Two Nucleons Near Unitarity with Perturbative Pions: Persistence vs Chiral Symmetry

H. W. Grießhammer

Yu Ping Teng (MSc)

Institute for Nuclear Studies
The George Washington University, DC, USA

How to root Nuclear Physics in QCD?
What is the underlying principle that makes simple structures emerge from complex nuclear dynamics?

Hypothesis (at least fpr perturbative pions):
Tensor-OPE does not enter before N^{3}LO.

König/hg/Hammer/van Kolck: *Phys. Rev. Lett.* **118** (2017) 202501 [1607.04623 [nucl-th]]

Teng/hg: MSc Thesis GW 2023 and [2410.09653 [nucl-th]]

The Only Magic In This Talk

1. Emergent Phenomena in Nuclear Physics: “Order From Chaos”

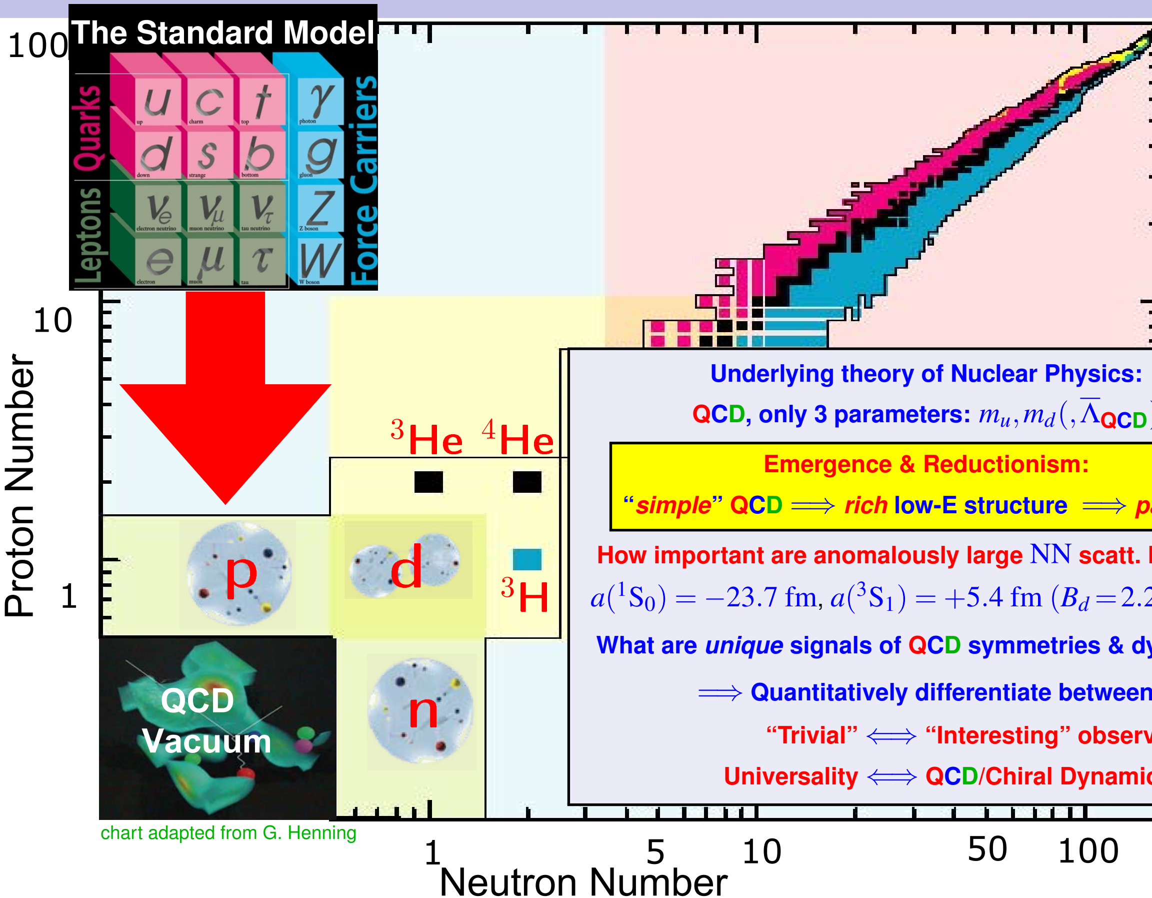


chart adapted from G. Henning

Underlying theory of Nuclear Physics:

QCD, only 3 parameters: $m_u, m_d (, \overline{\Lambda}_{\text{QCD}})$

Emergence & Reductionism:

“*simple*” QCD $\Longrightarrow$ *rich* low-E structure $\Longrightarrow$ *patterns*

How important are anomalously large NN scatt. lengths?:

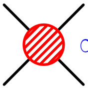
$a(^1\text{S}_0) = -23.7\text{ fm}, a(^3\text{S}_1) = +5.4\text{ fm } (B_d = 2.225\,\text{MeV})$

What are *unique* signals of QCD symmetries & dynamics?

$\Longrightarrow$ Quantitatively differentiate between:

“Trivial” $\Longleftrightarrow$ “Interesting” observables

Universality $\Longleftrightarrow$ QCD/Chiral Dynamics

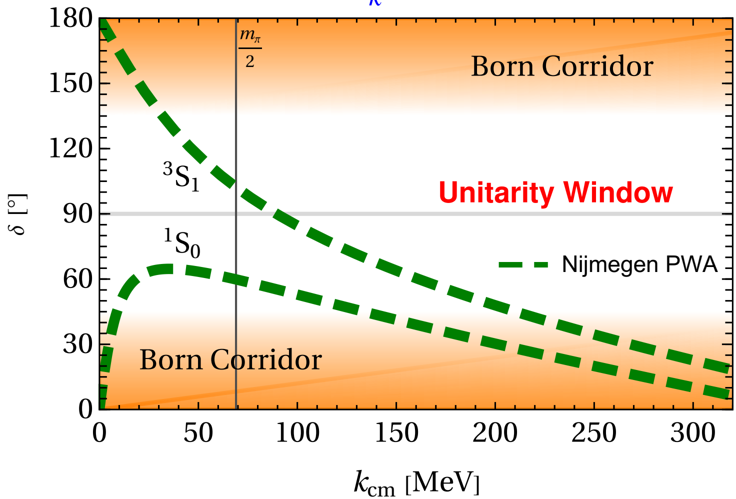


$\propto \frac{1}{\underbrace{k\cot\delta}_{\text{interaction}} \underbrace{-ik}_{\text{Unitarity}}} \to \begin{cases} \frac{1}{k\cot\delta}\left[1+\frac{i}{\cot\delta}+\dots\right] & \text{for } \cot\delta \gg |i| \text{ i.e. } \delta\to 0 \implies \\ \frac{1}{-ik}\left[1+\frac{\cot\delta}{i}+\dots\right] & \text{for } \cot\delta \ll |i| \text{ i.e. } \delta\to 90^\circ \implies \end{cases}$

Born approximation:
interactions small & perturbative,
their details & scales drive A_{NN}
no bound states

Unitarity Limit implies Universality:
interaction strong: *non-perturbative*,
details irrelevant, unitarity drives A_{NN};
LO: no scales in A_{NN},
bound state at $k=0$ (pole in A_{NN})

$\implies \sigma_{\mathrm{LO}} = \frac{4\pi}{k^2} \xrightarrow{k\to 0} \infty$ saturates **unitarity bound**: max allowed by probability conservation.



$30\ \mathrm{MeV} \lesssim k_{\mathrm{cm}} \lesssim [1.5\dots 2]m_\pi$:

NN well inside **Unitarity Window**:

$^1S_0$ & $^3S_1$: $|\cot\delta| \lesssim 1$ ($45^\circ \lesssim \delta \lesssim 135^\circ$)

$\implies$ LO nonperturbative

Outside: LO perturbative in

Born Corridors $|\cot\delta| \gtrsim 1$ ($|\delta| \lesssim 45^\circ$)

How much of Nuclear Physics does really depend on details of QCD?

