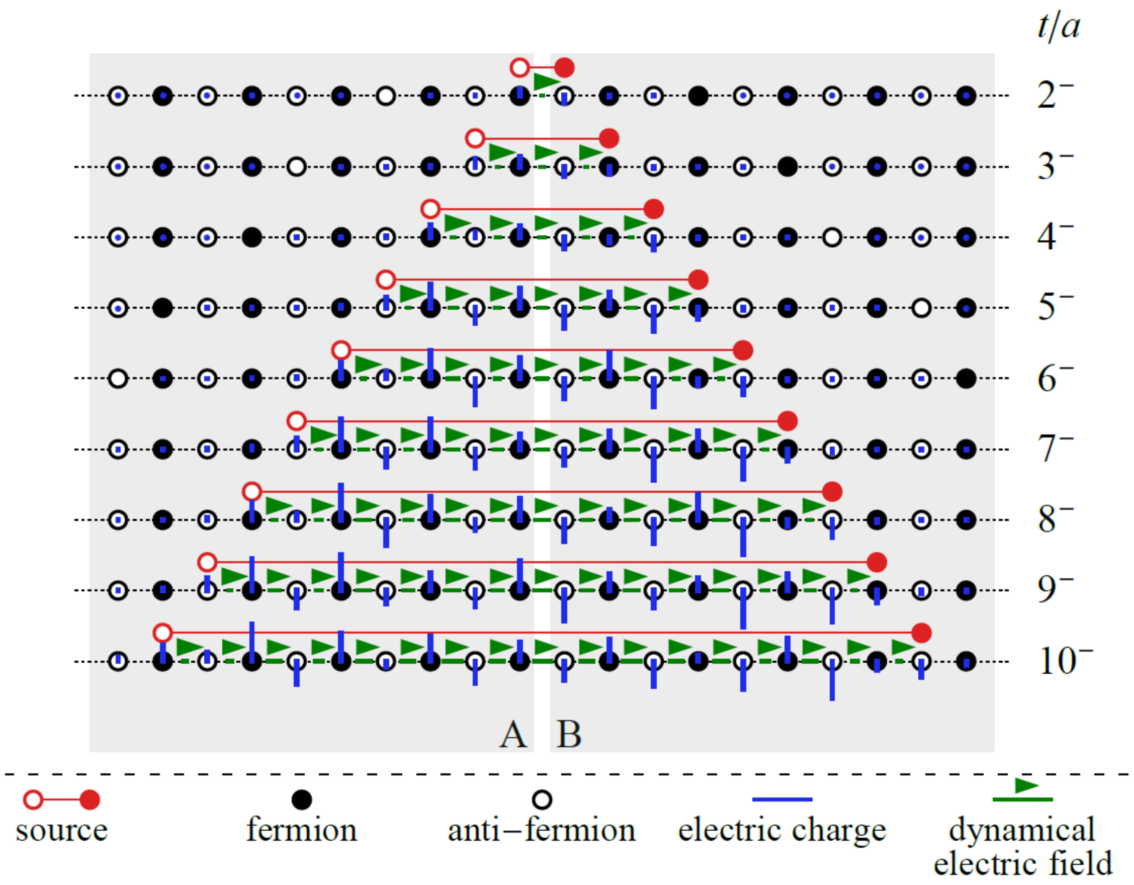
Quantum dynamics of entanglement and hadronization in jet production in the massive Schwinger model



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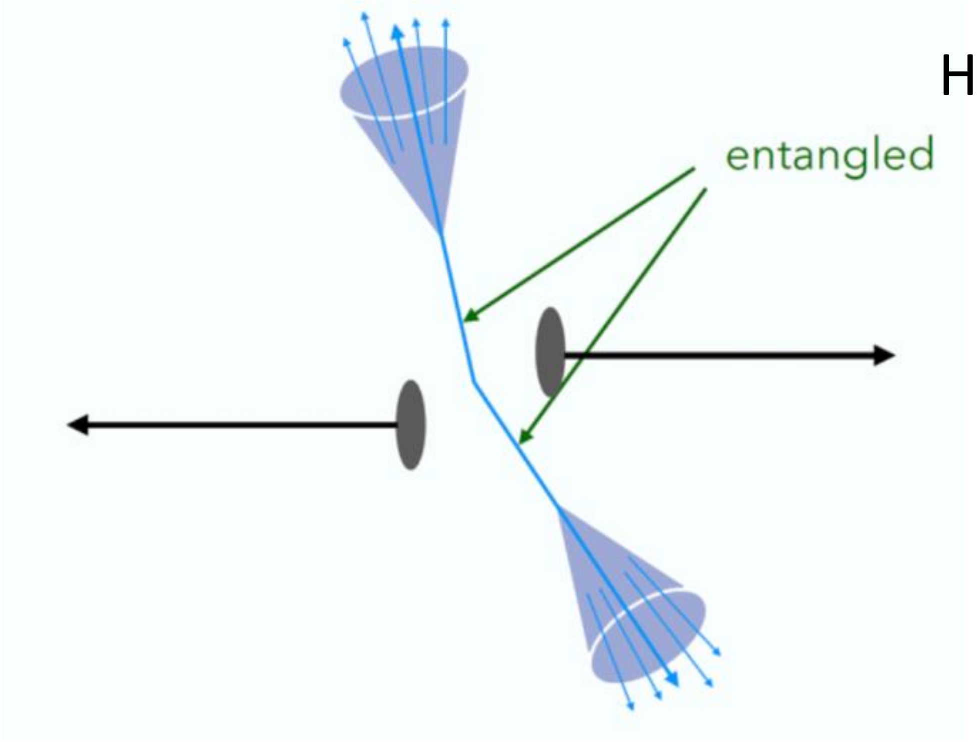
+ arXiv:2404.00087

with A.Florio, K.Ikeda, D.Kharzeev, V.Korepin, S. Shi and K.Yu

INT Program *Heavy ion physics in the EIC era*

Seattle, August 5, 2024

Motivation



How to understand entanglement in jet fragmentation?

Real-time quantum process requires

Real-time quantum simulation

Why Schwinger model?

- Simple enough for a first-principle quantum simulation
- Has a lot of similarity with QCD in 3+1

Outline

- Overview of the Schwinger model
- The setup for numerical simulation of jet fragmentation
- Observations: screening, vacuum modification, entanglement
- Properties of Schmidt states - hadronization
- Approach to thermalization reflected in local observables and entanglement

Schwinger model

Single-flavor (1+1)-dimensional QED:

$$\mathcal{L} = -\frac{1}{4}{F_{\mu\nu}}^2 + \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu + m)\psi$$

Features include:

- No magnetic field/no dynamical photons
- Linear potential between “quarks” – confinement
- Chiral condensate (spontaneous chiral symmetry breaking at *m*=0)

Massless case is exactly solvable, e.g. by bosonization:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_B^2\phi^2, \qquad m_B^2 = \frac{g^2}{\pi}$$

Schwinger model and jets: history

1974

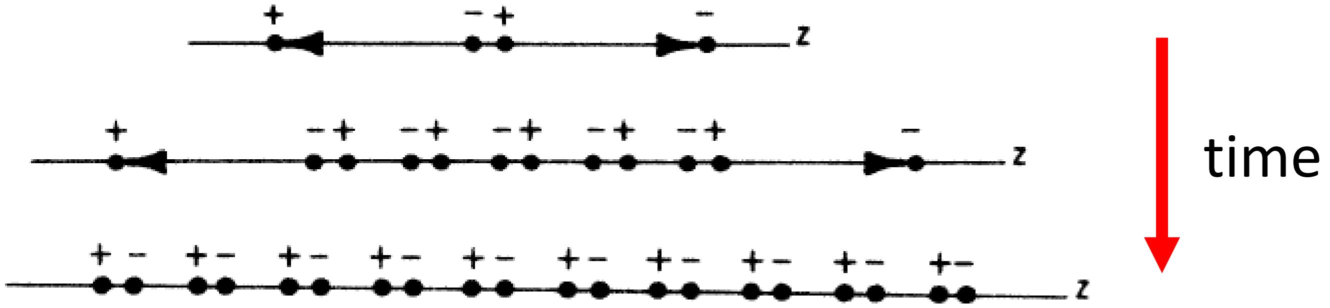
Vacuum polarization and the absence of free quarks

A. Casher,* J. Kogut,† and Leonard Susskind‡

Massless Schwinger model with external source:

$$j_0^{\text{ext}} = g\delta(z-t),\quad j_1^{\text{ext}} = g\delta(z-t)\qquad \text{for } z>0,$$

$$j_0^{\text{ext}} = -g\delta(z+t),\quad j_1^{\text{ext}} = g\delta(z+t)\qquad \text{for } z<0,$$



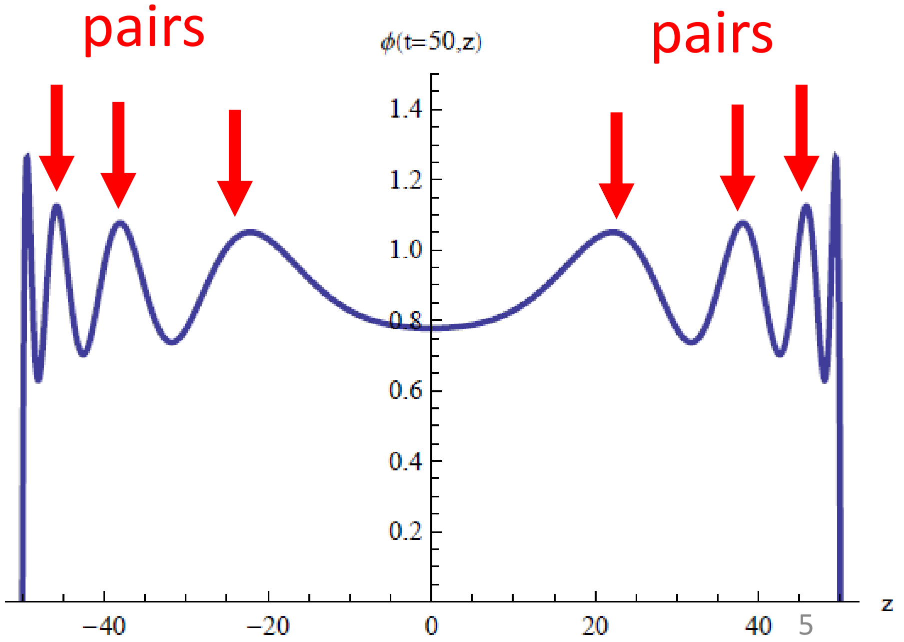
2012

Jet energy loss and fragmentation in heavy ion collisions

Dmitri E. Kharzeev[1,2] and Frashër Loshaj[1]

$$\phi(x) = \theta(t^2 - z^2)[1 - J_0(m\sqrt{t^2 - z^2})]$$

$$j^0 = \partial_z \phi$$



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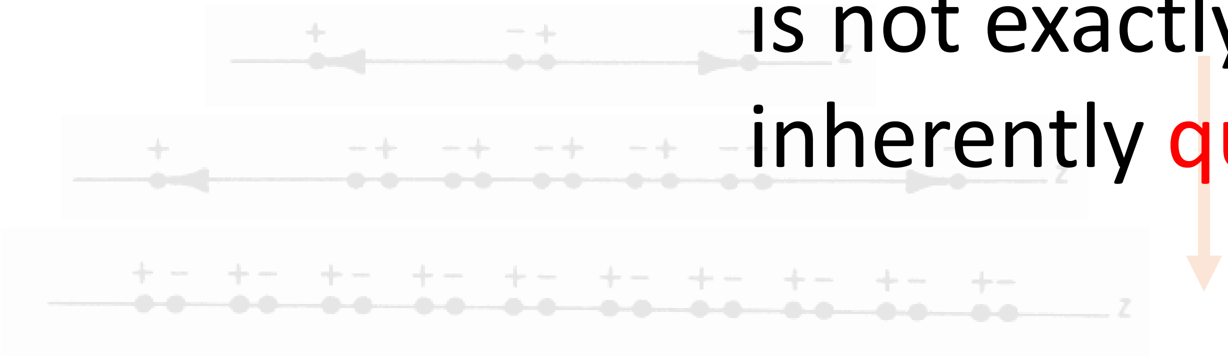

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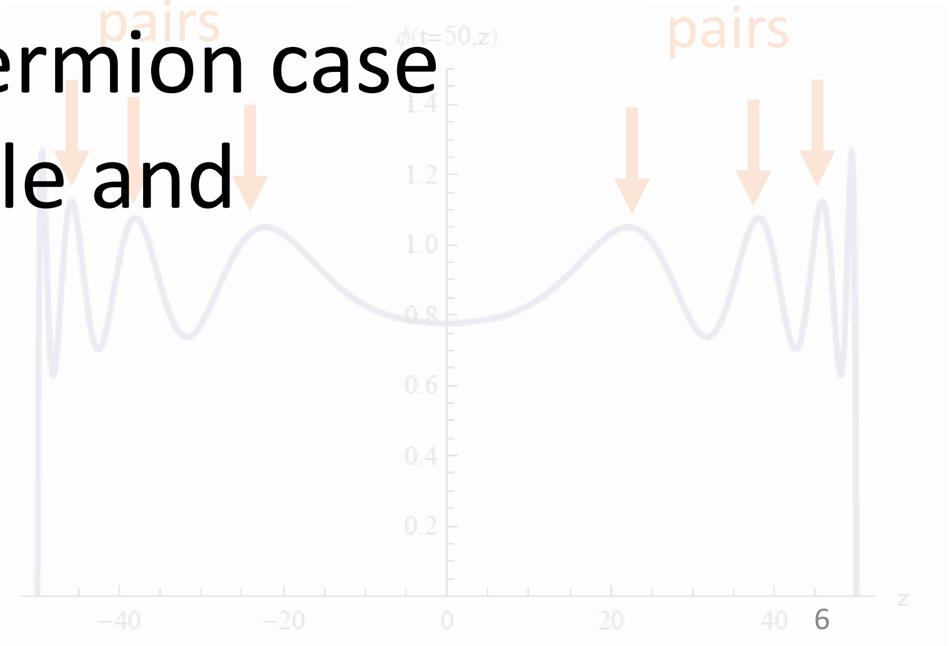

Classical treatment is sufficient in the exactly solvable massless case

However, massive fermion case is not exactly solvable and inherently quantum

The massive Schwinger model on the lattice

Continuum: $$H = \int dx \left[\frac{1}{2}E^2 + \bar{\psi}(-i\gamma^1\partial_1 + g\gamma^1 A_1 + m)\psi\right]$$

Temporal gauge
$A_0 = 0$

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Temporal gauge
$A_0 = 0$

Fermion $\psi(a\,n)$ → $$\frac{1}{\sqrt{a}}\begin{pmatrix}\chi_{2n}\\ \chi_{2n-1}\end{pmatrix}$$ Kogut-Susskind

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{ab}\delta(x-y)$$ → $$\{\chi_i, \chi_j^\dagger\} = \delta_{ij}$$

N lattice sites encode
N/2 physical sites

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N sites encode
N/2 physical sites

Gauge field $E(x = a\,n) \Rightarrow L_n$

Gauss law $\partial_1 E - gj^0 = 0 \Rightarrow L_n - L_{n-1} - q_n = 0, \quad q_i = \chi_i^\dagger \chi_i + \frac{(-1)^i - 1}{2}$

With open boundary conditions the electric field is fully determined by the fermionic one

Mapping to a spin chain (optional)

X, Y, Z – Pauli matrices

$$X_n \equiv \underset{1^{\text{st}}}{I} \otimes \cdots \otimes \underset{(n-1)^{\text{th}}}{I} \otimes \underset{n^{\text{th}}}{X} \otimes \underset{(n+1)^{\text{th}}}{I} \otimes \cdots \otimes I$$ etc.

$$\chi_n = \frac{X_n - iY_n}{2} \prod_{j=1}^{n-1} (-iZ_j),$$

$$\chi_n^\dagger = \frac{X_n + iY_n}{2} \prod_{j=1}^{n-1} (iZ_j),$$

Jordan-Wigner transformation

Spin chain Hamiltonian:

$$H^L = \frac{1}{4a} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n + \frac{ag^2}{2} \sum_{n=1}^{N-1} L_n^2$$

