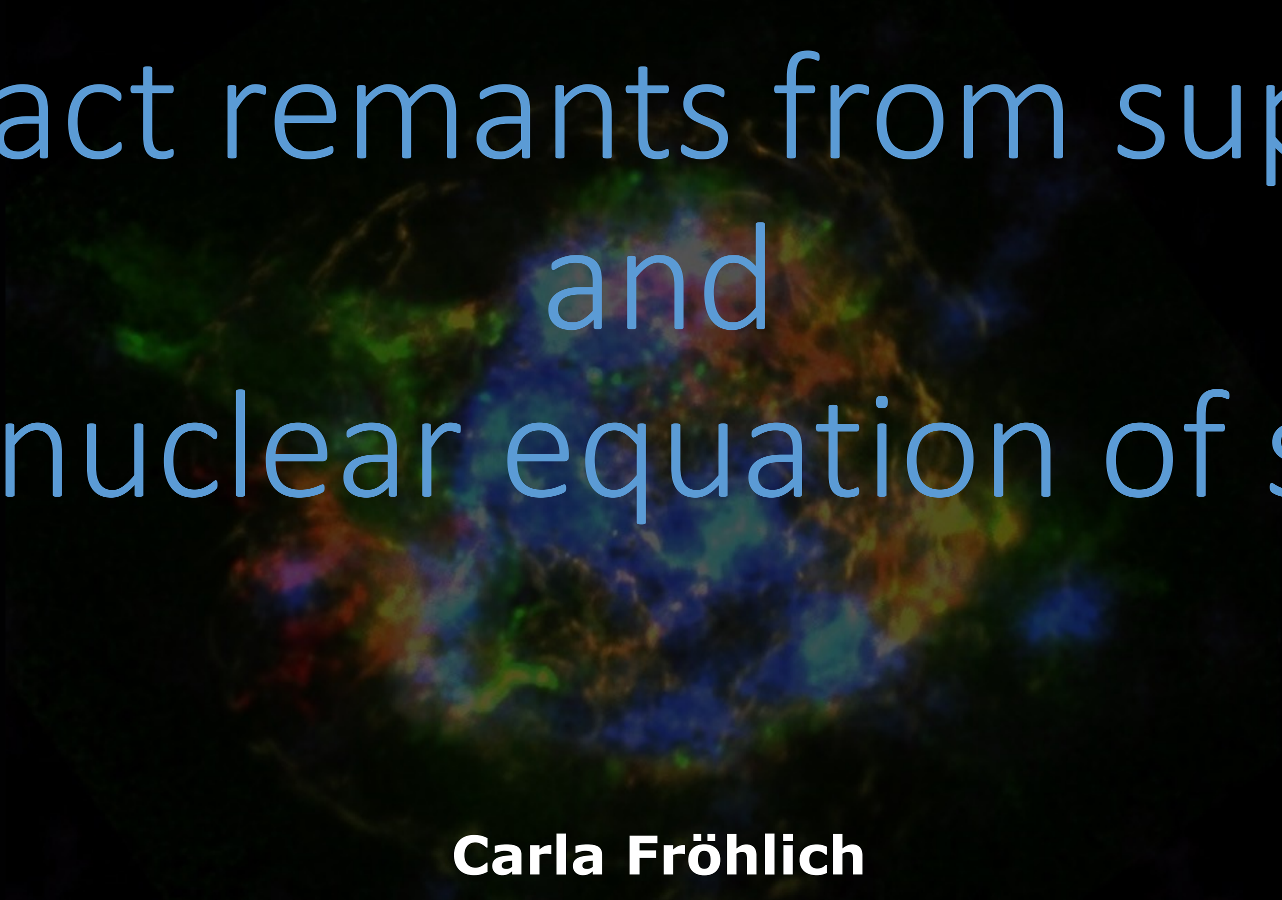
Statistical distributions of compact remants from supernovae and the nuclear equation of state



Carla Fröhlich
North Carolina State University

INT Workshop "Uncertainty Quantification" (8-12 July 2024)

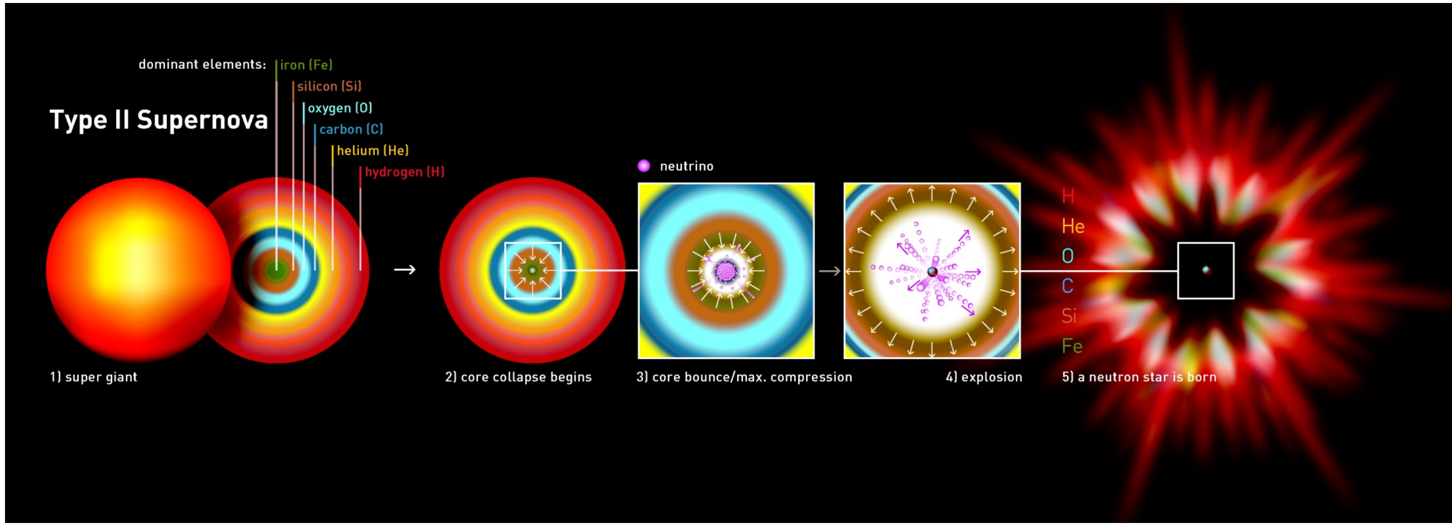
9 July 2024

Statistical distributions of compact remants from supernovae and the nuclear equation of state

Core-Collapse Supernovae (CCSNe)

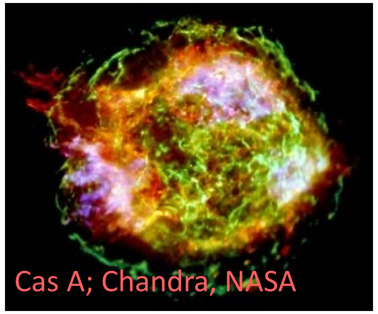
- Massive stars (> ~8-10Msun) at the end of their lives
    - After Si-burning
    - Onset of collapse (negative velocities in the core)

- Phases of a CCSN:
    - Collapse
    - Core-bounce
    - Prompt shock
    - Shock stall
    - Revival of shock / no revival
    - Explosion / no explosion



Janka (2012)

Core-collapse supernova simulations


Cas A; Chandra, NASA

- Multi-dimensional problem
- Multi-physics problem:
  - General relativity
  - Nuclear physics of dense matter
  - Neutrino transport
    (trapped, diffusive, free-streaming regimes)
- Multi-scale problem:
  - shock formation at ~200 km vs entire star 10^8 km
  - collapse and shock formation ~1 s vs shock breakout ~1 day

Core-collapse supernova simulations

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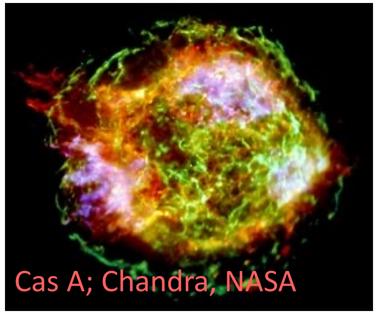
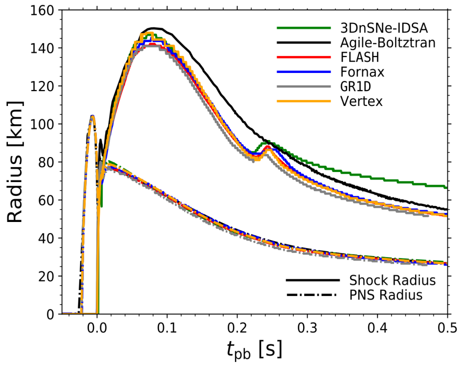


Cas A; Chandra, NASA

Simulation Status:

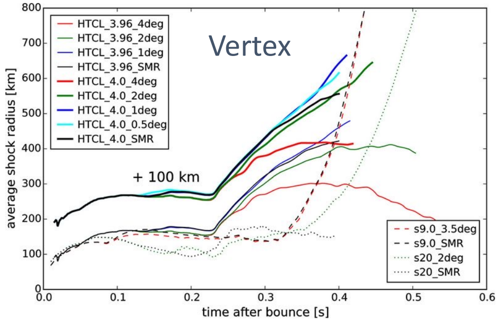
1D: in general no self-consistent explosions
~10 CPUh/model

