# Listening to the long ringdown:
## A novel way to pinpoint the equation of state in neutron-star cores

INT-24-89W: EOS Measurements with Next-Generation Gravitational-Wave Detectors
02 September 2024

*Based on*
2403.03246

*In collaboration with*
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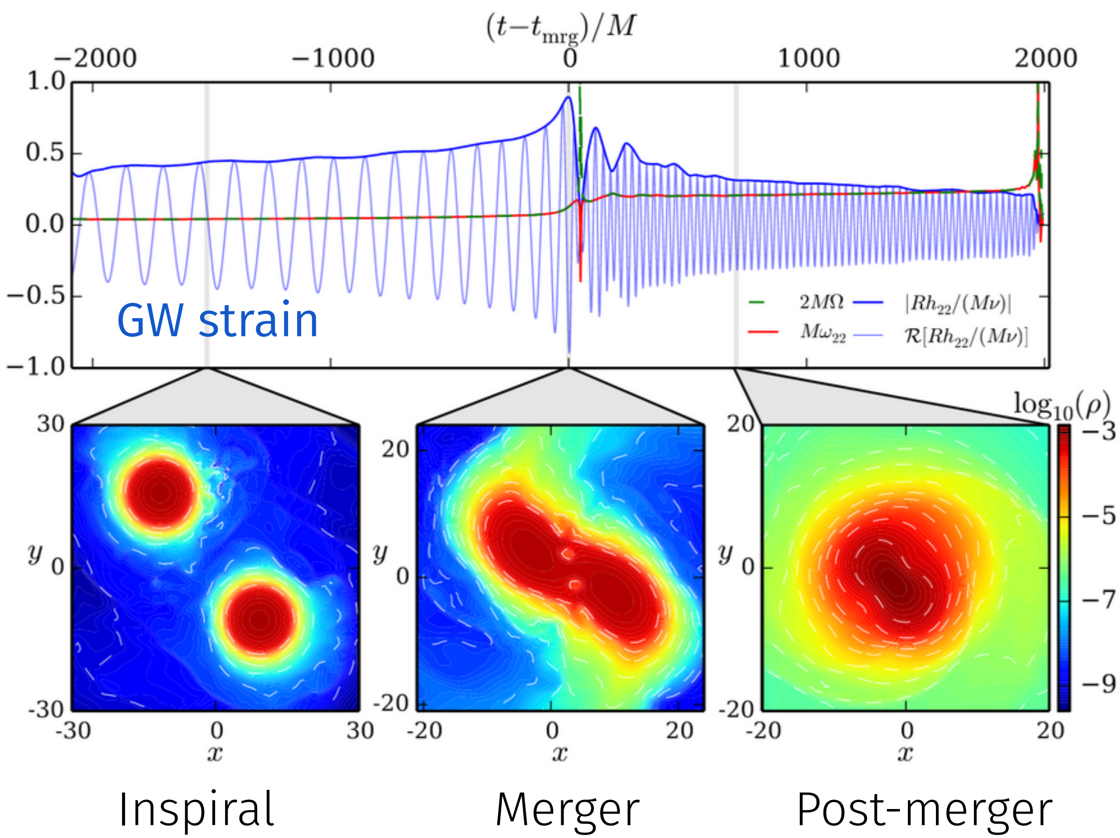
Tyler Gorda
Goethe-University Frankfurt

# Evolution of a binary neutron-star (NS) merger




Bernuzzi, Dietrich, Nagar, PRL 115, 091101 (2015); see also Takami, Rezzolla, Baiotti PRD 91, 064001 (2015), many others

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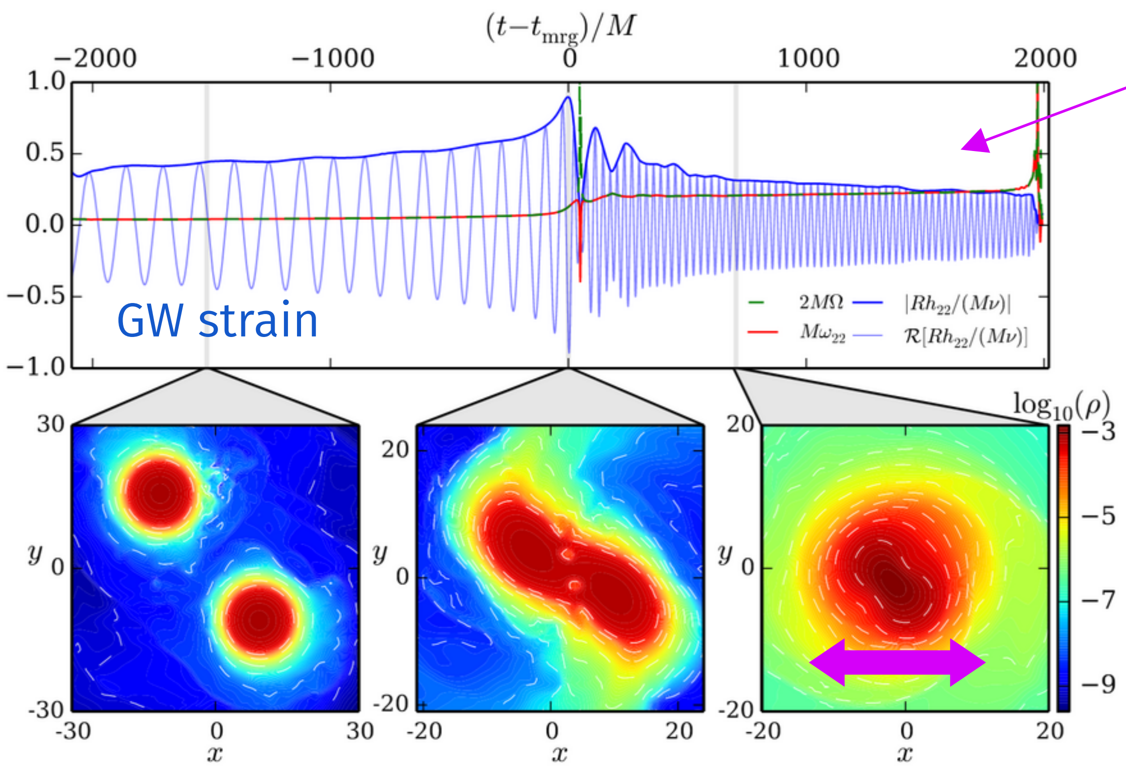
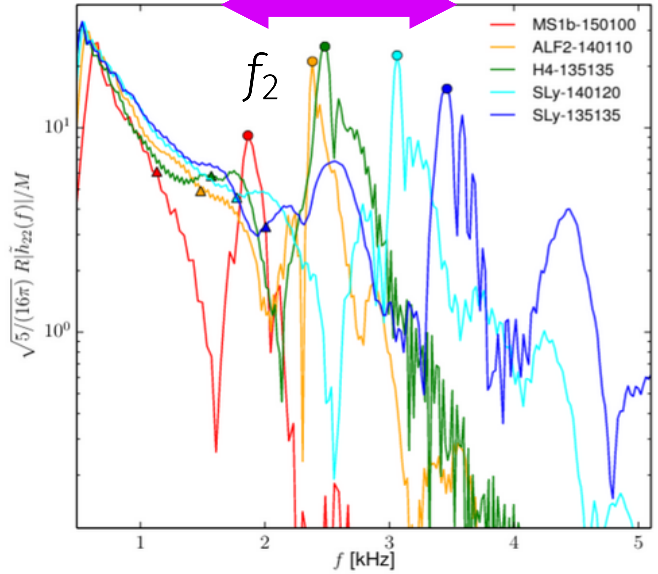



