DIS dijets production with finite-energy corrections and its correlation limit

Alina Czajka

National Centre for Nuclear Research (NCBJ), Warsaw, Poland

with Tolga Altinoluk, Guillaume Beuf and Cyrille Marquet (*to appear*)

Heavy Ion Physics in the EIC Era,
Institute for Nuclear Theory, Seattle, August 13, 2024

Eikonal approximation in the CGC

High-energy dense-dilute scattering in the CGC : Semiclassical and Eikonal approx.

Dense target represented by a **strong semiclassical gluon field** $\mathcal{A}^{\mu}(x) \propto 1/g$
$\Rightarrow$ Perturbative expansion in g needs improvement by all order resummation of $(g\,\mathcal{A}^{\mu}(x))^n$

Eikonal approx. : limit of **infinite boost** of $\mathcal{A}^{\mu}(x)$ along x^-:

- $\mathcal{A}^{\mu}(x)$ independent on x^- (static limit) due to Lorentz time dilation
  $\Rightarrow$ No p^+ transfer from the target
- Lorentz contraction of $\mathcal{A}^{\mu}(x)$ (shockwave limit)
  $\Rightarrow$ Partons from the projectile interact instantly in x^+ with the target, without transverse motion within the target
- Under a boost of parameter γ_t along the "$-$" direction, $\mathcal{A}^-$ is enhanced and $\mathcal{A}^+$ is suppressed: $\mathcal{A}^- = O(\gamma_t) \gg \mathcal{A}_\perp = O(1) \gg \mathcal{A}^+ = O(1/\gamma_t)$

Background field in the eikonal limit: $\mathcal{A}^{\mu}(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -}\mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

$\Rightarrow$ Only $\left(g\mathcal{A}^-(x^+, \mathbf{x})\right)^n$ needs all orders resummation $\Rightarrow$ Wilson line along x^+

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order $1/\gamma_t$ at the level of the boosted background field
- Of order $1/s$ at the level of a cross section

$\rightarrow$ They arise from relaxing either of the 3 approximations:

1. x^- dependence of $\mathcal{A}^\mu(x)$ beyond infinite Lorentz dilation
   $\rightarrow$ Treated as gradient expansion around a common x^- value:
   $\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)$
   $\Rightarrow$ Possibility of (small) p^+ exchange with the target
2. Target with finite width
   $\Rightarrow$ transverse motion of the projectile partons within the target
3. Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$

Note: Background quark field of the target also relevant at NEik.

CGC vs TMD approaches

Relaxing eikonal approximation in CGC

$\Downarrow$

3D structure of a nucleus

$\Downarrow$

Description of the nucleus in terms of Transverse Momentum Dependent distribution functions (TMDs)

CGC vs TMD approaches

connection between CGC and TMD $\Longleftrightarrow$ overlaping region between high-energy limit and correlation limit

For a process with a hard $\mathbf{P}$ and a not so hard $\mathbf{k}$ transverse momenta, and center-of-mass energy $\sqrt{s}$ $\longrightarrow$ **hierarchy of scales**

- CGC result: leading power (eikonal) in the limit $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$
  - subleading power corrections suppressed as $\left(\frac{\mathbf{P}^2}{s}\right)^n$
- TMD factorization: leading power (twist 2) in the limit $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$
  - subleading power corrections suppressed as $\left(\frac{|\mathbf{k}|}{|\mathbf{P}|}\right)^n$
- CGC vs TMD: $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$
  - what about power corrections in $\mathbf{P}^2/s$ or $|\mathbf{P}||\mathbf{k}|/s$ beyond the eikonal limit?

Consistency of both approaches shown in the double limit $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ at leading power (Dominguez, Marquet, Xiao, Yuan, 2011)

Power corrections in $|\mathbf{k}|/|\mathbf{P}|$ in the regime $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ studied from the CGC approach (Altinoluk, Boussarie, Kotko, 2019)

Full NEik quark propagator through a gluon background field

Propagator from y before the target to x after the target:

$$
\begin{aligned}
S_F(x,y) \;=\; & \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\; \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+ \\
& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \bigg\{ \int dz^- e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big) \\
& + \frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \bigg[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\, \mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\, \mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \bigg] \bigg\} \frac{(\check{\not{k}}+m)}{2k^+} \\
& + \text{NNEik}
\end{aligned}
$$

Altinoluk, Beuf, Czajka, Tymowska (2021); Altinoluk, Beuf (2022).
See also Kovchegov *et al.* (2016-$\cdots$); Chirilli (2019,2021).

- Generalized Eikonal contribution: also includes the NEik non-static corrections: overall z^- dependence of the Wilson line.

$$
\mathcal{U}_F(x^+,y^+;\mathbf{z},z^-) \;\equiv\; \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z) \right]^N
$$

- NEik contributions beyond the shockwave approx or due to $\mathcal{A}_\perp$.

Full NEik quark propagator through a gluon background field

Propagator from y before the target to x after the target:

$$S_F(x,y) = \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\, \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+$$
$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \bigg\{ \int dz^- e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big)$$
$$+\frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \Big[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\, \mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\, \mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \Big] \bigg\} \frac{(\check{\not{k}}+m)}{2k^+}$$
$$+\text{NNEik}$$

Altinoluk, Beuf, Czajka, Tymowska (2021); Altinoluk, Beuf (2022).
See also Kovchegov *et al.* (2016-$\cdots$); Chirilli (2019,2021).

NEik correction due to the overall transverse drift of the quark during its interaction with the target:

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty,z^+;\mathbf{z}\Big) \overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z}\Big)$$
$$= 2i \int dz^+\, z^+\, \mathcal{U}_F(+\infty,z^+;\mathbf{z})\, gt\cdot\mathcal{F}_j^{\ -}(z^+,\mathbf{z})\, \mathcal{U}_F(z^+,-\infty;\mathbf{z})$$

Full NEik quark propagator through a gluon background field

Propagator from y before the target to x after the target:

$$S_F(x,y) = \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\, \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not q}+m)}{2q^+}\gamma^+$$
$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big)$$
$$+ \frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \Big[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\, \mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\, \mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \Big] \Bigg\} \frac{(\check{\not k}+m)}{2k^+}$$
$$+\text{NNEik}$$

Altinoluk, Beuf, Czajka, Tymowska (2021); Altinoluk, Beuf (2022).
See also Kovchegov *et al.* (2016-$\cdots$); Chirilli (2019,2021).

