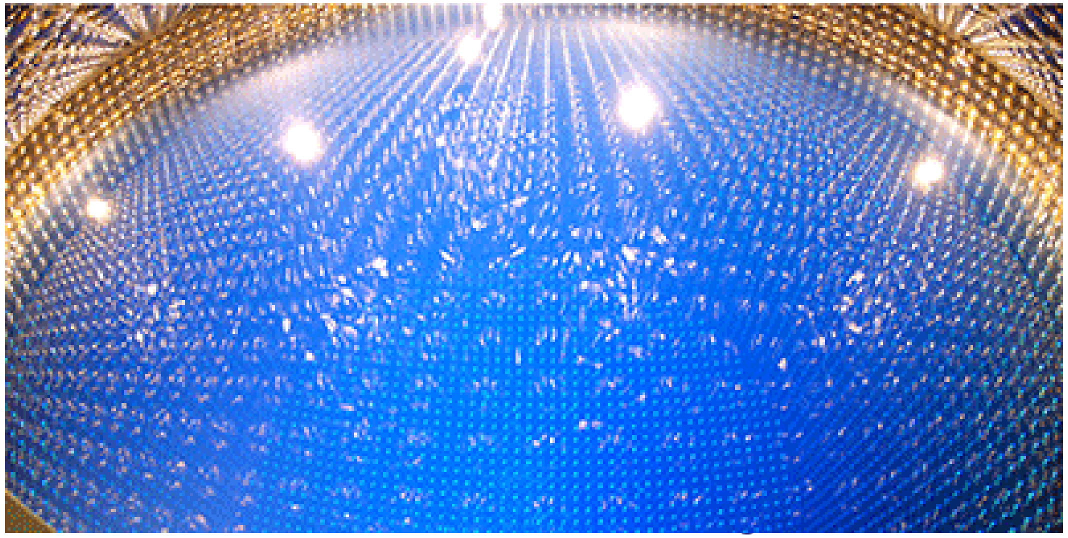
Scalar leptoquarks in loop-induced proton decays

Svjetlana Fajfer
Physics Department, University of Ljubljana and
Institute J. Stefan, Ljubljana, Slovenia



in collaboration with Ilja Doršner, Olcyr Sumensari, Mitja Šadl

Baryon Number Violation: From Nuclear Matrix Elements to BSM Physics
January 13-17, 2025

Motivation

theories: GUTs,…
experiments

Effective lagrangian describing baryon number violation

Lattice QCD for the hadronic part

Scalar Leptoquarks

Triple leptoquark interactions

Proton decays at tree-level

Proton decays at loop-level

Radiative $\Delta B = 1$ nucleon decays

Motivation

Proton Car is the national **car** brand of Malaysia. The brand was established in the early 1980's at the behest of the Malaysian government, ...

The word proton is **Greek for "first"**, and this name was given to the hydrogen nucleus by Ernest Rutherford in 1920.

History – Baryon Number Violation

lomb force law (the force between two charged particles falls off as $1/R^2$, where R is the distance between the particles) is a direct manifestation of the massless-

Proton stability was first formulated as a conservation law in 1929 by Weyl, who said (*1*), "It is plausible to anticipate that, of the two pairs of components of

M. Goldhaber is AUI Distinguished Scientist at Brookhaven National Laboratory, Associated Universities, Incorporated, Upton, New York 11973. P. Langacker is assistant professor in the Department of Physics, University of Pennsylvania, Philadelphia 19104. R. Slansky is a member of the staff in the The-

ter in the universe.

mber is conserved in the proton, which is state with nonzero nnot decay into any

masslessness of the photon is a ce result of quantum electrodynamics more generally, this connection is d a "local" symmetry (or gauge in

Is the Proton Stable?

M. Goldhaber, P. Langacker, R. Slansky

All existing experimental evidence is consistent with the absolute stability of the proton. The hypothesis of proton stability—that is, that a proton can never

way that requires electric charge conservation; violation would require a drastic alteration of the theory, including a violation of Coulomb's law, which is

- Weyl first proposed in 1929 that both electron and proton are in Dirac equation (!?)
- Number of protons and number of electrons are constant (electric charge conservation);
- Positron was discovered by Carl D. Anderson (1932);
- Stükelberg suggestion: new conservation low in addition to the electric charge conservation
  (Schwere Ladung - heavy charge- today **baryon number**)

Yamaguchi in 1959 suggested ”superweak” interaction leadng to $p \rightarrow e^+ e^+ e^-$
First experiment in 1960 (Backenstoss et al. using Cherenkov counters)

After 1965 Sakharov returned to fundamental science and began working on particle physics and particle cosmology.

He tried to explain the baryon asymmetry of the universe; in that regard, he was the first to give a theoretical motivation for proton decay.

According to the SM the proton, a type of baryon, is stable because baryon number (quark number) is conserved.

- Wigner: “It is conceivable, for instance, that a conservation law for the number of heavy particles (protons and neutrons) is responsible for the stability of the protons in the same way as the conservation law for charges is responsible for the stability of the electron. Without the conservation law in question, the proton could disintegrate, under emission of a light quantum, into a positron, just as the electron could disintegrate, were it not for the conservation law for the electric charge, into a light quantum and a neutrino."

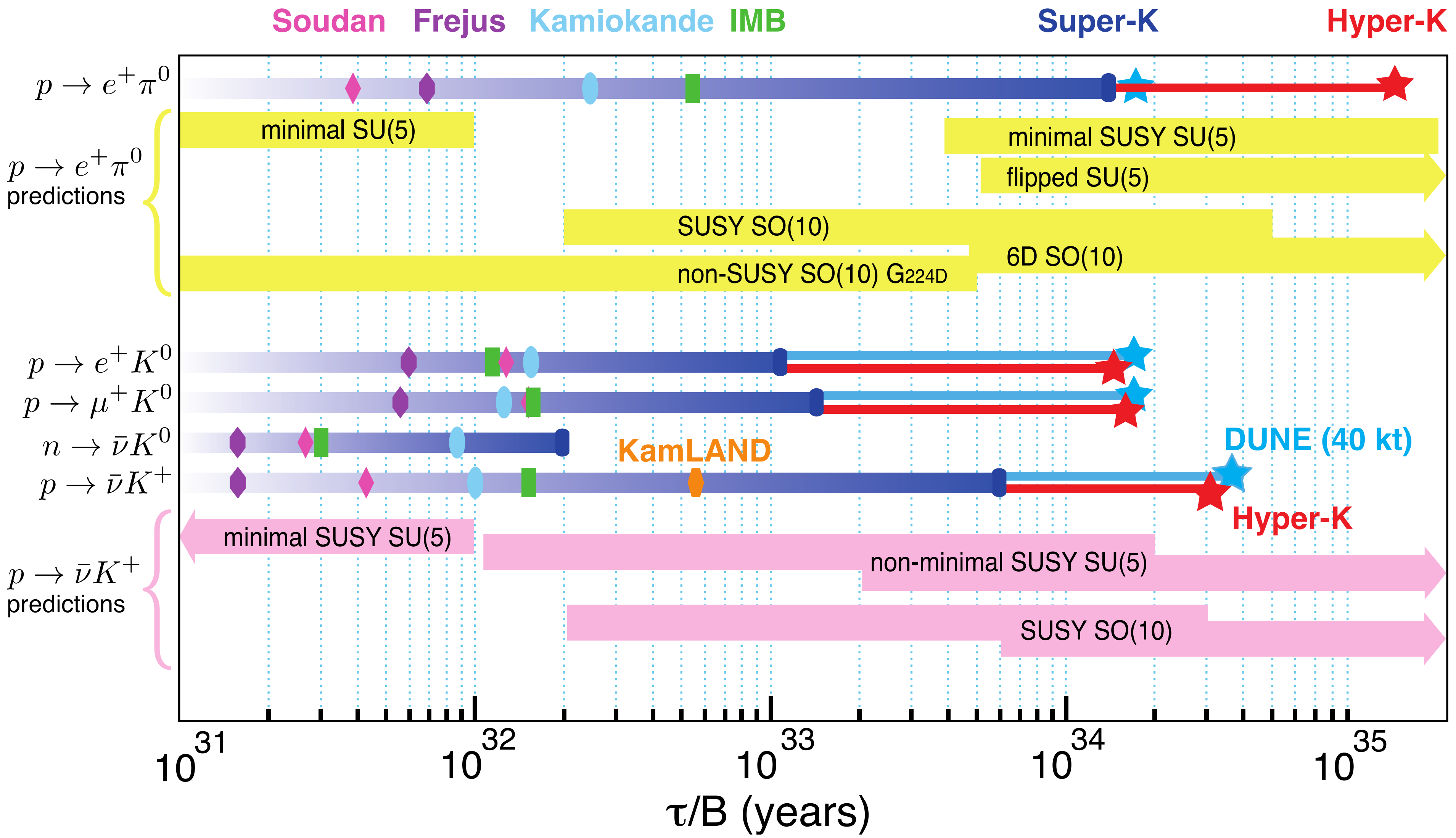
Wigner suggested proton decay in1949 $p \to \gamma e^+$

The first measurement gave bound $\tau_p > 10^{20}$ Years.