Inspiral Merger Post-merger

Power Spectral Density




- Peaks ( *"$f_2$ frequency"* ) associated with quasi-stationary evolution
- $f_2$ correlated with ***size of remnant*** and ***stiffness of equation of state***

Bernuzzi, Dietrich, Nagar, PRL 115, 091101 (2015); see also Takami, Rezzolla, Baiotti PRD 91, 064001 (2015), many others

# Generic equation-of-state approaches

- Post-merger phase has not yet been seen—too high frequency. 3rd generation detectors expected to see post-merger (≈180 BNSs/year with SNR > 8)

  see, e.g., Evans et al 2109.09882 (Cosmic Explorer technical report)

- Past works using individual "traditional" equation-of-state (EOS) models have seen correlation ***between*** $f_2$ and the ***underlying EOS***

  Bauswein, Stergioulas, PRD 91, 124056 (2015); Takami, Rezzolla, Baiotti PRD 91, 064001 (2015); Rezzolla, Takami, PRD 93, 124051 (2016); Bauswein, Nikolaos Stergioulas, J. Phys. G: Nucl. Part. Phys. 46 113002 (2019); Breschi, Bernuzzi, Godzieba+ PRL 128 (2022) …

- *Model-agnostic EOS inference studies* have been performed using information from the *inspiral phase* of binary NS mergers

- These rely on Bayes's theorem, which calculates a posterior weight for EOSs given prior EOS distribution and observational and theoretical data

$$P(\mathrm{EOS}|\mathrm{data}) = \frac{P(\mathrm{EOS})P(\mathrm{data}|\mathrm{EOS})}{P(\mathrm{data})}$$

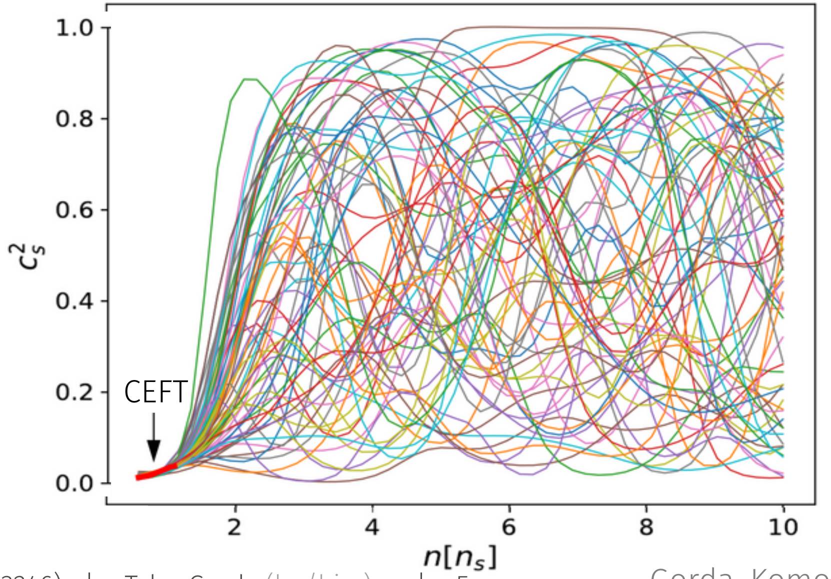
- Goal: Perform *model-agnostic analysis* to connect post-merger information and the EOS, and to *look for new correlations*

# Our generic, model-agnostic EOSs

- Use *Gaussian Process regression* to generate a large ensemble of model-agnostic EOSs. Draw hyperparameters: (based on Essick & Landry PRD 99 (2019) 8, 084049)

$$\bar{c}_s^2 \sim \mathcal{N}(0.5, 0.25^2), \ell \sim \mathcal{N}\left(1.0n_s, (0.2n_s)^2\right), \sigma^2 \sim \mathcal{N}(1.25, 0.25^2).$$

- Use them to generate model-agnostic $c_s^2(n)$; integrate to get pressure and energy density



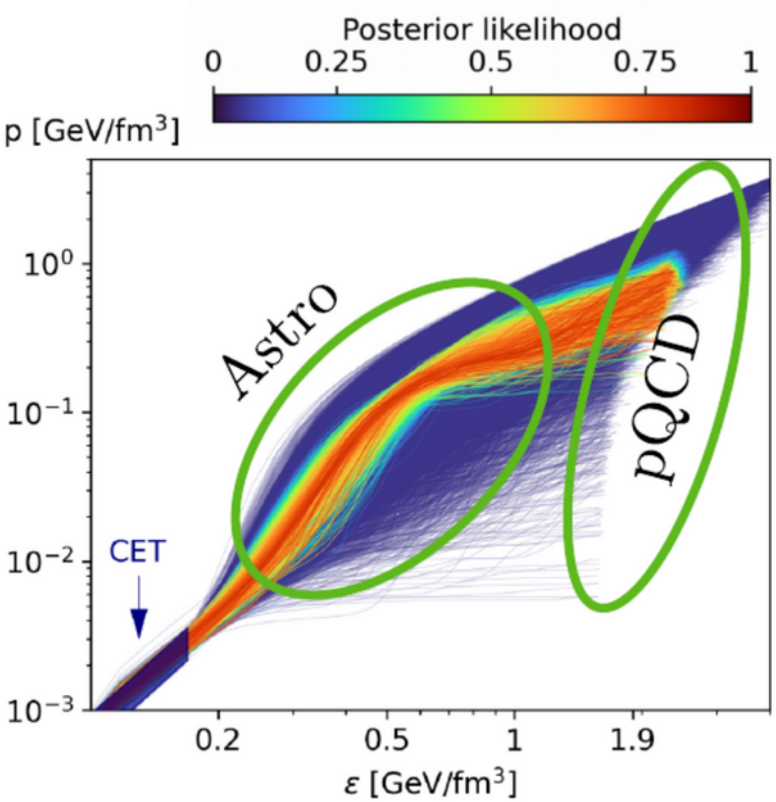



Gorda, Komoltsev, Kurkela, Astrophys.J. 950 (2023) 2, 107

# Constraints and EOS posterior

- Radio astronomy:
  - PSR J0348+0432 with $M = 2.01 \pm 0.04 M_\odot$
  - PSR J1624–2230 with $M = 1.928 \pm 0.017 M_\odot$
- Gravitational-waves:
  - 2d PDF for the tidal deformability and mass ratio at fixed chirp mass extracted from GW170817 by the LIGO-Virgo collaboration
- X-ray observations:
  - 2d PDF for mass and radius measurement of PSR J0740+6620 using NICER + XMM-Newton data.
- *Theory*
  - Chiral EFT and perturbative QCD calculations where valid + thermodynamic construction to apply pQCD at $10n_s$

Komoltsev, Kurkela PRL, 128 (2022) 20, 202701

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# Selecting a small, smart sample of model-agnostic EOSs (1)

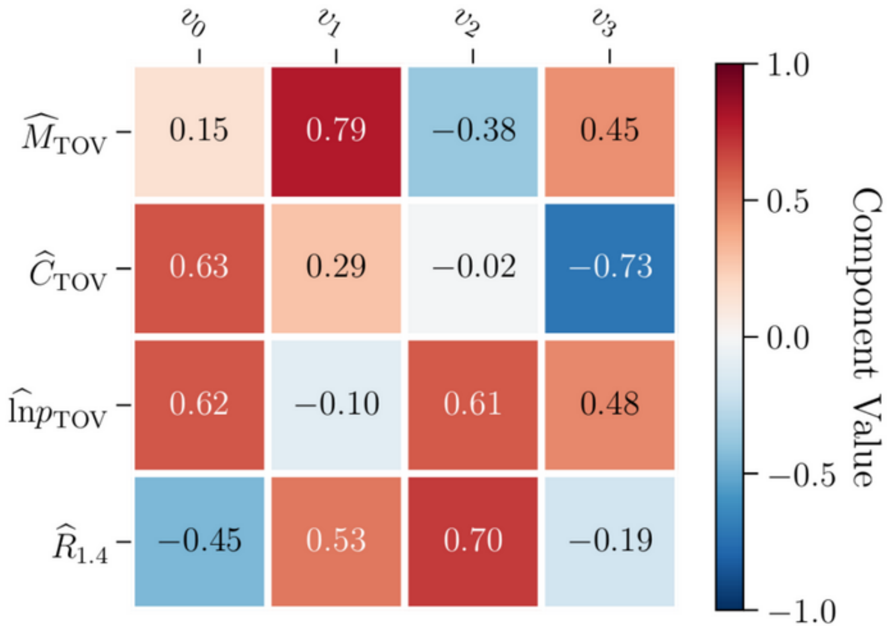
- >$10^5$ EOS simulations are **too expensive to simulate; need to select a small, smart sample**