NEik correction due to the transverse Brownian motion of the quark during its interaction with the target:

$$\mathcal{U}^{(2)}_F(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+;\mathbf{z}\Big) \overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{z}^j}} \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z}\Big)$$
$$= -\int dz^+ \int^{z^+} dz'^+\, (z^+-z'^+)\, \mathcal{U}_F(+\infty,z^+,\mathbf{z})\, gt\cdot\mathcal{F}_j^{\ -}(z^+,\mathbf{z})$$
$$\times\, \mathcal{U}_F(z^+,z'^+;\mathbf{z})\, gt\cdot\mathcal{F}_j^{\ -}(z'^+,\mathbf{z})\, \mathcal{U}_F(z'^+,-\infty;\mathbf{z})$$

Full NEik quark propagator through a gluon background field

Propagator from y before the target to x after the target:

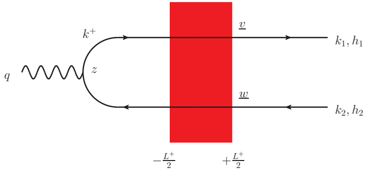
$$S_F(x,y) = \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\, \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+$$
$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ \int dz^- e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big)$$
$$+ \frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \Big[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\,\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \Big] \Bigg\} \frac{(\check{\not{k}}+m)}{2k^+}$$
$$+\text{NNEik}$$

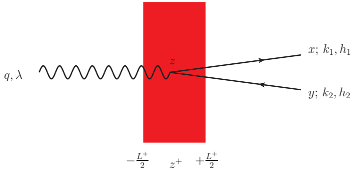
Altinoluk, Beuf, Czajka, Tymowska (2021); Altinoluk, Beuf (2022).
See also Kovchegov *et al.* (2016-· · ·); Chirilli (2019,2021).

NEik coupling between the light-front helicity of the quark and the longitudinal chromomagnetic field of the target $\mathcal{F}_{ij}$:

$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+;\mathbf{z}\Big)\, gt\cdot\mathcal{F}_{ij}(z^+,\mathbf{z})\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z}\Big)$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*





DIS dijet cross calculated at NEik accuracy, at LO in α_s in the CGC. (Altinoluk, Beuf, Czajka, Tymowska, (2023))

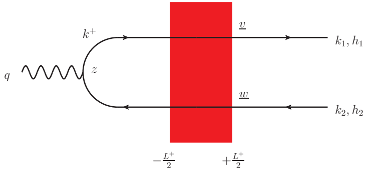
- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case.

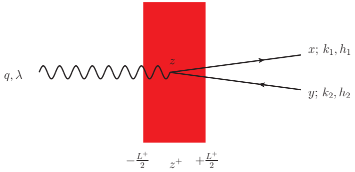
S-matrix at NEik accuracy: $S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$

$$S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = -2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+ v(2)\,\frac{(q^+ + k_1^+ - k_2^+)(q^+ + k_2^+ - k_1^+)}{4(q^+)^2}\int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\ \mathrm{K}_0\left(\hat{Q}\,|\mathbf{v}-\mathbf{w}|\right)\int db^-\ e^{ib^-(k_1^+ + k_2^+ - q^+)}\left[\mathcal{U}_F\left(\mathbf{v}, b^-\right)\mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right) - 1\right]$$

$$\hat{Q}^2 = m^2 + \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}Q^2\,.$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*





DIS dijet cross calculated at NEik accuracy, at LO in α_s in the CGC. (Altinoluk, Beuf, Czajka, Tymowska, (2023))

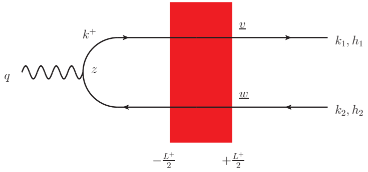
- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case.

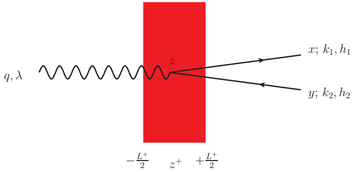
S-matrix at NEik accuracy: $S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$

$$S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, iQ\, \frac{ee_f}{2\pi}\, \bar{u}(1)\gamma^+ v(2)\, \frac{(k_1^+ - k_2^+)}{(q^+)^2} \int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1} \int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times \left[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right) - \frac{\left(\bar{Q}^2 - m^2\right)}{2\bar{Q}}\, |\mathbf{v}-\mathbf{w}|\, \mathrm{K}_1\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right)\right] \left[\mathcal{U}_F\left(\mathbf{v}, b^-\right) \overleftrightarrow{\partial_{b^-}} \mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right)\right]\Big|_{b^-=0}$$

$$\bar{Q}^2 = m^2 + \frac{k_1^+ k_2^+}{(q^+)^2}\, Q^2$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*





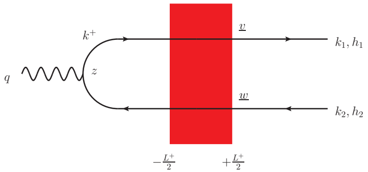
DIS dijet cross calculated at NEik accuracy, at LO in α_s in the CGC. (Altinoluk, Beuf, Czajka, Tymowska, (2023))

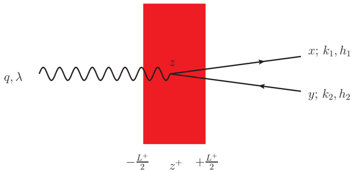
- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case.

S-matrix at NEik accuracy: $S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$

$$S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_2^+}{(q^+)^2}\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right)$$
$$\times\ \bar{u}(1)\gamma^+\left[\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) - i\,\mathcal{U}^{(2)}_{F}(\mathbf{v}) + \mathcal{U}^{(1)}_{F;j}(\mathbf{v})\left(\frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2} + \frac{i}{2}\,\partial_{\mathbf{w}^j}\right)\right]\mathcal{U}_F^\dagger(\mathbf{w})\,v(2)$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*





DIS dijet cross calculated at NEik accuracy, at LO in α_s in the CGC.
(Altinoluk, Beuf, Czajka, Tymowska, (2023))

- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case.

