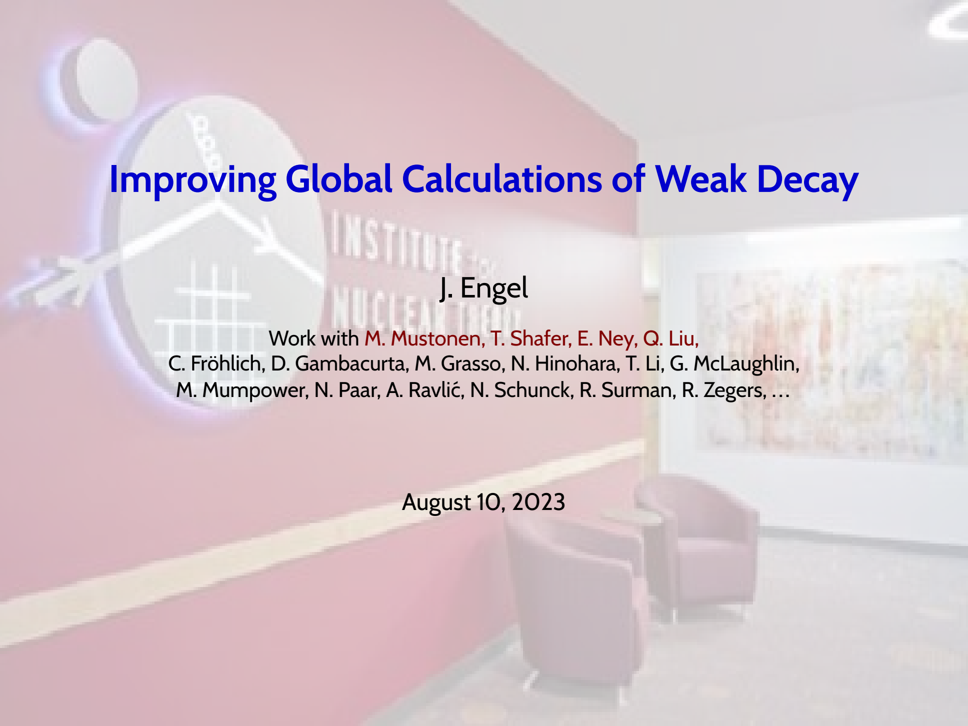


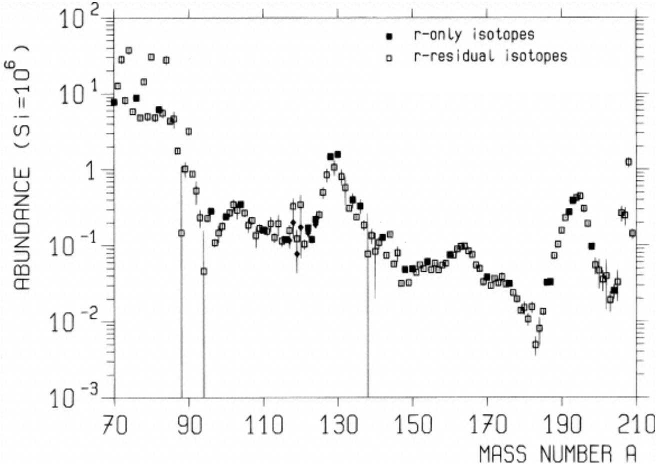
# Improving Global Calculations of Weak Decay

J. Engel

Work with M. Mustonen, T. Shafer, E. Ney, Q. Liu,
C. Fröhlich, D. Gambacurta, M. Grasso, N. Hinohara, T. Li, G. McLaughlin,
M. Mumpower, N. Paar, A. Ravlić, N. Schunck, R. Surman, R. Zegers, …

August 10, 2023

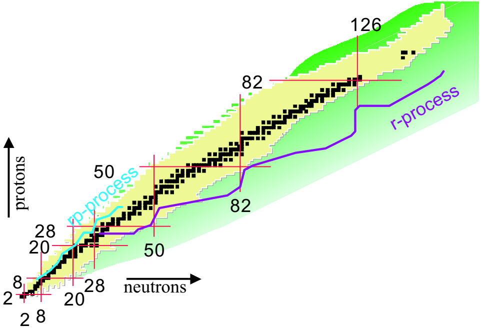
# R-Process Abundances



Blah, blah …

# Nuclear Landscape

To convincingly locate the site(s) of the *r* process, we need to know reaction rates, particularly $\beta$-decay rates, in many neutron-rich nuclei.



