Heavy-flavour jet substructure at the LHC

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Exploring the Nature of Matter

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Center for Frontiers in Nuclear Science

QCD Lagrangian and event structure

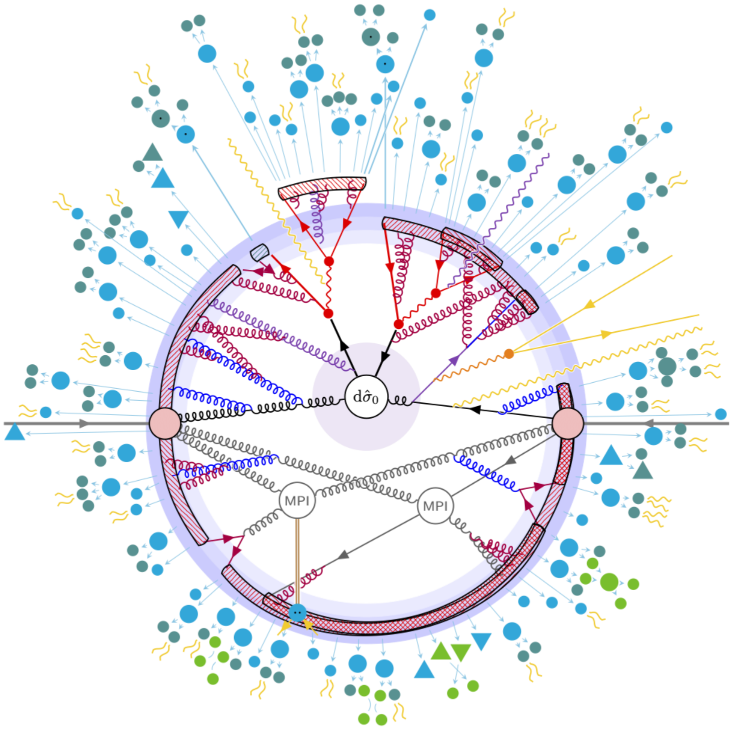


Image credit: 2203.11601

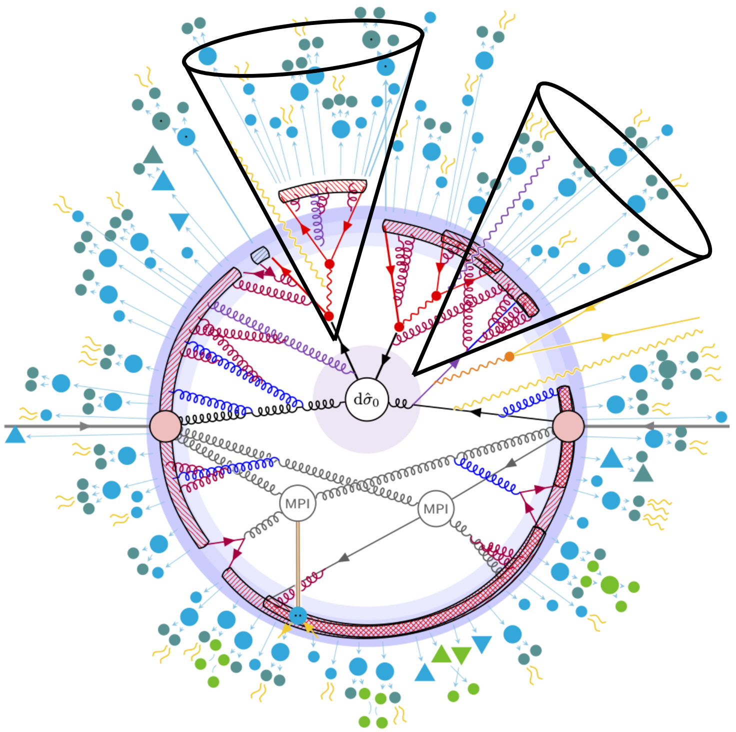
QCD Lagrangian is very simple:

$$\mathcal{L}_{QCD} = \sum_q \left(\bar{\psi}_{qi}\, i\gamma^\mu \left[\delta_{ij}\partial_\mu + ig \left(G^\alpha_\mu t_\alpha \right)_{ij} \right] \psi_{qj} - m_q \bar{\psi}_{qi}\psi_{qi} \right) - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$$

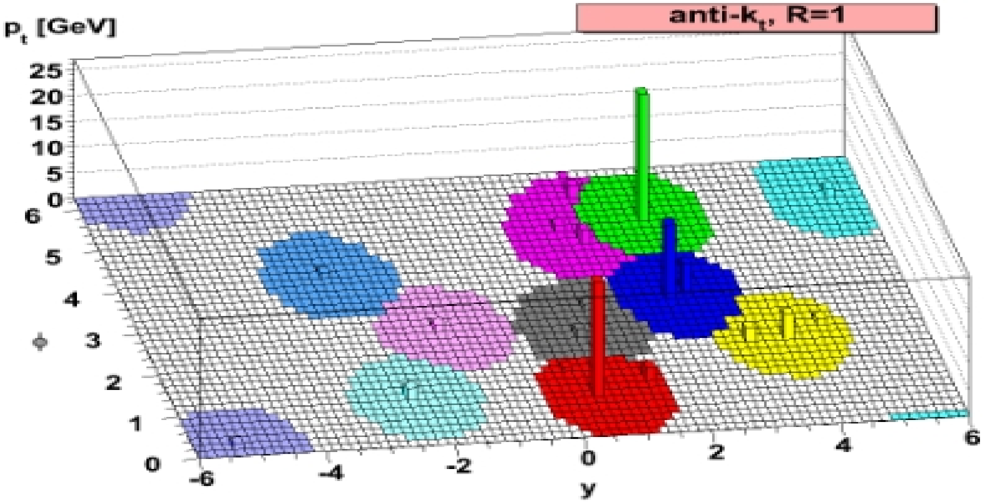
However, QCD is very complicated:

- Analytical calculations can be be performed only in high-energy limit where coupling is small
- Has a lot of open questions (e.g. the origin of confinement, CP-violation, collective behavior etc)
- Despite significant improvements in the accuracy of analytical calculations and lattice simulations, a lot still has to be done.

Jets and their connections to QCD



Jets are collimated clusters of particles which connect physics at the microscopic (much smaller than 1 fm) and the macroscopic scales (1 – 10 m)



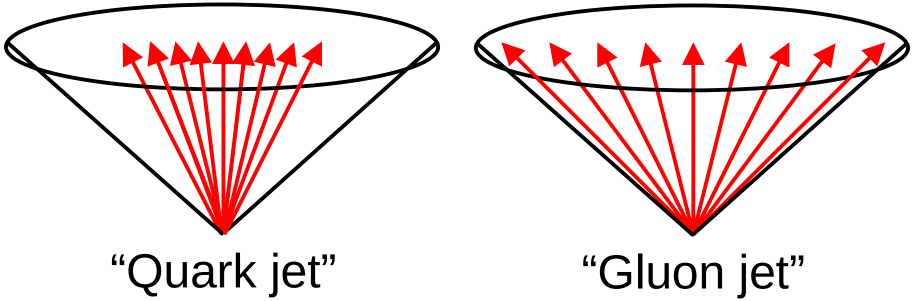
Observable definition: jet angularity

Jet angularity is defined as

$$\lambda^\alpha = \sum_i \frac{p_{ti}}{p_t}\left(\frac{\Delta R_{i,\text{jet}}}{R_0}\right)^\alpha, \quad \alpha > 0$$

- Sum runs over all particles inside the jet
- Jet radius R
- Rapidity-azimuth distance $\Delta R_{i,\text{jet}}$
- IRC (infrared and collinear) safe observable!

- LHA (Les Houches Angularity): $\alpha = 1/2$
- Jet Width: $\alpha = 1$
- Jet Thrust: $\alpha = 2$



CMS: 1808.07340, 2109.03340; ALICE: 2107.11303; ATLAS: 1702.00674

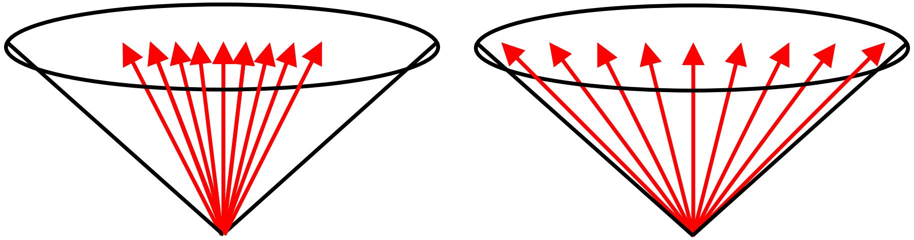
Observable definition: energy-correlation functions

Energy correlation functions (ECFs) are similar to jet angularities

$$e_2^\alpha = \sum_{i \neq j} \frac{p_{t_i} p_{t_j}}{p_t^2} \left(\frac{\Delta R_{ij}}{R_0} \right)^\alpha, \quad \alpha > 0$$

- Sum runs over all pairs of particles inside the jet
- Unlike jet angularity, does not depend on definition of the jet axis
- As a consequence, ECFs are recoil-insensitive

- If $\alpha = 2$ both ECF and Jet Thrust are proportional to jet mass
- ECFs are also related to N-subjettiness however ECFs are easier to use (do not require jet definition of jet axis



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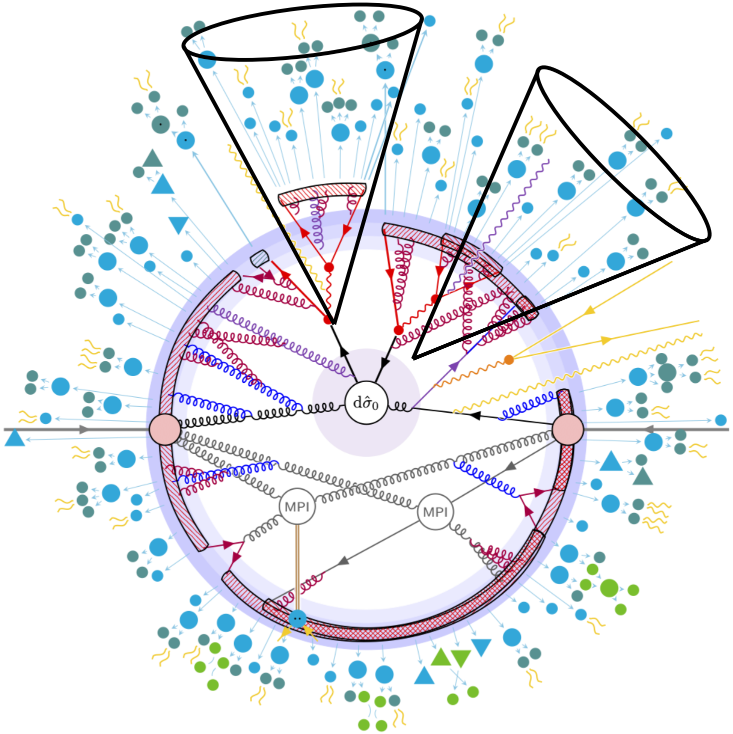
Impact of Multiple Partonic Interactions



- Protons are composite objects so several (semi-)hard partonic interactions can occur per one pp collision!

