(In-Medium) Similarity Renormalization Group and Effective Field Theory

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Similarity Renormaliztion Group

Similarity Renormalization Group

Basic Idea

continuous unitary transformation of the Hamiltonian to band-diagonal form w.r.t. a given "uncorrelated" many-body basis

- **flow equation** for Hamiltonian $H(s) = U(s)HU^{\dagger}(s)$:

$$\frac{d}{ds}H(s) = \left[\eta(s), H(s)\right], \qquad \eta(s) \equiv \frac{dU(s)}{ds}U^{\dagger}(s) = -\eta^{\dagger}(s)$$

- choose $\eta(s)$ to achieve desired behavior, e.g.,

$$\eta(s) \equiv \left[H_d(s), H_{od}(s)\right]$$

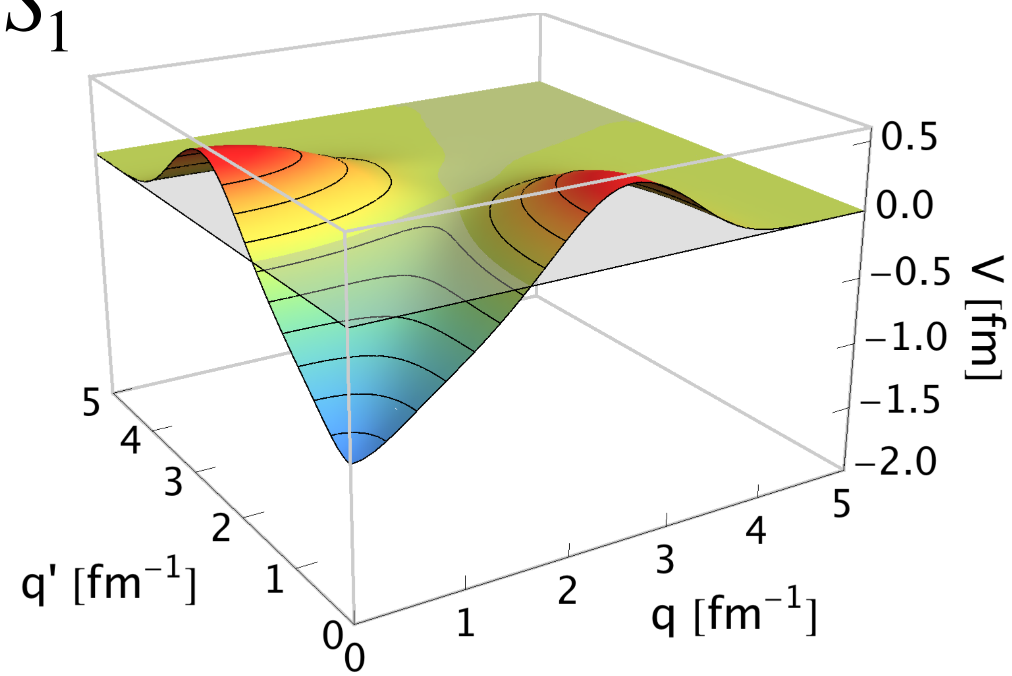
to **suppress** (suitably defined) **off-diagonal Hamiltonian**