Reines *et al.*, 1954 (*5*)	10^{21}	All (unbound proton)	(Rock)
	10^{22}	All (bound proton) (charged particle of energy > 100 MeV)	30
Reines *et al.*, 1957 (*13*)	4×10^{23}	All*	61
Backenstoss *et al.*, 1960 (*14*)	2.8×10^{26}	One relativistic e, μ, or π or secondary γ	800
Giamati and Reines, 1962 (*15*)	1×10^{26}	All*	585
Kropp and Reines, 1964 (*16*)	0.6×10^{28} to 4×10^{28}	Mode-dependent	585

- 1974: Grand unified theories, Georgi & Glashow SU(5)

Future experiments



Proton decays in effective Lagrangian approach

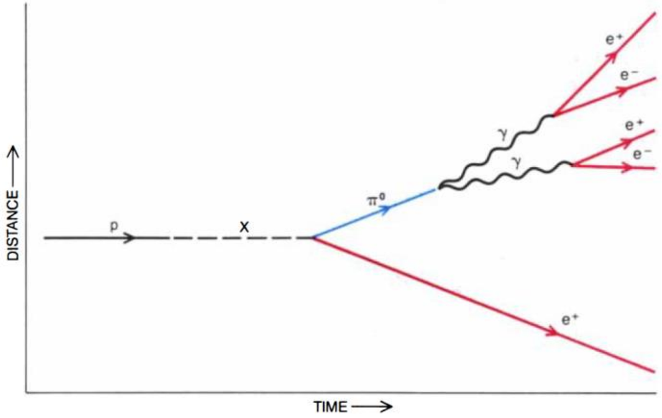
dimension 6

$$
\begin{aligned}
\mathcal{L}_{d=6} = & \frac{C_1}{\Lambda^2}\epsilon^{\alpha\beta\gamma}\epsilon_{il}\epsilon_{jk}(\overline{Q}^C_{i,\alpha}Q_{j,\beta})(\overline{Q}^C_{k,\gamma}L_l) \\
& + \frac{C_2}{\Lambda^2}\epsilon^{\alpha\beta\gamma}(\overline{Q}^C_{i,\alpha}\epsilon_{ij}Q_{j,\beta})(\overline{u}^C_\gamma \ell) \\
& + \frac{C_3}{\Lambda^2}\epsilon^{\alpha\beta\gamma}(\overline{d}^C_\alpha u_\beta)(\overline{Q}^C_{i,\gamma}\epsilon_{ij}L_j) \\
& + \frac{C_4}{\Lambda^2}\epsilon^{\alpha\beta\gamma}(\overline{d}^C_\alpha u_\beta)(\overline{u}^C_\gamma \ell) + \text{h.c.}\,,
\end{aligned}
$$

Q, L $\rightarrow$ $SU(2)_L$ quark, lepton doublets
u ,d, l $\rightarrow$ $SU(2)_L$ u, d, charged lepton singlets
C $\rightarrow$ charge conjugation

α, β, γ denote the colour, i, j, k, l the $SU(2)_L$ indices

See Aoki's talk



$$\left\langle P \left| \mathcal{O}^{\Gamma\Gamma'} \right| N \right\rangle = \left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_N} W_1^{\Gamma\Gamma'}(q^2) \right] P_{\Gamma'} u_N$$

$$\mathcal{O}^{\Gamma\Gamma'} = \left(\overline{q}^C P_\Gamma q\right) P_{\Gamma'} q \qquad \text{and} \qquad \Gamma, \Gamma' = R, L.$$

Lattice QCD
Aoki et al,1705.01338,
hep-lat/9911026,
hep-lat/0607002

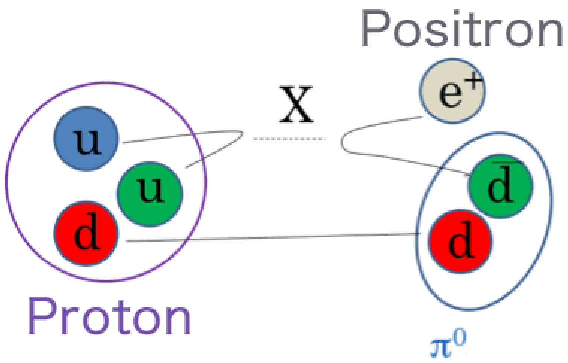
$$\Gamma(p \to \ell^+ \pi^0) = \frac{1}{32\pi}\left|\frac{C_1^p}{\Lambda^2}\right|^2 \left(W_0^{LL}(0)\right)^2 \left(m_p^2 - m_\pi^2 + m_\ell^2\right) \frac{\lambda^{1/2}(m_p^2, m_\ell^2, m_\pi^2)}{m_p^3}$$

Goal

The main goal of this work is to determine Wilson coefficents ot the dimension-6 operators, or in some cases dimension-9 operators using a model which generates

$$\Delta B = \pm 1$$

Leptoquarks are a natural possibility for this transition as well as $\Delta L = 2\,, \Delta B = 2$



Scalar and Vector Leptoquarks

- Grand Unified Theories (GUTs), Jogesh Pati and Abdus Salam in 1974

- In 1997, the H1 and ZEUS collaborations at HERA an excess of events, production of leptoquarks (electron-proton system, H1 mass 200 GeV).

- In 21st century
  - a) LHC Searches
  - b) Flavour Anomalies
  - c) Neutrino Mass Models

$(SU(3), SU(2), U(1))$	Spin	Symbol	Type	F
$(\overline{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	$LL\,(S_1^L)$	-2
$(\mathbf{3}, \mathbf{2}, 7/6)$	0	R_2	$RL\,(S_{1/2}^L),\ LR\,(S_{1/2}^R)$	0
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$\tilde{R}_2$	$RL\,(\tilde{S}_{1/2}^L),\ \overline{LR}\,(\tilde{S}_{1/2}^L)$	0
$(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$	0	$\tilde{S}_1$	$RR\,(\tilde{S}_0^R)$	-2
$(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$	0	S_1	$LL\,(S_0^L),\ RR\,(S_0^R),\ \overline{RR}\,(S_0^{\overline{R}})$	-2
$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$	0	$\bar{S}_1$	$\overline{RR}\,(\bar{S}_0^R)$	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	$LL\,(V_1^L)$	0
$(\overline{\mathbf{3}}, \mathbf{2}, 5/6)$	1	V_2	$RL\,(V_{1/2}^L),\ LR\,(V_{1/2}^R)$	-2
$(\overline{\mathbf{3}}, \mathbf{2}, -1/6)$	1	$\tilde{V}_2$	$RL\,(\tilde{V}_{1/2}^L),\ \overline{LR}\,(\tilde{V}_{1/2}^R)$	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$	1	$\tilde{U}_1$	$RR\,(\tilde{V}_0^R)$	0
$(\mathbf{3}, \mathbf{1}, 2/3)$	1	U_1	$LL\,(V_0^L),\ RR\,(V_0^R),\ \overline{RR}\,(V_0^{\overline{R}})$	0
$(\mathbf{3}, \mathbf{1}, -1/3)$	1	$\bar{U}_1$	$\overline{RR}\,(\bar{V}_0^R)$	0

Scalar leptoquarks → Yukawa-like couplings $\bar{\ell}\, P_{L,R}\, q\, \Phi$

$Q = I_3 + Y$

Vector leptoquarks → gauge bosons (in GUTs their masses at GUT scale) $\bar{\ell}\gamma_\mu\, P_{L,R}\, q\, V^\mu$

Dorsner, SF, Greljo, Kamenik, Košnik, 1603.04993 result from an interaction, which is mediated by a newly postulated vector boson that carries both color and flavor by a process that is analogous to muon capture. (No boson in the standard theory carries both color and flavor.) If the proton decays, at least one quark must be transformed into a lepton, since all lower-mass spin 1/2 systems contain at least one lepton. Such a vector boson is called a leptoquark. For example, a leptoquark with electric charge −1/3 (or −4/3), which also carries color, may couple to a current that transforms an up quark (or down quark) into a positron or positive muon. To complete the process

Vector LQ

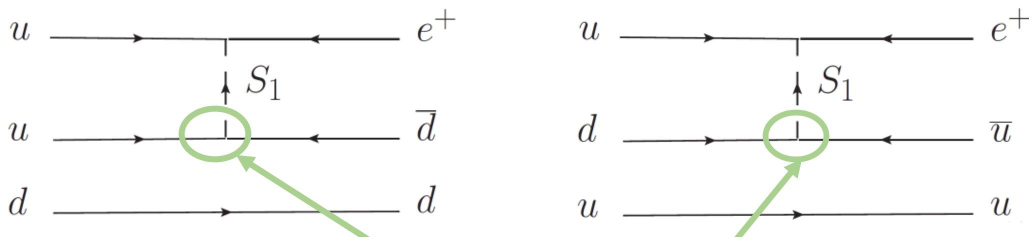
X,Y gauge bosons within GUT

$M_{X,Y} \sim M_{GUT}$

leptoquark couplings	diquark couplings	$SU(3)\times SU(2)\times U(1)$ representation of X
$X\bar{Q}e$, $XL\bar{u}$	–	$(3,2,7/6)$
$XL\bar{d}$	–	$(3,2,1/6)$
$X\bar{Q}\bar{L}$, $X\bar{u}\bar{e}$	XQQ, Xud	$(3,1,-1/3)_{\rm PD}$
$X\bar{Q}\bar{L}$	XQQ	$(3,3,-1/3)_{\rm PD}$
$X\bar{d}\bar{e}$	Xuu	$(3,1,-4/3)_{\rm PD}$

Scalar LQ

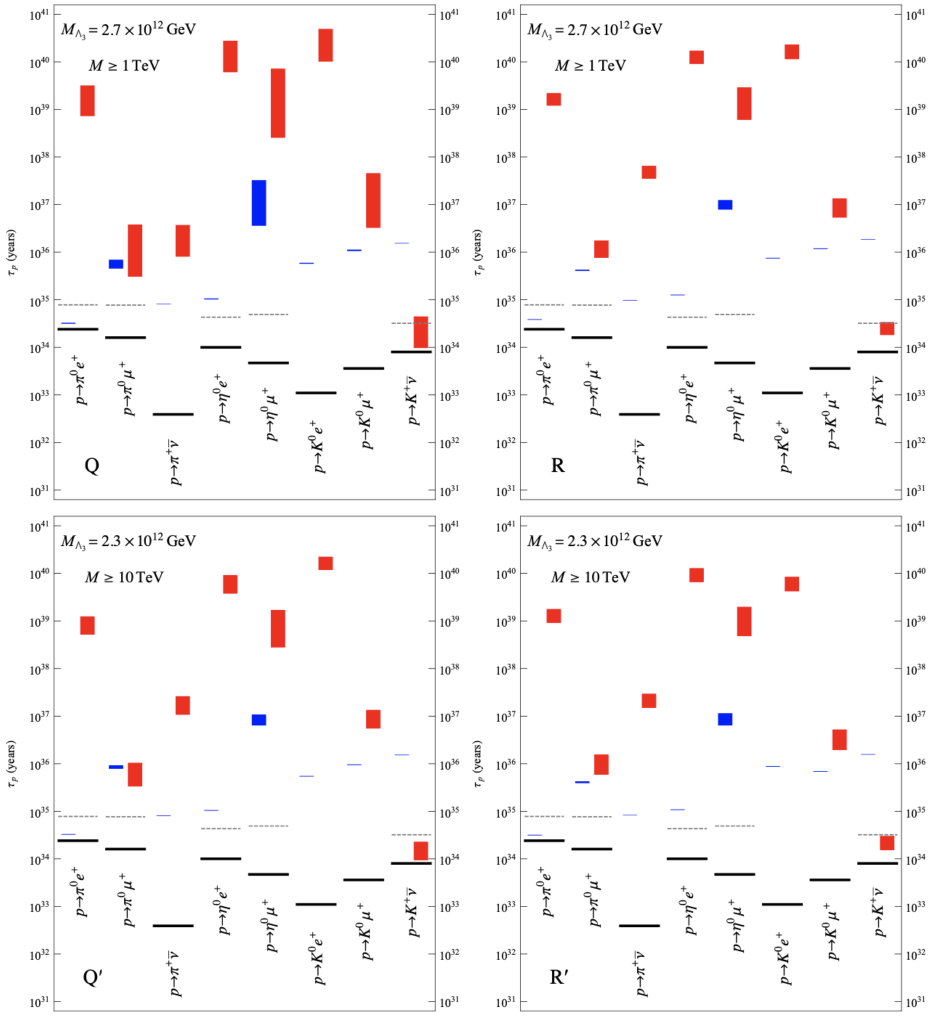
dim-6



Important: scalar LQ should have di-quark couplings that proton decays at the tree level (dim-6, dim-9,…)

e.g. Doršner, SF & Košnik, 1204.0674

If proton decay is seen, can that be a result of a gauge boson, or a scalar leptoquark?