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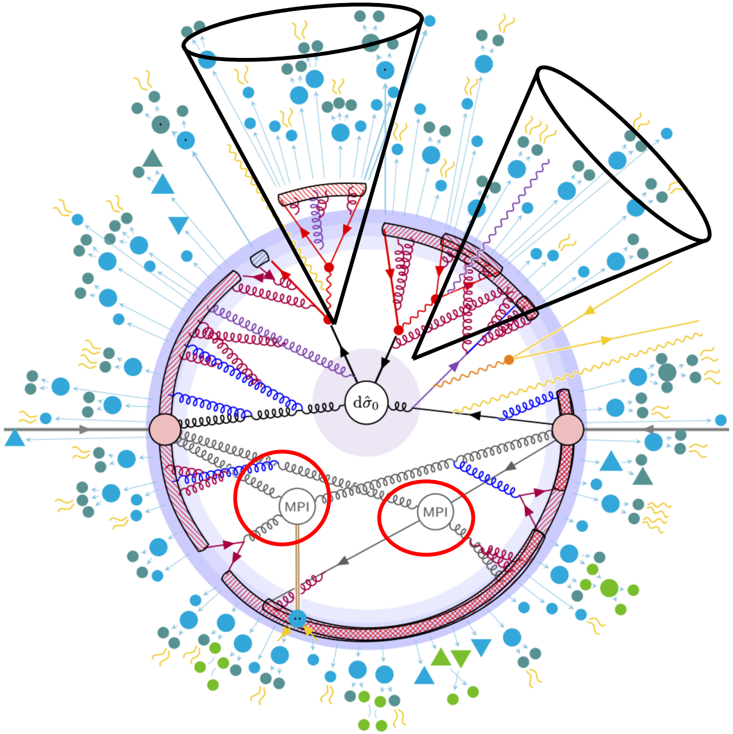

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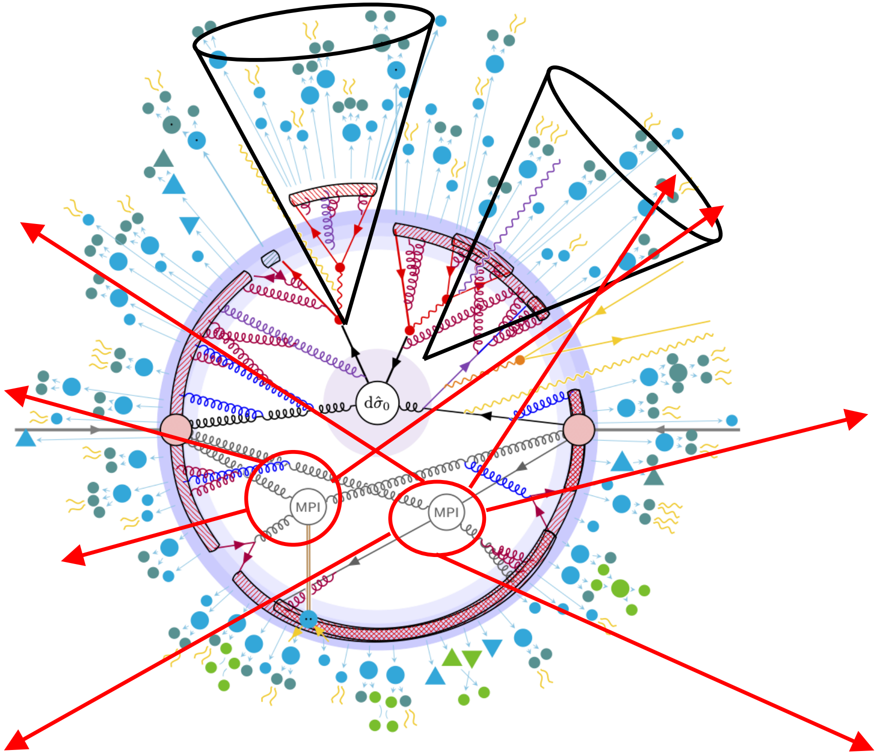
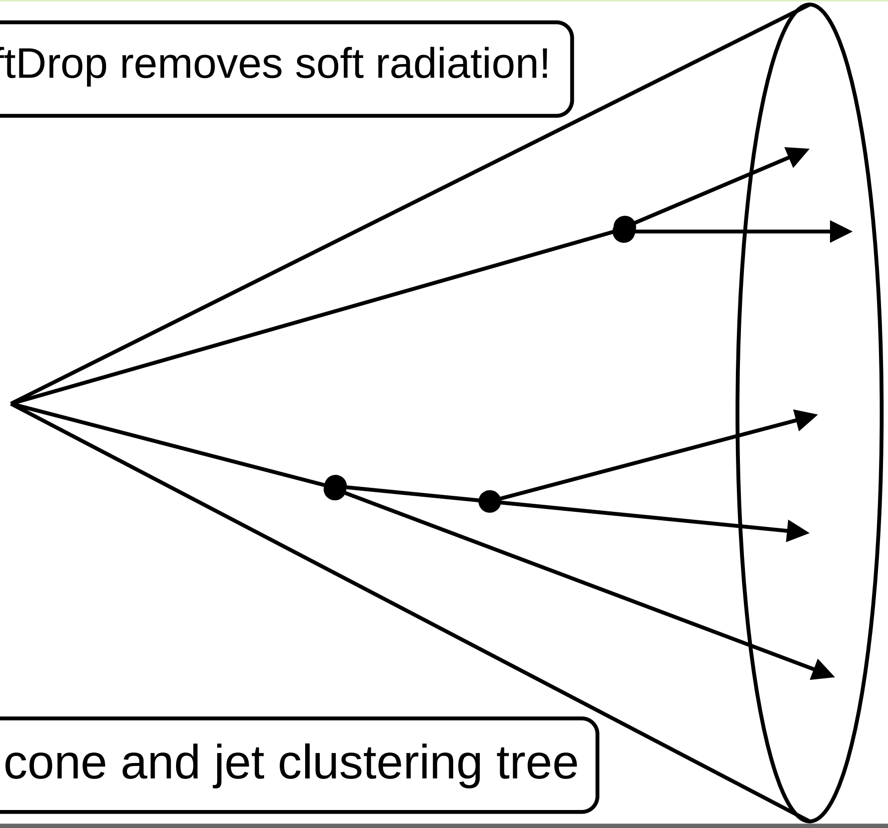


- Protons are composite objects so several (semi-)hard partonic interactions can occur per one pp collision!
- Such processes generally know as Multiple Partonic Interactions (MPI)
- MPI cause multiple uniform soft emissions which “contaminate” jet substructure

SoftDrop algorithm:

SoftDrop removes soft radiation!



Jet cone and jet clustering tree

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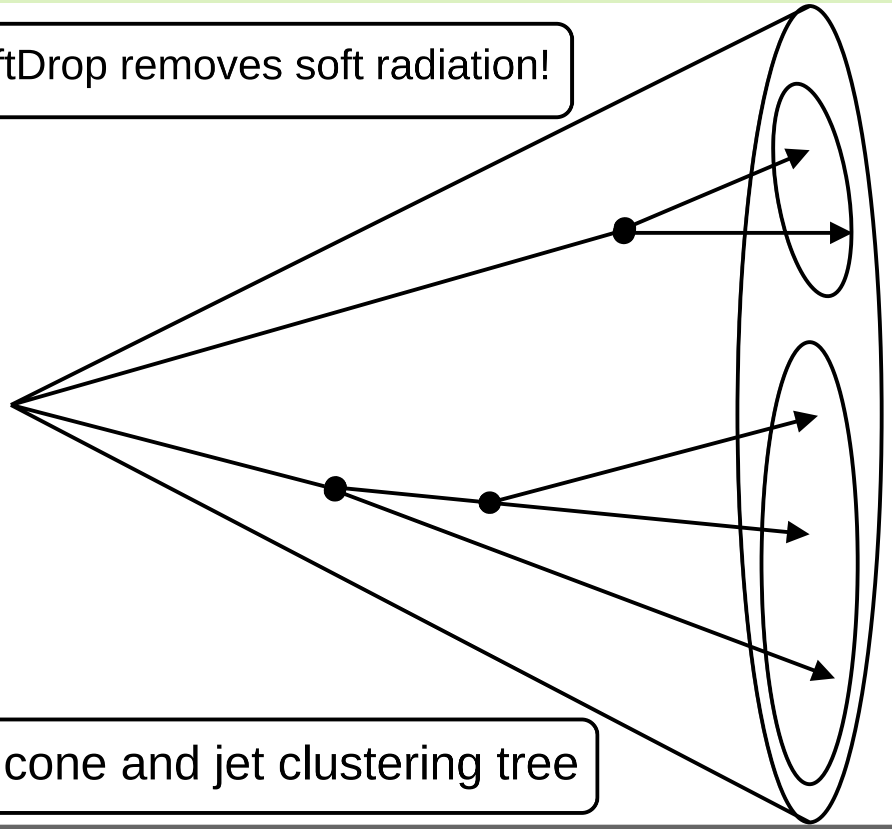

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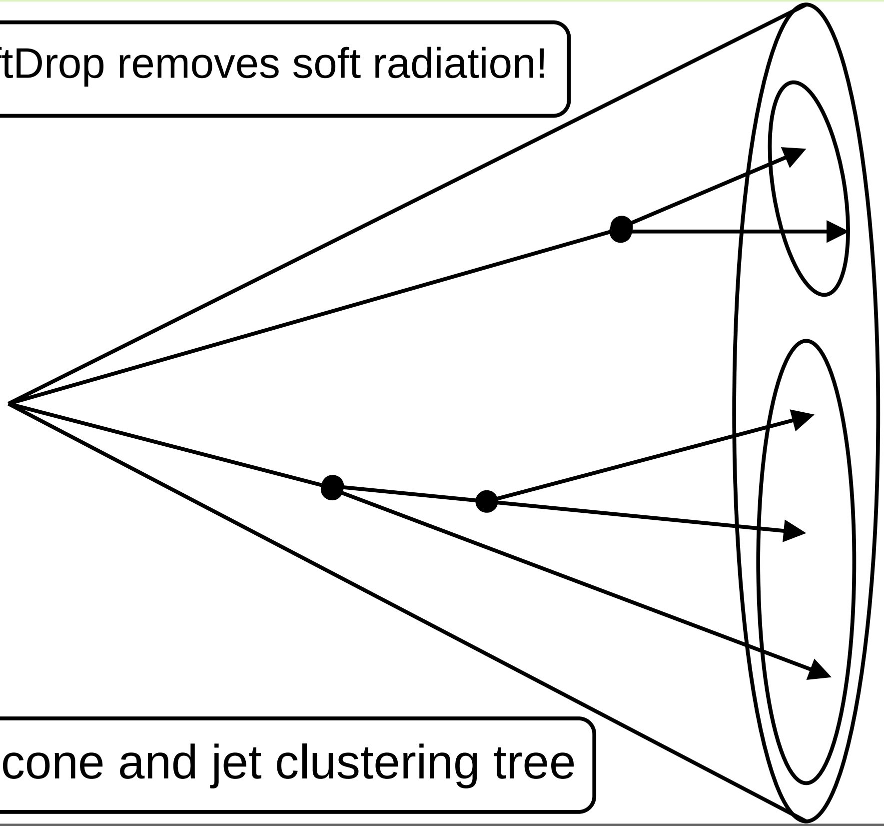