S-matrix at NEik accuracy: $S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$

$$S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_1^+}{(q^+)^2}\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right)$$
$$\times\ \bar{u}(1)\gamma^+\left[\mathcal{U}_F(\mathbf{v})\left(\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)\dagger}_{F;ij}(\mathbf{w}) - i\,\mathcal{U}^{(2)\dagger}_F(\mathbf{w}) + \left(\frac{i}{2}\overleftarrow{\partial_{\mathbf{v}^j}} - \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2}\right)\mathcal{U}^{(1)\dagger}_{F;j}(\mathbf{w})\right)\right]v(2)$$

Change of variables and back-to-back limit

Back-to-back limit of dijets is conveniently expressed in terms of:

(dijet momentum imbalance) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

$$\boxed{\mathbf{k}_1 = \mathbf{P} + z_1\mathbf{k}} \qquad \boxed{\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}}$$

back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to $\mathbf{k}$) $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$ and (conjugate to P) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

such that

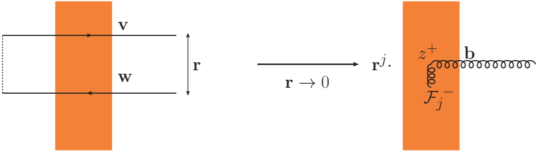
$$\boxed{\mathbf{v} = \mathbf{b} + z_2\,\mathbf{r}} \qquad \boxed{\mathbf{w} = \mathbf{b} - z_1\,\mathbf{r}}$$

back-to-back correlation limit: $|\mathbf{r}| \ll |\mathbf{b}|$

Small $\mathbf{r}$ expansion for the eikonal contribution (1)

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[\mathcal{U}_F\Big(\mathbf{b} + z_2\,\mathbf{r}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b} - z_1\,\mathbf{r}, b^-\Big) - 1\Big]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[z_2\mathbf{r}^j\left(\partial_j\mathcal{U}_F(\mathbf{b}, b^-)\right)\mathcal{U}_F^\dagger(\mathbf{b}, b^-) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b}, b^-)\left(\partial_j\mathcal{U}_F^\dagger(\mathbf{b}, b^-)\right) + O(\mathbf{r}^2)\Big]$$
$$= \mathbf{r}^j\, t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, b^-\Big)_{ab} (-ig)\mathcal{F}^{b}_{j}{}^{-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)$$

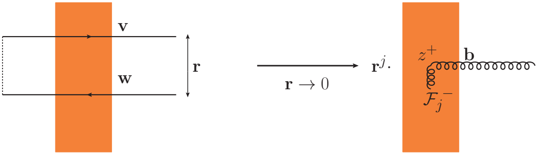


Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

Small $\mathbf{r}$ expansion for the eikonal contribution (1)

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$
\begin{aligned}
&\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r},b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r},b^-\Big)-1\Big] \\
=&\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[z_2\mathbf{r}^j\left(\partial_j\mathcal{U}_F(\mathbf{b},b^-)\right)\mathcal{U}_F^\dagger(\mathbf{b},b^-) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b},b^-)\left(\partial_j\mathcal{U}_F^\dagger(\mathbf{b},b^-)\right) + O(\mathbf{r}^2)\Big] \\
=&\,\mathbf{r}^j\,t^a\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}}\int_{z^+}\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b},b^-\Big)_{ab}(-ig)\mathcal{F}^{b}_{j}{}^{-}(z^+,\mathbf{b},b^-) + O(\mathbf{r}^2)
\end{aligned}
$$



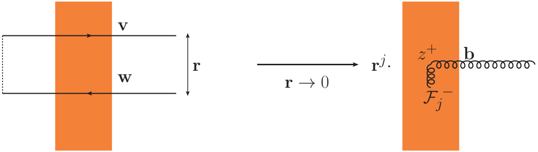
Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

However: the aim is to study the interplay between subleading power corrections
$\Rightarrow$ Terms of order $\mathbf{r}^2$ needed as well!

Small r expansion for the eikonal contribution (2)

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r}, b^-\Big) - 1\right]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Bigg[-i\Big(1+\frac{i(z_2-z_1)}{2}\mathbf{r}\cdot\mathbf{k}\Big)\mathbf{r}^j\, t^{a'} \int dv^+\, \mathcal{U}_A\left(+\infty, v^+; \mathbf{b}, b^-\right)_{a'a}\, g\mathcal{F}_j{}^-{}_a(v^+, \mathbf{b}, b^-)$$
$$-\frac{1}{2}\mathbf{r}^i\mathbf{r}^j\, t^{a'}t^{b'} \int dv^+ \int dw^+\, \mathcal{U}_A\left(+\infty, v^+; \mathbf{b}, b^-\right)_{a'a}\, g\mathcal{F}_i{}^-{}_a(v^+, \mathbf{b}, b^-)$$
$$\times\mathcal{U}_A\left(+\infty, w^+; \mathbf{b}, b^-\right)_{b'b}\, g\mathcal{F}_j{}^-{}_b(w^+, \mathbf{b}, b^-) \; + O\left(|\mathbf{r}|^3\right)\Bigg]$$



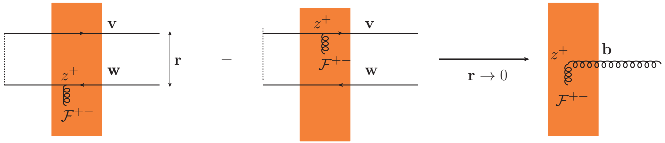
$\Rightarrow$ Order $|\mathbf{r}|^2$ correction: contributions with either one or two field strength $\mathcal{F}_\perp{}^-$