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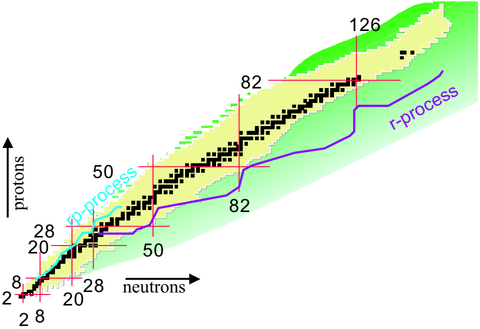
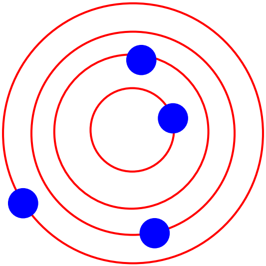


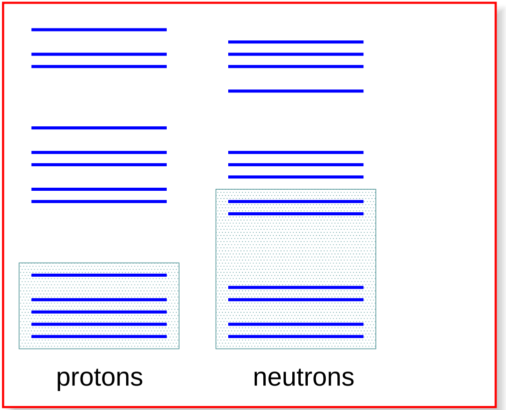
Eventually, we will be able to compute all these rates in an *ab initio* way (or ask ChatGPT v. 17 for them).

For now we still need approximate and phenomenological methods, based on density-functional theory, a fancy name for mean-field theory with a tailored interaction.

# Mean-Field Theory

## Ex: Hartree-Fock

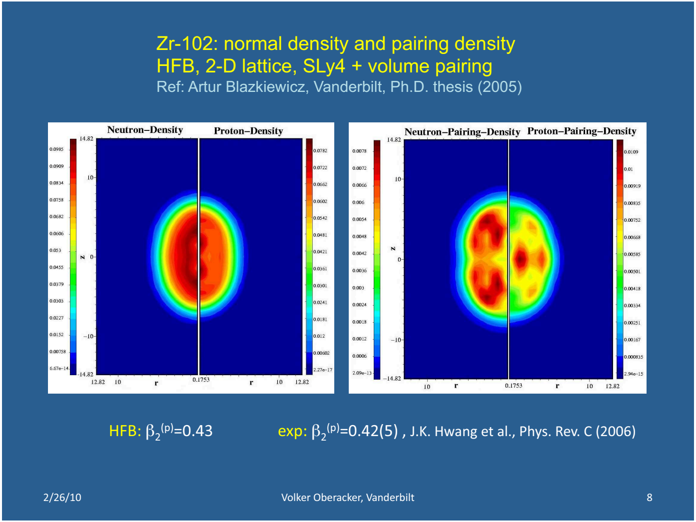




# Sophisticated Mean-Field Theory

## HFB with Skyrme Interactions

Gives you ground state density, pair density, etc. This is where Skyrme interactions have made their living.



Zr-102: normal density and pairing density
HFB, 2-D lattice, SLy4 + volume pairing
Ref: Artur Blazkiewicz, Vanderbilt, Ph.D. thesis (2005)

HFB: $\beta_2^{(p)}=0.43$ exp: $\beta_2^{(p)}=0.42(5)$ , J.K. Hwang et al., Phys. Rev. C (2006)

# QRPA

Self-consistent QRPA is time-dependent HFB with small harmonic perturbation. Decay matrix elements obtained from response of nucleus to harmonic perturbation $\mathcal{F}(t)$.