How much just from (corrections to) universal aspects around Unitarity?

(a) Expanding About the Unitarity Limit in EFT($\not\pi$)

König/hg/Hammer/van Kolck: PRL 2017 [1607.04623]
reviews: van Kolck [2003.09974]; Kievsky/... [2102.13504]

EFT($\not\pi$)/ERE: $\propto \frac{1}{-ik}\left[1+\frac{k\cot\delta = -\frac{1}{a}+\frac{r}{2}k^2+\dots}{ik}+\dots\right] \to \underbrace{\frac{1}{-ik}}_{\text{LO}}\left[1+i\underbrace{\left(\underbrace{\frac{1}{ka}}_{<1?!}-\underbrace{\frac{kr}{2}}_{<1?!}\right)}_{\text{NLO correction}}+\dots\right]$

***a priori* justified if** inverse scatt. length/ NN system size/ NN binding momentum $0 \leftarrow \frac{1}{a} \ll$ **typ. momentum** $k \ll \overline{\Lambda}_{\not\pi} \overset{???}{\sim} m_\pi \sim \frac{1}{r}$ **breakdown/ resolution scale.**

LO: No NN scale. $\Longrightarrow$ Nuclear Physics correlated to *just one* 3N *RG scale fixed by* B_3 *via Efimov effect*.

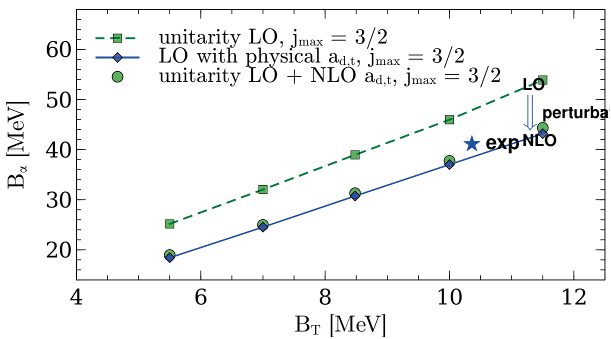
PARADIGM SHIFT: *Unitarity* de-emphasises details of NN & pions, emphasises 3N scale & Universality.

Information Theory in EFT: lossless compression into smallest number of parameters at given accuracy.

$\Longrightarrow$ Explore *Sweet Spot* for patterns, unique signals of QCD:

bound weakly enough to be insensitive to interaction details ($\frac{kr}{2} \ll 1$),

but strongly enough to be insensitive to exact large system size ($ka \gg 1$).

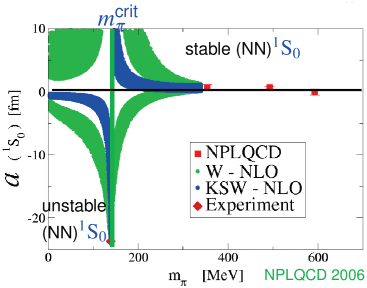


$B_{^3\text{H}} - B_{^3\text{He}}$ [MeV]
NLO: $[0.92 \pm 0.18]$
exp: 0.764

	Fermion Unitarity LO $\to$ NLO	exp ^{4}He/^{3}H
ground: B_4/B_3	$4.6 \to 3.8 \pm 0.2$	3.66
excited: B_4^*/B_3	$\sim 1.1 \to \sim 0.98 \pm 0.05$	0.96

Symm. Nucl. Matter	ρ_0 [fm^{-3}]	B/A [MeV]	E_{sym} [MeV]	L [MeV] slope of E_{sym}	K_∞ [MeV] compressib.
Kievsky/... EFT($\not\pi$)-inspired	0.15	-16	35	70	251
exp	0.16	-16	≈ 30	$[40\dots60]$	210

(b) Symmetries in the Unitarity Limit



NPLQCD 2006

χEFT cannot explain anomalous

scatt. lengths/shallow binding: Worlds with $a \lesssim \dfrac{1}{m_\pi}$

Noether Theorem 1918 [physics/0503066]:

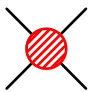
Symmetries and their breaking

all-important in Modern Physics.

(1) **Amplitude saturated at Unitarity Limit:** $\sigma = \dfrac{4\pi}{k^2}$ maximal (probability conservation).

(2) **Scale/Conformal Invariance** at Fixed Point: $\mathrm{k}\cot\delta = 0$.

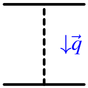
(3) **Wigner-$\mathrm{SU}(4)$ Symmetry of combined spin-isospin rotations** $\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix} \to U \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$ Wigner 1937 for heavy nuclei cf. Mehen/Stewart/Wise 1999

In NN:  $= \dfrac{4\pi}{M} \dfrac{1}{-\frac{1}{a} - ik} = A_{NN}(^3S_1) = A_{NN}(^1S_0)$ if $a(^3S_1) = a(^1S_0)$.

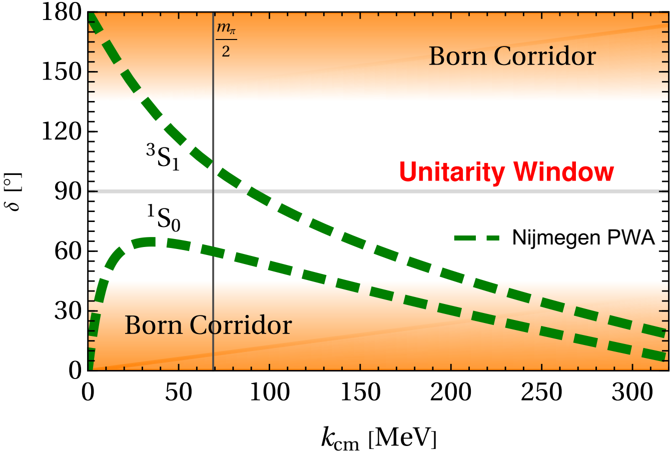
Theorists love unitarity limit as point of high symmetry: Wigner-$\mathrm{SU}(4)+$ scale-invariance.

Nature: Small breaking $\dfrac{1/a(^3S_1), 1/a(^1S_0)}{m_\pi \sim 1/r} \approx \dfrac{1/a(^3S_1) - 1/a(^1S_0)}{m_\pi \sim 1/r} \approx 0.3$ **in perturbation.**

(c) Why χEFT In the Unitarity Limit?



$$: -\frac{g_A^2}{4f_\pi^2}\,\frac{1}{\vec{q}^2+m_\pi^2}\left[\underbrace{\frac{1}{3}\,(\vec{\tau}_1\cdot\vec{\tau}_2)\,(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,\vec{q}^2}_{\text{central: Wigner-symmetric}}+\underbrace{(\vec{\tau}_1\cdot\vec{\tau}_2)\left((\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})-\frac{1}{3}\,(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,\vec{q}^2\right)}_{\text{tensor: Wigner-breaking, mixes } \mathrm{S}\leftrightarrow\mathrm{D},\ \mathrm{D}\to\mathrm{D}}\right]$$



$\Longrightarrow$ **Pions break** **scaling by f_π, m_π, Wigner by mixing.**

NN S waves well in **Unitarity Window** $|\cot\delta|<1$ for $30\ \mathrm{MeV}\lesssim k_{\mathrm{cm}}\lesssim[1.5\ldots2]m_\pi$.