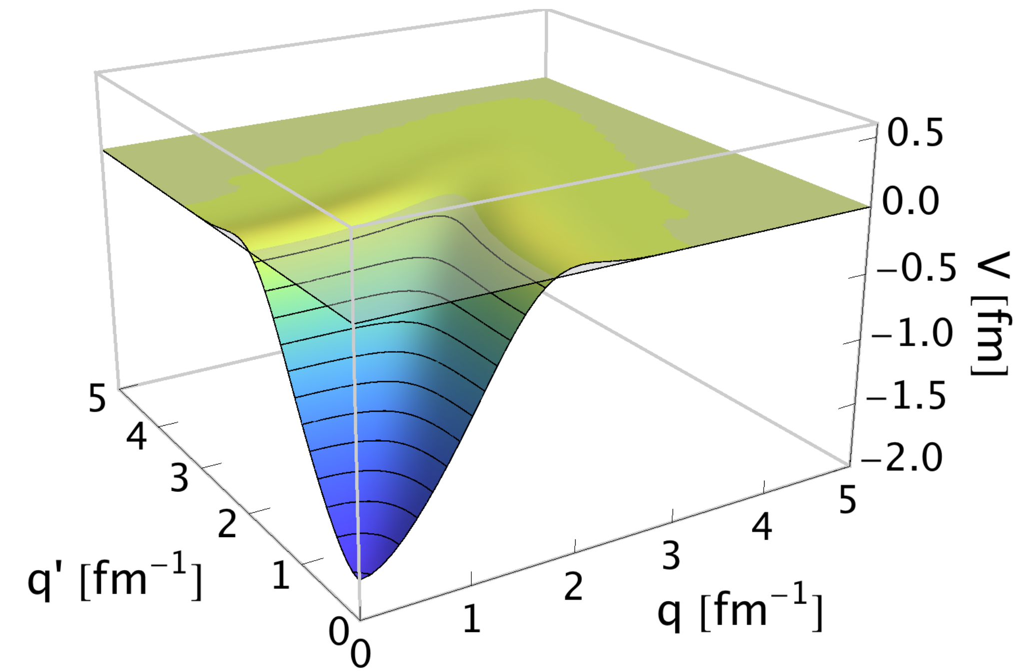
- **consistent evolution** for all **observables** of interest

