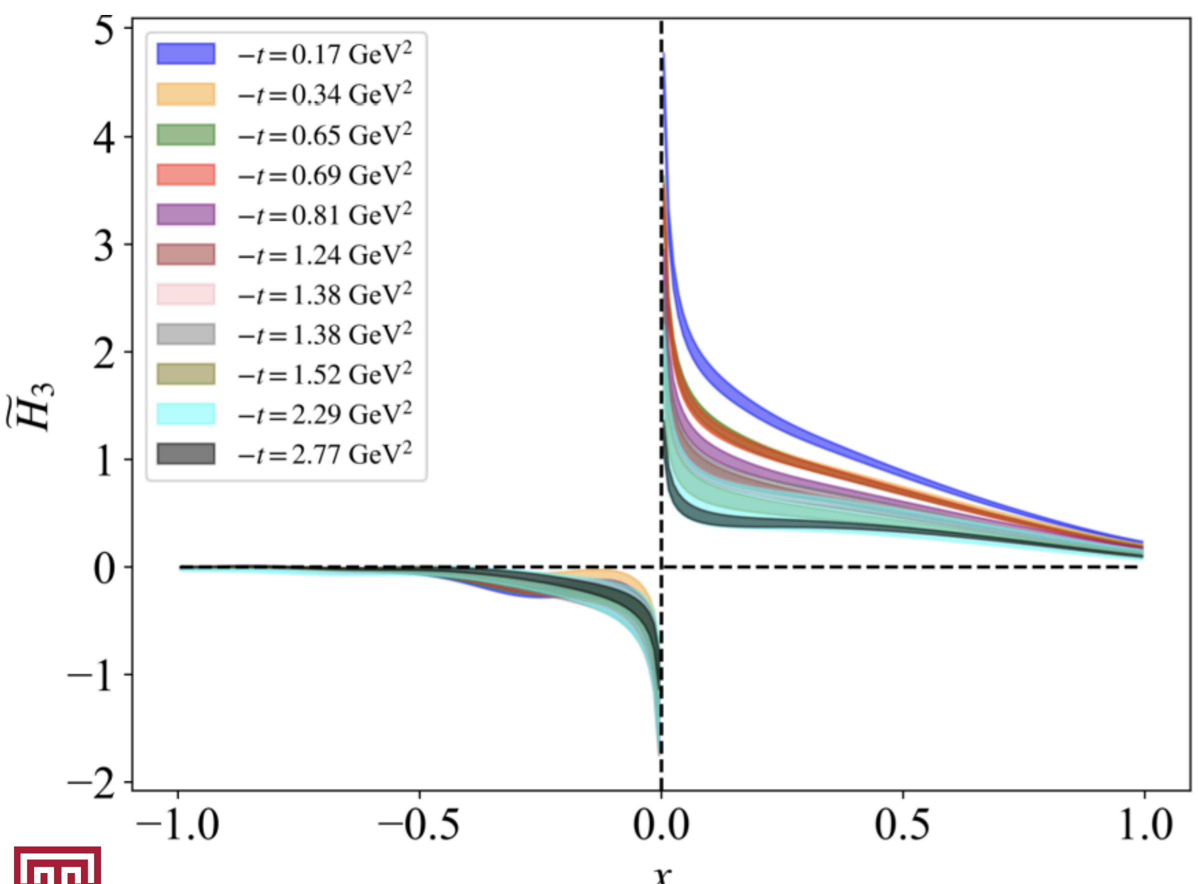
! Symmetric frame: each momentum requires separate computational resources

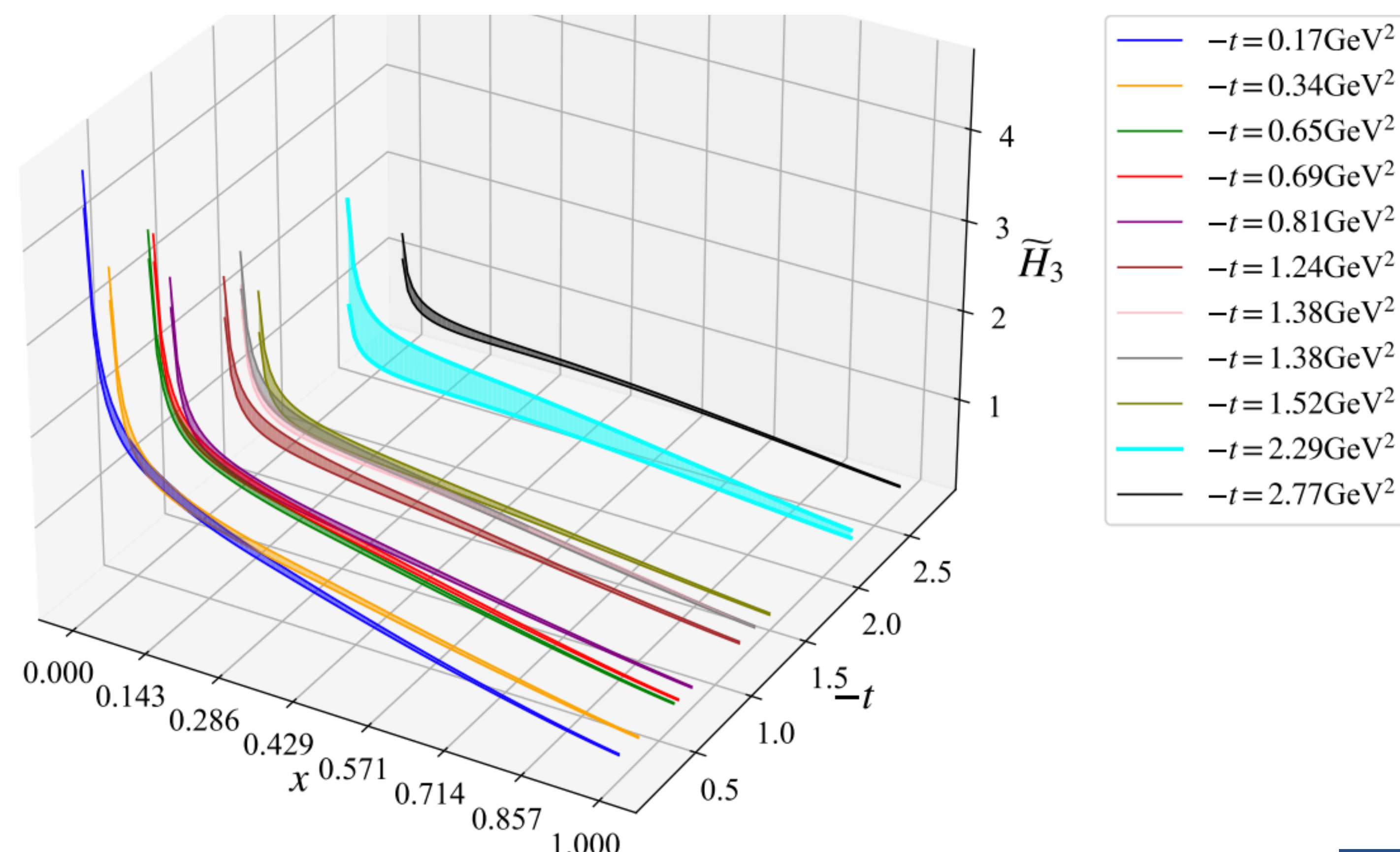
! Asymmetric frame: momenta grouped in 2 sets of runs [(Q,0,0), (Qx,Qy,0)]

Light-cone GPDs

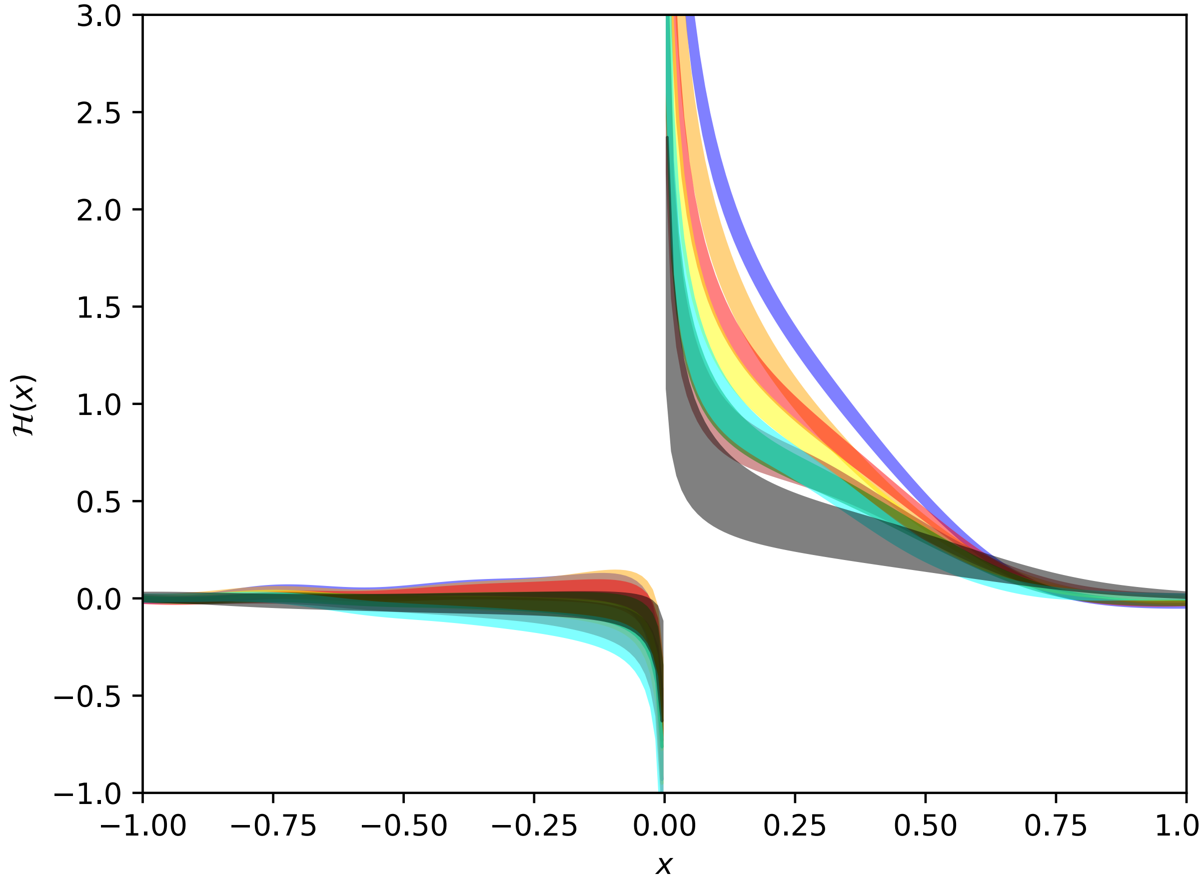


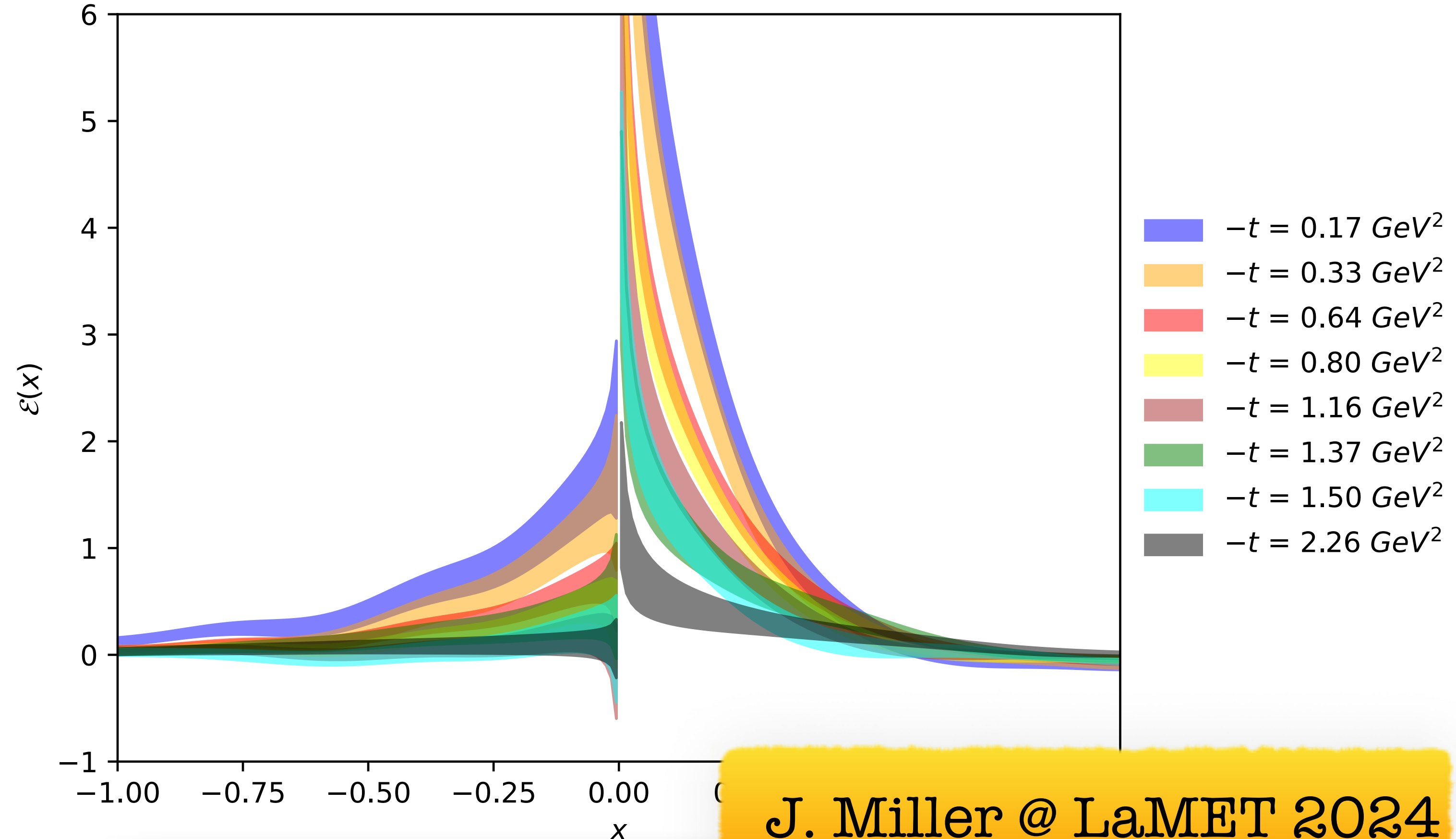


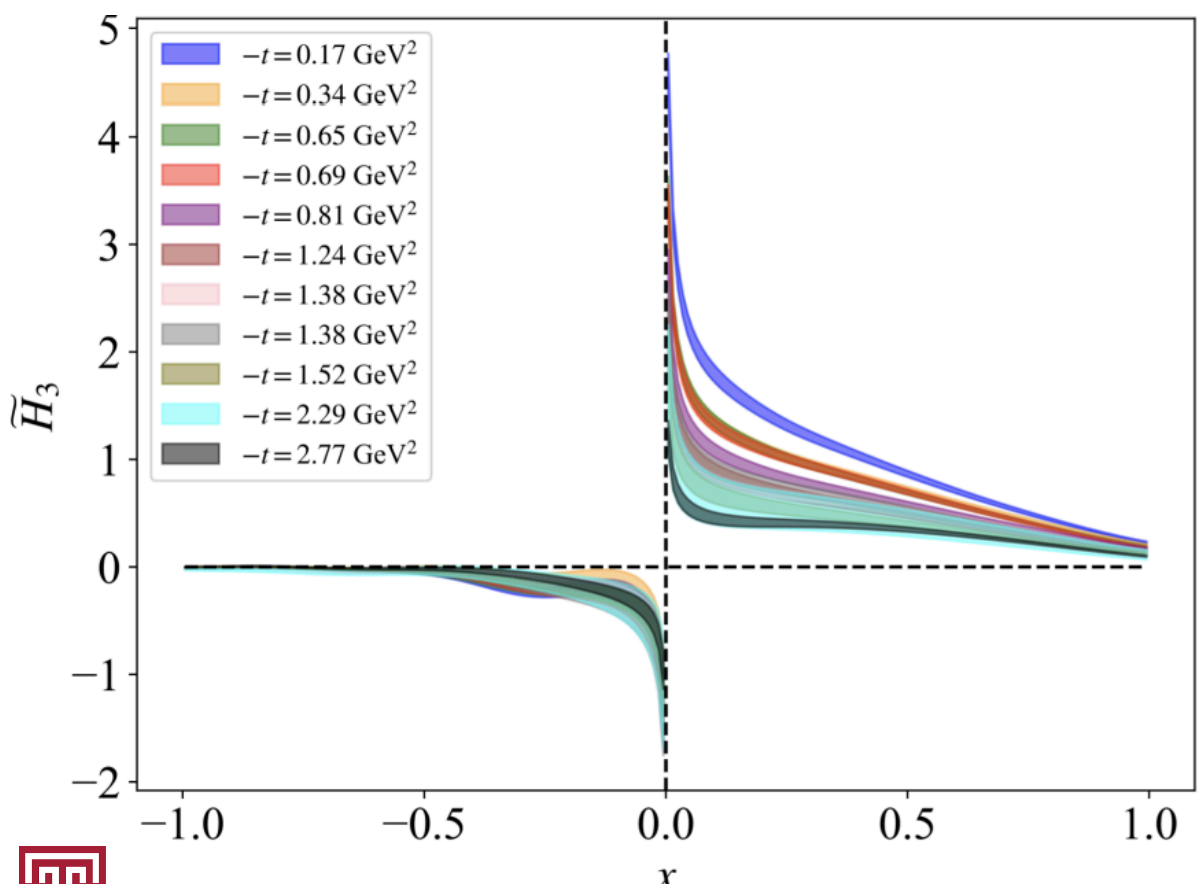




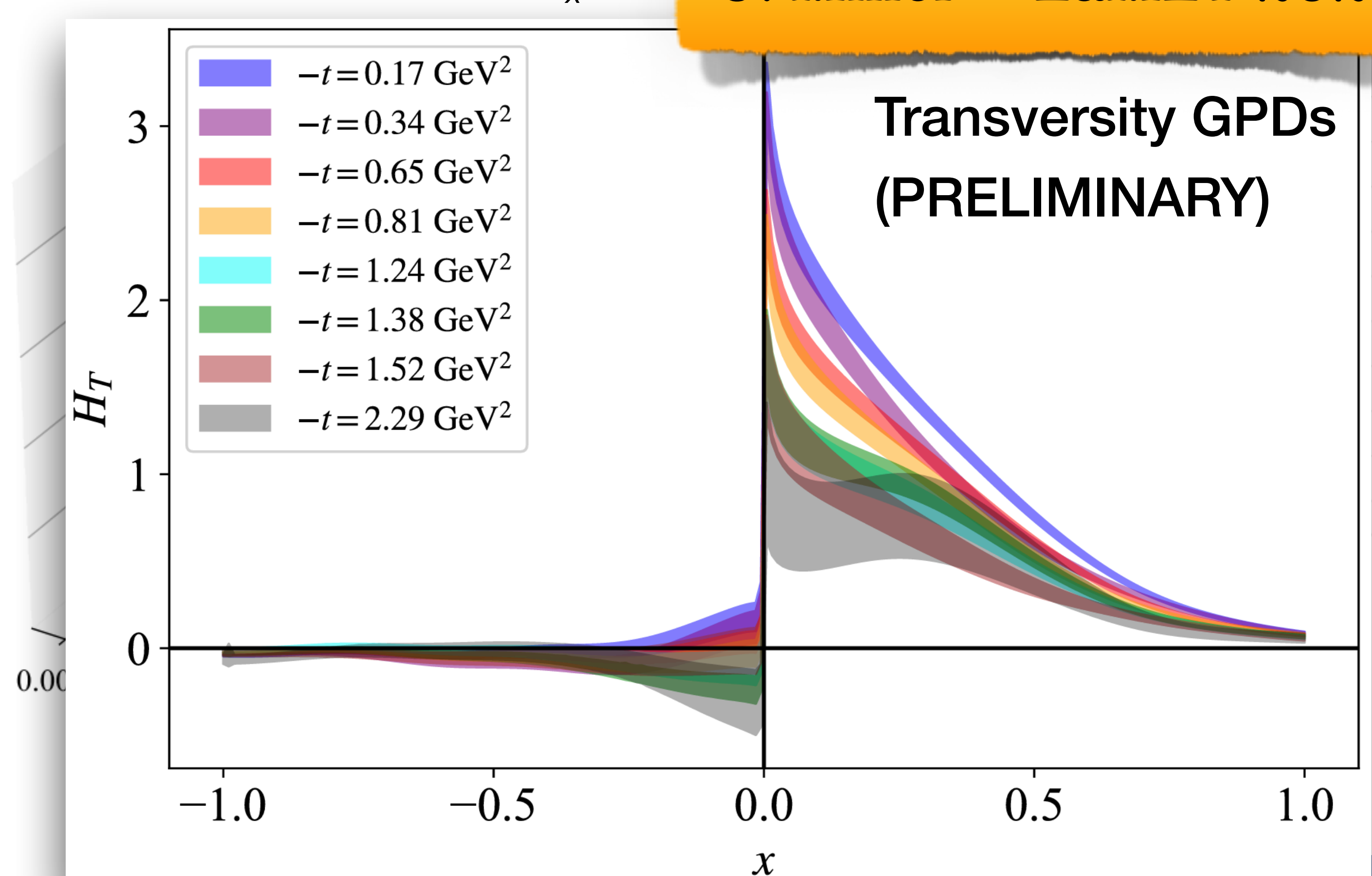
Light-cone GPDs







J. Miller @ LaMET 2024



Twist-3 GPDs

Update on:

PHYSICAL REVIEW D **108**, 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

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Also Josh Miller

(Graduate student at Temple)

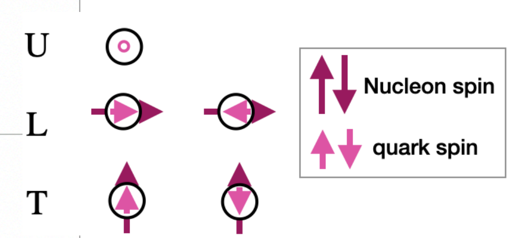
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
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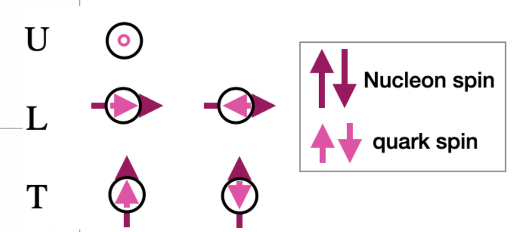

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★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

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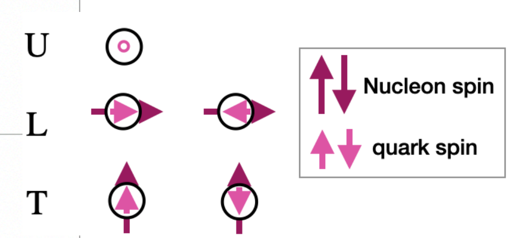

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While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

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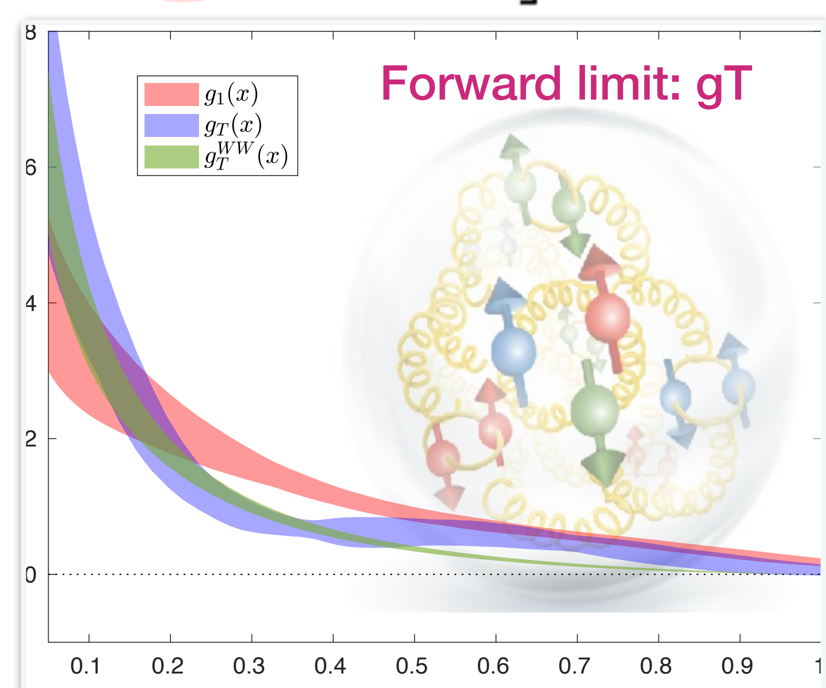
$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec z_\perp=\vec 0_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar u(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde H}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde E}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$



[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3)=\frac{1}{2}\int\frac{dz^3}{2\pi}e^{ik\cdot z}\langle p_f,\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec z_\perp=\vec 0_\perp}$$

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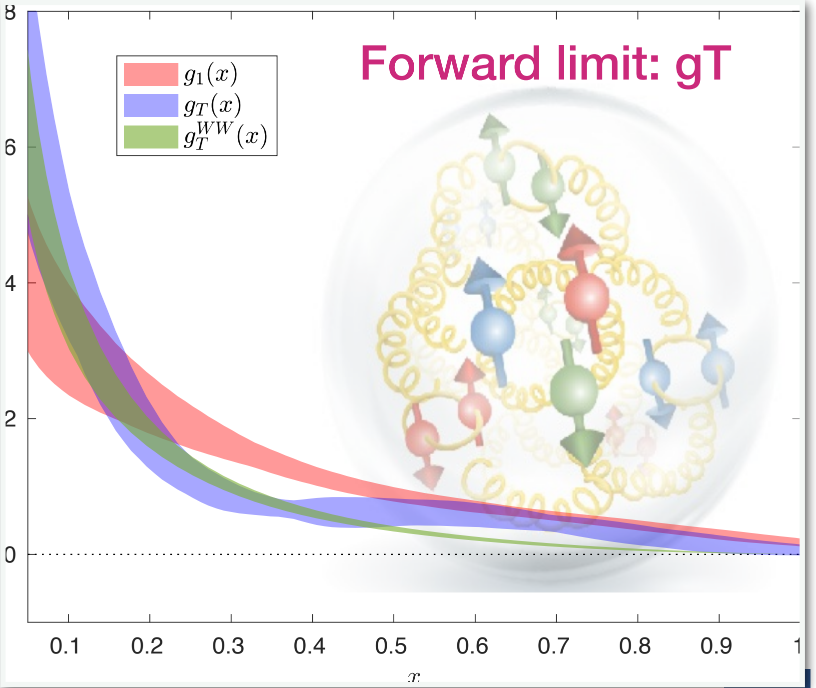
$$+\Delta^\mu_\perp\frac{\gamma_5}{2m}F_{\widetilde E+\widetilde G_1}(x,\xi,t;P^3)+\gamma^\mu_\perp\gamma_5F_{\widetilde H+\widetilde G_2}(x,\xi,t;P^3)$$
$$+\Delta^\mu_\perp\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde G_3}(x,\xi,t;P^3)+i\varepsilon^{\mu\nu}_\perp\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde G_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