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SoftDrop can be used to remove soft radiation from MPI:

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2. Check if one branch is much softer than the other one using the SoftDrop condition

$$\frac{\min(p_{ti}, p_{tj})}{p_{ti} + p_{tj}} > z_{\text{cut}} \left(\frac{\Delta R_{ij}}{R} \right)^{\beta}$$

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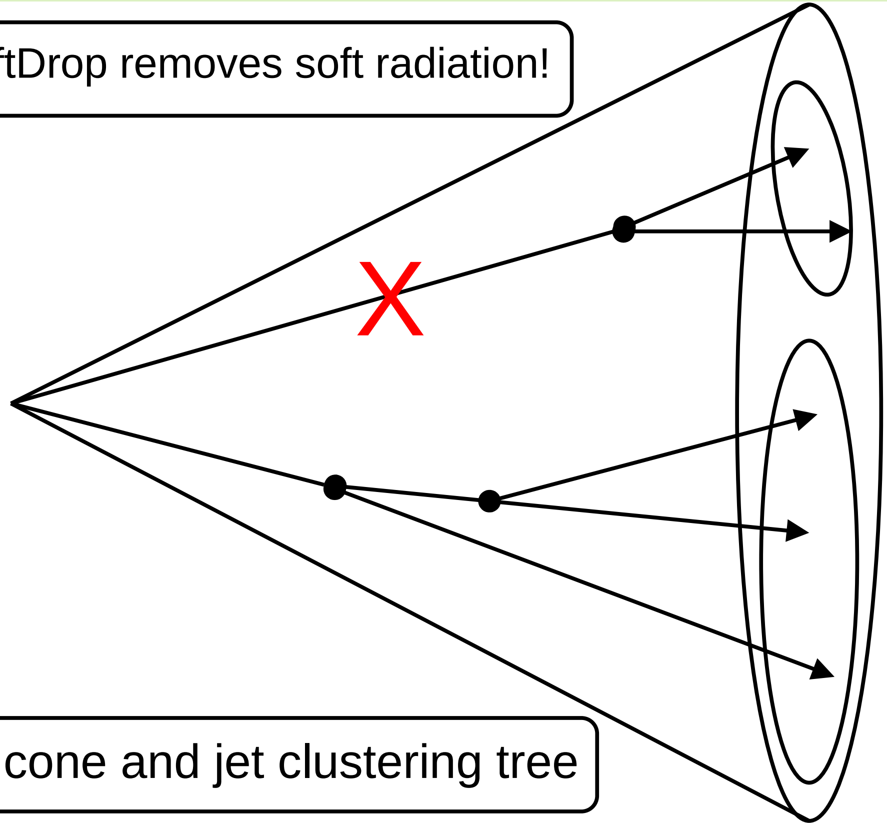

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Here z_{cut} and β control the intensity of grooming

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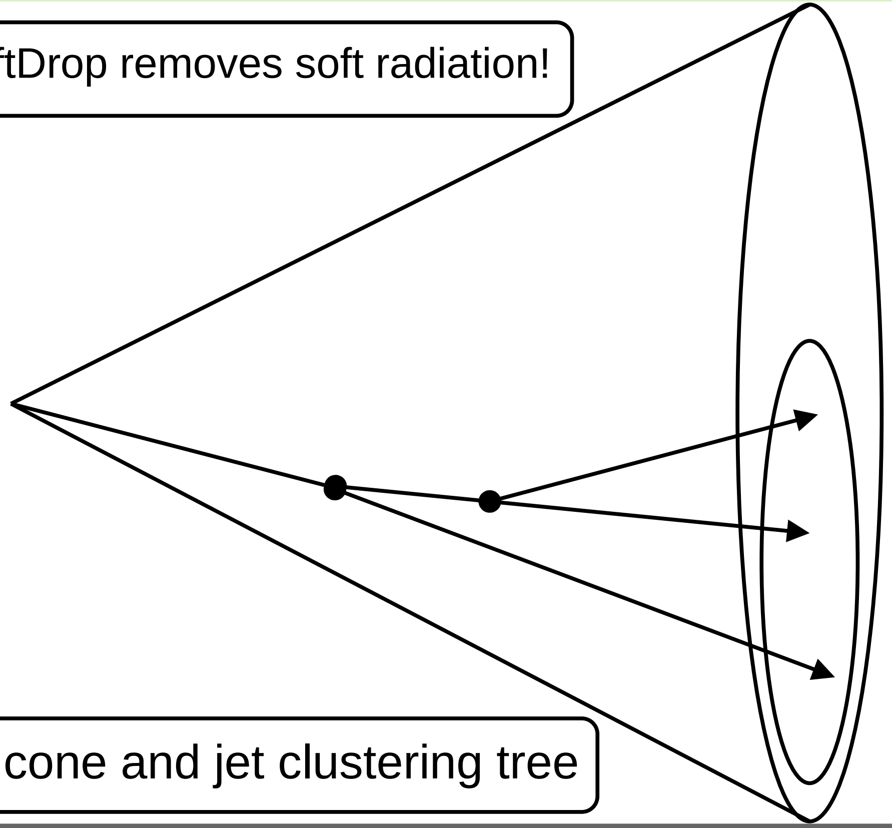

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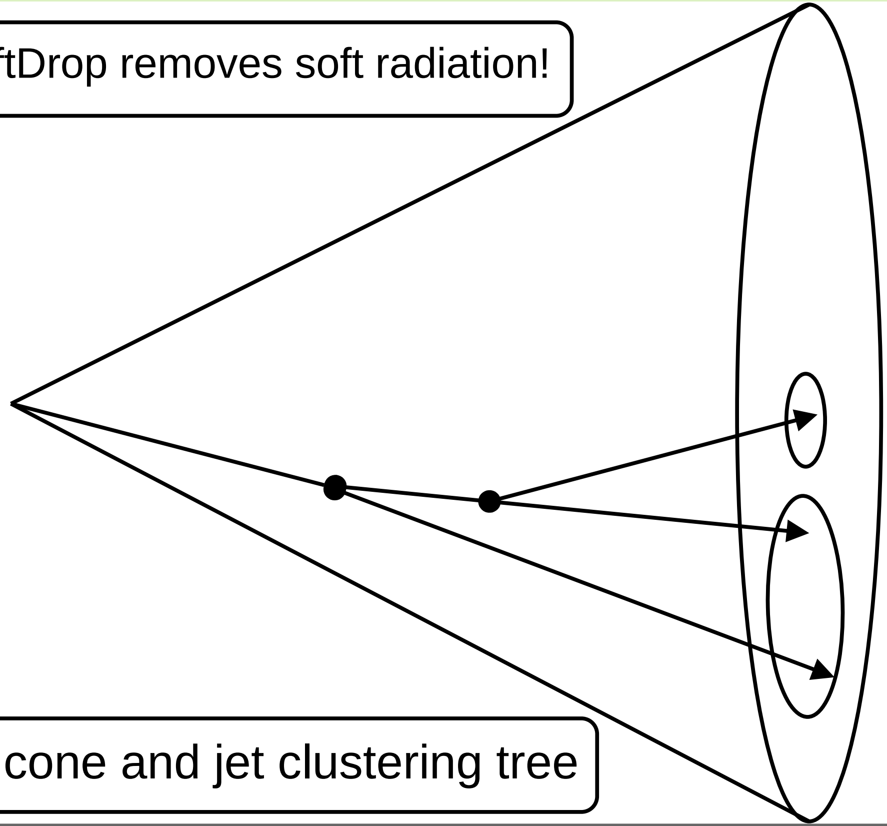

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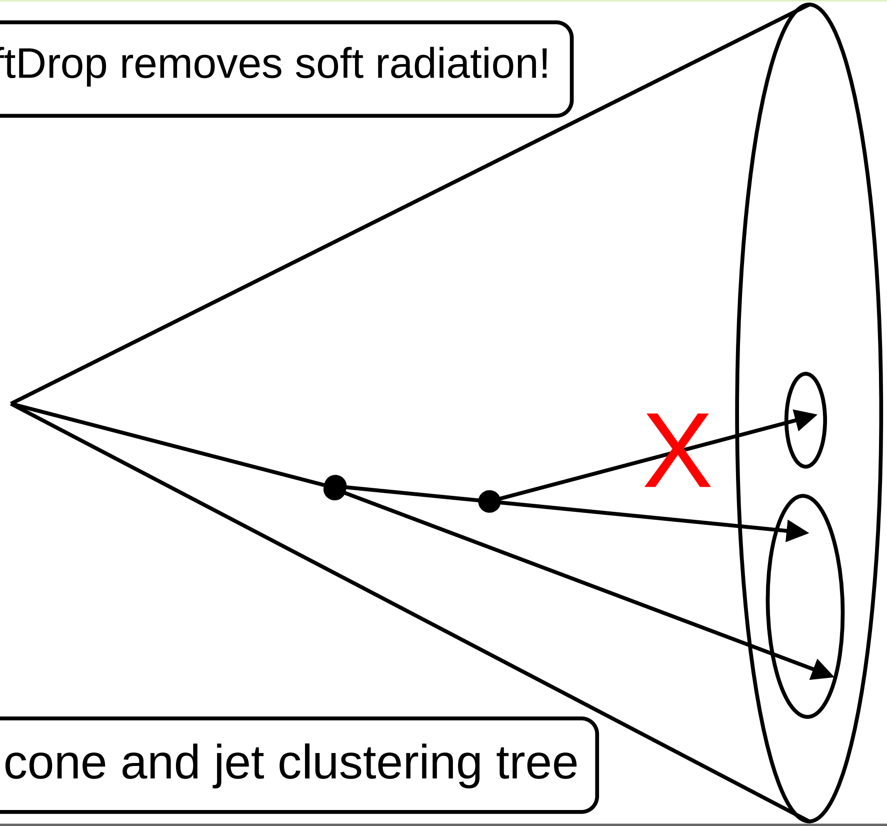

