Deblurring for Nuclei:
Triple-Differential Yields in Heavy-Ion Collisions

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Inverse Problems and Uncertainty Quantification in Nuclear Physics

Institute for Nuclear Theory Workshop 24-88W

Seattle, Washington, July 8-12, 2024

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

What is it?!

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n

Still not clear what the system is…

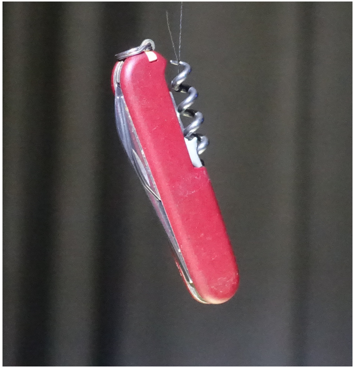
FRIB

Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

some control, v_n



full control, $\frac{d^3N}{dp^3}$

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Claim: You can go from center to right panel through deblurring

Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car

Deblurred

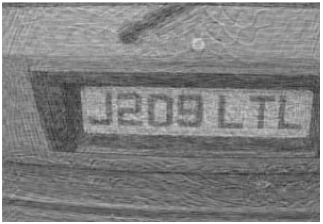
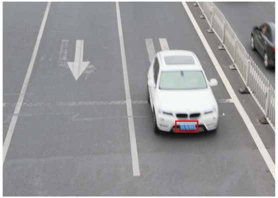


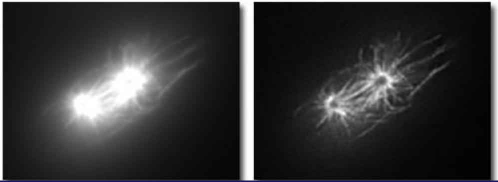
Photo of Parked Car

Fast Moving



Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon}\, n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\text{prod}} \simeq E_{\text{det}} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi)\, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ. Optical terminology: P - blurring or transfer function.

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Bayesian Deblurring

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - unknown complementary probability that ptcle has characteristic ξ while measured at ζ

Bayesian relation: number of times ptcle has characteristic in dξ while measured in dζ is

$$P(\zeta|\xi) \, N(\xi) \, \mathrm{d}\xi \, \mathrm{d}\zeta = Q(\xi|\zeta) \, n(\zeta) \, \mathrm{d}\xi \, \mathrm{d}\zeta$$

Hence $$N(\xi) = \frac{\int \mathrm{d}\zeta \, Q(\xi|\zeta) \, n(\zeta)}{\int \mathrm{d}\zeta' \, P(\zeta'|\xi)} \, , \quad Q(\xi|\zeta) = \frac{P(\zeta|\xi) \, N(\xi)}{\int \mathrm{d}\xi' \, P(\zeta|\xi') \, N(\xi')}$$

Richardson-Lucy method solves eqs iteratively till stabilization

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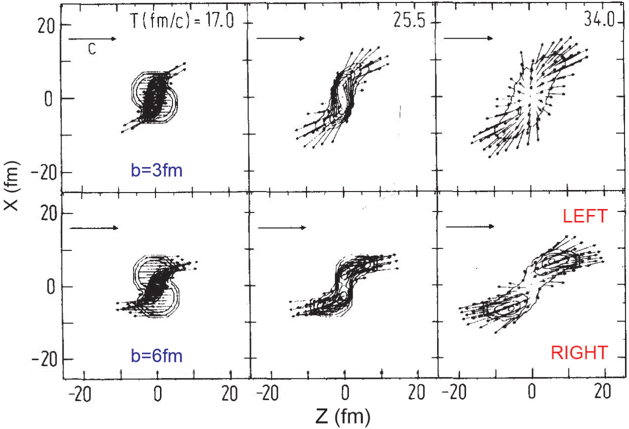
Side Focus in Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