★ Kinematic twist-3 contributions to pseudo- and quasi-GPDs to restore translation invariance

[V. Braun et al., JHEP 10 (2023) 134]



[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

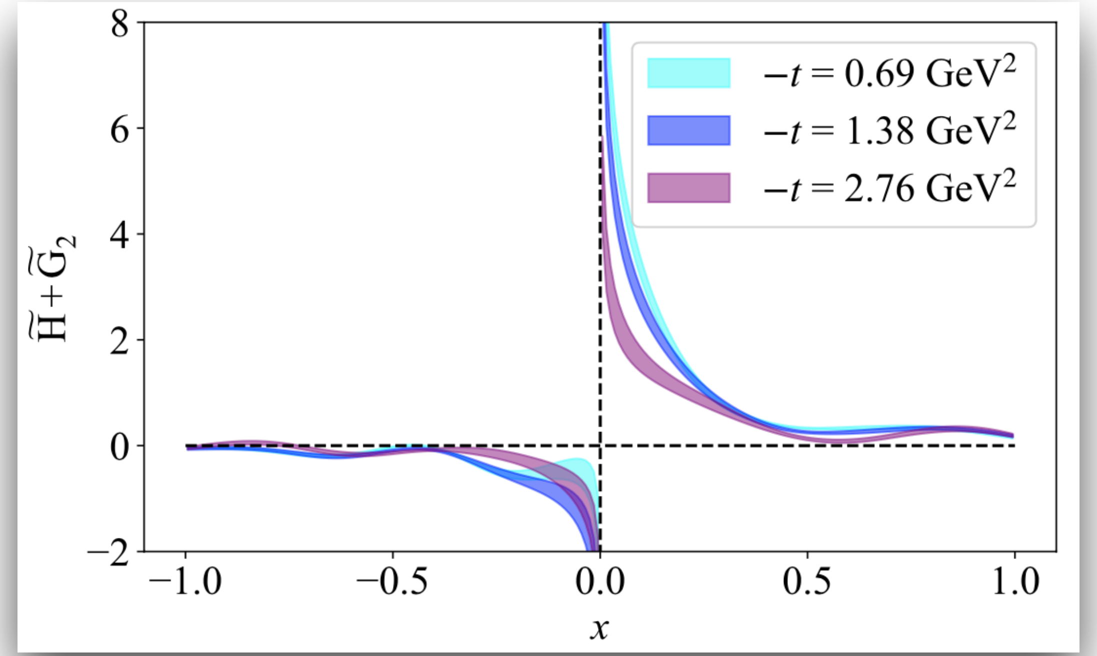
★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
±0.83	(0, 0, 0)	0	2	194	8	3104
±1.25	(0, 0, 0)	0	2	731	16	23392
±1.67	(0, 0, 0)	0	2	1644	64	210432
±0.83	(±2, 0, 0)	0.69	8	67	8	4288
±1.25	(±2, 0, 0)	0.69	8	249	8	15936
±1.67	(±2, 0, 0)	0.69	8	294	32	75264
±1.25	(±2, ±2, 0)	1.38	16	224	8	28672
±1.25	(±4, 0, 0)	2.76	8	329	32	84224

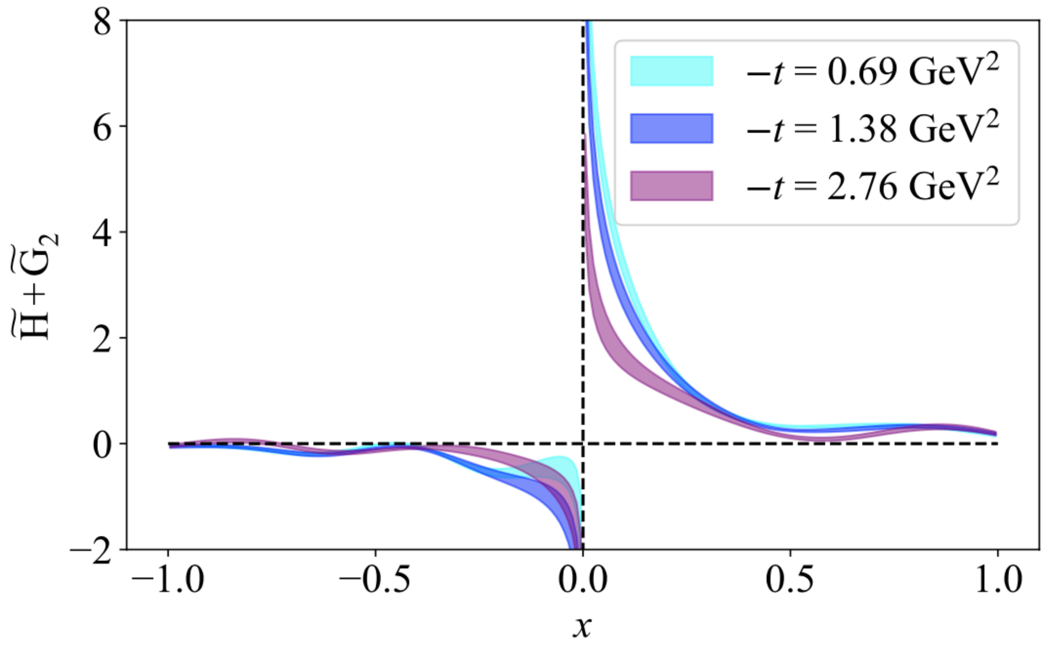
Symmetric frame computationally expensive

Zero skewness calculation

Lattice Results - light-cone GPDs

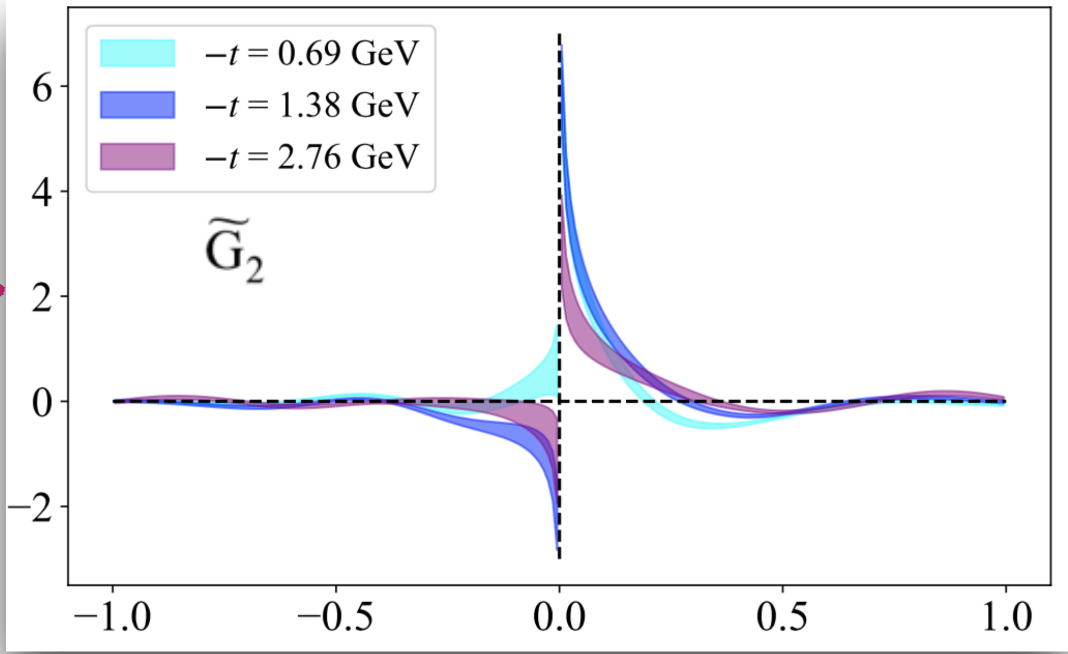


Lattice Results - light-cone GPDs

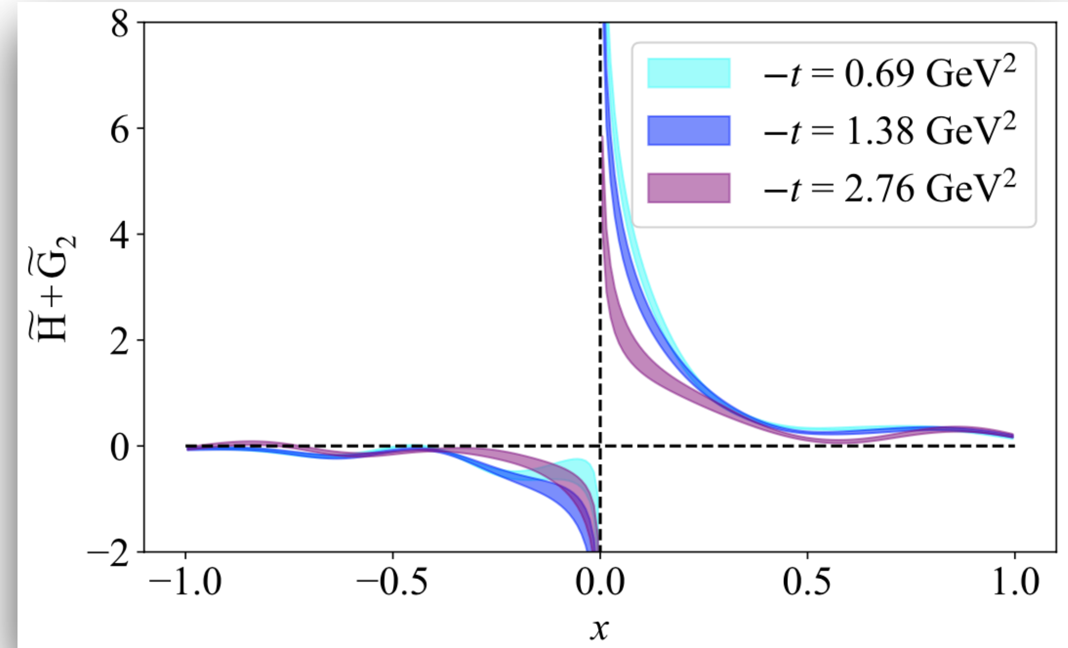


Isolating $\widetilde{G}_2$

using $\widetilde{H}$



Lattice Results - light-cone GPDs



Isolating $\widetilde{G}_2$

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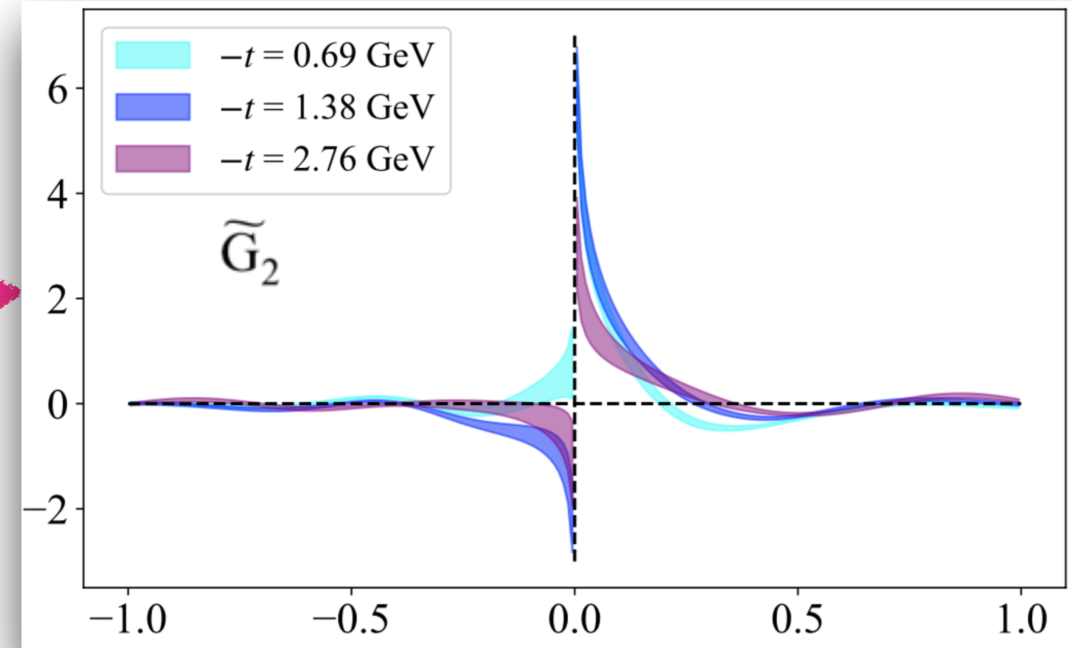

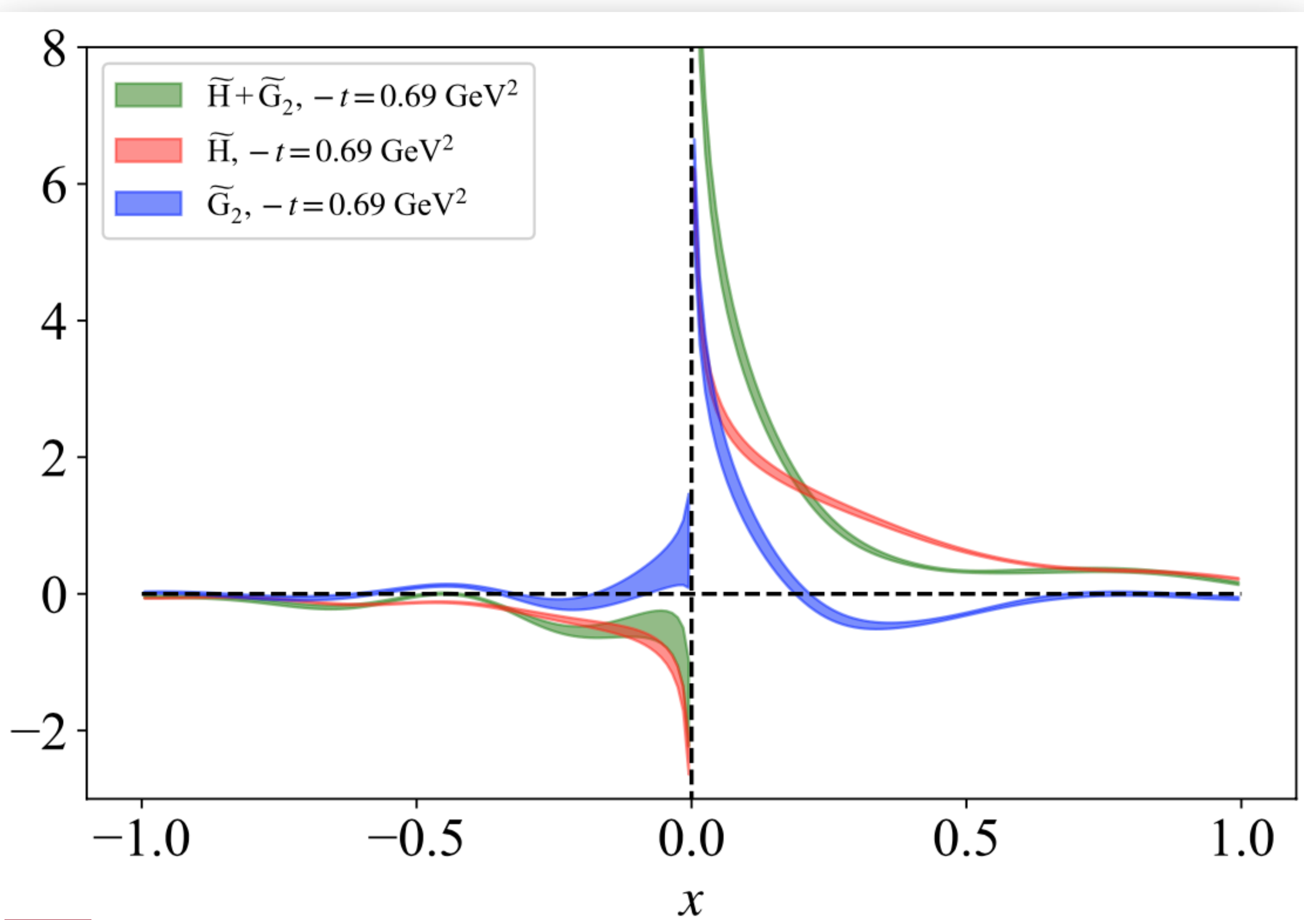

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x, \xi, t; P^3)$$

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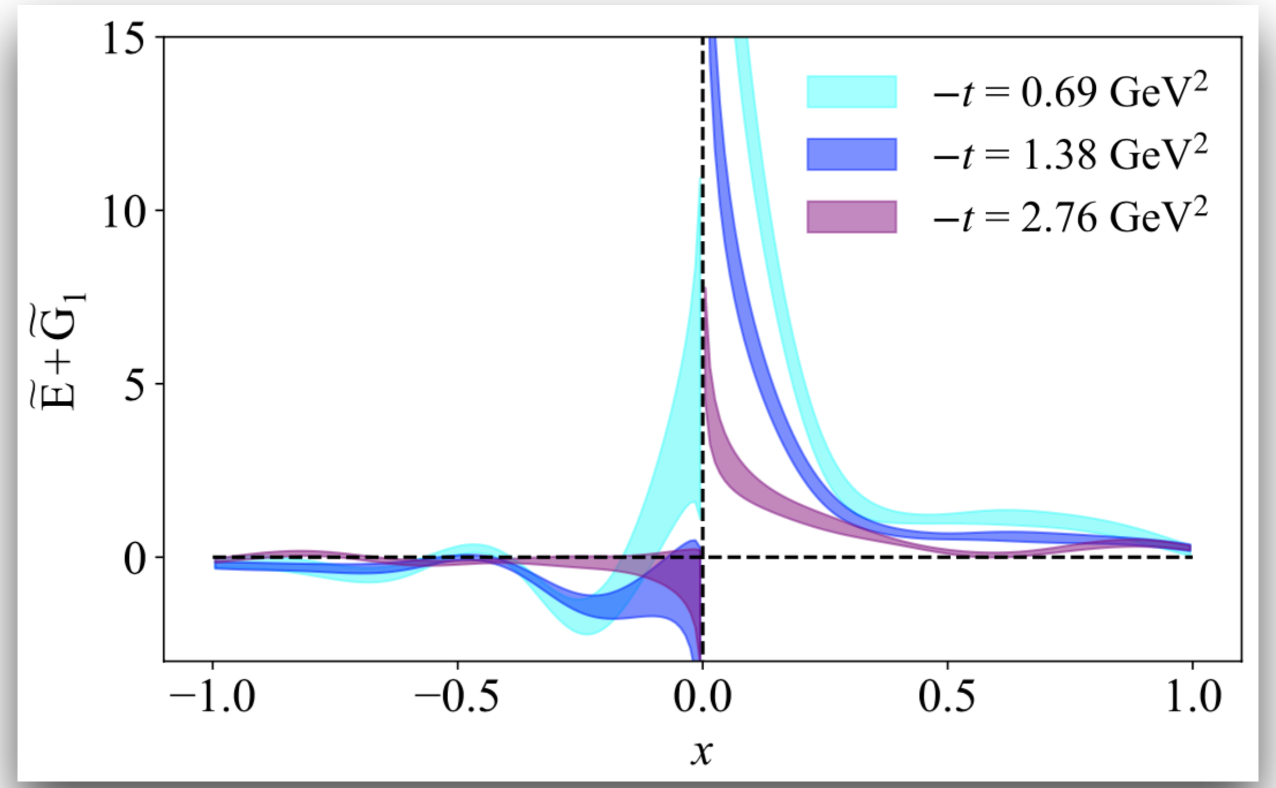

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

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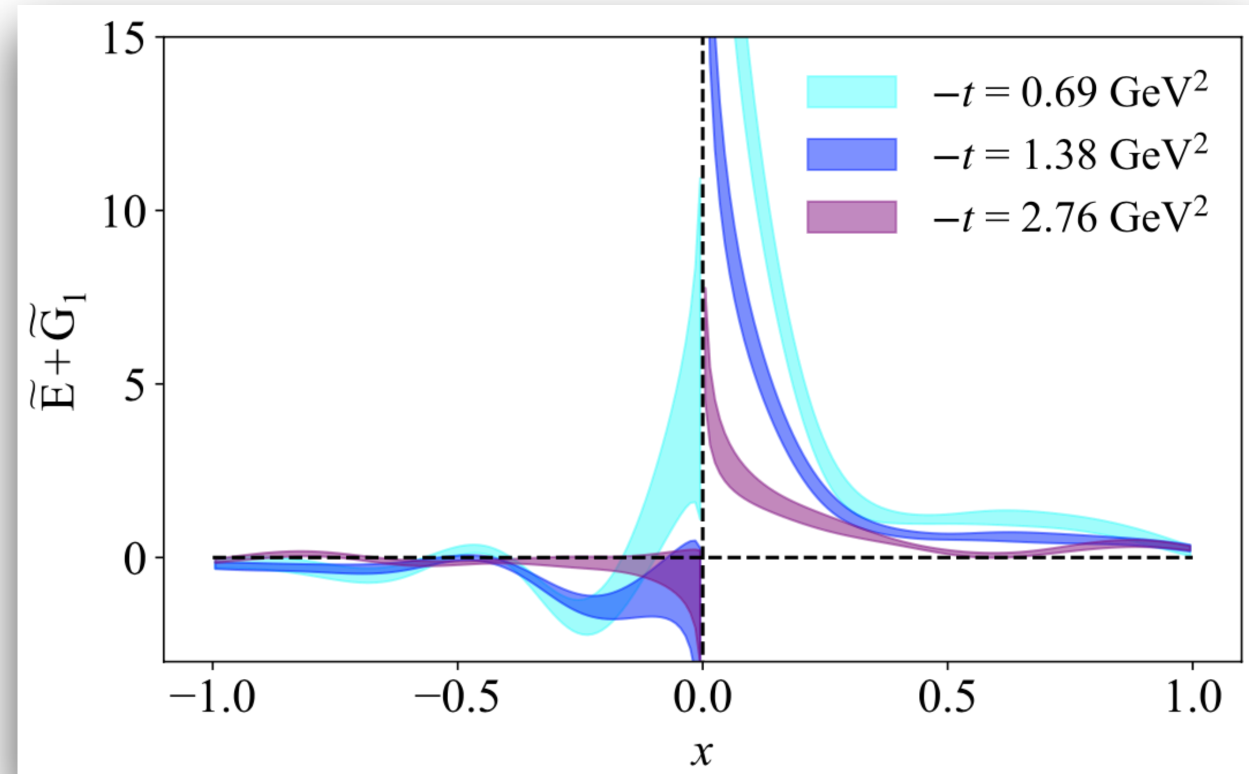

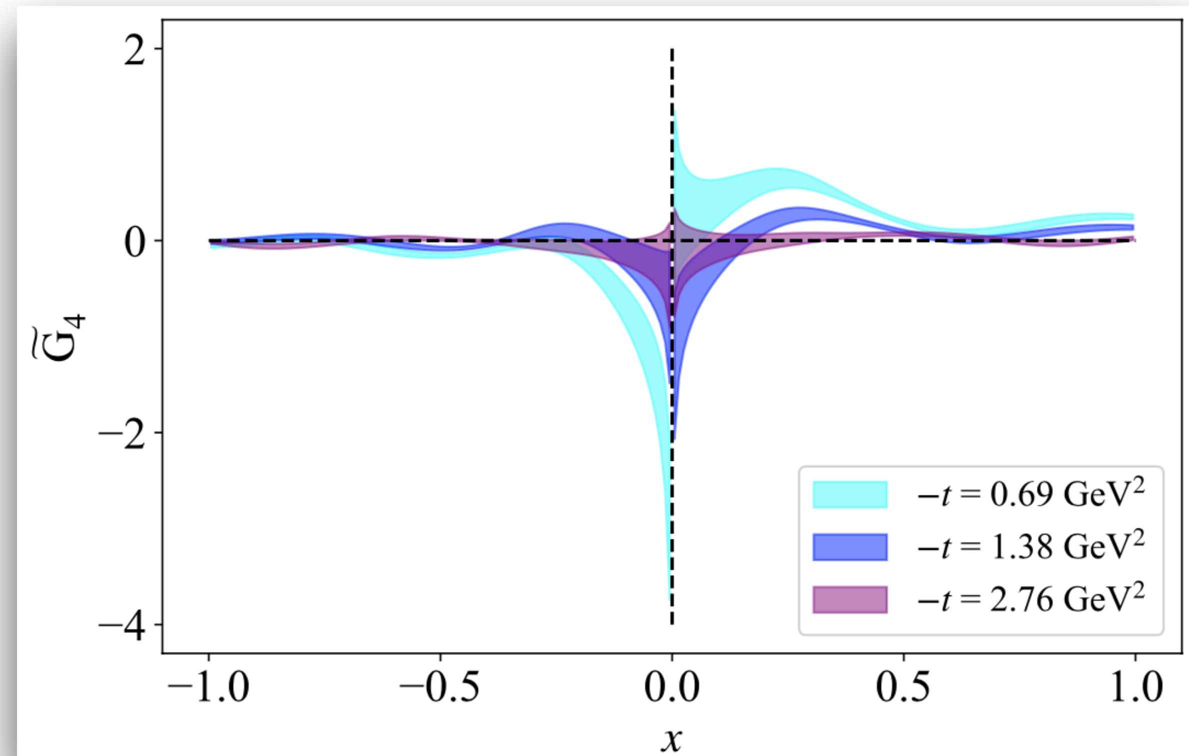

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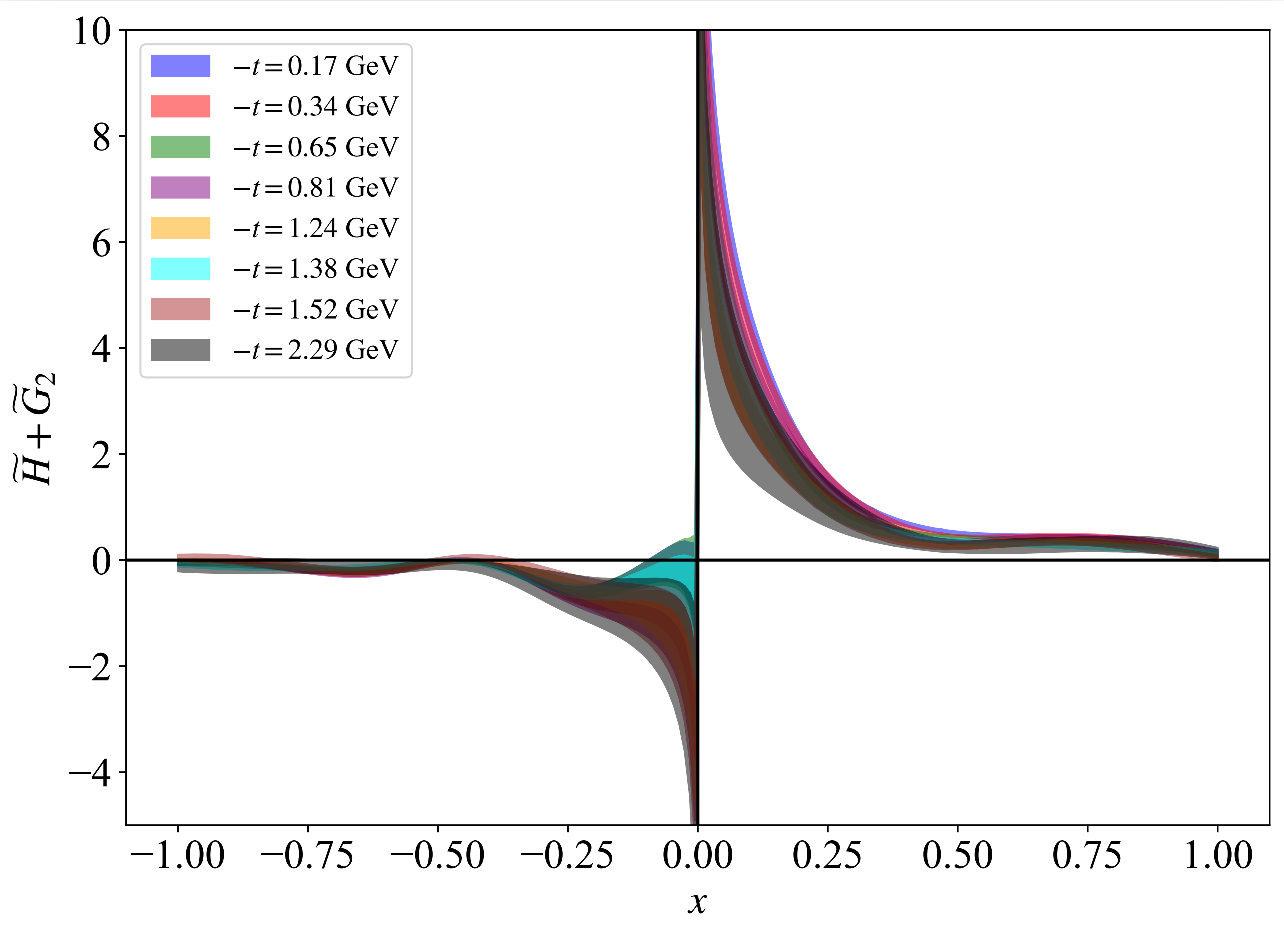
$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