Proton decay signatures via gauge boson and scalar leptoquark mediations within M ≥ 1 TeV and M ≥ 10 TeV scenarios. Black lines are current experimental limits, blue vertical bars are predictions for gauge boson mediation signatures, red vertical bars are predictions for the scalar leptoquark mediations, and gray dashed lines represent future experimental sensitivities after a ten-year period of data taking at 90 % C.L..

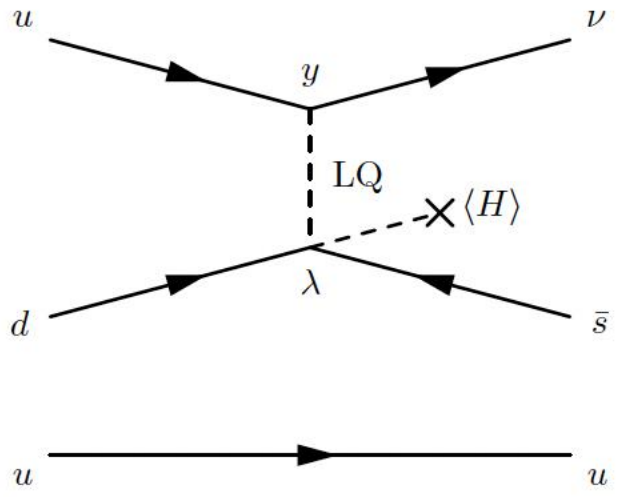
Model: SU(5) GUT, representations of dimensions 5, 10, 15, 24, and 35.

I. Doršner and S. Saad, 1910.09008; 2100.0678

Doršner, Džaferovi-Mašić, SF, Saad, 2401.16907

proton decay p → $K^+\nu$ suggests an exchange
of a scalar leptoquark, raising a potential to detect the p → $\pi^0\mu^+$ decay

Di-quark coupling dimension-5 operator



$$O^{(1)}_{ll} = \frac{1}{\Lambda} g^{ab} u^a_{R\alpha} d^b_{R\beta} (H^\dagger X_\gamma) \epsilon^{\alpha\beta\gamma}$$

$$O^{(2)}_{ll} = \frac{1}{\Lambda} g^{ab} u^a_{R\alpha} e^b_R (X_\beta \cdot X_\gamma) \epsilon^{\alpha\beta\gamma}$$

LQ: R_2, $\tilde{R}_2$

Arnold, Fornal & Wise, 1304.6119.

$n \to e^- \pi^+$

$p \to \pi^+ \nu$

Imposing a Z_3 discrete symmetry, with elements that are powers of exp[2πi(B − L)/3], one forbids these dimension five operators and, thus, prevents the proton from decaying in this class of models!

$n \to e^- K^+$

$p \to K^+ \nu$

$$\Gamma_p \simeq 2 \times 10^{-57} \left(\frac{50\ \text{TeV}}{m_V}\right)^4 \left(\frac{M_{PL}}{\Lambda}\right)^2 \text{GeV}$$

m_V <10000 TeV

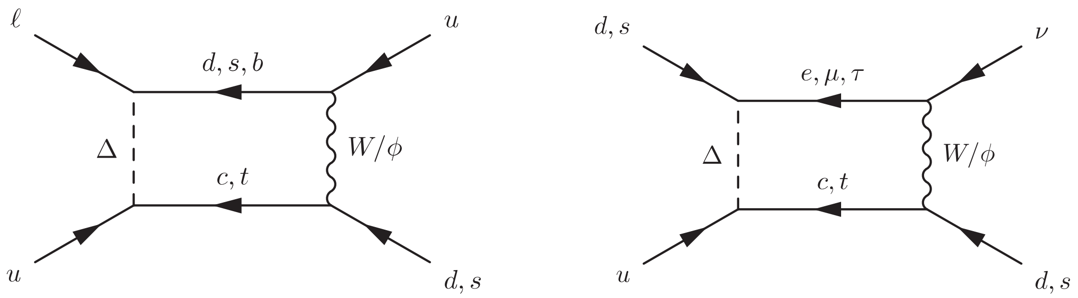
Dimension-6 opertors from the loops

Box-mediated operator

$(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \in \mathbf{45}$

$$\mathcal{L} \supset +\tilde{y}^{RR}_{1\,ij}\bar{d}^{C\,i}_{R}\tilde{S}_1 e^j_R + \tilde{z}^{RR}_{1\,ij}\bar{u}^{C\,i}_{R}\tilde{S}^*_1 u^j_R + \text{h.c.}$$

fully antisymmetric,
only different quark flavors
can couple.



Doršner, SF & Košnik, 1204.0674

$$O_H(d_\alpha, e_\beta) = a(d_\alpha, e_\beta) u^T L C^{-1} d_\alpha u^T L C^{-1} e_\beta$$

$$a(d_\alpha, e^C_\beta) = -\frac{G_F}{4\pi^2 m^2_W}\sum_{j,k}[U^\dagger_C(Y^{10*} - Y^{10\dagger})U^*_C]_{1j} \times [D^\dagger_C Y^{\bar{5}\dagger} E^*_C]_{k\beta} m_{u_j} V_{j\alpha} m_{d_k} V^*_{uk} J(x_\Delta, x_{u_j}, x_{d_k}),$$

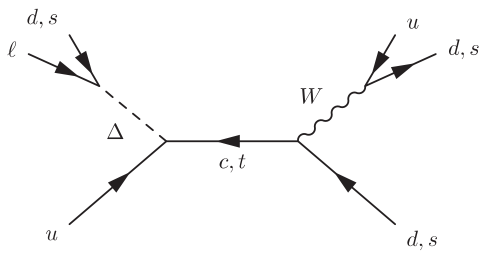
$$a(d_\alpha, d^C_\beta, \nu_i) = -\frac{G_F}{4\pi^2 m^2_W}\sum_{j}[U^\dagger_C(Y^{10*} - Y^{10\dagger})U^*_C]_{1j} \times [D^\dagger_C Y^{\bar{5}\dagger} E^*_C]_{\beta i} m_{u_j} V_{j\alpha} m_{\ell_i} J(x_\Delta, x_{u_j}, x_{\ell_i})$$

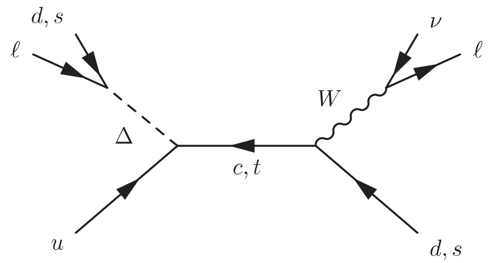
$x_{\ell_i}, x_{\ell_i} \ll 1,$

$$J(x, y, z) = \frac{(y-4)y\log y}{(y-1)(y-x)(y-z)} + \frac{(z-4)z\log z}{(z-1)(z-y)(z-x)} + \frac{(x-4)x\log x}{(x-1)(x-y)(x-z)}. \qquad (71)$$

$$J(x_\Delta, x_t, x_{\ell_i}) = \frac{1}{x_\Delta - x_t}\left[\frac{x_\Delta - 4}{x_\Delta - 1}\log x_\Delta - \frac{x_t - 4}{x_t - 1}\log x_t\right]$$

Tree-level dimension-nine operator from $(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{4/3}) \in \mathbf{45}$





$$\mathcal{L}_9 = \sum_{U=c,t} \frac{-8G_F V_{U\alpha} V^*_{u\gamma}}{m_U m^2_\Delta} [U_C (Y^{10} - Y^{10T})^\dagger U^*_C]_{1U} \times [D^\dagger_C Y^{\bar{5}\dagger} E^*_C]_{\beta i} \epsilon_{abc} (\overline{u^C_a} \gamma^\mu L d_{b\alpha})(\overline{d^C_{c\beta}} R \ell_i) \times (\overline{d_{k\gamma}} \gamma_\mu L u_k).$$

$$\mathcal{M}_9^{p\to\pi^0 e^+_i} = \sum_{U=c,t} \frac{8iG_F}{m^2_\Delta m_U} [U_C (Y^{10} - Y^{10T})^\dagger U^*_C]_{1U} \times [D^\dagger_C Y^{\bar{5}\dagger} E^*_C]_{1i} V_{Ud} V^*_{ud} \epsilon_{abc} \langle \pi^0 \ell^+_i | (\overline{\ell^C_i} R u_a) \times (\overline{d_k} R d_c)(\overline{u^C_k} L d_b) | p \rangle + \text{tensor terms.}$$