Small $\mathbf{r}$ limit for the non-static NEik correction

For the open decorated dipole due to the dynamics of the target:

$$\left[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r},b^-\Big)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r},b^-\Big)\right]\Big|_{b^-=0}$$
$$=\int_{z^+}\Big\{\mathcal{U}_F(\mathbf{b})\mathcal{U}_F^\dagger\Big(z^+,-\infty;\mathbf{b}\Big)igt\cdot\mathcal{F}^{+-}(z^+,\mathbf{b})\mathcal{U}_F^\dagger\Big(+\infty,z^+;\mathbf{b}\Big)$$
$$-\mathcal{U}_F\Big(+\infty,z^+;\mathbf{b}\Big)(-ig)t\cdot\mathcal{F}^{+-}(z^+,\mathbf{b})\mathcal{U}_F\Big(z^+,-\infty;\mathbf{b}\Big)\mathcal{U}_F^\dagger(\mathbf{b})+O(|\mathbf{r}|)\Big\}$$
$$=2it^{a'}\int_{z^+}\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{a'a}\,g\mathcal{F}_a^{+-}(z^+,\mathbf{b})+O(|\mathbf{r}|)$$

Involves the longitudinal chromoelectric field $\mathcal{F}^{+-}$ instead of the transverse field $\mathcal{F}_j^{\ -}$



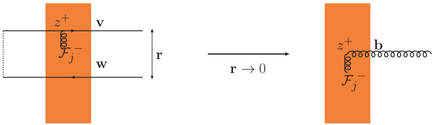
Note: Similar result for the NEik corrections with $\mathcal{U}^{(3)}_{F;ij}$, but with $\mathcal{F}_{ij}$ instead of $\mathcal{F}^{+-}$

Small $\mathbf{r}$ limit for the NEik corrections in $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_{F}$

Terms with $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_{F}$ decorating the quark line (remembering that $|\mathbf{r}| \sim 1/|\mathbf{P}|$):

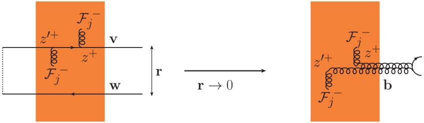
$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[-\Big(\mathbf{P}^j + \frac{(z_1 - z_2)}{2}\mathbf{k}^j\Big)\mathcal{U}^{(1)}_{F;j}(\mathbf{b}+z_2\mathbf{r})\mathcal{U}^{\dagger}_{F}(\mathbf{b}-z_1\mathbf{r}) + \frac{i}{2}\mathcal{U}^{(1)}_{F;j}(\mathbf{b}+z_2\mathbf{r})\,\partial_j\mathcal{U}^{\dagger}_{F}(\mathbf{b}-z_1\mathbf{r}) - i\,\mathcal{U}^{(2)}_{F}(\mathbf{b}+z_2\mathbf{r})\mathcal{U}^{\dagger}_{F}(\mathbf{b}-z_1\mathbf{r})\Big]$$

$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big\{ \Big[-\mathbf{P}^j + \frac{(z_2-z_1)}{2}\mathbf{k}^j - iz_2\mathbf{P}^j(\mathbf{r}\cdot\mathbf{k})\Big]\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\mathcal{U}^{\dagger}_{F}(\mathbf{b}) + \Big[\frac{i}{2}\delta^{ij} + \mathbf{P}^j\mathbf{r}^i\Big]\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\,\partial_i\mathcal{U}^{\dagger}_{F}(\mathbf{b}) - i\,\mathcal{U}^{(2)}_{F}(\mathbf{b})\mathcal{U}^{\dagger}_{F}(\mathbf{b}) + O(|\mathbf{r}|)\Big\}$$



$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\mathcal{U}^{\dagger}_{F}\big(\mathbf{b}\big) = 2it^{a'}\int_{z^+} z^+\,\mathcal{U}_A\big(+\infty, z^+;\mathbf{b}\big)_{a'a}\, g\mathcal{F}^{a\ -}_{j}(z^+,\mathbf{b})$$

Small $\mathbf{r}$ limit for the NEik corrections in $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_F$



$$\mathcal{U}^{(2)}_F(\mathbf{b})\mathcal{U}^\dagger_F(\mathbf{b}) = -t^{a'}t^{b'}\int_{z^+,z'^+} (z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{a'a}\, g\mathcal{F}^{a\ -}_{j}(z^+,\mathbf{b})$$
$$\times\,\mathcal{U}_A\Big(+\infty,z'^+;\mathbf{b}\Big)_{b'b}\, g\mathcal{F}^{b\ -}_{j}(z'^+,\mathbf{b})$$

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\,\partial_i\mathcal{U}^\dagger_F(\mathbf{b}) = -\,2t^{a'}t^{b'}\int dz^+\int dz'^+\, z^+\,\mathcal{U}_A(+\infty,z^+;\mathbf{b})_{a'a}\, g\mathcal{F}^{a\ -}_{j}(z^+,\mathbf{b})$$
$$\times\,\mathcal{U}_A(+\infty,z'^+;\mathbf{b})_{b'b}\, g\mathcal{F}^{b\ -}_{i}(z'^+,\mathbf{b})$$

Like in the GEik term: contributions with either 1 or 2 $\mathcal{F}_\perp^{\ -}$,
but now with an extra factor z^+ or $(z^+-z'^+)$: NEik suppression with the target width.

Similar results for decorations on the antiquark line instead.

Back-to-back cross section: (Generalized) Eikonal piece

Squaring the single $\mathcal{F}_{\perp}^{-}$ part of the Generalized Eikonal contribution in the back-to-back limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\Bigg|_{\text{Gen.Eik}}^{\mathcal{F}_{\perp}^{-}\mathcal{F}_{\perp}^{-}} = g^2(ee_f)^2Q^2(q^+ + k_1^+ - k_2^+)^2(q^+ - k_1^+ + k_2^+)^2\frac{k_1^+k_2^+}{4(q^+)^6}\Bigg[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\hat{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\hat{Q}^2]^4}$$
$$+16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\hat{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\Bigg](2q^+)\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}$$
$$\times\left\langle \mathcal{F}_i^{a\,-}\left(z'^+,\mathbf{b}',-\frac{\Delta b^-}{2}\right)\left[\mathcal{U}_A^\dagger\left(+\infty,z'^+;\mathbf{b}',-\frac{\Delta b^-}{2}\right)\mathcal{U}_A\left(+\infty,z^+;\mathbf{b},\frac{\Delta b^-}{2}\right)\right]_{ab}\mathcal{F}_j^{b\,-}\left(z^+,\mathbf{b},\frac{\Delta b^-}{2}\right)\right\rangle$$