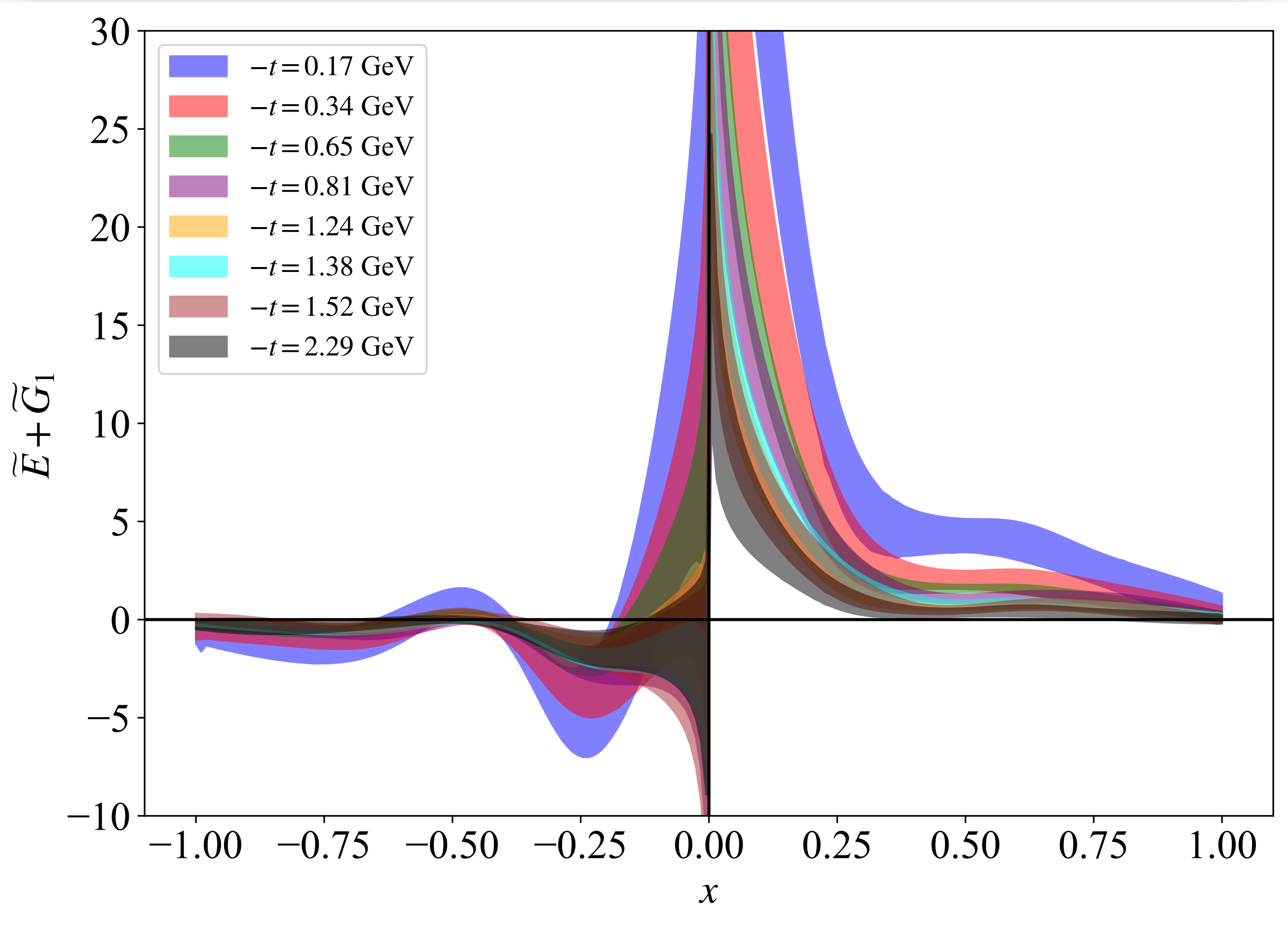
★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Preliminary Results

Axial twist-3 GPDs via asymmetric frame calculation

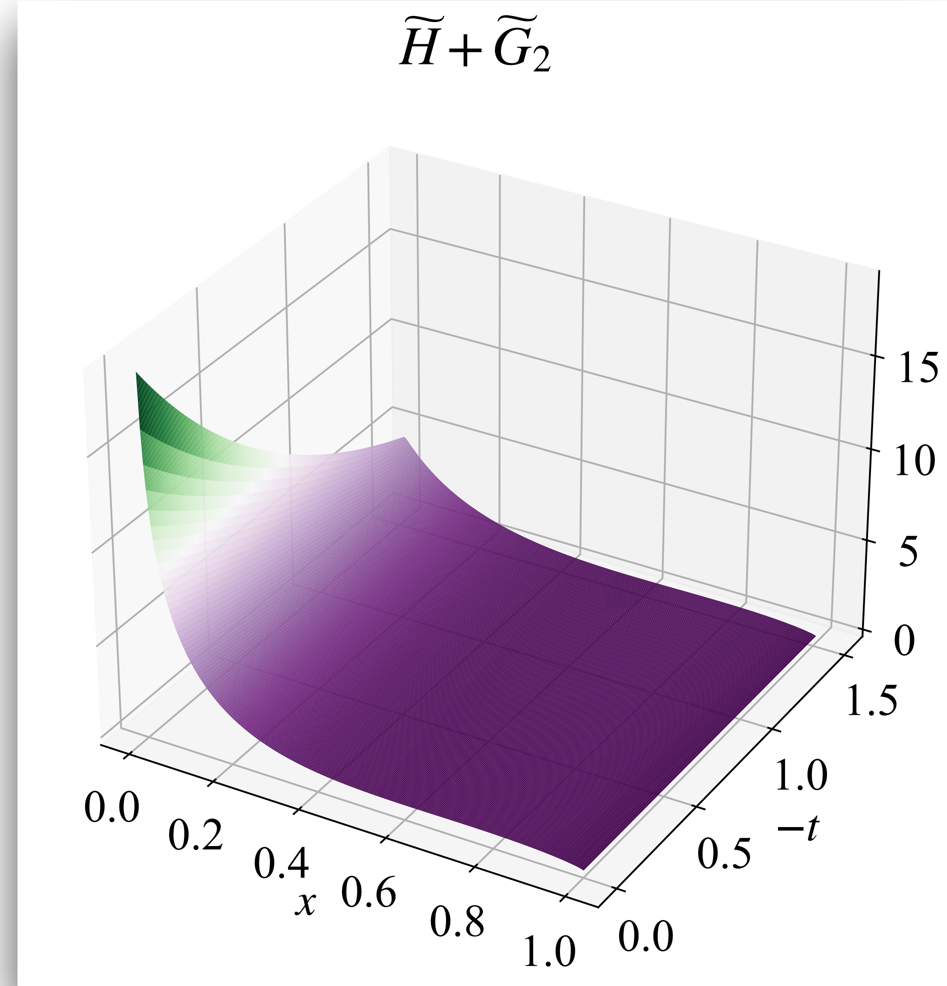




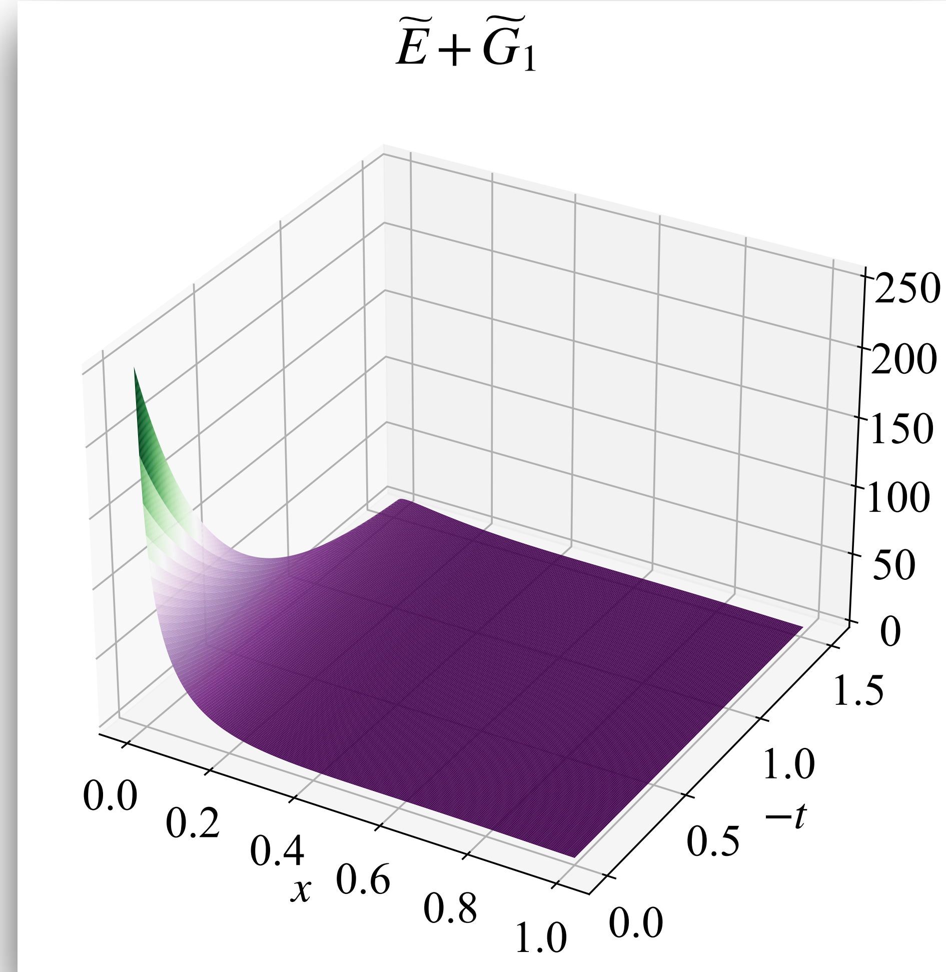
★ Parametrization of -t dependence

$$\text{GPD}(x,-t,0) = A x^{\alpha_0 - \alpha_1 t} (1-x)^{\beta}$$

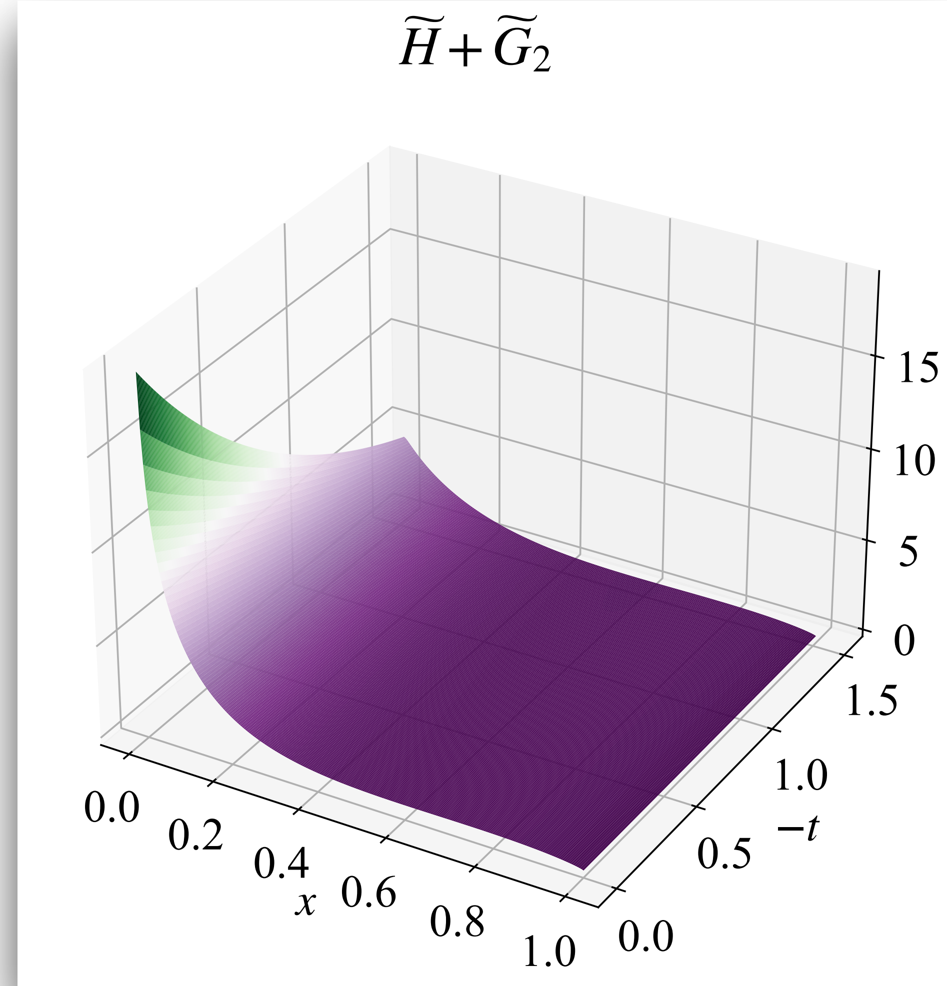
Ademollo & Del Giudice Gatto & Preparata

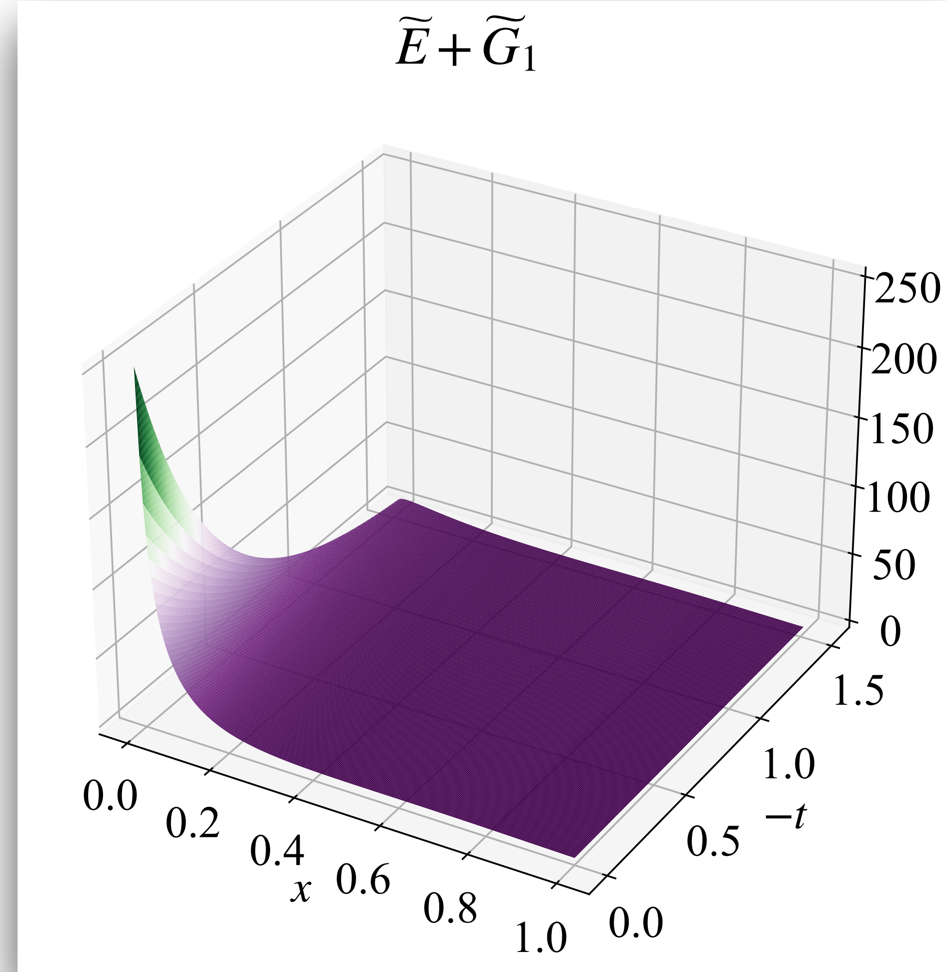


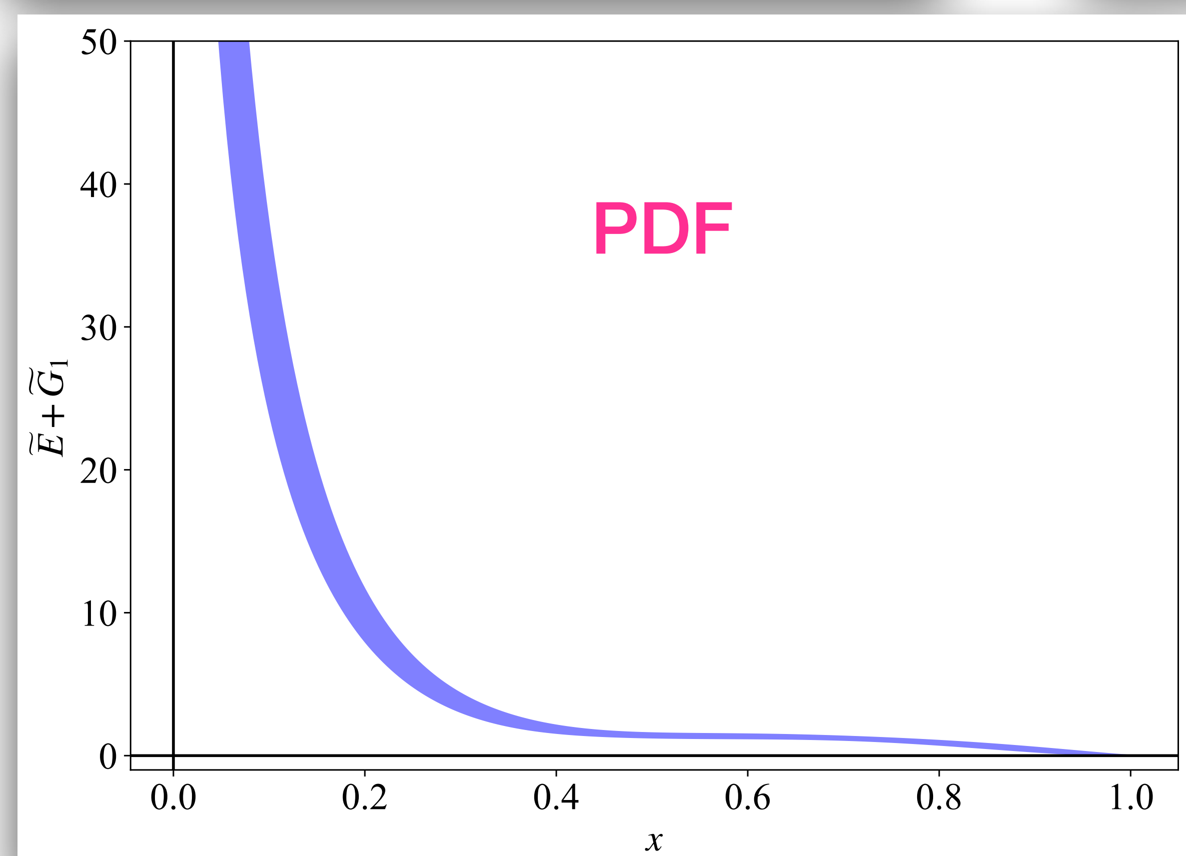
$\widetilde{H}+\widetilde{G}_2$
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0
x
1.5
1.0
0.5
0.0
$-t$

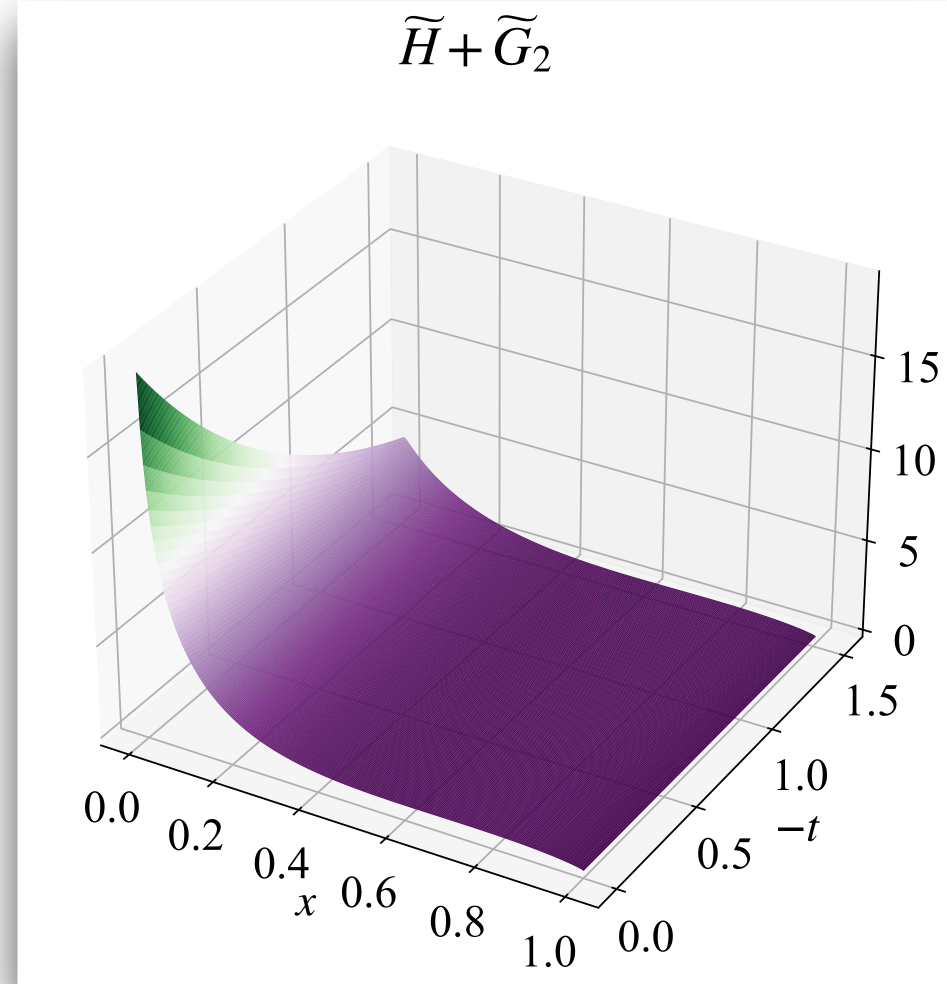


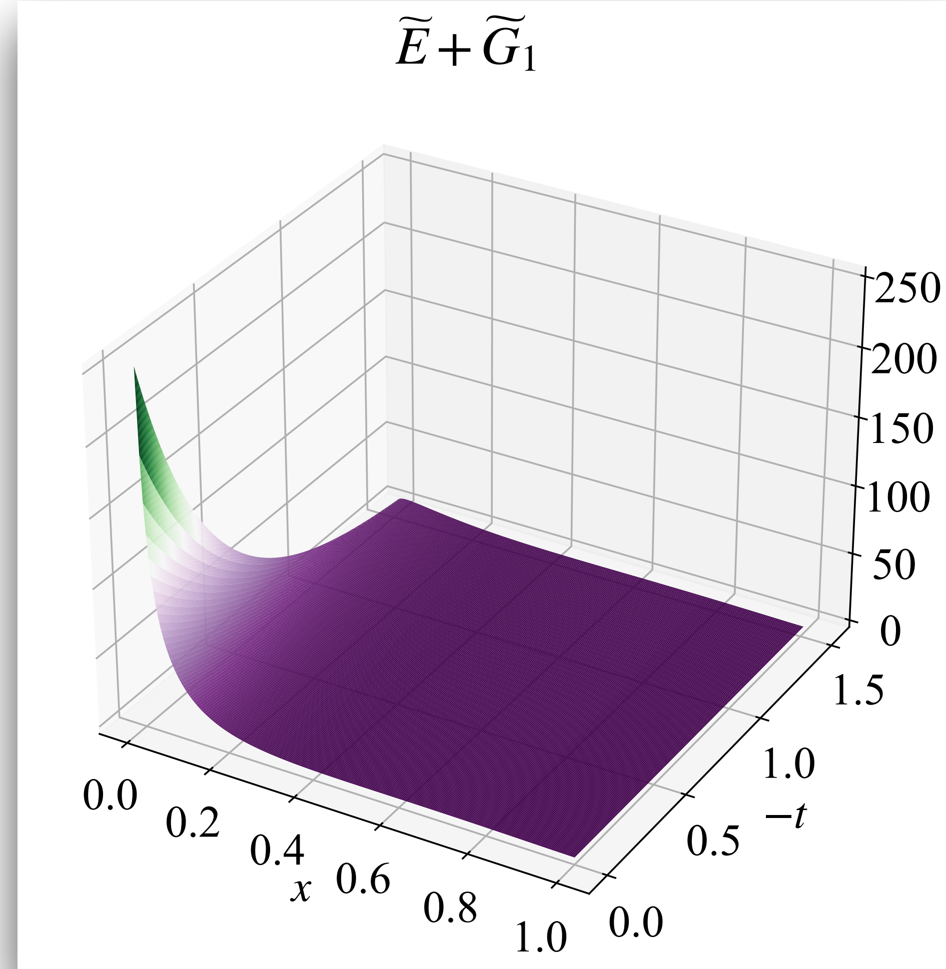
$\widetilde{E}+\widetilde{G}_1$
250
200
150
100
50
0
0.0
0.2
0.4
0.6
0.8
1.0
x
1.5
1.0
0.5
0.0
$-t$