- Focus on a ***few variables that characterize the highest-density (TOV) part of the EOS***, and one to break degeneracy at lower densities:

$$\left(M_{\mathrm{TOV}}, C_{\mathrm{TOV}} \equiv M_{\mathrm{TOV}}/R_{\mathrm{TOV}}, \ln p_{\mathrm{TOV}}, R(1.4M_{\odot})\right)$$

- Consider the 4d distribution of these variables, ***normalized*** as $\hat{x} \equiv (x - \bar{x})/\sigma_x$ to have mean 0 and variance 1 — but still have ***covariances***

- Find the ***principal components*** in this 4d space that ***capture the majority of the variance*** ($v_0,\ldots,v_3$) — eigenvectors & eigenvalues of covariance matrix

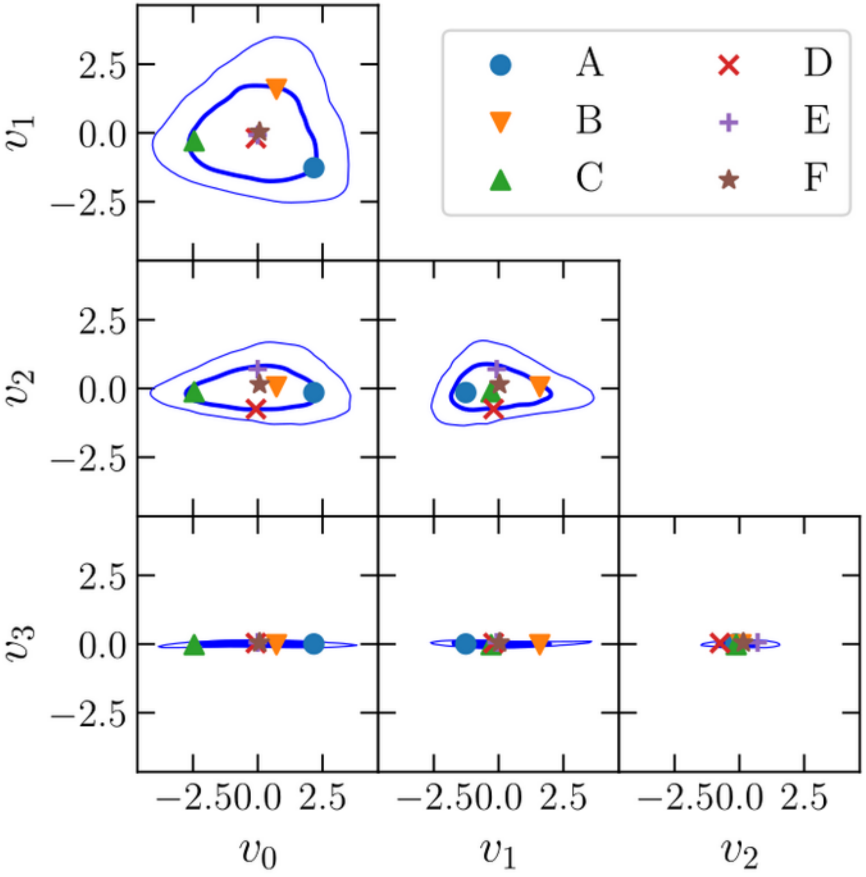


# Selecting a small, smart sample of model-agnostic EOSs (2)

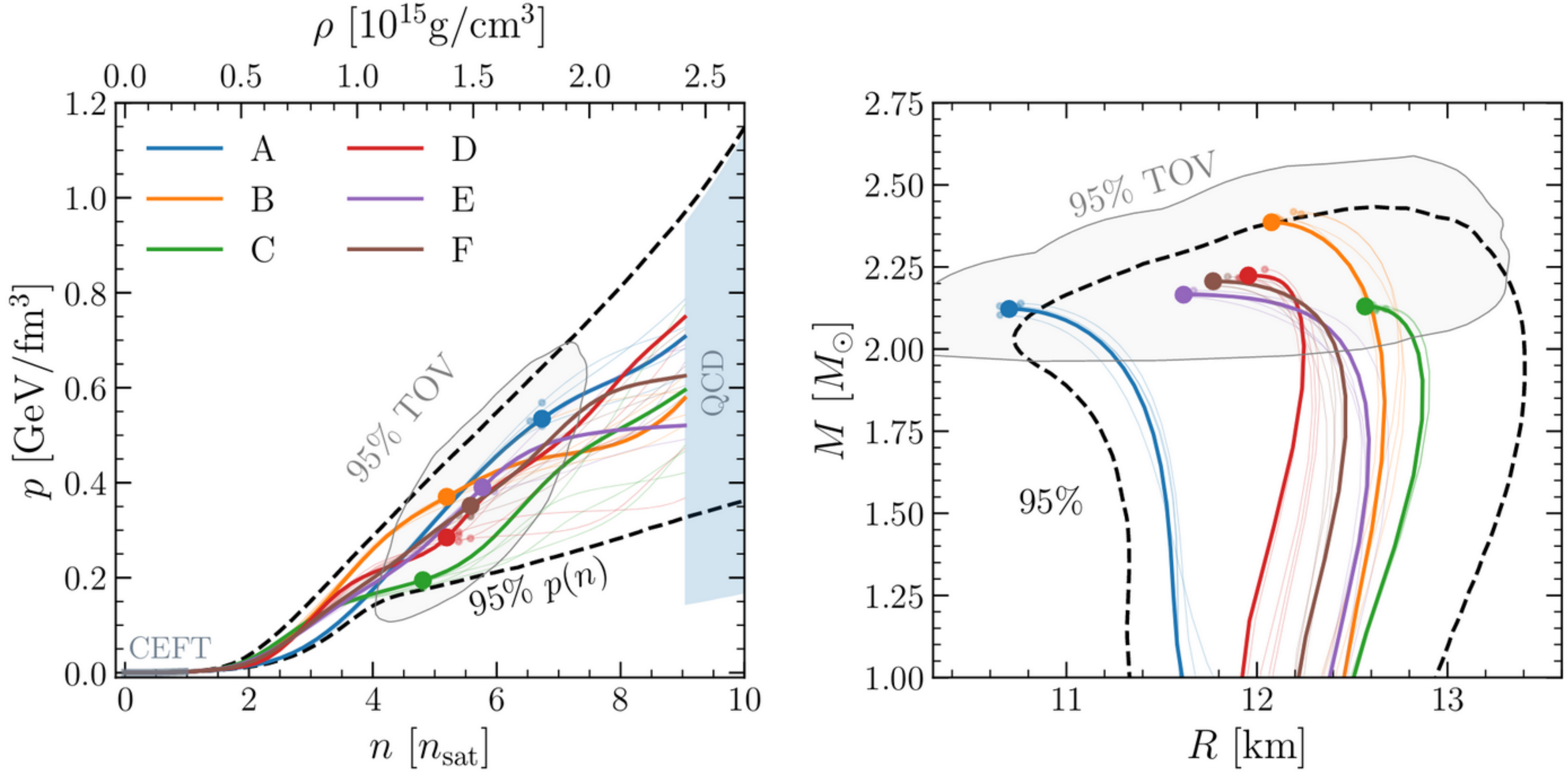
- We find the 4d distribution in the principal components is ***primarily 3d***, with ***prominent triangular shape*** in $(v_0, v_1)$ plane