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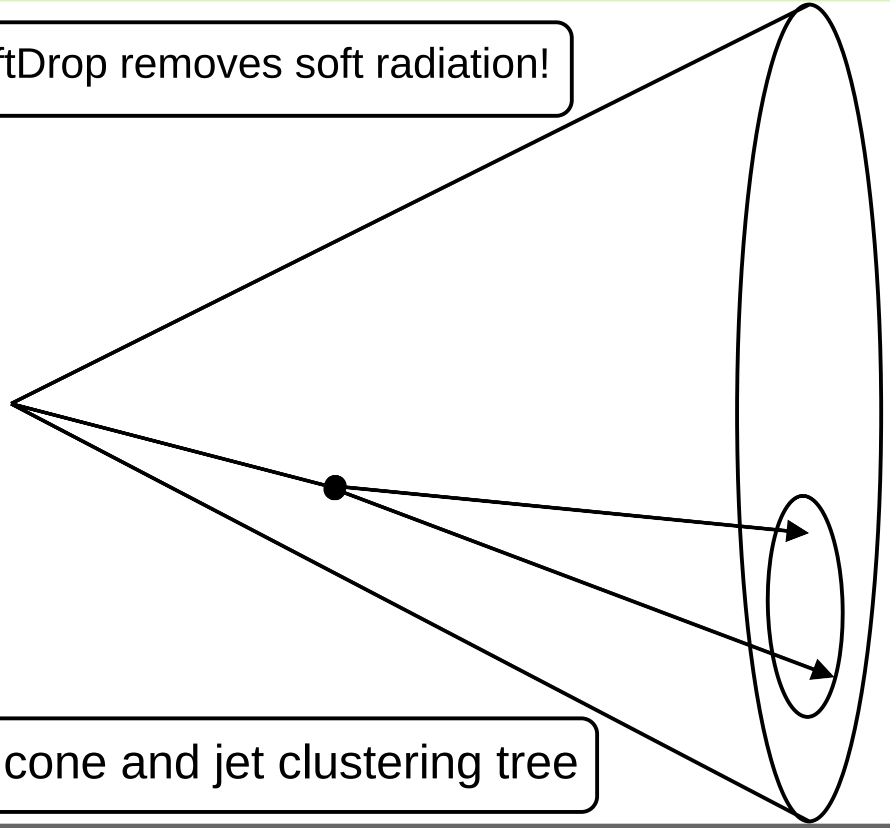

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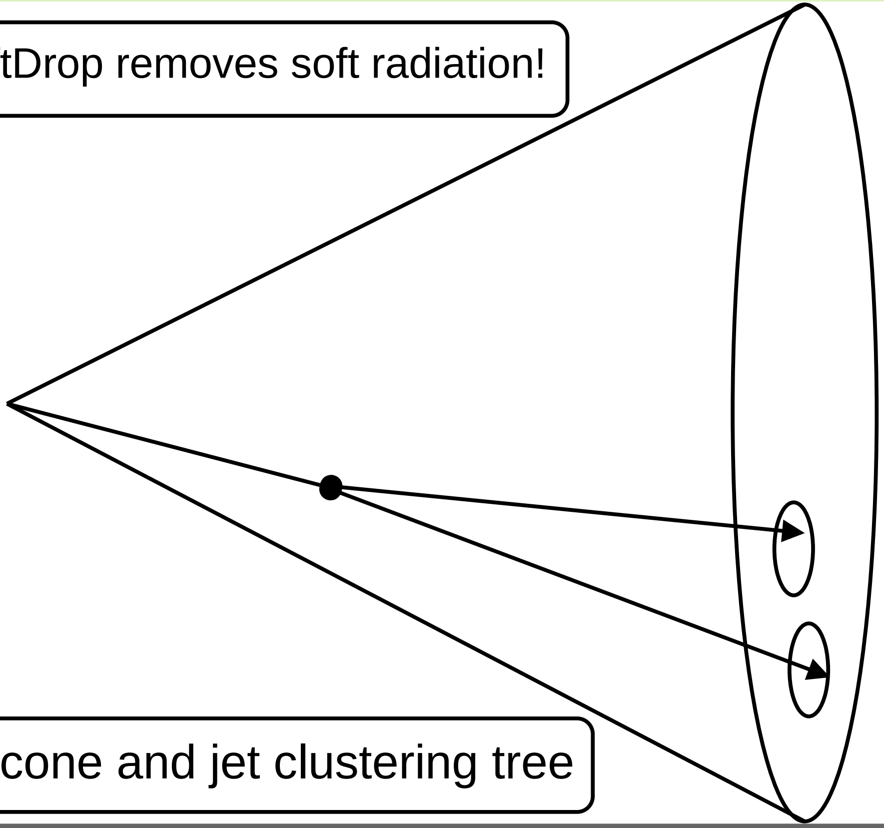

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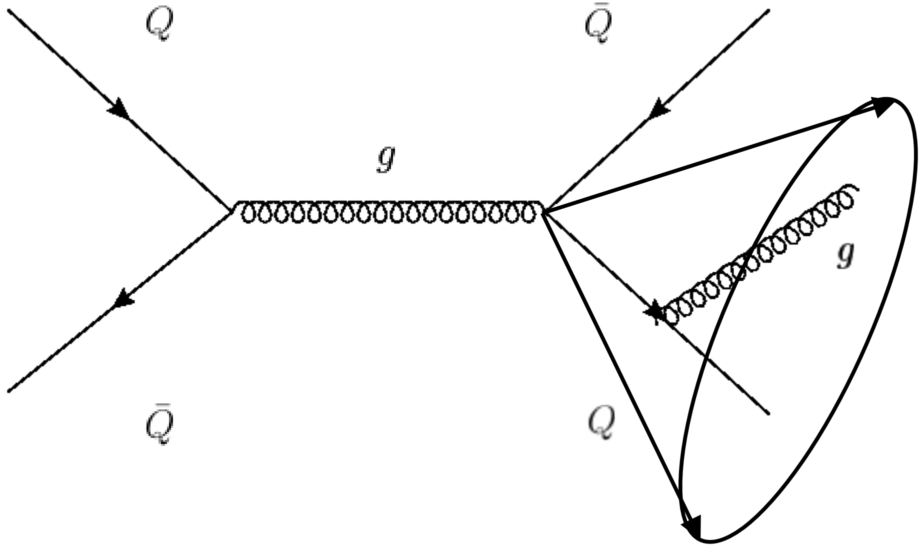
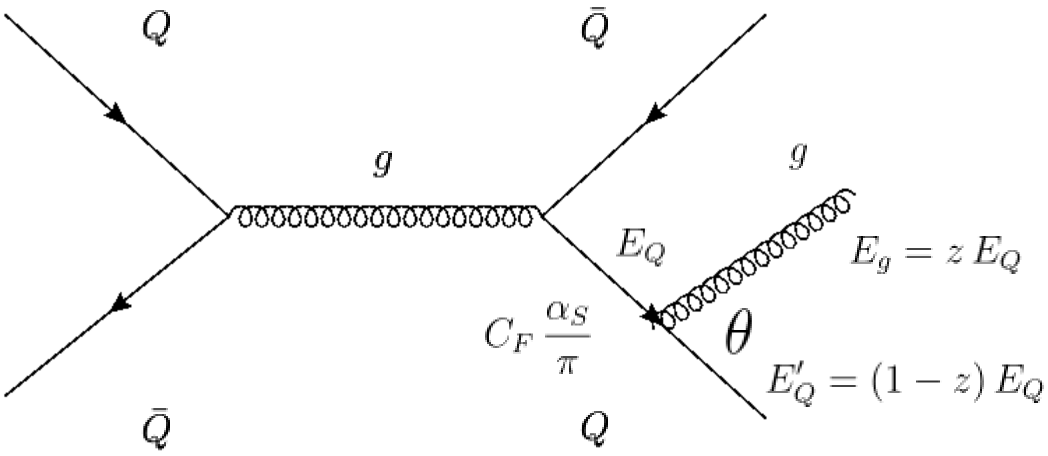
Fixed order (FO) jet substructure calculations



$$Q + \bar{Q} \longrightarrow Q + \bar{Q} + g$$

- A “standard” 2-to-2 QCD process cannot be used for jet substructure calculations (no substructure!)
- So one needs to take 2-to-2 process and add more emissions to it
- Jet substructure can be studied already for the 2-to-3 processes
- It is called fixed-order (FO) calculation.
- However, higher order corrections e.g. 2-to-4, 2-to-5 etc. in general are difficult to calculate

Resummation: Kinoshita–Lee–Nauenberg (KLN) theorem



$$Q + \bar{Q} \longrightarrow Q + \bar{Q} + g$$

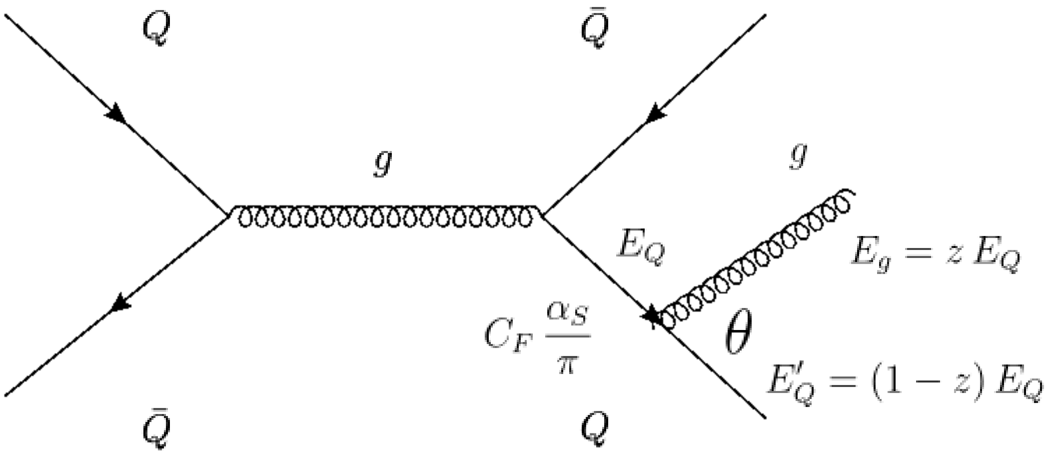
Indistinguishable final states (containing soft and/or collinear particles) are degenerated which causes singularities

Sum over all final states removes singularities

2-to-3 cross section gets a di-log enhancement:

$$d\sigma \sim d(\log \theta^2)\, d(\log z)$$

Resummation: leading log (LL)