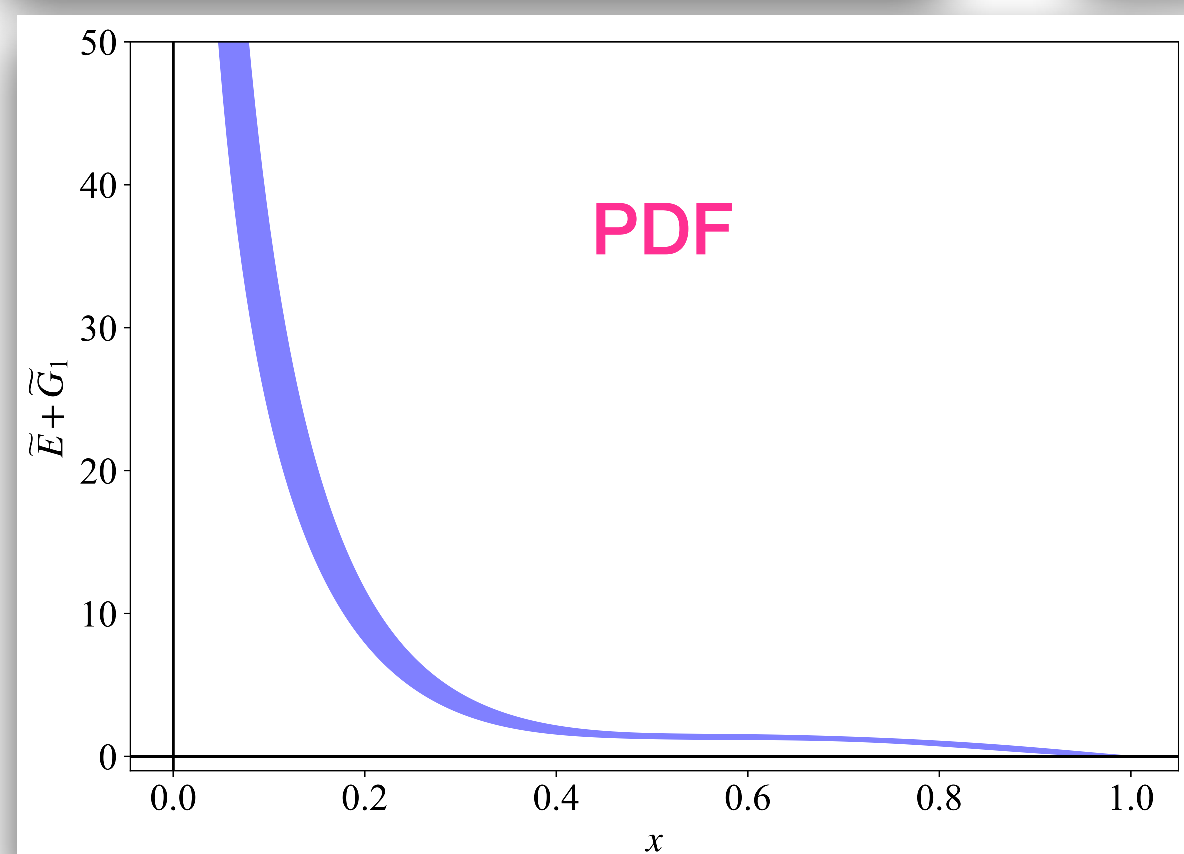

$\widetilde{H}+\widetilde{G}_2$
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.5
1.0
1.5
$-t$


$\widetilde{E}+\widetilde{G}_1$
250
200
150
100
50
0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.5
1.0
1.5
$-t$


PDF
$\widetilde{E}+\widetilde{G}_1$
50
40
30
20
10
0
0.0
0.2
0.4
0.6
0.8
1.0
x







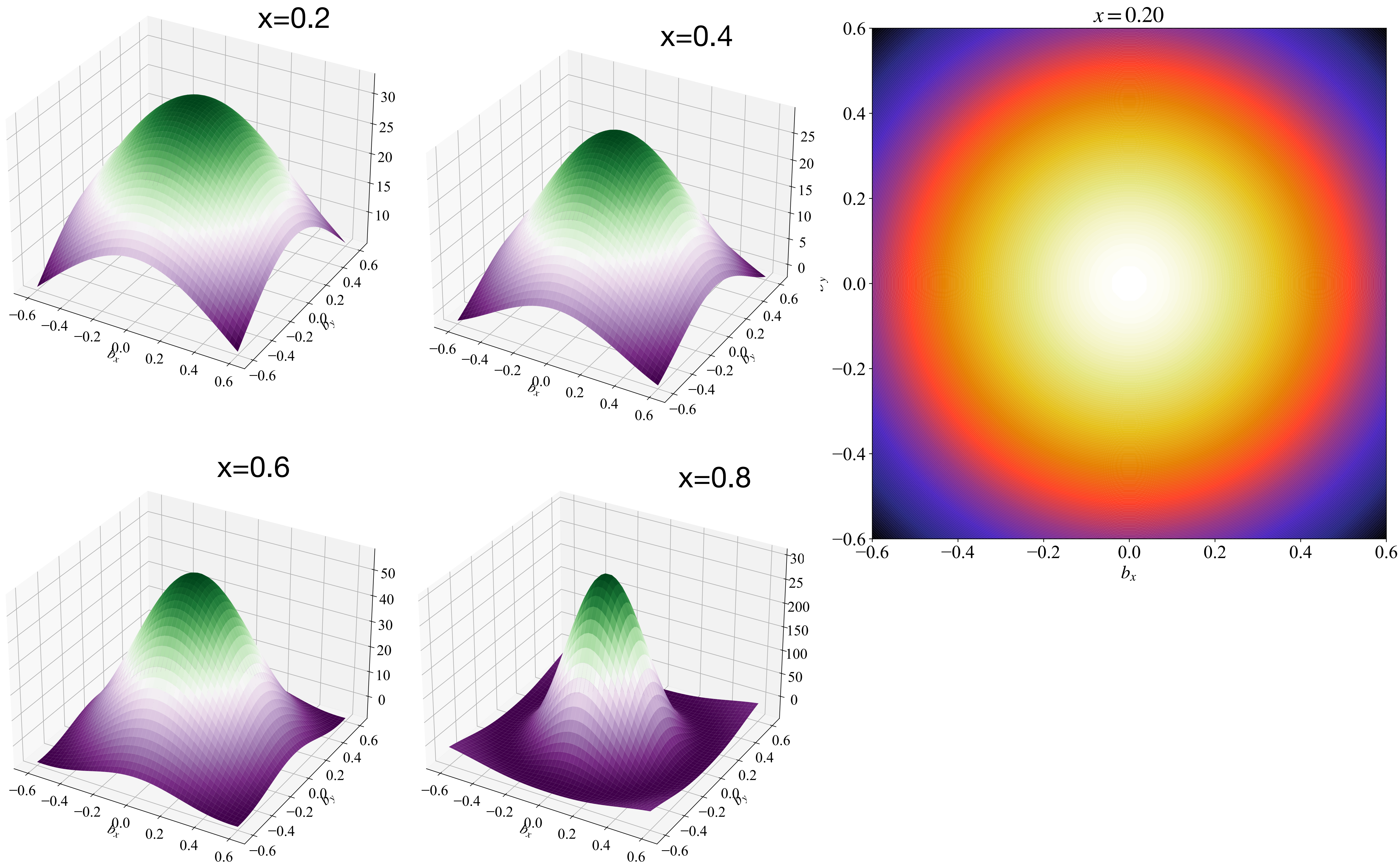
★ **At ξ=0 we obtain GPDs in transverse plane via Fourier transform**

$$q(x,\mathbf{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

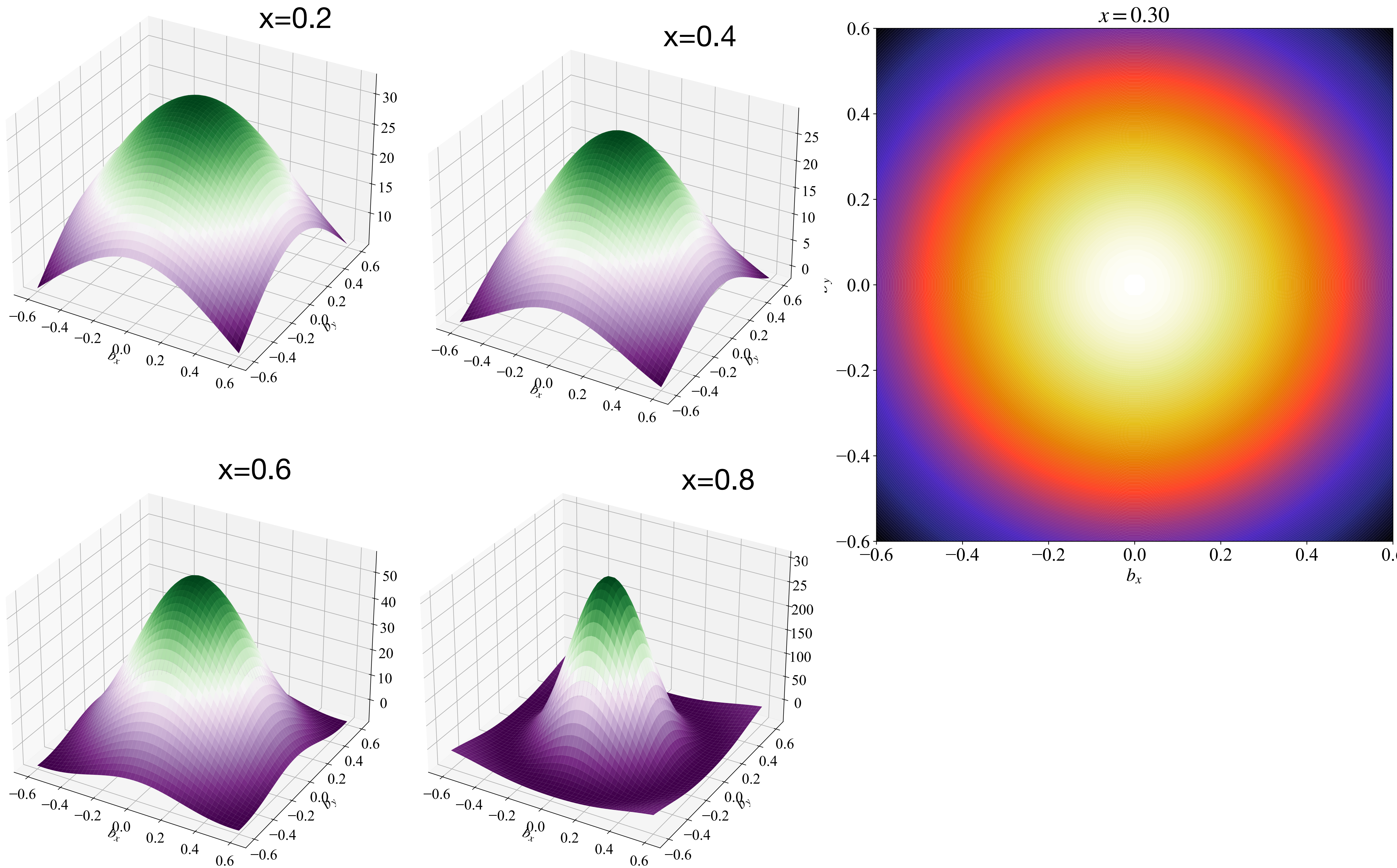
$$= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_q(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp},$$

$b_\perp$: transverse distance from the (transverse) center of momentum

Impact parameter space $\widetilde{H} + \widetilde{G}_2$



Impact parameter space $\widetilde{H} + \widetilde{G}_2$



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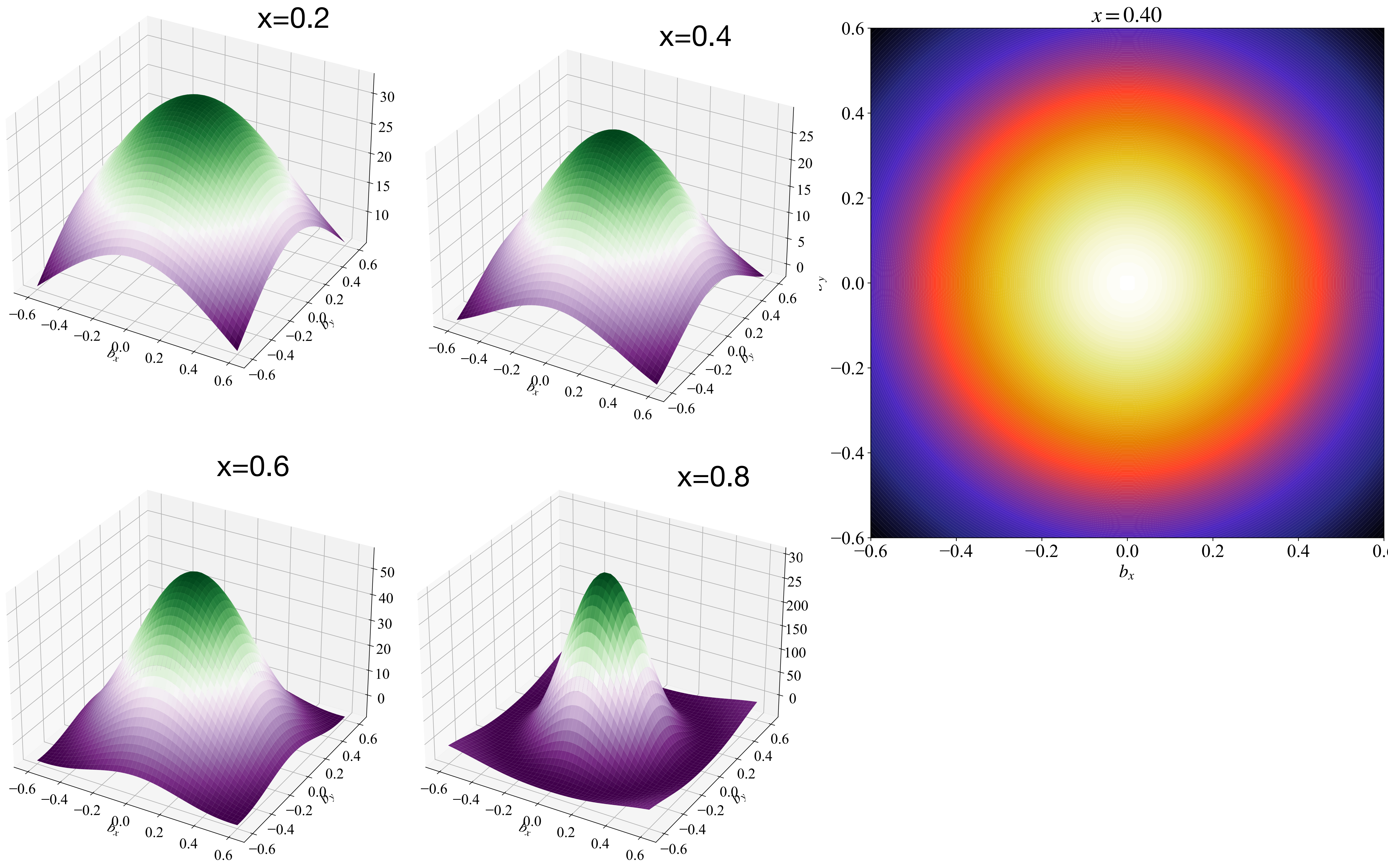
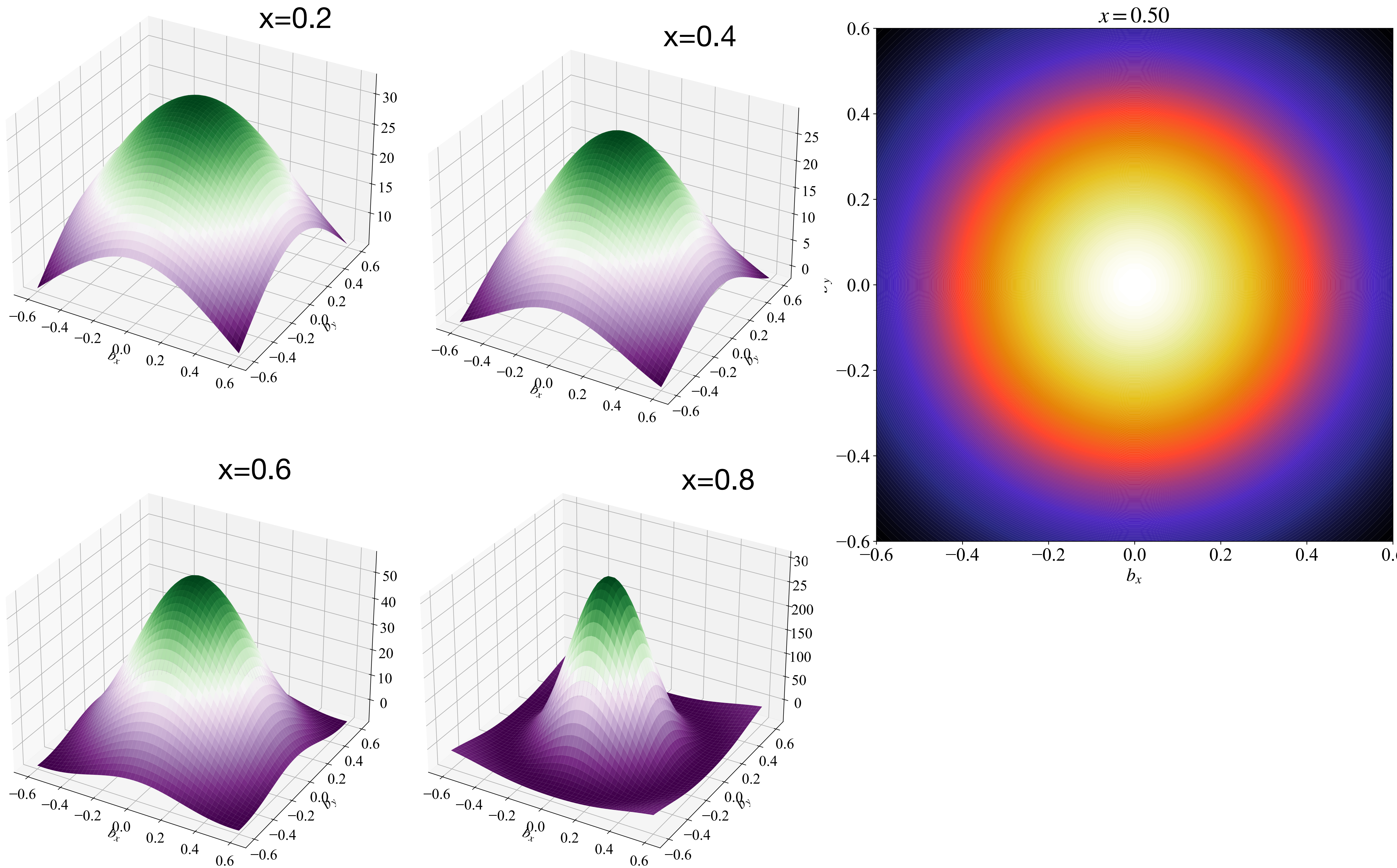


Impact parameter space $\widetilde{H} + \widetilde{G}_2$



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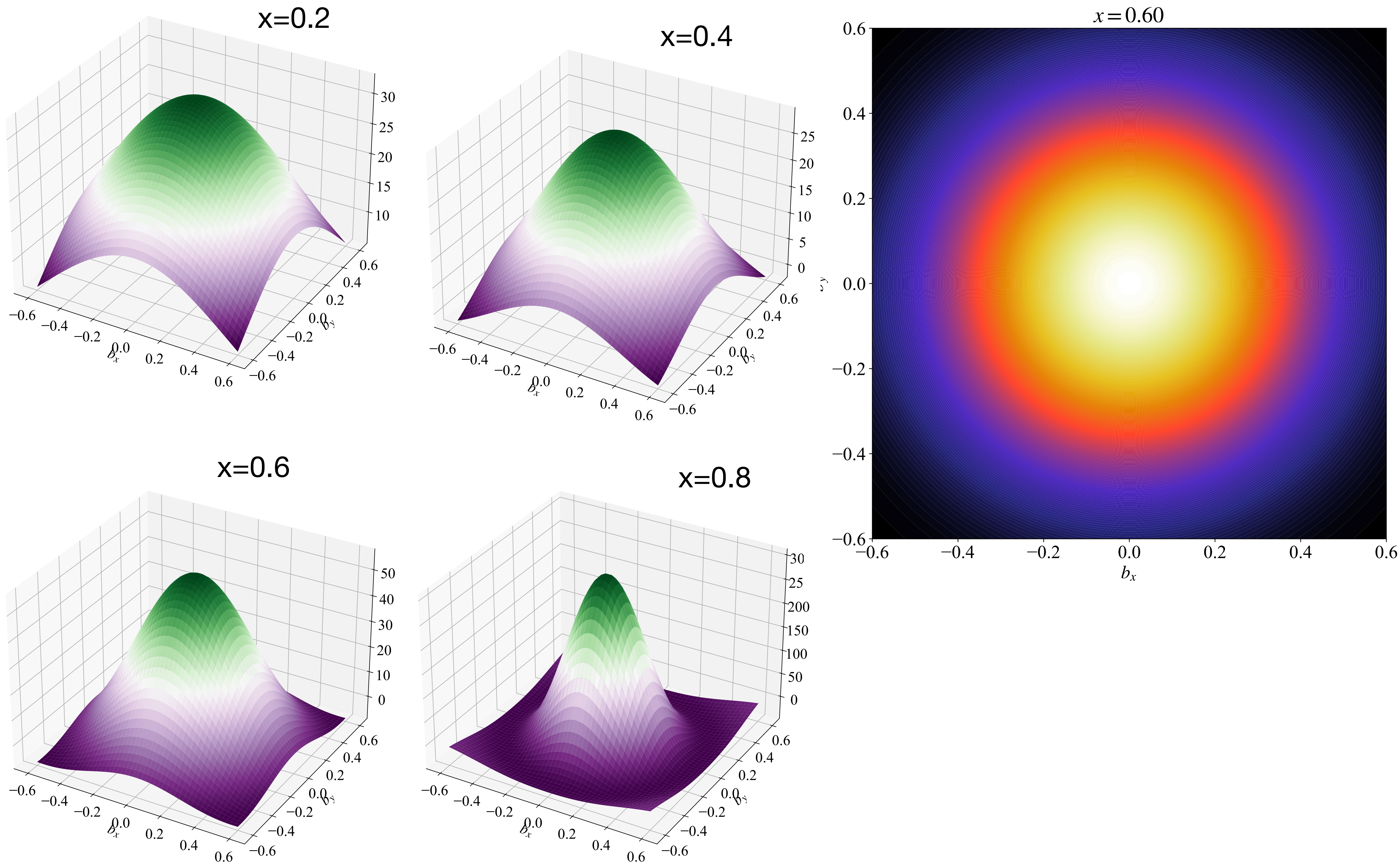