- Identify 6 “golden EOSs” near the 68% credible region of the 3d part (+1 in center) to simulate – we select the highest-likelihood EOSs out of the 30 closest EOSs

- We find that the golden EOS selection is robust to using the full 4d or reduced 3d metric



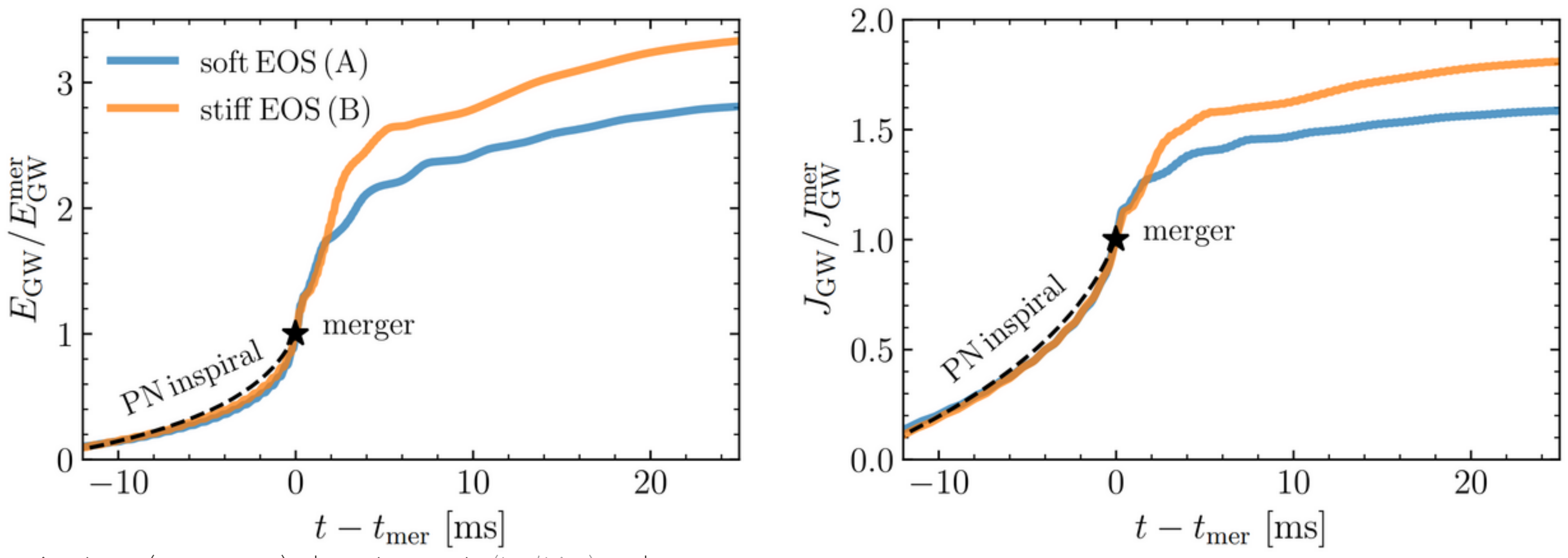
# The six "golden EOSs"



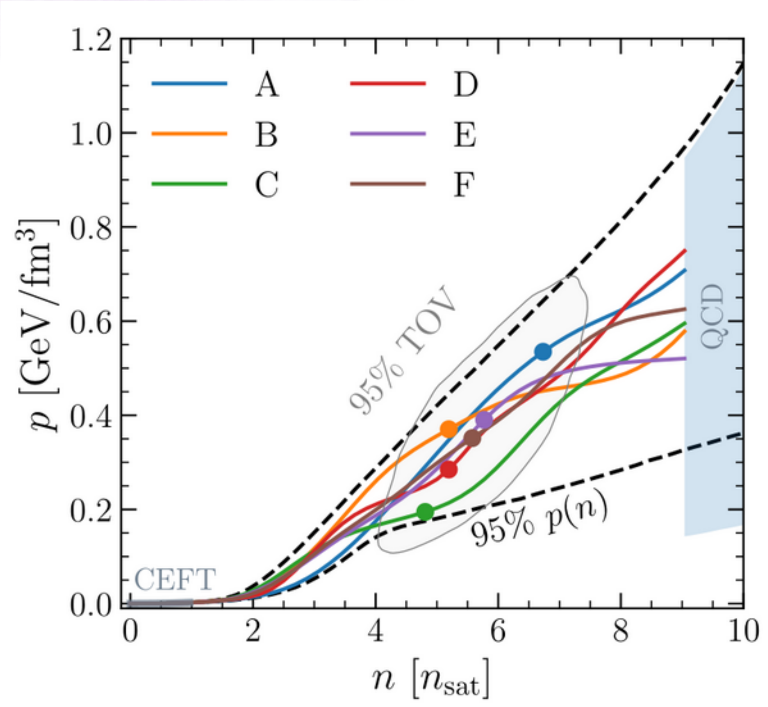
# Binary NS mergers and post-merger quantities

- Six EOSs are manageable, but need to restrict BNS parameter space. Simulate $q = M_2/M_1 = 1.0, 0.85, 0.7$ for GW170817 chirp mass $\mathcal{M}_{\rm chirp} = 1.18 M_\odot$
- Add $T$-dependence via simple hybrid EOS construction with fixed $\Gamma_{\rm th}$=1.75 (but we find that varying $\Gamma_{\rm th}$=1.5, 2.0 has no effect) Figura, Lu, Burgio, Li, Schulze 2005.08691 (PRD)
- Compute (normalized) radiated Energy and Angular momentum in post-merger:

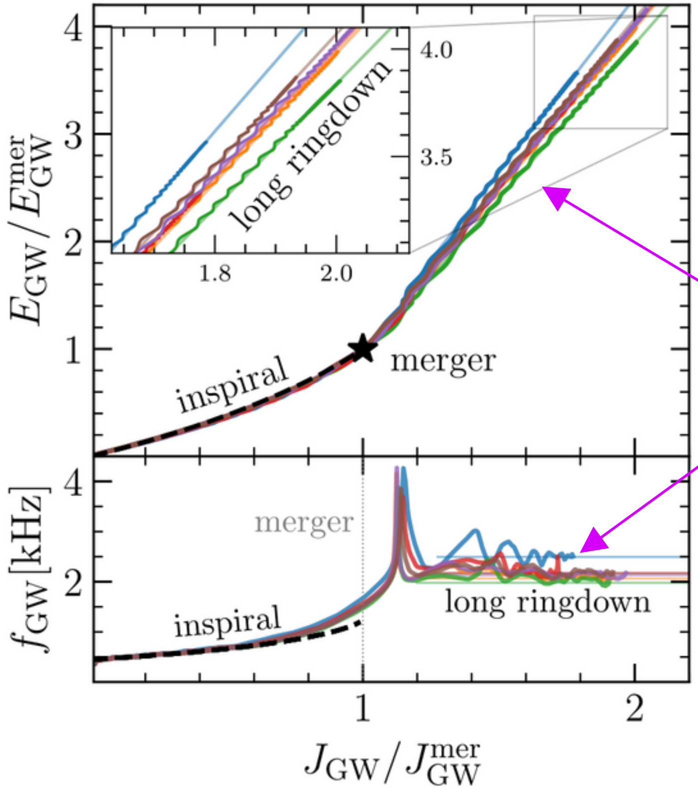
Normalize by merger values



# Correlation observed in the "Long ringdown" of the remnant




$q$ = 1 results:




Slope given by $f_{GW}$
if pure quadrupole
(good approximation)

cf. Bernuzzi, Radice, Ott+ PRD 94(2), (2016)

Striking linear relation between emitted GW energy $E$ and angular momentum $J$

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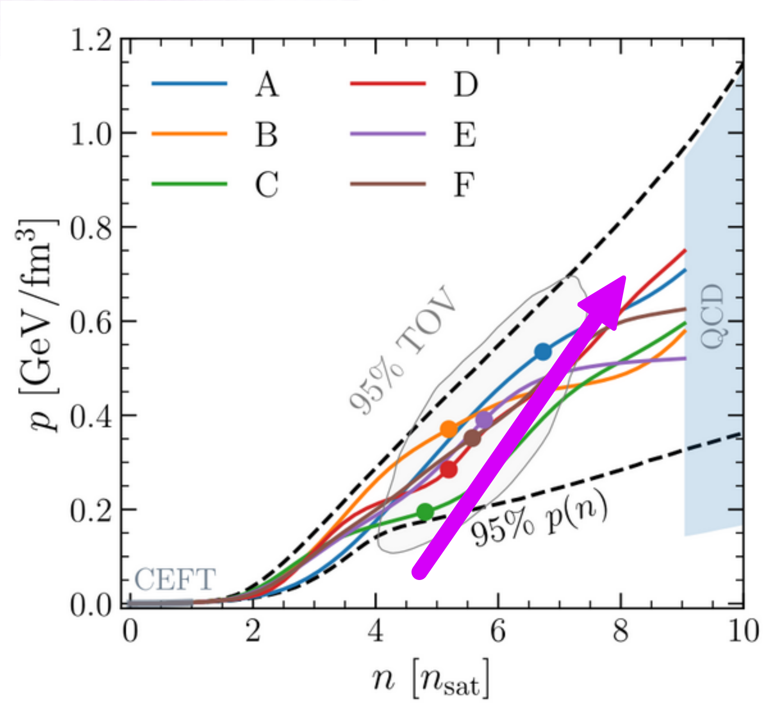


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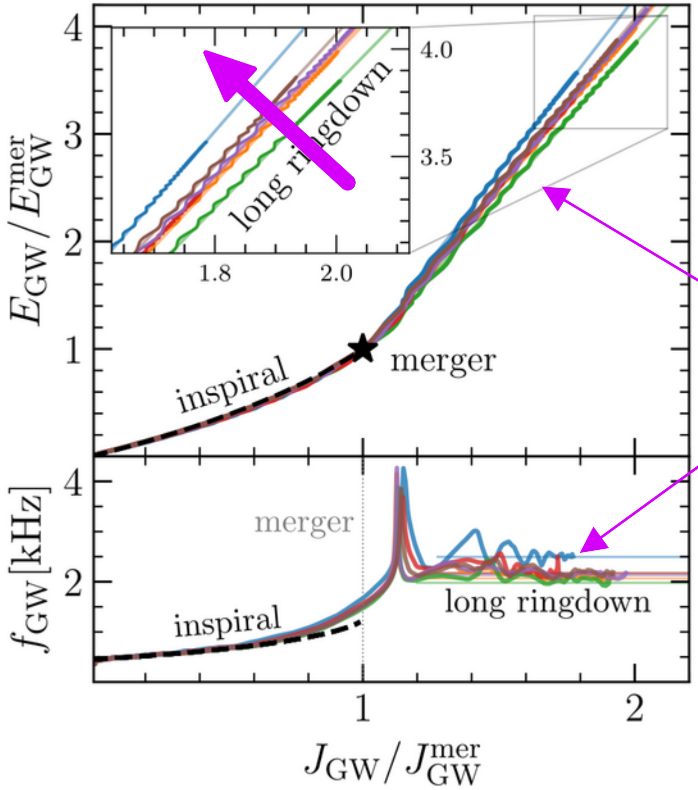


Slope given by $f_{GW}$ if pure quadrupole (good approximation)

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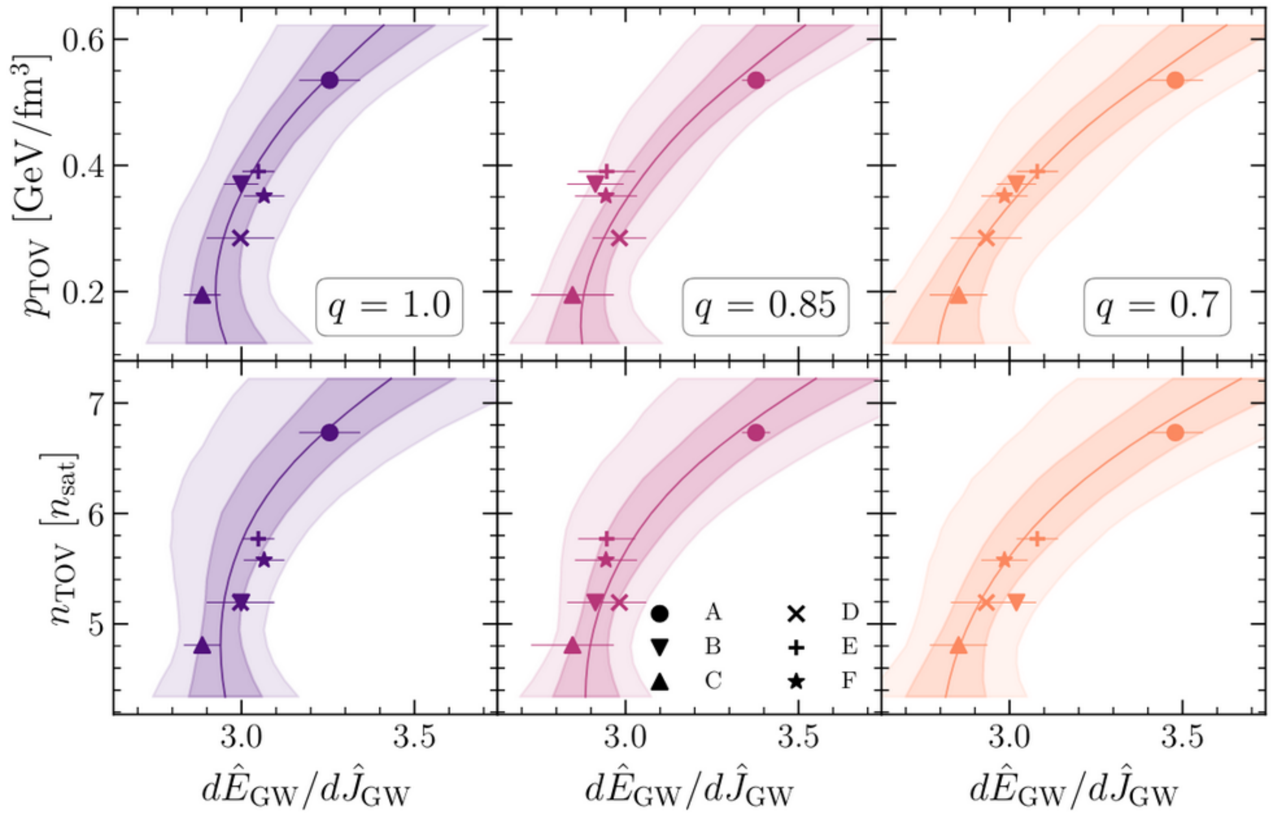
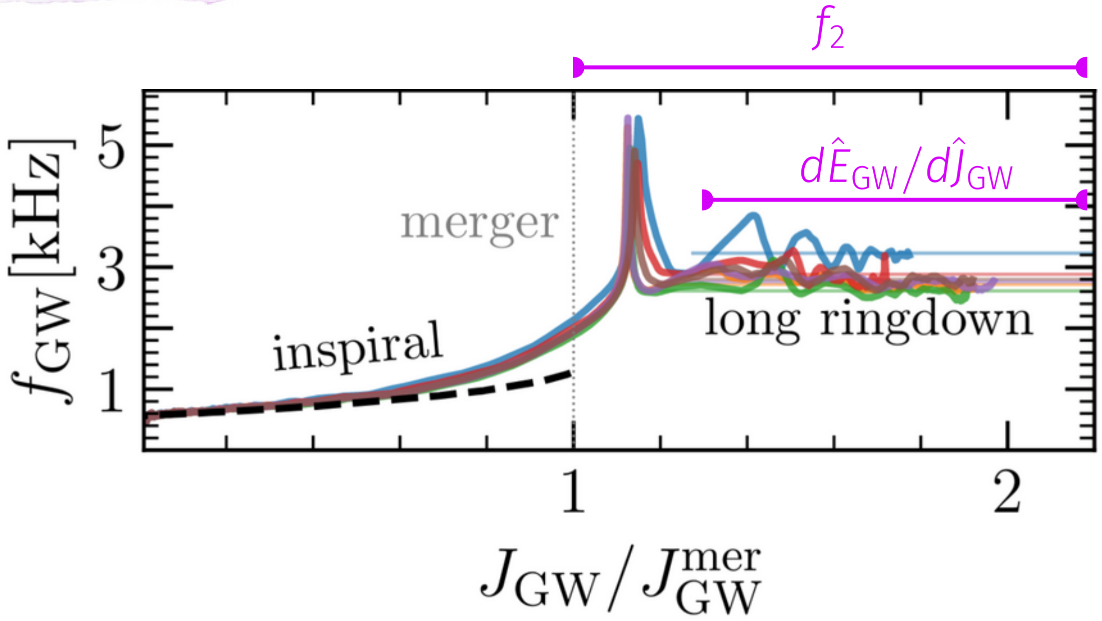
Find that TOV properties correlated with this linear slope