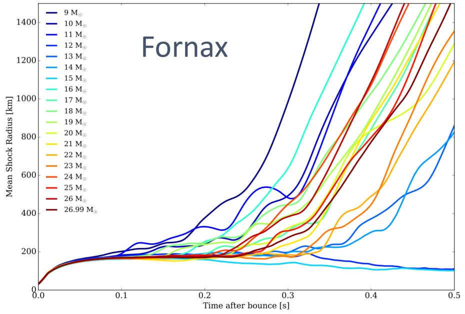
O'Conor+18



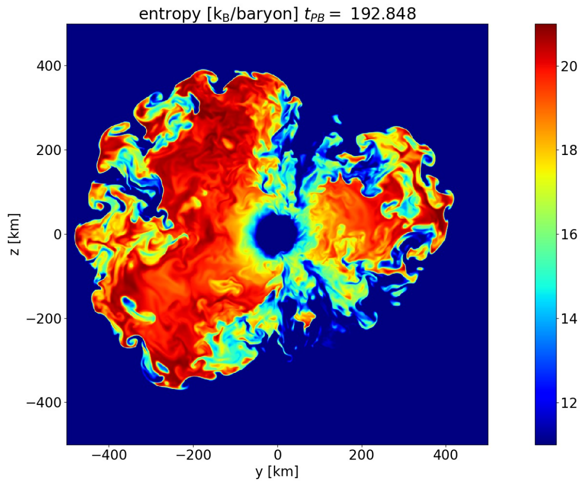
2D: models have converged

3D: mixed results
~ Mio CPUh/model





The paths forward



- Self-consistent 3D simulations
  - The ultimate goal
  - Computationally expensive → can do O(10)

- Effective models
  - Simplify part of the problem, but have free parameters
  - Physically reliable
  - Computationally efficient → can do O(1000)

- The two paths are complimentary (3D-1D-3D feedback loop)
- Both paths are needed for current open science questions

Effective CCSN Models

- Parametrize a multi-dimensional aspect in 1D simulations
  - Mixing above the PNS, enhanced neutrino heating, etc
- Calibrate parametrization, then apply to many models
  - Eg a suitable model should reproduce observables of SN1987A
  - Predictive within the framework

- PUSH: Parametrized neutrino heating — Perego+15, Ebinger+19, Curtis+19, Ebinger+20, Ghosh+23
- PHOT-B: Parametrized neutrino heating — Ugliano+12, Ertl+15, Sukhbbold+16
- STIR: Parametrized mixing above PNS — Couch+20

O'Connor+13; Mueller+15; Pejcha15; Fryer+12,22; …

PUSH: An effective CCSN Model

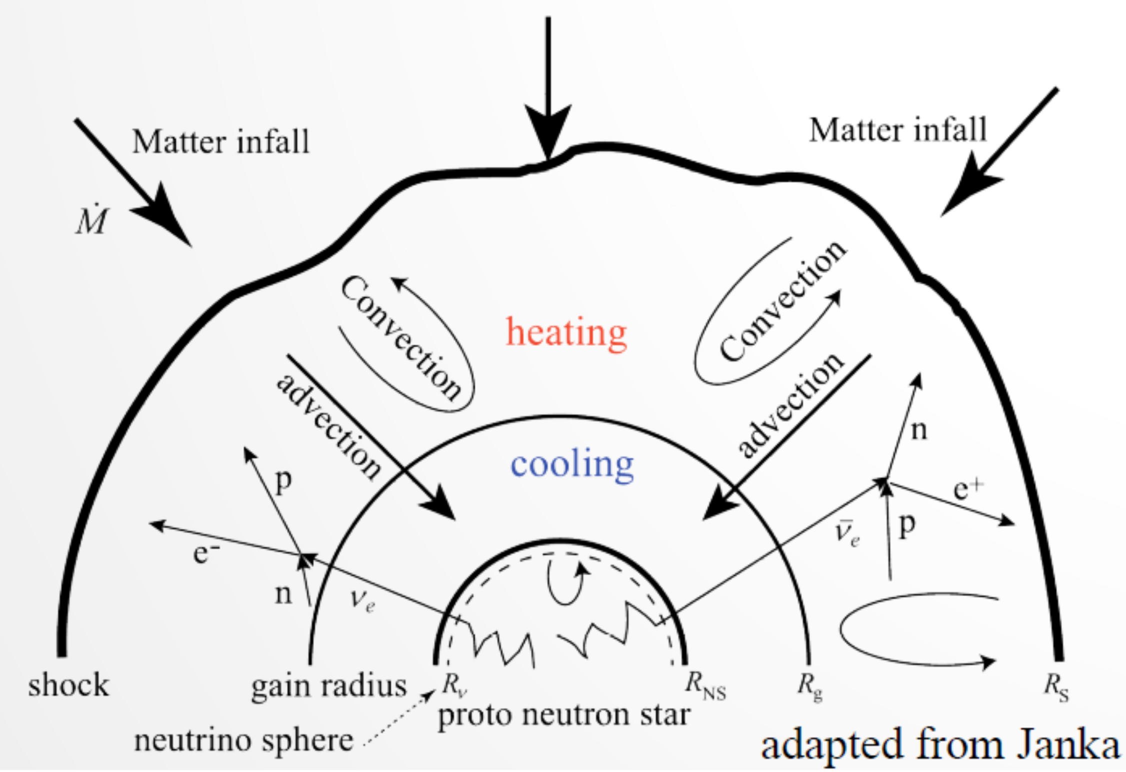
- Neutrino-driven (convection-aided) mechanism
  - Neutrinos are emitted from hot PNS, deposit energy behind the shock
  - Material behind the shock is unstable to convection → enhanced neutrino heating

- Additional (artificial) heating term:

$$\frac{dE_{\rm tot}}{dt} = \frac{dE_{\nu_e+\bar{\nu}_e}}{dt} + \frac{dE_{\rm push}}{dt}$$

Standard heating from electron (anti-) neutrinos

Additional heating in PUSH



adapted from Janka

Simulation Setup: PUSH

- General relativistic hydrodynamics: Agile

Liebendoerfer+02

Simulation time: up to 15sec (typically ~8sec)

- Neutrino transport:
  - IDSA and advanced spectral leakage (ASL)

Lieberdoerfer+09; Perego+16

Electron fraction is evolved during collapse and explosion

- Nuclear EOS: 6 different nuclear EOSs
  - DD2, SFHo, SFHx, BHBλφ, TM1, NL3

Hempel+02; Typel+10

Mass cut emerges from the simulation
→ ejecta and explosion energy are not independent "knobs to turn"

→Predictive (within the framework) for outcome (NS or BH), explosion energy, etc

Statistical distributions of compact remants from supernovae and the nuclear equation of state

(as relevant for supernova simulations)