Doršner, SF & Košnik, 1204.0674

Proton decay to charged leptons

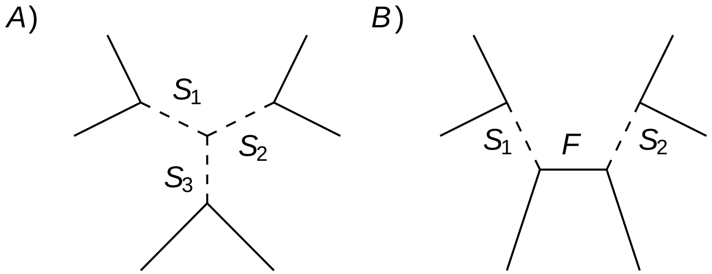
Hambye & Heeck 1712.04871

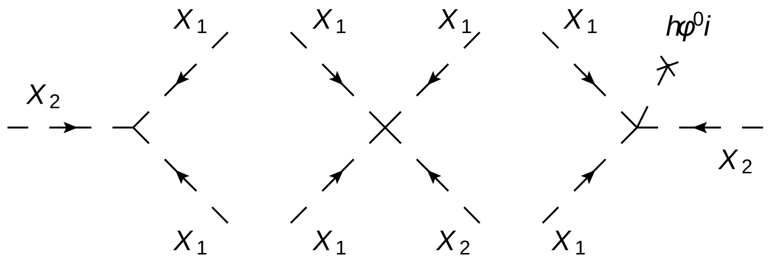
channel	$(\Delta L_e, \Delta L_\mu)$	limit/years
$p \to e^+ e^+ e^-$	$(1,0)$	793×10^{30}
$p \to e^+ \mu^+ \mu^-$	$(1,0)$	359×10^{30}
$p \to \mu^+ e^+ e^-$	$(0,1)$	529×10^{30}
$p \to \mu^+ \mu^+ \mu^-$	$(0,1)$	675×10^{30}
$p \to \mu^+ \mu^+ e^-$	$(-1,2)$	359×10^{30}
$p \to e^+ e^+ \mu^-$	$(2,-1)$	529×10^{30}

$$\Gamma(p \to \ell_\alpha^+ \ell_\beta^+ \ell_\gamma^-) \simeq \frac{|\langle H \rangle|^2 \beta_h^2 m_p^5}{6144\, \pi^3 \Lambda^{12}} \simeq \frac{(100\,\text{TeV}/\Lambda)^{12}}{10^{33}\,\text{yrs}}$$

Dimension-nine operators

$$\begin{aligned}
\mathcal{O}_1^9 &= (QQ)_1(\bar{L}\bar{L})_1(\ell d)\,, & \mathcal{O}_2^9 &= (QQ)_1(\bar{L}\ell)(\bar{L}d)\,,\\
\mathcal{O}_3^9 &= (QL)_1(\bar{L}d)(\bar{L}d)\,, & \mathcal{O}_4^9 &= (\bar{\ell}Q)(\bar{L}d)(\ell d)\,,\\
\mathcal{O}_5^9 &= (\bar{L}\bar{L})(ud)(\ell d)\,, & \mathcal{O}_6^9 &= (\bar{L}u)(\bar{L}d)(\ell d)\,,\\
\mathcal{O}_7^9 &= (\bar{L}d)(\bar{L}\ell)(ud)\,, & \mathcal{O}_8^9 &= (\bar{L}d)(\bar{L}d)(\ell u)\,,\\
\mathcal{O}_9^9 &= (QL)_3((\bar{L}d)(\bar{L}d))_3\,, & \mathcal{O}_{10}^9 &= (QL)_1(\bar{L}\bar{L})_1(dd)\,,\\
\mathcal{O}_{11}^9 &= (QL)_3(\bar{L}\bar{L})_3(dd)\,, & \mathcal{O}_{12}^9 &= (\bar{\ell}Q)(\bar{L}\ell)(dd)\,,\\
\mathcal{O}_{13}^9 &= (\bar{L}\bar{L})(u\ell)(dd)\,, & \mathcal{O}_{14}^9 &= (\bar{L}u)(\bar{L}\ell)(dd)\,,\\
\mathcal{O}_{15}^9 &= (\bar{\ell}L)(\bar{L}d)(dd)\,, & \mathcal{O}_{16}^9 &= (\bar{\ell}\bar{\ell})(\ell d)(dd)\,.
\end{aligned}$$

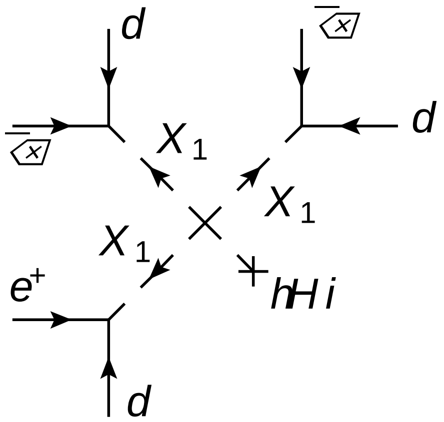




Many authors discussed

C. Faroughy et al., 1409.5438, Kovalenko & Schmidt,hep-ph/0210187
Klapdor-Kleingrothaus et al., hep-ph/0210156,
Hambye & Heeck, 1712.04871, J. Heeck & Takhistov, 1910.07647
Foncesca & et al., 1802.04814, Murgui &Wise, 2105.14029

From Arnold et al., 1212.4556

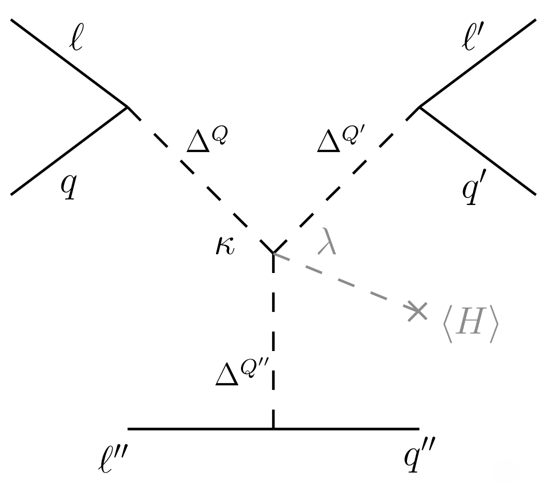


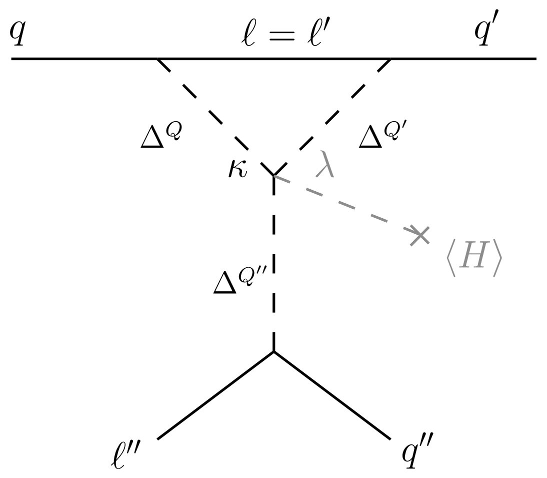
$$X_1 \to \tilde{R}_2$$

Murgui &Wise, 2105.14029 found that if LQ X_1
Is in the same representation that this coupling vanishes.

Interaction which leads to proton decay, $p \to \pi^+\pi^+e^-\nu\nu$,
For $X_1 \in (\bar{3},2,-1/6)$.

Triple-leptoquark interactions for tree- and loop-level proton decays





I. Doršner, SF & O. Sumensari, 2202.08287

Triple-LQs - scalars only!

- the assumption scalar leptoquarks of interest couple solely to the quark-lepton pairs

- Two different proton decay topologies

- with or without a Higgs vacuum expectation value

-Δ_Q, $\Delta_{Q'}$, and $\Delta_{Q''}$ are scalar leptoquark mass eigenstates with electric charges Q, Q', and Q'', respectively.

Classification

scalars

Leptoquark multiplets	Yukawa interactions
$R_2 = (\mathbf{3}, \mathbf{2}, 7/6)$	$-(y^L_{R_2})_{ij}\, \bar{u}_{R\,i} R_2 i\tau_2 L_j + (y^R_{R_2})_{ij}\, \bar{Q}_i R_2 e_{R\,j} + \text{h.c.}$
$\tilde{R}_2 = (\mathbf{3}, \mathbf{2}, 1/6)$	$-(y^L_{\tilde{R}_2})_{ij}\, \bar{d}_{R\,i} \tilde{R}_2 i\tau_2 L_j + \text{h.c.}$
$S_1 = (\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	$(y^L_{S_1})_{ij}\, \bar{Q}^C_i i\tau_2 S_1 L_j + (y^R_{S_1})_{ij}\, \bar{u}^C_{R\,i} S_1 e_{R\,j} + \text{h.c.}$
$S_3 = (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	$(y^L_{S_3})_{ij}\, \bar{Q}^C_i i\tau_2 (\vec{\tau} \cdot \vec{S}_3) L_j + \text{h.c.}$
$\tilde{S}_1 = (\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	$(y^R_{\tilde{S}_1})_{ij}\, \bar{d}^C_{R\,i} \tilde{S}_1 e_{R\,j} + \text{h.c.}$

Scalar leptoquark multiplets and their interactions with the SM quark-lepton pairs.

The SM extended with up to three different scalar leptoquark multiplets, denoted with Δ, Δ′, and Δ′′ and study all possible cubic and quartic contractions Δ-Δ′-Δ′′ and Δ-Δ′-Δ′′-H, yield to 3-LQ interactions and 3-LQ ⟨H⟩.

	$SU(3)\times SU(2)\times U(1)$ level	$SU(3)\times U(1)_{\rm em}$ level
(a)	$\kappa\tilde{R}_2^T i\tau_2\tilde{R}_2 S_1^*$	$-2\kappa\epsilon_{abc}\tilde{R}_{2a}^{-1/3}\,\tilde{R}_{2b}^{2/3}\,S_{1c}^{-1/3}$
(b)	$\kappa R_2^T i\tau_2\tilde{R}_2\tilde{S}_1^*$	$\kappa\epsilon_{abc}\left(R_{2a}^{5/3}\,\tilde{R}_{2b}^{-1/3}\,\tilde{S}_{1c}^{-4/3}-R_{2a}^{2/3}\,\tilde{R}_{2b}^{2/3}\,\tilde{S}_{1c}^{-4/3}\right)$
(c)	$\lambda H^\dagger i\tau_2(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2 R_2 S_1^*$	$\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}\left(-S_{3a}^{-1/3}\,R_{2b}^{2/3}\,S_{1c}^{-1/3}+\sqrt{2}S_{3a}^{-4/3}\,R_{2b}^{5/3}\,S_{1c}^{-1/3}\right)$
(d)	$\lambda H^\dagger i\tau_2(\vec{\tau}\cdot\vec{S}_3)^*(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2 R_2$	$\lambda v\sqrt{2}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{-1/3}\,S_{3b}^{-4/3}\,R_{2c}^{5/3}-S_{3a}^{-4/3}\,S_{3b}^{2/3}\,R_{2c}^{2/3}\right)$
(e)	$\lambda H^T i\tau_2 R_2 S_1^*\tilde{S}_1^*$	$-\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}R_{2a}^{5/3}\,S_{1b}^{-1/3}\,\tilde{S}_{1c}^{-4/3}$
(f)	$\lambda H^T(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2 R_2\tilde{S}_1^*$	$\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{2/3}\,R_{2b}^{2/3}\,\tilde{S}_{1c}^{-4/3}+S_{3a}^{-1/3}\,R_{2b}^{5/3}\,\tilde{S}_{1c}^{-4/3}\right)$
(g)	$\lambda H^T(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2\tilde{R}_2 S_1^*$	$\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{2/3}\,\tilde{R}_{2b}^{-1/3}\,S_{1c}^{-1/3}+S_{3a}^{-1/3}\,\tilde{R}_{2b}^{2/3}\,S_{1c}^{-1/3}\right)$
(h)	$\lambda H^\dagger(\vec{\tau}\cdot\vec{S}_3)^*(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2\tilde{R}_2$	$\lambda v\sqrt{2}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{2/3}\,S_{3b}^{-1/3}\,\tilde{R}_{2c}^{-1/3}+S_{3a}^{-4/3}\,S_{3b}^{2/3}\,\tilde{R}_{2c}^{2/3}\right)$