Similarity Renormalization Group

momentum space matrix elements

$^3S_1$



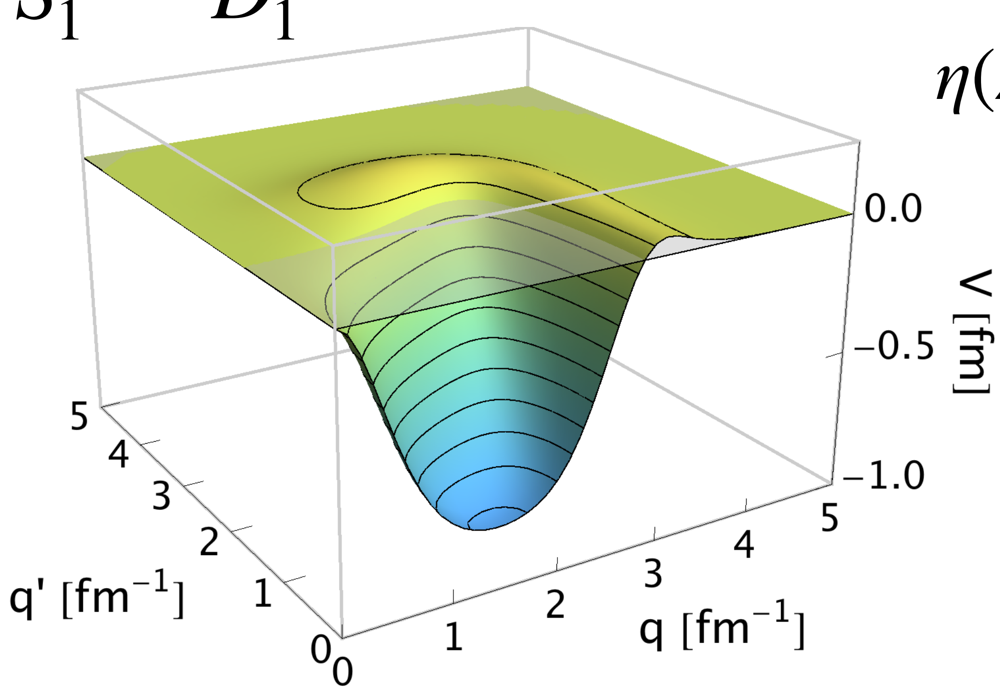


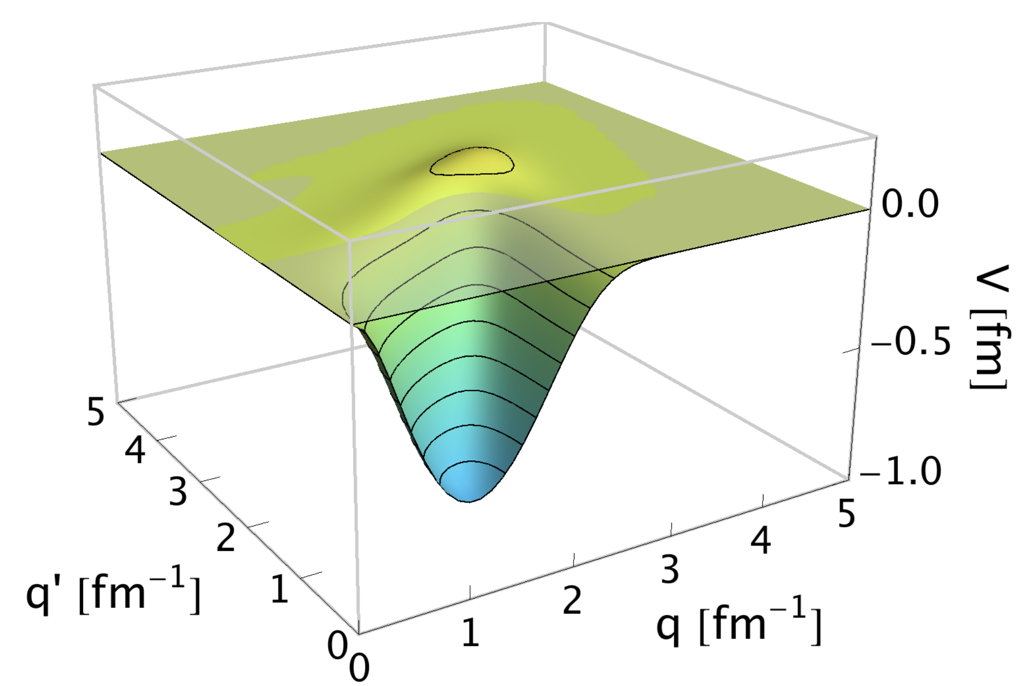
$\lambda = \infty$

$\lambda = 1.8\,\mathrm{fm}^{-1}$

$$\eta(\lambda) \equiv \frac{1}{2\mu}\left[\mathbf{q}^2, H(\lambda)\right]$$

$^3S_1 - {}^3D_1$





Induced Interactions

- SRG is a **unitary** transformation in **A-body space**
- up to **A-body interactions** are **induced** during the flow:

$$\frac{dH}{d\lambda} = [[\sum a^\dagger a, \underbrace{\sum a^\dagger a^\dagger a a}_{2-\text{body}}], \underbrace{\sum a^\dagger a^\dagger a a}_{2-\text{body}}]] = \ldots + \underbrace{\sum a^\dagger a^\dagger a^\dagger a a a}_{3-\text{body}} + \ldots$$

- **state-of-the-art:** evolve in three-body space, truncate induced four- and higher many-body forces
- **flow parameter dependence** of eigenvalues is a **diagnostic** for size of omitted induced interactions

What is “Magic” about EM1.8/2.0?

Roland Wirth
(now at DWD,
Offenbach,
Germany)

What is "Magic" about EM1.8/2.0?

- The **"magic" NN+3N interaction**, EM1.8/2.0 yields excellent ground-state energies across the nuclear chart, all the way to ^{208}Pb [Simonis et. al., PRC **96**, 014303; Hu et al., Nat. Phys. **18**, 1196]

- **Construction:** [Hebeler et. al., PRC 83, 031301(R)]
  - **NN:** Entem-Machleidt N^{3}LO @ 500 MeV cutoff, SRG evolved to $\lambda = 1.8\,\mathrm{fm}^{-1}$
  - **3N:** N^{2}LO, nonlocal regulator with $\Lambda_{3\mathrm{N}} = 2.0\,\mathrm{fm}^{-1}$, c_D and c_E fit to ^{3}H g.s. energy and ^{4}He charge radius

- **Assumption:** Induced 3N terms can be **absorbed into c_D and c_E**

- **Test:** Evolve EM2.0/2.0 to $\lambda = 1.8\,\mathrm{fm}^{-1}$and project 3N force onto N^{2}LO topologies

Projecting 3N Forces

- Use chiral N²LO operators $O_{1,3,4,D,E}$ with $\Lambda_{3\mathrm{N}} = 2.0\,\mathrm{fm}^{-1}$ as a **basis for 3N force**
  - represented as **three-body Jacobi HO matrices**
- **Frobenius inner product:**