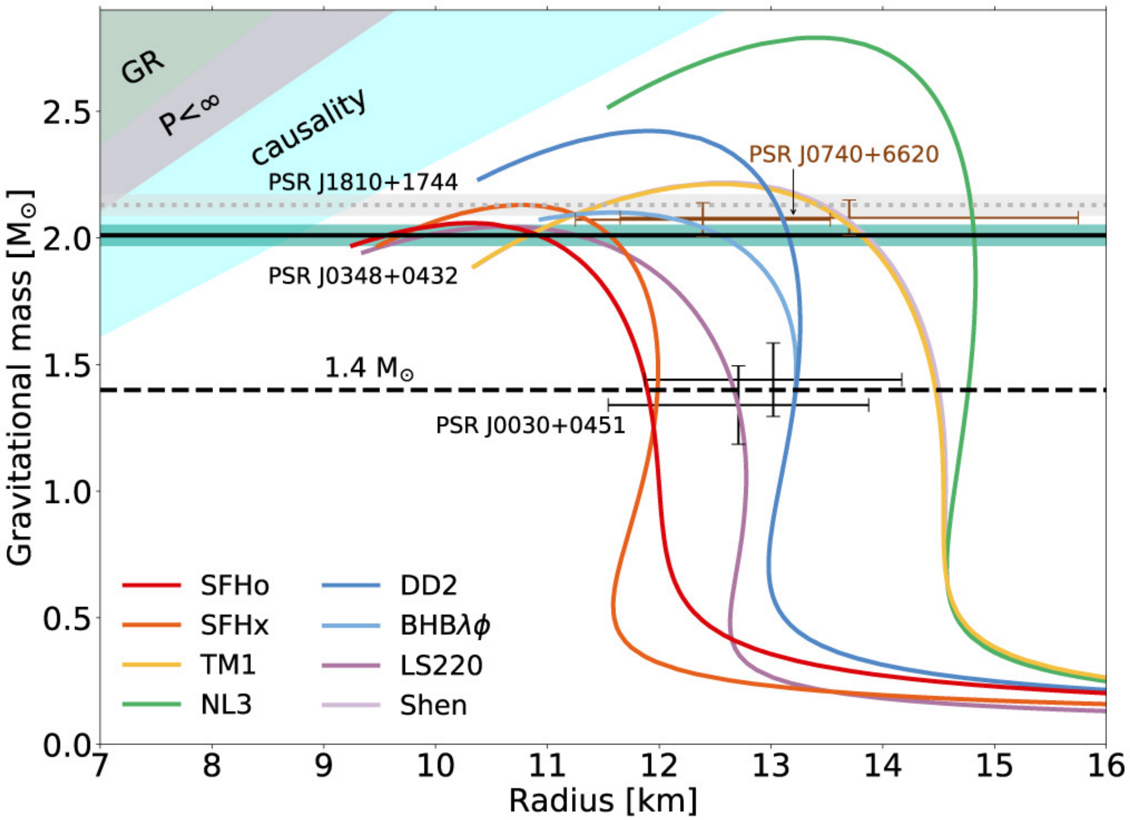
Supernova nuclear EOS

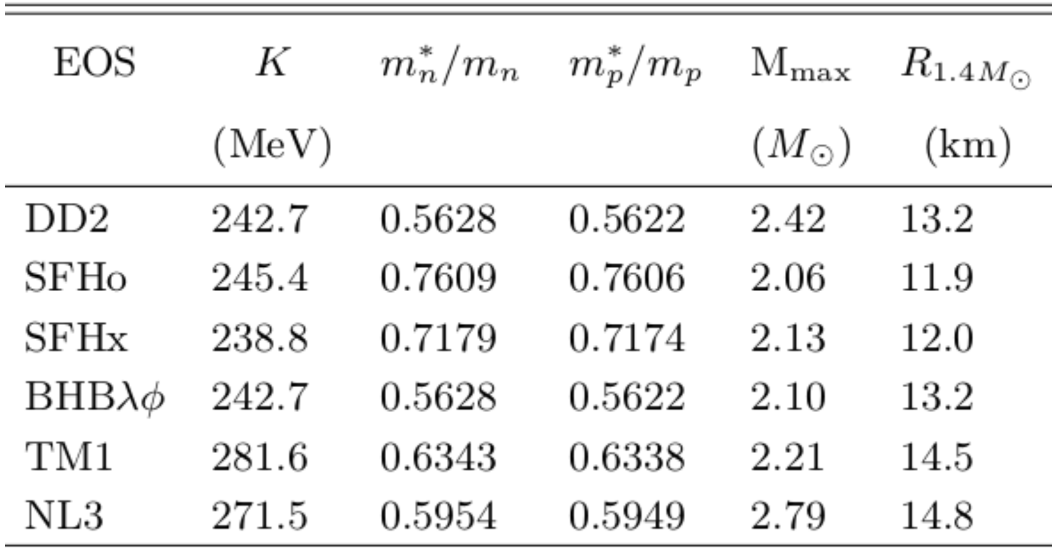
- Nuclear physics input to astrophysical simulations
  - Thermodynamic quantities
  - Nuclear composition

- Challenges:
  - Finite temperature: $T = 0 – 100$ MeV
  - No weak equilibrium: $Y_e = 0 – 0.6$
  - Wide density range: $\rho = 10^4 – 10^{15}$ g/cm^3

  → **In tabular form**: ~1 million points in (T, Y_e, ρ)

Tabulated EOS





EOS	K (MeV)	m_n^*/m_n	m_p^*/m_p	$\mathrm{M_{max}}$ $(M_\odot)$	$R_{1.4M_\odot}$ (km)
DD2	242.7	0.5628	0.5622	2.42	13.2
SFHo	245.4	0.7609	0.7606	2.06	11.9
SFHx	238.8	0.7179	0.7174	2.13	12.0
BHB$\lambda\phi$	242.7	0.5628	0.5622	2.10	13.2
TM1	281.6	0.6343	0.6338	2.21	14.5
NL3	271.5	0.5954	0.5949	2.79	14.8

Statistical distributions of compact remants from supernovae and the nuclear equation of state

Or, what can we do with >1500 supernova simulations?

How to compare simulated and observed data

- Typical setup:
  - Collect input-output pairs from Nature $\{((x)_i, (\tilde{M}_{\mathrm{BH}})_i)\}_{i=1}^{n}$
  - Calibrate the simulator $\arg\min\limits_{s} \frac{1}{n} \sum_{i=1}^{n} \left\{ (\tilde{M}_{\mathrm{BH}})_i - \eta((x)_i; s) \right\}^2$

observed

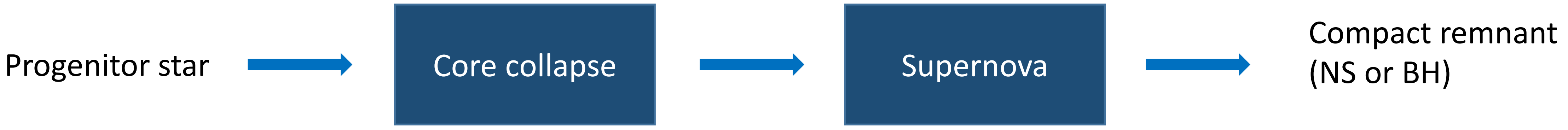
simulated

x … initial mass & metallicity

S … EOS

From progenitor to compact remnant

Simulation:



Progenitor star

Core collapse

Supernova

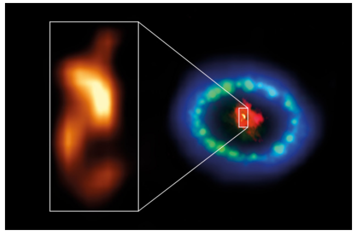
Compact remnant
(NS or BH)

From progenitor to compact remnant

Simulation:

Progenitor star → Core collapse → Supernova → Compact remnant (NS or BH)

SN1987A:



But NS is missing?!

From progenitor to compact remnant

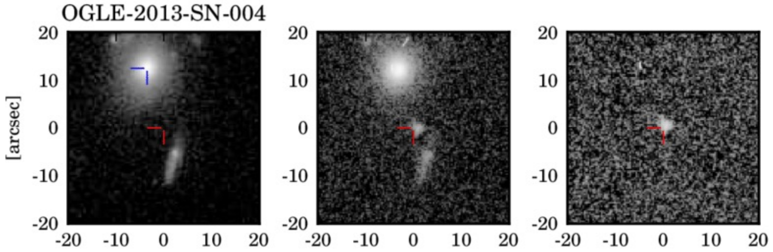
Simulation:

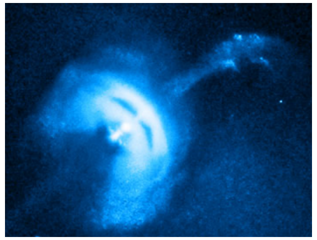
Progenitor star → Core collapse → Supernova → Compact remnant (NS or BH)

Nature:

Stars

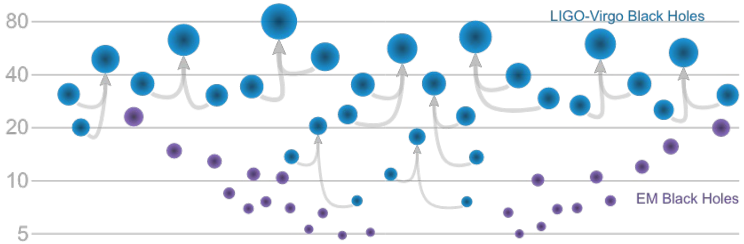
Supernovae





Neutron stars

Black holes



How to compare simulated and observed data

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observed simulated

x … initial mass & metallicity

S … EOS

- Here:
  - We cannot observe such input-output pairs from Nature
  - We cannot calibrate on the joint distribution $p(x, \tilde{M}_{\mathrm{BH}})$ nor the conditional distribution $p(\tilde{M}_{\mathrm{BH}} \mid x)$
  - Instead obtain marginal distributions: $p(x)$ and $p(\tilde{M}_{\mathrm{BH}})$

Simulated data

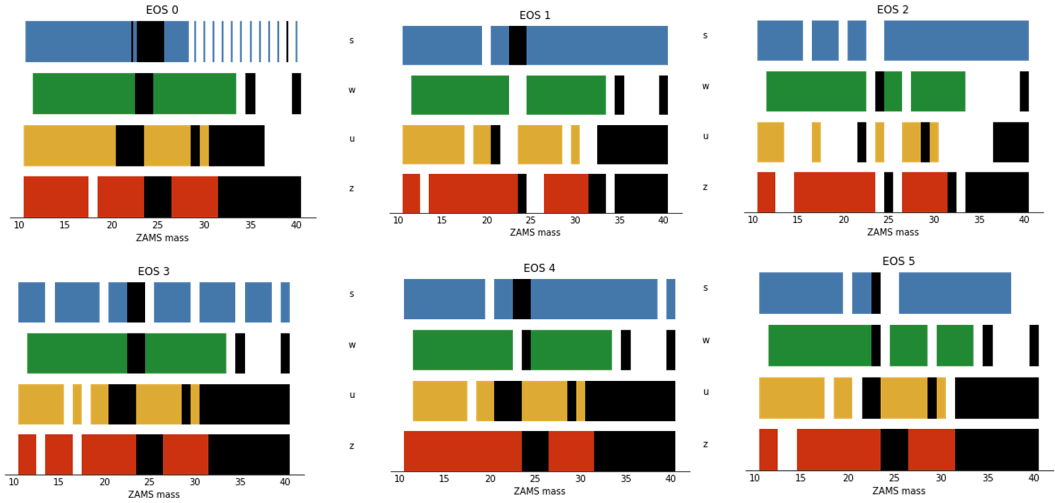
- Simulations of core-collapse supernovae using PUSH
- Each simulation predicts as output as remnant mass $\eta(x,s)$
  - Either a neutron star (NS) of a given mass
  - Or a black hole (BH) of a given mass