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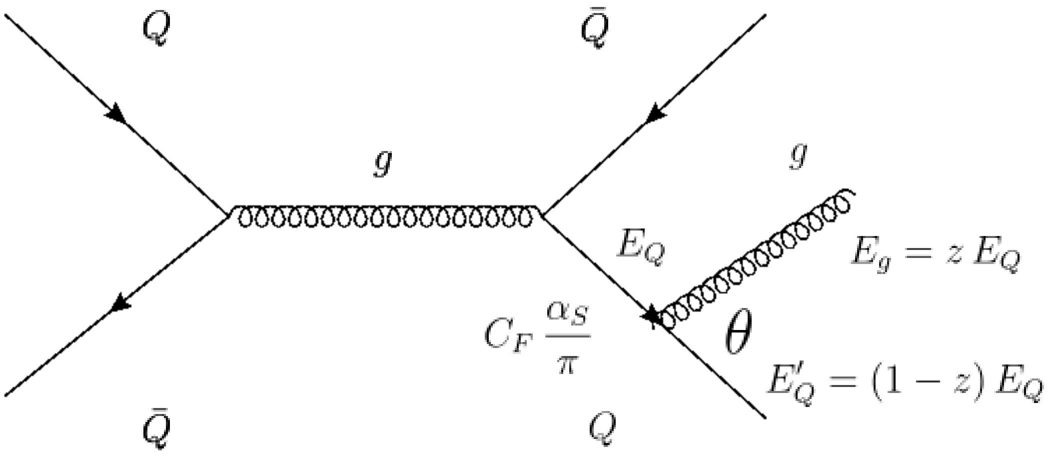
Let's define a simple IRC safe jet substructure observable:

$$\tau = z\theta^2$$

In case of multiple emissions:

$$\tau = \sum_{i=\mathrm{gluon}} z_i \theta_i^2$$

Resummation: leading log (LL)



$$Q + \bar{Q} \longrightarrow Q + \bar{Q} + g$$

Multiple gluon emissions exponentiate:

$$P_q(x < \tau) = \exp\left(-\frac{\alpha_s}{\pi}\frac{C_F}{2}\log^2\tau\right)$$

Similar expression can be obtained for quark emissions:

$$P_g(x < \tau) = \exp\left(-\frac{\alpha_s}{\pi}\frac{C_A}{2}\log^2\tau\right)$$

Note that both expressions are finite if $\tau \to 0$ whereas FO result diverges!

For more details see: 1709.06195

Resummation: next-to-leading log (NLL)

In general:

$$P_{q/g} = 1 + \alpha_S \left(c_{22}L^2 + c_{21}L + ...\right) + \alpha_S^2 \left(c_{24}L^4 + c_{23}L^3 + ...\right) + ...$$

Both LL and NLL resummation can be performed separately for quark and gluon production channels!

Therefore, resummed expressions can be used to define “quark” and “gluon” jets!

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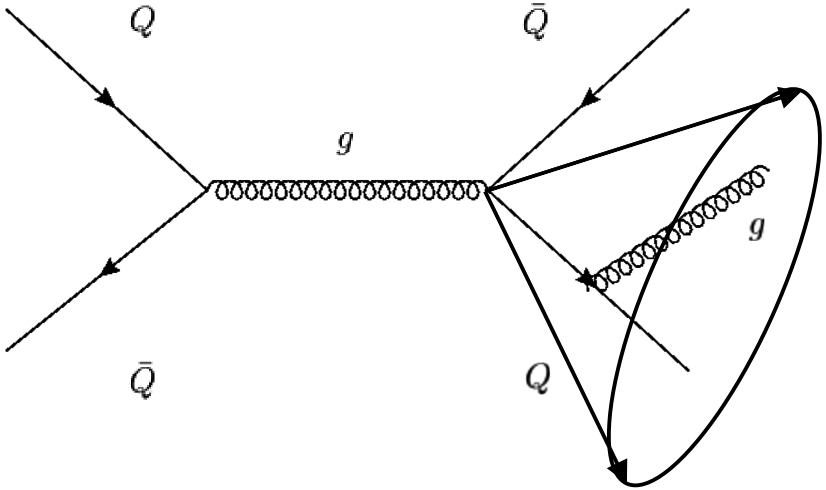
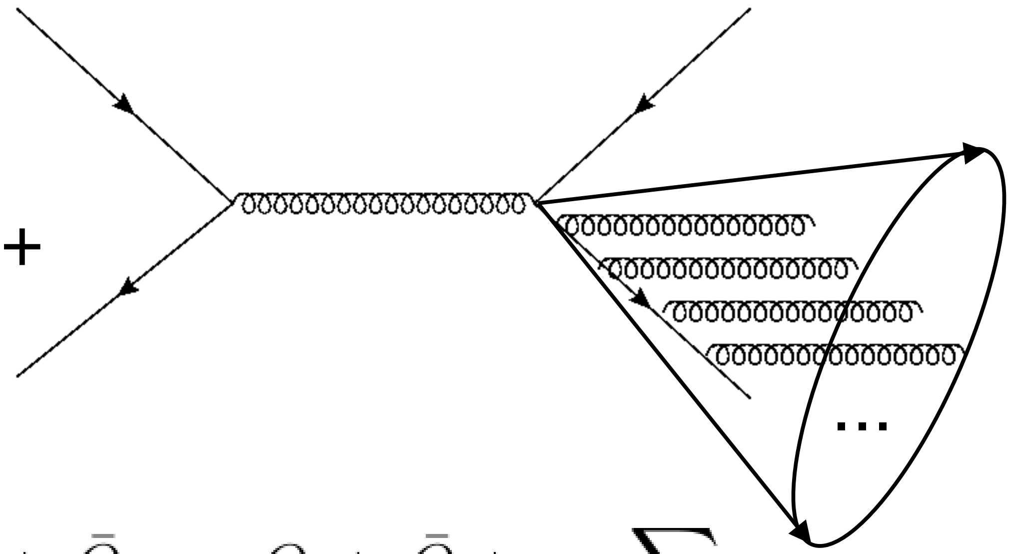
Resummation and matching to fixed order (FO) results

FO calculations



Resummation

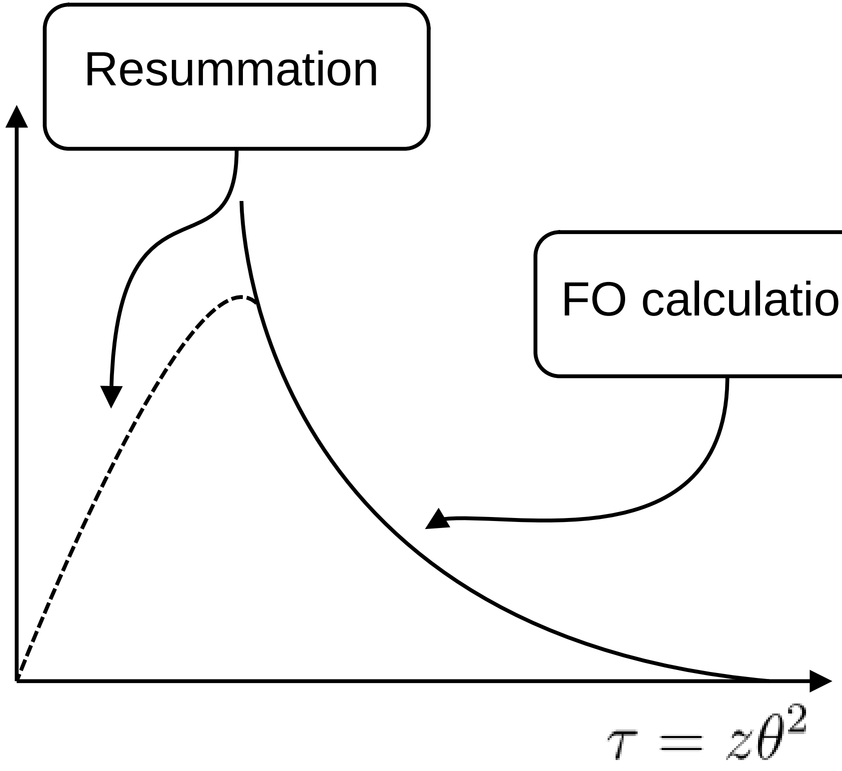
+



$$Q + \bar{Q} \longrightarrow Q + \bar{Q} + g$$

$$Q + \bar{Q} \longrightarrow Q + \bar{Q} + \sum_{soft,\, collinear} g$$

Resummation and matching to fixed order (FO) results



Matching to FO results:

- Excludes double counting between overlapping phase space regions.
- Provides finite results at small values of observable of interest
- Matching “quark” and “gluon” jet contributions can be done separately which leads to NLL' accuracy level

CAESAR approach by Banfi, Salam and Zanderighi

CAESAR allows to automate resummation for each observable that can be parametrized as

$$\Sigma_{\rm res}(v) = \sum_{\delta} \Sigma^{\delta}_{\rm res}(v)\,, \ \ \text{with}$$

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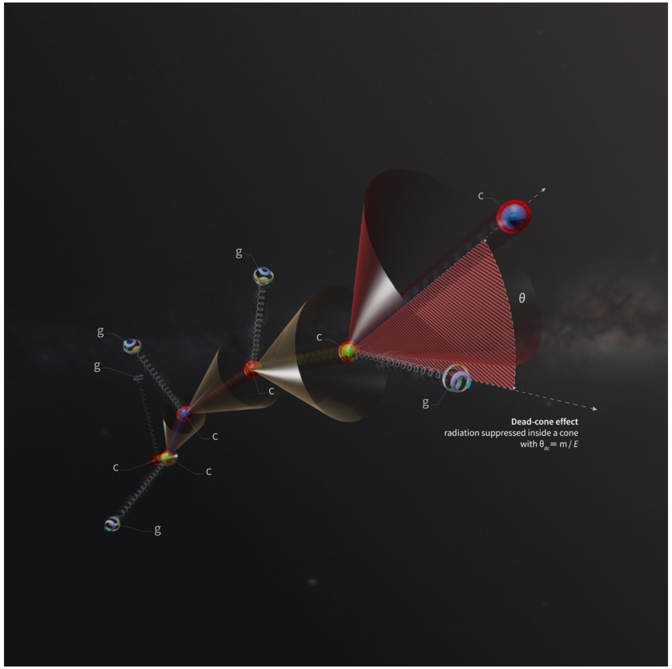
Impact of the quark mass: Dead cone effect