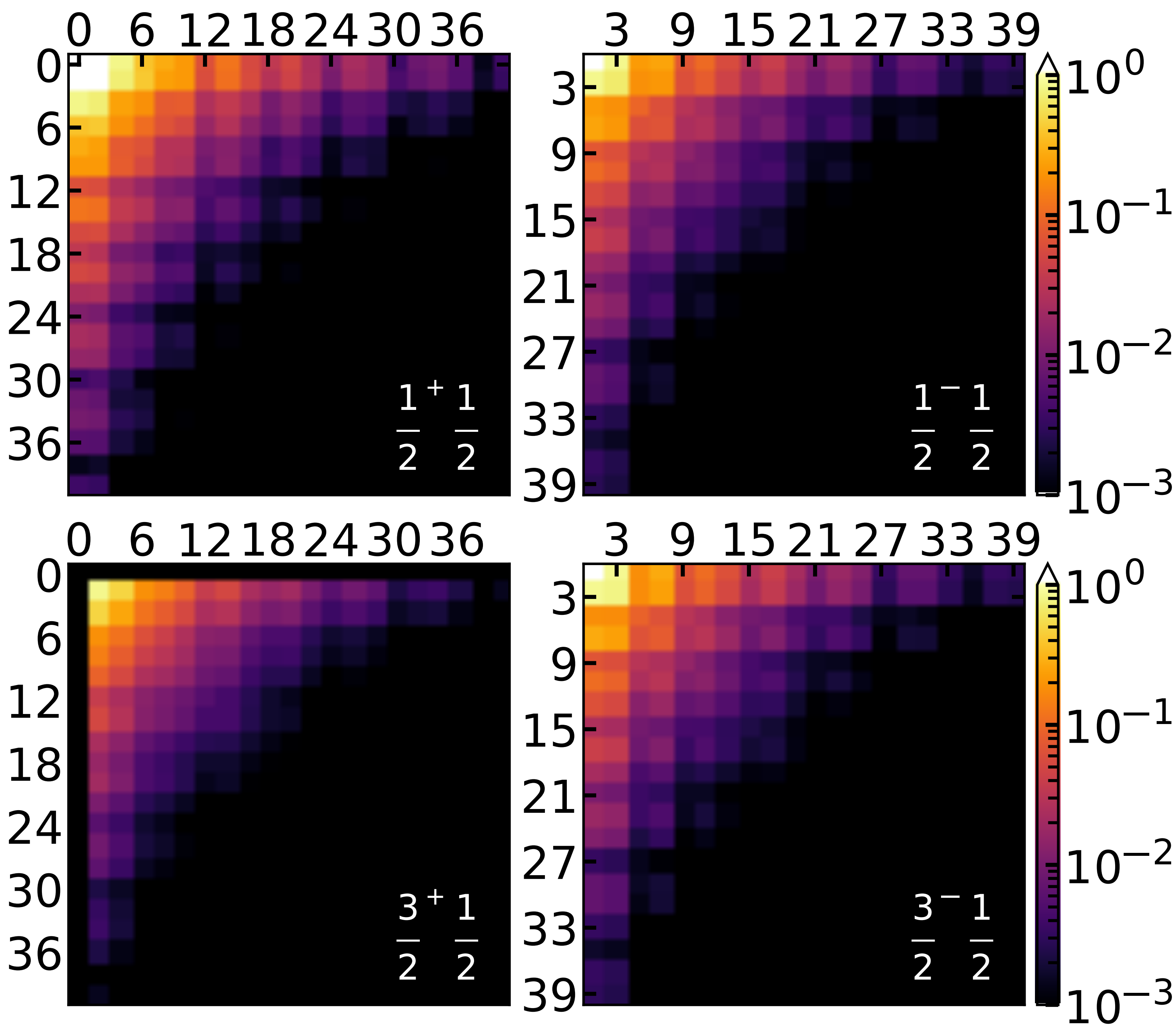
$$\left\langle U, V \right\rangle \equiv \sum_{J^\pi T} \mathrm{tr}\left(U^\dagger_{J^\pi T} V_{J^\pi T} \right)$$