# Slope is correlated with the TOV pressure and density




Use bilinear model : $\frac{d\hat{E}_{\rm GW}}{d\hat{J}_{\rm GW}} = \beta_0 + \beta_1 n_{\rm TOV} + \beta_2 p_{\rm TOV} + \beta_3 q + \beta_4 n_{\rm TOV} \cdot q + \beta_5 p_{\rm TOV} \cdot q + \beta_6 n_{\rm TOV} p_{\rm TOV}$

# Long ring-down slope and $f_2$ are not the same




$f_2$ picks up power even during the transient first few ms

We find $d\hat{E}_{GW}/d\hat{J}_{GW}$ **better correlated with the TOV point** (though both are well correlated)

# Use correlation in a simple mock measurement

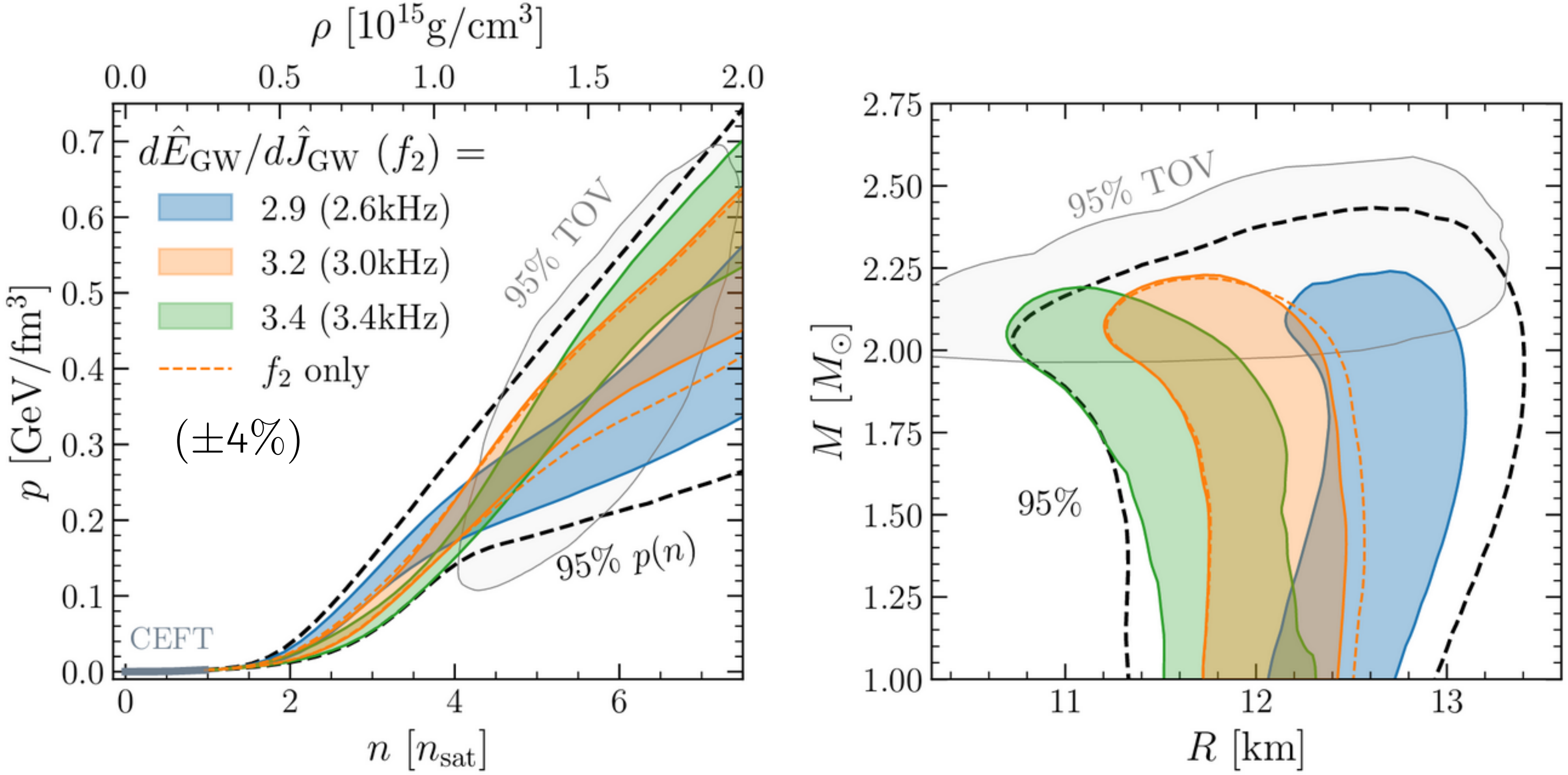
New component of likelihood in Bayes's Theorem

*Joint likelihood* from measurement, assuming a multivariate Gaussian in $f_2$ and slope

$$P(\text{data}|\text{EOS}, q) = \int df_2 \, d\left(d\widehat{E}_{\text{GW}}/d\widehat{J}_{\text{GW}}\right) \times P_{\text{meas}}(\text{data}|f_2, d\widehat{E}_{\text{GW}}/d\widehat{J}_{\text{GW}}, q) \times P_{\text{mod}}(d\widehat{E}_{\text{GW}}/d\widehat{J}_{\text{GW}}, f_2|\text{EOS}, q) \,,$$

Bilinear model(s)