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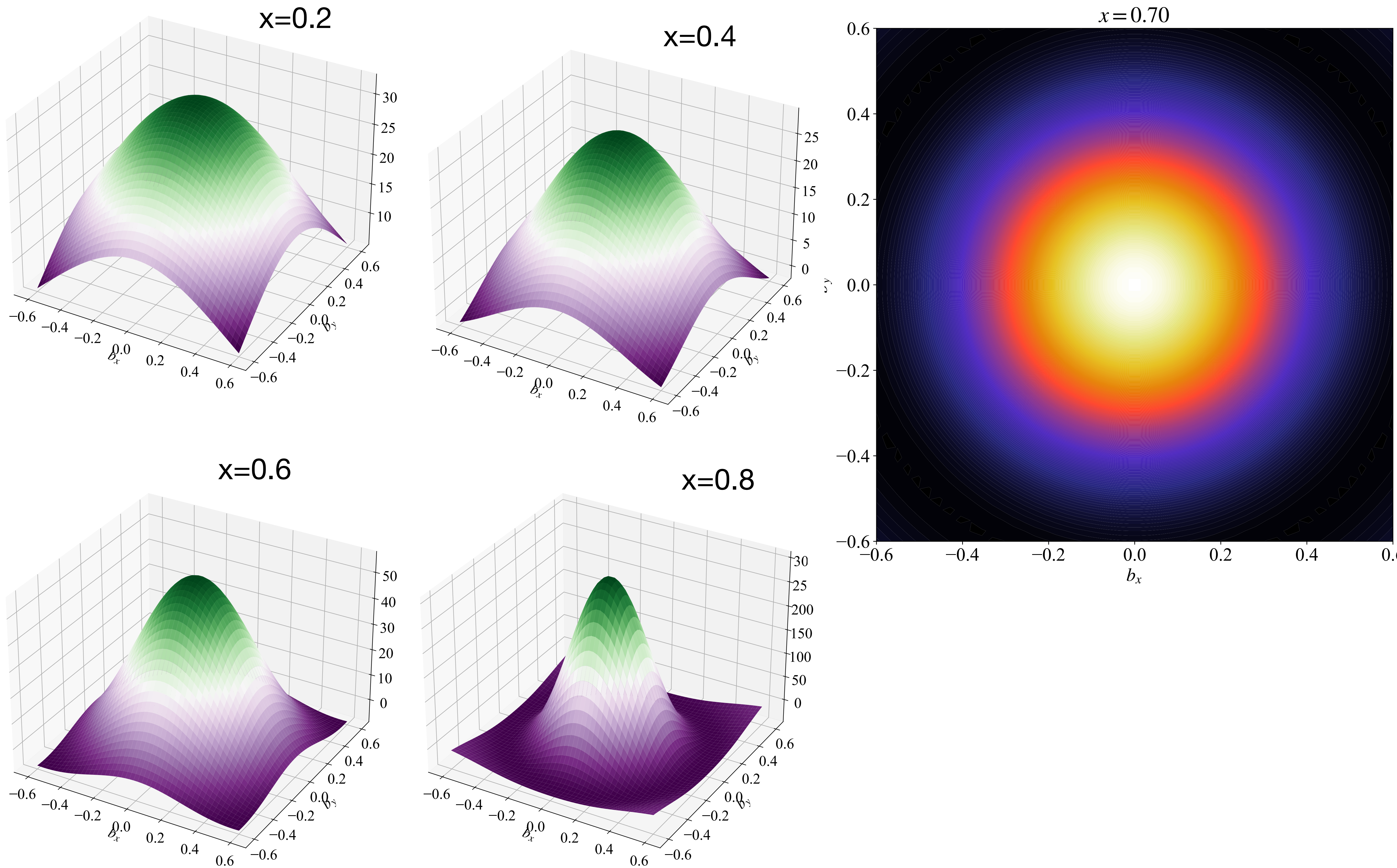
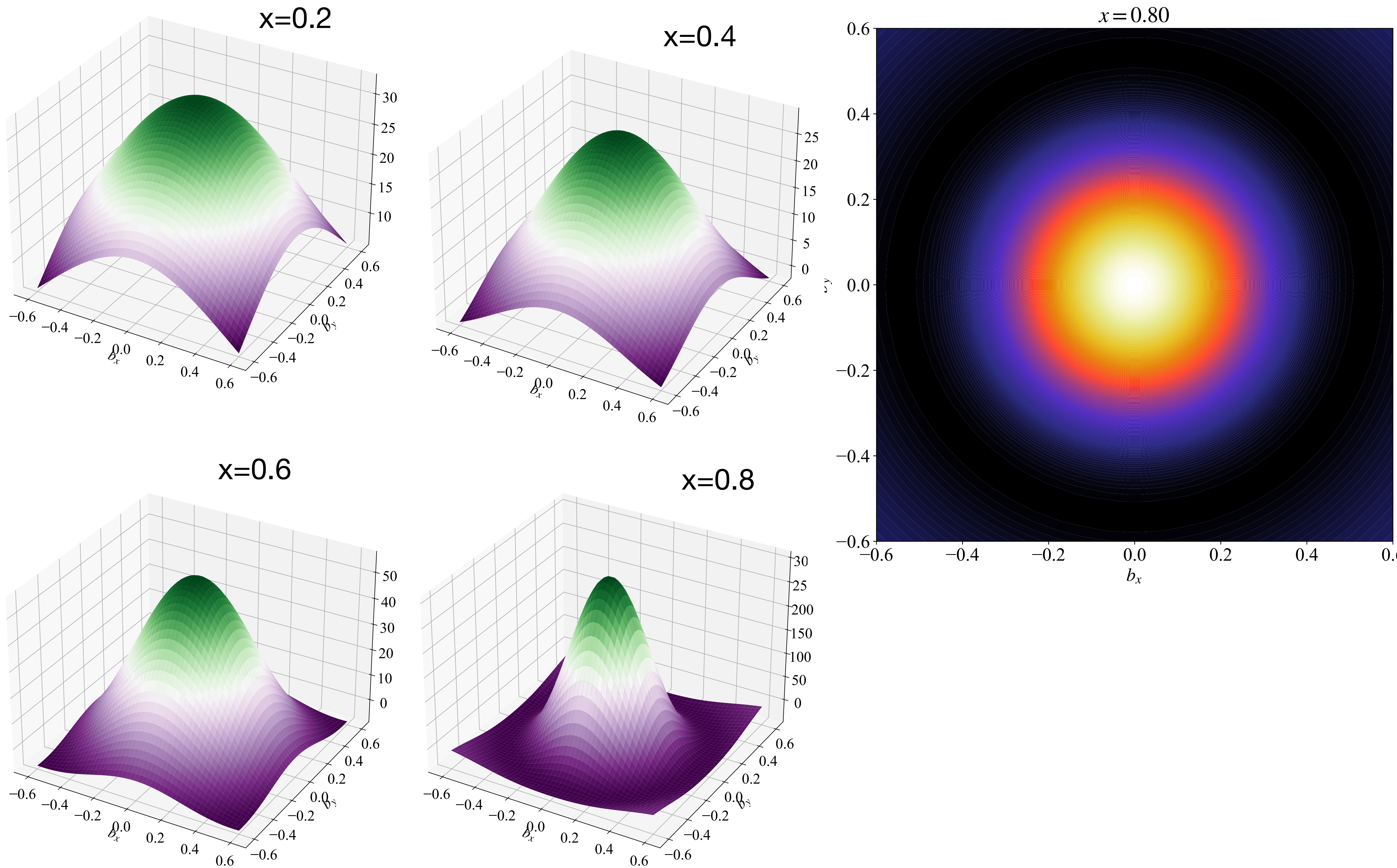
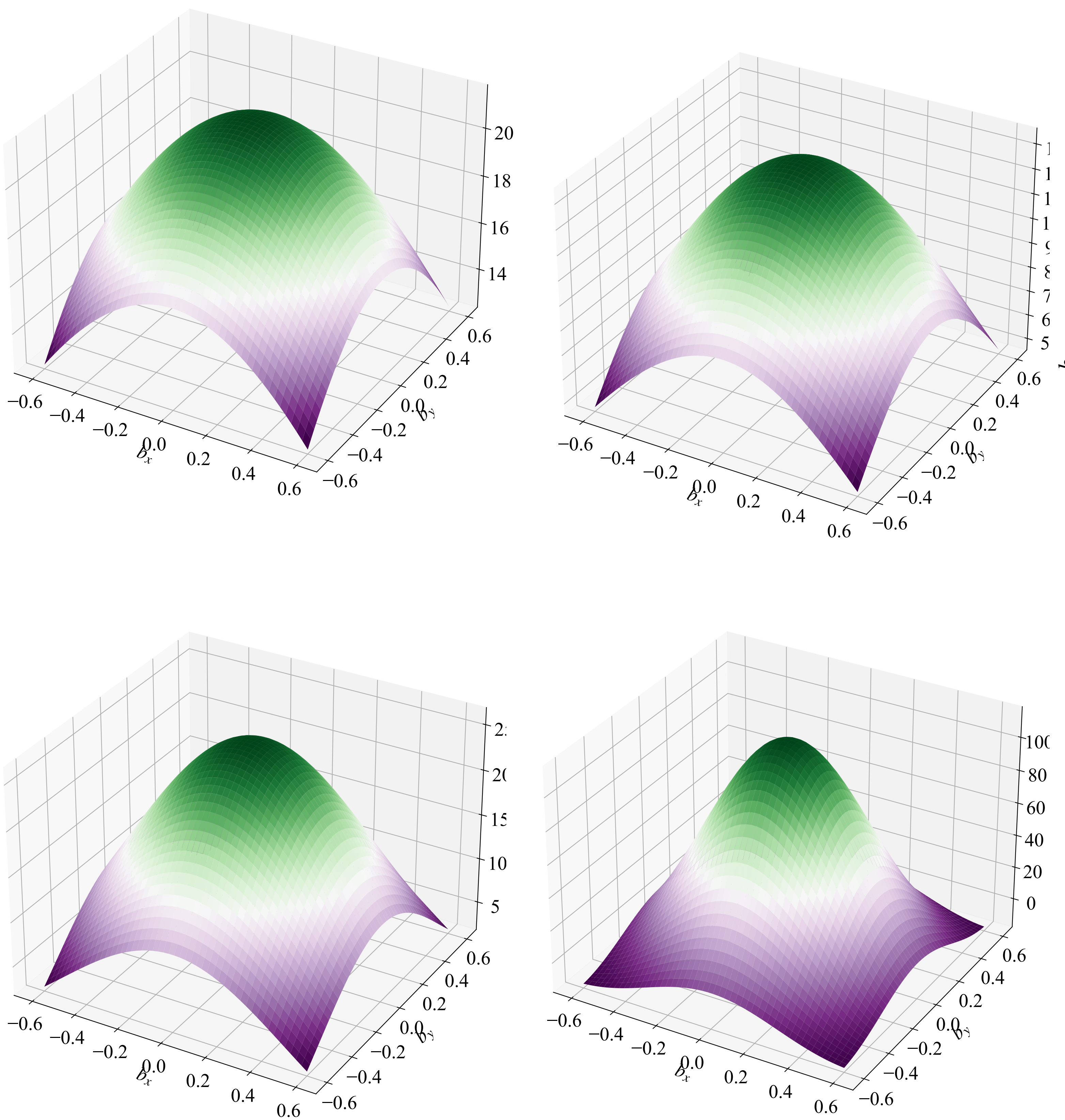
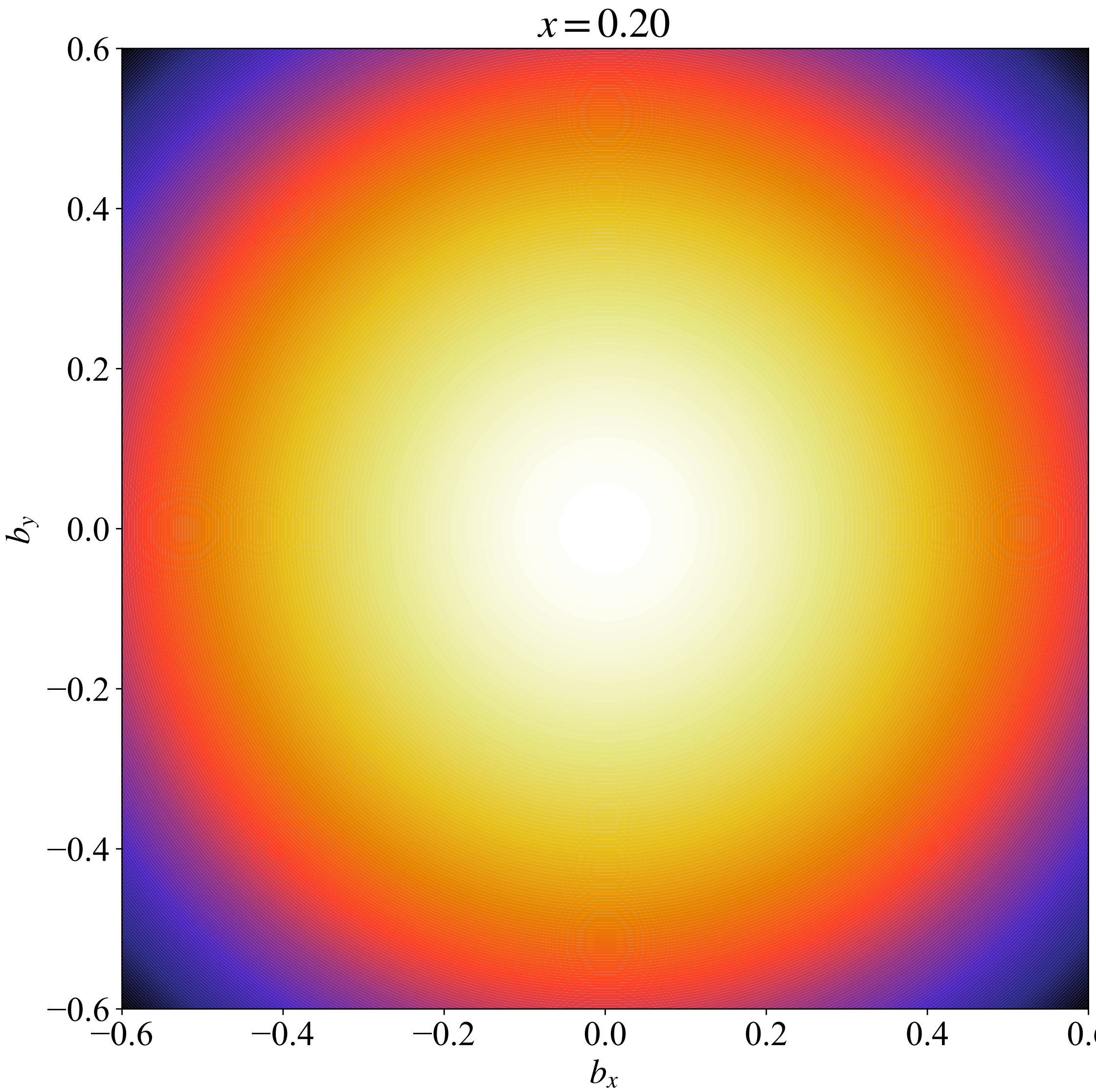


Impact parameter space $\widetilde{H} + \widetilde{G}_2$



Impact parameter space $\widetilde{E} + \widetilde{G}_1$





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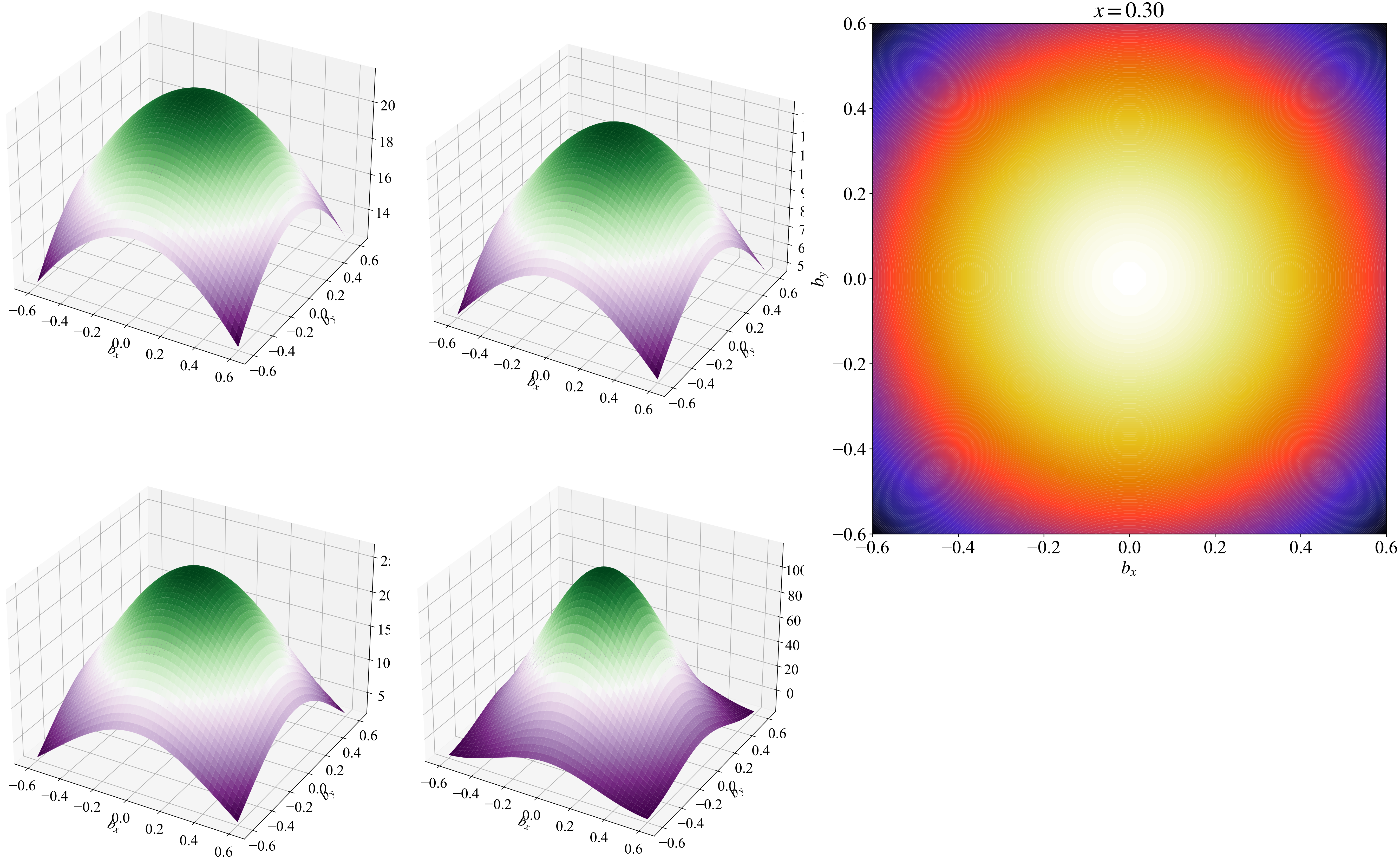
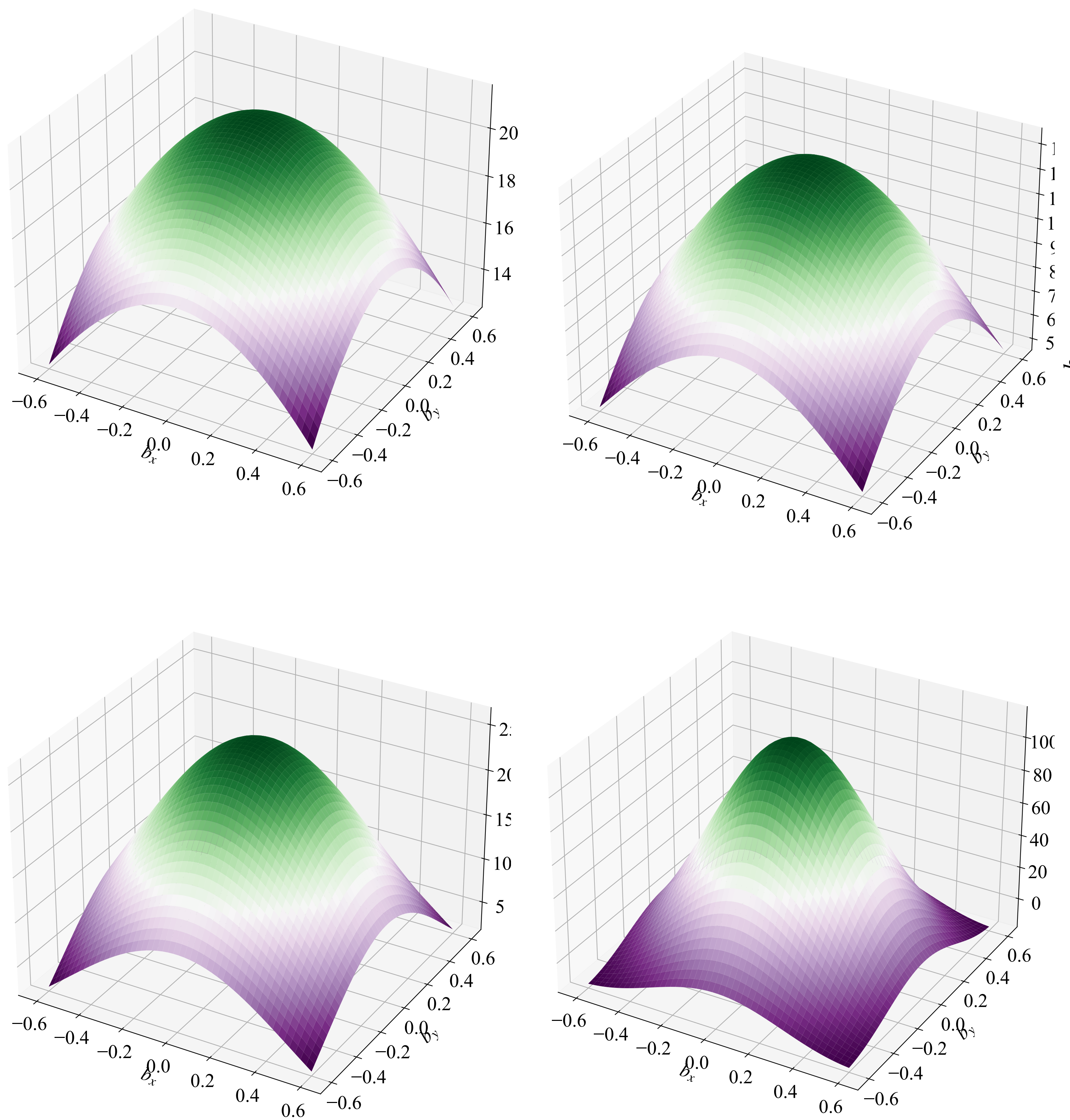
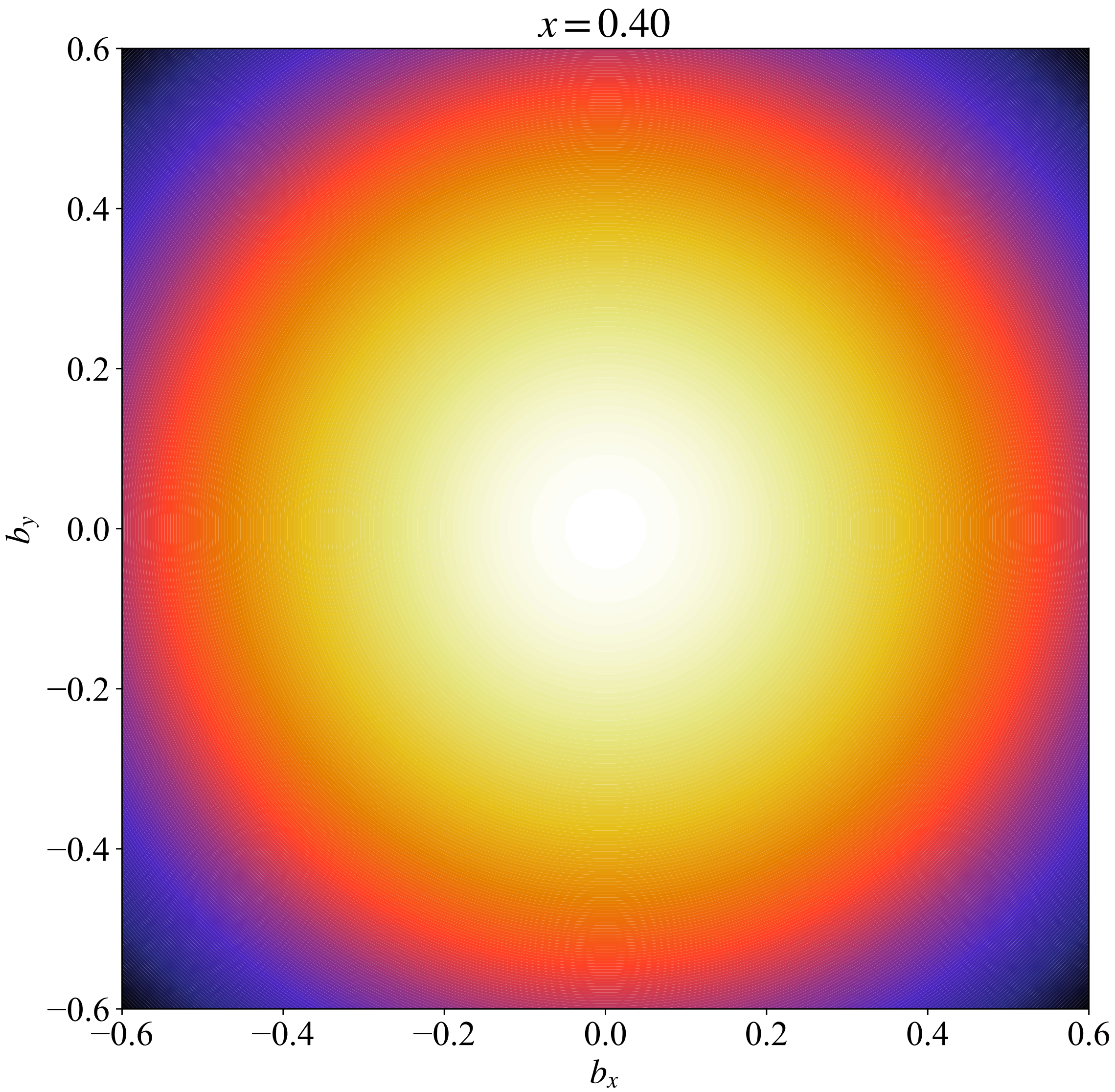


Impact parameter space $\widetilde{E} + \widetilde{G}_1$





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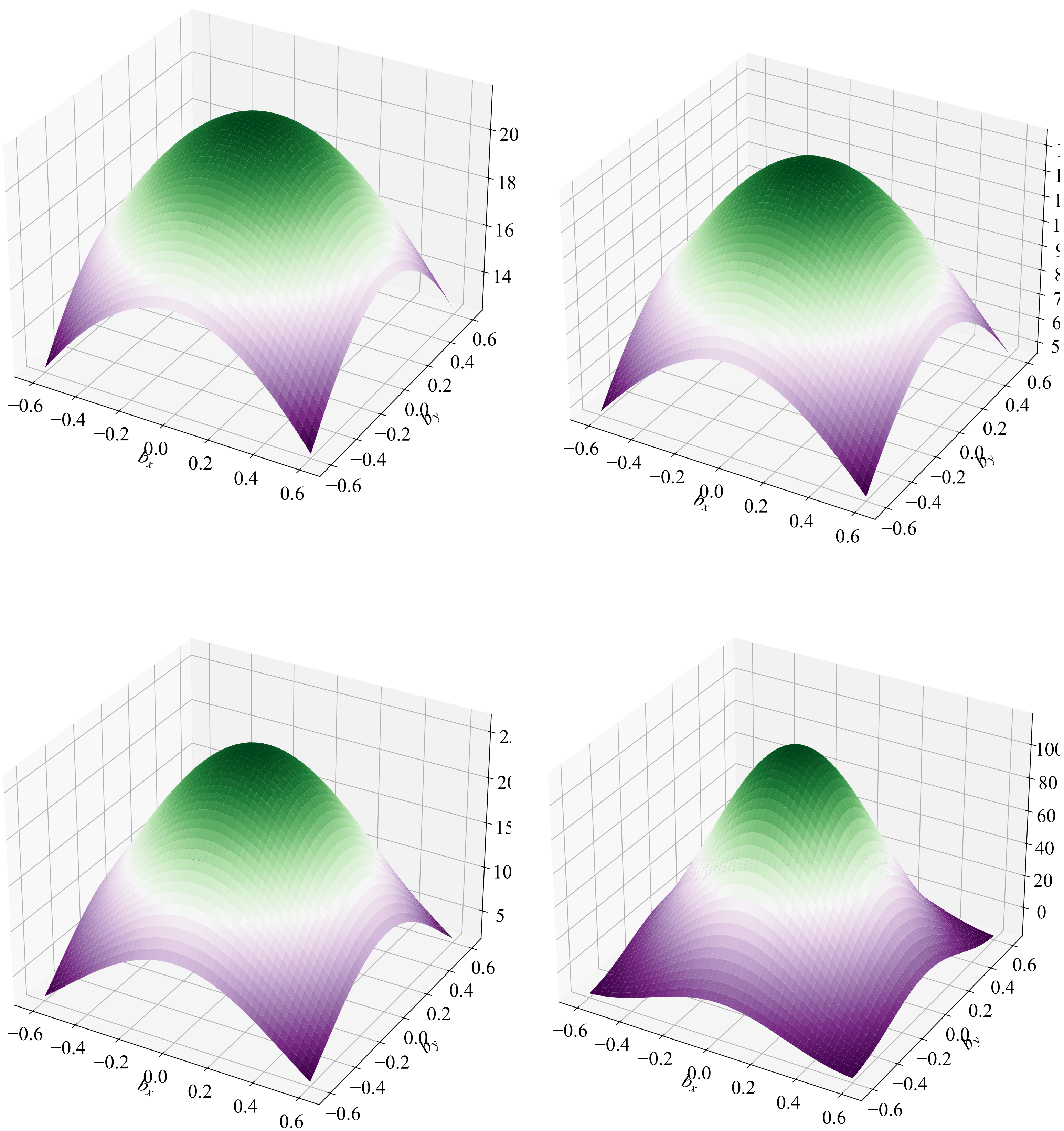