Mentioned in
Kovalenko and Schmidt, hep-ph/0210187
Crivellin and Schnell, 2105.04844

$$S_3^{1/3}=S_3^3,\ S_3^{4/3}=(S_3^1-iS_3^2)/\sqrt{2},\ S_3^{-2/3}=(S_3^1+iS_3^2)/\sqrt{2}$$

Cubic and quartic leptoquark multiplet contractions at the SU(3)×SU(2)× U(1) level and the associated triple-leptoquark interactions at the SU(3) × U(1)$_{\rm em}$ level

$\tilde{R}_2$-$\tilde{R}_2$-$\tilde{R}_2$-H^*, S_1-S_1-R_2^*-H,
$\tilde{R}_2$-$\tilde{R}_2$-S_3^*, S_1-S_1-$\tilde{R}_2^*$-H^*

vanish

symmetric under the exchange of two identical electric charge eigenstates in direct conflict with the antisymmetric nature in the colour SU(3) space.

Way out: to accommodate them in different representations.

	Contractions	Operators	Proton decay (tree)	Proton decay (one-loop)
(a)	$\tilde{R}_2$-$\tilde{R}_2$-S_1^*	$ddd\bar{e}\nu\bar{\nu}$	$p \to \pi^+\pi^+e^-\nu\bar{\nu}$	–
		$ddue\bar{e}\bar{\nu}$	$p \to \pi^+e^+e^-\nu$	$p \to \pi^+\nu$
(b)	R_2-$\tilde{R}_2$-$\tilde{S}_1^*$	$ddde\bar{e}\bar{e}$	$p \to \pi^+\pi^+e^-e^+e^-$	–
		$ddue\bar{e}\bar{\nu}$	$p \to \pi^+e^+e^-\nu$	$p \to \pi^+\nu$
(c)	S_1-S_3-R_2^*-H	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$
		$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
		$uuuee\bar{\nu}$	$p \to \pi^-e^+e^+\nu$	–
(d)	S_3-S_3-R_2^*-H	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	–
		$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(e)	S_1-$\tilde{S}_1$-R_2^*-H^*	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(f)	S_3-$\tilde{S}_1$-R_2^*-H^*	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$
		$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(g)	S_1-S_3-$\tilde{R}_2^*$-H^*	$ddu\nu\bar{\nu}\nu$	$p \to \pi^+\nu\bar{\nu}\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$
		$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(h)	S_3-S_3-$\tilde{R}_2^*$-H^*	$ddu\nu\bar{\nu}\nu$	$p \to \pi^+\nu\bar{\nu}\bar{\nu}$	$p \to \pi^+\bar{\nu}$
		$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	–
		$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$

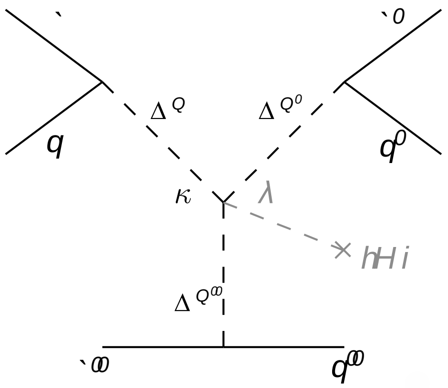
non-trivial Δ-Δ'-Δ'' and Δ-Δ'-Δ''-H contractions,

d = 9 effective operators, and corresponding proton decay

The effective operators in scenarios (a) and (b) conserve B + L, while the ones appearing in the remaining scenarios conserve B − L, where B and L are baryon and lepton numbers, respectively.

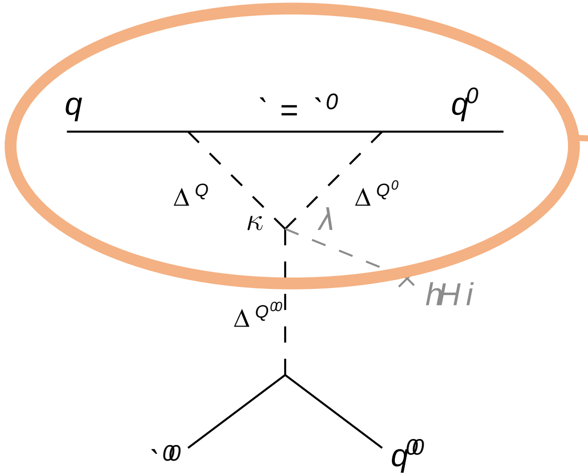
Phenomenological analysis

Tree level proton decays



$$\Gamma(p \to e^+ e^+ e^-) \simeq \frac{m_p}{(16\pi)^3} \left(\frac{m_p^5 v}{\Lambda^6} \right)^2 |\lambda\, y_{ue}^2\, y_{de}|^2$$

Loop-level proton decay



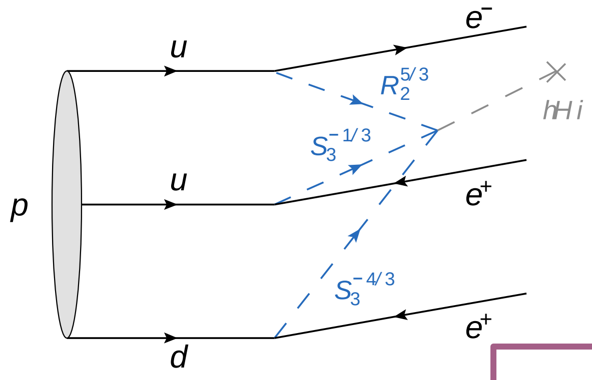
Effective di-quark coupling

$$y_{ud} \simeq \frac{1}{16\pi^2} \frac{m_f v}{\Lambda^2} \lambda\, y_{ue} y_{de}^*$$

$$\Gamma(p \to \pi^0 e^+) \simeq \frac{m_p}{16\pi} \left(\frac{m_p^2}{\Lambda^2} \right)^2 |y_{ud}\, y_{ue}|^2$$

Comparison tree and loop level proton decay width

an example



comparison of the existing data

$$\begin{cases} p \to e^- e^+ e^+ \\ p \to \pi^0 e^+ \end{cases}$$

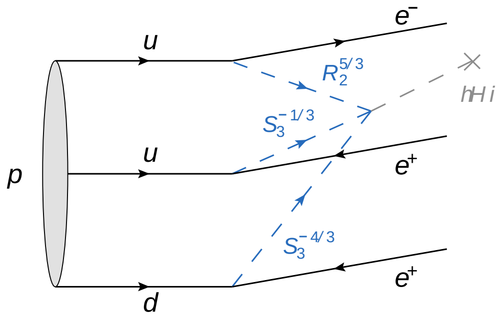
$$\frac{\Gamma(p \to e^+ e^+ e^-)}{\Gamma(p \to \pi^0 e^+)} \simeq \frac{1}{\pi^2}\left(\frac{m_p^3}{m_f\, \Lambda^2}\right)^2 \simeq 10^{-7}\left(\frac{m_e}{m_f}\right)^2\left(\frac{1\,\mathrm{TeV}}{\Lambda}\right)^4,$$

The loop-induced processes are more sensitive probes of the triple-leptoquark interactions than the tree-level ones!

$$\Gamma(p \to \pi^0 \ell^+) \sim \frac{1}{10}\Gamma(p \to \ell^+ \ell^- \ell^+)$$

Tree-level leptoquark mediation of $p \to e^- e^+ e^+$

$$\mathcal{L}_{\rm eff}^{(d=9)} \supset \sum_{X=L,R} \epsilon_{abc}\, C_X\, (\bar{u}_a^C P_L e)(\bar{d}_b^C P_L e)(\bar{e} P_X u_c) + \text{h.c.}\,,$$



S_3-S_3-R_2^*-H

$$C_L = \frac{2\sqrt{2}\lambda v}{m_{S_3}^4 m_{R_2}^2} (V^* y_{S_3}^L) y_{S_3}^L (V y_{R_2}^R)^*\,,$$

$$C_R = -\frac{2\sqrt{2}\lambda v}{m_{S_3}^4 m_{R_2}^2} (V^* y_{S_3}^L) y_{S_3}^L (y_{R_2}^L)^*\,.$$

$$\epsilon_{abc}\langle 0|(\bar{u}_a^C P_R d_b) P_L u_c|p\rangle = \alpha_p P_R u_p \qquad \alpha_p = -0.0144(3)(21)\,\text{GeV}^3$$

$$\epsilon_{abc}\langle 0|(\bar{u}_a^C P_L d_b) P_L u_c|p\rangle = \beta_p P_L u_p \qquad \beta_p = +0.0144(3)(21)\,\text{GeV}^3$$

Lattice QCD — Aoki et al. 1705.01338

Decay width

$$\Gamma(p \to e^+ e^+ e^-) = \frac{m_p^5}{6(16\pi)^3} \left(\beta_p^2 |C_L|^2 + \alpha_p^2 |C_R|^2\right)$$

$\tau(p \to e^+ e^+ e^-) > 3.4 \times 10^{34}$ years

experiment SuperKamiokande
Takenaka et al., 2010.16098

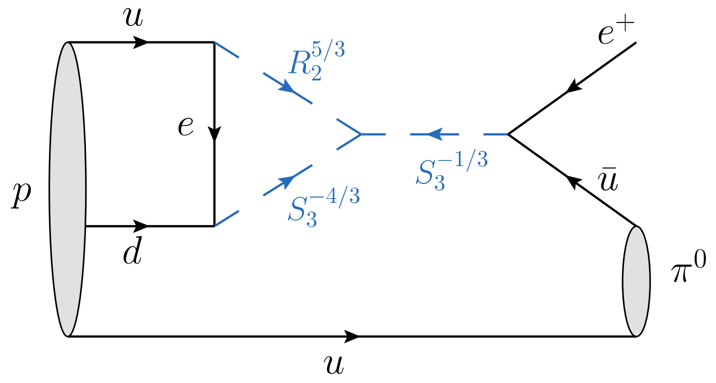
assumptions $y_{S_3}^L = y_{R_2}^R = y_{R_2}^L = \lambda = 1$