Kinetic term | Mass term | Nonlocal electric field term

$$L_n = \sum_{i=1}^{n} q_i$$

$$q_{n,t} = \frac{\langle Z_n \rangle_t + (-1)^n}{2a}$$

Adding the jets

$$j^0_{\text{ext}}(x,t) = g[\delta(\Delta x - \Delta t) - \delta(\Delta x + \Delta t)]\theta(\Delta t)$$
$$j^1_{\text{ext}}(x,t) = g[\delta(\Delta x - \Delta t) + \delta(\Delta x + \Delta t)]\theta(\Delta t)$$

$$\Delta x \equiv x - x_0$$
$$\Delta t \equiv t - t_0$$

$$H = \int dx \left[\bar{\psi}(-i\gamma^1\partial_1 + g\gamma^1 A_1 + m)\psi + \frac{1}{2}E^2 + j^1_{\text{ext}}(x,t)A_1\right]$$

$$\Downarrow$$

$$H^L(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1} + Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}(L_{\text{dyn},n} + L_{\text{ext},n}(t))^2.$$

$$L_{\text{dyn},n} = \sum_{i=1}^{n} q_i$$
$$L_{\text{ext},n}(t) = -\theta\left(t - t_0 - \left|x - x_0 + \frac{a}{2}\right|\right)$$

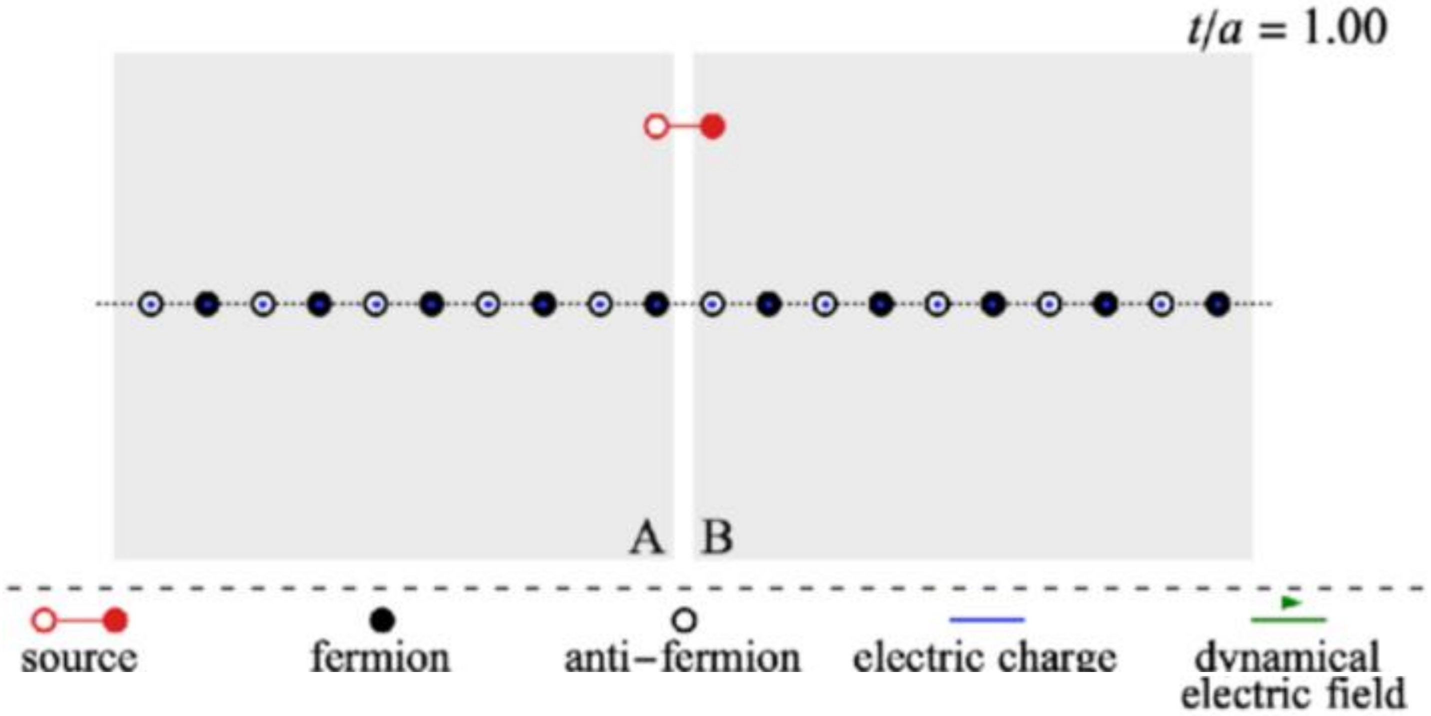
Numerical procedure

Start from the ground state of the Hamiltonian:

$$H(t=0)|\Psi(t=0)\rangle = E_0|\Psi(t=0)\rangle$$

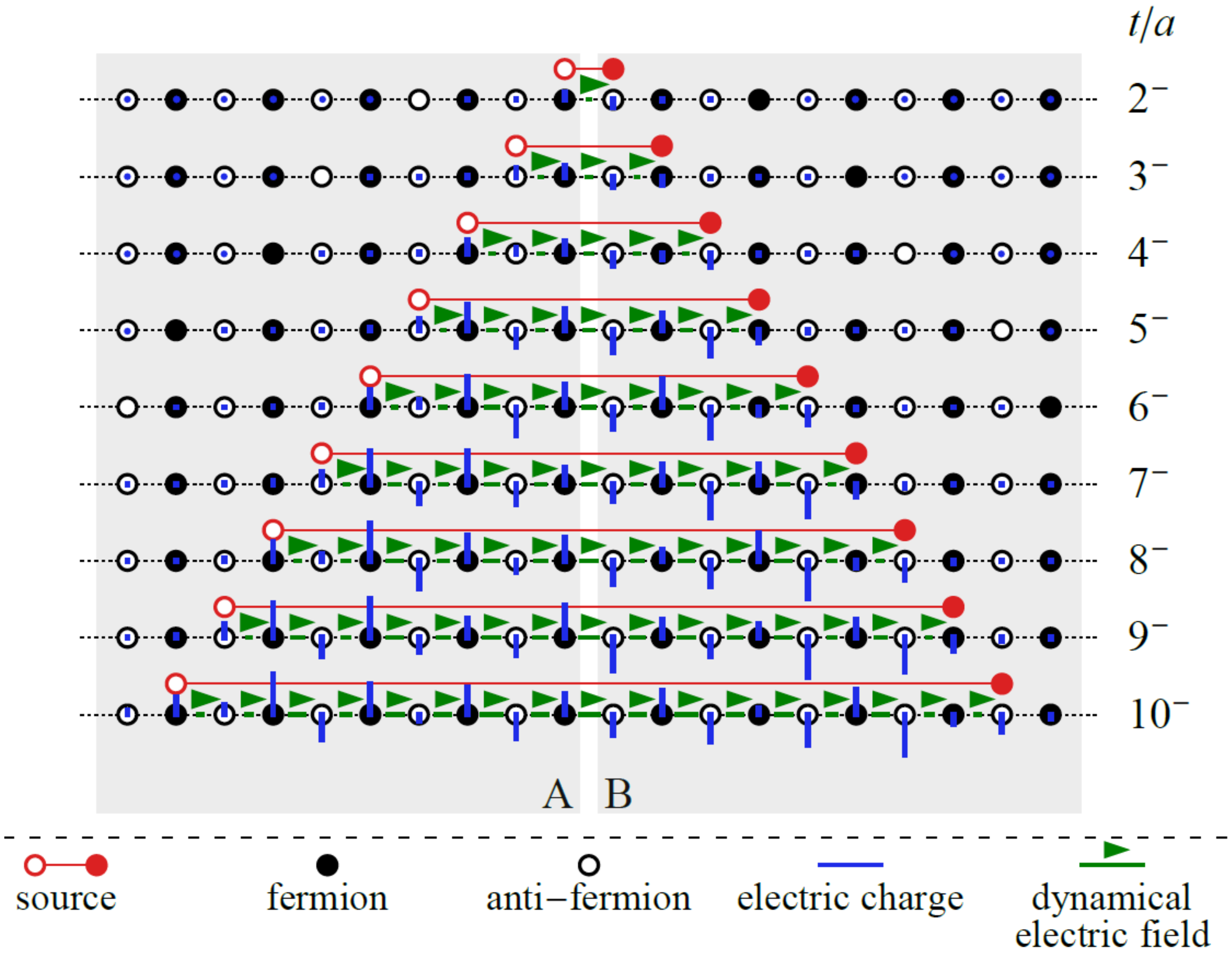
Switch on the external source and time evolve:

$$|\Psi_t\rangle = \mathcal{T}e^{-i\int_0^t dt' H(t')}|\Psi_0\rangle$$



Numerical time evolution using classical exact diagonalization or tensor networks mimics simulation on a quantum device

Screening, chiral condensate and entanglement



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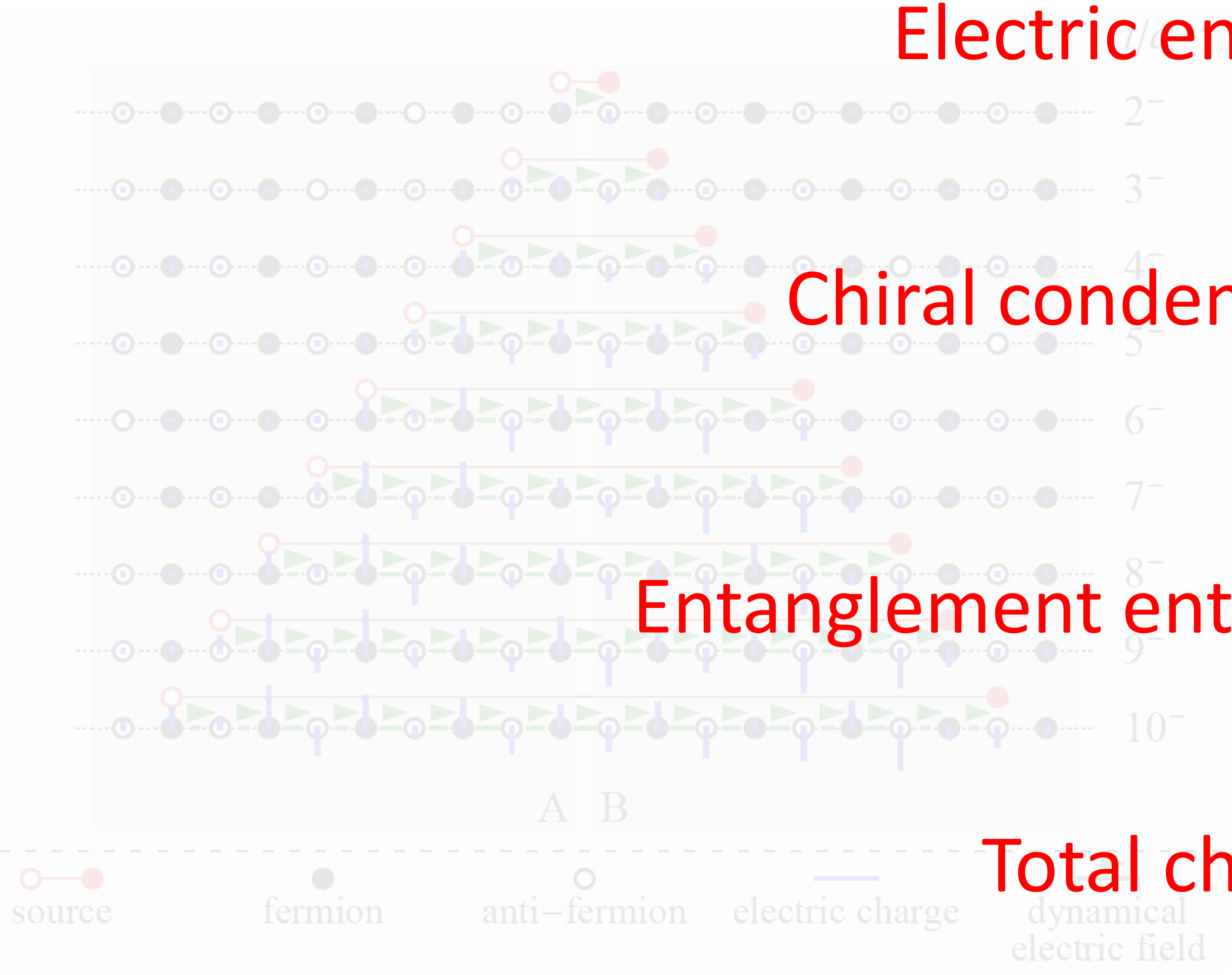
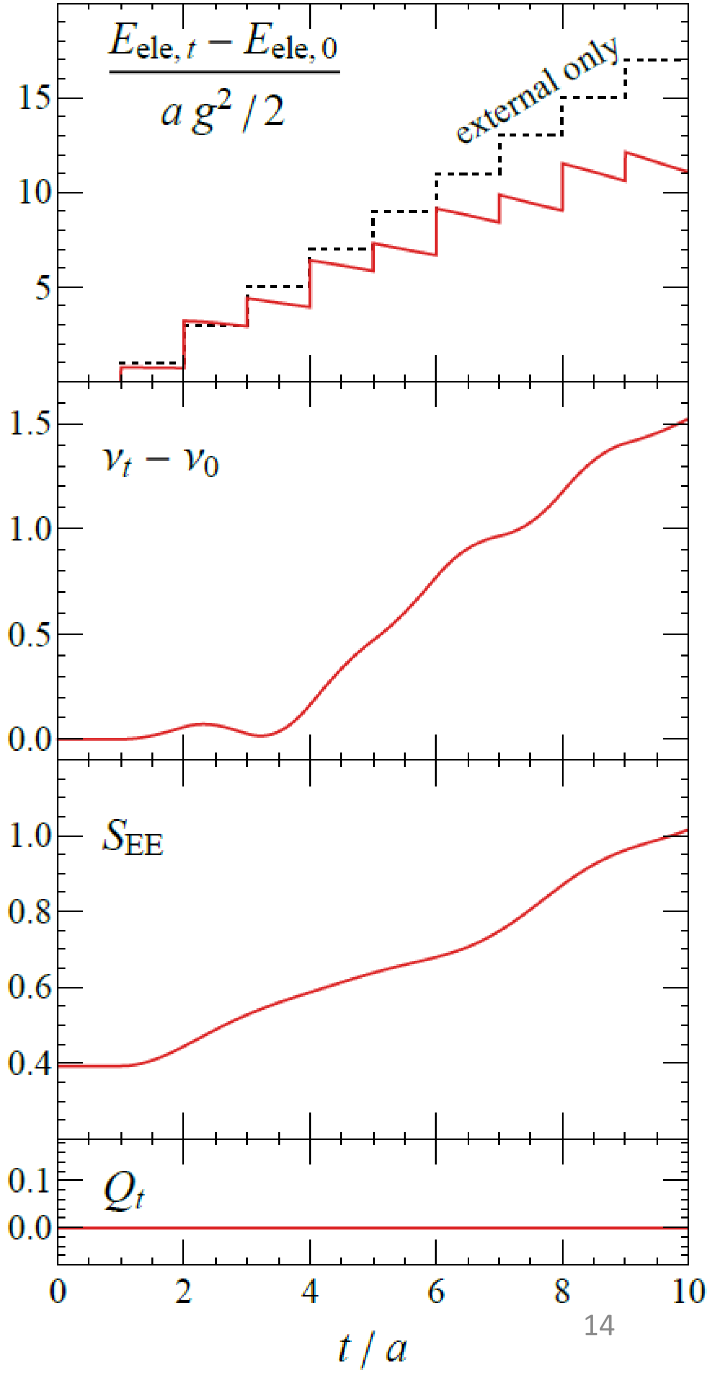