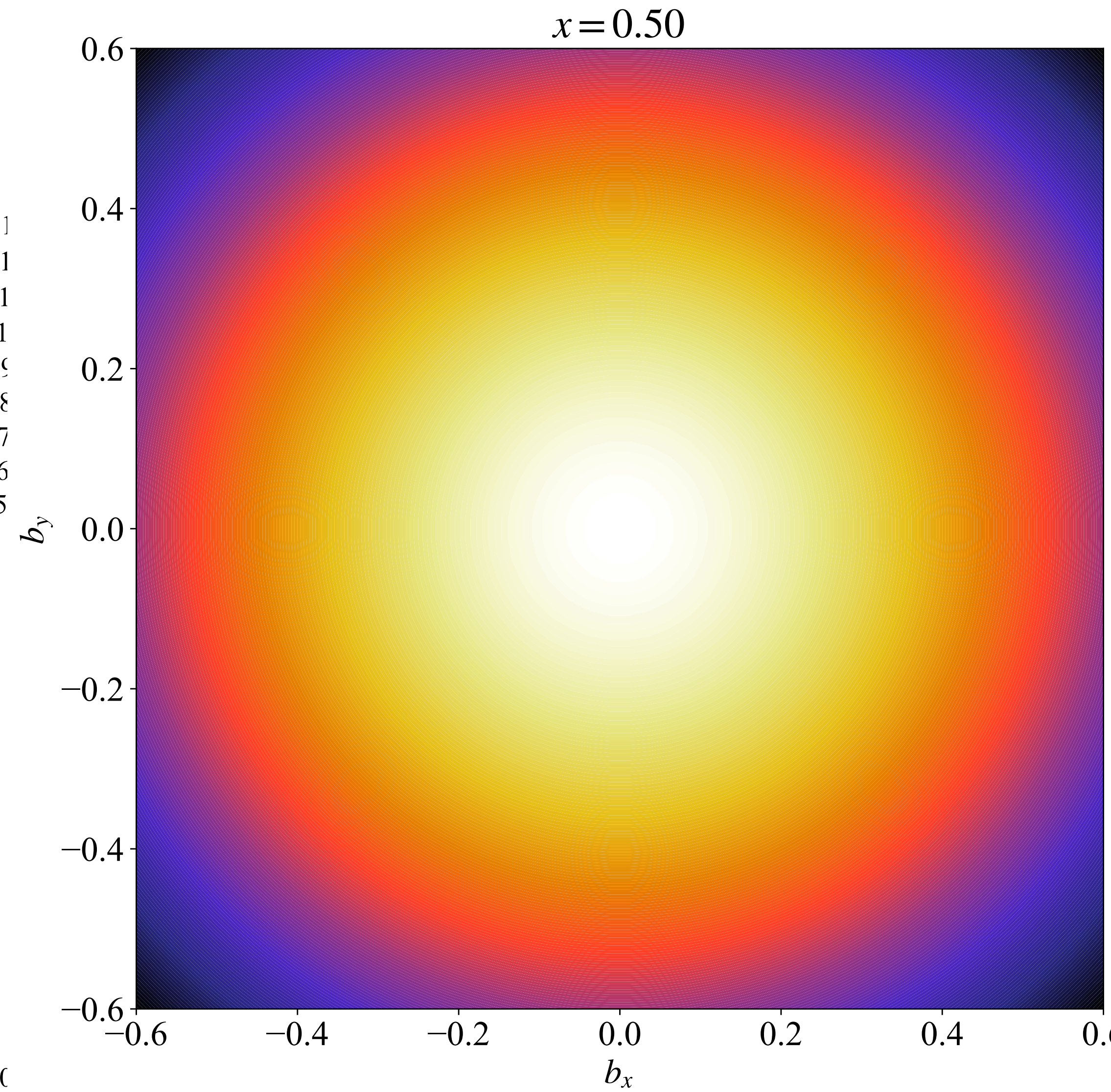

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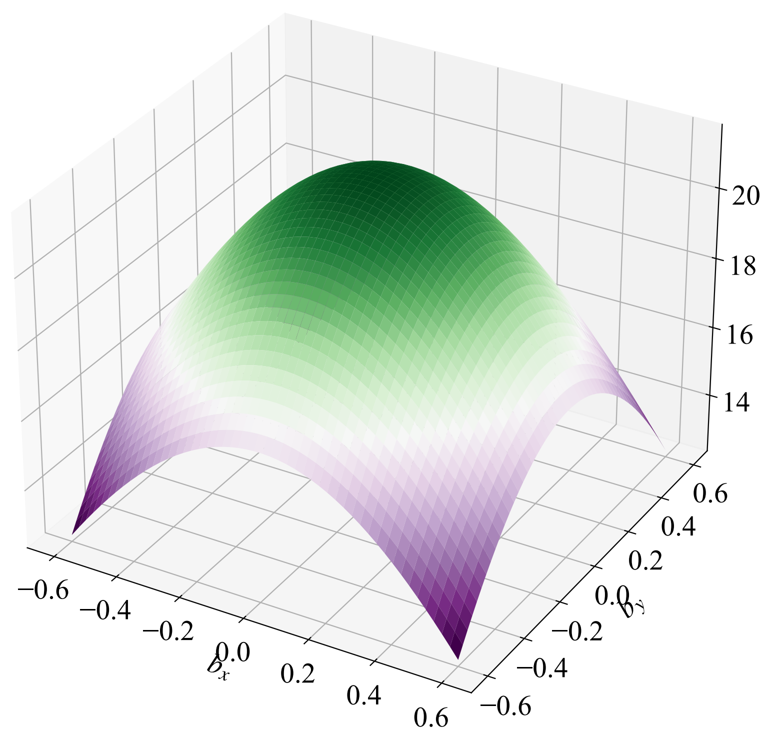
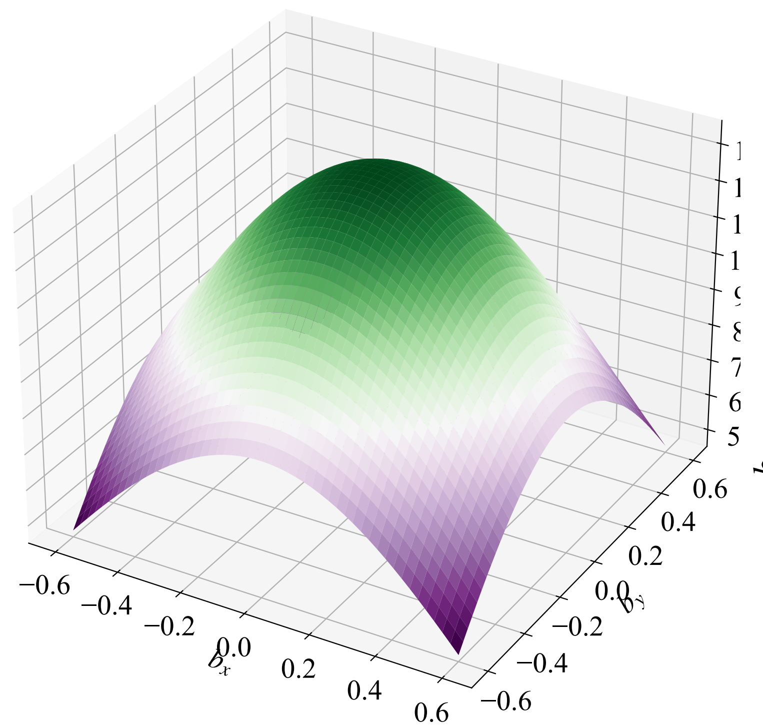
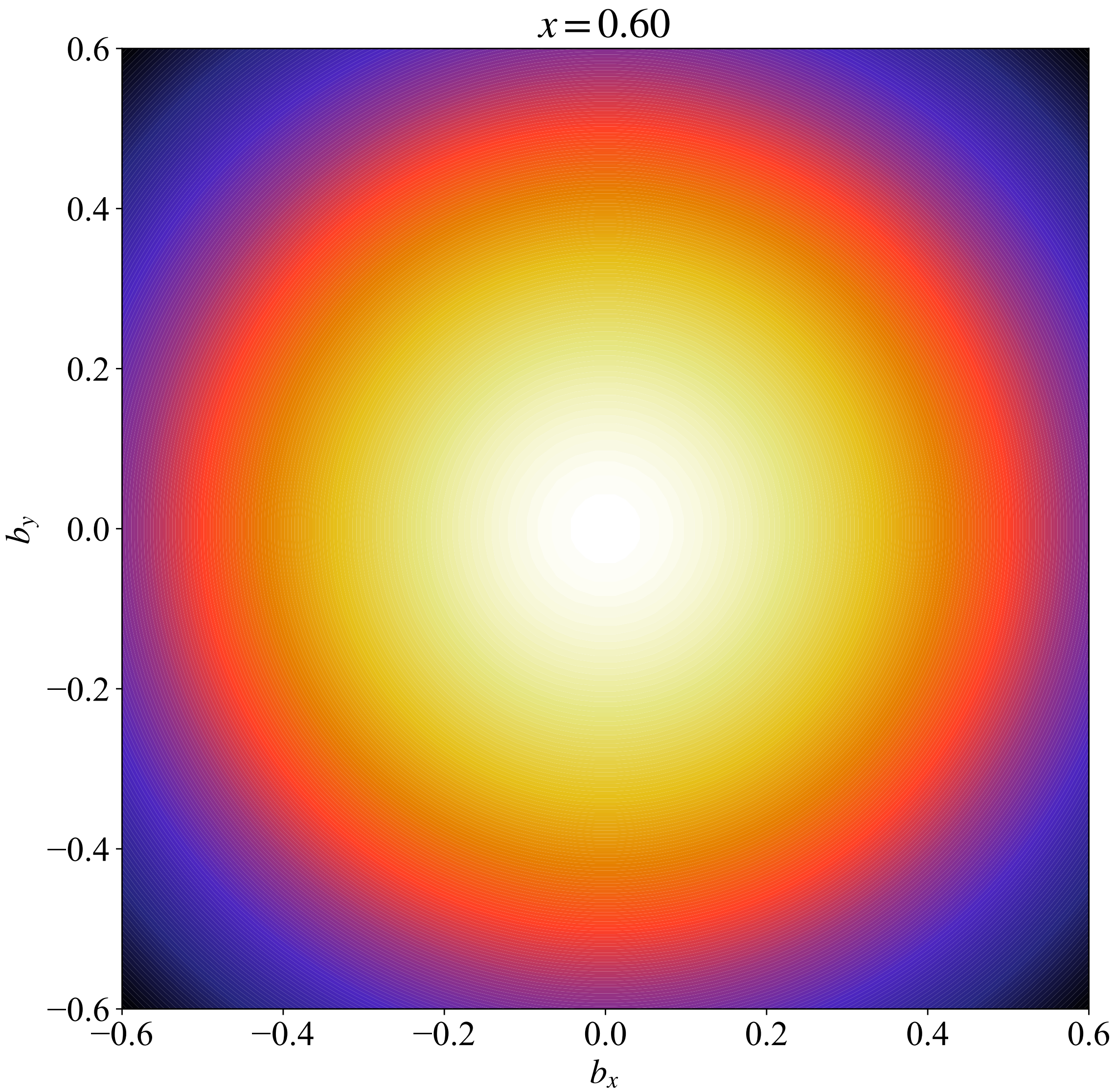
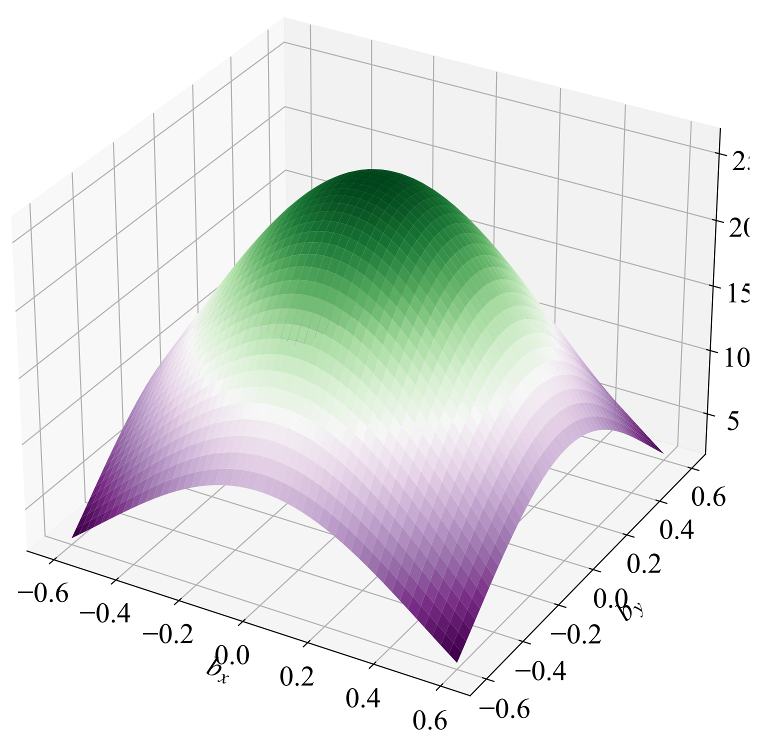
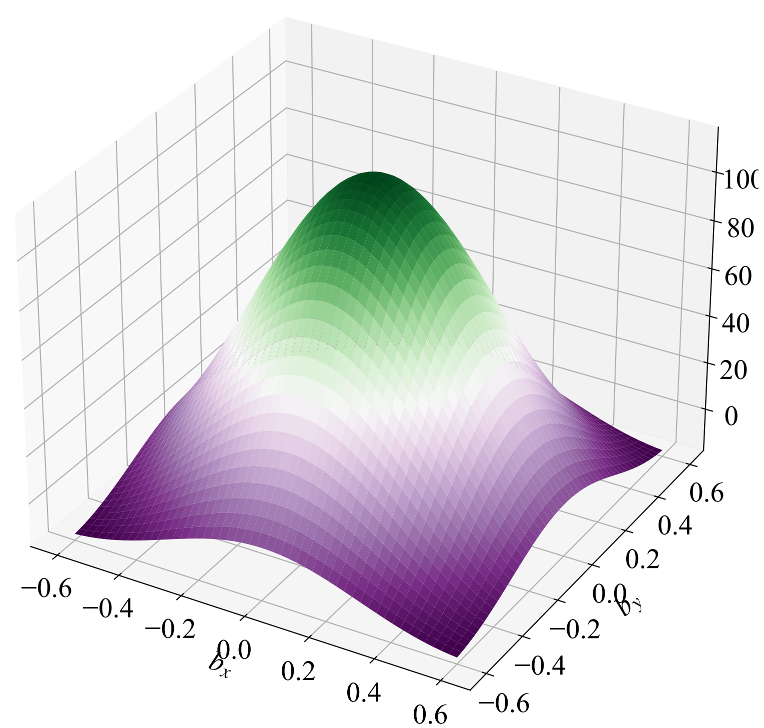


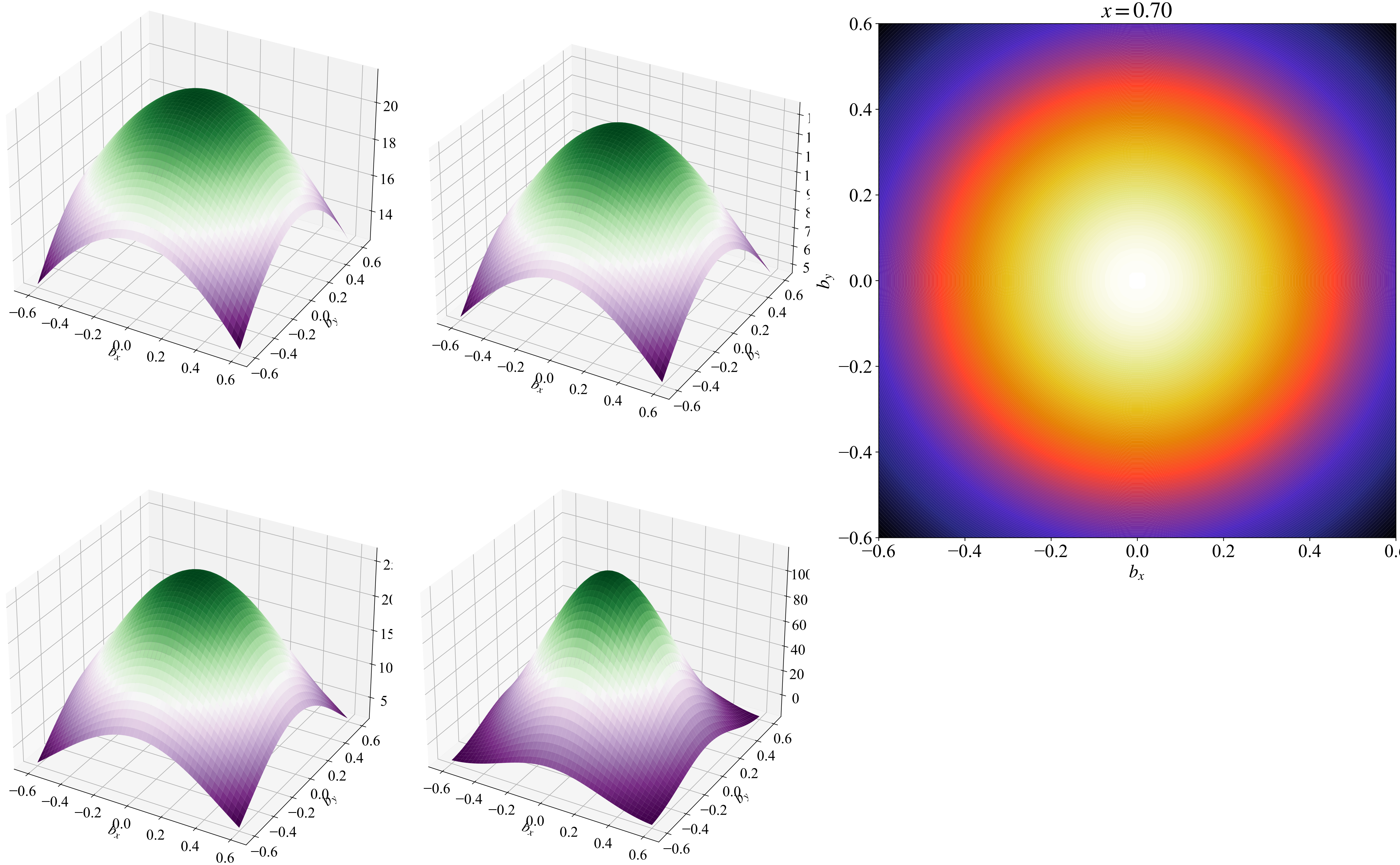




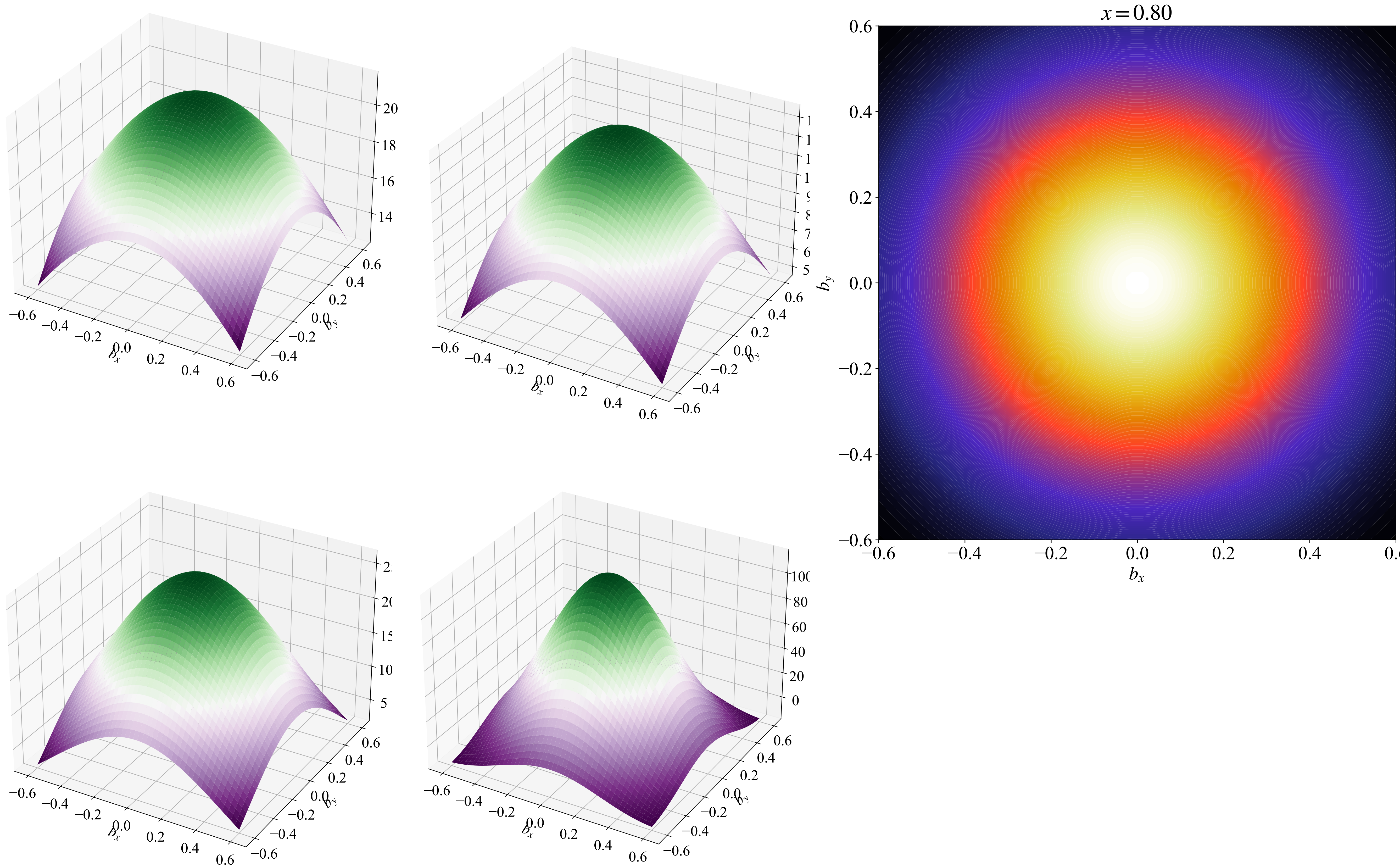




Impact parameter space $\widetilde{E} + \widetilde{G}_1$



Impact parameter space $\widetilde{E} + \widetilde{G}_1$



Synergistic efforts

How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number:
DE-SC0023646

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. **Lattice QCD** calculations of GPDs and related structures
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

How to lattice QCD data fit into the overall effort for hadron tomography

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

PHENOMENOLOGY
LATTICE QCD
THEORY

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★ Three bridge faculty positions will be created in nuclear theory

Stony Brook & Temple

QGT TC Publications

https://qgtcollab.github.io/publications.html

- **24 publications (PRL, PRC, PRD, PLB, JHEP, Rev. Mod. Phys.)**
- **16 preprints**
- **also proceedings**

- **Fangcheng He, Ismail Zahed**,
  Gravitational form factors of light nuclei: Impulse approximation,
  Phys.Rev.C 109 (Apr 2024)
- **Florian Hechenberger, Kiminad A. Mamo, Ismail Zahed**,
  Threshold photoproduction of $|eta_c$ and $|eta_b$ using holographic QCD,
  Phys.Rev.D 109 (Apr 2024)
- **Kiminad A. Mamo, Ismail Zahed**,
  String-based parametrization of nucleon GPDs at any skewness: a comparison to lattice QCD,
  Unpublished (Apr 2024)
- **Kemal Tezgin, Brean Maynard, Peter Schweitzer**,
  Chiral-odd GPDs in the bag model,
  Unpublished (Apr 2024)
- **Sebastian Grieninger, Kazuki Ikeda, Ismail Zahed**,
  Quasi-parton distributions in massive QED2: Towards quantum computation,
  Unpublished (Apr 2024)
- **Wei-Yang Liu, Ismail Zahed**,
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  JHEP 05 150 (Feb 2023)

Summary

★ Impressive progress in the calculation of Mellin moments of GPDs

★ Novel methods to access x dependence complementary to Mellin moments

★ New methods applicable beyond leading twist.
Several improvements needed, e.g.,
mixing with quark-gluon-quark correlator

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Future calculations have the potential to transform the field of GPDs

★ Synergy with phenomenology is an exciting prospect!
QGT Collaboration will be instrumental in such effort

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Thank you

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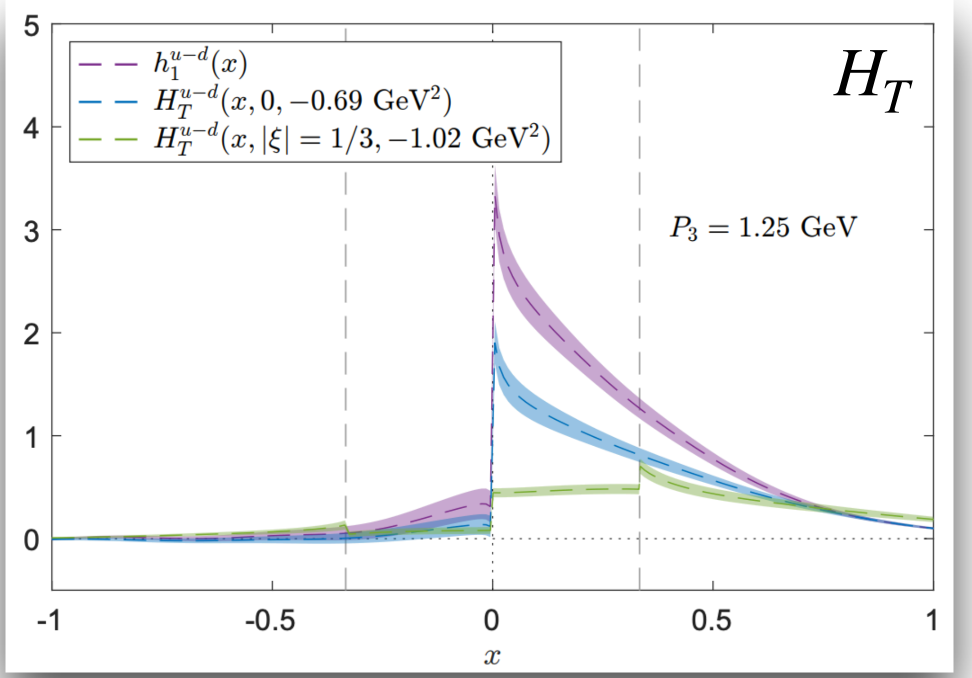
U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING
NERSC
ASCR Leadership Computing Challenge ALCC