$$\mathcal{F}(t) = Fe^{-i\omega t} + F^{\dagger}e^{i\omega t}$$

$$F = f_{kl}a_k^{\dagger}a_l$$

Response of density to perturbation given by *response function* $R(\omega)$ :

$$\delta\rho(\omega) \sim R(\omega)f$$

$R$ contains information about transitions to excited states:

$$S(\omega) = \mathrm{Im}\left(f^{\dagger}R(\omega)f\right) = -\pi\sum_{\nu}|\langle\nu|F|0\rangle|^2\,\delta(\omega-\Omega_{\nu})\,.$$

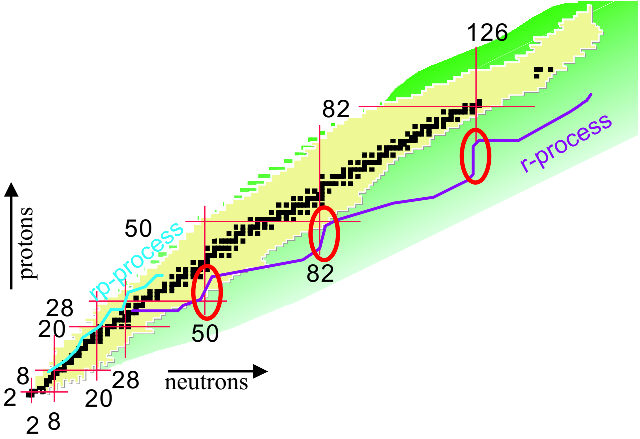
S is the “strength function.”

In our case $F$ is one of the operators that cause $\beta$ decay: Gamow-Teller operator or a forbidden operators.

# Initial Skyrme Application: Spherical QRPA

## Even Isotopes Only

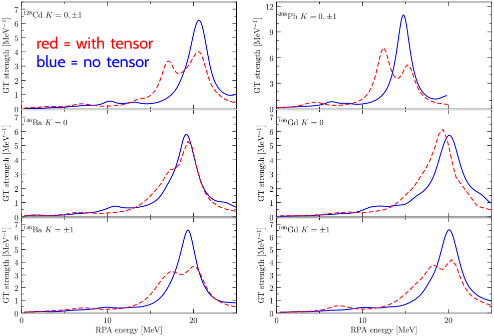
Traditional matrix formulation of QRPA



Closed shell nuclei are spherical.

# 2014: Skyrme QRPA in Deformed Nuclei

## Finite-Amplitude Method (FAM) — Nakatsukasa et al.



Strength functions computed directly from linear response, in orders of magnitude less time than with matrix QRPA.

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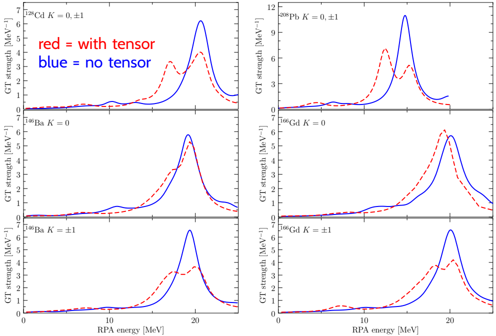
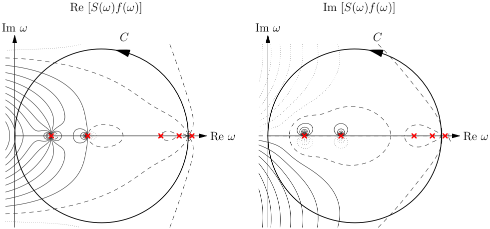


Beta-decay rates obtained by integrating strength with phase-space weighting function in contour around excited states below threshold.