$m_{S_3} = m_{R_2} = \Lambda$

$$p \to e^+ e^+ e^- : \qquad \Lambda \geq 1.6 \times 10^2\ \text{TeV}$$

Loop-level leptoquark mediation of $p \to \pi^0 e^+$

$$\mathcal{L}_{\text{eff}}^{(d=6)} \supset C_{LL}^{udeu}\ (\overline{u}^{C} P_L d)\ (\overline{e}^{C} P_L u) + C_{LR}^{udeu}\ (\overline{u}^{C} P_L d)\ (\overline{e}^{C} P_R u) + \text{h.c.}$$



Explicit loop computation

General scenario $\mathcal{L}_{\text{scalar}} \supset \lambda\, v\, \varepsilon_{abc}\, \Delta_a^{Q}\, \Delta_b^{Q'}\, \Delta_c^{Q''} + \text{h.c.}$

LQ interactions with quarks and leptons

$$\mathcal{L}_{\text{yuk.}} \supset \overline{q}\,(y_R P_R + y_L P_L)\,\ell\,\Delta^{Q} + \overline{q'^{C}}\,(y'_R P_R + y'_L P_L)\,\ell\,\Delta^{Q'*} + \text{h.c.}$$

The loop diagram corresponds to a loop-induced diquark coupling of the $\Delta_{Q''}$ leptoquark

$$\mathcal{L}_{\text{qq}'} = \varepsilon_{abc}\,\overline{q_a^{C}}\,(y_{qq'}^{L} P_L + y_{qq'}^{L} P_R)\,q'_b\,\Delta_c^{Q''} + \text{h.c.}$$

$$y_{qq'}^{L} = \frac{\lambda v}{16\pi^2 m_\Delta^2}\left(m_\ell\, y'_L y_R^* - \frac{m_q}{4} y'_R y_R^* - \frac{m_{q'}}{4} y'_L y_L^*\right)$$

$$y_{qq'}^{R} = \frac{\lambda v}{16\pi^2 m_\Delta^2}\left(m_\ell\, y'_R y_L^* - \frac{m_q}{4} y'_L y_L^* - \frac{m_{q'}}{4} y'_R y_R^*\right)$$

Chirality flip in the internal lepton and external quark lines

Ingredients

$$C_{LL}^{udeu} = \frac{\sqrt{2}\lambda}{8\pi^2}\frac{v m_e}{\Lambda^4}(V^* y_{S_3}^L)\left[y_{S_3}^L (V y_{R_2}^R)^* + \frac{m_d}{4m_e} y_{S_3}^L (y_{R_2}^L)^*\right],$$

$$C_{LR}^{udeu} = \frac{\lambda}{32\pi^2}\frac{v m_u}{\Lambda^4}(V^* y_{S_3}^L)^2 (y_{R_2}^L)^* .$$

$$\left\langle \pi^0 \middle| \mathcal{O}^{\Gamma\Gamma'} \middle| p \right\rangle = \left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_p} W_1^{\Gamma\Gamma'}(q^2)\right] P_{\Gamma'} u_p$$

$$\mathcal{O}^{\Gamma\Gamma'} = \left(\overline{u}^C P_\Gamma d\right) P_{\Gamma'} u \quad \Gamma, \Gamma' = R, L.$$

Form factors

$$\langle \pi^+ | \left(\overline{u}^C P_\Gamma d\right) P_{\Gamma'} d | p \rangle = \sqrt{2} \langle \pi^0 | \left(\overline{u}^C P_\Gamma d\right) P_{\Gamma'} u | p \rangle$$

$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left[(W_0^{LL})^2 |C_{LL}^{udeu}|^2 + (W_0^{RL})^2 |C_{LR}^{udeu}|^2\right]$$

$W_0^{LL} = 0.134(5)\,\mathrm{GeV}^2 \quad W_0^{LR} = -0.131(4)\,\mathrm{GeV}^2$ Lattice QCD, Aoki et al., 1705.01338

assuming $y_{S_3}^L = y_{R_2}^L = y_{R_2}^R = \lambda = 1$ and $m_{S_3} = m_{R_2} = \Lambda$

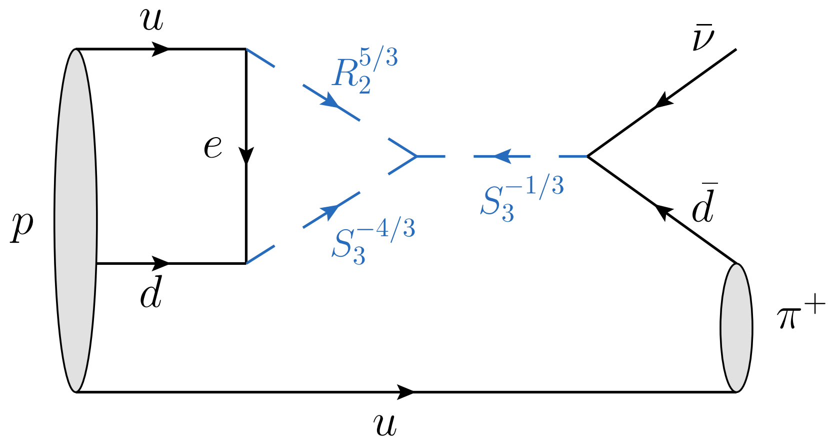
$$p \to \pi^0 e^+ : \quad \Lambda \geq 1.8 \times 10^4\,\mathrm{TeV}$$

$p \to \pi^+ \bar{\nu}$

$$\mathcal{L}_{\text{eff}}^{(d=6)} \supset C_{LL}^{ud\nu d}\ (\overline{u}^{C} P_L d)\ (\overline{\nu}^{C} P_L d) + C_{RL}^{ud\nu d}\ (\overline{u}^{C} P_R d)\ (\overline{\nu}^{C} P_L d) + \text{h.c.}$$

$$C_{LL}^{ud\nu d} = -\frac{\sqrt{2}\lambda}{8\pi^2}\frac{v m_e}{\Lambda^4}(y_{S_3}^L)^2\left[(V y_{R_2}^R)^* + \frac{m_d}{4m_e}(y_{R_2}^L)^*\right]$$

$$C_{RL}^{ud\nu d} = -\frac{\lambda}{32\pi^2}\frac{v m_u}{\Lambda^4}(V^* y_{S_3}^L)^2 (y_{R_2}^L)^* \,.$$



$$\Gamma(p \to \pi^+ \nu) = \frac{m_p}{16\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left[(W_0^{LL})^2 |C_{LL}^{ud\nu d}|^2 + (W_0^{RL})^2 |C_{RL}^{ud\nu d}|^2\right]$$

assuming $y_{S_3}^L = y_{R_2}^L = y_{R_2}^R = \lambda = 1$ and $m_{S_3} = m_{R_2} = \Lambda$

$$p \to \pi^+ \bar{\nu}\ : \qquad \Lambda \geq 1.2 \times 10^4\ \text{TeV}$$

Radiative nucleon decays with $\Delta B = 1$

SF, Šadl, 2304.00825

Decay mode	$\Gamma^{-1}/10^{30} yr$
$p \to e^+\pi^0$	16000
$p \to \mu^+\pi^0$	7700
$n \to \nu\,\pi^0$	1100
$p \to e^+\gamma$	670
$p \to \mu^+\gamma$	478
$n \to \nu\,\gamma$	550

Existing bounds

$$\mathcal{L}_{p\gamma p} = e\bar{p}\left(\not{A} + \frac{a_p}{4\,m_p}\sigma^{\alpha\beta}F_{\alpha\beta}\right)p, \qquad a_p = 1.793.$$

$$\mathcal{L}_{n\gamma n} = e\bar{n}\left(\frac{a_n}{4\,m_n}\sigma^{\alpha\beta}F_{\alpha\beta}\right)n \qquad a_n = -1.913.$$

$$\mathcal{L}^{p\ell}_{\rm mix} = \varepsilon_p(\bar{p}\ell + \bar{\ell}p), \quad \mathcal{L}^{n\nu}_{\rm mix} = \varepsilon_n(\bar{n}\nu + \bar{\nu}n)$$

After the diagonalisation of the mass matrices, in the limit $\varepsilon_p \ll (m_p - m_\ell)$ or $\varepsilon_n \ll m_n$ such interaction leads to the contributions

$$\mathcal{L}^{\rm eff}_{p\to\ell\gamma} = -\frac{a_p e}{4\,m_p}\frac{\varepsilon_p}{m_p - m_\ell}\,\bar{\ell}\sigma^{\alpha\beta}F_{\alpha\beta}\,p + {\rm h.c.}$$

$$\mathcal{L}^{\rm eff}_{n\to\bar{\nu}\gamma} = -\frac{a_n e\,\varepsilon_n}{4\,m_n^2}\bar{\nu}\sigma^{\alpha\beta}F_{\alpha\beta}\,n + {\rm h.c.}$$

Rest of the "ingredients"

example

$$\mathcal{L}_{S_1} = -(y_1^{LL}U)_{ij}\bar{d}_L^{C\,i}S_1\nu_L^j + (V^T y_1^{LL})_{ij}\bar{u}_L^{C\,i}S_1 e_L^j + y_{1\,ij}^{RR}\bar{u}_R^{C\,i}S_1 e_R^j + y_{1\,ij}^{\overline{RR}}\bar{d}_R^{C\,i}S_1\nu_R^j + (V^T z_1^{LL})_{ij}\bar{u}_L^{C\,i}S_1^* d_L^j - (z_1^{LL}V^\dagger)_{ij}\bar{d}_L^{C\,i}S_1^* u_L^j + z_{1\,ij}^{RR}\bar{u}_R^{C\,i}S_1^* d_R^j + \text{h.c.}\,,$$