- basis is **not orthogonal** - introduce **metric** $G_{ij} \equiv \left\langle O_i, O_j \right\rangle$
- compute $\mathbf{y} = \left(\left\langle O_1, V \right\rangle, \ldots \left\langle O_E, V \right\rangle \right)^T$ and solve

$$G\mathbf{c} = \mathbf{y}$$

- $\mathbf{c}$ contains the **LECs of the projected interaction**

Structure of N²LO Topologies



$$|\langle EJ^\pi T|\, V_{123}(\lambda=\infty)\,|E'J^\pi T\rangle|$$

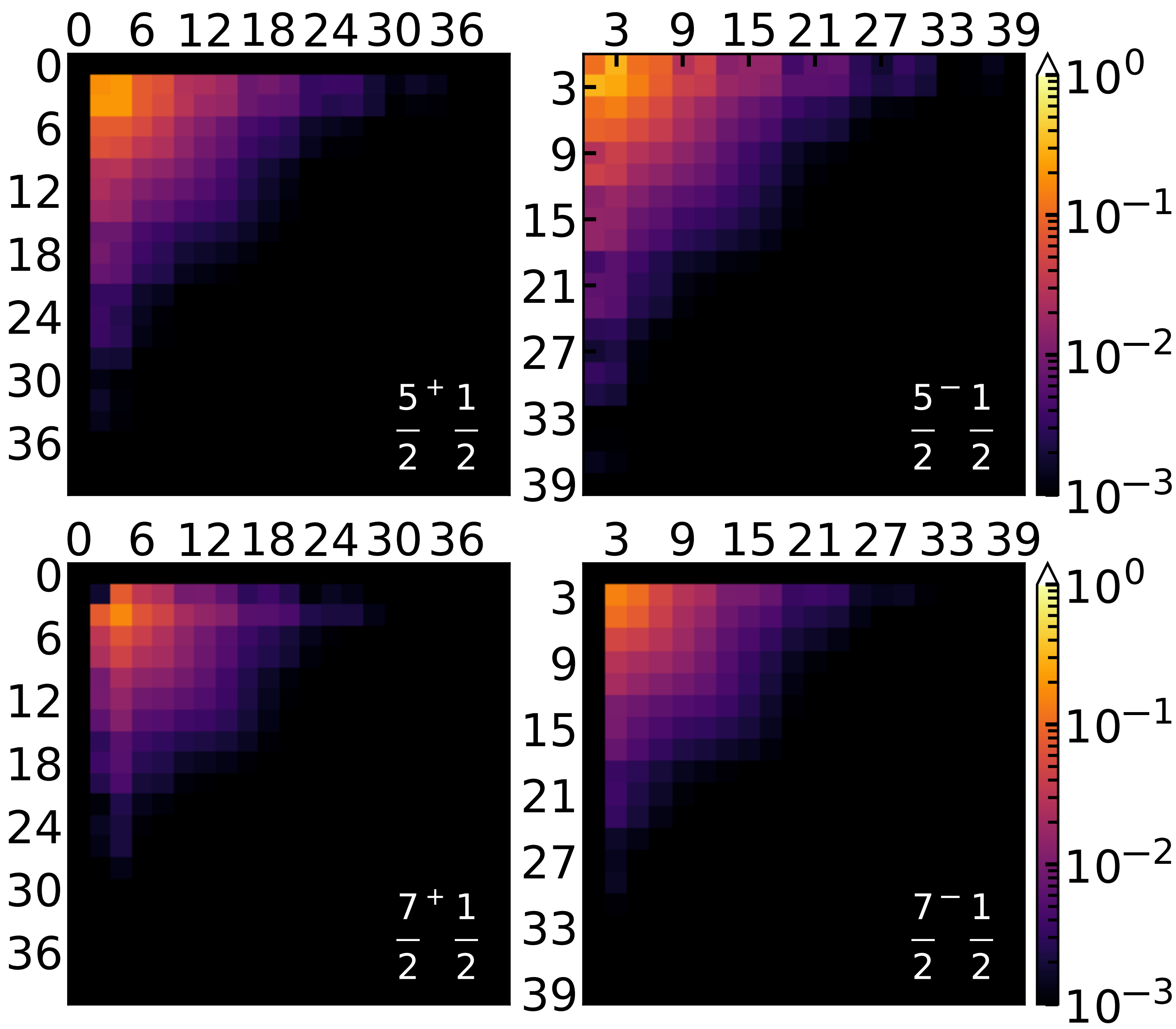
$$\hbar\omega = 36\,\mathrm{MeV}$$

$$n_{\mathrm{reg}} = 4$$

$$\Lambda = 2.0\,\mathrm{fm}^{-1}$$

- low E, low J
- c_D similar to c_3
- c_E is S-wave only

Structure of N^2LO Topologies



$$|\langle EJ^\pi T | V_{123}(\lambda = \infty) | E' J^\pi T \rangle|$$

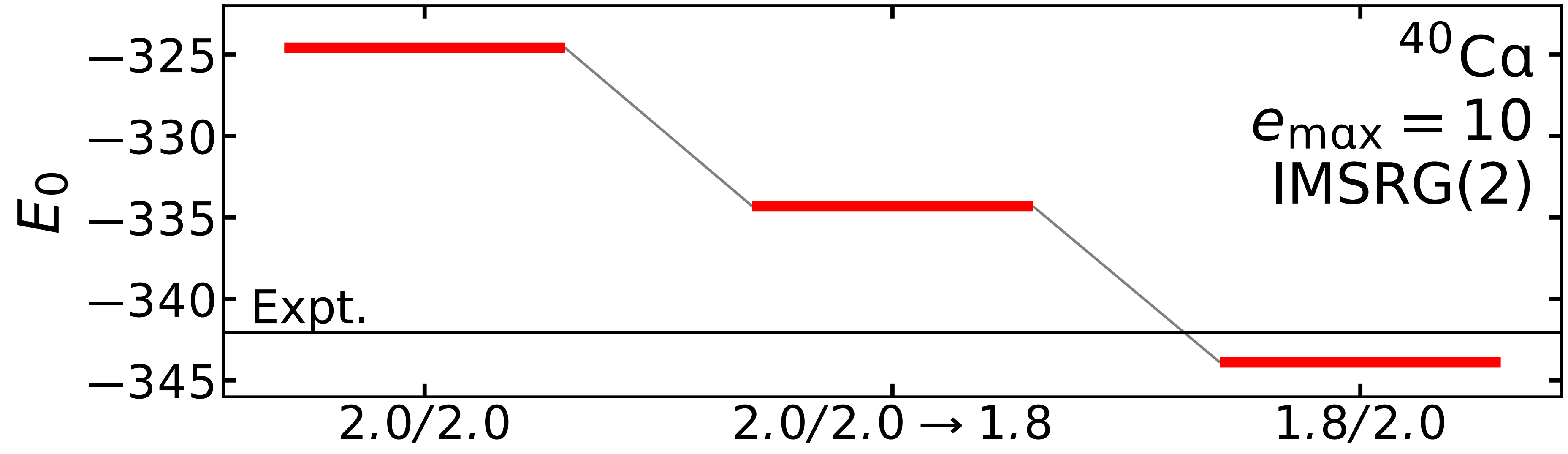
$$\hbar\omega = 36\,\mathrm{MeV}$$

$$n_{\mathrm{reg}} = 4$$

$$\Lambda = 2.0\,\mathrm{fm}^{-1}$$