# Global Skyrme Fit for Even Nuclei

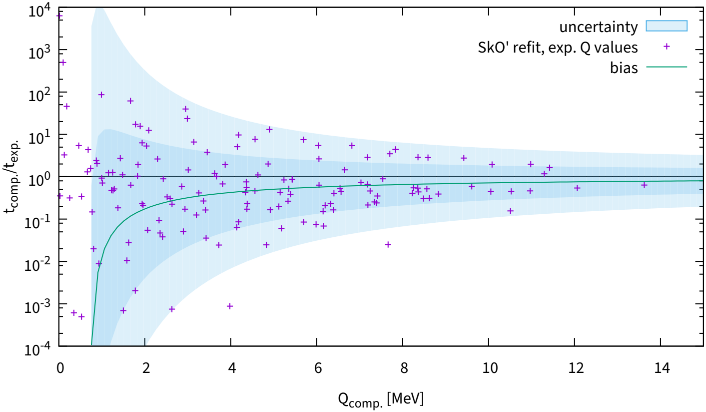
Mika Mustonen

## Fit the charge-changing time-odd functional

$$\begin{aligned}\mathcal{H}^{\text{c.c.}}_{\text{odd}} =& C^{s}_{1}\,\boldsymbol{s}^{2}_{11} + C^{\Delta s}_{1}\,\boldsymbol{s}_{11}\cdot\nabla^{2}\boldsymbol{s}_{11} + C^{T}_{1}\boldsymbol{s}_{11}\cdot\boldsymbol{T}_{11} + C^{j}_{1}\boldsymbol{j}^{2}_{11} \\ &+ C^{\nabla j}_{1}\,\boldsymbol{s}_{11}\cdot\boldsymbol{\nabla}\times\boldsymbol{j}_{11} + C^{F}_{1}\,\boldsymbol{s}_{11}\cdot\boldsymbol{F}_{11} + C^{\nabla s}_{1}\,\left(\boldsymbol{\nabla}\cdot\boldsymbol{s}_{11}\right)^{2} + V_{0}\times pn\text{ pair.}\end{aligned}$$

Included 7 GT resonance energies, 2 spin-dipole resonance energies, 7 $\beta$-decay rates in selected spherical and well-deformed nuclei from light to heavy.

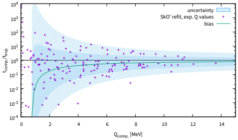
# Results


uncertainty
SkO' refit, exp. Q values
bias
$t_{comp.}/t_{exp.}$
$10^{4}$
$10^{3}$
$10^{2}$
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
0
2
4
6
8
10
12
14
$Q_{comp.}$ [MeV]

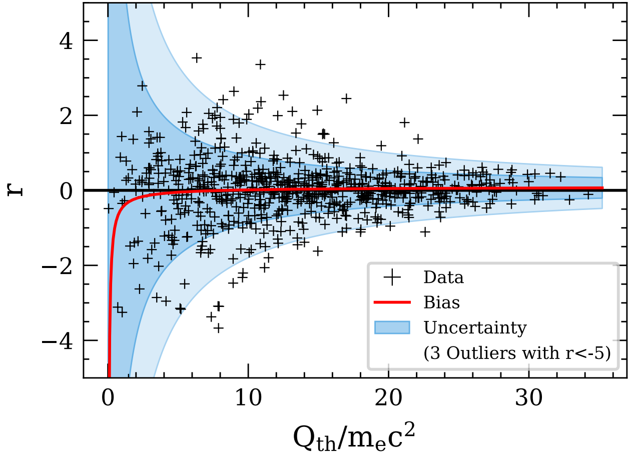
# Results with All Nuclei

Evan Ney

Figured out how to adapt FAM to treat odd-A and odd-odd nuclei.