# Long ringdown measurement constrains the whole EOS

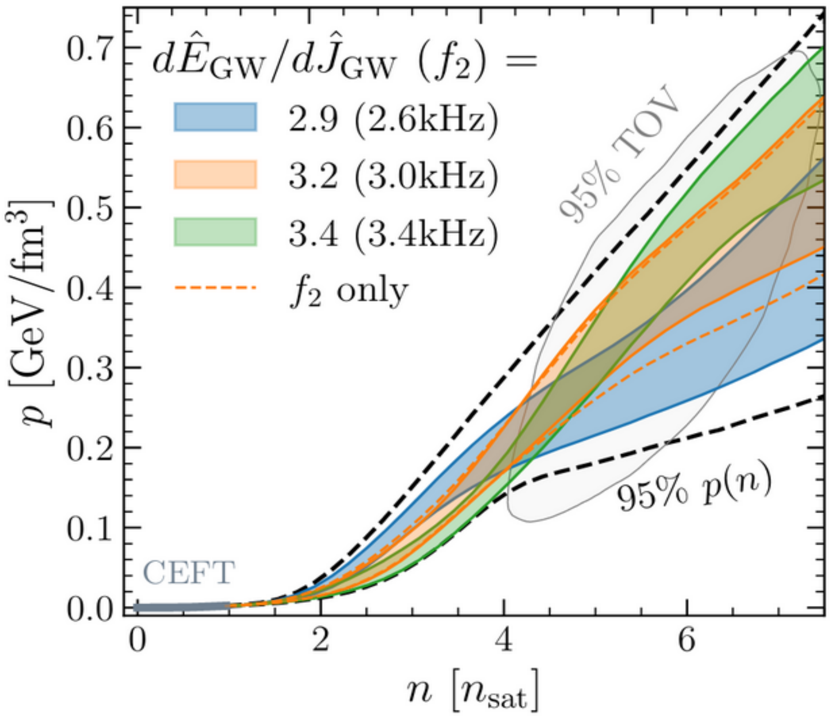



Measurement improves upon constraints from $f_2$ alone due to better correlation (here assuming flat prior for $q \in [0.7, 1]$)

# Summary + Outlook

✔ Identified a long-ringdown phase in BNS mergers, which will constrain the EOS

- Principal component analysis allows us to *capture model-agnostic ensemble behavior*
- *Identified linear post-merger relation* between GW energy and angular momentum *correlated with TOV properties*
- Yields *improved constraints on the NS-matter EOS* beyond those from $f_2$ alone



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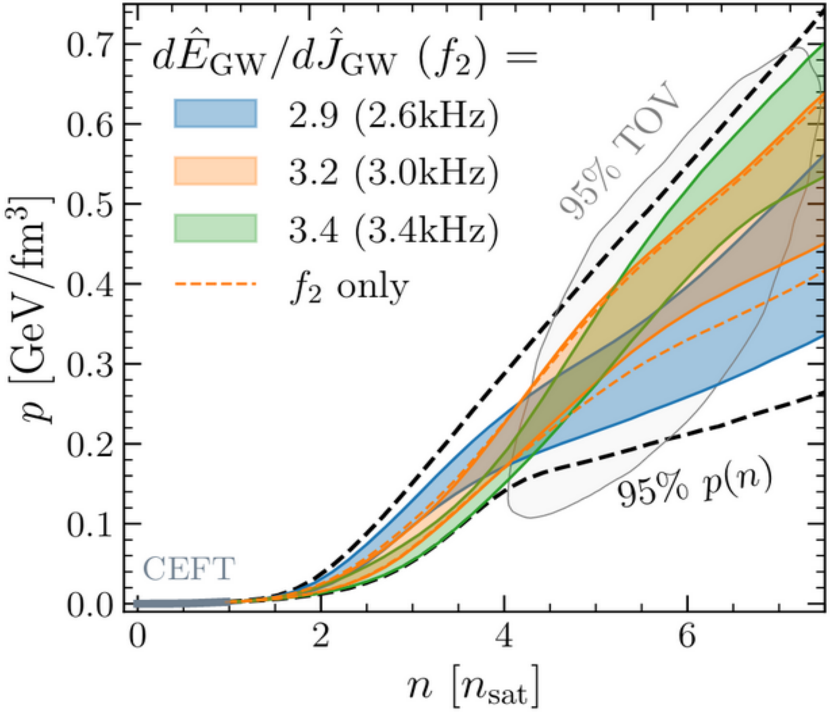
? Do phase transitions affect this observed correlation?

? How constraining is the long-ringdown slope when performing a full GW pipeline analysis?



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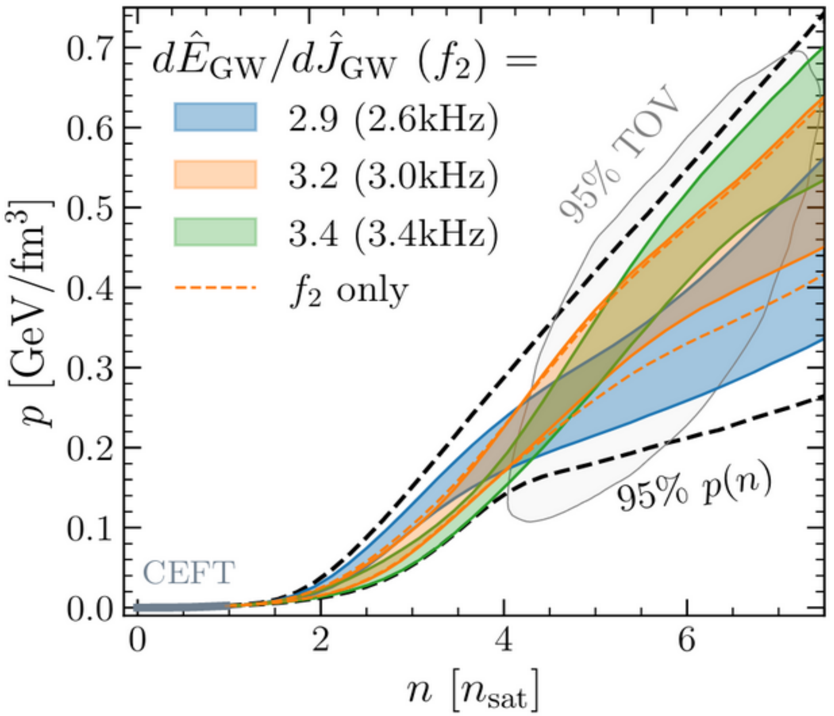
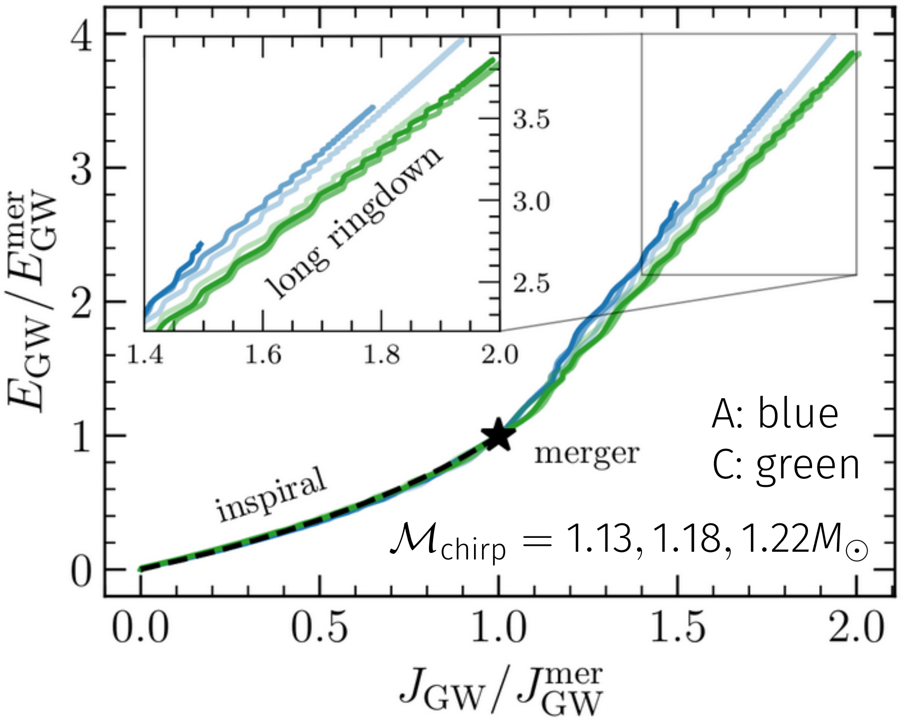