$\Longrightarrow$ **How to embed pions/χiral symmetry inside Unitarity Window?**

Window's upper limit close to scale

$$\overline{\Lambda}_{\mathrm{NN}}=\frac{16\pi f_\pi^2}{g_A^2 M}\approx 300\ \ \mathrm{MeV}$$

where OPE becomes nonperturbative KSW 1999 FMS 2000.

Explore transition "no$\longrightarrow$nonperturbative pions" via Perturbative ("KSW") Pions (only χEFT known to be consistent).

> **χEFT(pπ)$_{\mathrm{UE}}$: χEFT with Perturbative Pions in the Unitarity Expansion:** $Q\sim\frac{1}{ka},\frac{kr}{2},\frac{k,m_\pi}{\overline{\Lambda}_{\mathrm{NN}}}\ll 1$

$\Longrightarrow$ Apply Unitarity Expansion to N^{2}LO amplitudes already computed analytically

by Rupak/Shoresh PRC60 (2000) 0540004 [nucl-th/9902077] (^{1}S$_0$) and Fleming/Mehen/Stewart NPA677 (2000) 313 [nucl-th/9911001] (^{1}S$_0$, ^{3}S$_1$).

3. Unitarity With Perturbative Pions in NN

(a) χEFT(pπ)$_{\text{UE}}$ at N^{2}LO with $Q \sim \frac{1}{ka}, \frac{kr}{2}, \frac{k, m_\pi}{\Lambda_{\text{NN}}} \ll 1$

based on Rupak/Shoresh [nucl-th/9902077] (^{1}S$_0$),
Fleming/Mehen/Stewart [nucl-th/9911001] (^{1}S$_0$, ^{3}S$_1$)
mod. for unitarity Teng/hg MSc thesis GW 2023, [2410.09653]

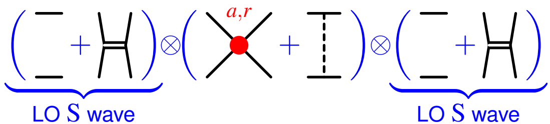
$\mathcal{O}(Q^{-1})$ **(LO):** Nonperturbative; no scale, perfect Wigner, pure S wave state.

$A^{(S)}_{-1} = \frac{4\pi \mathrm{i}}{M} \frac{1}{k} =$

$\mathcal{O}(Q^{0})$ **(NLO):** Scaling and Wigner broken by contacts determined to reproduce PWA values of a, r.

Non-iterated OPE: central only, does not break Wigner but scaling; first non-analyticity: branch point $\pm\mathrm{i}\frac{m_\pi}{2}$.

$A^{(S)}_{0} =$



$\Longrightarrow$ **Unitarity, Wigner-SU(4) spin-isospin symmetry align naturally for Perturbative Pions at NLO.**

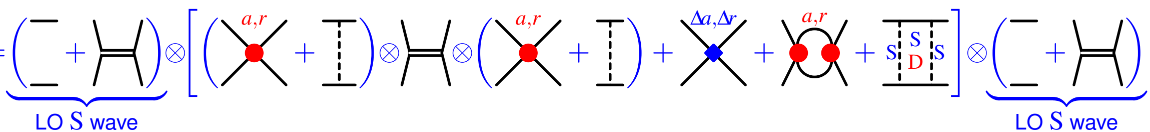
$\mathcal{O}(Q^{1})$ **(N^{2}LO):** Contacts adjusted to keep a, r at PWA values; multiplied with non-iterated OPE (central only).

Once-iterated OPE added: first and second non-analyticity: branch points $\pm\mathrm{i}\frac{m_\pi}{2}, \pm\mathrm{i}m_\pi$.

$A_{1\text{sym}}$: Central $\mathrm{S} \to \mathrm{S} \to \mathrm{S}$ does not break Wigner but scaling: identical in $^1\mathrm{S}_0$ and $^3\mathrm{S}_1$.

$A_{1\text{break}}$: Tensor $\mathrm{S} \to \mathrm{D} \to \mathrm{S}$ breaks Wigner and scaling: only in $^3\mathrm{S}_1$.

$A^{(S)}_{1} =$



$\Longrightarrow$ **Is breaking of Wigner-SU(4) spin-isospin symmetry for Perturbative Pions at N^{2}LO indeed small?**

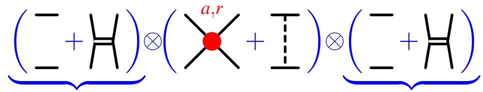
(b) Analytic Answers Shorter By Unitarity

based on Rupak/Shoresh [nucl-th/9902077] ($^1S_0$), Fleming/Mehen/Stewart [nucl-th/9911001] ($^1S_0$, $^3S_1$)
mod. for unitarity Teng/hg MSc thesis GW 2023, [2410.09653]

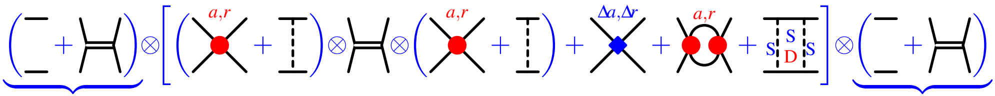
LO: $A^{(S)}_{-1}(k) = \frac{4\pi\mathrm{i}}{M}\,\frac{1}{k}$ is only S wave

NLO: $$A^{(S)}_{0}(k) = -\frac{4\pi}{Mk}\left(\frac{1}{ka}-\frac{kr}{2}\right) - \frac{g_A^2}{4f_\pi^2}\left(1-\frac{m_\pi^2}{4k^2}\overbrace{\ln[1+\frac{4k^2}{m_\pi^2}]}^{1\pi\text{ cut}}\right)$$

Non-iterated OPE does not break Wigner.



N^{2}LO:



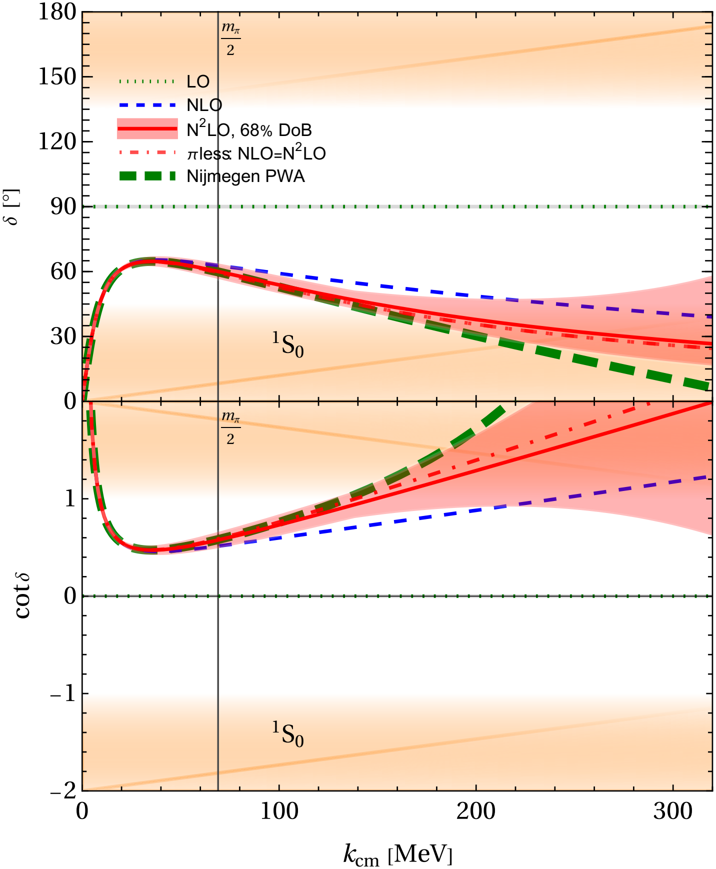
Once-iterated OPE breaks Wigner: $S \to D \to S$