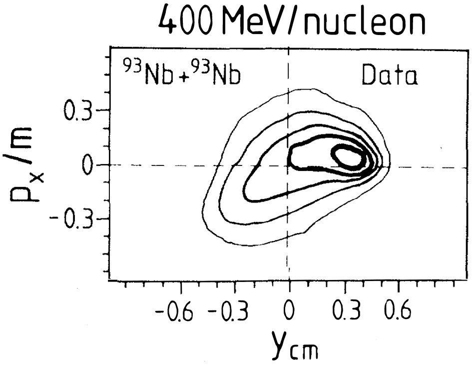
Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

1984 Claim

Gustafsson *et al.* PRL 18(84)1590 Plastic Ball Group claims to see preferential emission away from the beam axis, in $d^3N_{ch}/dy\, d^2p^\perp$ for 400 MeV/nucl Nb + Nb collisions, when determining reaction plane from flow tensor, $\mathbf{S}^{\perp z} = \sum_\nu \mathbf{p}_\nu^\perp\, p^z/2m_\nu$



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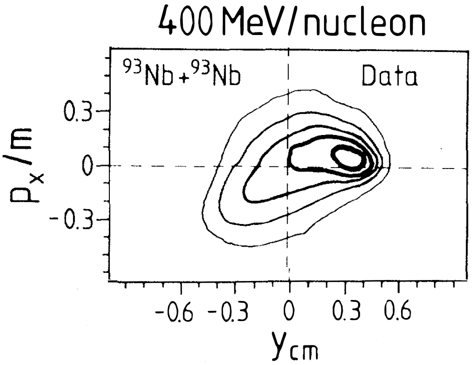
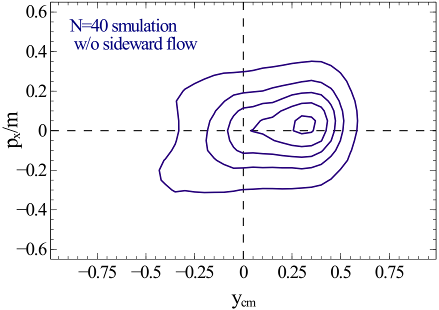
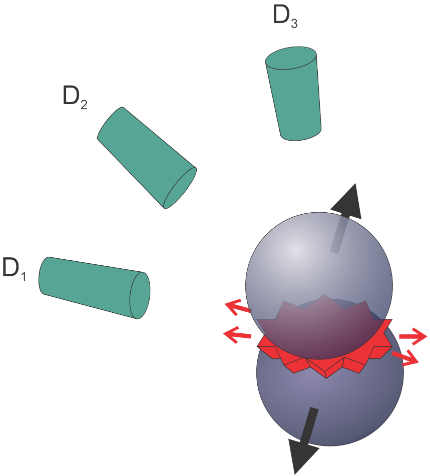




The observation can be explained by particle self-correlation, w/o invoking transverse collective movement

Estimating Reaction-Plane Direction w/o Self-Correlation



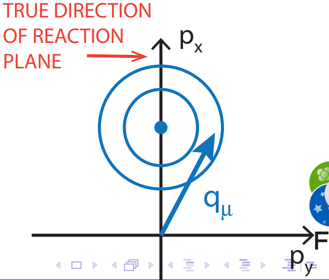
Plane direction f/particle μ estimated with

$$\mathbf{q}_\mu = \frac{1}{N} \sum_{\nu \neq \mu} \omega_\nu \, \mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction, blurring features



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Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

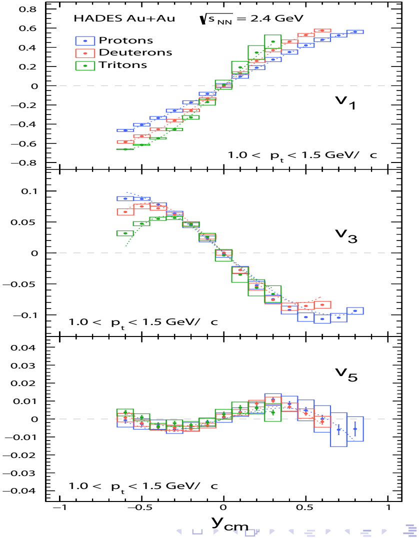
$$\langle \mathbf{p}_\mu^\perp \cdot \mathbf{q}_\mu \rangle \simeq p^\perp \langle q^x \rangle \langle \cos \phi \rangle$$