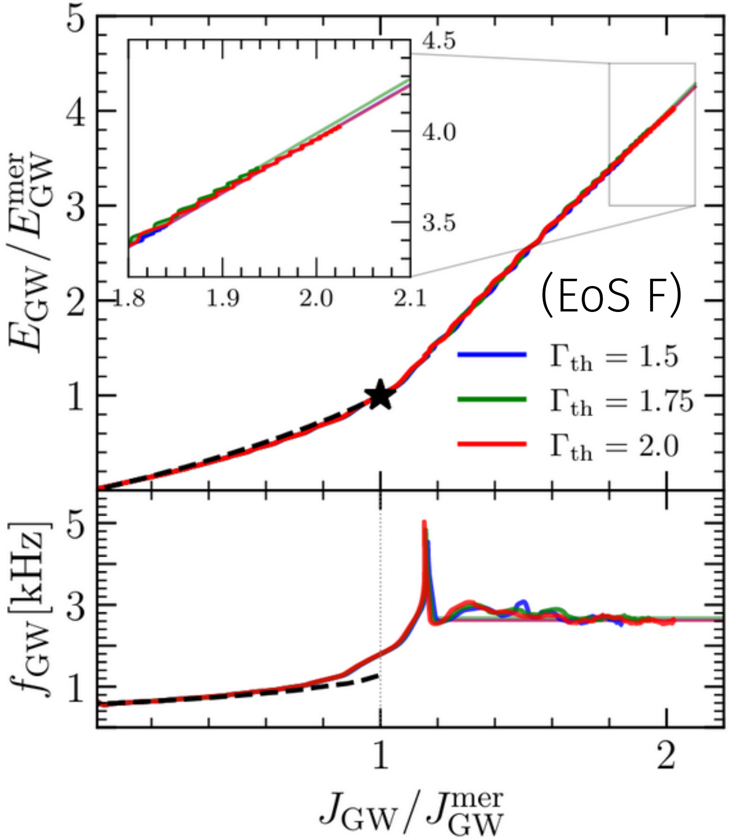

Thanks for your attention!

*Here there be details…*

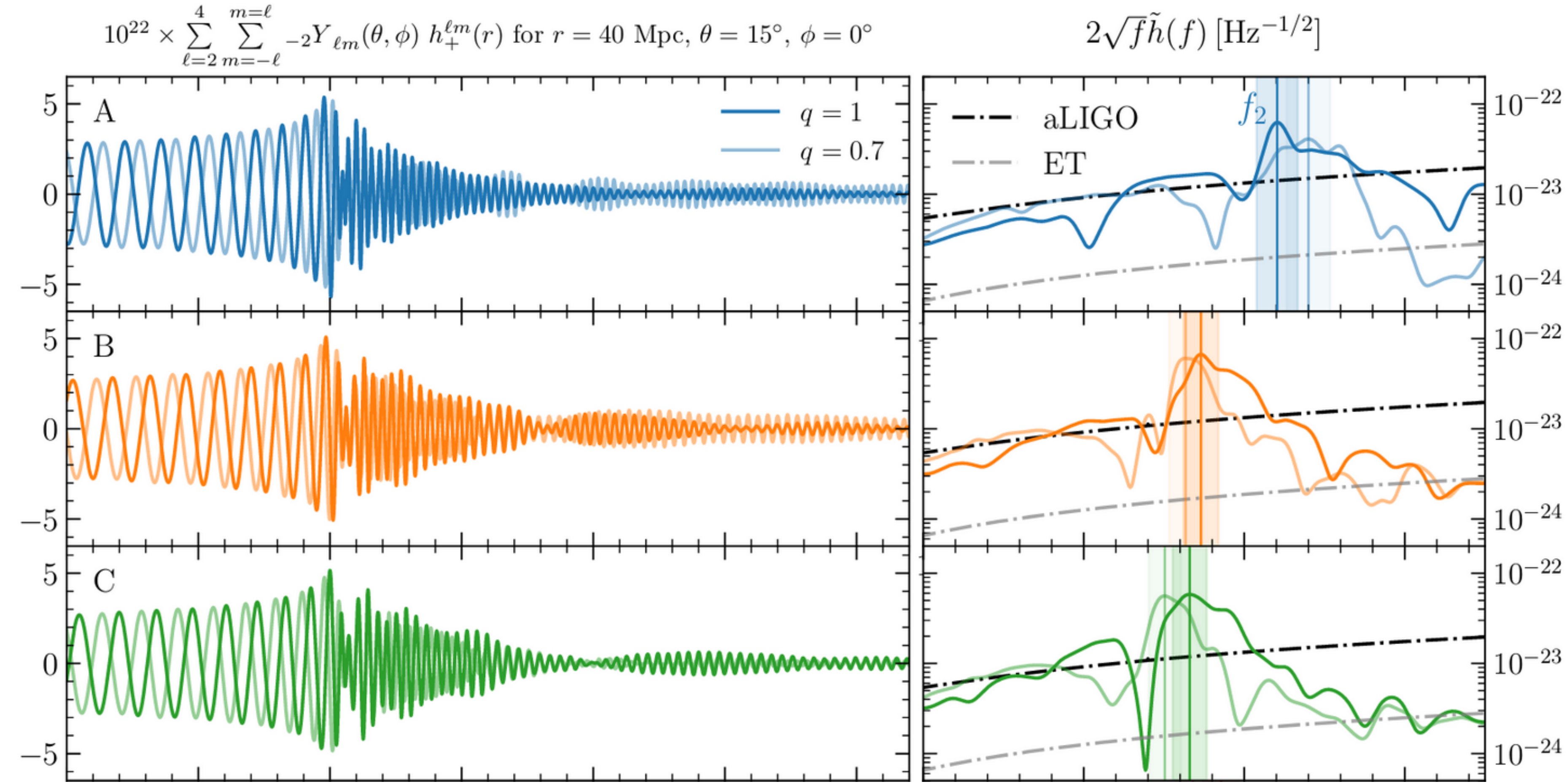
# Slope insensitive chirp mass and thermal effects




Thermal effects ($\Gamma_{\rm th}$)



# Sample waveforms



# Simulation details:

- FUKA code initial data solver; initial separation ≈ 45 km

  Papenfort, Tootle, Grandclément, Most, Rezzolla, PRD 104, 024057 (2021)

- Evolution with Einstein-Toolkit, including the fixed-mesh box-in-box refinement framework Carpet

  Haas+ The Einstein Toolkit. Zenodo. (http://einsteintoolkit.org) (2020).
  Schnetter, Hawley, Hawke, Class. Quantum Grav. 21, 1465–1488 (2004)

  - six refinement levels; finest grid spacing of 295 m;
  - impose reflection symmetry across orbital plane
  - computational domain outer boundary at ±1512 km

- Hybrid EoS construction w/ fixed $\Gamma_{\rm th} \equiv (p_{\rm th}/\varepsilon_{\rm th}) + 1$

- Extract $\psi_4 \equiv \ddot{h}_+ + i\ddot{h}_\times$ with sampling rate ≈ 634 kHz from a spherical surface with radius ≈ 574 km (spin-weighted spherical-harmonic modes with $l \leq 4$)