$$A_1^{(^1S_0)}(k) \equiv A^{(S)}_{1\text{sym}}(k) = \frac{\left[A_0^{(S)}(k)\right]^2}{A_{-1}^{(S)}(k)} + \frac{g_A^2}{2f_\pi^2}\left[\frac{4}{3am_\pi} - \frac{m_\pi}{k}\left(\frac{1}{ka}-\frac{kr}{2}\right)\right]$$
$$-\frac{g_A^2 M m_\pi}{16\pi f_\pi^2}\left\{A_0^{(S)}(k)\,\frac{m_\pi}{k}\,\underbrace{\arctan[\frac{2k}{m_\pi}]}_{1\pi\text{ cut}} + \frac{g_A^2}{2f_\pi^2}\left[\frac{1}{12}+\left(\frac{m_\pi^2}{4k^2}-\frac{1}{3}\right)\ln 2 - \underbrace{F_\pi(\frac{k}{m_\pi})}_{1,2\pi\text{ cut}}\right]\right\}$$

$$A_1^{(^3S_1)}(k) = A^{(S)}_{1\text{sym}}(k) + A^{(S)}_{1\text{break}}(k)$$

$$A^{(S)}_{1\text{break}}(k) = -\frac{\left[A_0^{(SD)}(k)\right]^2}{A_{-1}^{(S)}} + \frac{g_A^2}{f_\pi^2}\,\frac{g_A^2 M m_\pi}{16\pi f_\pi^2}\left\{\frac{571-352\ln 2}{210} - \left(1+\frac{3m_\pi^2}{2k^2}+\frac{9m_\pi^4}{16k^4}\right)\overbrace{F_\pi(\frac{k}{m_\pi})}^{1,2\pi\text{ cut}} + \frac{2m_\pi^2}{5k^2}(\ln 4-1)\right.$$
$$\left. + \frac{3m_\pi^4}{16k^4} - \frac{3}{2}\left[\frac{k}{m_\pi}+\frac{m_\pi}{k}-\frac{m_\pi^3}{8k^3}-\frac{3m_\pi^5}{16k^5}\right]\overbrace{\arctan[\frac{k}{m_\pi}]}^{2\pi\text{ cut}} + \frac{3}{16}\left(\frac{m_\pi^4}{k^4}+\frac{3m_\pi^6}{4k^6}\right)\overbrace{\ln[\frac{16(k^2+m_\pi^2)}{4k^2+m_\pi^2}]}^{1,2\pi\text{ cut}}\right\}$$

$$F_\pi(x) := \frac{1}{8x^3}\left(\underbrace{\arctan[2x]\ln[1+4x^2] - \mathrm{Im}\left[\mathrm{Li}_2[\frac{2\mathrm{i}x+1}{2\mathrm{i}x-1}]\right.}_{1\pi\text{ cut}} \left. -2\,\underbrace{\mathrm{Li}_2[\frac{1}{2\mathrm{i}x-1}]}_{2\pi\text{ cut}}\right]\right)$$



$^1S_0$: central OPE $\Longrightarrow$ Wigner-symmetric.

f_π, m_π break scaling.

Strict perturbation in "basic interaction part"

$$k\cot\delta = 0_{\mathrm{LO}} + k\cot\delta_{\mathrm{NLO}} + k\cot\delta_{\mathrm{N^2LO}}$$
$$= 0_{\mathrm{LO}} - \frac{4\pi}{M}\left[\frac{A_0}{A_{-1}^2} - \frac{A_0^2}{A_{-1}^3} + \frac{A_1}{A_{-1}^2}\right].$$

$\Longrightarrow$ Get δ from $k\cot\delta$.

Advantage: $k\cot\delta(k\to 0)\to 0 \Rightarrow \delta(0) = \begin{cases}0\\ \pi\end{cases}$.

$^1S_0$ is **"boring"** partial wave: no tensor int.

Bayesian truncation uncertainty at 68% DoB.

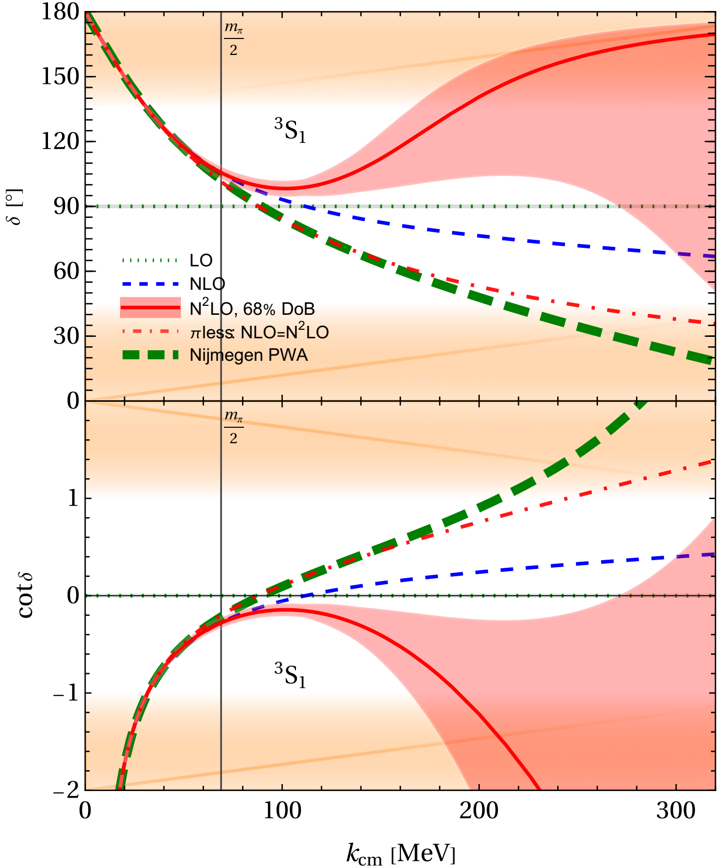
$\Longrightarrow$ Converges order-by-order $\lesssim 300$ MeV.

Agrees within uncertainties with PWA for $\lesssim 250$MeV (even outside Unitarity Window).

Compare to EFT($\not\pi$): minuscule impact of π.

(d) Perturbative Pions at N^2LO: $^3S_1$

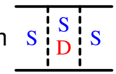
perturbative pions to N^2LO: Fleming/Mehen/Stewart 2000
unitarity for them: Teng/hg MSc Thesis GW 2023, [2410.09653]



$^3S_1$: pions break Wigner-$SU(4)$ & scale inv.

$^3S_1$ is **“interesting”** partial wave:

tensor-OPE $\Longrightarrow$ SD mixing from



$$k\cot\delta = -\frac{4\pi}{M}\left[\frac{A_0}{A_{-1}^2} - \frac{A_0^2}{A_{-1}^3} + \frac{A_1}{A_{-1}^2} + \frac{A_{SD}^2}{A_{-1}^3}\right]$$

$\Longrightarrow$ Terrible convergence (already in FMS):

Converges order-by-order $\lesssim 80$ MeV.

Agrees within uncertainties with PWA only for $\lesssim 70$ MeV (not even in Unitarity Window).