- Simulation is a mapping $\zeta(x): x \rightarrow \eta(x,s)$
- Assume: mapping mimics physical reality for ideal EOS s_0

$$\zeta(x) \approx \eta(x,s_0)$$

Simulated data

- Binary outcome: explosion / no explosion
  - Explosion → NSs of a given mass
  - No explosion → BHs of a given mass



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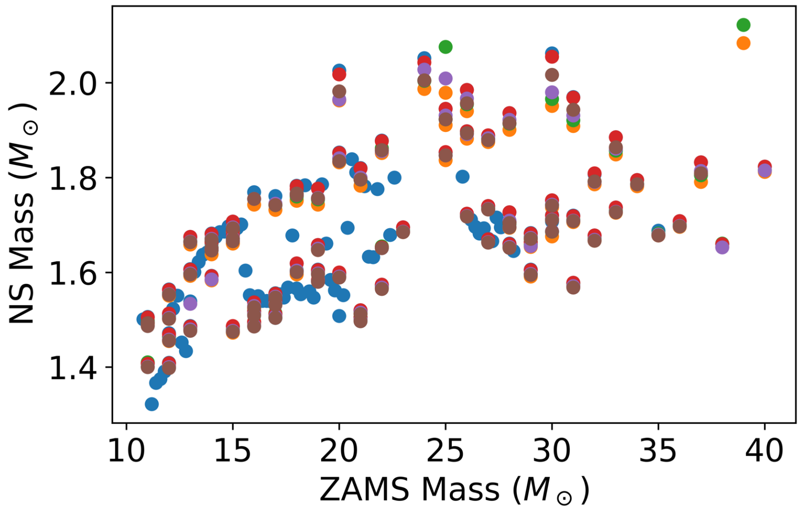

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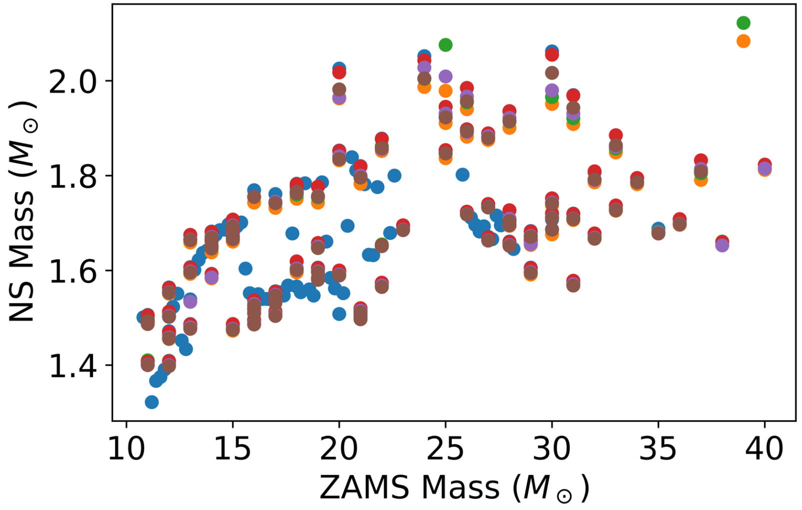


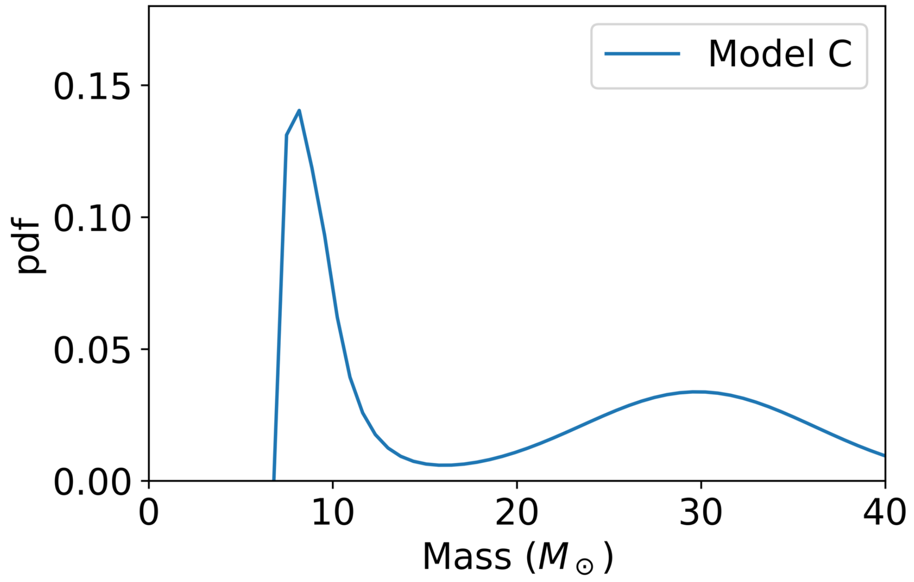
- Distribution of mass:
  - Kroupa initial mass function

$$\xi(M_{\mathrm{ZAMS}}, z) = \begin{cases} 0.035 M_{\mathrm{ZAMS}}{}^{-1.3} & \text{for } M_{\mathrm{ZAMS}} < 0.5 \\ 0.019 M_{\mathrm{ZAMS}}{}^{-2.2} & \text{for } 0.5 \leq M_{\mathrm{ZAMS}} < 1.0 \\ 0.019 M_{\mathrm{ZAMS}}{}^{-2.7} & \text{for } M_{\mathrm{ZAMS}} \geq 1.0 \end{cases}$$

- Distribution of metallicity:
  - uniform

Data from Nature: Observations of Black Holes (BHs)

- Black holes: $p(M_{\mathrm{BH}}) = C {M_{\mathrm{BH}}}^{-\alpha} \Theta({M_{\mathrm{BH}}}^{+} - M_{\mathrm{BH}}) S({M_{\mathrm{BH}}}^{-}, \delta_m)$



${M_{\mathrm{BH}}}^{-} = 6.8$

${M_{\mathrm{BH}}}^{+} = 75,$

$\delta_m = 3.$

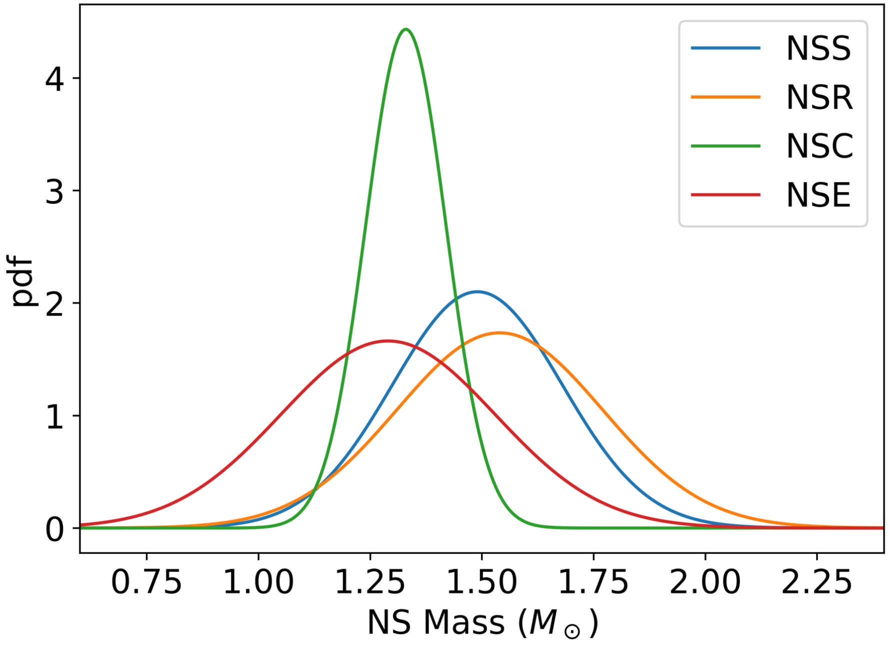
$\alpha = 7.1$

- Model C from Abott et al (2019)
- No BHs below M_{BH}^-
- Truncated power law from M_{BH}^- to M_{BH}^+
- Gaussian distribution of high-mass BHs from pair-instability SNe ←

We drop this term because PUSH does not caputre PISNe

Data from Nature: Observations of Neutron Stars (NSs)

- Neutron stars: $$p(M_{\rm NS}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(M_{\rm NS}-M_0)^2/(2\sigma^2)}$$
  - Split into different astrophysical systems
    - For slow pulsars: $M_0 = 1.49 \text{ and } \sigma = 0.19$
    - For recycled pulsars: $M_0 = 1.54 \text{ and } \sigma = 0.23$
    - For NSs in binaries with another degenerate object (concentric orbits): $M_0 = 1.33 \text{ and } \sigma = 0.09$.
    - For NSs in binaries with another degenerate object (eccentric orbits): $M_0 = 1.29 \text{ and } \sigma = 0.24$