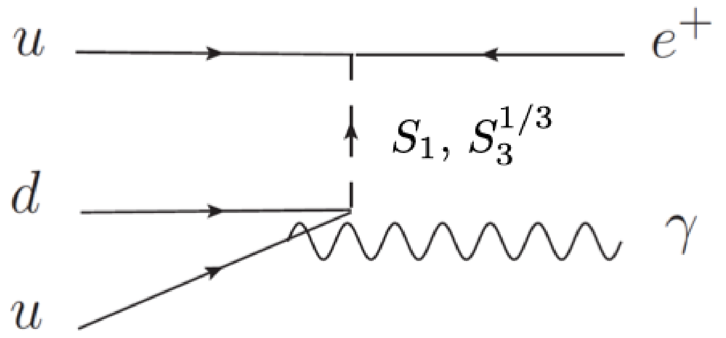
$$\begin{aligned} C_1^{p,S_1} &= (V^T z_1^{LL})_{11}(V^T y_1^{LL})_{11}, \\ C_2^{p,S_1} &= (V^T z_1^{LL})_{11}(y_1^{RR})_{11}, \\ C_3^{p,S_1} &= (V^T y_1^{LL})_{11}(z_1^{RR})_{11}, \\ C_4^{p,S_1} &= (z_1^{RR})_{11}(y_1^{RR})_{11}. \end{aligned}$$

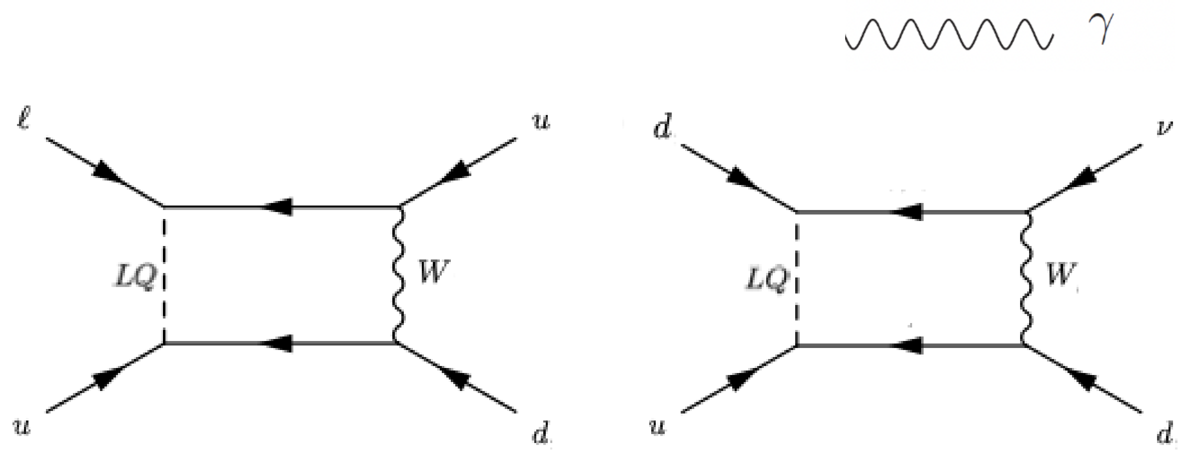
Wilson coefficents for p $\rightarrow$ e transitions

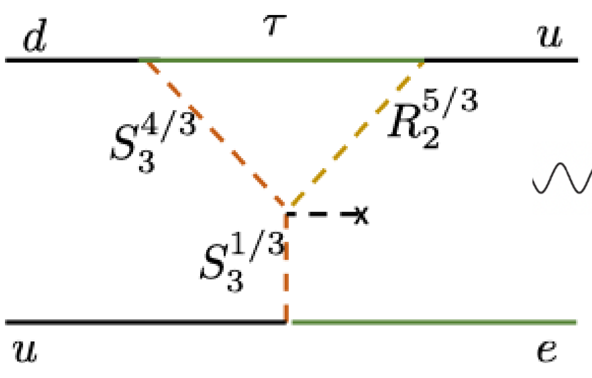
$$\varepsilon_1^{p,S_1} = \frac{C_1^p\beta_p}{m_{S_1}^2}, \quad \varepsilon_2^{p,S_1} = -\frac{C_2^p\alpha_p}{m_{S_1}^2}, \quad \varepsilon_3^{p,S_1} = \frac{C_3^p\alpha_p}{m_{S_1}^2}, \quad \varepsilon_4^{p,S_1} = -\frac{C_4^p\beta_p}{m_{S_1}^2}, \quad \varepsilon_1^{n,S_1} = -\frac{C_1^n\beta_n}{m_{S_1}^2}. \tag{19}$$

Possible decay mechanism

attaching photon everywhere







$$\Gamma(p \to \ell^+ \gamma) = C_{\gamma\ell}\, \Gamma(p \to \ell^+ \pi^0)$$

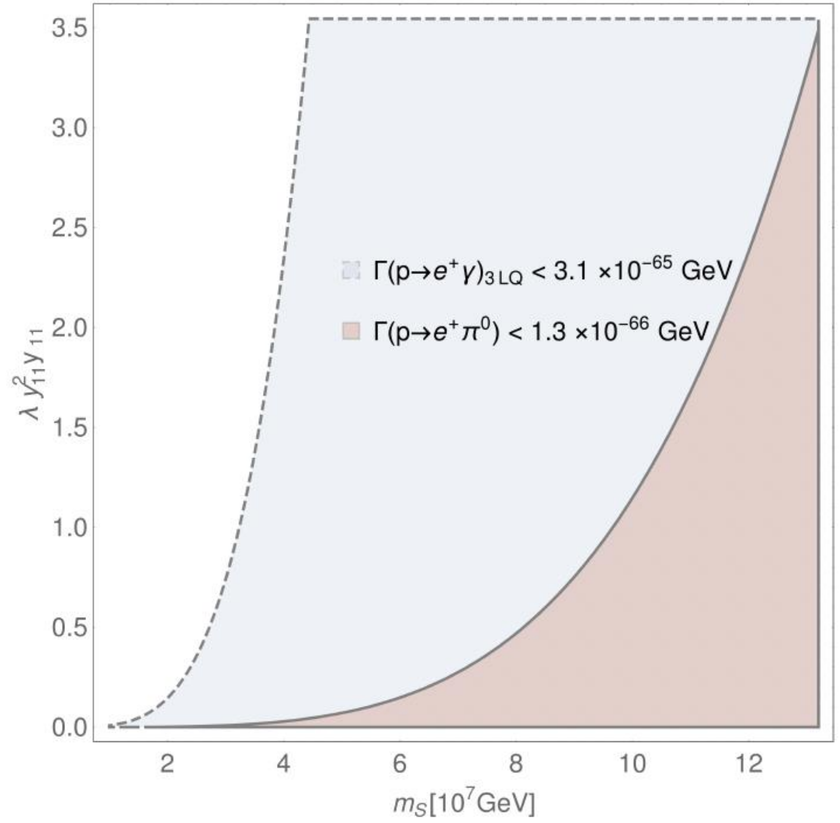
$$C_{\gamma\ell} = e^2\, a_p^2\, \frac{\beta_p^2}{W_0^{\rm LL}(0)^2} F(m_p, m_\ell, m_\pi)$$

$$F(m_p, m_\ell, m_\pi) =$$

$$= \frac{m_p^4 \left(1 - \left(\frac{m_\ell}{m_p}\right)^2\right)^3}{(m_p - m_\ell)^2 \lambda^{1/2}(m_p^2, m_\ell^2 m_\pi^2)(m_p^2 + m_\ell^2 - m_\pi^2}$$

$$\langle 0|(ud)_R u_L|p\rangle = \alpha_p P_L u_p,\ \langle 0|(ud)_L u_R|p\rangle = -\alpha_p P_R u_p,$$
$$\langle 0|(ud)_L u_L|p\rangle = \beta_p P_L u_p,\ \langle 0|(ud)_R u_R|p\rangle = -\beta_p P_R u_p,$$

$$\alpha_p = -0.0144(3)(21)\ {\rm GeV}^3$$
$$\beta_p = +0.0144(3)(21)\ {\rm GeV}^3$$



$$\begin{aligned}\Gamma(p \to e^+ \gamma) &\simeq 3.8 \times 10^{-3}\, \Gamma(p \to e^+ \pi^0),\\ \Gamma(p \to \mu^+ \gamma) &\simeq 4.6 \times 10^{-3}\, \Gamma(p \to \mu^+ \pi^0),\\ \Gamma(n \to \bar\nu\, \gamma) &\simeq 3.8 \times 10^{-3}\, \Gamma(n \to \bar\nu \pi^0).\end{aligned}$$

Conclusions

- we study a phenomenological impact of triple-leptoquark interactions on proton stability;

- there are two different decay topologies under the assumption that scalar leptoquarks of interest couple solely to the quark-lepton pairs;

- the tree - level topology has been analysed in the literature before in the context of baryon number violation while the one-loop level one has not been featured in any scientific study to date;

- we demonstrate that it is the one-loop level topology that is producing more stringent bounds on the scalar leptoquark masses of the two, if and when they coexist;

$$p \to e^+ e^+ e^- : \qquad \Lambda \geq 1.6 \times 10^2 \,\text{TeV}$$

$$p \to \pi^0 e^+ : \qquad \Lambda \geq 1.8 \times 10^4 \,\text{TeV}$$

- we also specify the most prominent proton decay signatures due to the presence of all non-trivial cubic and quartic contractions involving three scalar leptoquark multiplets, where in the latter case one of the scalar multiplets is the SM Higgs doublet;

- We revisited radiative nucleon decays exploring that the photon radiation from a hadron and charged lepton can be related nucleons' anomalous magnetic moments;

- The braching ratio for radiative decays are $BR(p \to \ell^+\gamma) \simeq 10^{-3} BR(p \to \ell^+\pi^0)$.