Credit: CERN

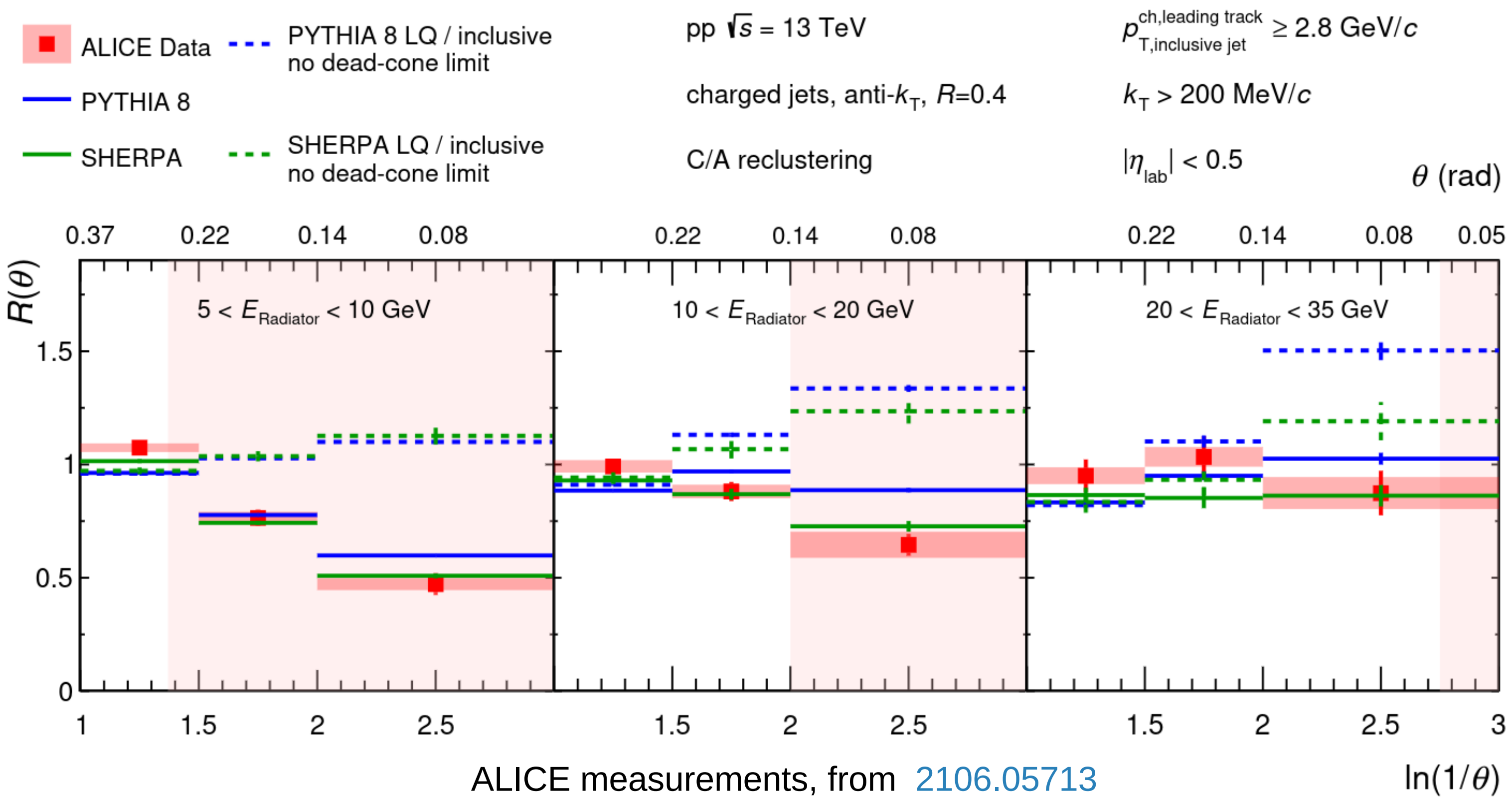
$$d\sigma \approx C_F \frac{\alpha_S}{\pi} \frac{(2\sin\theta/2)^2 d(2\sin\theta/2)^2}{\left[(2\sin\theta/2)^2 + \theta_D^2\right]^2} \frac{dz}{z} \approx C_F \frac{\alpha_S}{\pi} \frac{\theta^2 d\theta^2}{\left[\theta^2 + \theta_D^2\right]^2} \frac{dz}{z}$$

$$\theta_D = \lim_{E_g \to 0} (2m_Q/\sqrt{s}) = m_Q/E_Q$$

$$d\sigma \sim d(\log\theta^2)\, d(\log z) \qquad d\sigma \sim \left(\frac{\theta}{\theta_D}\right)^2 d\left(\frac{\theta}{\theta_D}\right)^2 d(\log z)$$

- Dead cone is a general property of gauge theories (e.g. it also exist in QED)
- The QCD predictions were made a long time ago Dokshitzer et al 91, Ellis et al 96
- The direct observation was made only a few years ago 2106.05713 by ALICE
- Jet substructure observables can be used to probe the dead cone effect

Impact of the quark mass: Dead cone effect



ALICE measurements, from 2106.05713

Collinear and quasi-collinear limits

Standard ECFs is defined as

$$e_2^\alpha = \sum_{i\neq j} \frac{p_{t_i} p_{t_j}}{p_t^2} \left(\frac{\Delta R_{ij}}{R_0}\right)^\alpha$$

An alternative definition can be written as a dot product

$$\dot{e}_2^\alpha = \sum_{i\neq j} \frac{p_{t_i} p_{t_j}}{p_t^2} \left(\frac{2 p_i \cdot p_j}{p_{t_i}\, p_{t_j}\, R_0^2}\right)^{\alpha/2}$$

In soft and collinear limit both definitions coincide

In our case we work in quasi-collinear limit (hep-ph/0201036) where we keep ratio m_i/p_{ti} fixed

$$p_i \cdot p_j = m_{ti}\, m_{tj} \cosh(y_i - y_j) - p_{ti}\, p_{tj} \cos(\phi_i - \phi_j)$$

In the quasi-collinear limit one gets

$$\dot{e}_2^\alpha \simeq \sum_{i\neq j} \frac{p_{ti} p_{tj}}{p_t^2} \left(\frac{m_i^2}{p_{ti}^2 R_0^2} + \frac{m_j^2}{p_{tj}^2 R_0^2} + \frac{\Delta R_{ij}^2}{R_0^2}\right)^{\alpha/2}$$

Hence the dead cone transition must happen around $\left(\frac{m}{p_t R_0}\right)^\alpha$

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Five definitions of jet angularities:

Standard jet angularity is defined as

$$\lambda^\alpha = \sum_i \frac{p_{ti}}{p_t}\left(\frac{\Delta R_i}{R_0}\right)^\alpha$$

One can define reference axis in two different ways

$$n_0 = (\cosh y, \cos\phi, \sin\phi, \sinh y)\,,$$
$$n = \left(\frac{m_t}{p_t}\cosh y, \cos\phi, \sin\phi, \frac{m_t}{p_t}\sinh y\right),$$

which coincide in case of massless particles

So we get four-more definitions

$$\dot{\lambda}_0^\alpha = \sum_i \frac{p_{ti}}{p_t}\left(\frac{2p_i\cdot n_0}{p_{ti}R_0^2}\right)^{\frac{\alpha}{2}}\,,$$ massless axis, all particles

$$\mathring{\lambda}_0^\alpha = \sum_{i\neq n} \frac{p_{ti}}{p_t}\left(\frac{2p_i\cdot n_0}{p_{ti}R_0^2}\right)^{\frac{\alpha}{2}}$$ massless axis, no alignment

$$\dot{\lambda}^\alpha = \sum_i \frac{p_{ti}}{p_t}\left(\frac{2p_i\cdot n}{p_{ti}R_0^2}\right)^{\frac{\alpha}{2}}$$ massive axis, all particles

$$\mathring{\lambda}^\alpha = \sum_{i\neq n} \frac{p_{ti}}{p_t}\left(\frac{2p_i\cdot n}{p_{ti}R_0^2}\right)^{\frac{\alpha}{2}}$$ massive axis, no alignment

Jet angularities in quasi-collinear limit:

So we get four-more definitions

$$\dot{\lambda}_0^\alpha = \sum_i \frac{p_{ti}}{p_t}\left(\frac{2p_i\cdot n_0}{p_{ti}R_0^2}\right)^{\frac{\alpha}{2}}, \quad \xrightarrow{\text{quasi-collinear limit}} \quad \dot{\lambda}_0^\alpha \simeq \sum_{i\neq n}\frac{p_{t_i}}{p_t}\left(\frac{m_i^2}{p_{t_i}^2R_0^2}+\frac{\Delta R_i^2}{R_0^2}\right)^{\frac{\alpha}{2}} + \left(\frac{m_n^2}{p_t^2R_0^2}\right)^{\frac{\alpha}{2}}$$

$$\mathring{\lambda}_0^\alpha = \sum_{i\neq n} \frac{p_{ti}}{p_t}\left(\frac{2p_i\cdot n_0}{p_{ti}R_0^2}\right)^{\frac{\alpha}{2}} \quad \xrightarrow{\text{quasi-collinear limit}} \quad \mathring{\lambda}_0^\alpha = \sum_{i\neq n}\frac{p_{t_i}}{p_t}\left(\frac{m_i^2}{p_{t_i}^2R_0^2}+\frac{\Delta R_i^2}{R_0^2}\right)^{\frac{\alpha}{2}}$$

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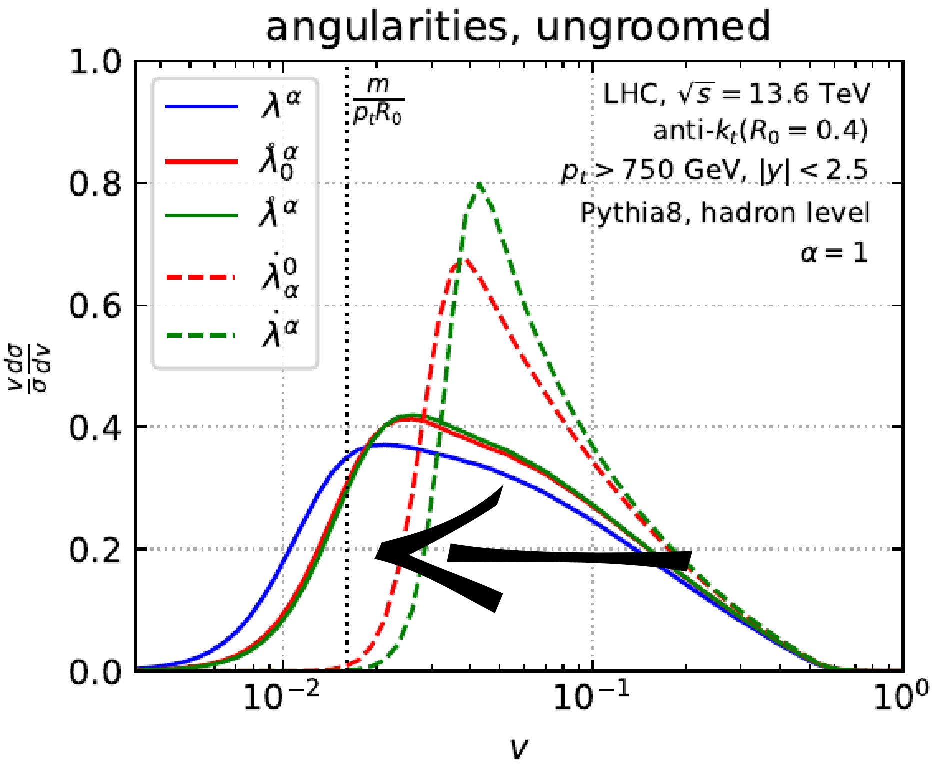
Jet angularities MC-simulations:



$$\dot{\lambda}_0^\alpha \simeq \sum_{i\neq n} \frac{p_{t_i}}{p_t}\left(\frac{m_i^2}{p_{t_i}^2 R_0^2} + \frac{\Delta R_i^2}{R_0^2}\right)^{\frac{\alpha}{2}} + \boxed{\left(\frac{m_n^2}{p_t^2 R_0^2}\right)^{\frac{\alpha}{2}}}$$