Electric energy

Chiral condensate

Entanglement entropy

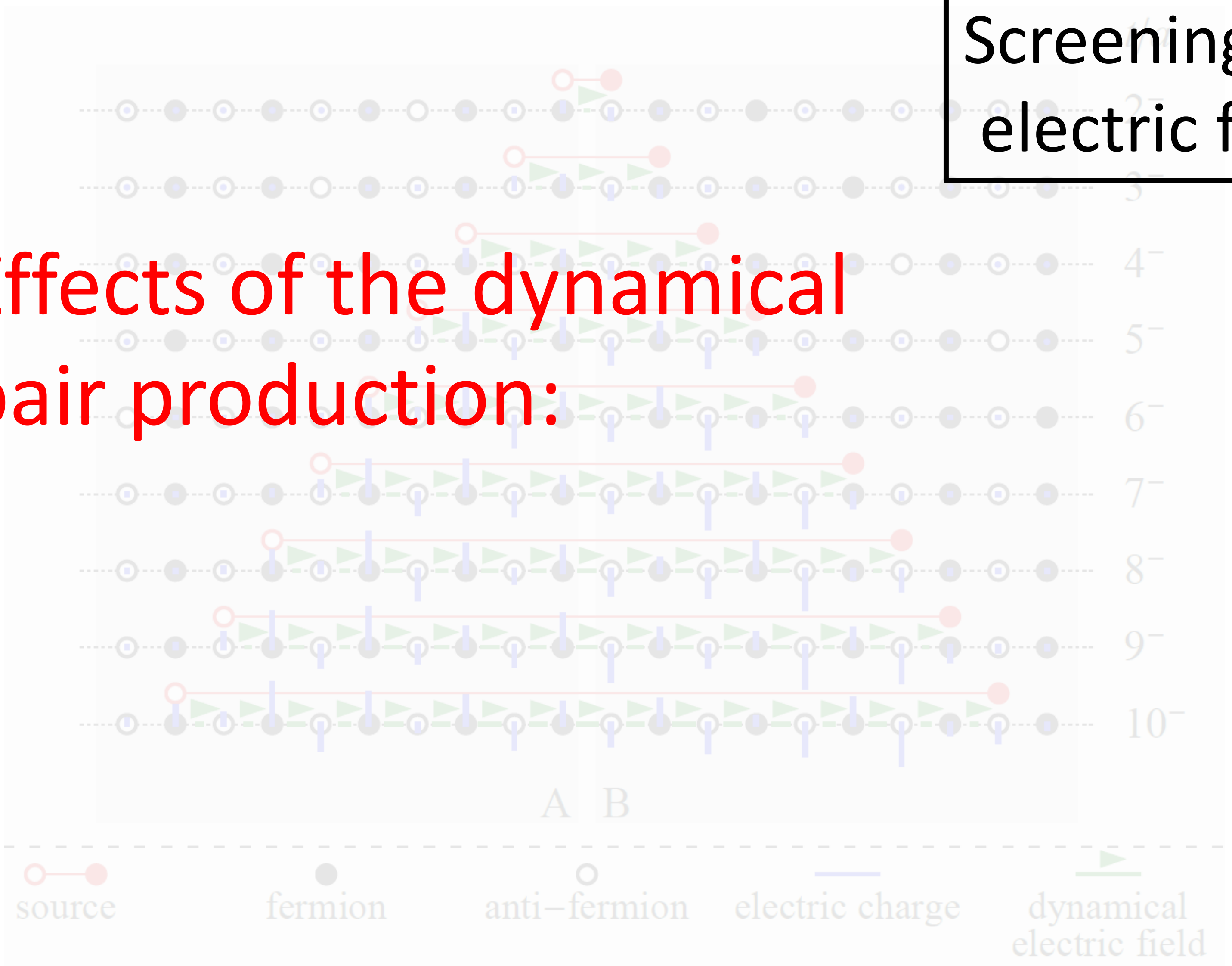
Total charge

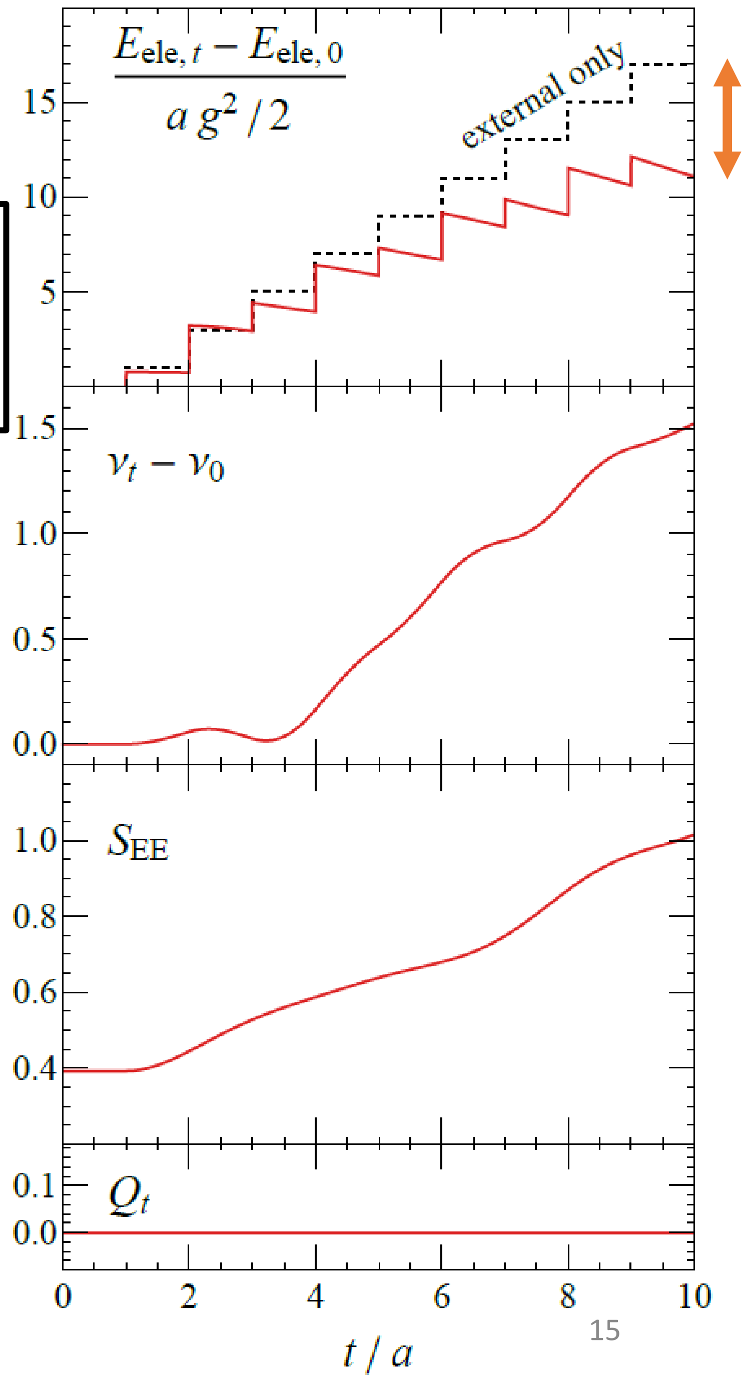


Screening, chiral condensate and entanglement

Screening the electric field

Effects of the dynamical pair production:





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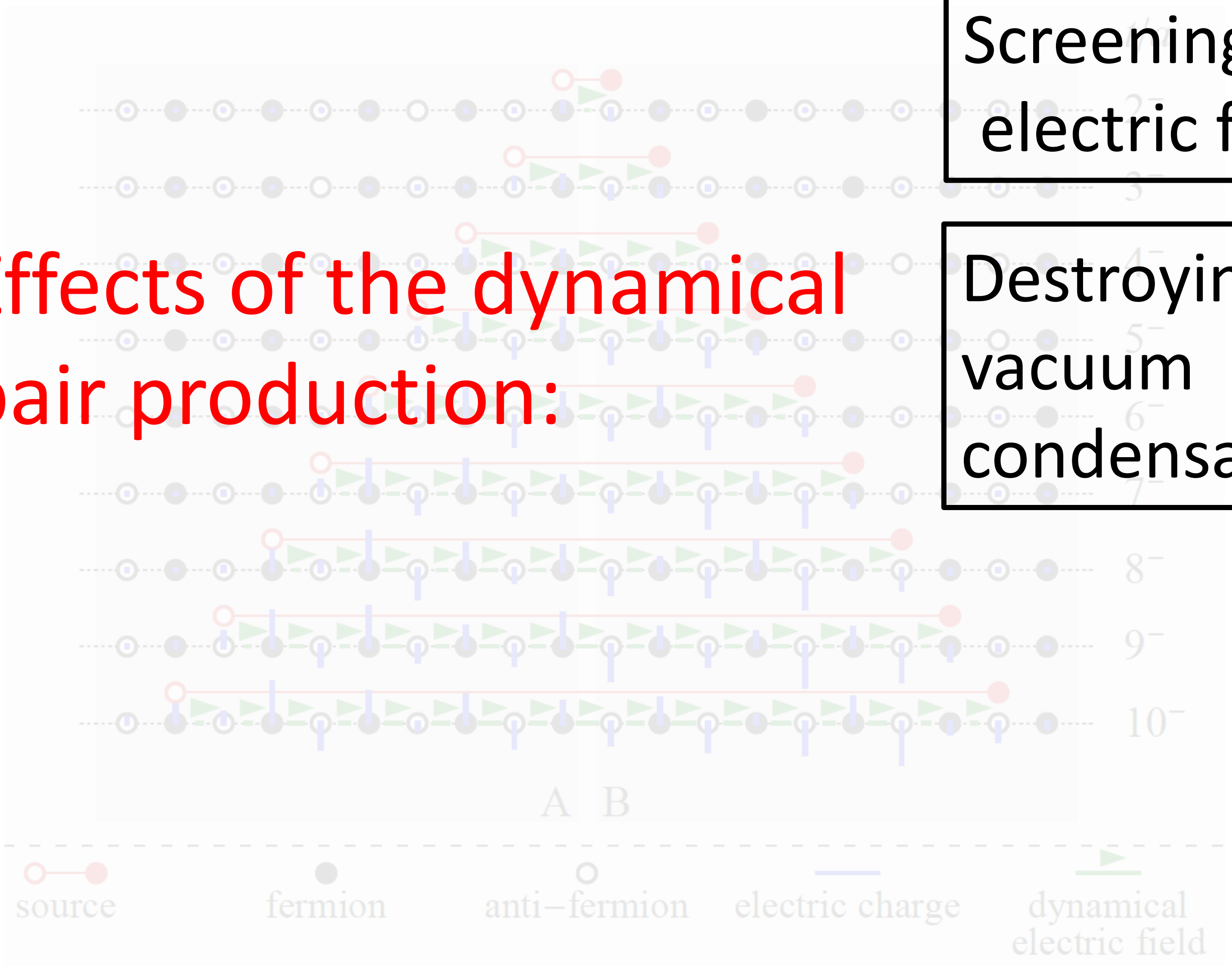
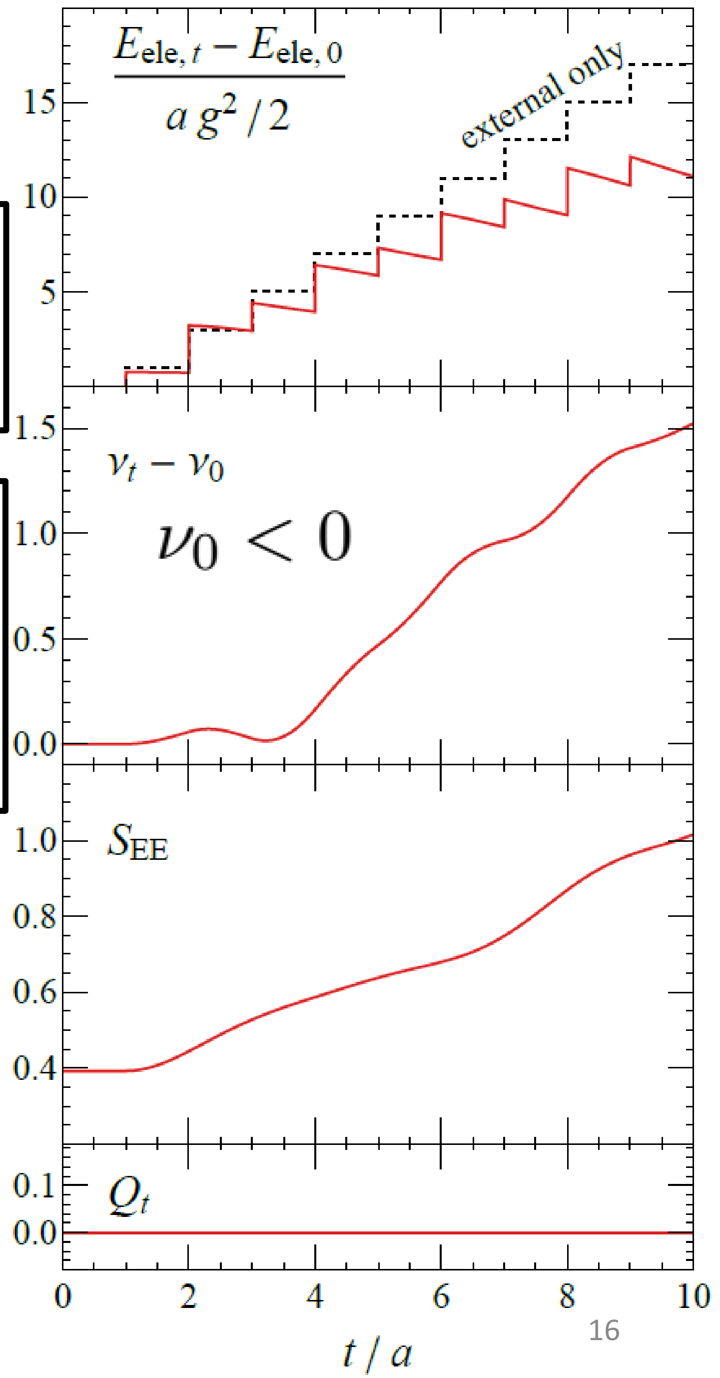


Effects of the dynamical pair production:

Screening the electric field

Destroying vacuum condensate



Screening, chiral condensate and entanglement

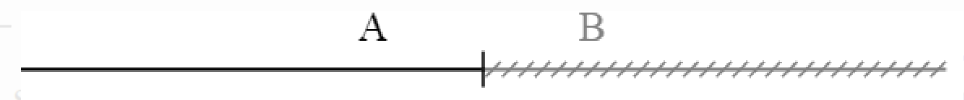
Effects of the dynamical pair production:

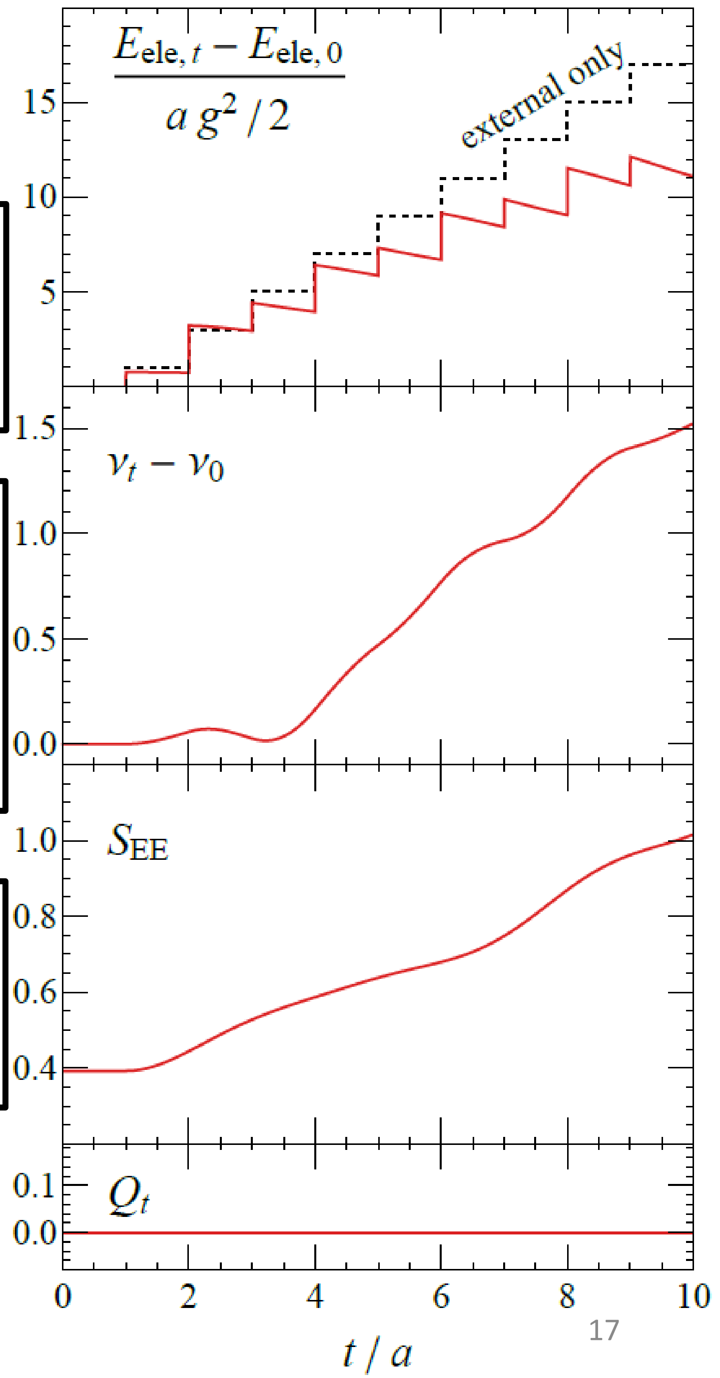
Screening the electric field

Destroying vacuum condensate

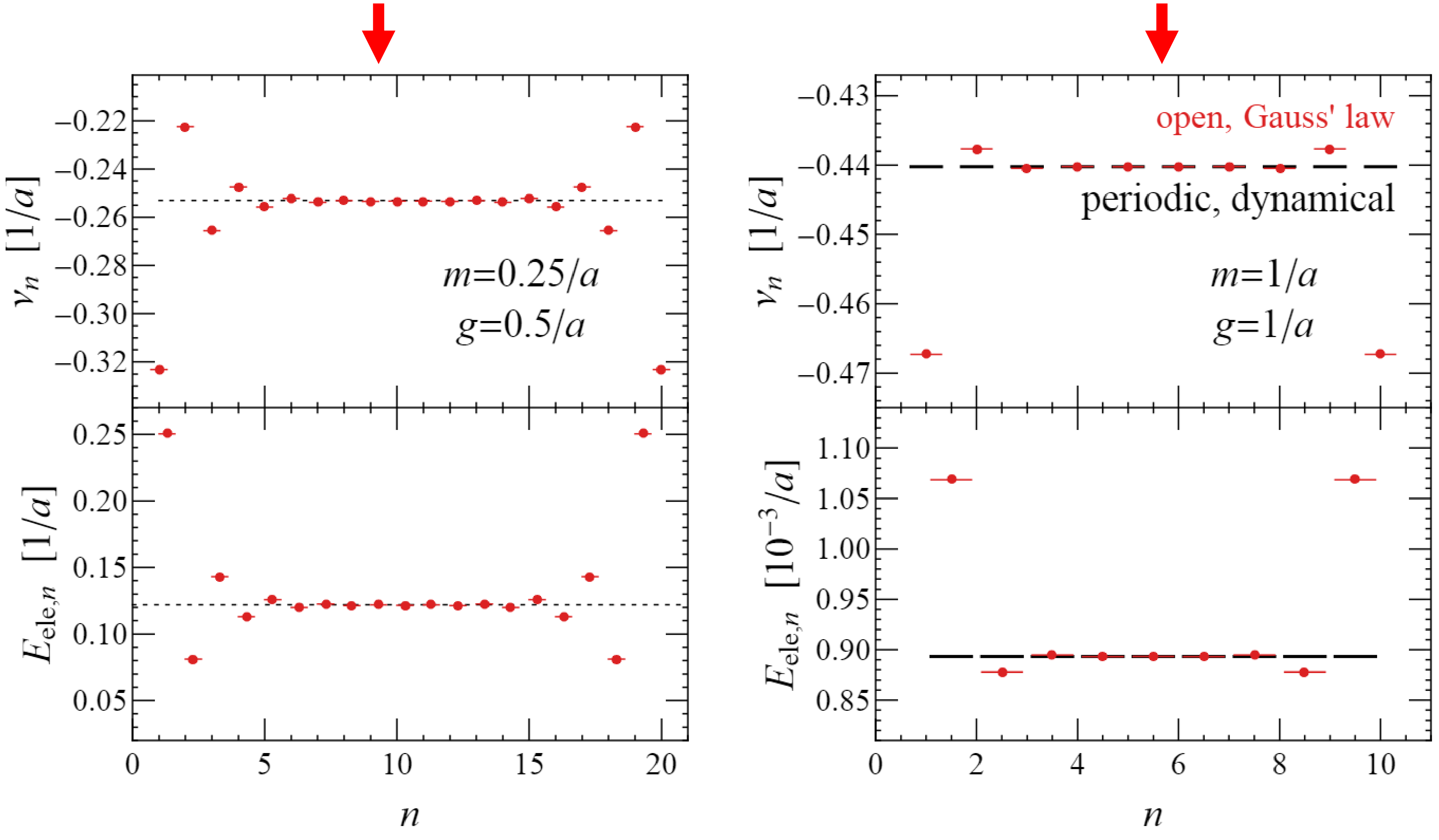
Entangling the jets

$$\rho_A = \mathrm{Tr}_B \rho, \qquad S_{EE} = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$





Boundary effects and boundary conditions



Compare expectation values of local operators in the ground state

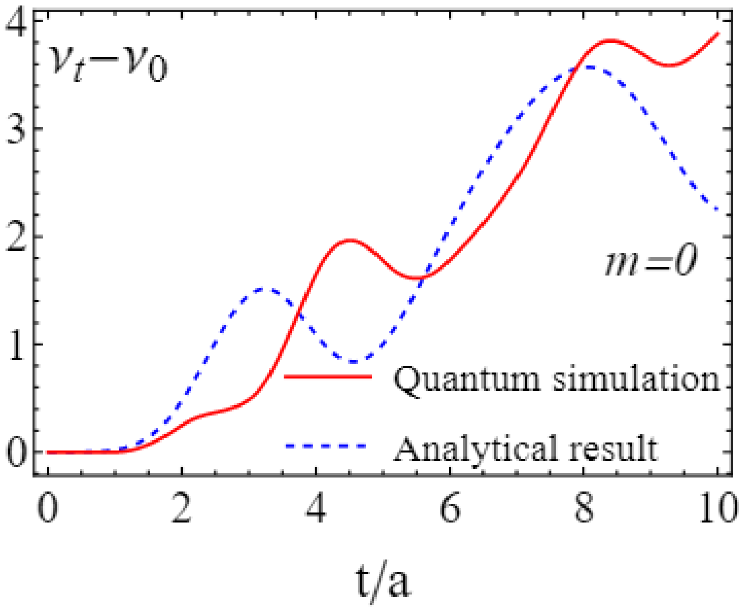
Massless fermion benchmark

Bosonization: $$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_B^2 (\phi + \phi_{ext})^2 \, , \qquad m_B^2 = \frac{g^2}{\pi}$$

$$\phi(t,x) = \sqrt{\pi}\theta(t^2 - x^2)\left[1 - J_0\left(m_B\sqrt{t^2 - x^2}\right)\right]$$

Casher, Kogut, Susskind (1974)
Kharzeev, Loshaj (2011)

$$\bar{\psi}\psi(x) = -\frac{e^\gamma}{2\pi} m_B \cos[2\sqrt{\pi}\phi(x)]$$



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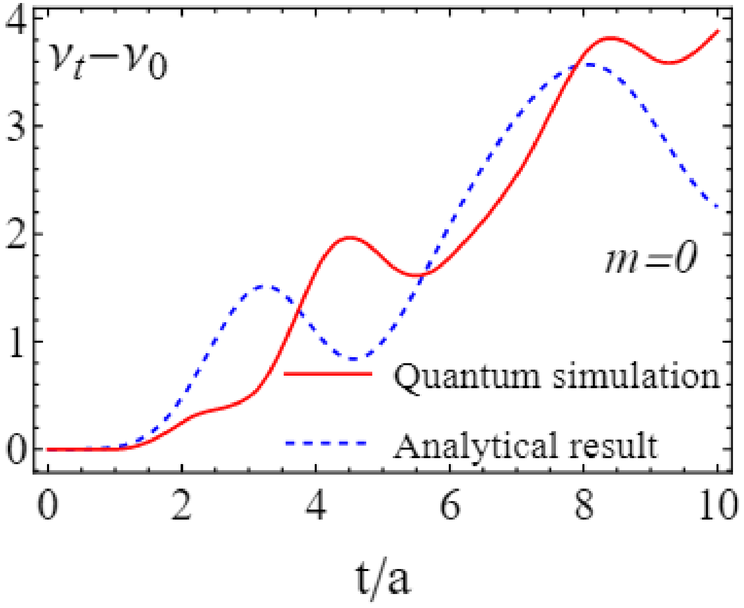

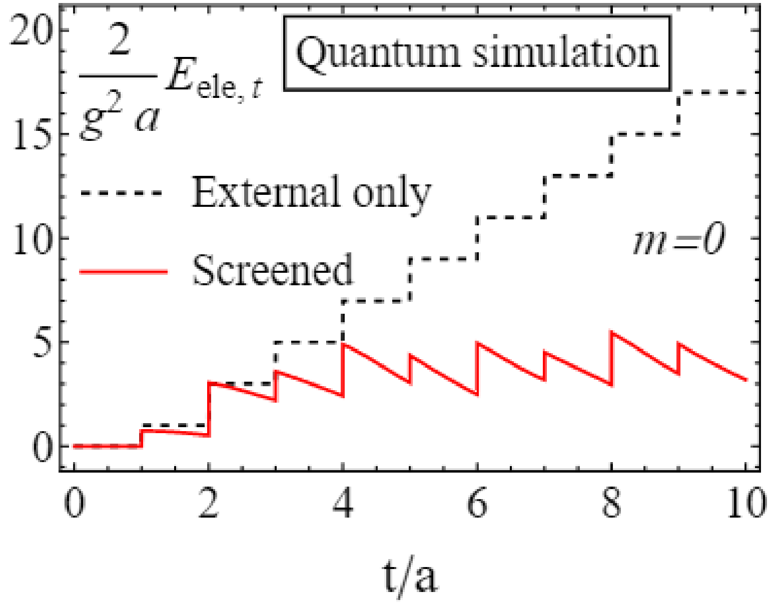

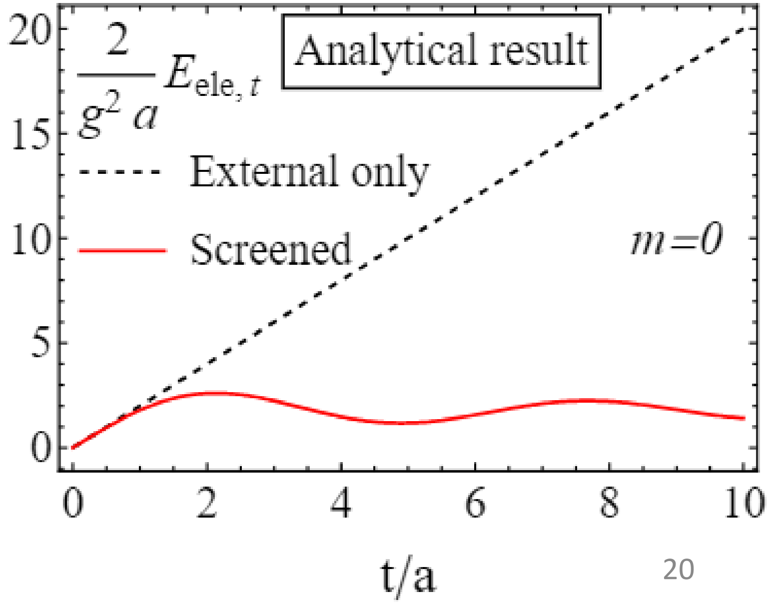

Massless fermion as a benchmark

Bosonization: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_B^2 (\phi + \phi_{ext})^2 \, , \qquad m_B^2 = \frac{g^2}{\pi}$

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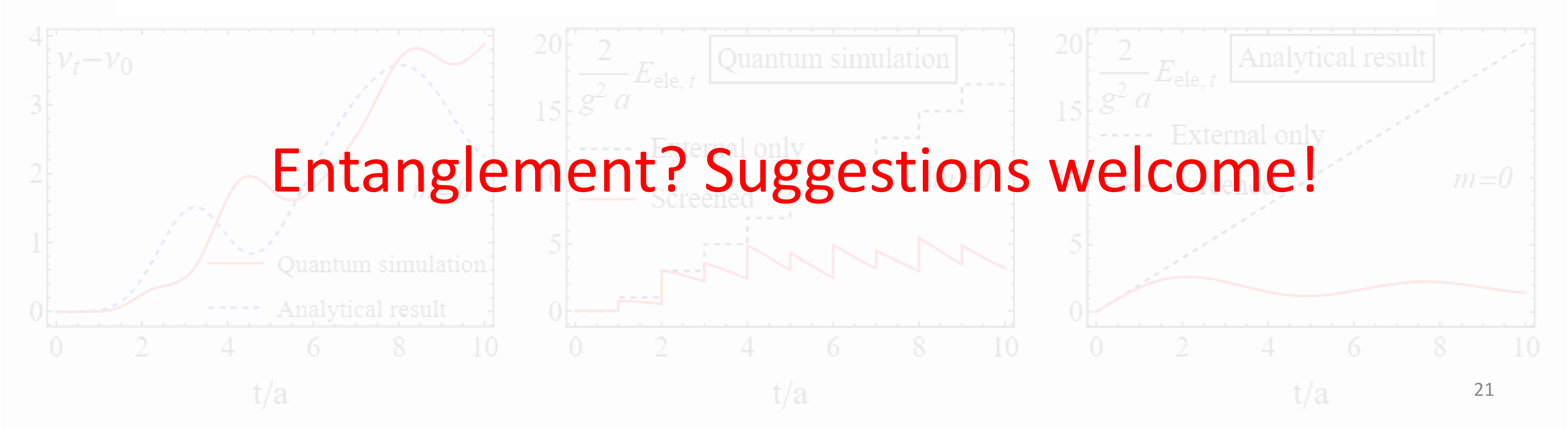


Entanglement? Suggestions welcome!

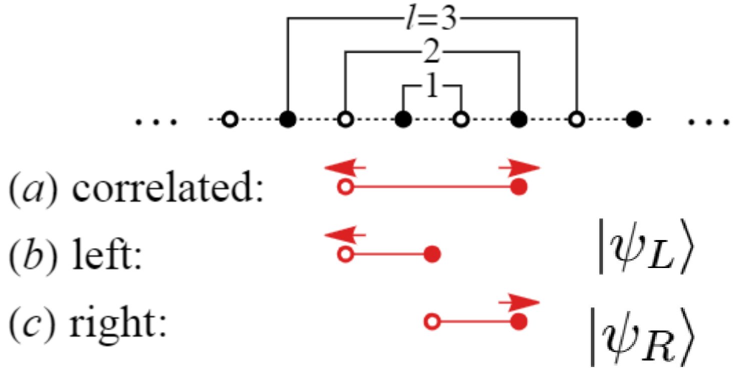
Correlations to probe entanglement

$$\eta_s = \text{arctanh}\frac{(\ell - 1/2)a}{t}$$

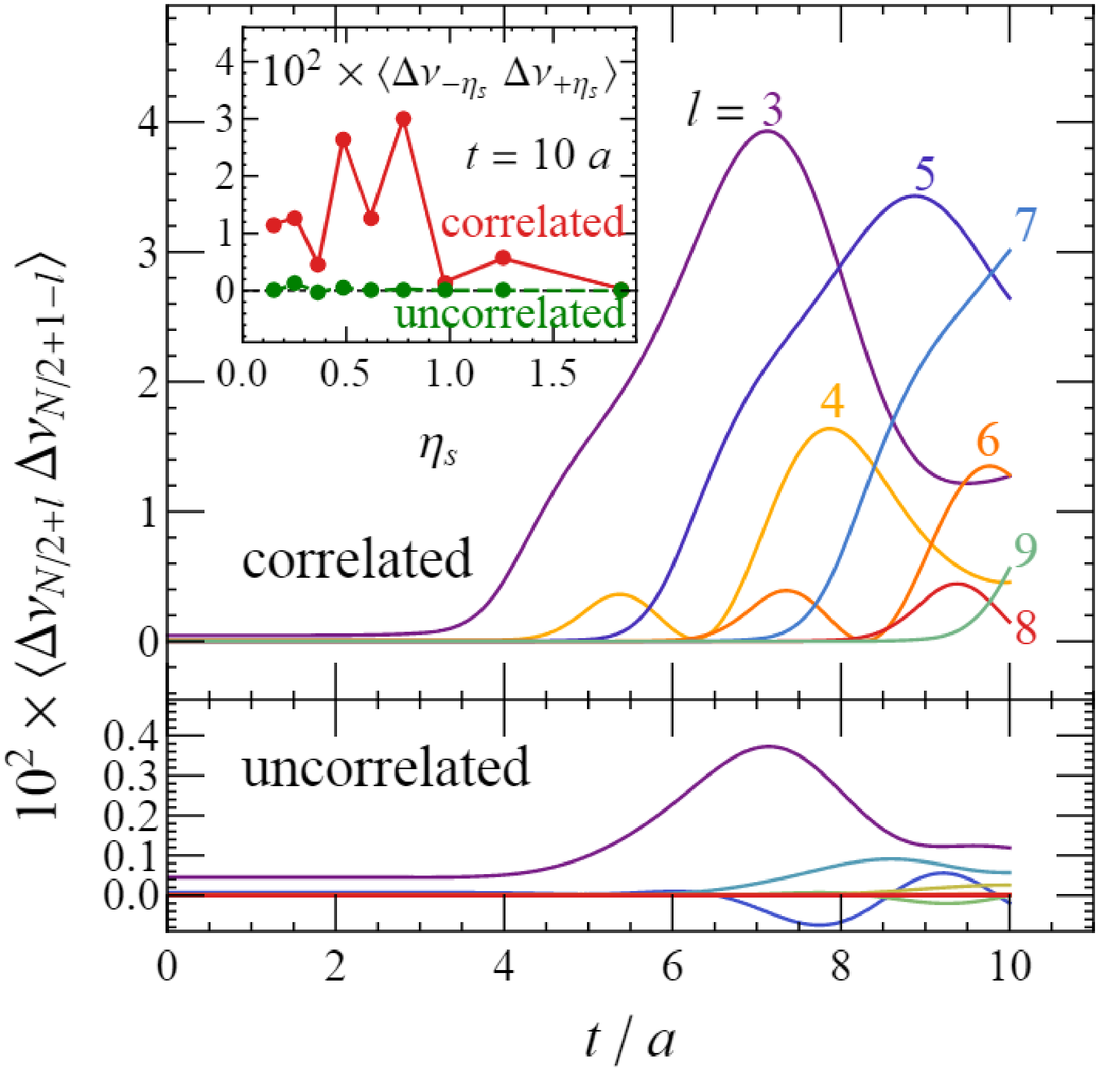
Consider correlation function

$$\langle \Delta\nu_{N/2+\ell}\, \Delta\nu_{N/2+1-\ell} \rangle\,, \qquad \Delta\nu_n \equiv \nu_n - \langle \nu_n \rangle_{\rm vac}$$