- low E, low J
- c_D similar to c_3
- c_E is S-wave only

Evolving from $2.0\,\mathrm{fm}^{-1}$ to $1.8\,\mathrm{fm}^{-1}$

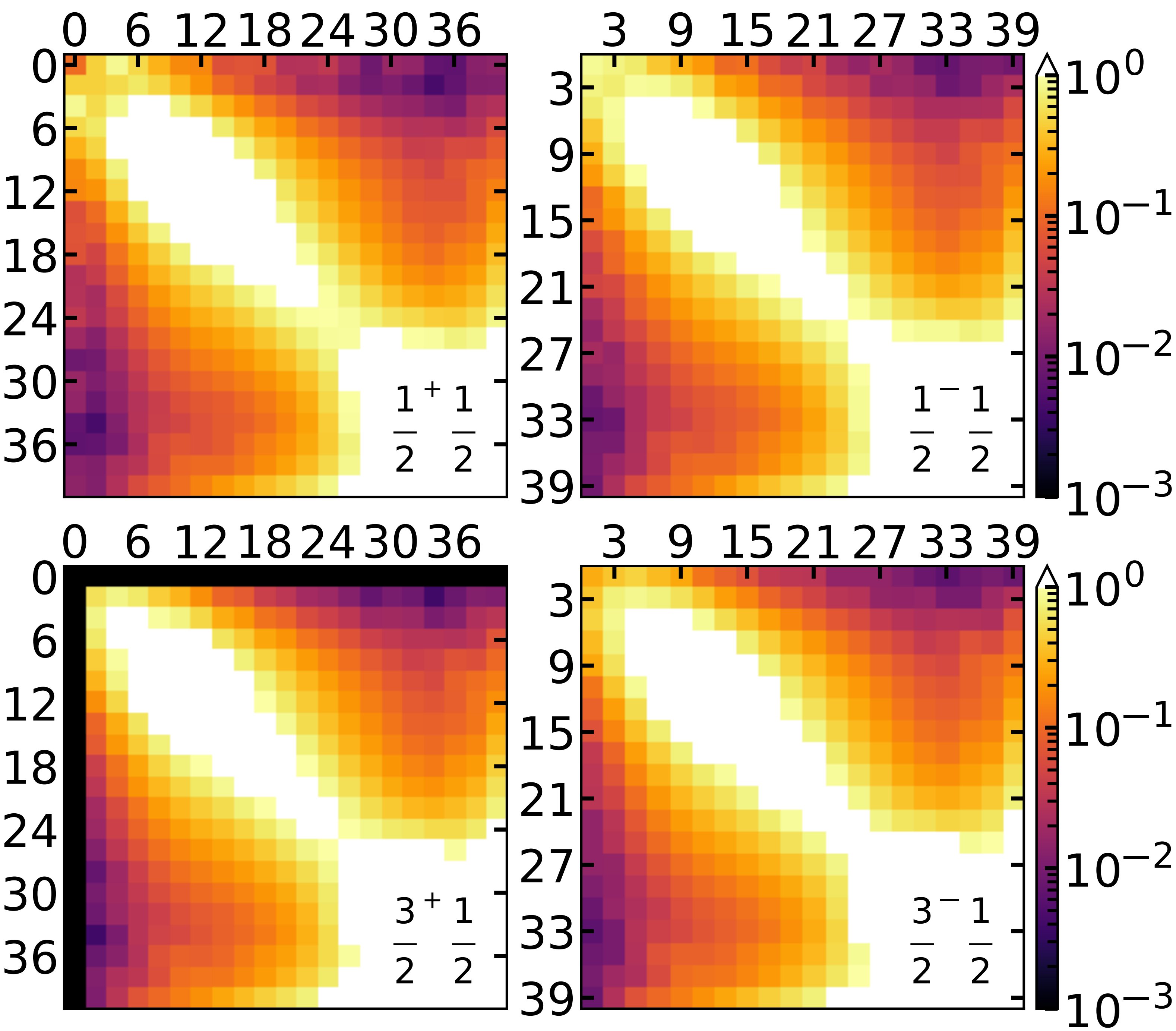


- SRG evolution in **three-body space**
- unitary transformation obeys

$$\frac{dU}{d\lambda} = -\frac{4}{\lambda^5}\eta(\lambda)U(\lambda)$$

- can use $U(\lambda)$ to separate induced 3N interaction, $V_{2\to3}(\lambda)$, from evolved initial 3N interaction $V_{3\to3}(\lambda)$