for different $p^\perp$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301

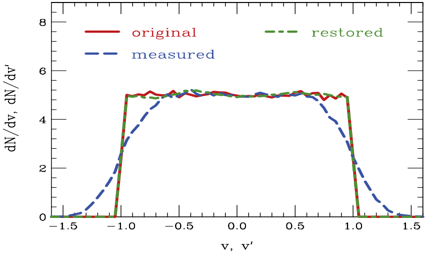


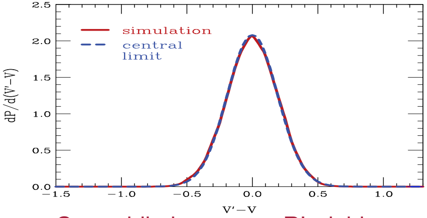
Schematic 1D Model

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~v_n~~?

First 1D deblurring test. Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like $\mathrm{d}N/\mathrm{d}v$ in projectile cm. Task: Measuring ptcles in lab, determine $\mathrm{d}N/\mathrm{d}v$. Cm velocity V' estimated from remaining ptcles, so V' & $\mathrm{d}N/\mathrm{d}v'$ smeared:

$$\frac{\mathrm{d}N}{\mathrm{d}v'} = \int \mathrm{d}V' \, \frac{\mathrm{d}P}{\mathrm{d}V'} \, \frac{\mathrm{d}N}{\mathrm{d}v}$$





PD&Kurata-Nishimura PRC105(2022)034608

→Central-limit smear + RL deblur

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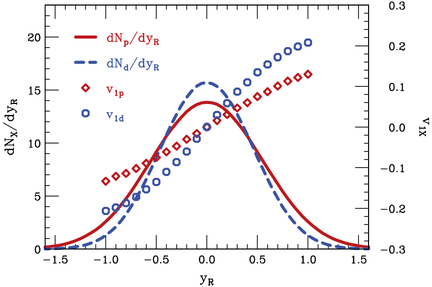
3D Model for Collisions

Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$
Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl

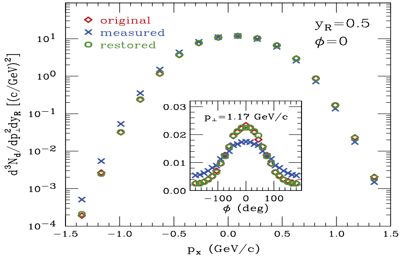


$$\frac{dN}{d\phi'} = \int d\Phi' \, \frac{dP}{d\Phi'} \, \frac{dN}{d\phi}$$

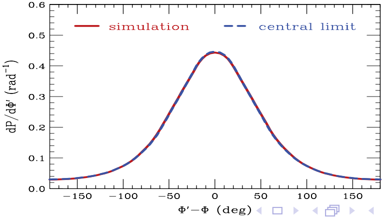
$$\phi' + \Phi' = \phi + \Phi$$

RL deblur + central-limit
Strong anisotropies restored!

Triple differential spectrum in reaction plane:



Uncertainty in reaction plane:



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Ar + KCl @ 1.8 GeV/nucl

Ströbele PRC 27(83)1349

495 events from Streamer Chamber, $b \lesssim 2.4$ fm

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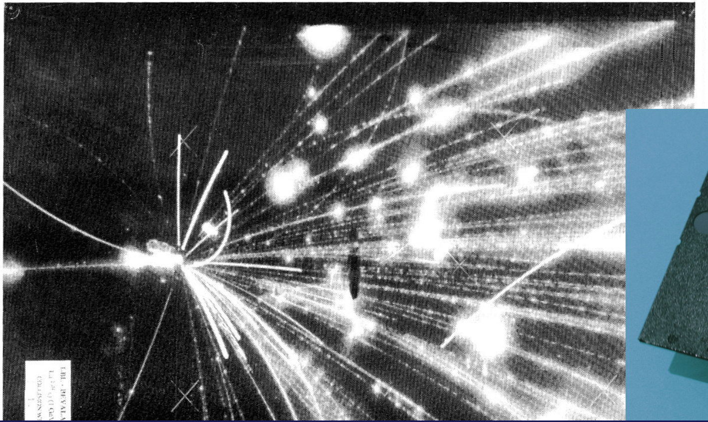
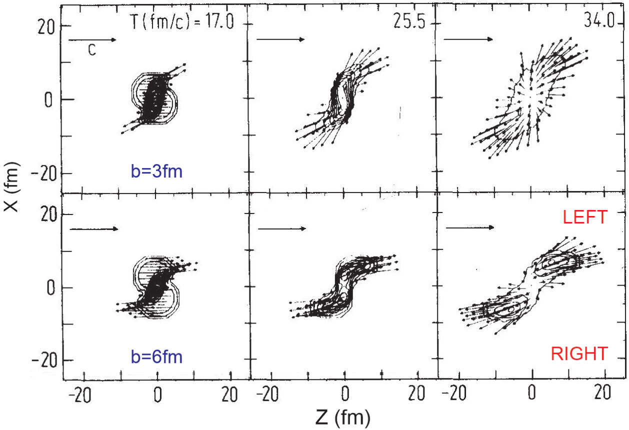


Reminder: Hydrodynamic Calculations

Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

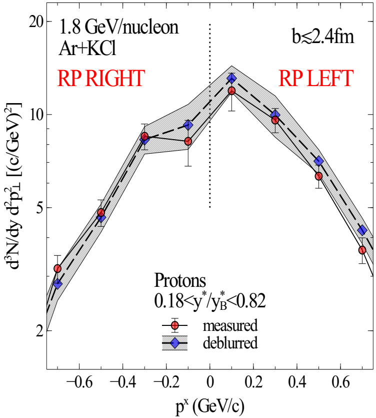
Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

Side-Focus in Ar + KCl 1.8 GeV/nucl?

protons



Particles in the forward hemisphere, $y^* \sim 0.5 y_B^*$

PD, Ströbele, Nzabahimana PRC108(23)L051603

Side-Focus in Ar + KCl 1.8 GeV/nucl?

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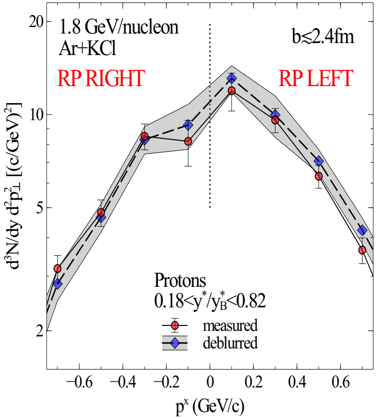
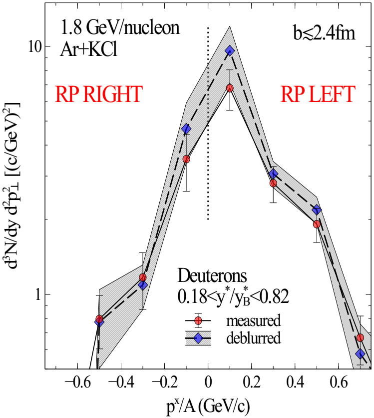


deuterons



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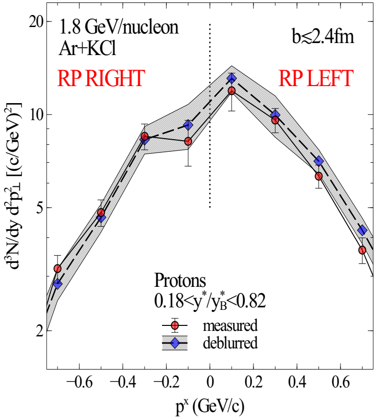