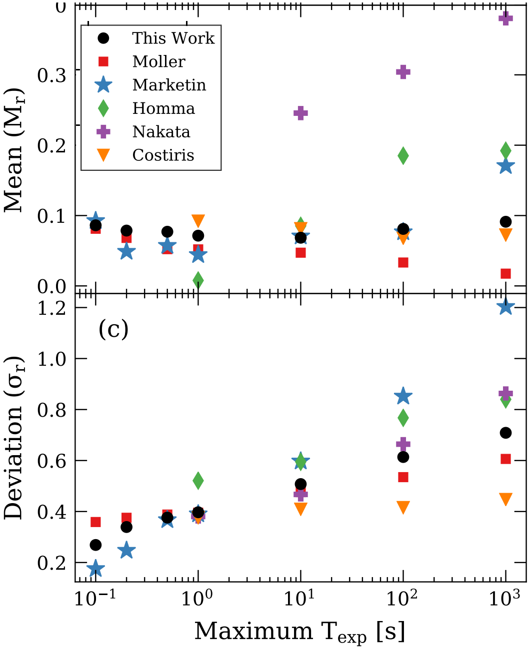


Even-even results



$$r = \log_{10}\left(\frac{t_{th}}{t_{exp}}\right)$$

# Comparison



$$r = \log_{10}\left(\frac{t_{\text{th}}}{t_{\text{exp}}}\right)$$

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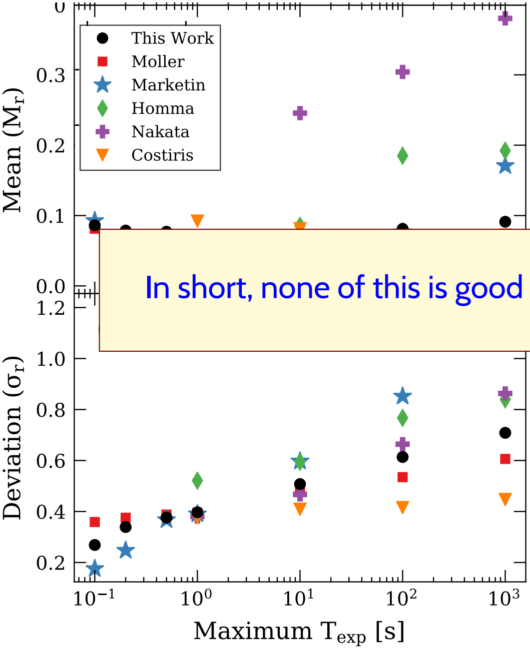
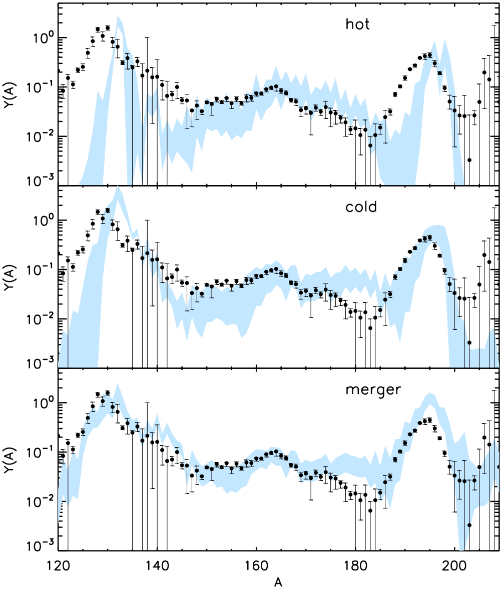


In short, none of this is good enough.

$$r = \log_{10}\left(\frac{t_{th}}{t_{exp}}\right)$$

# What's at Stake Here?

## Significance of Factor-of-Two Uncertainty

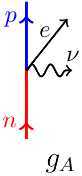


See Kelsey's talk, too!

# Improvement I: Two-Body Current

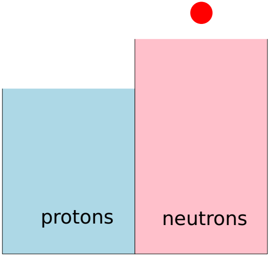
Evan Ney

Leading order:



Usual $\beta$-decay current

Consider very simple wave function



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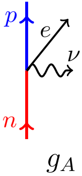

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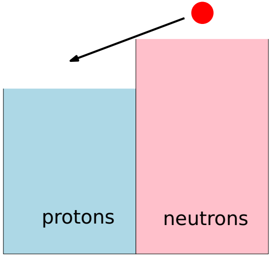

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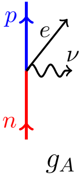

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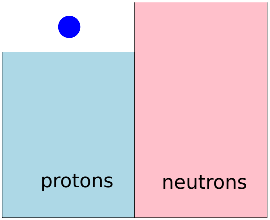

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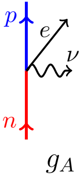

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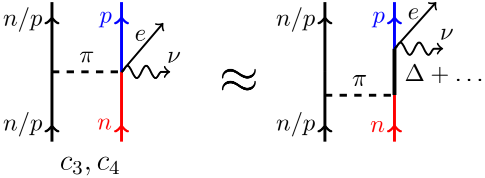
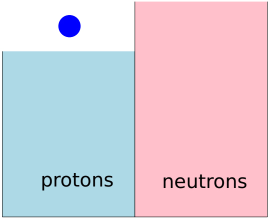
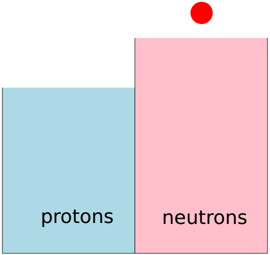