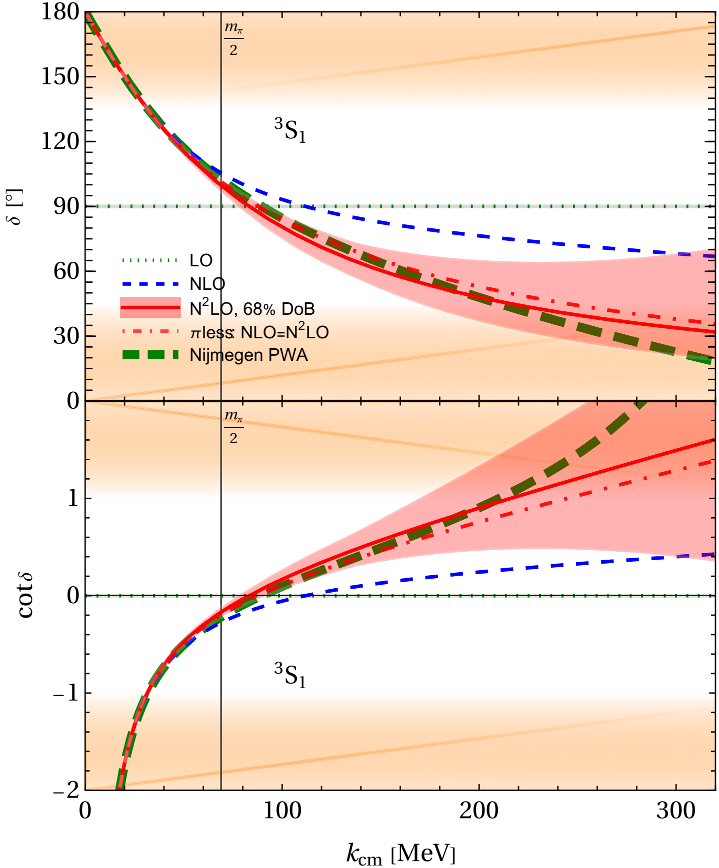
Compare to EFT($\not\pi$): huge impact of pion.

Source of discrepancy: tensor int. (via SD).

(central is identical in $^3S_1$&$^1S_0$)

(d) Perturbative Pions at N^2LO: ${}^3S_1$

perturbative pions to N^2LO: Fleming/Mehen/Stewart 2000
unitarity for them: Teng/hg MSc Thesis GW 2023, [2410.09653]



${}^3S_1$: pions break Wigner-$SU(4)$ & scale inv.

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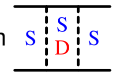


Broken Wigner-$SU(4)$ spoils convergence!

Idea: Use Wigner-$SU(4)$-symmetric pion part.

$\Longrightarrow$ Only ${}^1S_0$-${}^3S_1$ differences of a & r break Wigner-$SU(4)$.

RG-invariant, mildly χsymmetry-breaking.

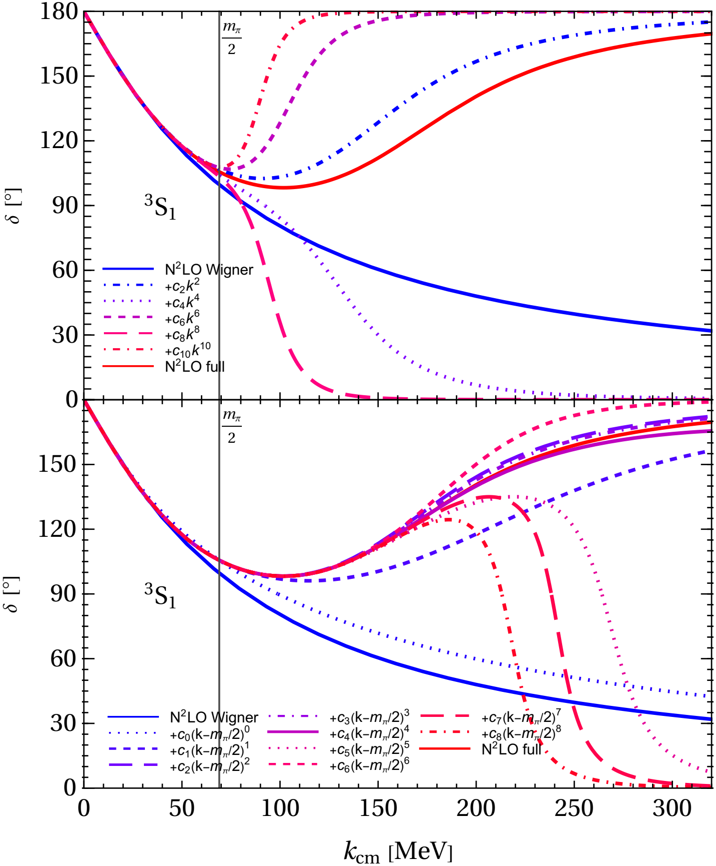
$\Longrightarrow$ Converges order-by-order $\gtrsim 300$ MeV.

Agrees within uncertainties with PWA for $\gtrsim 300$ MeV (even outside Unitarity Window).

Compare to EFT(π̸): tiny impact of pion.

$\Longrightarrow$ **All very similar to ${}^1S_0$.**

(e) Whence the Hockey Stick in ${}^3S_1$?



Expand Wigner-breaking in Taylor:

$$A^{(S)}_{\rm sym}(k) + \sum \frac{1}{n!}[A^{(S)}_{\rm break}]^{(n)}(k_0)(k-k_0)^n$$

Expand about 0:

$k \lesssim \frac{m_\pi}{2}$: convergent, Wigner-breaking tiny

$k \gtrsim \frac{m_\pi}{2}$: no convergence

$\Longrightarrow$ ERE not the problem.

Sorry, no Cohen/Hansen 1999.

Expand about 1st branch point scale $\frac{m_\pi}{2}$:

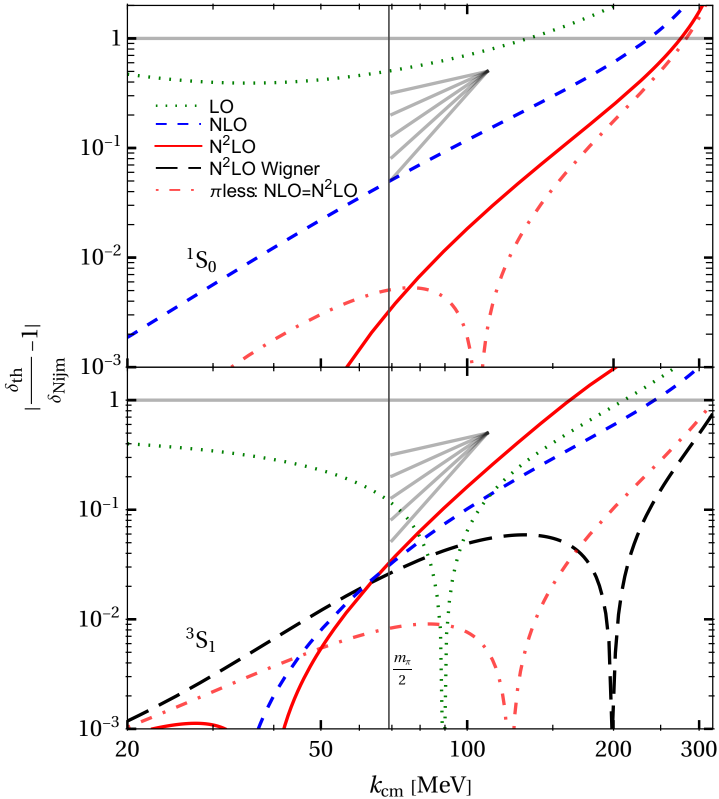
$k \lesssim \frac{m_\pi}{\sqrt{2}}$: convergent, Wigner-breaking tiny

(larger distance to branch point)

$k \lesssim \frac{3}{2}m_\pi$ (>2nd br. pt. scale): convergent

$k \gtrsim \frac{3}{2}m_\pi$: asymptotic (optimal: incl. k^4)

Landau/Páez/Bordeianu: Comp. Phys., Lepage 1997
Teng/hg [2410.09653]



$$\frac{\delta(\text{N}^n\text{LO}) - \delta(\text{PWA})}{\delta(\text{PWA})} \sim \left(\frac{k, m_\pi}{\overline{\Lambda}}\right)^{n+1}$$

at N^{n}LO with empirical breakdown scale $\overline{\Lambda}$.