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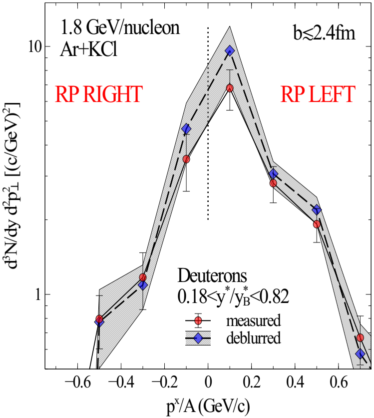

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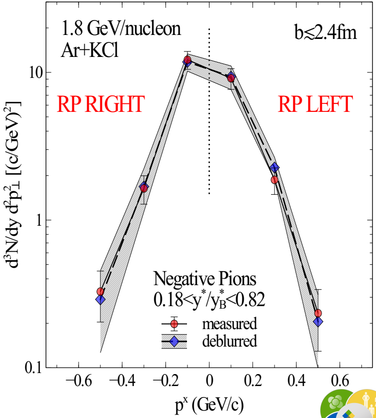

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PD, Ströbele, Nzabahimana PRC108(23)L051603

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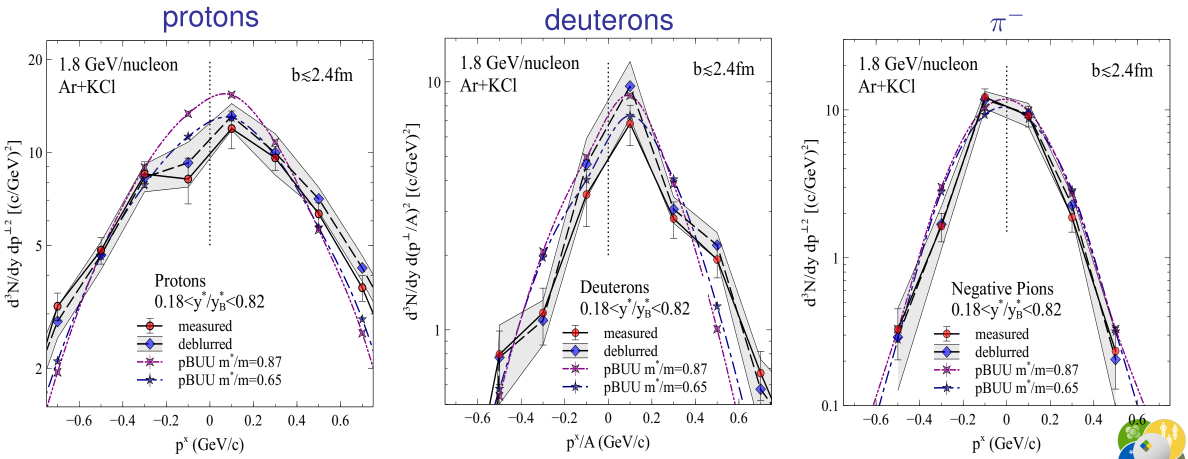
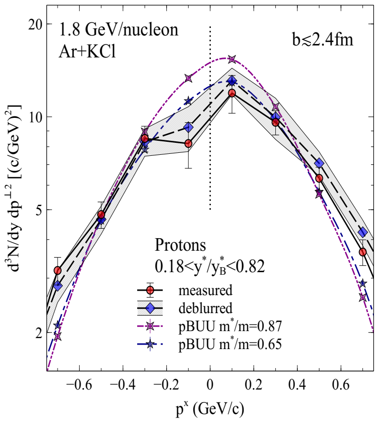
Side-Focus: Experiment vs Theory



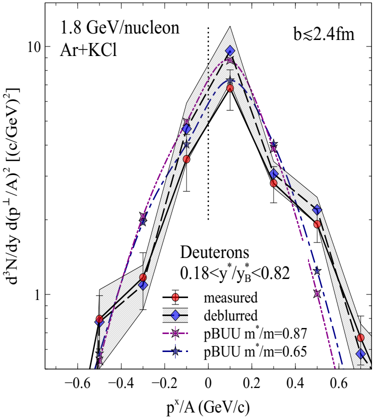
Transport Calculations: sensitivity to mean-field *p*-dependence in Ar+KCl