There are also contact terms…

Consider very simple wave function





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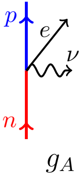

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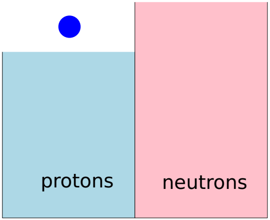

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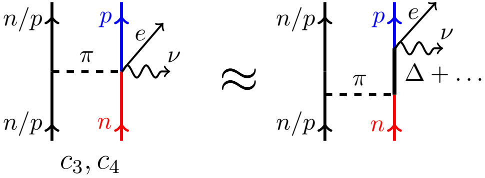

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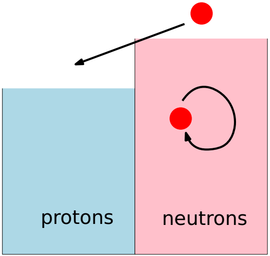

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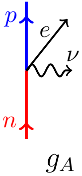

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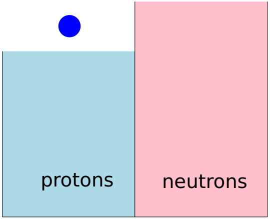

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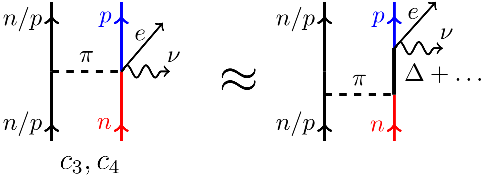

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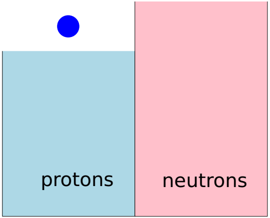
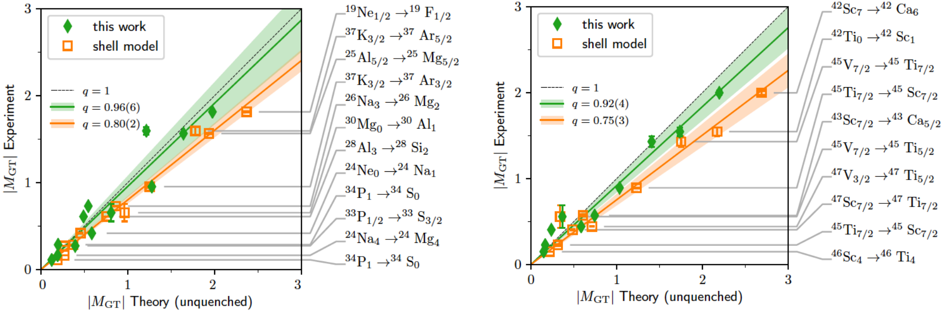


# Quenching in the *sd* and *pf* Shells



IMSRG calculation, Gysbers et al

Some quenching from correlations omitted by the shell model.

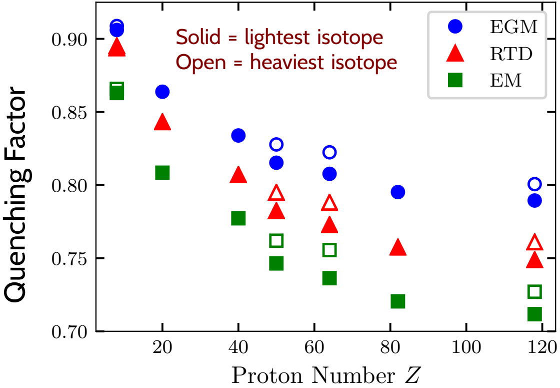
But a lot comes from the two-body current.

In these $A < 50$ nuclei, $\beta$-decay quenching doesn't much depend on $Z$ and $N$. But what about in heavier nuclei?