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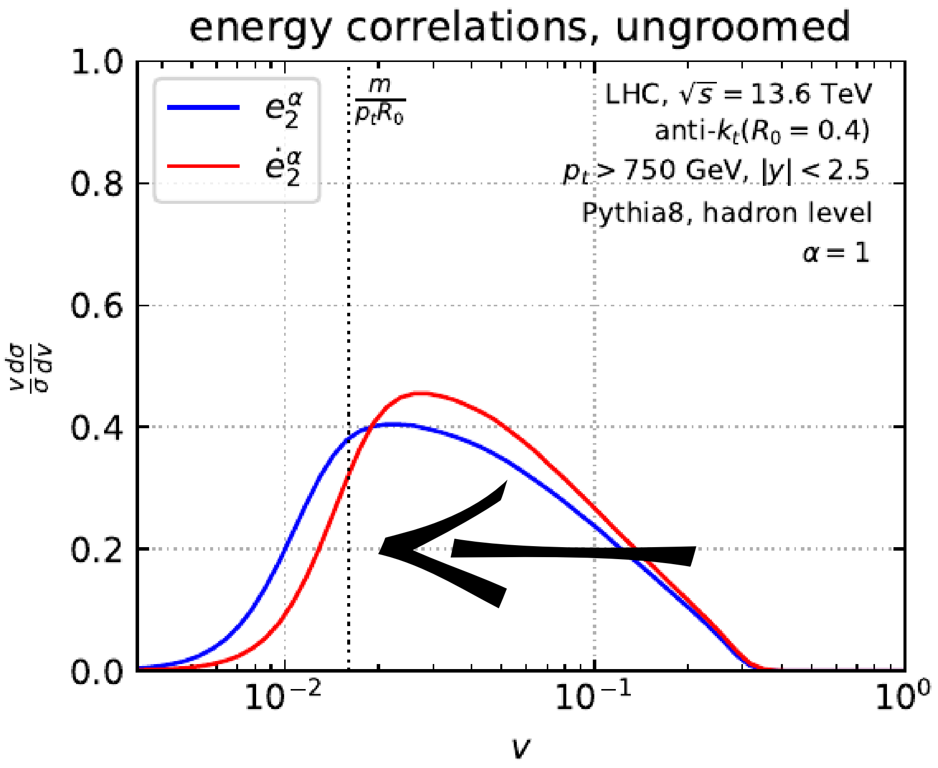
$$\mathring{\lambda}^\alpha \simeq \sum_{i\neq n} \frac{p_{t_i}}{p_t}\left(\frac{m_i^2}{p_{t_i}^2 R_0^2} + \frac{m_n^2}{p_t^2 R_0^2} + \frac{\Delta R_i^2}{R_0^2}\right)^{\frac{\alpha}{2}}$$

ECFs MC-simulations:

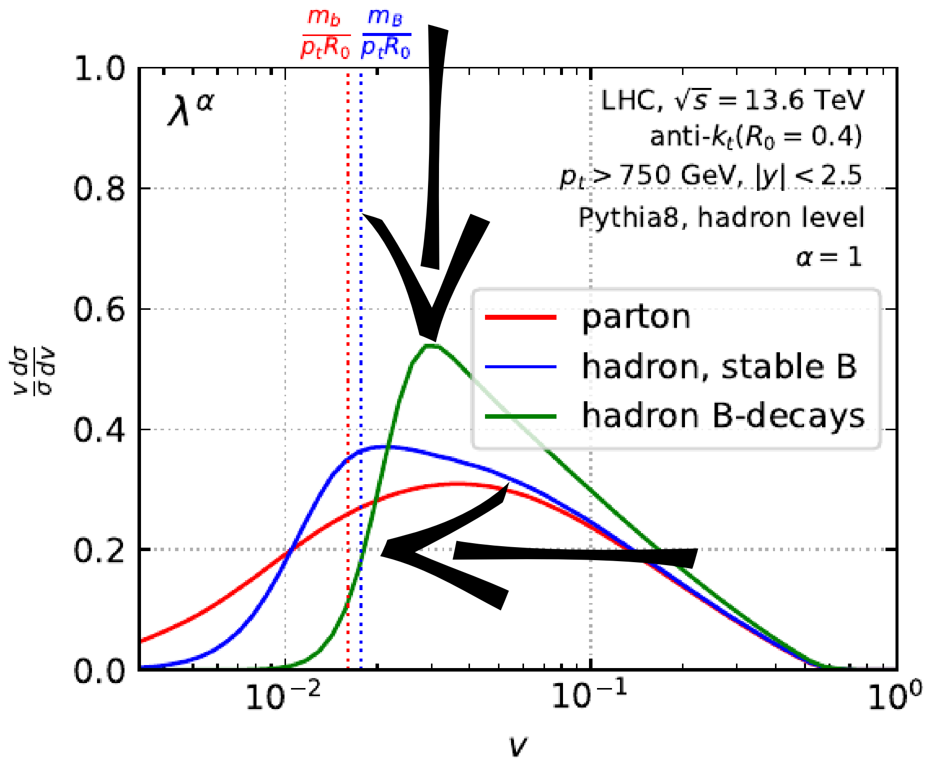


Since ECFs do not depend on the definition of the jet axis, they do not acquire ‘’pathological’’ terms caused by particles aligned to the jet axis

$$e_2^\alpha = \sum_{i \neq j} \frac{p_{t_i} p_{t_j}}{p_t^2} \left(\frac{\Delta R_{ij}}{R_0} \right)^\alpha$$

$$\dot{e}_2^\alpha \simeq \sum_{i \neq j} \frac{p_{ti} p_{tj}}{p_t^2} \left(\frac{m_i^2}{p_{ti}^2 R_0^2} + \frac{m_j^2}{p_{tj}^2 R_0^2} + \frac{\Delta R_{ij}^2}{R_0^2} \right)^{\alpha/2}$$

The role of the decay of B-hadrons



- Since b-quark mass differs from masses of B-hadrons we expect dead-cone transition to change its position.
- Decays of B-hadrons "spoil" analytical calculations.
- B-hadrons must be reconstructed (kept stable in Pythia) to avoid drastic modifications, our observation agree with Lee, Shrivastava, Vaidya

Fix-order analysis (an example)

Most of the ingredients (radiators) needed for resummation can be obtained by performing fix order calculations

In quasi-collinear limit matrix elements factorize as

$$|\mathcal{M}|^2 \simeq \frac{8\pi\alpha_S z(1-z)}{k_t^2 + z^2 m^2} P_{gq}(z, k_t^2)|\mathcal{M}_0|^2.$$

The splitting functions now depend on the quark mass

$$P_{gq}(z, k_t^2) = C_F \left(\frac{1 + (1-z)^2}{z} - \frac{2m^2 z(1-z)}{k_t^2 + z^2 m^2} \right)$$

The master equation for the radiator is given by

$$\begin{aligned} \mathcal{R}_V^{(\mathrm{f.o.})}(v, \xi) &= -\frac{\alpha_S(\mu^2)}{2\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2 + z^2 m^2} \int_0^1 dz P_{gq}(z, k_t^2) \Big[\Theta(v - V(k_t, \eta)) - 1 \Big] \\ &= \frac{\alpha_S(\mu^2)}{2\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2 + z^2 m^2} \int_0^1 dz P_{gq}(z, k_t^2) \Theta(V(k_t, \eta) - v) \end{aligned}$$

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$$= \frac{\alpha_S(\mu^2)}{2\pi}\int_0^{Q^2}\frac{dk_t^2}{k_t^2+z^2m^2}\int_0^1 dz P_{gq}(z,k_t^2)\Theta(V(k_t,\eta)-v)$$

where $V(k_t,\eta)$ is observable parametrization, e.g. $V=\frac{k_t^\alpha z^{1-\alpha}}{Q^\alpha}$, and $\xi=\frac{m^2}{Q^2}$ $x=\frac{\xi}{v^{\frac{2}{\alpha}}}$,

which gives

$$\mathcal{R}_{\lambda^\alpha}^{(\text{f.o.})}(v,\xi) = \frac{\alpha_S(\mu^2)\ C_F}{\pi}\left(\frac{1}{\alpha}\log^2 v - \frac{\alpha}{4}\log^2(1+x) + \frac{3}{2\alpha}\log v + \left(\frac{3}{4}-\frac{\alpha}{2}\right)\log(1+x) - \frac{\alpha}{2}\,\text{Li}_2\left(\frac{x}{1+x}\right)\right.$$
$$\left. + \frac{\alpha-2}{\alpha+2}x\ {}_2F_1\left(1,1+\frac{\alpha}{2};2+\frac{\alpha}{2};-x\right) + \frac{x}{4(\alpha+1)}\ {}_2F_1\left(1,1+\alpha;2+\alpha;-x\right) + \frac{7}{4\alpha}\right)$$

Fix-order analysis (an example)

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without mass dependence we get a "standard" result

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Fix-order analysis (an example)

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mass dependence introduces additional logs

$$\mathcal{R}^{(\mathrm{f.o.})}_{\lambda^\alpha}(v,\xi) = \frac{\alpha_S(\mu^2)C_F}{\pi}\left(\frac{1}{\alpha}\log^2 v + \frac{3}{2\alpha}\log v + \frac{7}{4\alpha} + \mathcal{O}(x)\right)$$

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Fix-order analysis for groomed jets (an example)