Side-Focus: Experiment & Theory

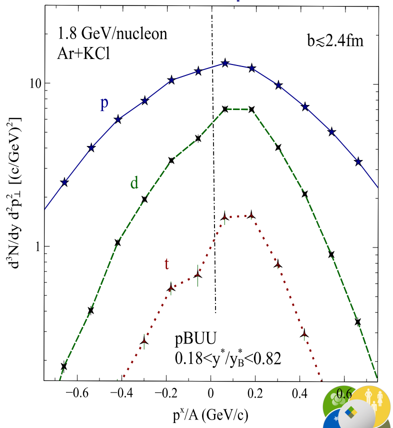
protons



deuterons



H Isotopes



Transverse boost $v^x \sim 0.1\,c$

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What's Behind Deblurring's Success?

Singular value decomposition f/forward conditional probability:

$$P_{ij} = \sum_{n} \sigma_n \, U_{ni} \, V_{nj} \qquad \Rightarrow \qquad Q_{ji} \stackrel{?}{=} \sum_{n} \sigma_n^{-1} \, V_{nj} \, U_{ni}$$

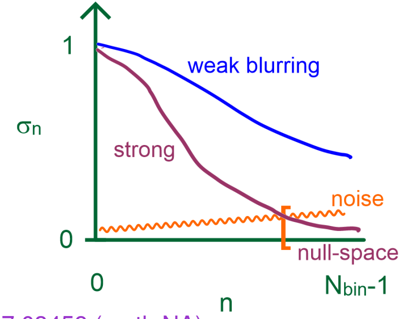
i - measurement, j - reality, Q - backward conditional probability. Instability??

Plain Reaction-Plane Deblurring:

$$U_n(\varphi) = V_n(\varphi) = \begin{cases} \frac{1}{\sqrt{2\pi}}, & n = 0 \\ \frac{\cos(n\varphi)}{\sqrt{\pi}}, & n > 0 \end{cases}$$

$$\sigma_n = \langle \cos(n\Delta\Phi) \rangle$$

with $\Delta\Phi$ estimated-true reaction plane deviation



Detector effects yield more complicated vectors

Positivity + regularization stabilize restoration!

Hansen *et al. Deblurring Images* 2006; Mamba&PD arXiv:2407.03458 (math.NA)

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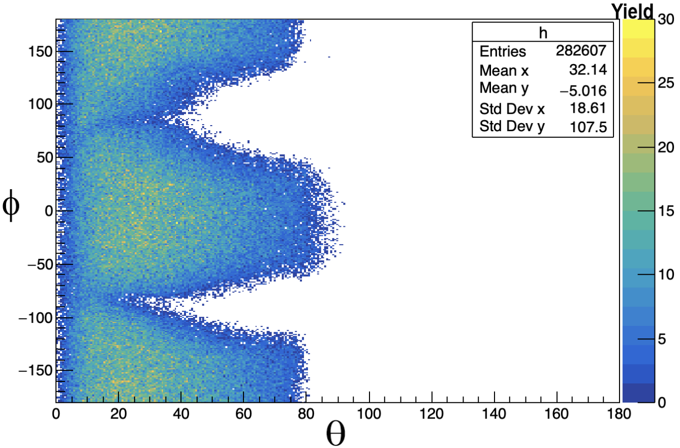
Restoration with Inefficiencies