# *Z*- and *N*-Dependence of Quenching from Currents

## Integrated GT Strength

Three sets of chiral parameters, no contact



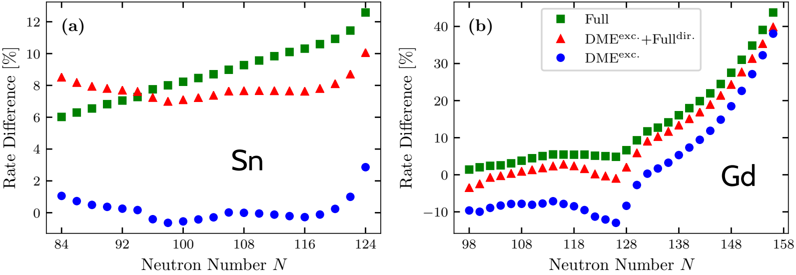
EGM - E. Epelbaum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. A 747, 362 (2005).
RTS - M. C. M. Rentmeester, R. G. E. Timmermans, and J. J. de Swart, Phys. Rev. C 67, 044001 (2003)
EM - D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001(R) (2003).

$$\boxed{g_A = 1 \quad \longrightarrow \quad q = .79}$$

# Effect on $\beta$-Decay Rates

Difference from rate with one-body operator, with $g_A = 1.0$

Focus on green squares

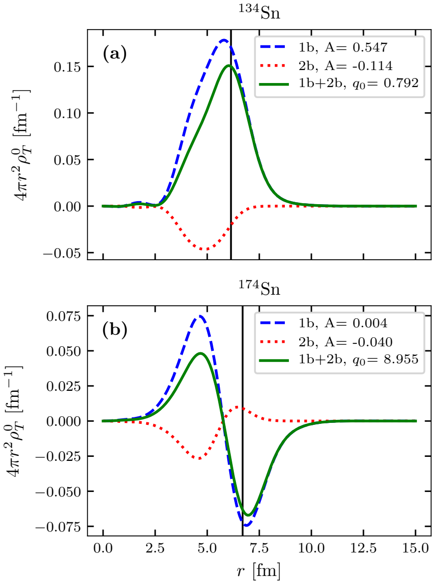


Two-body current quenches rates less in neutron-rich nuclei, and can even increase them near the drip line.

Why?

# Enhancement of Low-Lying Strength

## Can occur in neutron-rich isotopes



Two body currents do the most when density is large. Have little effect beyond the nuclear surface.

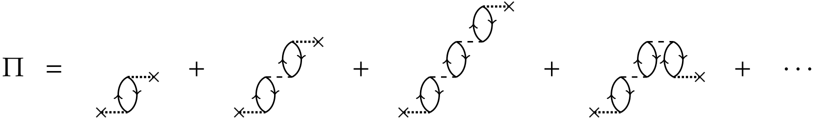
← Typical transition in $^{134}$Sn

← Unusual (and lowest-lying) transition in $^{174}$Sn

# Improvement II: Beyond QRPA

## Quasiparticle-Vibration Coupling

RPA response function



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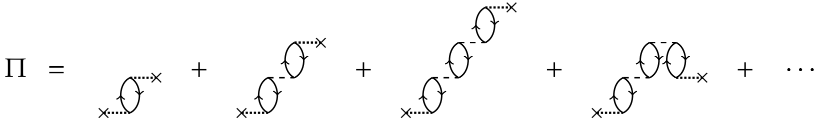
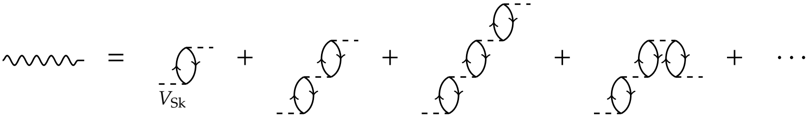


Phonon-exchange potential



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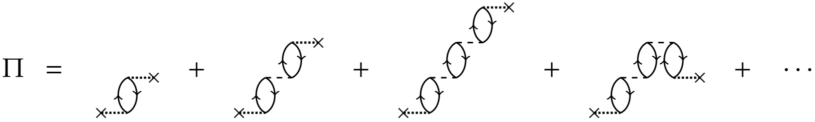