Statistical distributions
of compact remants from supernovae
and
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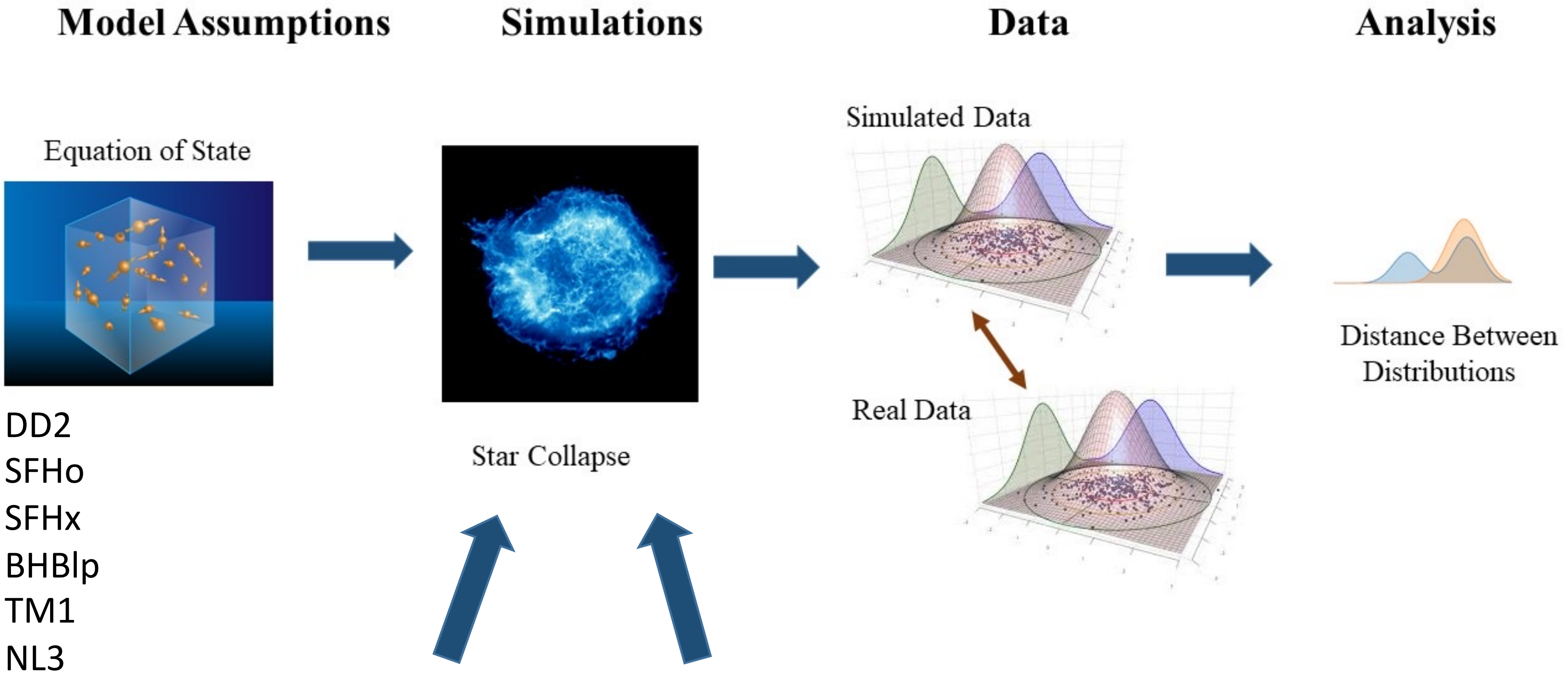
Overview of methodology



Model Assumptions

Simulations

Data

Analysis

Equation of State

DD2
SFHo
SFHx
BHBlp
TM1
NL3

Star Collapse

Simulated Data

Real Data

Distance Between Distributions

Initial stellar mass: 11-40 $M_\odot$

Initial stellar metallicity: 1, 10^{-4}, 0 $Z_\odot$

Measurement Error

- Observed mass distributions are subject to observational error

- We convert the ensemble of measurements to a distribution via the central limit theorem

- Error e can be drawn from a Gaussian distribution with mean zero and standard deviation σ for each population

$$p(e;\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-e^2/2\sigma^2}$$

  - Standard deviation:
    - For BHs: from 90% confidence interval in Abbott et al (2019)

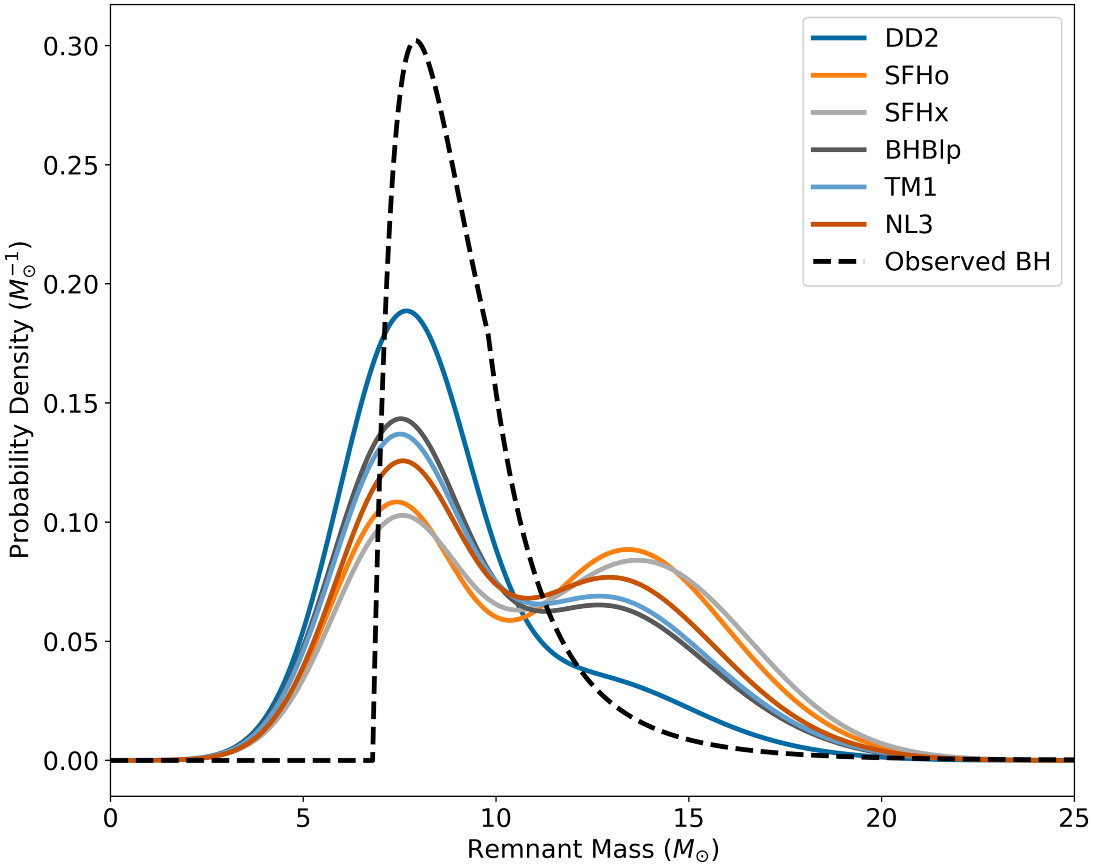
$$\sigma_{\mathrm{BH}}(M_{\mathrm{BH}}) = 0.120213 M_{\mathrm{BH}} + 0.355936$$