Strict Eikonal result found by neglecting Δb^- in the fields:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\Bigg|_{\text{Strict.Eik}}^{\mathcal{F}_{\perp}^{-}\mathcal{F}_{\perp}^{-}} = (2q^+)2\pi\delta(k_1^++k_2^+-q^+)(ee_f)^2g^2 4z_1^3z_2^3Q^2$$
$$\times\Bigg[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\Bigg]$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

- **Correlator related to twist-2 gluon TMDs in the target**, with momentum fraction $\mathrm{x}=0$ and transverse momentum $\mathbf{k}$, with a *future staple* gauge link.
- **Kinematical twist 3 corrections**, suppressed by an extra $|\mathbf{k}|/|\mathbf{P}|$ in the back-to-back dijet limit $|\mathbf{k}| \ll |\mathbf{P}|$
- Not shown here: **Genuine twist 3 corrections**, involving a correlator of the type $\langle\mathcal{F}_{\perp}^{-}\mathcal{F}_{\perp}^{-}\mathcal{F}_{\perp}^{-}\rangle$

Back-to-back cross section: (Generalized) Eikonal piece

Squaring the single $\mathcal{F}_{\perp}^{-}$ part of the Generalized Eikonal contribution in the back-to-back limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Gen.Eik}}^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = g^2(ee_f)^2Q^2(q^+ + k_1^+ - k_2^+)^2(q^+ - k_1^+ + k_2^+)^2\frac{k_1^+k_2^+}{4(q^+)^6}\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\hat{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\hat{Q}^2]^4}\right.$$
$$\left. + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\hat{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right](2q^+)\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}$$
$$\times\left\langle \mathcal{F}_i^{a\,-}\left(z'^+,\mathbf{b}',-\frac{\Delta b^-}{2}\right)\left[\mathcal{U}_A^\dagger\left(+\infty,z'^+;\mathbf{b}',-\frac{\Delta b^-}{2}\right)\mathcal{U}_A\left(+\infty,z^+;\mathbf{b},\frac{\Delta b^-}{2}\right)\right]_{ab}\mathcal{F}_j^{b\,-}\left(z^+,\mathbf{b},\frac{\Delta b^-}{2}\right)\right\rangle$$

Strict Eikonal result found by neglecting Δb^- in the fields:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Strict.Eik}}^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = (2q^+)2\pi\delta(k_1^++k_2^+-q^+)(ee_f)^2g^24z_1^3z_2^3Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle\mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

- Difference between Gen. Eik and strict Eik. : involves correlator $\langle\mathcal{F}_\perp^-\mathcal{F}_\perp^-\mathcal{F}^{+-}\rangle$ or $\mathbf{k}\langle\mathcal{F}_\perp^-\mathcal{F}^{+-}\rangle$

$\Rightarrow$ NEik correction but twist 4: beyond our accuracy here!

Back-to-back cross section: twist 2 TMDs contribution

Including all contributions of the form $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$, of order Eik or NEik, and twist 2 or twist 3:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times \left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j + \mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+ z_1 z_2} + \text{NNEik}\right]$$
$$\times \left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty, z'^+;\mathbf{b}')\mathcal{U}_A(+\infty, z^+;\mathbf{b})\right]_{ab} \mathcal{F}_j^{b\,-}(z^+,\mathbf{b}) \right\rangle$$

- NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$ contribution factorize from each other!
- Result of the same form can be obtained for transverse photon case.

Back-to-back cross section: twist 2 TMDs contribution

Including all contributions of the form $\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \rangle$, of order Eik or NEik, and twist 2 or twist 3:

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}}\Bigg|^{\mathcal{F}_\perp^{\ -}\mathcal{F}_\perp^{\ -}} = (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times \left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i\mathbf{k}^j + \mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} \left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + \text{NNEik}\right]$$
$$\times \left\langle \mathcal{F}_i^{a\ -}(z'^+, \mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}')\mathcal{U}_A(+\infty, z^+; \mathbf{b})\right]_{ab} \mathcal{F}_j^{b\ -}(z^+, \mathbf{b})\right\rangle$$

- NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \rangle$ contribution factorize from each other!
- Result of the same form can be obtained for transverse photon case.

Notation: TMD $\langle \mathcal{F}^{\mu\nu} \mathcal{F}^{\rho\sigma} \rangle$ correlator in unpolarized target:

$$\Phi^{\mu\nu;\rho\sigma}(\text{x}, \mathbf{k}) \equiv \frac{1}{\text{x}P_{tar}^-} \frac{1}{(2\pi)^3} \int d^2\mathbf{z}\, e^{-i\mathbf{k}\cdot\mathbf{z}} \int dz^+\, e^{i\text{x}P_{tar}^- z^+} \left\langle P_{tar} \middle| \mathcal{F}_a^{\mu\nu}(0) \left[\mathcal{U}_A^\dagger(+\infty, 0; 0)\, \mathcal{U}_A(+\infty, z^+; \mathbf{z})\right]_{ab} \mathcal{F}_b^{\rho\sigma}(z^+, \mathbf{z}) \middle| P_{tar} \right\rangle$$
$$= \frac{1}{(2\pi)^3} \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} e^{i\text{x}P_{tar}^-(z^+ - z'^+)} \left\langle \mathcal{F}_a^{\mu\nu}(z'^+, \mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}')\mathcal{U}_A(+\infty, z^+; \mathbf{b})\right]_{ab} \mathcal{F}_b^{\rho\sigma}(z^+, \mathbf{b}) \right\rangle,$$

Back-to-back cross section: twist 2 TMDs contribution

Including all contributions of the form $\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \rangle$, of order Eik or NEik, and twist 2 or twist 3:

- NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_\perp^{\ -} \mathcal{F}_\perp^{\ -} \rangle$ contribution factorize from each other!
- Result of the same form can be obtained for transverse photon case.