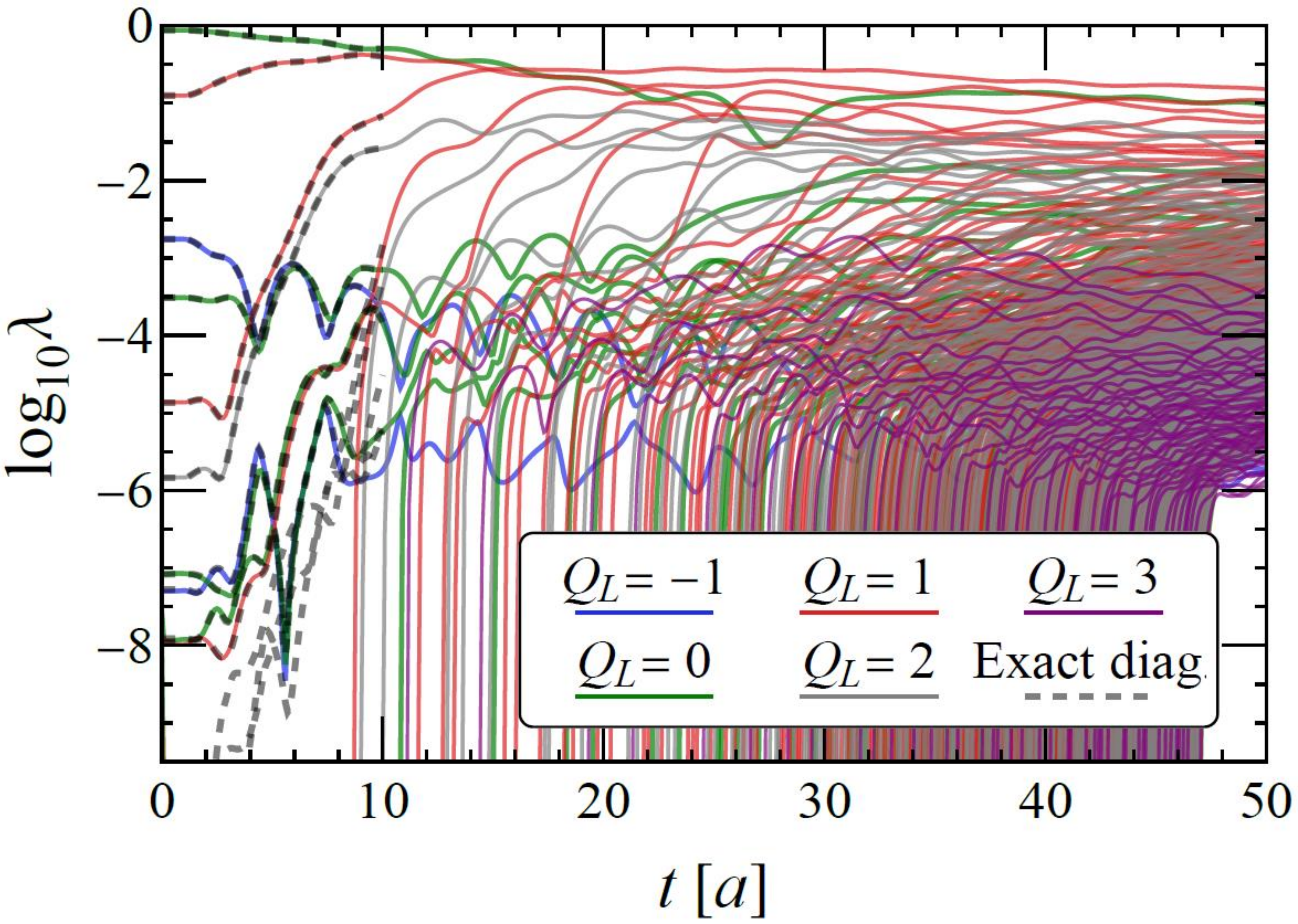
Compare the original setup with uncorrelated: $|\psi_{\rm uncorr}\rangle = \frac{1}{\sqrt{2}}|\psi_{\rm L}\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|\psi_{\rm R}\rangle$



$$\langle\langle \psi_{\rm uncorr}|O|\psi_{\rm uncorr}\rangle\rangle \equiv \int \langle \psi_{\rm uncorr}|O|\psi_{\rm uncorr}\rangle \frac{d\varphi}{2\pi}$$



Entanglement spectrum



Schmidt decomposition:

$$|\Psi(t)\rangle = \sum_{i=1}^{2^{N/2}} \sqrt{\lambda_i(t)}|\psi_i^L(t)\rangle \otimes |\psi_i^R(t)\rangle$$

$$\rho_L(t) = \sum_{i=1}^{2^{N/2}} \lambda_i(t)|\psi_i^L(t)\rangle\langle\psi_i^L(t)|$$

$$S_{EE}(t) = -\sum_{i=1}^{2^{N/2}} \lambda_i \ln \lambda_i$$

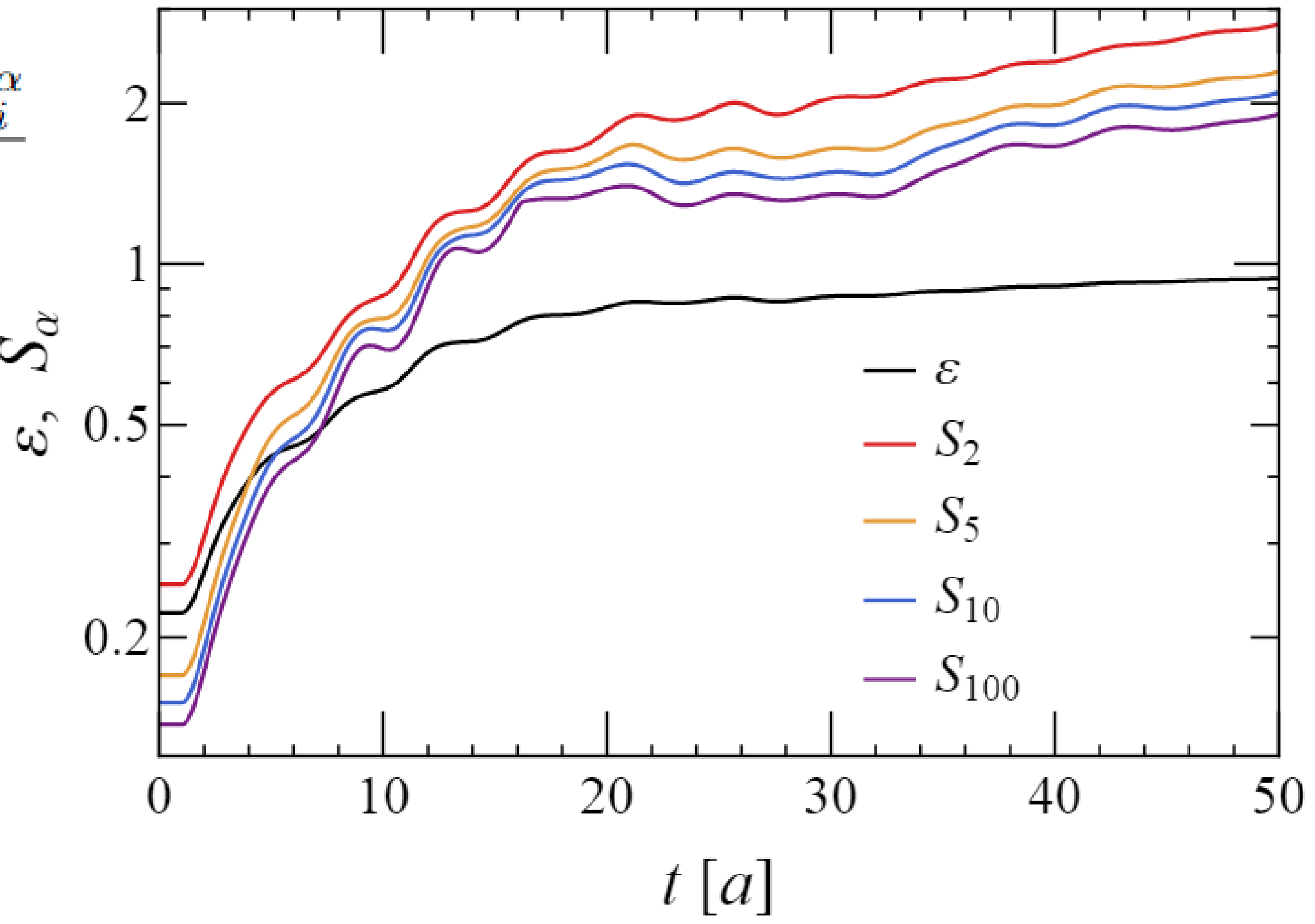
Symmetry-resolved:

$$\sum_{n=1}^{N/2} q_n|\psi_i^L\rangle \equiv Q_L|\psi_i^L\rangle$$

Renyi entropies and entangleness

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}$$

$$\mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1-\sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}$$



Differentiate between pure state (PS) and maximally entangled state (MES):

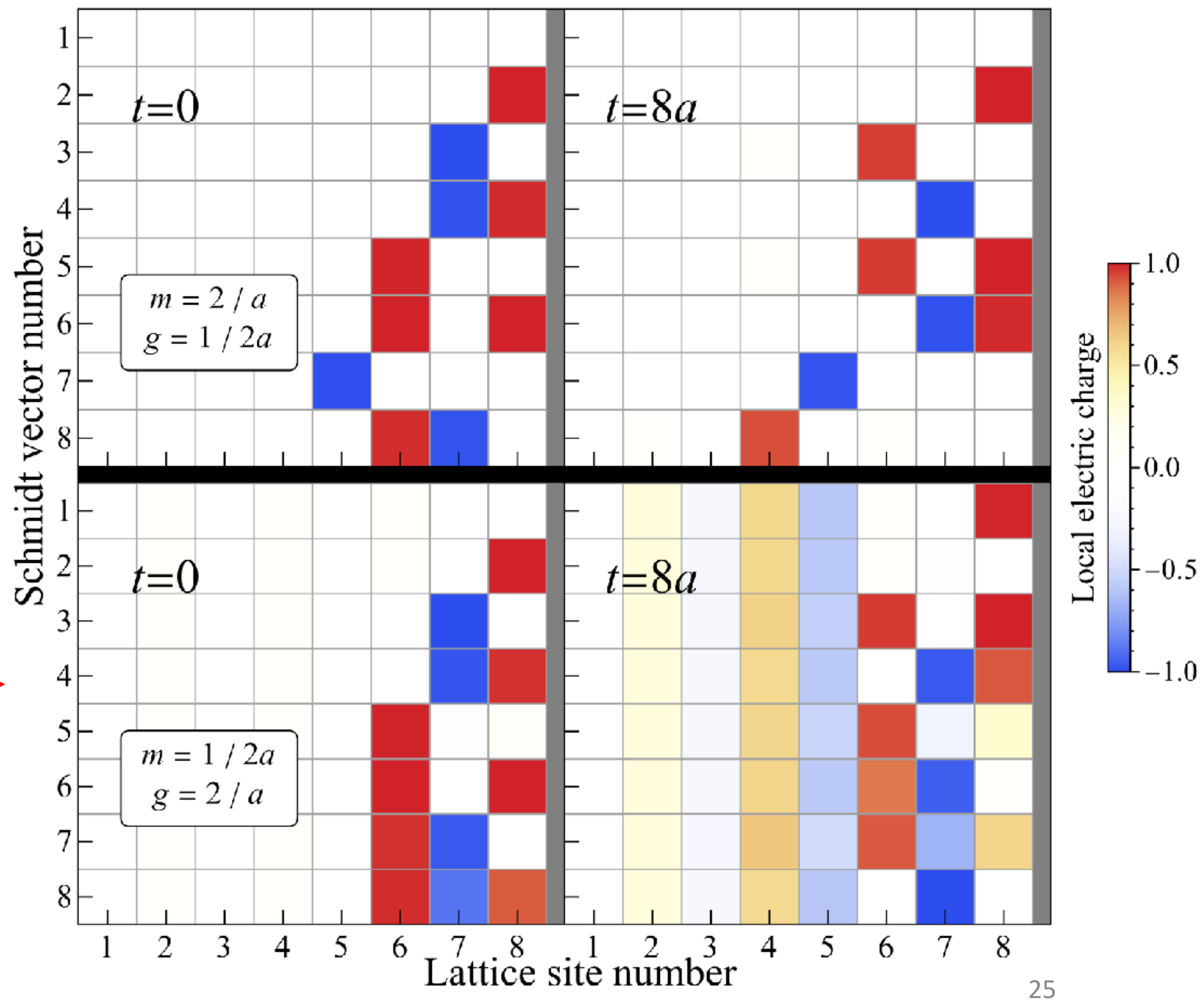
$$S_\alpha[\mathrm{PS}] = 0\ , \qquad \mathcal{E}[\mathrm{PS}] = 0$$

$$S_\alpha[\mathrm{MES}] = \frac{N \ln 2}{2}\ \forall\alpha\ , \qquad \mathcal{E}[\mathrm{MES}] = 1$$

Charge distribution in Schmidt vectors

Weak coupling:

Strong coupling:



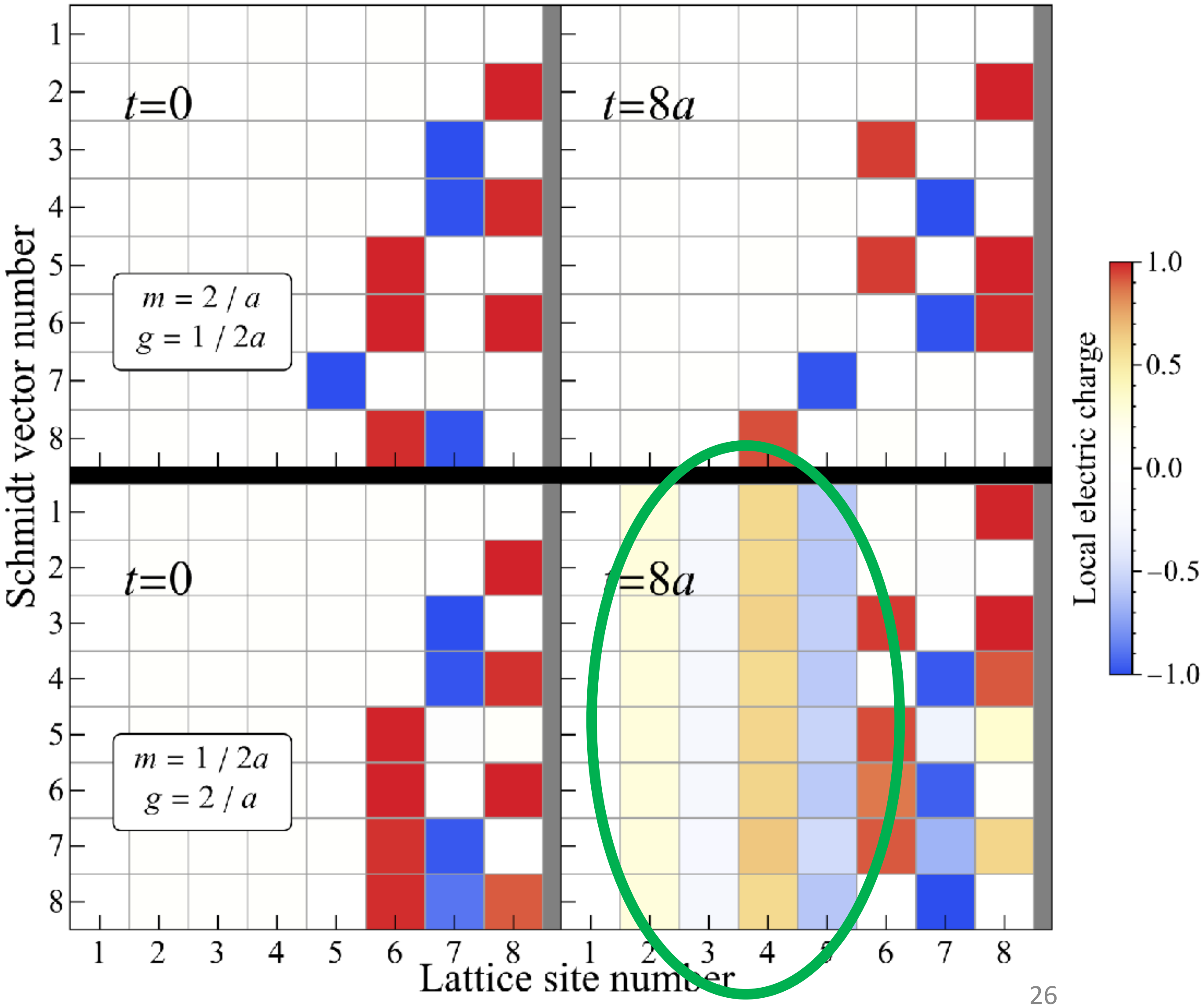
Charge distribution in Schmidt vectors

Weak coupling:

Charge is concentrated

Strong coupling:

Charge is distributed



Charge distribution in Schmidt vectors

Weak coupling:

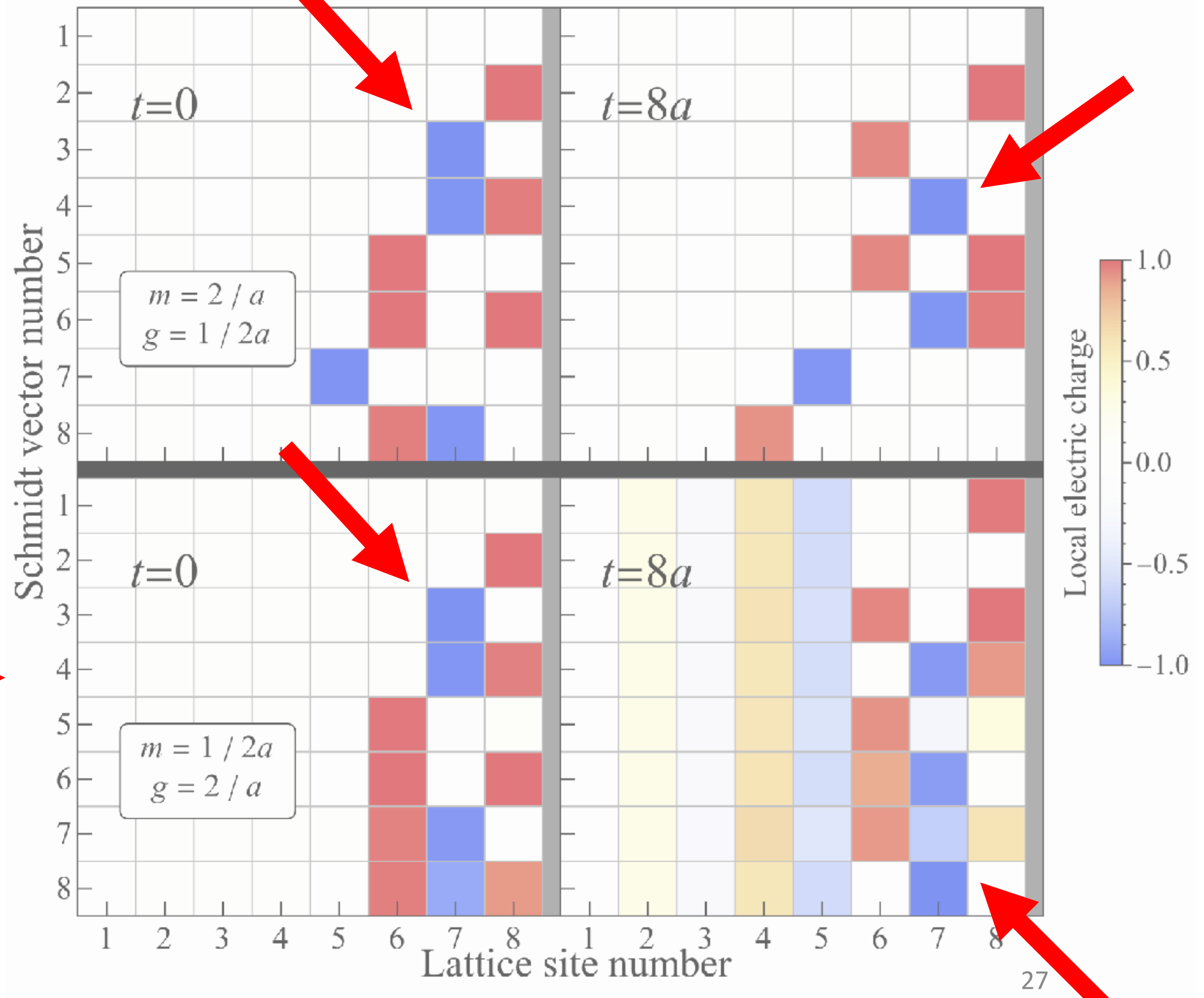
Charge is concentrated

Some screening

Strong coupling:

Charge is distributed

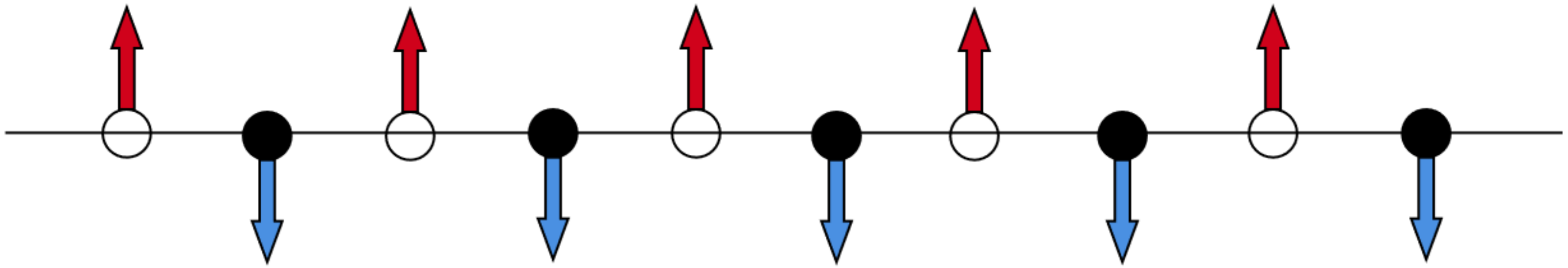
Stronger screening



Fermionic Fock basis

$$|\mathcal{N}\rangle = |1010\ldots10\rangle$$

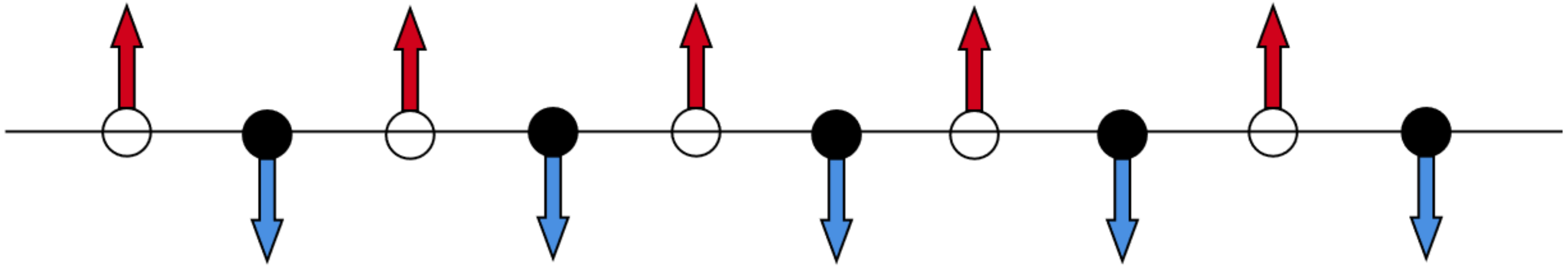
Neel state



N=10 example

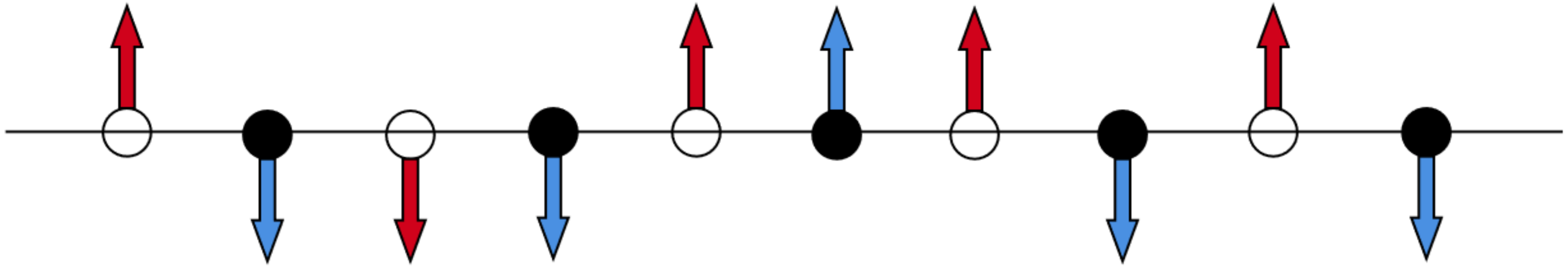
Fermionic Fock basis

$|\mathcal{N}\rangle = |1010\ldots10\rangle$ Neel state



$N=10$ example

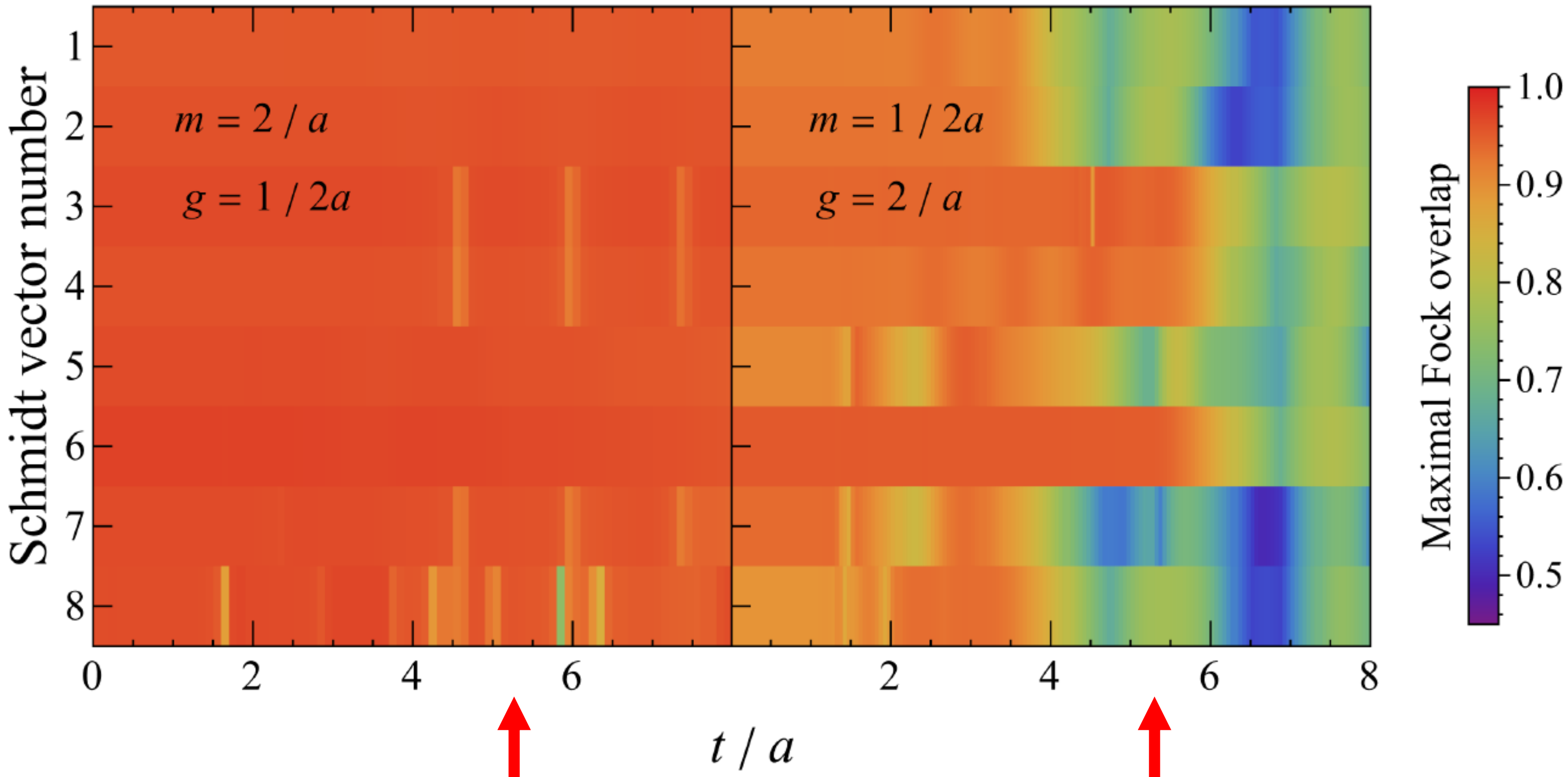
$|i\,j\rangle = X_i X_j|\mathcal{N}\rangle$ 1-pair excitation