In case of the groomed jets one gets additional phase space constraints

$$\bar{\mathcal{R}}_V^{(\mathrm{f.o.})} = \frac{\alpha_S(\mu^2)C_F}{2\pi}\int_0^{Q^2}\frac{dk_t^2}{k_t^2+z^2m^2}\int_0^1 dz P_{qg}(z,k_t^2)\Theta(V(k_t,\eta)-v)\Theta\left(\min(z,1-z)-z_{\mathrm{cut}}\right)$$

$$= \frac{\alpha_S(\mu^2)C_F}{2\pi}\int_0^{Q^2}\frac{dk_t^2}{k_t^2+z^2m^2}\int_{z_{\mathrm{cut}}}^{1-z_{\mathrm{cut}}} dz P_{qg}(z,k_t^2)\Theta(V(k_t,\eta)-v)$$

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SoftDrop introduces logarithms of z_{cut} and complicates analytic structure

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NLL resummation

At our accuracy level $\mathcal{R}_V(v,\xi) \xrightarrow{\text{NLL}} R(v,\xi)$ so we can reuse our fixed-order results!

$$\Sigma_{\text{res}}(v) = \sum_{\delta} \Sigma^{\delta}_{\text{res}}(v)\,, \text{ with}$$

$$\Sigma^{\delta}_{\text{res}}(v) = \int d\mathcal{B}_\delta \frac{d\sigma_\delta}{d\mathcal{B}_\delta} \exp\left[-\sum_{l\in\delta} R_l^{\mathcal{B}_\delta}(L)\right] \mathcal{P}^{\mathcal{B}_\delta}(L)\mathcal{S}^{\mathcal{B}_\delta}(L)\mathcal{F}^{\mathcal{B}_\delta}(L)\mathcal{H}^{\delta}(\mathcal{B}_\delta)\,,$$

where

- Born cross section $\frac{d\sigma_\delta}{d\mathcal{B}_\delta}$
- Soft function $\mathcal{S}$
- Ratio of PDFs $\mathcal{P}$
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- Collinear radiator R_l
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$$\mathcal{F} = e^{-\gamma_E R'}/\Gamma(1+R') \qquad R' = \partial R/\partial L \qquad L = -\log v$$

NLL resummation

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However there are additional complications due to the running coupling.
For example,

$$R_b(v,\xi) = \int_0^1 dz \int_{z^2m^2}^{Q^2} \frac{dk_t^2}{k_t^2} P_{gq}\left(z, k_t^2 - z^2m^2\right) \frac{\alpha_S^{\text{CMW}}(k_t^2)}{2\pi} \Theta\left(V(k_t,\eta) - v\right)$$

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$$K^{(n_f)} = C_{\text{A}}\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n_f$$

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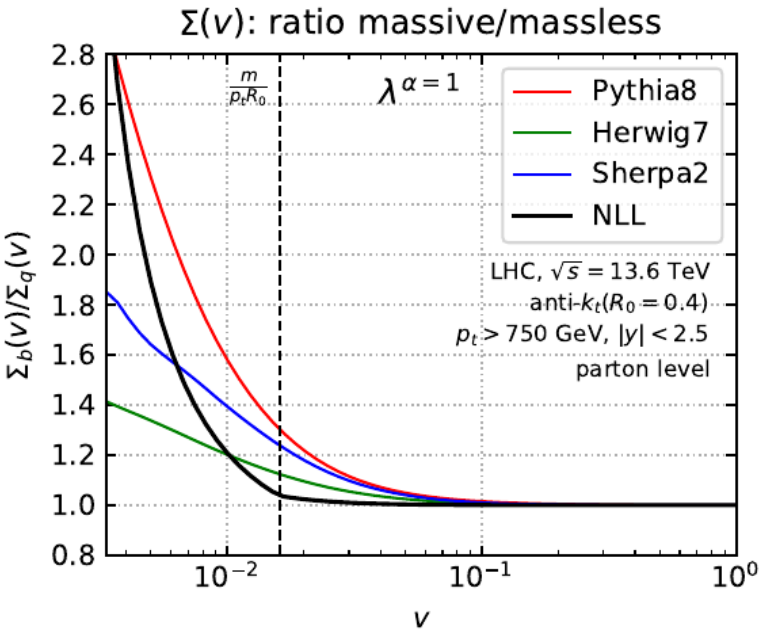
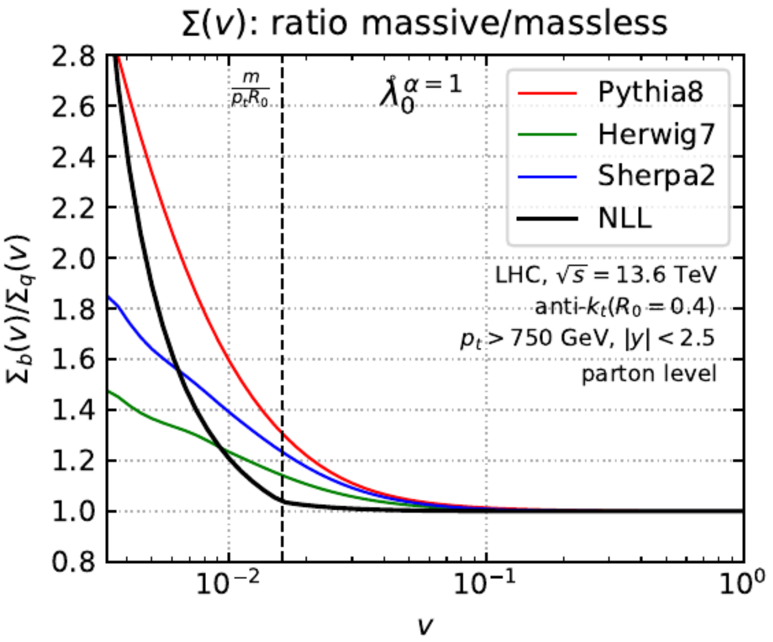
The threshold effects are implemented as

$$\alpha_S(k_t^2) = \alpha_S^{(5)}(k_t^2)\Theta(k_t^2 - m^2) + \alpha_S^{(4)}(k_t^2)\Theta(m^2 - k_t^2)$$

NLL resummation vs. MC (ungroomed)

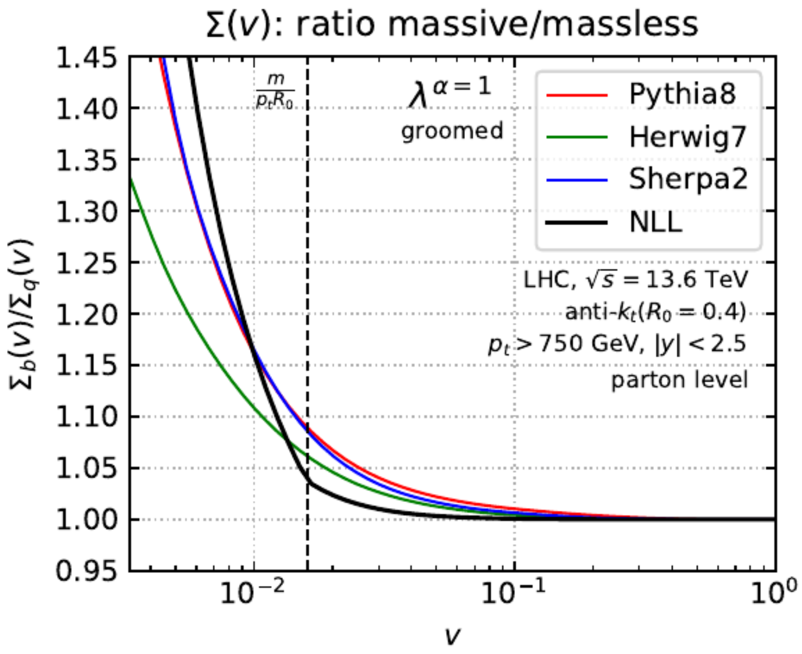


$$\lambda^{\alpha} = \sum_i \frac{p_{ti}}{p_t} \left(\frac{\Delta R_i}{R_0} \right)^{\alpha}$$



$$\mathring{\lambda}_0^{\alpha} = \sum_{i \neq n} \frac{p_{ti}}{p_t} \left(\frac{2 p_i \cdot n_0}{p_{ti} R_0^2} \right)^{\frac{\alpha}{2}}$$

NLL resummation vs. MC (groomed)



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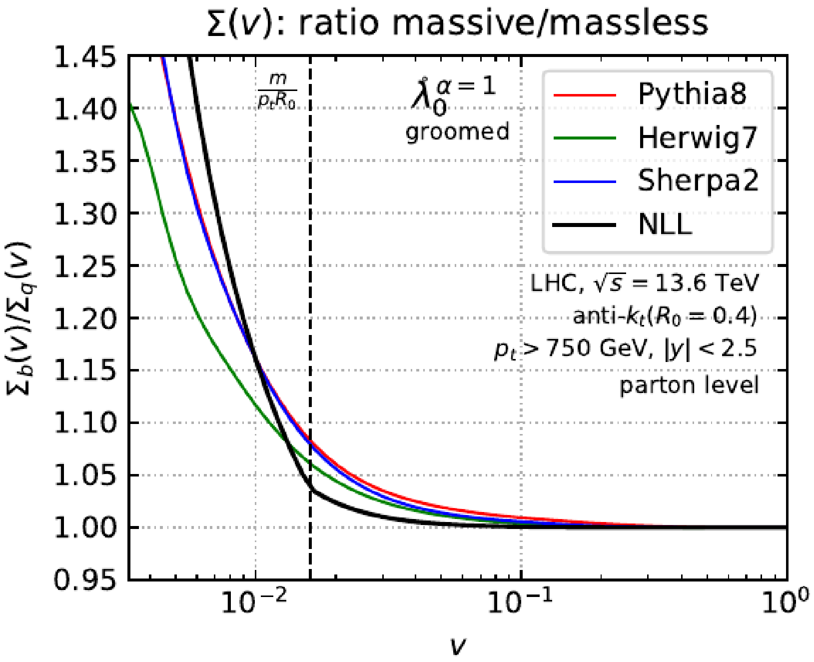

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Summary

- We explored 7 different observable definitions (2 ECFs and 5 jet angularities).
- Corresponding calculations are available as fix order results and NLL summation.
- NLL predictions show clear transition around the dead-cone boundary, however, for MC simulations the transition happens much earlier.
- Non-perturbative corrections have large impact which, however, can be somewhat reduced by SoftDrop grooming.
- B-hadrons must be reconstructed in order to get meaningful results.
- Our results show behavior similar to SCET result by Lee, Shrivastava, Vaidya arXiv:1901.09095 .

Next steps:

- Implement our results into CAESAR plugin to SHERPA (which allows automated usage similar to “standard” MC).
- Perform detailed phenomenological study (currently we were considering high-pT jets, however, one may consider jets with lower-pT to enhance the dead-cone contribution).
- Incorporation of non-perturbative effects into our framework (e.g. by using parton-to-hadron transition matrices as in 2404.04168 , 2112.09545, 2104.06920).

THANK YOU FOR LISTENING!