Notation: TMD $\langle \mathcal{F}^{\mu\nu} \mathcal{F}^{\rho\sigma} \rangle$ correlator in unpolarized target:

$$\Phi^{\mu\nu;\rho\sigma}(\mathrm{x},\mathbf{k}) \equiv \frac{1}{\mathrm{x}P^-_{tar}} \frac{1}{(2\pi)^3} \int d^2\mathbf{z}\, e^{-i\mathbf{k}\cdot\mathbf{z}} \int dz^+\, e^{i\mathrm{x}P^-_{tar}z^+} \left\langle P_{tar} \middle| \mathcal{F}_a^{\mu\nu}(0) \left[\mathcal{U}_A^\dagger\left(+\infty,0;0\right) \mathcal{U}_A\left(+\infty,z^+;\mathbf{z}\right) \right]_{ab} \mathcal{F}_b^{\rho\sigma}(z^+,\mathbf{z}) \middle| P_{tar} \right\rangle$$
$$= \frac{1}{(2\pi)^3} \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} e^{i\mathrm{x}P^-_{tar}(z^+-z'^+)} \left\langle \mathcal{F}_a^{\mu\nu}(z'^+,\mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}') \mathcal{U}_A(+\infty,z^+;\mathbf{b}) \right]_{ab} \mathcal{F}_b^{\rho\sigma}(z^+,\mathbf{b}) \right\rangle,$$

$$\left.\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\right|^{\mathcal{F}_\perp^{\ -}\mathcal{F}_\perp^{\ -}} = \alpha_{\mathrm{em}}\, \alpha_s\, e_f^2\, \mathcal{C}^{ij}_{T,L}(z_1,\mathbf{P},\mathbf{k}) \left\{ \left[1 + \frac{(\mathbf{P}^2+\bar{Q}^2)}{z_1 z_2 W^2}\, \partial_{\mathrm{x}} + \mathrm{NNEik} \right] \left[\mathrm{x}\, \Phi^{i-;j-}(\mathrm{x},\mathbf{k}) \right] \right\} \Big|_{\mathrm{x}=0}$$
$$= \alpha_{\mathrm{em}}\, \alpha_s\, e_f^2\, \mathcal{C}^{ij}_{T,L}(z_1,\mathbf{P},\mathbf{k}) \left\{ \left[\mathrm{x}\, \Phi^{i-;j-}(\mathrm{x},\mathbf{k}) \right] \Big|_{\mathrm{x}=\frac{(\mathbf{P}^2+\bar{Q}^2)}{z_1 z_2 W^2}} + \mathrm{NNEik} \right\}$$

Non-zero value of momentum fraction x in the twist 2 gluon TMDs recovered from NEik corrections!

Standard parametrization of $\Phi^{i-;j-}(\mathrm{x},\mathbf{k})$ (with the target mass M):

$$\Phi^{i-;j-}(\mathrm{x},\mathbf{k}) = \frac{\delta^{ij}}{2}\, f_1^g(\mathrm{x},\mathbf{k}) + \left[\mathbf{k}^i\mathbf{k}^j - \frac{\mathbf{k}^2}{2}\,\delta^{ij} \right] \frac{1}{2M^2}\, h_1^{\perp g}(\mathrm{x},\mathbf{k})$$

Back-to-back cross section: twist 3 TMDs from NEik

From the interference between the non-static NEik correction and the strict Eikonal amplitudes: terms in $\langle \mathcal{F}^{+-} \mathcal{F}_{\perp}^{-} \rangle$

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|_{NEik}^{\mathcal{F}_{\perp}^{-}\mathcal{F}^{+-}} = \alpha_{\text{em}}\, \alpha_s\, e_f^2\, \frac{1}{2q^+} \mathcal{C}^{j}_{T,L}(z_1, \mathbf{P}) \Big[\text{x}\, \Phi^{j-;+-}(\text{x}, \mathbf{k}) + \text{x}\, \Phi^{+-;j-}(\text{x}, \mathbf{k}) \Big]\Bigg|_{\text{x}=0}$$

$\Rightarrow$ NEik. correction beyond the static approximation for the target involves a **twist-3 gluon TMD**, (Mulders, Rodrigues (2001)) with momentum fraction $\text{x} = 0$.

From the interference between the NEik correction with $\mathcal{U}^{(3)}_{F;ij}$ and the strict Eikonal amplitude: terms in $\langle \mathcal{F}^{ij} \mathcal{F}_{\perp}^{-} \rangle$

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|_{NEik}^{\mathcal{F}_{\perp}^{-}\mathcal{F}^{ij}} = \alpha_{\text{em}}\, \alpha_s\, e_f^2\, \frac{1}{2q^+} \mathcal{C}^{ijl}_{T,L}(z_1, \mathbf{P}) \Big[\text{x}\, \Phi^{l-;ij}(\text{x}, \mathbf{k}) + \text{x}\, \Phi^{ij;l-}(\text{x}, \mathbf{k}) \Big]\Bigg|_{\text{x}=0}$$

- Contribution from another type of **twist-3 gluon TMD**, (Mulders, Rodrigues (2001)), still with $\text{x} = 0$.
- Note: term absent in in the γ^*_L: $\mathcal{C}^{ijl}_{L}(z_1, \mathbf{P}) = 0$ due to Dirac algebra.