    - For NSs: average width of 90% confidence intervals of NS observations

$$\sigma_{\mathrm{NS}} = 0.1.$$

Results: Probability Density Functions

Black holes



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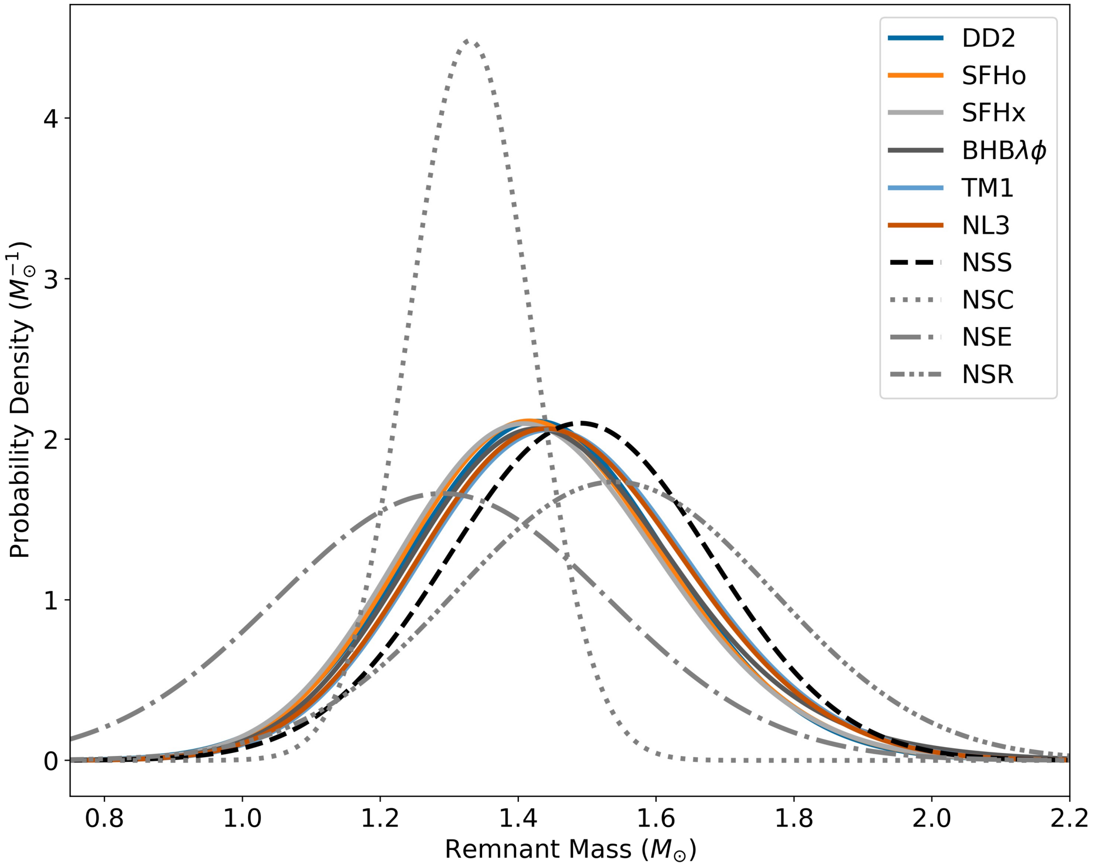


Statistical distances between *p* and p_s

- Use two different distance measures:
  - **Kullback-Leibler (KL) divergence**

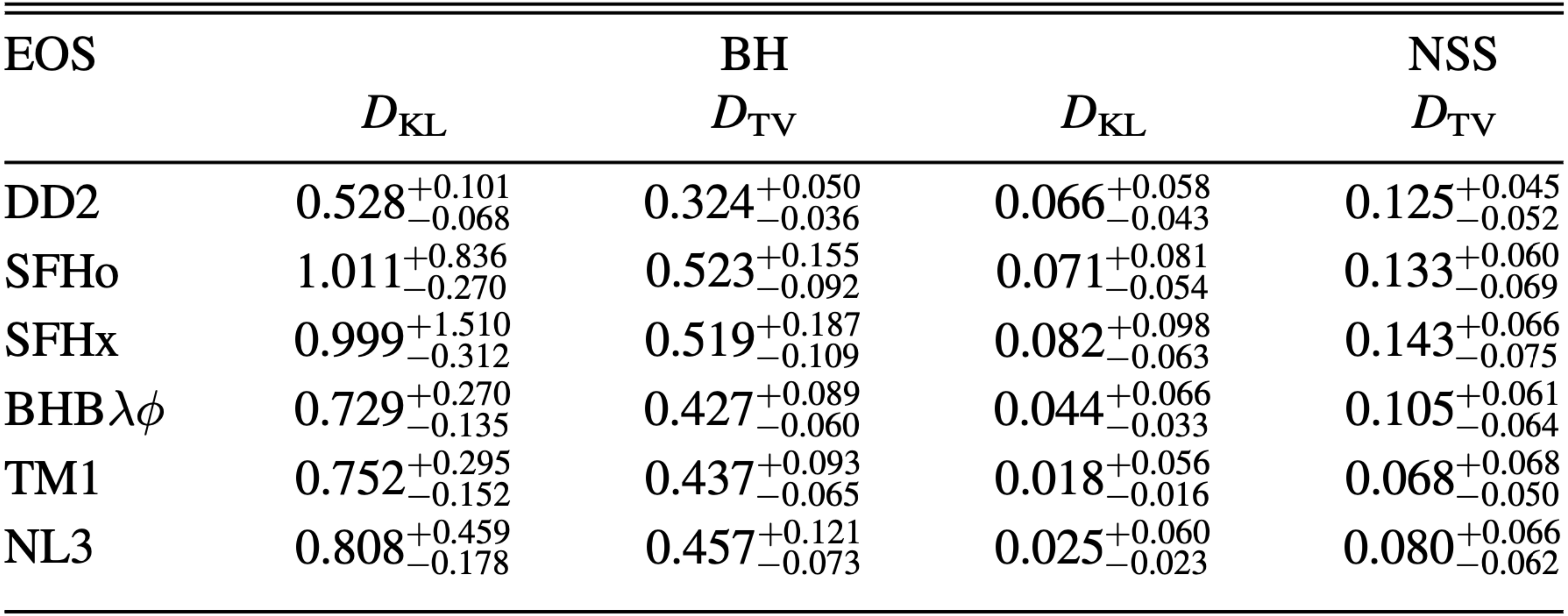
    $$D_{\mathrm{KL}}(p\,||\,p_s) = \int p(\tilde{M}_{\mathrm{BH}}) \log\left(\frac{p(\tilde{M}_{\mathrm{BH}})}{p_s(\tilde{M}_{\mathrm{BH}})}\right) \mathrm{d}\tilde{M}_{\mathrm{BH}}$$

    - Measures the total entropy between *p* and p_s
  - **Total variation (TV) distance**

    $$D_{\mathrm{TV}}(p, p_s) = \frac{1}{2}\int |p(\tilde{M}_{\mathrm{BH}}) - p_s(\tilde{M}_{\mathrm{BH}})| \mathrm{d}\tilde{M}_{\mathrm{BH}}$$

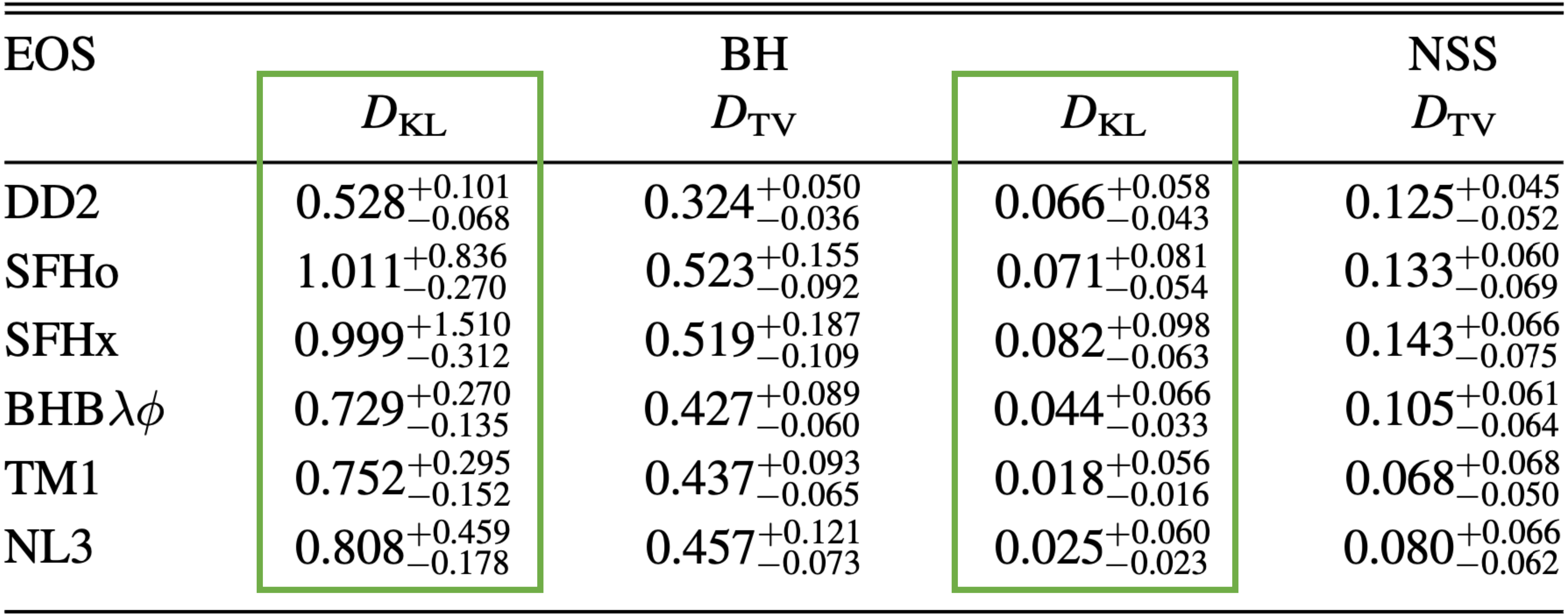
    - Measures the maximum distance between the probabilities assigned to an event by two probability distributions