Miscellaneous

Transversity GPDs

Standard parametrization

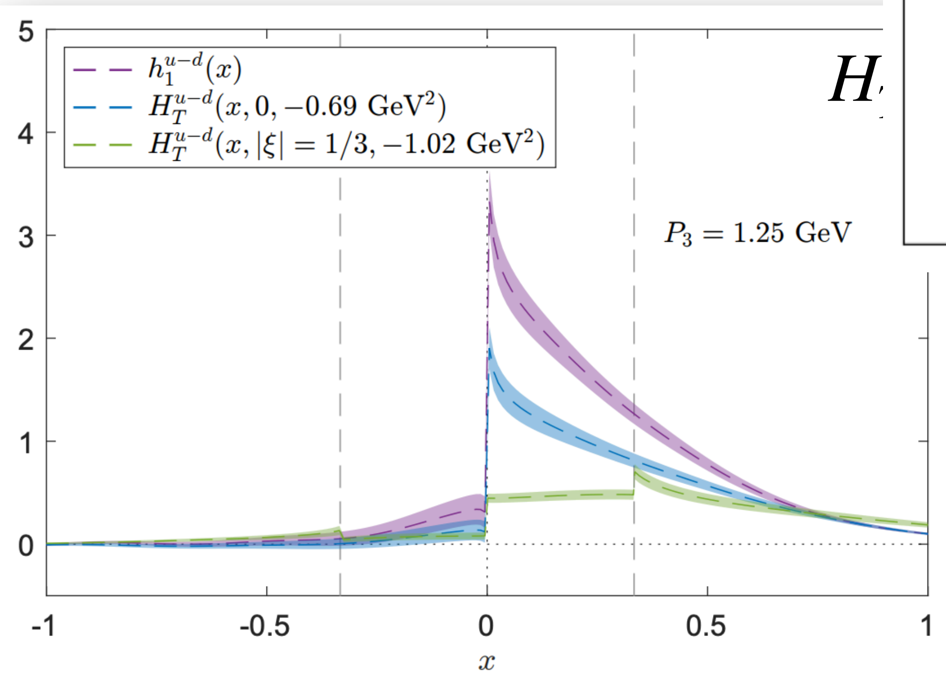


Transversity GPDs

Lorentz covariant parametrization

$$
\begin{aligned}
F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = {} & P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 \\
& + M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10} \\
& + i\epsilon^{\mu\nu Pz}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}
\end{aligned}
$$

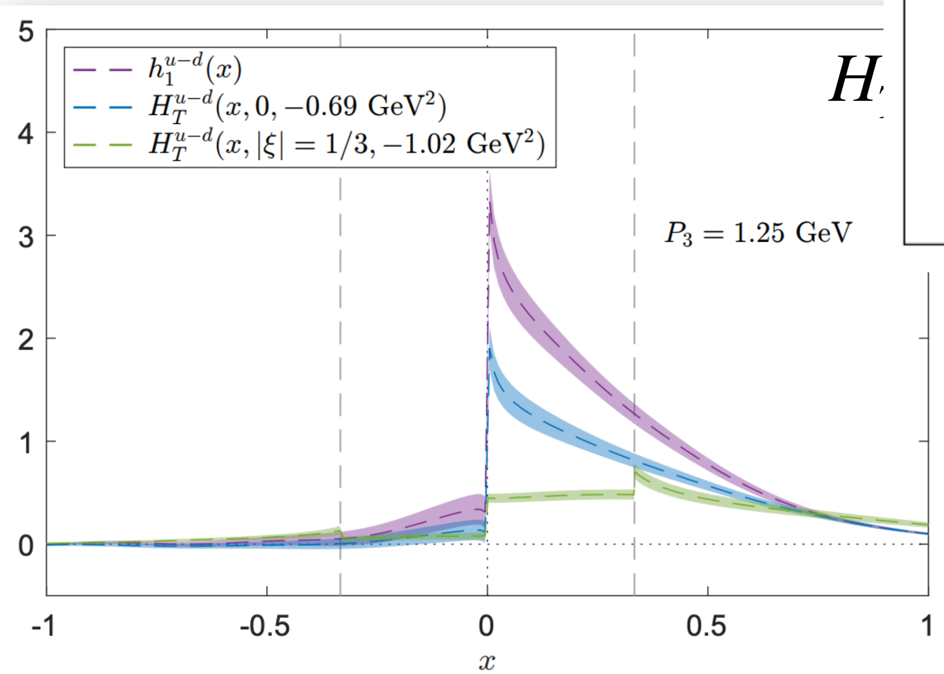
Standard parametrization



$$
\begin{aligned}
\Pi^s_{01}(\Gamma_0) &= K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{(P_3^2+E(E+m))\,z\Delta_2}{16m^2} + A_{T12}\frac{(P_3^2-E(E+m))\,z\Delta_2}{8m^2}\right) \\
\Pi^s_{01}(\Gamma_1) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2+4m(E+m)\right)}{8m^3} + A_{T10}\frac{(4(E+m)^2+4P_3^2+\Delta_1^2-\Delta_2^2)}{16m^2}\right. \\
&\qquad \left. + A_{T11}\frac{P_3\left(8E(E+m)-\Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right) \\
\Pi^s_{01}(\Gamma_2) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right) \\
\Pi^s_{01}(\Gamma_3) &= i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right) \\
\Pi^s_{02}(\Gamma_0) &= K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{(P_3^2+E(E+m))\,z\Delta_1}{16m^2} + A_{T12}\frac{(E(E+m)-P_3^2)\,z\Delta_1}{8m^2}\right) \\
\Pi^s_{02}(\Gamma_1) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right) \\
\Pi^s_{02}(\Gamma_2) &= i\,K\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2+4m(E+m)\right)}{8m^3} + A_{T10}\frac{(4E(E+m)-\Delta_1^2)}{8m^2}\right. \\
&\qquad \left. + A_{T11}\frac{P_3\left(8E(E+m)-\Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right) \\
\Pi^s_{02}(\Gamma_3) &= i\,K\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)
\end{aligned}
$$

Transversity GPDs

Standard parametrization



Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5$$
$$+ M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu Pz}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

$$\Pi^s_{01}(\Gamma_0) = K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{(P_3^2 + E(E+m))\,z\Delta_2}{16m^2} + A_{T12}\frac{(P_3^2 - E(E+m))\,z\Delta_2}{8m^2}\right)$$

$$\Pi^s_{01}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2 + 4P_3^2 + \Delta_1^2 - \Delta_2^2\right)}{16m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right)$$

$$\Pi^s_{02}(\Gamma_0) = K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{(P_3^2 + E(E+m))\,z\Delta_1}{16m^2} + A_{T12}\frac{(E(E+m) - P_3^2)\,z\Delta_1}{8m^2}\right)$$

$$\Pi^s_{02}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m) - \Delta_1^2\right)}{8m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right)$$

On-going work

$$\Pi^s_{02}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)$$

Decomposition

★ Requirement: four independent matrix elements

$P_3\,[\,\mathrm{GeV}\,]$	$\vec{q}\,[\,\frac{2\pi}{L}\,]$	$-t\,[\,\mathrm{GeV}^2\,]$
±0.83	(0, 0, 0)	0
±1.25	(0, 0, 0)	0
±1.67	(0, 0, 0)	0
±0.83	(±2, 0, 0)	0.69
±1.25	(±2, 0, 0)	0.69
±1.67	(±2, 0, 0)	0.69
±1.25	(±2, ±2, 0)	1.38
±1.25	(±4, 0, 0)	2.76

★ Average kinematically equivalent matrix elements

$$\Pi^1(\Gamma_0) = C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_y}{4m^2} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y(E+m)}{2m^2}\right),$$

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right)$$

$$\Pi^1(\Gamma_2) = i\,C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{\Delta_x\Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x\Delta_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x\Delta_y(E+m)}{4m^2P_3}\right),$$

$$\Pi^1(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_x}{4m^2} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x(E+m)}{2m^2}\right),$$

$$\Pi^2(\Gamma_1) = i\,C\left(-F_{\widetilde{H}+\widetilde{G}_2}\frac{\Delta_x\Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x\Delta_y(E+m)}{8m^3} - F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x\Delta_y(E+m)}{4m^2P_3}\right),$$

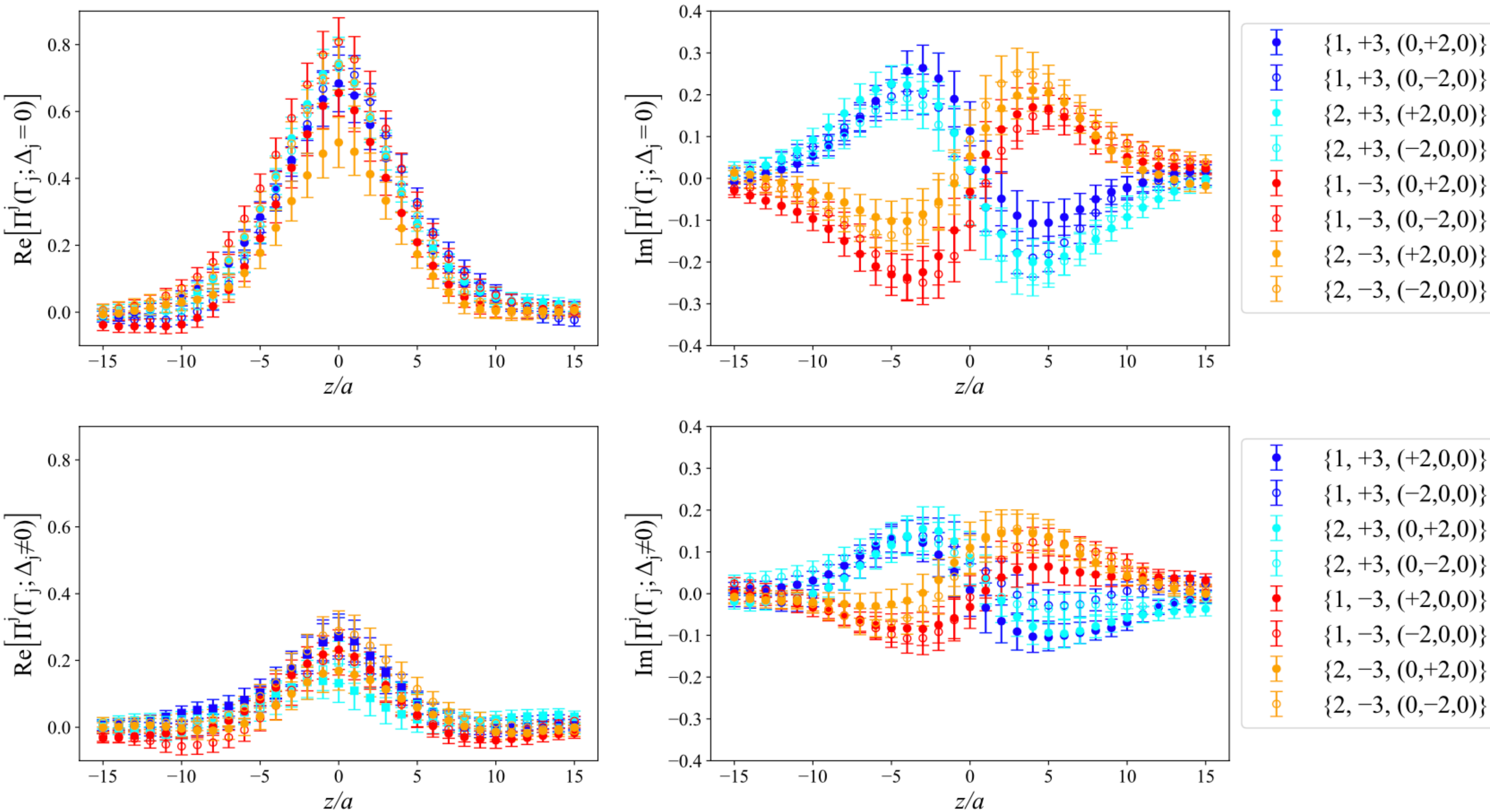
$$\Pi^2(\Gamma_2) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_x^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_y^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\mathrm{sign}[P_3]\,\Delta_x^2(E+m)}{4m^2P_3}\right)$$

$$\Pi^2(\Gamma_3) = C\left(-F_{\widetilde{G}_3}\frac{E\Delta_y(E+m)}{2m^2P_3}\right),$$

Lattice Results - Matrix Elements

★ **Bare matrix elements**

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\text{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right)$$



Lattice Results - Matrix Elements

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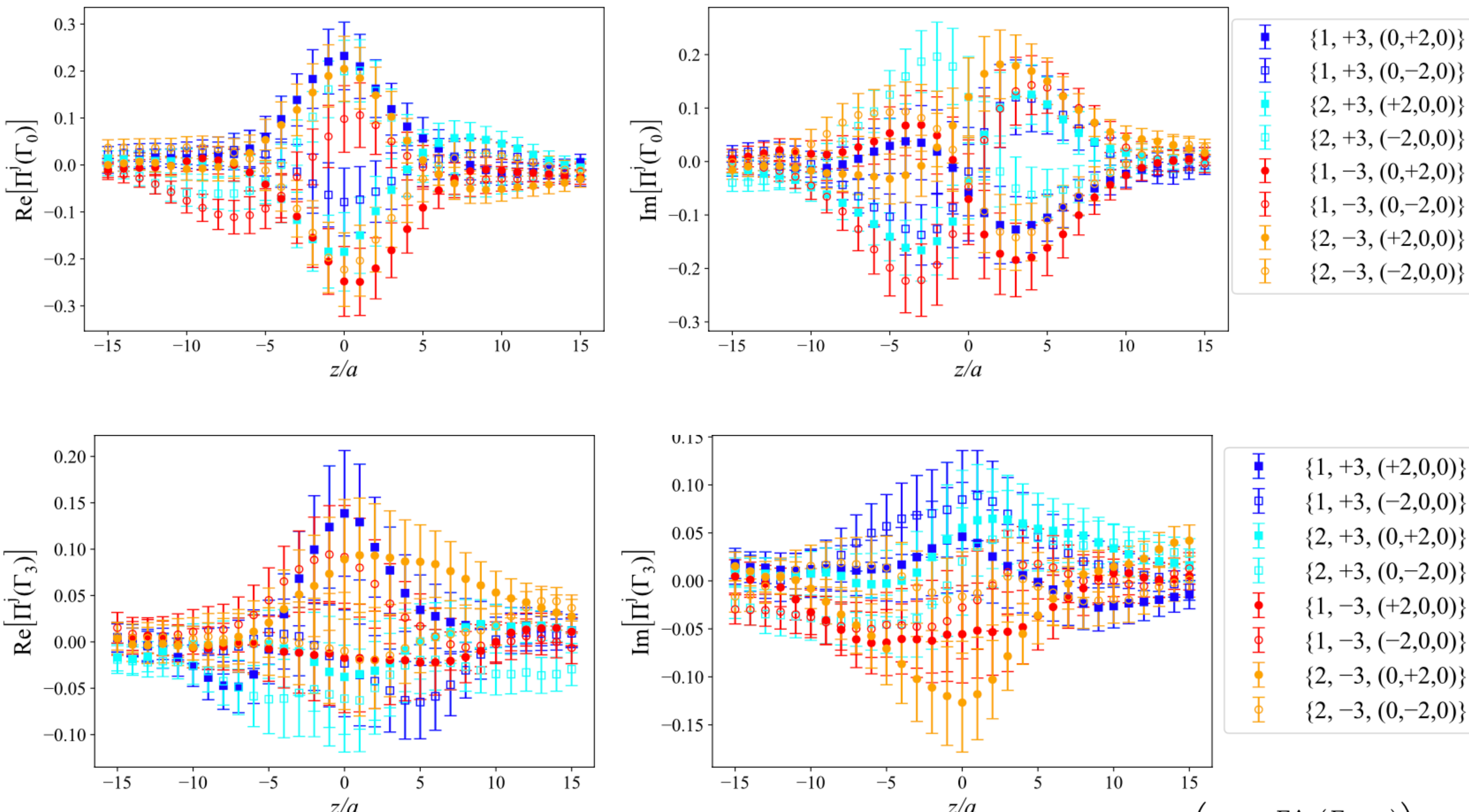


$$\Pi^1(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E\Delta_x(E+m)}{2m^2 P_3} \right)$$

★ Suppressed signal compared to $\gamma_+ \gamma_5$ operators

Consistency checks

★ Norms satisfied   encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

Consistency checks

★ Norms satisfied encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2(\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

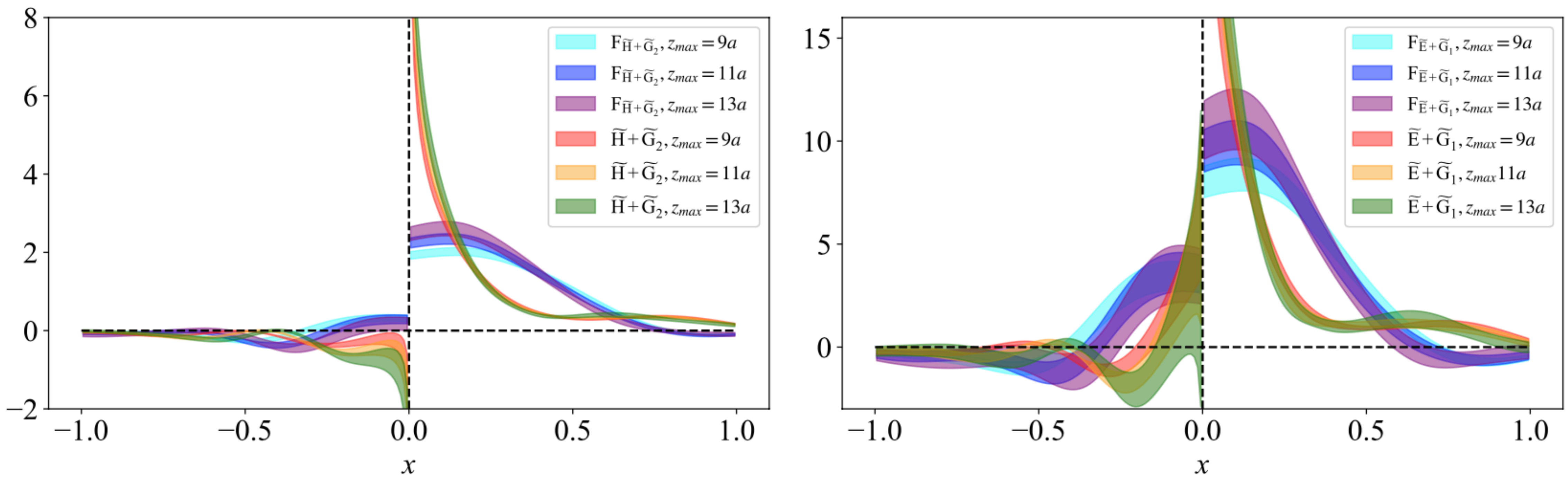


FIG. 10. $z_{\max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

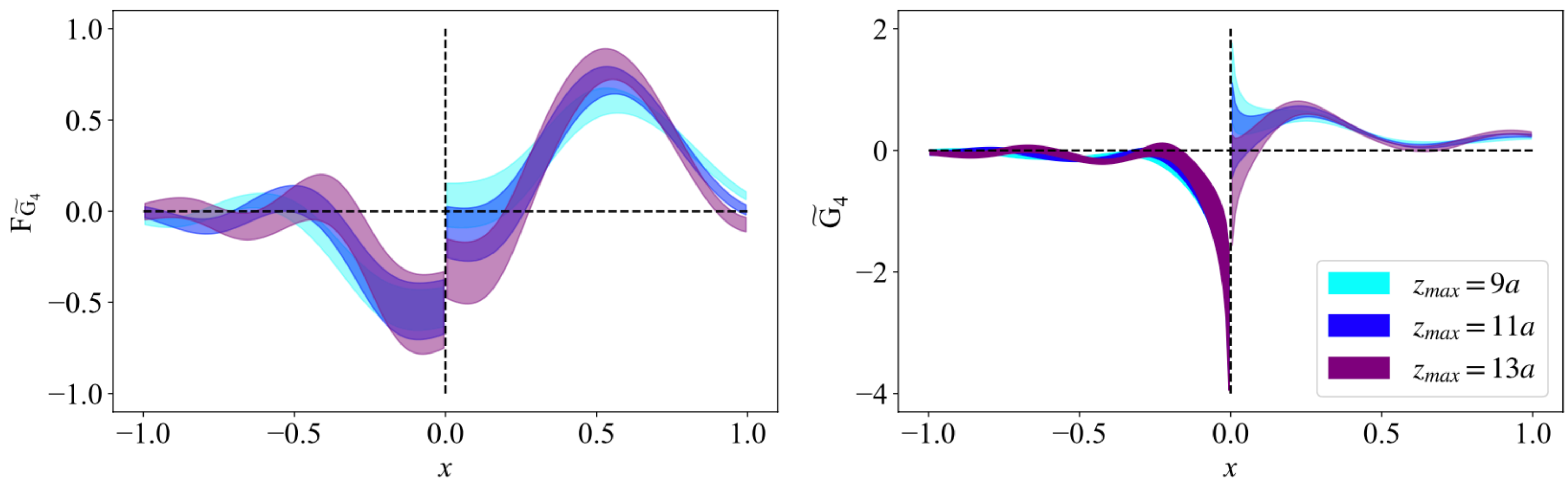
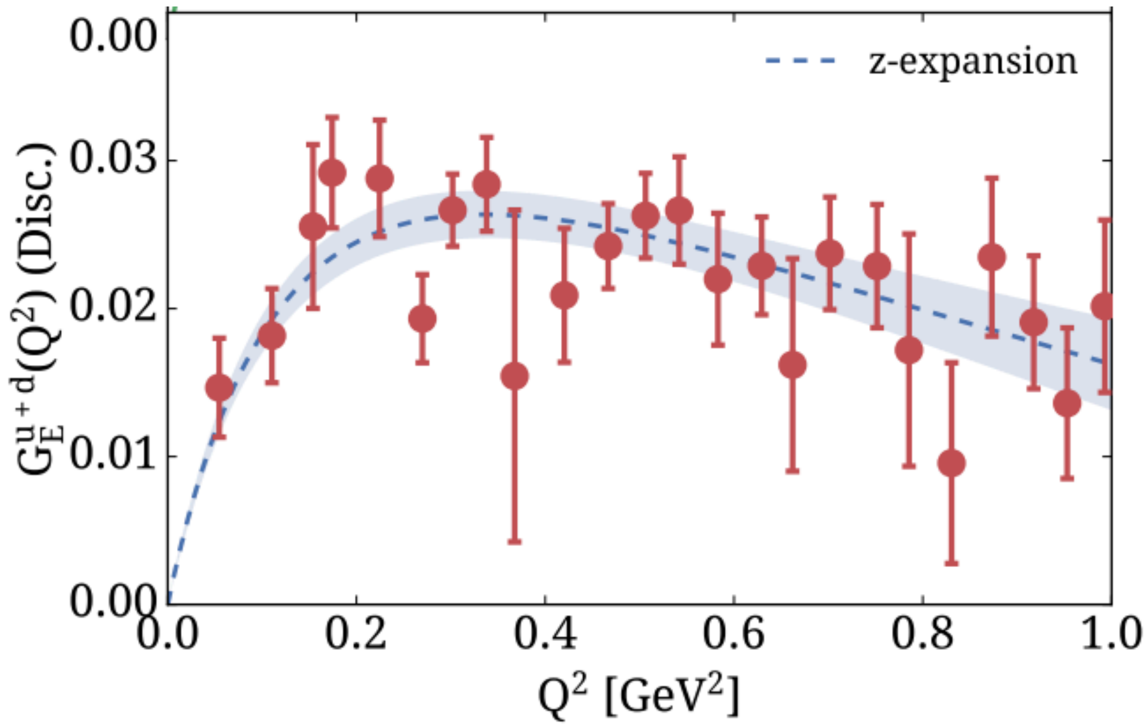


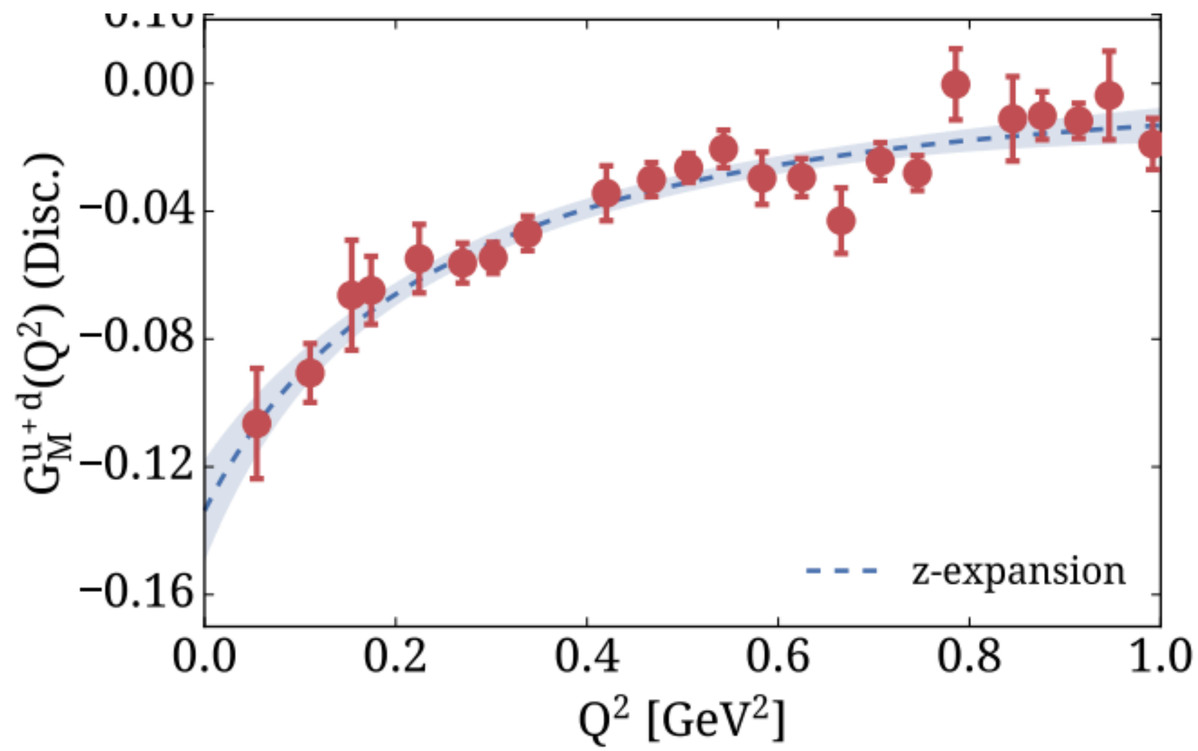
FIG. 11. $z_{\max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

E/M form factors

★ Disconnected contributions non-negligible

[(ETMC) Alexandrou et al., PRD 100 (2019) 1, 014509]