$|3\,6\rangle$ example

…

Hadronization in real time



Look at the maximal overlap:

$$\max |\langle \psi_L^i(t)|jk\ldots\rangle|$$

Schmidt vector

Fock basis state

Weak coupling

Strong coupling

Towards thermalization

Electric field

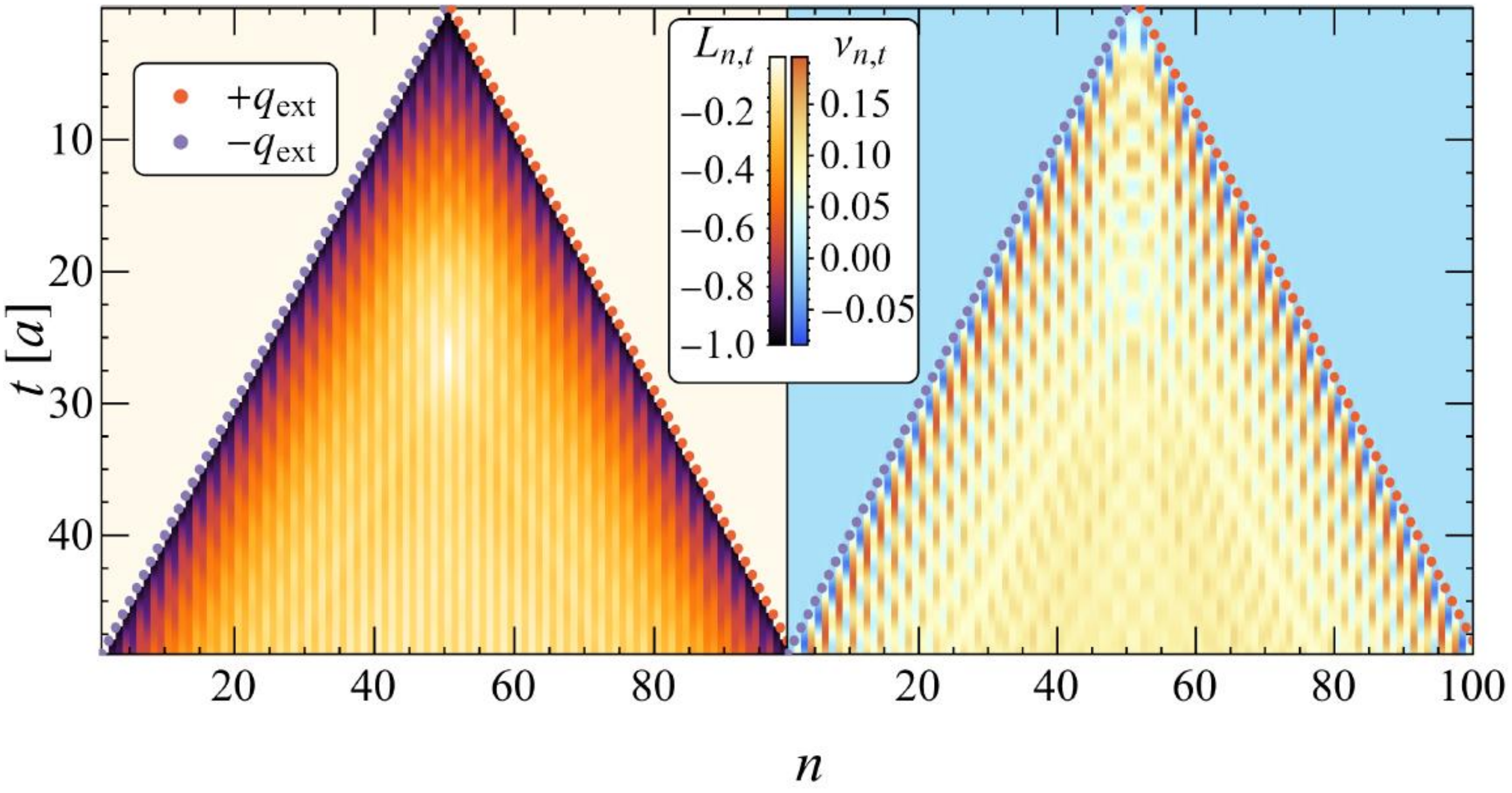
$$L_n = L_{\text{dyn},n} + L_{\text{ext},n}(t)$$

$$L_{\text{dyn},n} = \sum_{i=1}^{n} q_i$$

$$L_{\text{ext},n}(t) = -\theta\left(\frac{t-t_0}{a} - \left|n - \frac{N}{2}\right|\right)$$

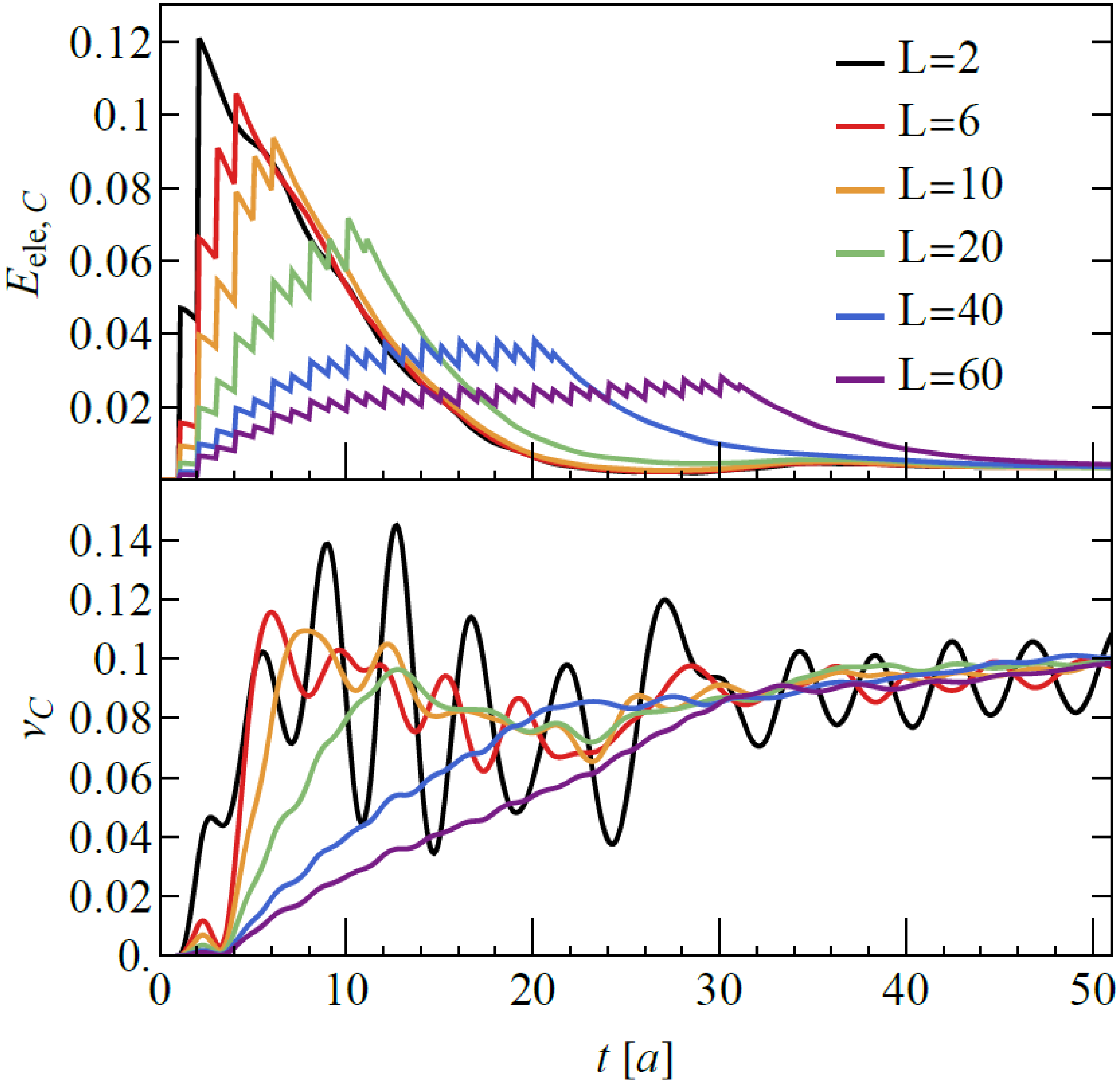
Chiral condensate

$$\nu_{n,t} \equiv \frac{(-1)^n}{a} q_n - \frac{1}{2a}$$



Equilibration in the middle?

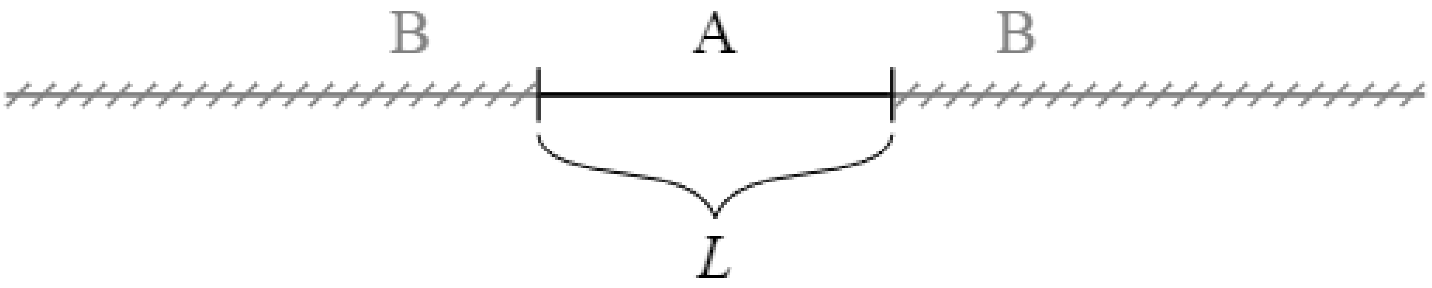
Averaged observables equilibrate



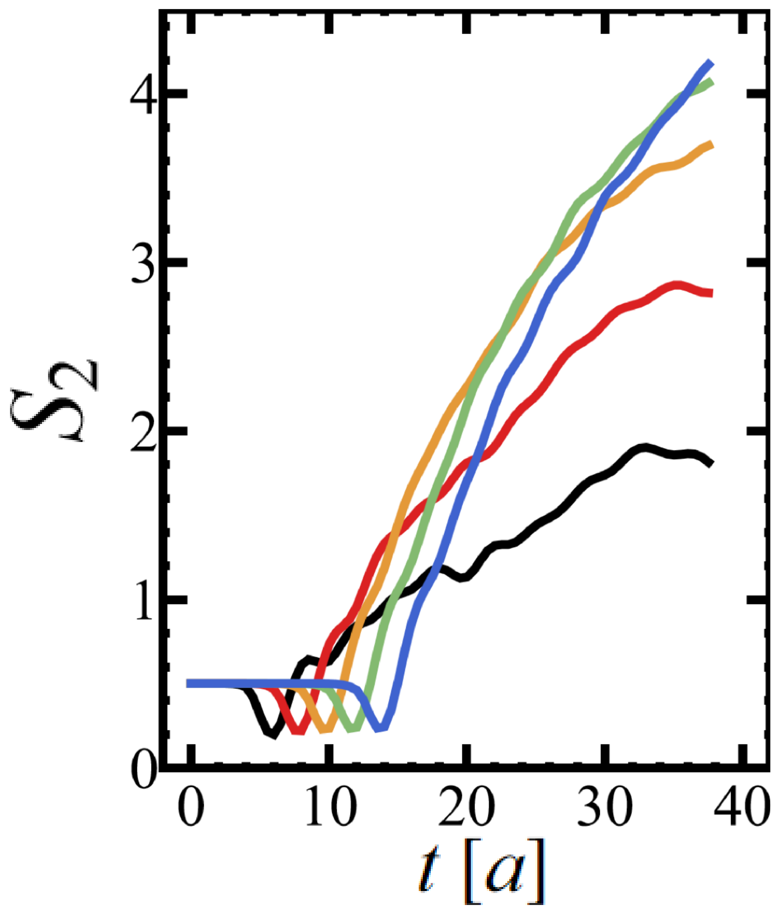
$$E_{\mathrm{ele,\ C}}(L) = \frac{1}{L}\,\frac{ag^2}{2}\sum_{n=N/2-L/2+1}^{N/2+L/2}\left(L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t)\right)^2$$

$$\nu_C(L) \equiv \frac{1}{L}\sum_{n=N/2-L/2+1}^{N/2+L/2}(\nu_{n,t} - \nu_{n,0}).$$

Renyi entropy of the central region



$$S_2(L) = -\log \mathrm{Tr}(\rho_A^2)$$



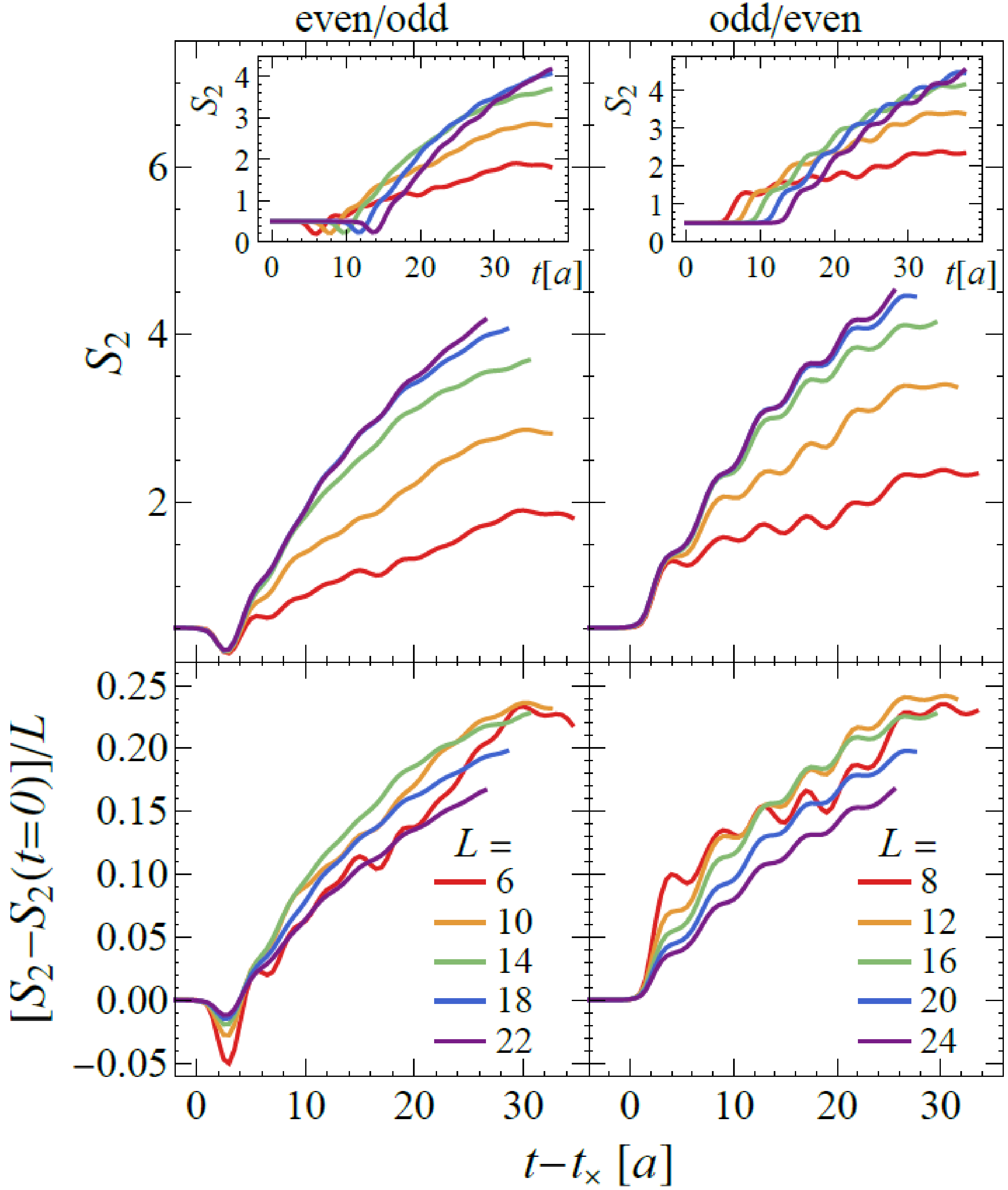
Study as a function of *L*

Ground state: "area law" (*L*-independent)

Typical state, e.g. thermal: "volume law" (linear in *L*)