Results



EOS	BH		NSS	
	D_{KL}	D_{TV}	D_{KL}	D_{TV}
DD2	$0.528^{+0.101}_{-0.068}$	$0.324^{+0.050}_{-0.036}$	$0.066^{+0.058}_{-0.043}$	$0.125^{+0.045}_{-0.052}$
SFHo	$1.011^{+0.836}_{-0.270}$	$0.523^{+0.155}_{-0.092}$	$0.071^{+0.081}_{-0.054}$	$0.133^{+0.060}_{-0.069}$
SFHx	$0.999^{+1.510}_{-0.312}$	$0.519^{+0.187}_{-0.109}$	$0.082^{+0.098}_{-0.063}$	$0.143^{+0.066}_{-0.075}$
BHB$\lambda\phi$	$0.729^{+0.270}_{-0.135}$	$0.427^{+0.089}_{-0.060}$	$0.044^{+0.066}_{-0.033}$	$0.105^{+0.061}_{-0.064}$
TM1	$0.752^{+0.295}_{-0.152}$	$0.437^{+0.093}_{-0.065}$	$0.018^{+0.056}_{-0.016}$	$0.068^{+0.068}_{-0.050}$
NL3	$0.808^{+0.459}_{-0.178}$	$0.457^{+0.121}_{-0.073}$	$0.025^{+0.060}_{-0.023}$	$0.080^{+0.066}_{-0.062}$

Results



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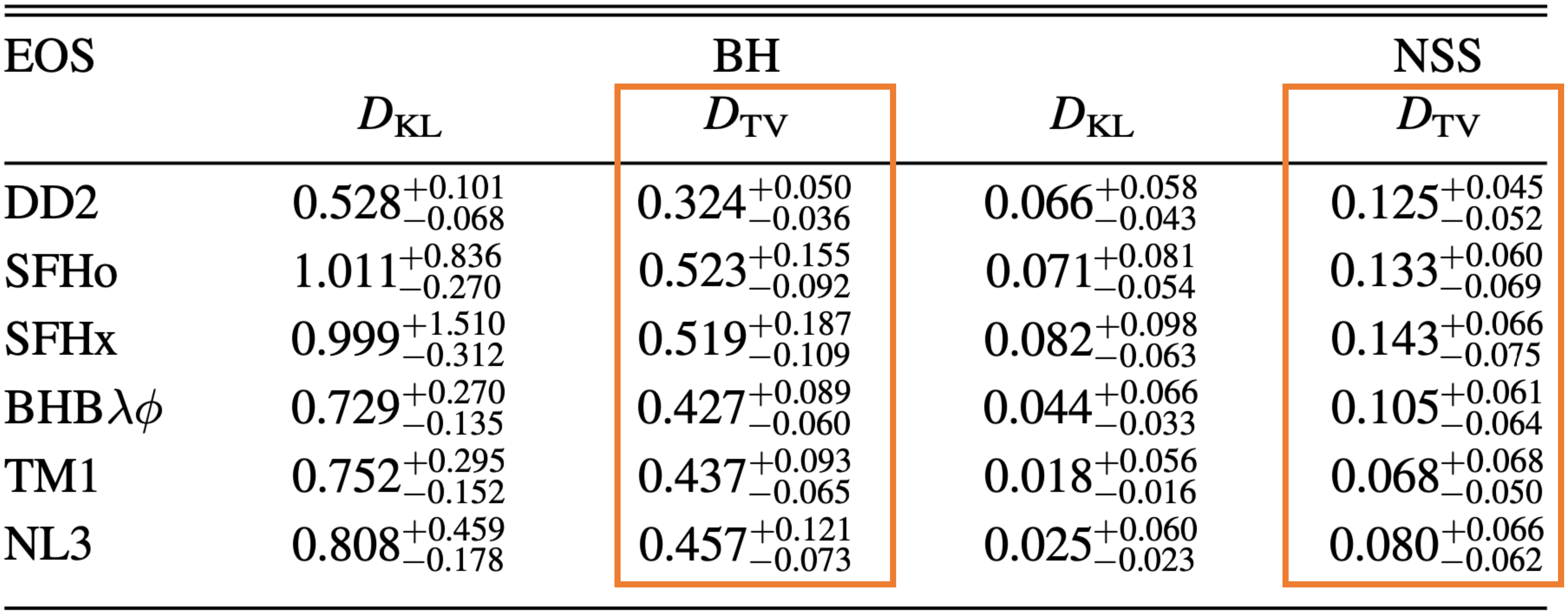
D_{KL} can suffer from stability issues due to small density values, eg at the tails of the distribution

Results



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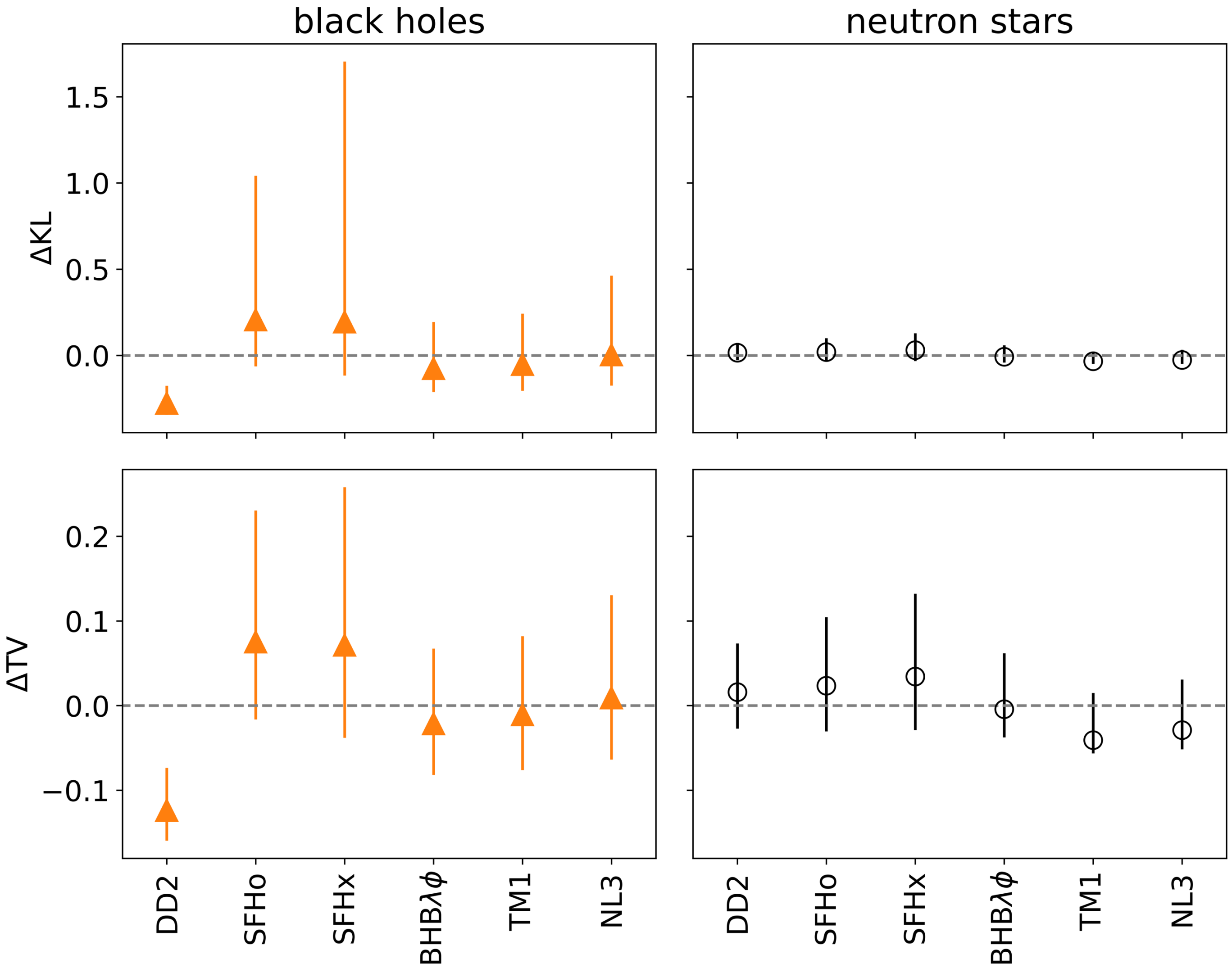
normalized to 1

Results: Distances between distributions

$$\Delta = D_{EOS} - \overline{D}$$

$\Delta < 0$ → more favored
$\Delta > 0$ → less favored

Error bars:
95% confidence interval



Conclusions…

- Rankings are slightly different between BHs and NSs
  - Results for NSs are not statistically significant
  - We show results for NSS (most similar to our simulation setup)

- DD2 is most favored
  - Caveat: DD2 was used to calibrate the PUSH parameters

- SFHo and SFHx are mildly disfavored
- BLBλφ, TM1, NL3 are mildly favored

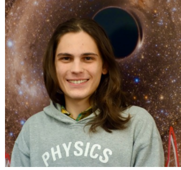
… and improvements

- Include progenitors with ZAMS masses of 8 -11 $M_\odot$
    - We miss NSs at the low mass end (~1.4 Msun)
- Include binary stars (currently assume isolated stars as progenitors)
- Influence of the PUSH calibration on the results
    - Re-calibrated PUSH using TM1 instead of DD2 → no relevant difference
- Understand the influence of the progenitors used
    - All progenitors are from the same stellar evolution code
- Use more observables: NS radius, Ni mass, explosion energy, …