SΠRIT@RIKEN Time-Projection Chamber

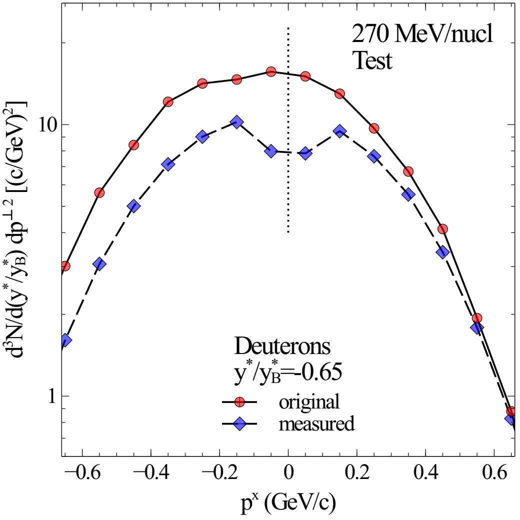
Sn + Sn @ 270 MeV/nucl

Proton distribution in lab angles



Strong azimuthally-asymmetric inefficiencies for slow particles and at small polar angles

Simulated Restoration f/SΠRIT TPC in Backward CM Hemisphere



Preliminary (minimal statistics)

Flow model ran forward through efficiency simulator for the SΠRIT TPC: not only particles lost but also reaction-plane effects

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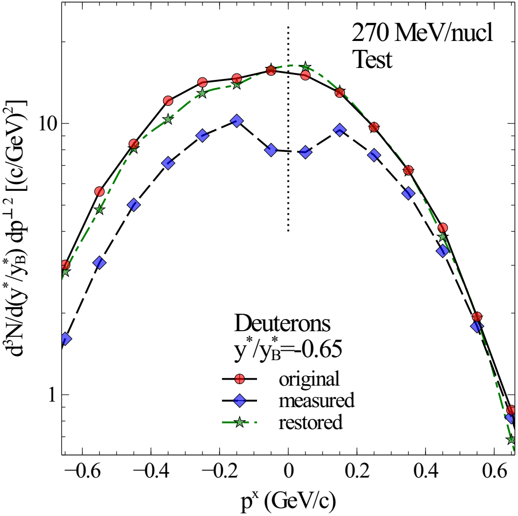

Preliminary (minimal statistics)

Flow model ran forward through efficiency simulator for the SΠRIT TPC: not only particles lost but also reaction-plane effects - restored through deblurring

Conclusions

- Reaction-plane fluctuations made us concentrate at intermediate energies on azimuthal moments w/unclear physics content for higher orders
- Deblurring, common in optics, enables accessing 3D distributions associated w/true reaction plane, when reaction-plane effects are strong enough
- Side focus in Ar + KCl collisions at 1.8 GeV/nucl with $v^x \sim 0.1\, c$, visible with just $\sim$ 500 collision events, is just an example of what may be achieved!
- Other nuclear problems where deblurring started producing results: $^{26}\text{O} \rightarrow ^{24}\text{O} + n + n$ decay, source-imaging from 2-particle correlations in HIC

PD&Kurata-Nishimura PRC105(22)034608; Nzabahimana *et al.* PRC107(23)064315;
PD *et al.* PRC108(23)L051603; Adamczewski-Musch *et al.* PRL125(20)262301 - v_n reconstruction
Berkowitz Physics 15(22)s26
https://www.energy.gov/science/np/articles/deblurring-can-reveal-3d-features-heavy-ion-collisions