$^1S_0$ and Wigner-symmetric $^3S_1$:

consistent slopes and

$\overline{\Lambda} \approx 270\ \text{MeV} \approx \overline{\Lambda}_{NN}$ OPE scale.

Full $^3S_1$:

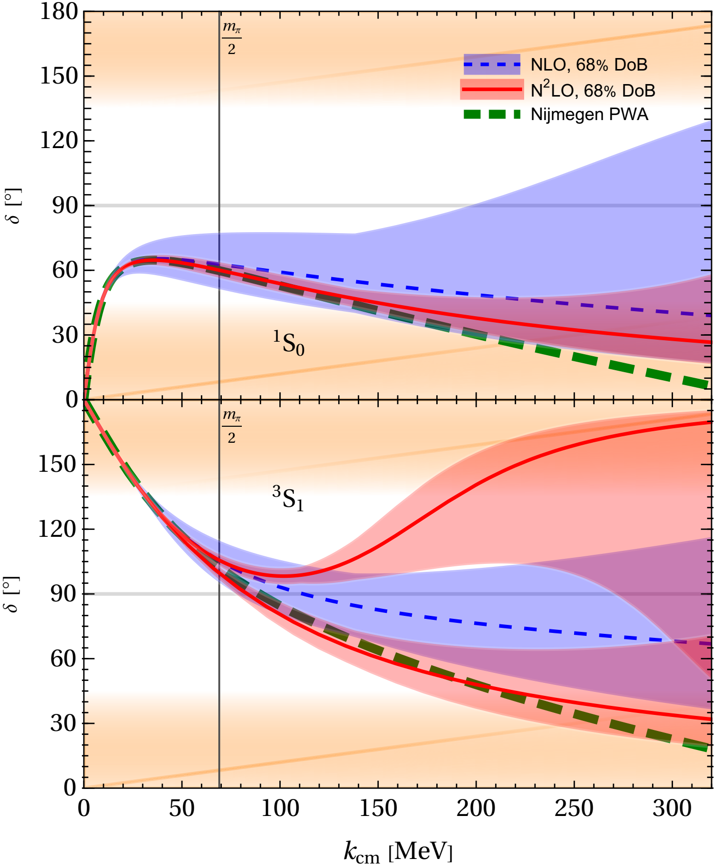
N^{2}LO worse than NLO for $\gtrsim 70\ \text{MeV}$.

Picture obscured by points where

theory & PWA identical (“artificial zero”),

or PWA close to zero (“artificial ∞”).

(g) NLO & N^{2}LO Bayesian Truncation Uncertainties



Apply "$\max$" criterion to $\cot\delta$ order-by-order:

Unitarity: $k\cot\delta_{LO} = 0 \Rightarrow$ "$-ik$" sets scale.

Bayesian N^{2}LO truncation uncertainty at k:

$$\pm Q^3 \max\left\{\frac{\cot\delta_0(k)-\cot\delta_0(0)}{Q}; \frac{\cot\delta_1(k)}{Q^2}\right\}$$

with $Q = \dfrac{\max\{k;m_\pi\}}{\overline{\Lambda}_{NN} \sim 300\ \text{MeV}}$

NLO: rescaled to 68% DoB,

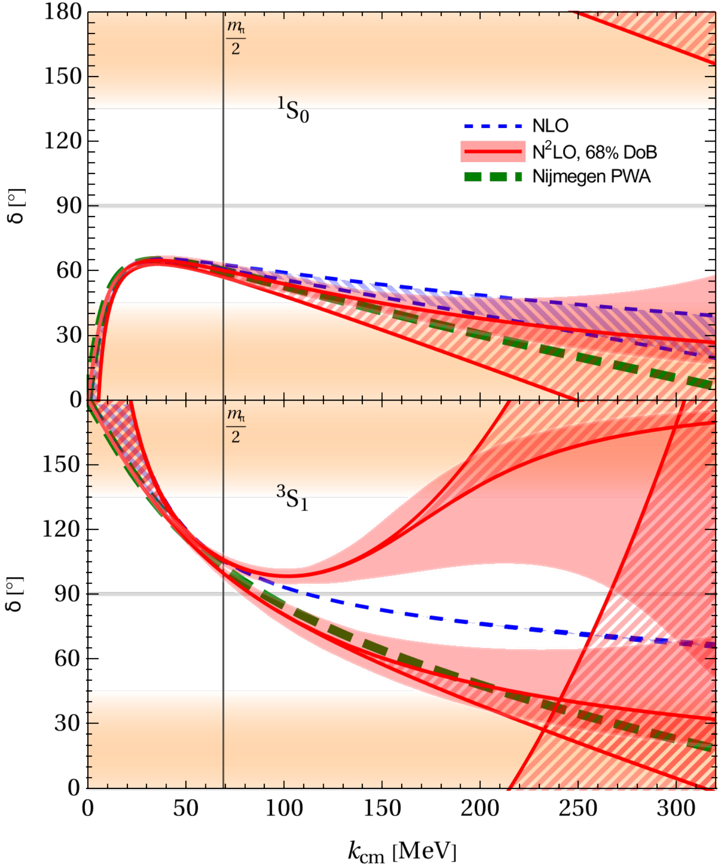
assuming uniform&log-uniform prior.

Only Wigner-symmetric forms have

N^{2}LO uncertainties consistent with NLO,

and NLO&N^{2}LO consistent with PWA.

Teng/hg [2410.09653]



So far:

$$k\cot\delta = 0_{\text{LO}} + k\cot\delta_{\text{NLO}} + k\cot\delta_{\text{N}^2\text{LO}}$$
$$= -\frac{4\pi}{M}\left[\frac{A_0}{A_{-1}^2} - \frac{A_0^2}{A_{-1}^3} + \frac{A_1}{A_{-1}^2}\right]$$

is fundamental, derive $\delta(k)$ from it.

$$\xrightarrow{k\to 0} 0_{\text{LO}} + \left(-\frac{1}{a} + \frac{r}{2}k^2\right)_{\text{N}^{1+2}\text{LO}} + \mathcal{O}(k^4)$$

constructed to reproduce ERE.

Hatched: difference to

directly from amplitude KSW 1999,FMS 2000

$$\delta(k) = \frac{\pi}{2}\Big|_{-1} + \left(\frac{1}{ka} - \frac{kr}{2}\right)\Big|_0 + \dots$$

$\to \infty$ for $k \to 0$ outside Unitarity Window.