E/M form factors

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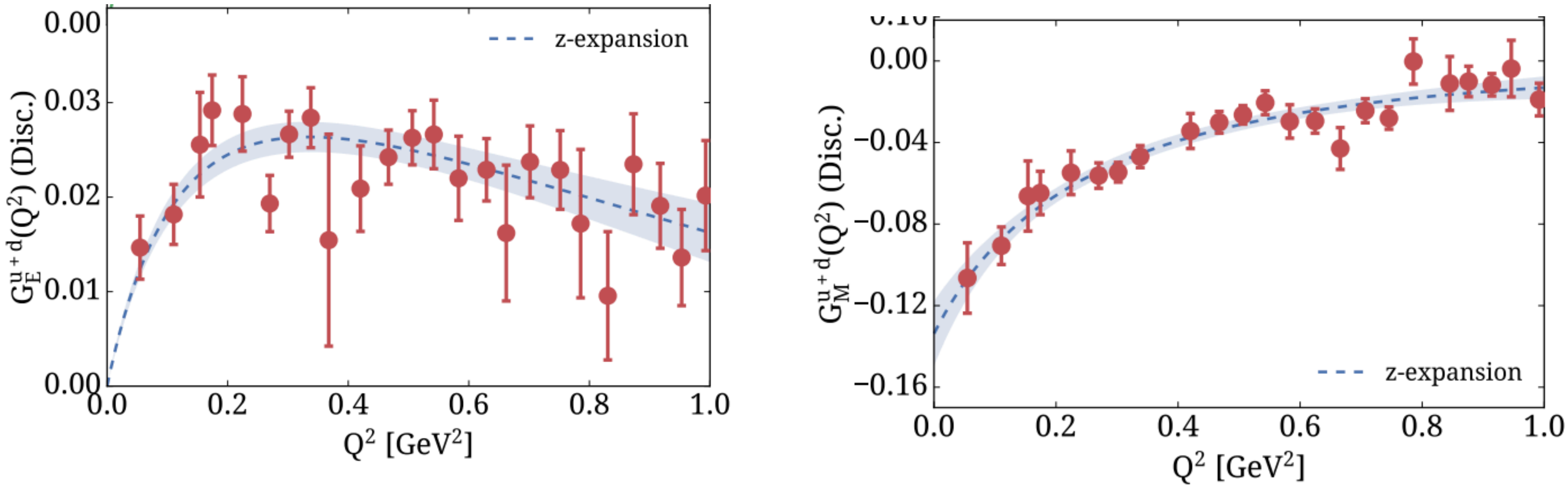
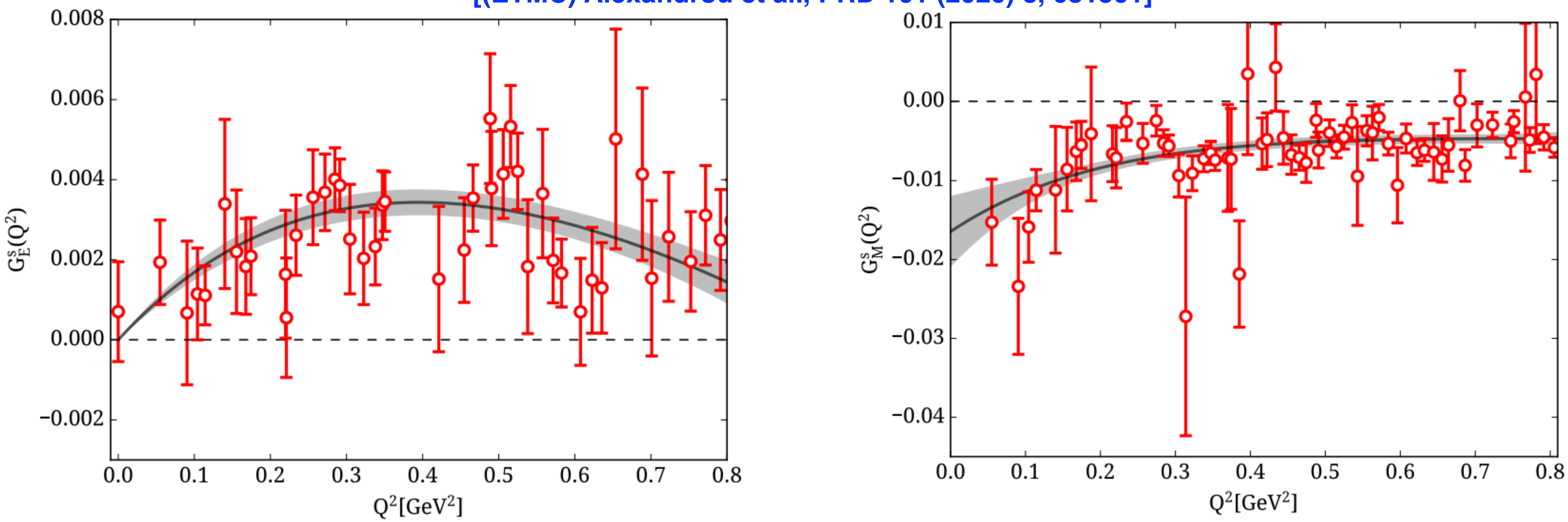


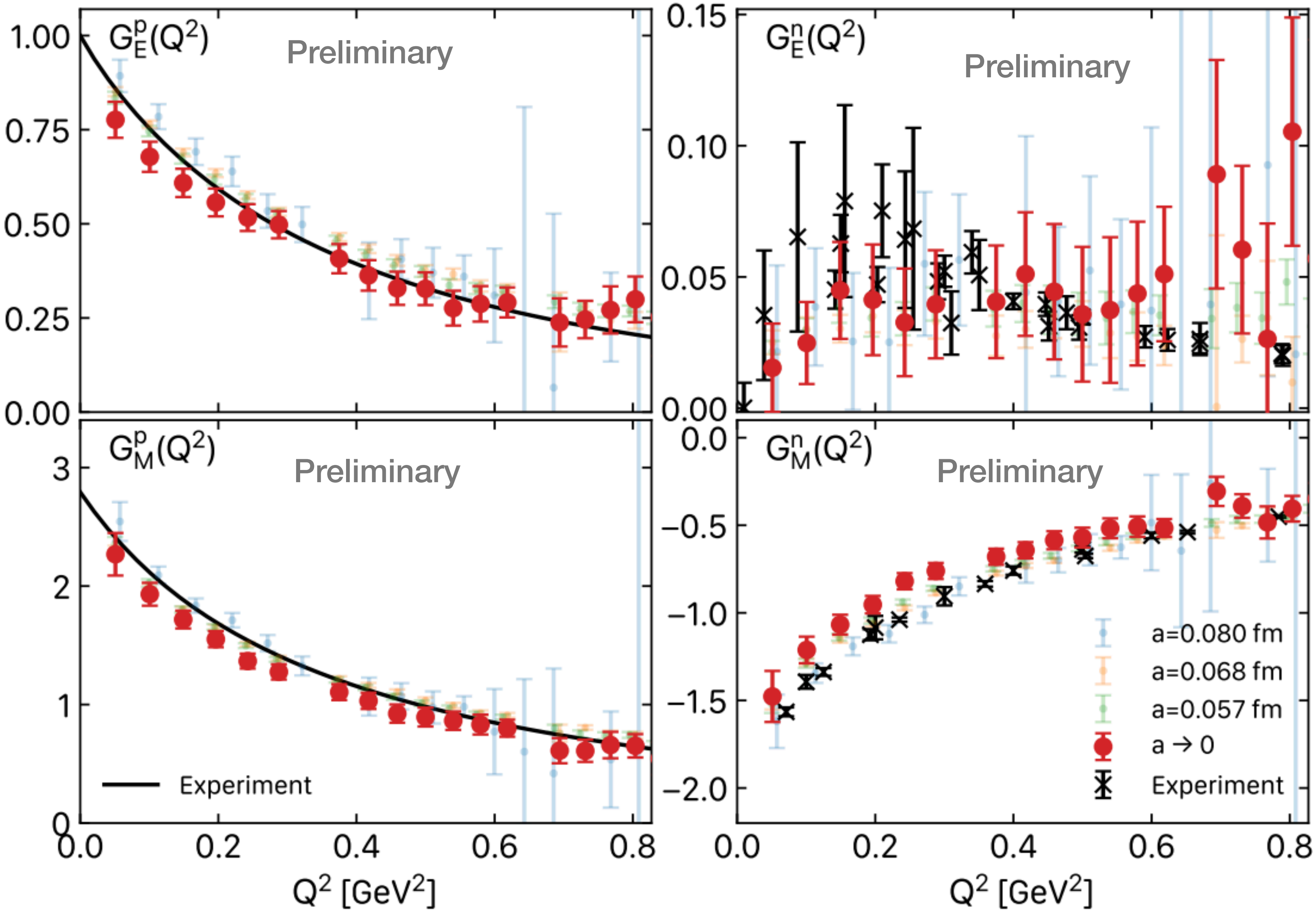
[(ETMC) Alexandrou et al., PRD 101 (2020) 3, 031501]



E/M form factors

★ Towards the continuum limit

[(ETMC) C. Alexandrou et al., PoS(LATTICE2022)114 (2023)]



★ Results are promising

★ Next step: extraction of radii (coming soon)

Axial form factors

[(ETMC) Alexandrou et al., (PRD) arXiv:2309.05774]

★ Matrix elements (including disconnected)

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

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- excited states (T_{sink} up to ~1.6 fm)
- Q^2 parametrization (dipole, z-expansion)

$$G(Q^2) = \sum_{k=0}^{k_{\max}} a_k\, z^k(Q^2)$$

- continuum limit

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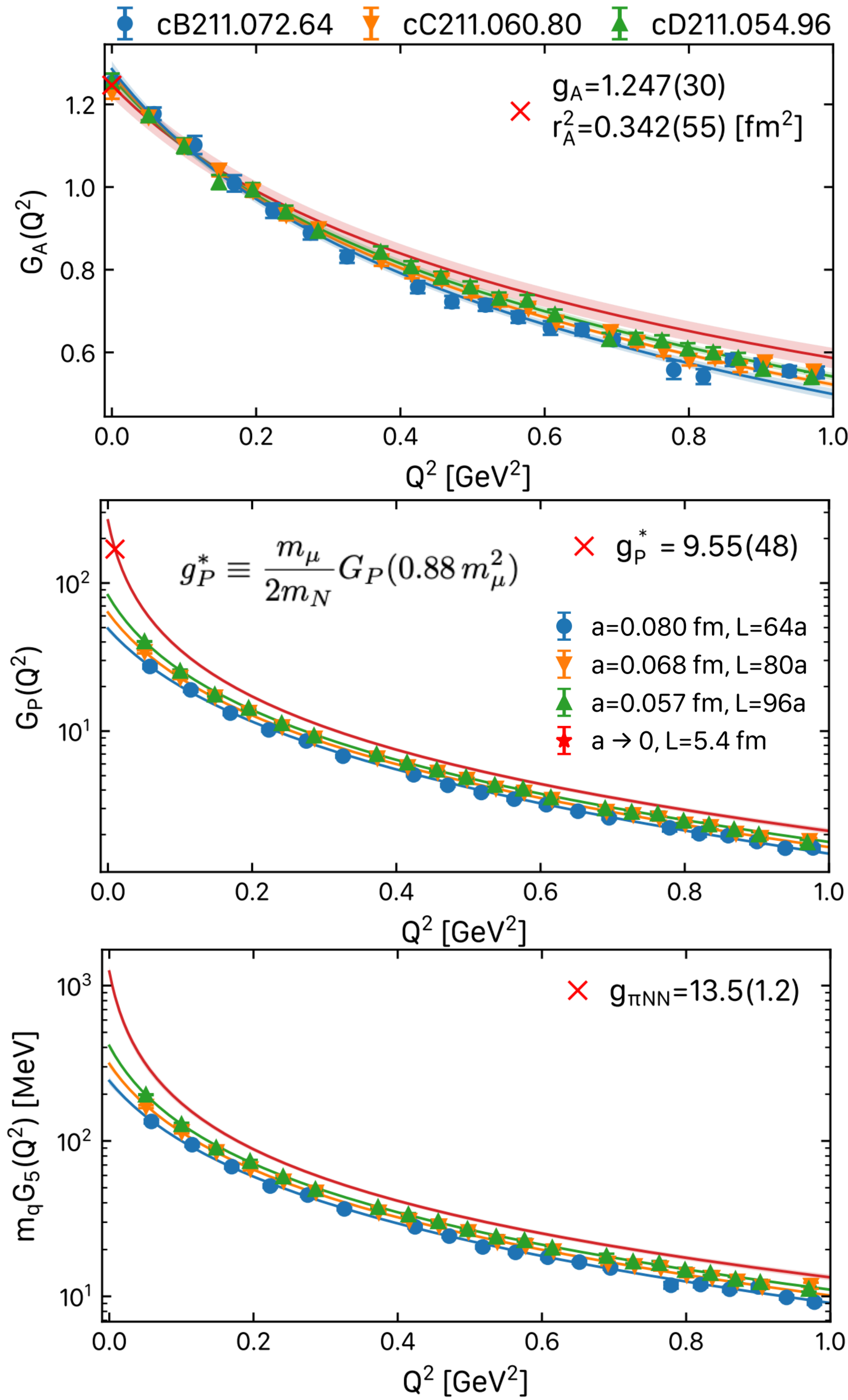

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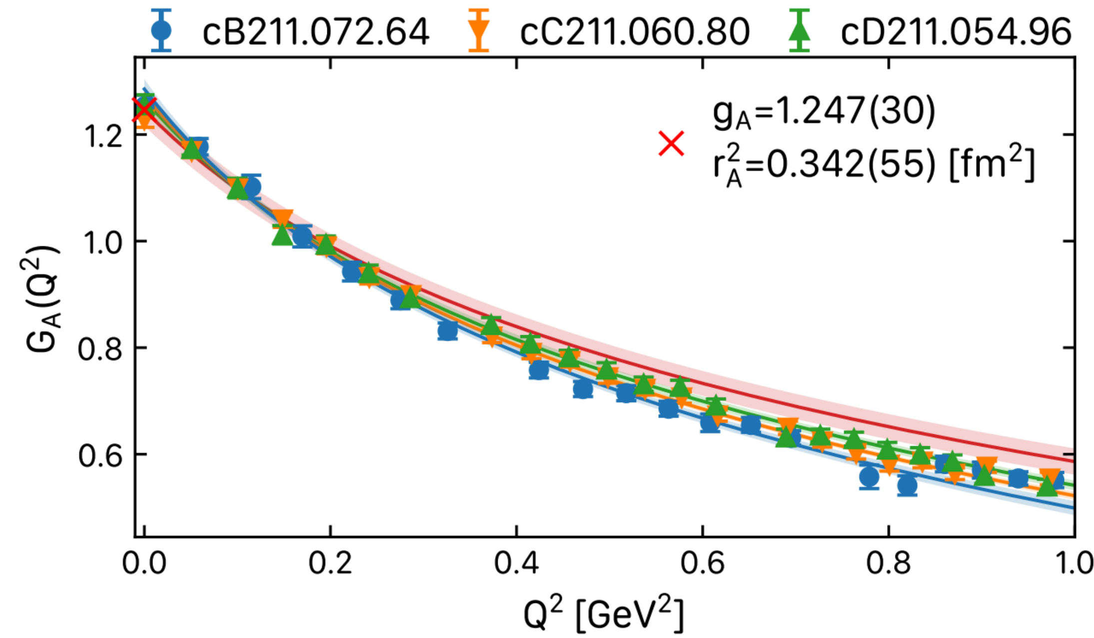

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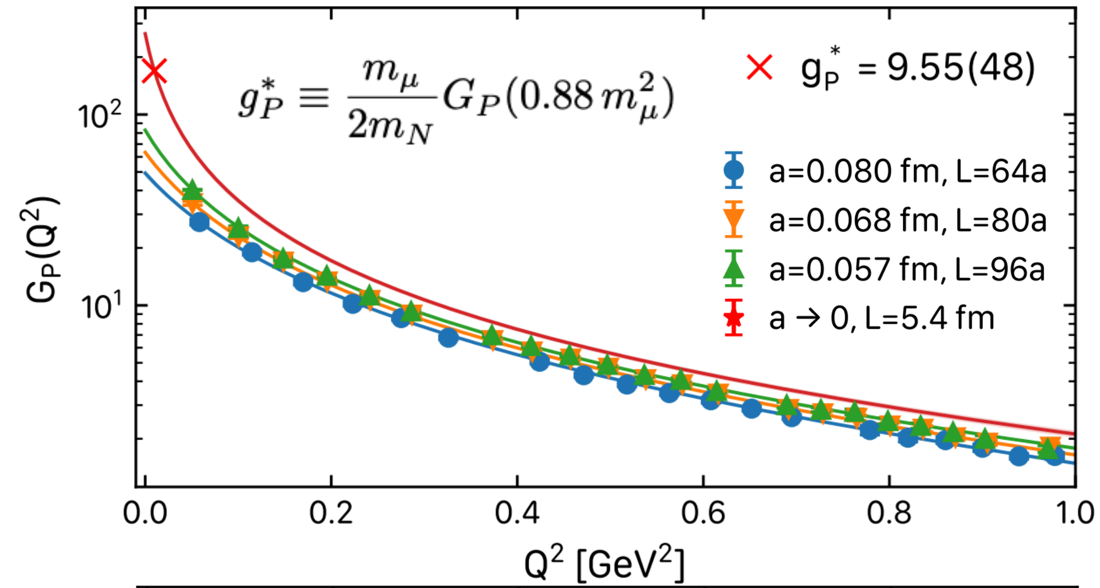
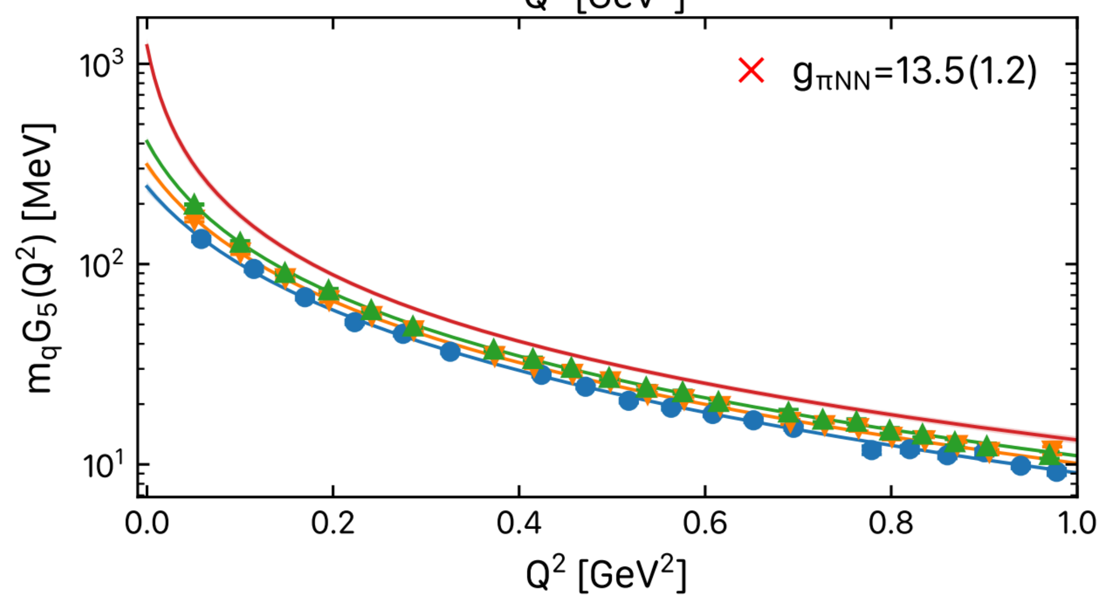


★ Pion pole dominance supported by lattice data of Gp

$$G_P(Q^2) = \frac{4m_N^2}{Q^2+m_\pi^2}G_A(Q^2)\Big|_{Q^2\to -m_\pi^2}$$



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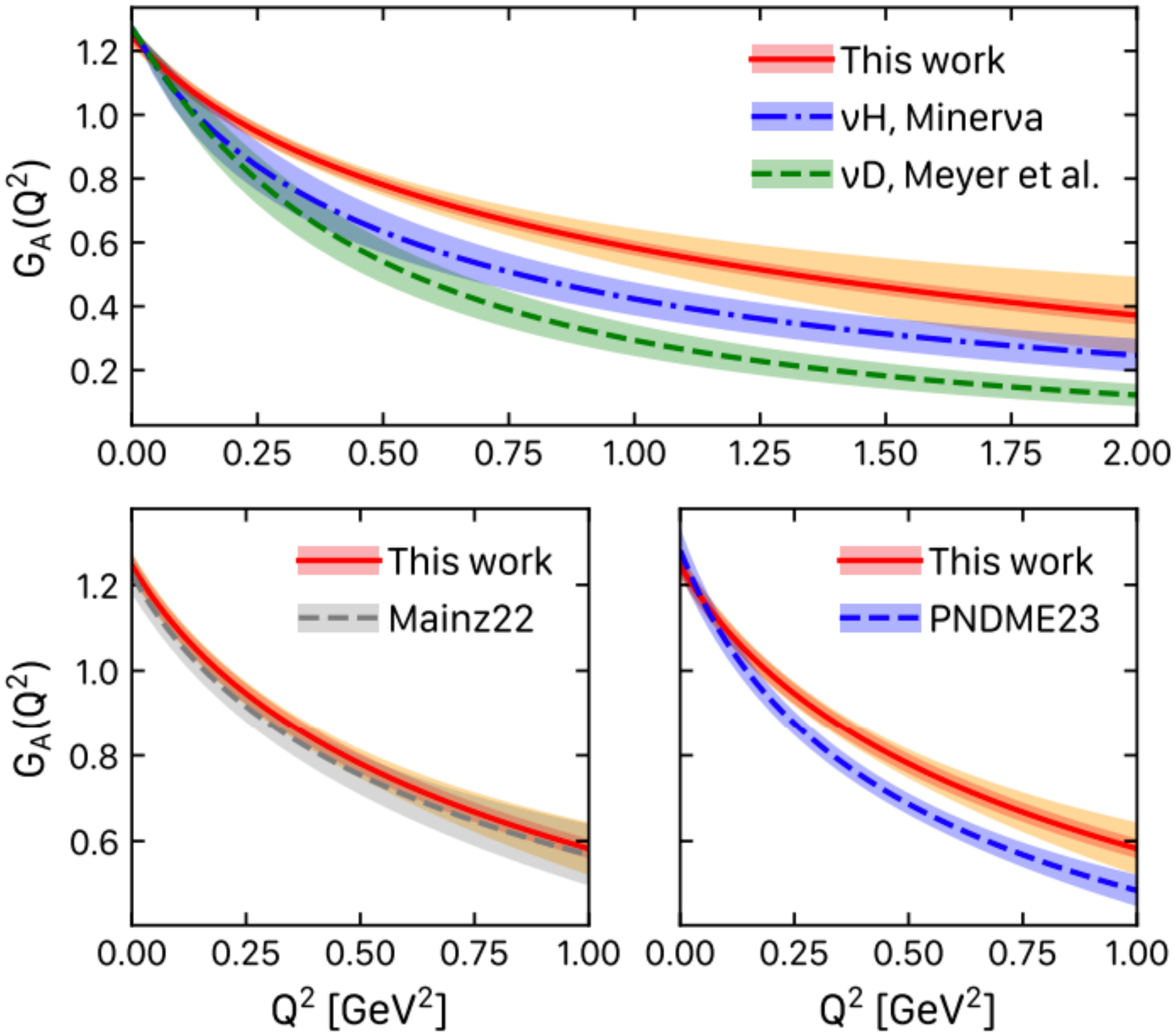

★ Results closer to the new Minerva antineutrino-hydrogen data

[T. Cai et al., Nature 614, 48 (2023)]

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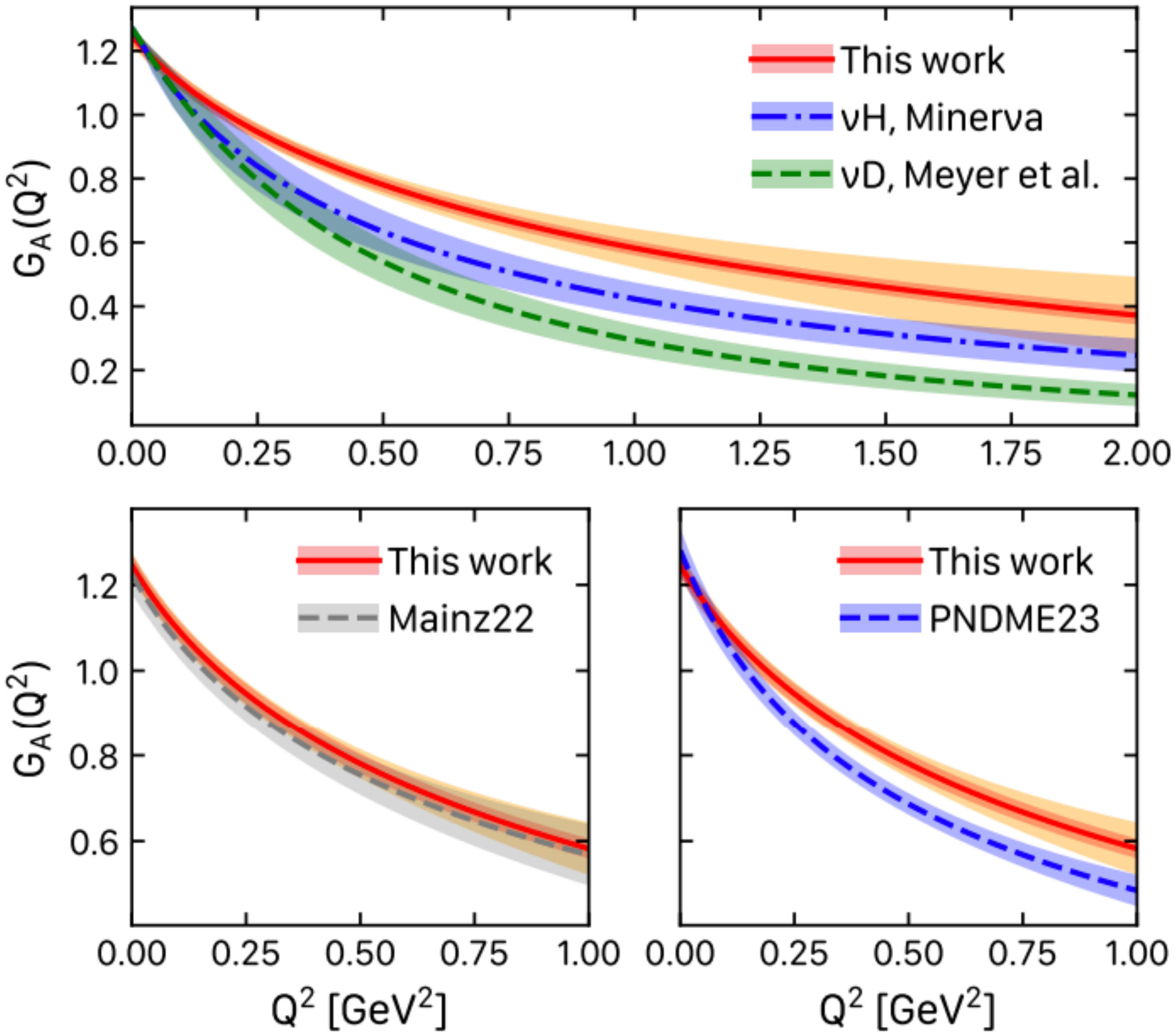

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★ Axial radius

- dipole fit

$$G(Q^2) = \frac{g}{(1+\frac{Q^2}{m^2})^2} \qquad r^2 = \frac{12}{m^2}$$

- z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\rm max}} a_k\, z^k(Q^2) \qquad r^2 = -\frac{3a_1}{2a_0 t_{\rm cut}}$$