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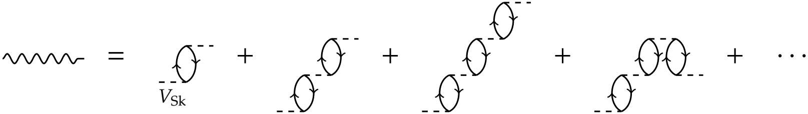
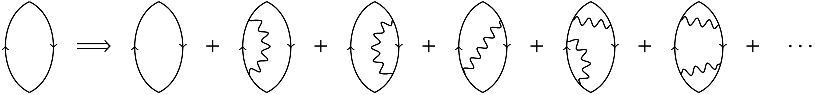


Modification of particle-hole bubble by QVC



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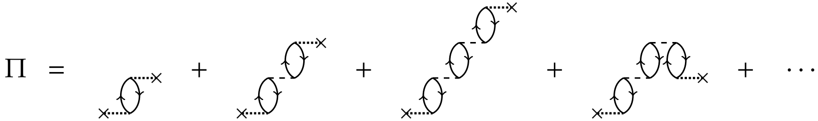
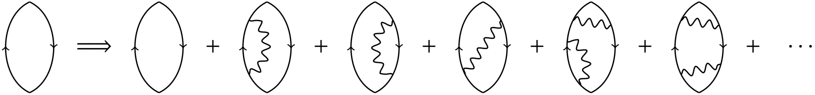


Has been applied in spherical nuclei, but never deformed ones. We figured out how to build it into the FAM.

Modification of particle-hole bubble by QVC

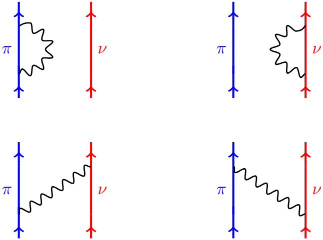


# QVC Results

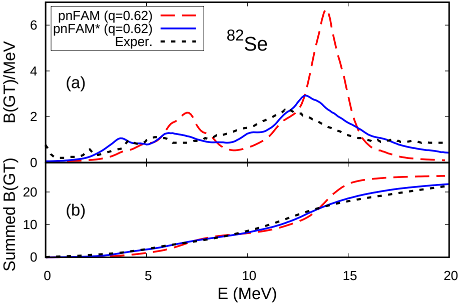
## Called pnFAM* Here

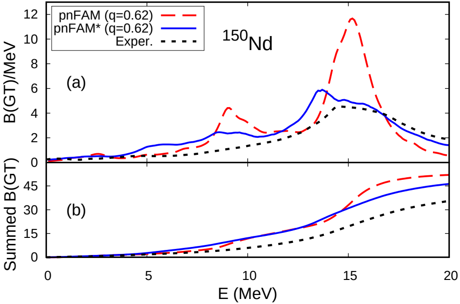
GT Distributions $(g_A = 1)$

Phonon-exchange diagrams: phonons are like-particle excitations



With Q. Liu





# $\beta$ Decay, No “Isoscalar Pairing”

| Isotope | $\beta$ | $t_{1/2}^{\text{Exp.}}(s)$ | $t_{1/2}^{\text{pnFAM}}(s)$ | $t_{1/2}^{\text{pnFAM*}}(s)$ |
|---|---|---|---|---|
| $^{78}$Zn | 0.12 | 1.47 | 408 | 3.77 |
| $^{168}$Gd | 0.31 | 3.03 | 381 | 37.1 |
| $^{152}$Ce | 0.29 | 1.40 | 93.1 | 19.0 |
| $^{156}$Nd | 0.32 | 5.49 | 470 | 53.5 |
| $^{164}$Sm | 0.33 | 1.42 | 142 | 17.2 |
| $^{154}$Ce | 0.30 | 0.30 | 19.2 | 7.26 |
| $^{112}$Mo | -0.18 | 0.15 | 1.92 | 2.47 |
| $^{94}$Kr | -0.22 | 0.21 | 1.48 | 3.23 |
| $^{112}$Ru | -0.21 | 1.75 | 93 | 27.0 |
| $^{106}$Mo | -0.20 | 8.73 | 62.8 | 38.0 |
| $^{100}$Zr | -0.19 | 7.1 | 124 | 41.9 |
| $^{96}$Sr | -0.21 | 1.07 | 23.8 | 20.0 |

# Next

All these developments will require us to refit pieces of the Skyrme functional and constants in the current, and to do UQ.

They also increase computation time a lot, and we will need more efficient calculations.

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# Next

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*Thanks for Listening.*