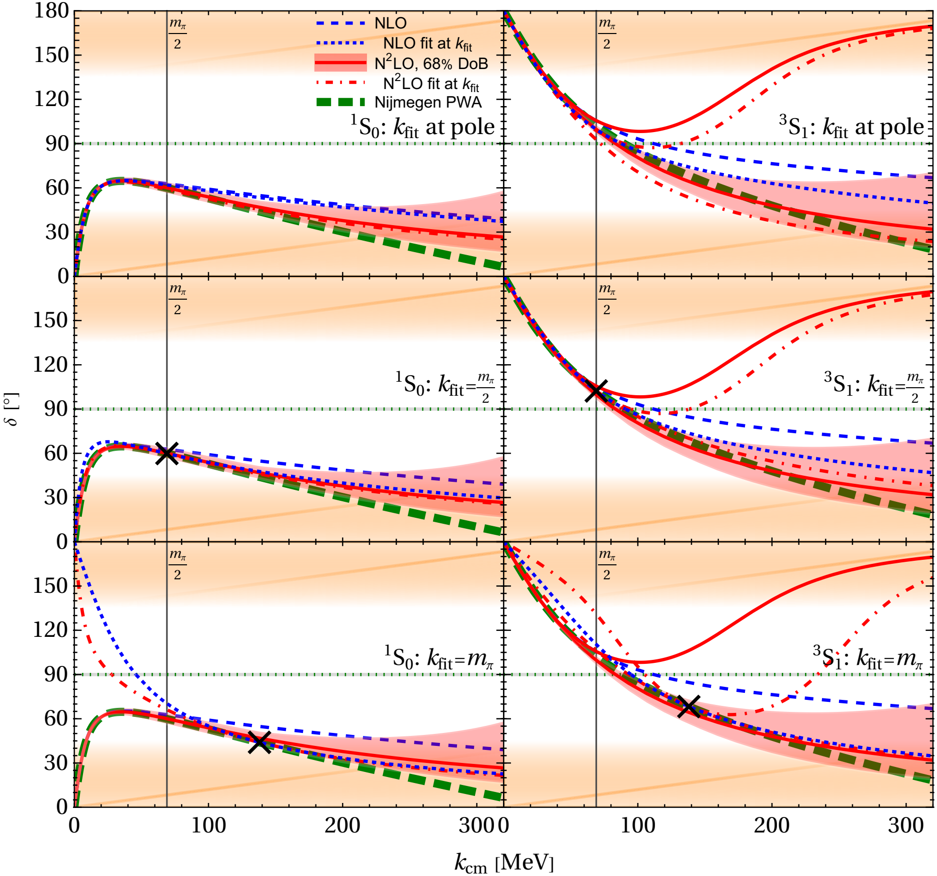
Methods agree inside Unitarity Window

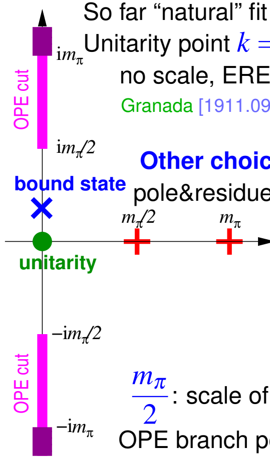
$\frac{1}{ka}, \frac{kr}{2} < 1$ (must in centre $|\cot\delta| \to 0$):

Independent assessment of

truncation uncertainty, consistent with Bayes.

(i) Different Renormalisation/Parameter-Determination Points





So far "natural" fit at Unitarity point $k=0$: no scale, ERE

Granada [1911.09637]

Other choices:

pole&residue: ok

$\frac{m_\pi}{2}$: scale of 1st OPE branch point

No cure to hockey-stick.

Uncertainties & breakdown scale very similar.

m_π: 2nd OPE branch point

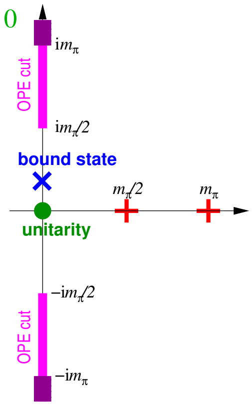
No cure to hockey-stick.

Low-k $^1S_0$ bad inside Unitarity Window.

(j) Virtual/Real Bound-State Pole Positions and Residues

$$\frac{1}{A_{-1}(\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots)+A_0(\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots)+A_1(\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots)+\dots} \overset{!}{=} 0$$

$$\frac{1}{Z} = \mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}k}\left(k\cot\delta(k) - \mathrm{i}k\right)\Big|_{k=\mathrm{i}\gamma_{-1}+\mathrm{i}\gamma_0+\mathrm{i}\gamma_1+\dots}$$



For $k_{\text{fit}} = 0$, pions cannot correct a, r since we force the ERE values Granada [1911.09637]

$\Longrightarrow$ pole at binding momentum $\mathrm{i}\gamma = \frac{\mathrm{i}}{a}\left(1+\frac{r}{2a}+\frac{r^2}{2a^2}+\mathcal{O}(\frac{r^3}{a^3})\right)$

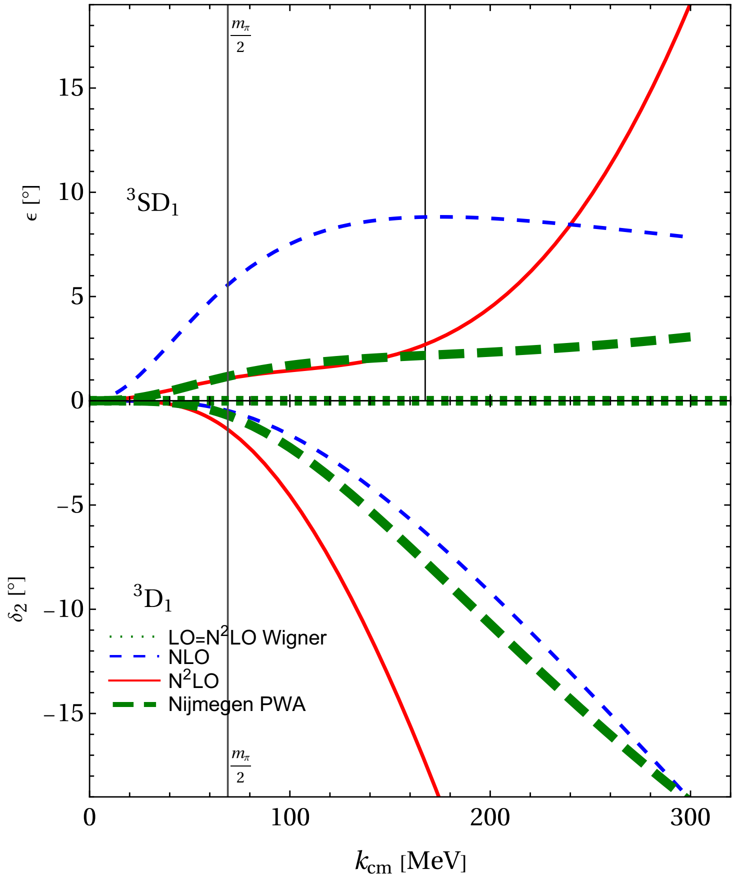
with residue $Z = 1+\frac{r}{a}+\frac{3r^2}{2a^2}+\mathcal{O}(\frac{r^3}{a^3})$.

For general k_{fit}, match to $k\cot\delta_{\text{PWA}}(k_{\text{fit}})$, $\frac{\mathrm{d}}{\mathrm{d}k}k\cot\delta_{\text{PWA}}(k_{\text{fit}})$. $\Longrightarrow$ Predict a, r.

	$^1S_0$			$^3S_1$		
k_{fit}	scatt. length a [fm]	eff. range r [fm]	(bind. mom.,residue) (γ [MeV], Z)	scatt. length a [fm]	eff. range r [fm]	(bind. mom.,residue) (γ [MeV], Z)
ERE	$-23.735(6)*$	$2.673(9)*$	$(-7.892, 0.9034)$	$5.435(2)*$	$1.852(2)*$	$(+47.7023, 1.689)*$
pole	-23.7104	2.7783		5.6128	2.3682	
$\frac{m_\pi}{2}$ NLO	-38.988	3.3270	$(-4.86, 0.925)$	4.9310	2.4966	$(+55., 1.9)$
$\frac{m_\pi}{2}$ N²LO	-25.428	2.7281	$(-7.34, 0.910(2))$	4.7768	2.4492	$(+57(3)., 1.9(2))$
$\frac{m_\pi}{2}$ sym.				5.4625	1.6124	$(+43.0(5), 1.42(4))$
m_π NLO	$+9.2856$	4.2285	$(+28., 1.8)$	$3.3442\dagger$	$3.1886\dagger$	$(+114., 3.)$ ⚡
m_π N²LO	$+34.3335$	2.8956	$(+6.01, 1.10)$	$1.8376\dagger$	$3.3741\dagger$	$(+387(330)., 7(9).)$ ⚡
m_π sym.				4.5344	1.7006	$(+54(1)., 1.5(1))$

Bayesian N²LO uncertainties $*$: input ⚡: cannot converge: $Q \sim \frac{1}{ka}, \frac{kr}{2} \ll 1 \Longrightarrow \frac{r}{2a} \ll 1$ ⚡

(k) $^3SD_1$ Mixing: Full vs. Wigner



No other channels close to Unitarity Window:

$|\delta_{l\geq1}| < 25°$ ($|\cot\delta_{l\geq1} > 2|$).