Parametrization (from Lorcé, Pasquini (2013)):

$$\Phi^{j-;+-}(\text{x}, \mathbf{k}) + \Phi^{+-;j-}(\text{x}, \mathbf{k}) = \frac{2\mathbf{k}^j}{P^-_{tar}}\, f^{\perp g}(\text{x}, \mathbf{k})$$

$$\Phi^{l-;ij}(\text{x}, \mathbf{k}) + \Phi^{ij;l-}(\text{x}, \mathbf{k}) = \epsilon^{ij}\, \epsilon^{ln}\, \frac{2\mathbf{k}^n}{P^-_{tar}}\, \bar{g}^{\perp g}(\text{x}, \mathbf{k})$$

Final result: $\langle \mathcal{F}\mathcal{F} \rangle$ contributions to the cross sections

$$\frac{d\sigma_{\gamma^*_{T,L}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|^{\mathcal{FF}}_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2\, \alpha_s \bigg\{ \mathcal{C}^{f_1^g}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, f_1^g(\text{x},\mathbf{k}) + \mathcal{C}^{h_1^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, h_1^{\perp g}(\text{x},\mathbf{k})$$
$$+ \frac{1}{W^2}\, \mathcal{C}^{f^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, f^{\perp g}(\text{x},\mathbf{k}) + \frac{1}{W^2}\, \mathcal{C}^{\bar{g}^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, \bar{g}^{\perp g}(\text{x},\mathbf{k}) \bigg\}\bigg|_{\text{x}=\frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1 z_2\, W^2}}$$

Contribution from longitudinal photon exchange:

$$\mathcal{C}^{f_1^g}_{L}(z_1,\mathbf{P},\mathbf{k}) = \frac{8Q^2\, z_1^2 z_2^2}{[\mathbf{P}^2+\bar{Q}^2]^4} \bigg\{ \mathbf{P}^2 + (z_2-z_1)(\mathbf{k}\cdot\mathbf{P}) \bigg[-1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]} \bigg] \bigg\} + O\bigg(\frac{Q^2\mathbf{k}^2}{\mathbf{P}^8}\bigg)$$

$$\mathcal{C}^{h_1^{\perp g}}_{L}(z_1,\mathbf{P},\mathbf{k}) = \frac{4Q^2\, z_1^2 z_2^2}{[\mathbf{P}^2+\bar{Q}^2]^4}\, \frac{\mathbf{k}^2}{M^2} \bigg\{ \bigg(\frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2} - 1 \bigg) \mathbf{P}^2$$
$$+ (z_2-z_1)(\mathbf{k}\cdot\mathbf{P}) \bigg[-1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]} \bigg(\frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2} - 1 \bigg) \bigg] \bigg\} + O\bigg(\frac{Q^2\mathbf{k}^2}{\mathbf{P}^8}\bigg)$$

$$\mathcal{C}^{f_1^{\perp g}}_{L}(z_1,\mathbf{P},\mathbf{k}) = -\, 32Q^2\, z_1 z_2\, \frac{(z_2-z_1)(\mathbf{k}\cdot\mathbf{P})[\mathbf{P}^2+m^2]}{[\mathbf{P}^2+\bar{Q}^2]^4} + O\bigg(\frac{Q^2\mathbf{k}^2}{\mathbf{P}^6}\bigg)$$

$$\mathcal{C}^{\bar{g}^{\perp g}}_{L}(z_1,\mathbf{P},\mathbf{k}) = 0$$

Final result: $\langle \mathcal{F}\mathcal{F} \rangle$ contributions to the cross sections

$$\frac{d\sigma_{\gamma^*_{T,L}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|^{\mathcal{F}\mathcal{F}}_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2\, \alpha_s \Bigg\{ \mathcal{C}^{f_1^g}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, f_1^g(\text{x},\mathbf{k}) + \mathcal{C}^{h_1^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, h_1^{\perp g}(\text{x},\mathbf{k}) + \frac{1}{W^2}\, \mathcal{C}^{f^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, f^{\perp g}(\text{x},\mathbf{k}) + \frac{1}{W^2}\, \mathcal{C}^{\bar{g}^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, \bar{g}^{\perp g}(\text{x},\mathbf{k}) \Bigg\}\Bigg|_{\text{x}=\frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1 z_2\, W^2}}$$

Contribution from transverse photon exchange:

$$\mathcal{C}^{f_1^g}_{T}(z_1,\mathbf{P},\mathbf{k}) = -\frac{2\big[(z_1^2+z_2^2)\bar{Q}^2-m^2\big]}{[\mathbf{P}^2+\bar{Q}^2]^4}\left\{\mathbf{P}^2 + (z_2-z_1)(\mathbf{k}\cdot\mathbf{P})\left[-1+\frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]}\right]\right\} + \frac{(z_1^2+z_2^2)}{[\mathbf{P}^2+\bar{Q}^2]^2}\left[1+\frac{2(z_2-z_1)(\mathbf{k}\cdot\mathbf{P})}{[\mathbf{P}^2+\bar{Q}^2]}\right] + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^6}\right)$$

$$\mathcal{C}^{h_1^{\perp g}}_{T}(z_1,\mathbf{P},\mathbf{k}) = -\frac{\big[(z_1^2+z_2^2)\bar{Q}^2-m^2\big]}{[\mathbf{P}^2+\bar{Q}^2]^4}\,\frac{\mathbf{k}^2}{M^2}\left\{\left(\frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2}-1\right)\mathbf{P}^2 + (z_2-z_1)(\mathbf{k}\cdot\mathbf{P})\left[-1+\frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]}\left(\frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2}-1\right)\right]\right\} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^6}\right)$$

$$\mathcal{C}^{f_1^{\perp g}}_{T}(z_1,\mathbf{P},\mathbf{k}) = \frac{4(z_2-z_1)(\mathbf{k}\cdot\mathbf{P})}{[\mathbf{P}^2+\bar{Q}^2]^4}\left\{\left[\mathbf{P}^2+\bar{Q}^2+(z_1^2+z_2^2)Q^2\right]\left[\mathbf{P}^2-\bar{Q}^2\right]+2m^2Q^2\right\} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^4}\right)$$

$$\mathcal{C}^{\bar{g}^{\perp g}}_{T}(z_1,\mathbf{P},\mathbf{k}) = \frac{4(z_2-z_1)(\mathbf{k}\cdot\mathbf{P})}{[\mathbf{P}^2+\bar{Q}^2]^2} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^4}\right)$$

Back-to-back cross section: 3 $\mathcal{F}$ correlators

Leftover contributions in $\langle \mathcal{F}_\perp^{\;-} \mathcal{F}_\perp^{\;-} \mathcal{F}_\perp^{\;-} \rangle$, of order Eik or NEik, starting at twist 3.