Supported by US Department of Energy under Grant US DE-SC0019209

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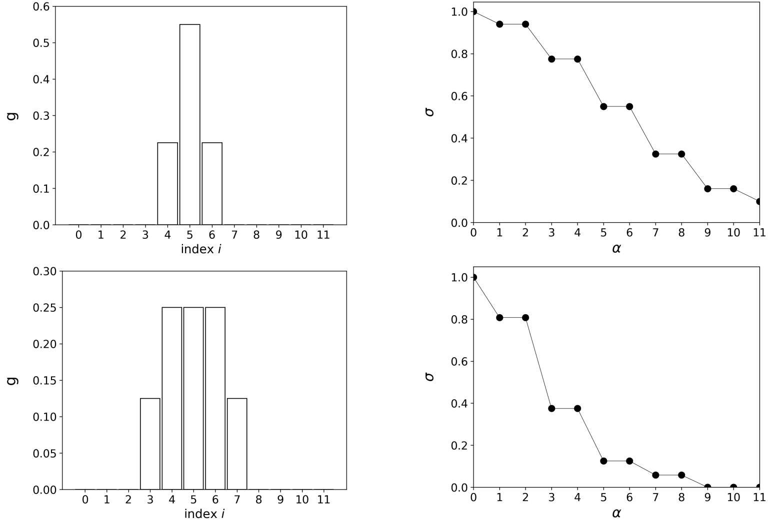
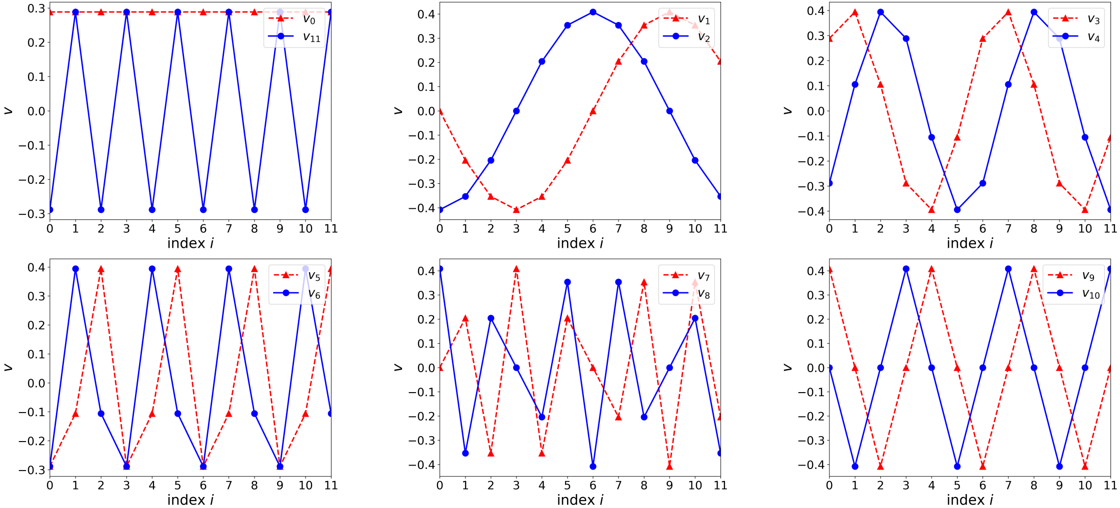
Blurring Function & Singular-Value Spectrum



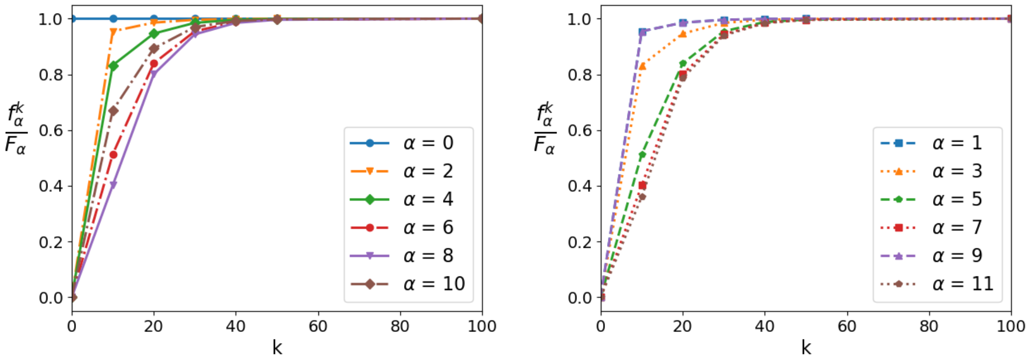
$$P_{ij} = \sum_\alpha \sigma_\alpha \, U_{\alpha i} \, V_{\alpha j}$$

Mamba&PD
arXiv:2407.03458

Singular Vectors, Ordered by Singular Values



Expansion-Coefficient Relaxation w/Iterations



Regularization Works w/Low Contrast