$^3SD_1$ mixing, $^3D_1$ coupled to $^3S_1$, come only from tensor/Wigner-breaking.

In Unitarity Expansion very similar to FMS:

$k \gtrsim 70$ MeV:

No order-by-order convergence, convergence to PWA elusive.

Zero by Wigner at N^{2}LO.

Natural size at N^{3}LO at $k \approx m_\pi$:
$90° \times (Q \approx 0.5)^3 \approx 10°$

$\Longrightarrow$ Not inconsistent.

SD & DD contacts at N^{3}LO

$\Longrightarrow$ Reproducing PWA possible.

4. Concluding Hypothesis

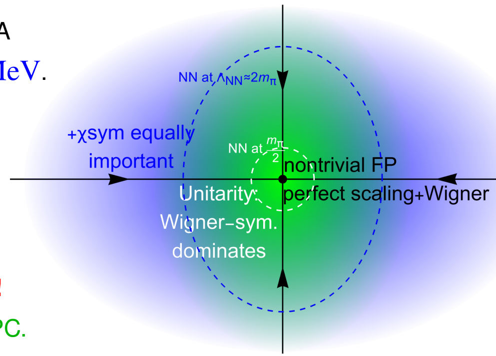
χ**EFT with Perturbative Pions in Unitarity Expansion** $Q \sim \frac{1}{ka}, \frac{kr}{2}, \frac{m_\pi}{\overline{\Lambda}_{\text{NN}}} \ll 1$.

Chiral Physics: m_π, f_π, $(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})(\tau_1 \cdot \tau_2)$ seem opposed to Wigner, but NN/few-N projection forces into it.

> **Hypothesis (at least for Perturbative Pions): Tensor/Wigner-$\text{SU}(4)$ symmetry-breaking part of One-Pion Exchange is *super-perturbative* in few-N systems, i.e. does *not enter before* N^{3}LO.**
>
> $\Longleftrightarrow$ ***Persistence:* Footprint of Symmetries in Unitarity Limit extends far into $p_{\text{typ}} \gtrsim m_\pi$,**
> **more relevant than χiral symmetry in few-N?! $\Longleftrightarrow$ Better lossless compression of Information!**

Evidence: NN S-waves at N^{2}LO converge order-by-order and to PWA
inside all of **Unitarity Window** $30\ \text{MeV} \lesssim k \lesssim \overline{\Lambda}_{\text{NN}} \approx 300\ \text{MeV}$.
Successful extension of EFT($\not{\pi}$) to pions.

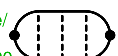
Appeal: Fine-Tuning Problem $\Longrightarrow$ Highly Symmetric Solution?
Unitarity: Nontrivial RG FP of NN has high degree of symmetry:
Universality/scaling $+$ **Wigner-$\text{SU}(4)$** weakly broken in vicinity.
χiral symmetry not explicit at FP: less protected? $\Longrightarrow$ **Quantify!**
No Wigner in meson/1N sector $\Longrightarrow$ no change to χPT, HBχPT PC.



"Coincidence": N^{2}LO Perturbative Pions overpredict $^3\text{SD}_1$ mixing, $^3\text{D}_1$ $\Longrightarrow$ Zero without tensor int. at N^{2}LO.

Some Crucial Tests: If either fails without good reason, Hypothesis falsified.

N^{3}LO cf. Beane/Kaplan/Vuorinen 2009, Kaplan 2020

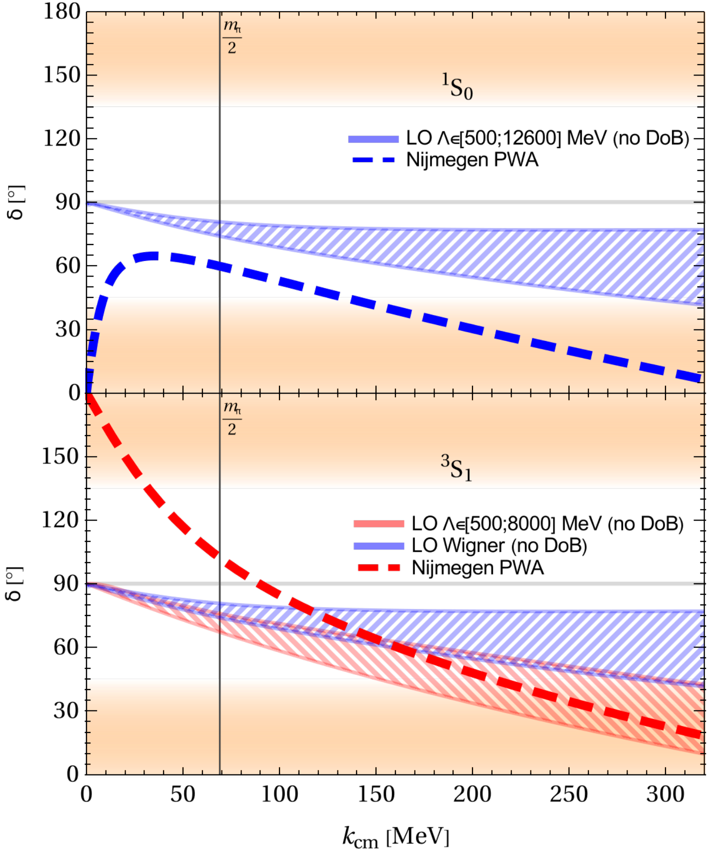


$\text{d}\pi \to \text{d}\pi$, $\gamma\text{d} \to \pi\text{d}$
cf. Borasoy/hg 2003

Nd scattering
cf. Bedaque/hg 2000

Nonperturbative Pions to N^{2}LO in strict perturbation LO: hg 2023

(a) Nonperturbative Pions at LO: Maybe Not Hopeless



LO, 1 mom.-indep. CT, Gaußian regulator.

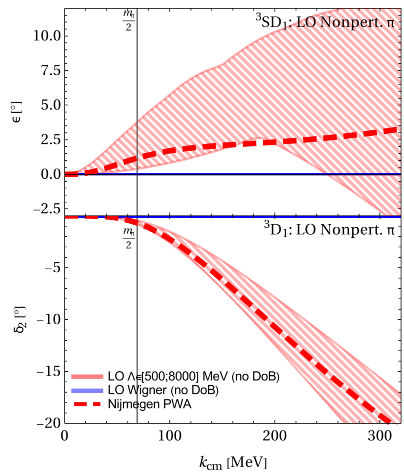
Already deviates from Unitarity $\delta = 90°$.

$\Longrightarrow$ Explicit scale breaking at LO,

$r = \begin{cases} ^1S_0/\text{Wigner } [1\ldots2]\text{fm}; ^3S_1\ [1.2\ldots2.5]\text{fm} \\ \text{PWA } 2.767(9)\text{fm} \quad 1.852(2)\text{fm} \end{cases}$

Tensor/Wigner-breaking less compatible with unitarity than central/Wigner-symmetric.

Not Bayesian DoBs but cutoff-variation.



The efficient person gets the job done right. The effective person gets the right job done.