Various terms in the cross section, with a correlator of the form:

$$2\mathrm{Re}\,\mathrm{tr}_F\big(t^{a'}t^{b'}t^{c'}\big) \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{v^+,w^+,v'^+} \left[1 + i\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+ z_1 z_2}\,\phi(v^+ - v'^+, w^+ - v'^+)\right]$$
$$\times \Big\langle \mathcal{U}_A(+\infty, v'^+;\mathbf{b}')_{c'c}\, g\mathcal{F}_l^{c\,-}(v'^+,\mathbf{b}')\, \mathcal{U}_A(+\infty, v^+;\mathbf{b})_{a'a}\, g\mathcal{F}_i^{a\,-}(v^+,\mathbf{b})\, \mathcal{U}_A(+\infty, w^+;\mathbf{b})_{b'b}\, g\mathcal{F}_j^{b\,-}(w^+,\mathbf{b}) \Big\rangle$$

with several possible functions ϕ in the NEik correction.

In the TMD formalism, such 3 fields correlator should in principle depend on two different momentum fractions x_1 and x_2 via phase factors. However: not yet studied in the gluon case in the TMD literature.

Not entirely clear at the moment how to resum the NEik corrections unambiguously in order to recover this dependence on x_1 and x_2.

Note: $\mathrm{tr}_F\big(t^{a'}t^{b'}t^{c'}\big) = \big(d^{a'b'c'} + i\,f^{a'b'c'}\big)/4$

- Antisymmetric piece in $f^{a'b'c'}$: Eikonal contribution cancel $\Rightarrow$ starts at NEik only
- Symmetric piece in $d^{a'b'c'}$: both Eik and NEik terms. But still unclear how to interpret in terms of dependence on x_1 and x_2.

Summary

To further understand the interplay between CGC and TMD, we studied the NEik DIS dijet cross-section in the back-to-back jets limit, including twist 3 power corrections. Various types of contributions are obtained:

- $\langle \mathcal{F}_i^{\ -} \mathcal{F}_j^{\ -} \rangle$: twist 2 gluon TMDs f_1^g and $h_1^{\perp g}$
  - Factorization of kinematic twist 3 and of NEik corrections
  - NEik correction is the first order correction in the Taylor expansion of the TMDs around $\mathrm{x} = 0$
    $\Rightarrow \mathrm{x}$ dependence of the TMDs recovered by resumming terms of all powers beyond the eikonal approximation
- Twist 3 gluon TMDs found as contributions to the cross section at NEik and twist 3 order:
  - $f^{\perp g}$, of the type $\langle \mathcal{F}_i^{\ -} \mathcal{F}^{+-} \rangle$
  - $\bar{g}^{\perp g}$, of the type $\langle \mathcal{F}_l^{\ -} \mathcal{F}_{ij} \rangle$, for γ_T^* case
- 3 fields twist 3 correlators $\langle \mathcal{F}_i^{\ -} \mathcal{F}_j^{\ -} \mathcal{F}_l^{\ -} \rangle$: beyond TMD partonic distributions
  - Symmetric part in $d^{a'b'c'}$ already appear in Eikonal contributions
  - NEik corrections should be related to the dependence on x_1, x_2
  - Antisymmetric part in $f^{a'b'c'}$ starts only at NEik order

Recap: Corrections beyond the static approximation

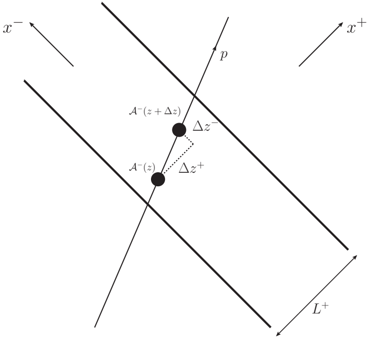
- Relative z^- dependence along the same propagator : NNEik, higher twist ?
- Relative z^- dependence between Wilson lines in the amplitude: NEik, twist 3
  $\rightarrow f^{\perp g}$ twist 3 gluon TMD
- Relative z^- dependence between amplitude and cc. amplitude: NEik, twist 4
  $\rightarrow$ Associated with the overall k^+ exchange with the target
- Overall z^- dependence at cross section level:
  $\rightarrow$ Disappear when performing target average, in (semi-)inclusive processes.

All these non-static corrections can be written as insertions of the longitudinal chromoelectric field $\mathcal{F}^{+-}$ of the target.

More about NEik corrections beyond the static approx

Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator:

$$\mathcal{A}^-(z^- + \Delta z^-) - \mathcal{A}^-(z^-) \simeq \Delta z^- \, \partial_- \mathcal{A}^-(z^-)$$



- Slow z^- dependence from time dilation:

$$\partial_- \mathcal{A}^- \propto \frac{1}{\gamma_t} \mathcal{A}^-$$

- Small Δz^- displacement of the trajectory within the target width L^+:

$$\Delta z^- \sim \frac{p^-}{p^+} \Delta z^+ \leq \frac{p^-}{p^+} L^+ = O\left(\frac{1}{\gamma_t}\right)$$

Double power suppression, beyond static approx and beyond shockwave approx:

$\Rightarrow$ NNEik effect within a single propagator!

More about NEik corrections beyond the static approx

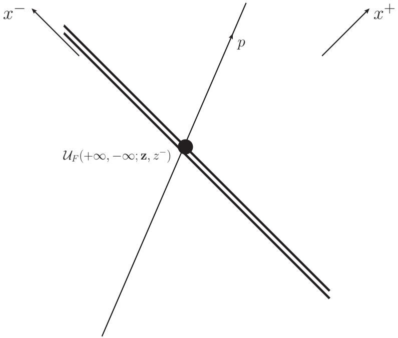
Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator is NNEik.

However, dependence on average z^- is suppressed only once.

$\Rightarrow$ Use Wilson lines with overall z^- dependence

$$\partial_- \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \propto \frac{1}{\gamma_t} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-)$$

$\rightarrow$ Accounts for NEik effects beyond static approx



More about NEik corrections beyond the static approx

Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator is NNEik.

However, dependence on average z^- is suppressed only once.

$\Rightarrow$ Use Wilson lines with overall z^- dependence

$$\partial_- \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \propto \frac{1}{\gamma t} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-)$$

$\rightarrow$ Accounts for NEik effects beyond static approx

In particular: NEik corrections induced by the difference in z^- between different Wilson lines.