Thanks

d = 9 effective operators

$$\mathcal{L}_{(a)} \supset \frac{2\kappa\epsilon_{abc}}{m_{\tilde{R}_2}^4 m_{S_1}^2} (y_{\tilde{R}_2}^L)_{1j}^* (\bar{\nu}_L^j d_{R\,a}) \, (y_{\tilde{R}_2}^L)_{1k}^* (\bar{e}_L^k d_{R\,b})$$

$$\times \Big[(V^* y_{S_1}^L)_{1i} (\bar{u}_{L\,c}^C e_L^i) - (y_{S_1}^L)_{1i} (\bar{d}_{L\,c}^C \nu_L^i) + (y_{S_1}^R)_{1i} (\bar{u}_{R\,c}^C e_R^i) \Big] + \text{h.c.}\,,$$

$$\mathcal{L}_{(b)} \supset \frac{\kappa\epsilon_{abc}}{m_{\tilde{S}_1}^2 m_{R_2}^2 m_{\tilde{R}_2}^2} \Big\{ \Big[(V y_{R_2}^R)_{1j}^* (\bar{e}_R^j u_{L\,a}) - (y_{R_2}^L)_{1j}^* (\bar{e}_L^j u_{R\,a}) \Big] (y_{\tilde{R}_2}^L)_{1k}^* (\bar{\nu}_L^k d_{R\,b})$$

$$+ \Big[(y_{R_2}^L)_{1j}^* (\bar{\nu}_L^j u_{R\,a}) + (y_{R_2}^R)_{1j}^* (\bar{e}_R^j d_{L\,a}) \Big] (y_{\tilde{R}_2}^L)_{1k}^* (\bar{e}_L^k d_{R\,b}) \Big\} (y_{\tilde{S}_1}^R)_{1i} (\bar{d}_{L\,c}^C e_R^i) + \text{h.c.}\,,$$

$$\mathcal{L}_{(c)} \supset \frac{\lambda\epsilon_{abc} v}{\sqrt{2} m_{S_3}^2 m_{R_2}^2 m_{S_1}^2} \Big[(V^* y_{S_1}^L)_{1i} (\bar{u}_{L\,c}^C e_L^i) - (y_{S_1}^L)_{1i} (\bar{d}_{L\,c}^C \nu_L^i) + (y_{S_1}^R)_{1i} (\bar{u}_{R\,c}^C e_R^i) \Big]$$

$$\times \Big\{ \Big[(V^* y_{S_3}^L)_{1j} (\bar{u}_{L\,a}^C e_L^j) + (y_{S_3}^L)_{1j} (\bar{d}_{L\,a}^C \nu_L^j) \Big] \Big[(y_{R_2}^L)_{1k}^* (\bar{\nu}_L^k u_{R\,b}) + (y_{R_2}^R)_{1k}^* (\bar{e}_R^k d_{L\,b}) \Big]$$

$$- \sqrt{2} (y_{S_3}^L)_{1j} (\bar{d}_{L\,a}^C e_L^j) \Big[(V y_{R_2}^R)_{1k}^* (\bar{e}_R^k u_{L\,b}) - (y_{R_2}^L)_{1k}^* (\bar{e}_L^k u_{R\,b}) \Big] \Big\} + \text{h.c.}\,,$$

$$\mathcal{L}_{(d)} \supset \frac{2\sqrt{2}\lambda\epsilon_{abc} v}{m_{S_3}^4 m_{R_2}^2} (y_{S_3}^L)_{1j} (\bar{d}_{L\,a}^C e_L^j)$$

$$\times \Big\{ (V^* y_{S_3}^L)_{1k} (\bar{u}_{L\,b}^C \nu_L^k) \Big[(y_{R_2}^L)_{1i}^* (\bar{\nu}_L^i u_{R\,c}) + (y_{R_2}^R)_{1i}^* (\bar{e}_R^i d_{L\,c}) \Big]$$

$$+ \Big[(y_{S_3}^L)_{1k} (\bar{d}_{L\,b}^C \nu_L^k) + (V^* y_{S_3}^L)_{1k} (\bar{u}_{L\,b}^C e_L^k) \Big] \Big[(y_{R_2}^L)_{1i}^* (\bar{e}_L^i u_{R\,c}) - (V y_{R_2}^R)_{1i}^* (\bar{e}_R^i u_{L\,c}) \Big] \Big\}$$

$$+ \text{h.c.}$$

$$\mathcal{L}_{(e)} \supset \frac{-\lambda\epsilon_{abc}v}{\sqrt{2}m^2_{\tilde{S}_1}m^2_{R_2}m^2_{S_1}}(y^R_{\tilde{S}_1})_{1i}(\bar{d}^C_{R\,c}e^i_R)\Big[(y^L_{R_2})^*_{1j}(\bar{e}^j_L u_{R\,a}) - (V y^R_{R_2})^*_{1j}(\bar{e}^j_R u_{L\,a})\Big]$$
$$\times\Big[(V^* y^L_{S_1})_{1k}(\bar{u}^C_{L\,b}e^k_L) + (y^R_{S_1})_{1k}(\bar{u}^C_{R\,b}e^k_R) - (y^L_{S_1})_{1k}(\bar{d}^C_{L\,b}\nu^k_L)\Big] + \text{h.c.}\,,$$

$$\mathcal{L}_{(f)} \supset \frac{\lambda\epsilon_{abc}v}{\sqrt{2}m^2_{R_2}m^2_{S_3}m^2_{\tilde{S}_1}}(y^R_{\tilde{S}_1})_{1i}(\bar{d}^C_c P_R e^i)$$
$$\times\Big\{2(V^* y^L_{S_3})_{1j}(\bar{u}^C_{L\,a}\nu^j_L)\Big[(y^L_{R_2})^*_{1k}(\bar{\nu}^k_L u_{R\,b}) + (y^R_{R_2})^*_{1k}(\bar{e}^k_R d_{L\,b})\Big]$$
$$+\Big[(y^L_{S_3})_{1j}(\bar{d}^C_{L\,a}\nu^j_L) + (V^* y^L_{S_3})_{1j}(\bar{u}^C_{L\,a}e^j_L)\Big]\Big[(y^L_{R_2})^*_{1k}(\bar{e}^k_L u_{R\,b}) - (V y^R_{R_2})^*_{1k}(\bar{e}^k_R u_{L\,b})\Big]\Big\}$$

$$\mathcal{L}_{(g)} \supset \frac{\lambda\epsilon_{abc}v}{\sqrt{2}m^2_{S_3}m^2_{\tilde{R}_2}m^2_{S_1}}\Big[(V^* y^L_{S_1})_{1i}(\bar{u}^C_{L\,c}e^i_L) - (y^L_{S_1})_{1i}(\bar{d}^C_{L\,c}\nu^i_L) + (y^R_{S_1})_{1i}(\bar{u}^C_{R\,c}e^i_R)\Big]$$
$$\times\Big\{\Big[(y^L_{S_3})_{1j}(\bar{d}^C_{L\,a}\nu^j_L) + (V^* y^L_{S_3})_{1j}(\bar{u}^C_{L\,a}e^j_L)\Big](y^L_{\tilde{R}_2})^*_{1k}(\bar{e}^k_L d_{R\,b})$$
$$+\,2(V^* y^L_{S_3})_{1j}(\bar{u}^C_{L\,a}\nu^j_L)\,(y^L_{\tilde{R}_2})^*_{1k}(\bar{\nu}^k_L d_{R\,b})\Big\} + \text{h.c.}\,,$$

$$\mathcal{L}_{(h)} \supset \frac{2\sqrt{2}\lambda\epsilon_{abc}v}{m^4_{S_3}m^2_{\tilde{R}_2}}\Big\{(y^L_{S_3})_{1j}(\bar{d}^C_{L\,a}e^j_L)\,(V^* y^L_{S_3})_{1k}(\bar{u}^C_{L\,b}\nu^k_L)\,(y^L_{\tilde{R}_2})^*_{1i}(\bar{e}^i_L d_{R\,c})$$
$$-\,(y^L_{\tilde{R}_2})^*_{1i}(\bar{\nu}^i_L d_{R\,c})\,(V^* y^L_{S_3})_{1j}(\bar{u}^C_{L\,a}\nu^j_L)\Big[(y_{S_3})^L_{1k}(\bar{d}^C_{L\,b}\nu^k_L) + (V^* y^L_{S_3})_{1k}(\bar{u}^C_{L\,b}e^k_L)\Big]\Big\} + \text{h.c.}$$

SCALARS			FERMIONS		
$SU(5)$	Standard Model	(b_3, b_2, b_1)	$SU(5)$	Standard Model	(b_3, b_2, b_1)
$\Lambda = 5_H$	$\Lambda_1\left(1,2,+\frac{1}{2}\right)$	$\left(0,\frac{1}{6},\frac{1}{10}\right)$	$F_i = \overline{5}_{Fi}$	$L_i\left(1,2,-\frac{1}{2}\right)$	$\left(0,1,\frac{3}{5}\right)$
	$\Lambda_3\left(3,1,-\frac{1}{3}\right)$	$\left(\frac{1}{6},0,\frac{1}{15}\right)$		$d_i^c\left(\overline{3},1,+\frac{1}{3}\right)$	$\left(1,0,\frac{2}{5}\right)$
$\phi = 24_H$	$\phi_0\,(1,1,0)$	$(0,0,0)$	$T_i = 10_{Fi}$	$Q_i\left(3,2,+\frac{1}{6}\right)$	$\left(2,3,\frac{1}{5}\right)$
	$\phi_1\,(1,3,0)$	$\left(0,\frac{1}{3},0\right)$		$u_i^c\left(\overline{3},1,-\frac{2}{3}\right)$	$\left(1,0,\frac{8}{5}\right)$
	$\phi_3\left(3,2,-\frac{5}{6}\right)$	$\left(\frac{1}{6},\frac{1}{4},\frac{5}{12}\right)$		$e_i^c\,(1,1,+1)$	$\left(0,0,\frac{6}{5}\right)$
	$\phi_{\overline{3}}\left(\overline{3},2,+\frac{5}{6}\right)$	$\left(\frac{1}{6},\frac{1}{4},\frac{5}{12}\right)$	$\Sigma = 15_F$	$\Sigma_1(1,3,+1)$	$\left(0,\frac{4}{3},\frac{6}{5}\right)$
	$\phi_8\,(8,1,0)$	$\left(\frac{1}{2},0,0\right)$		$\Sigma_3\left(3,2,+\frac{1}{6}\right)$	$\left(\frac{2}{3},1,\frac{1}{15}\right)$
$\Phi = 35_H$	$\Phi_1\left(1,4,-\frac{3}{2}\right)$	$\left(0,\frac{5}{3},\frac{9}{5}\right)$		$\Sigma_6\left(6,1,-\frac{2}{3}\right)$	$\left(\frac{5}{3},0,\frac{16}{15}\right)$
	$\Phi_3\left(\overline{3},3,-\frac{2}{3}\right)$	$\left(\frac{1}{2},2,\frac{4}{5}\right)$	$\overline{\Sigma} = \overline{15}_F$	$\overline{\Sigma}_1\,(1,3,-1)$	$\left(0,\frac{4}{3},\frac{6}{5}\right)$
	$\Phi_6\left(\overline{6},2,+\frac{1}{6}\right)$	$\left(\frac{5}{3},1,\frac{1}{15}\right)$		$\overline{\Sigma}_3\left(\overline{3},2,-\frac{1}{6}\right)$	$\left(\frac{2}{3},1,\frac{1}{15}\right)$
	$\Phi_{10}\left(\overline{10},1,+1\right)$	$\left(\frac{5}{2},0,2\right)$		$\overline{\Sigma}_6\left(\overline{6},1,+\frac{2}{3}\right)$	$\left(\frac{5}{3},0,\frac{16}{15}\right)$