E. Bianchi, L. Hackl, M. Kieburg, M. Rigol, and L. Vidmar,
PRX Quantum **3** (2022)

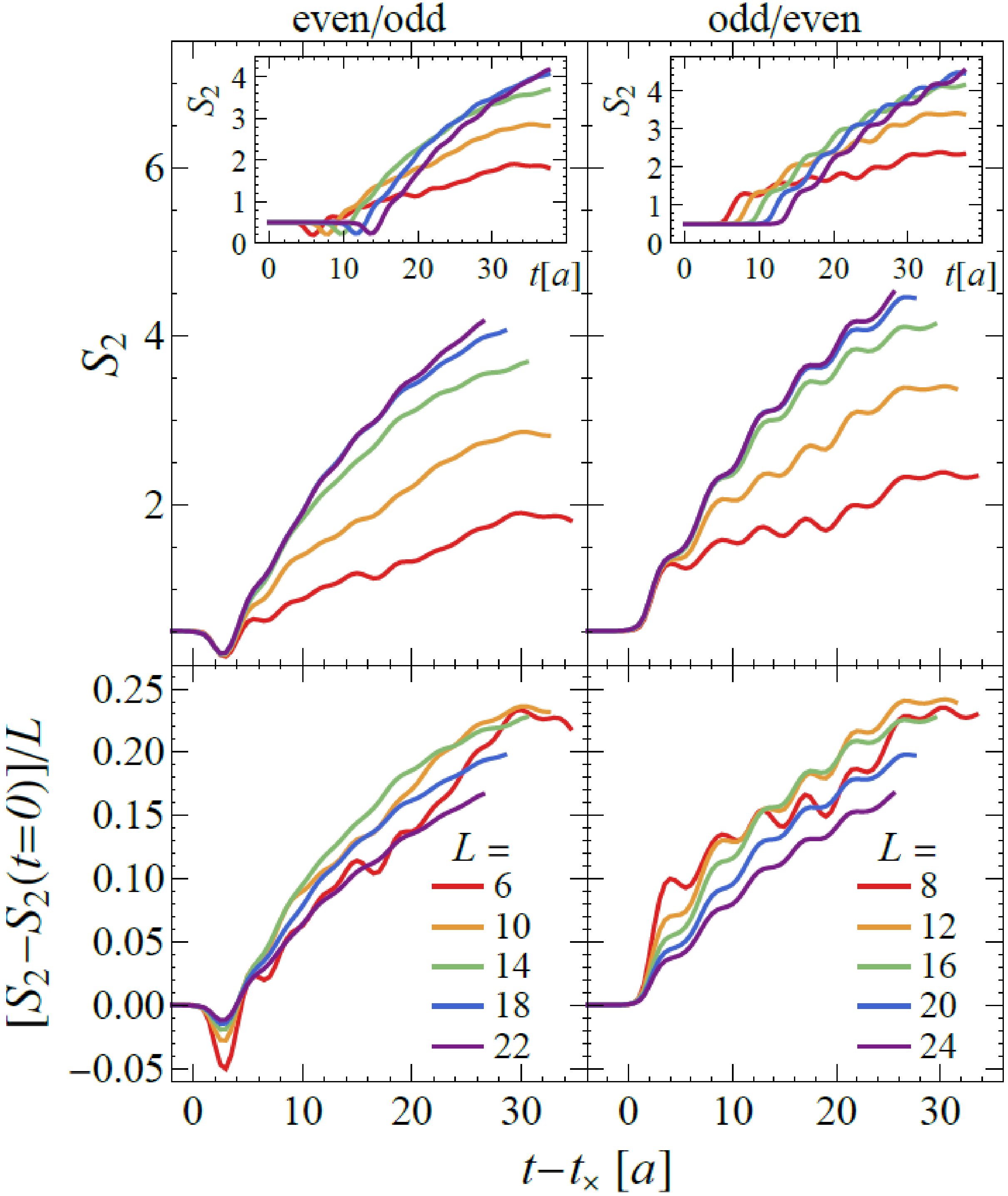
Area and volume laws of entanglement



Adjust by the jet arrival time

area law at early times

Area and volume laws of entanglement



Adjust by the jet arrival time

area law at early times

Rescale by the subsystem size

volume law at late times

Stopping the jets

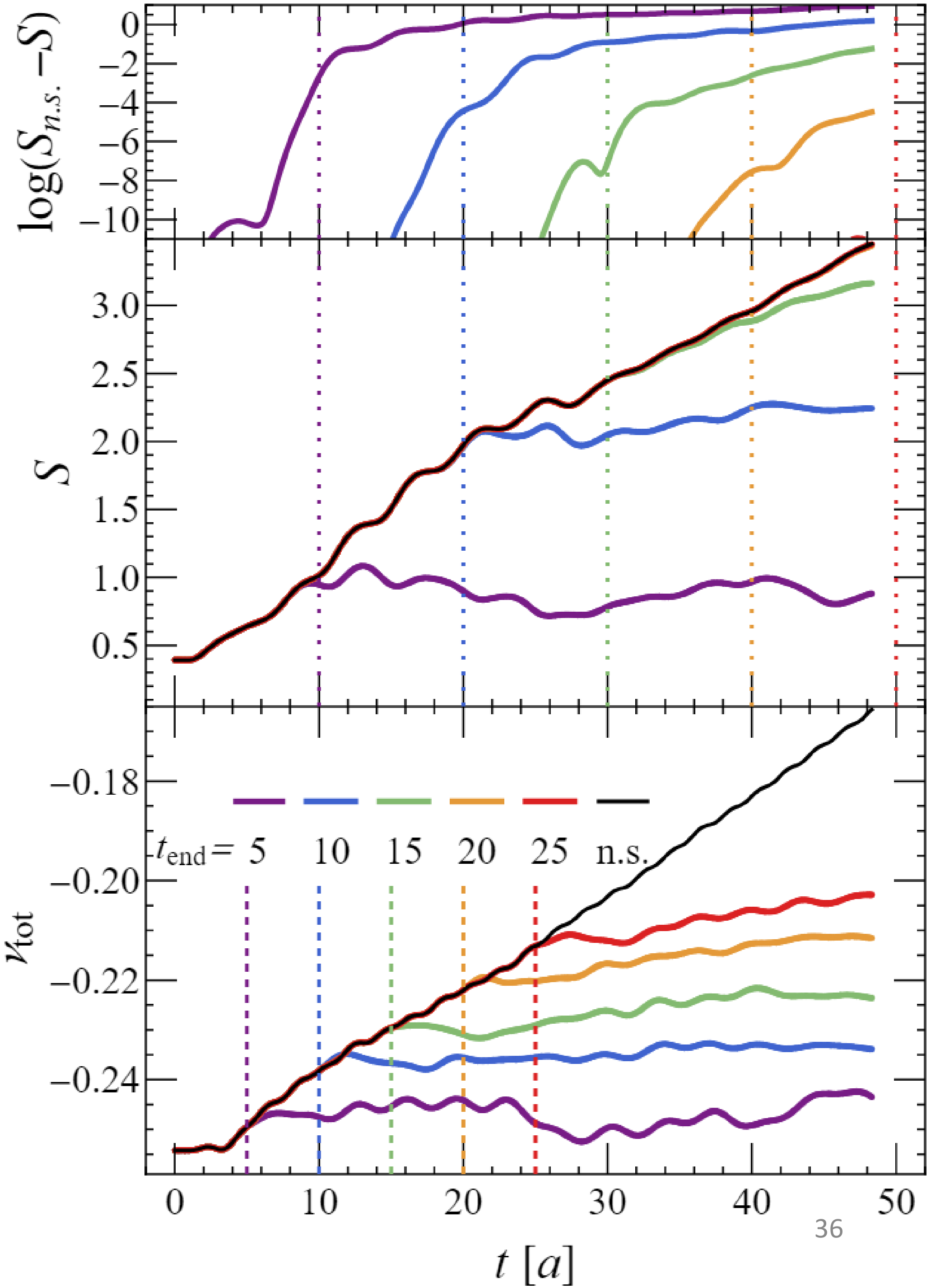
Modify the external source:

$$L_{\mathrm{ext},n}(t) = \begin{cases} -\theta\left(\frac{t-t_0}{a} - \left|n - \frac{N}{2}\right|\right), & t \leq t_{end} \\ -\theta\left(\frac{t_{end}-t_0}{a} - \left|n - \frac{N}{2}\right|\right), & t > t_{end} \end{cases}$$

instead of

$$L_{\mathrm{ext},n}(t) = -\theta\left(\frac{t-t_0}{a} - \left|n - \frac{N}{2}\right|\right)$$

Compare different t_{end}



Stopping the jets

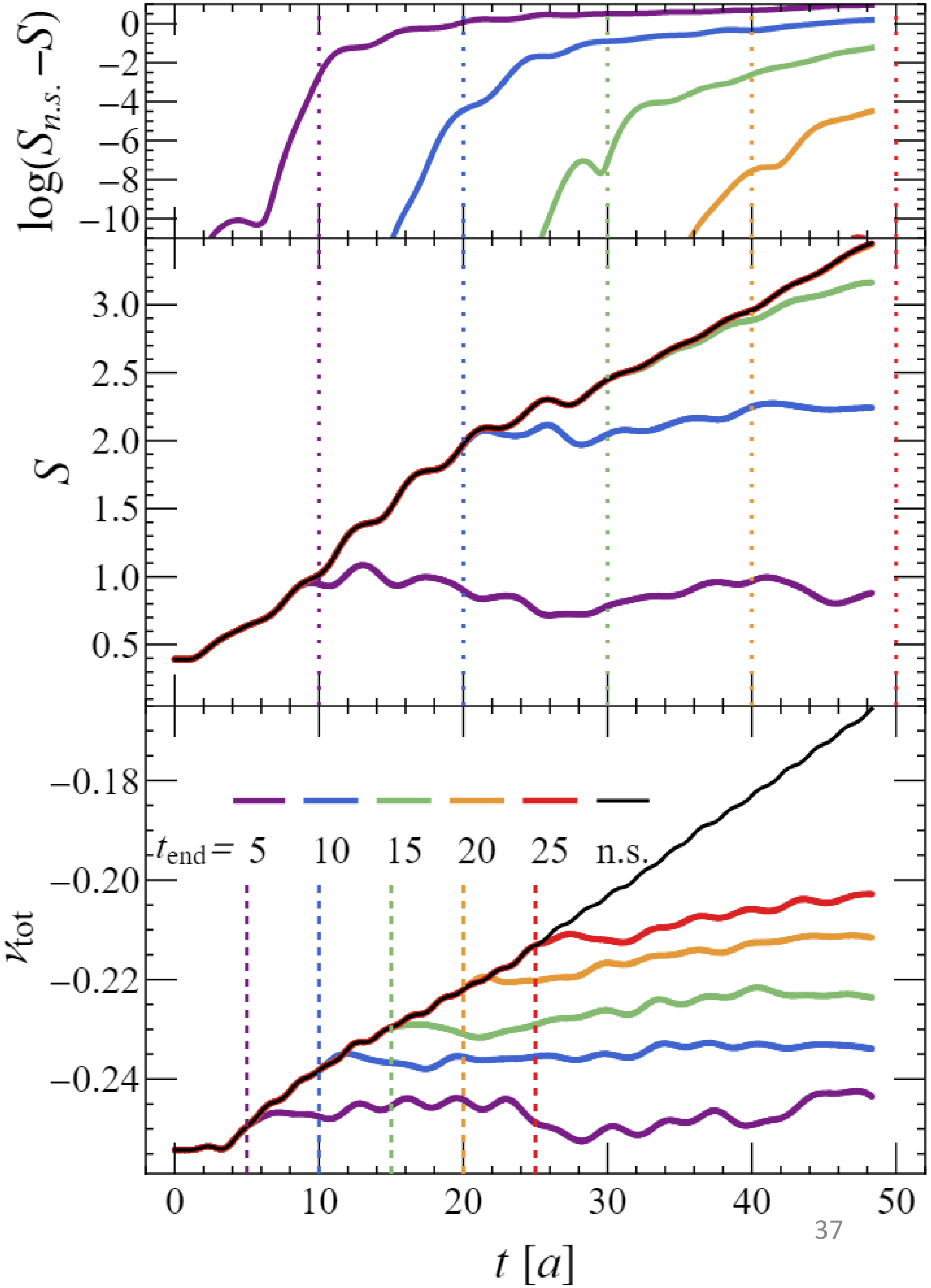
Modify the external source:

$$L_{\text{ext},n}(t) = \begin{cases} -\theta\left(\frac{t-t_0}{a} - \left|n - \frac{N}{2}\right|\right), & t \le t_{end} \\ -\theta\left(\frac{t_{end}-t_0}{a} - \left|n - \frac{N}{2}\right|\right), & t > t_{end} \end{cases}$$

instead of

$$L_{\text{ext},n}(t) = -\theta\left(\frac{t-t_0}{a} - \left|n - \frac{N}{2}\right|\right)$$

Condensate responds instantaneously

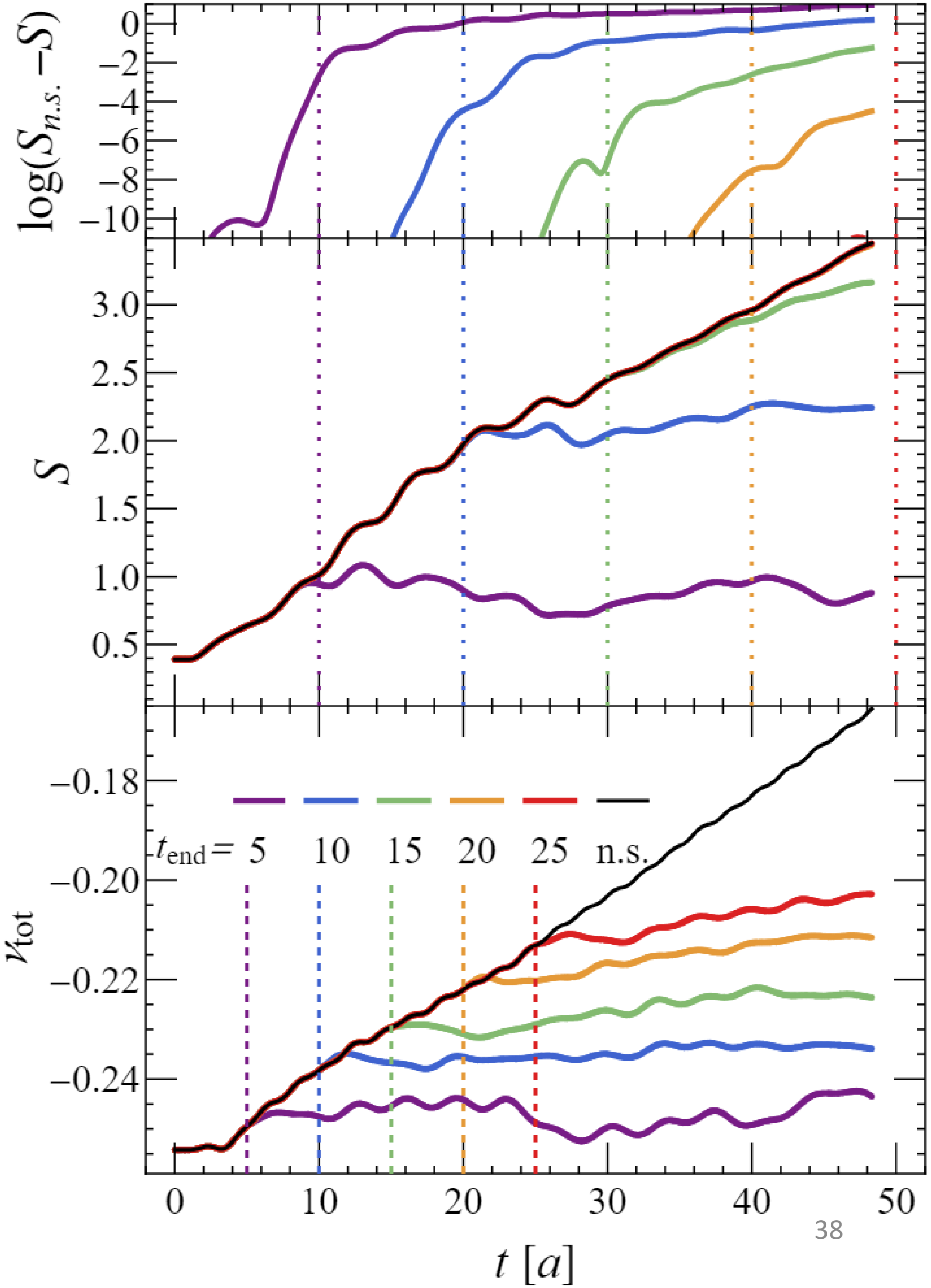


Stopping the jets

Entanglement entropy does not deviate until $t \sim 2\,t_{end}$

Information propagates back to the boundary at about speed of light

Condensate responds instantaneously



Conclusion

- Dynamical pair production leads to electric field screening and modification of the vacuum condensate
- Electric field and chiral condensate equilibrate in the central region
- Entanglement between jets steadily grows with contributions from many Schmidt states
- At large coupling we observe a dynamical transition of Schmidt states from fermionic Fock states to bosonic Fock states
- Second Renyi entropy in the central region exhibits a transition from the area law to the volume law

Backup

System size (in)dependence with exact diagonalization