More (future) observables

Gravitational Waves and the nuclear EOS

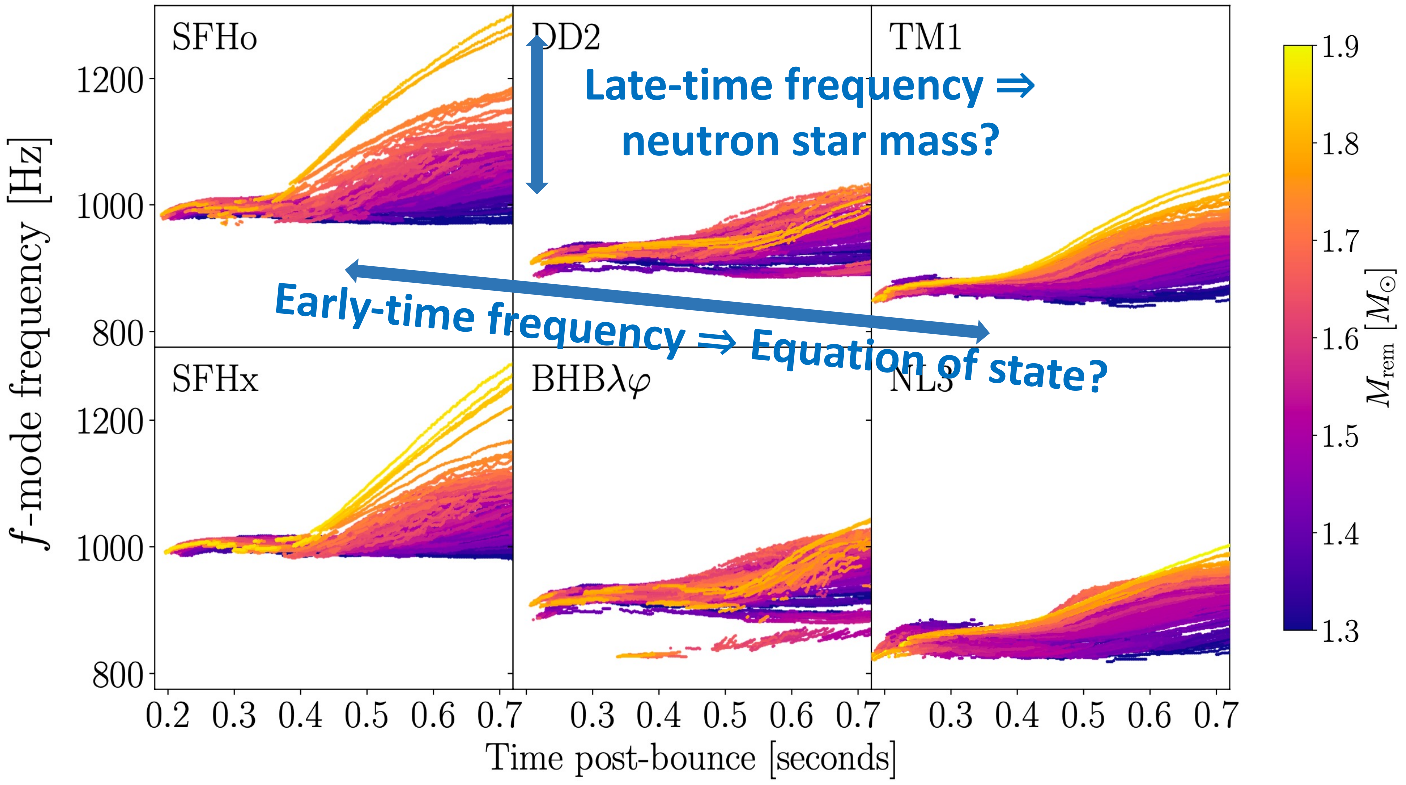
Wolfe+23



Noah Wolfe:
undergraduate at NCSU
→ PhD at MIT

Gravitational Wave Eigenfrequencies from Neutrino-driven Core-collapse Supernovae

Noah E. Wolfe[1,2,3], Carla Fröhlich[1], Jonah M. Miller[4,5], Alejandro Torres-Forné[6,7], and Pablo Cerdá-Durán[6,7]



Late-time frequency ⇒ neutron star mass?

Early-time frequency ⇒ Equation of state?

- Linear perturbation analysis of general-relativistic hydrodynamics background

  Torres-Forné+18, Morozova+18, Torres-Forné+19, Sotani+20

- Calculate time-frequency evolution (no amplitudes) from spherically-symmetric proto-neutron star background

- Identify frequencies that characterize astrophysical properties of proto-neutron star
  - Universal relations for PNS surface gravity

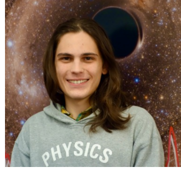
    Torres-Forne+19
  - Multi-messenger observations of core-collapse

    Warren+20, Nakamura+22
  - Parameter estimation

    Bizouzard+21, Powell+22

Gravitational Waves and the nuclear EOS

Wolfe+23



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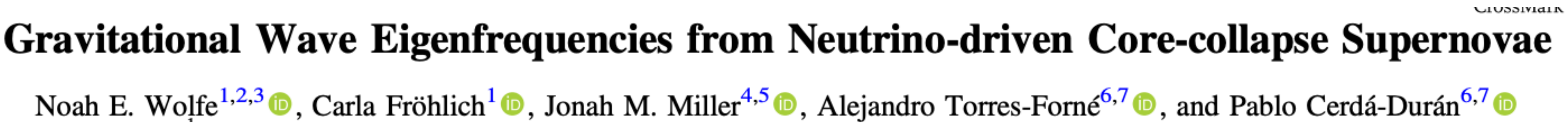

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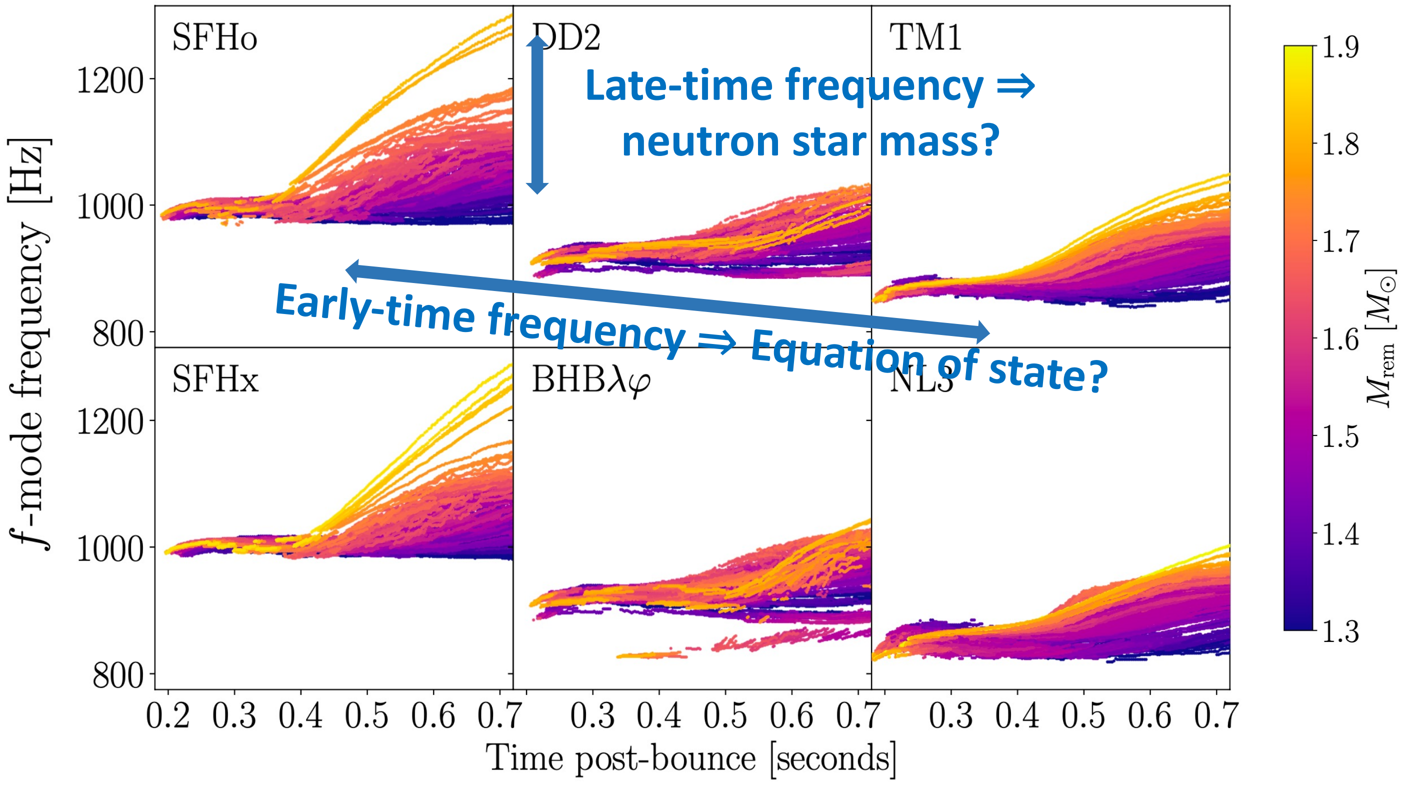

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