Structure of Induced Interactions



$$|\langle E J^\pi T \,|\, V_{2\to3} \,|\, E' J^\pi T \rangle|$$

$$\hbar\omega = 36\,\mathrm{MeV}$$

$$\lambda = 1.8\,\mathrm{fm}^{-1}$$

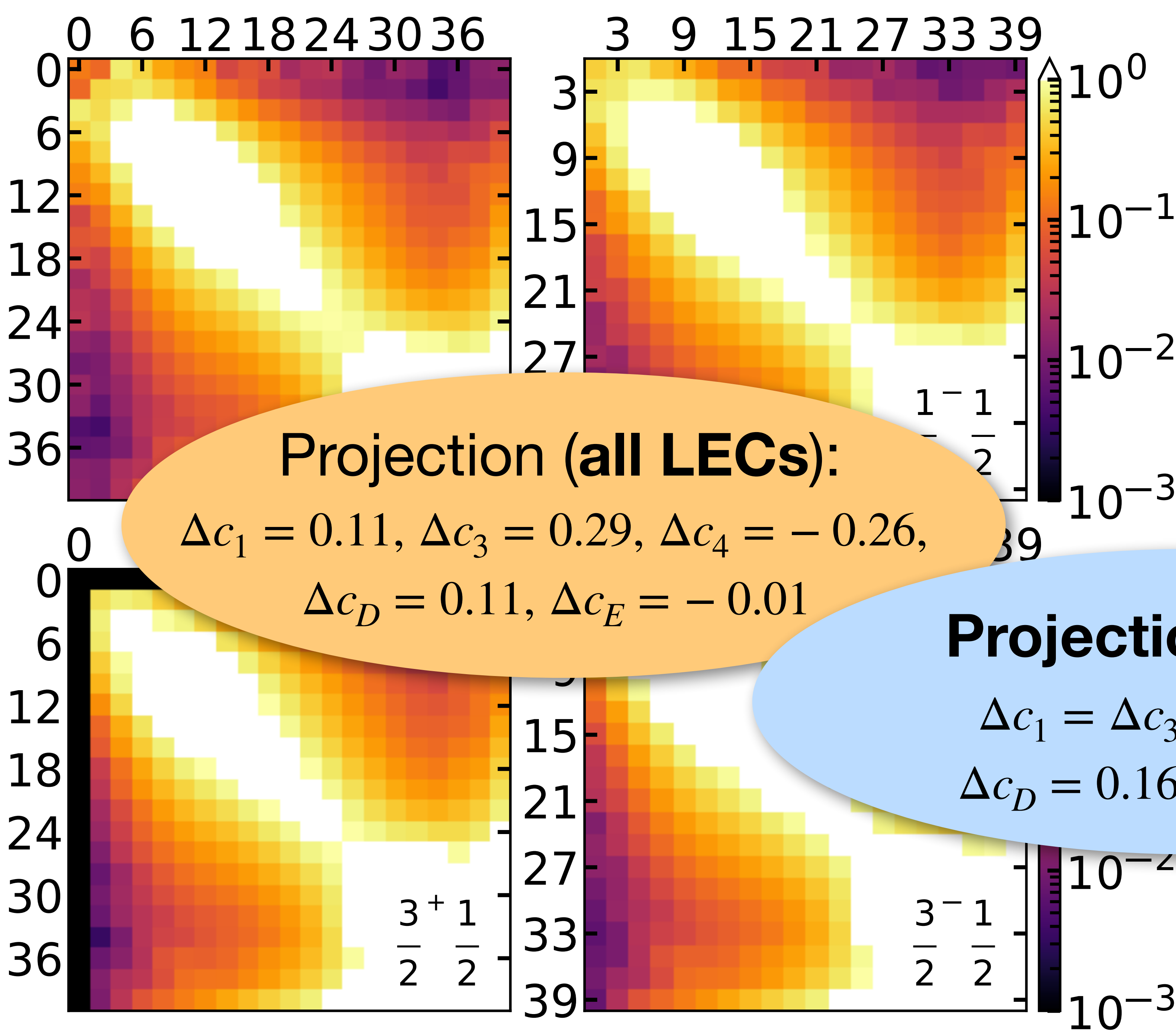
- contributions from **all energies** (up to model space truncation)
- **dominant diagonal**
- **different from N²LO topologies**

Structure of Induced Interactions

$$|\langle EJ^\pi T\,|\,\Delta V_{2\to3}\,|\,E'J^\pi T\rangle|$$

$$\hbar\omega = 36\,\mathrm{MeV}$$

$$\lambda = 1.8\,\mathrm{fm}^{-1}$$



Projection (**all LECs**):

$\Delta c_1 = 0.11,\ \Delta c_3 = 0.29,\ \Delta c_4 = -0.26,$

$\Delta c_D = 0.11,\ \Delta c_E = -0.01$

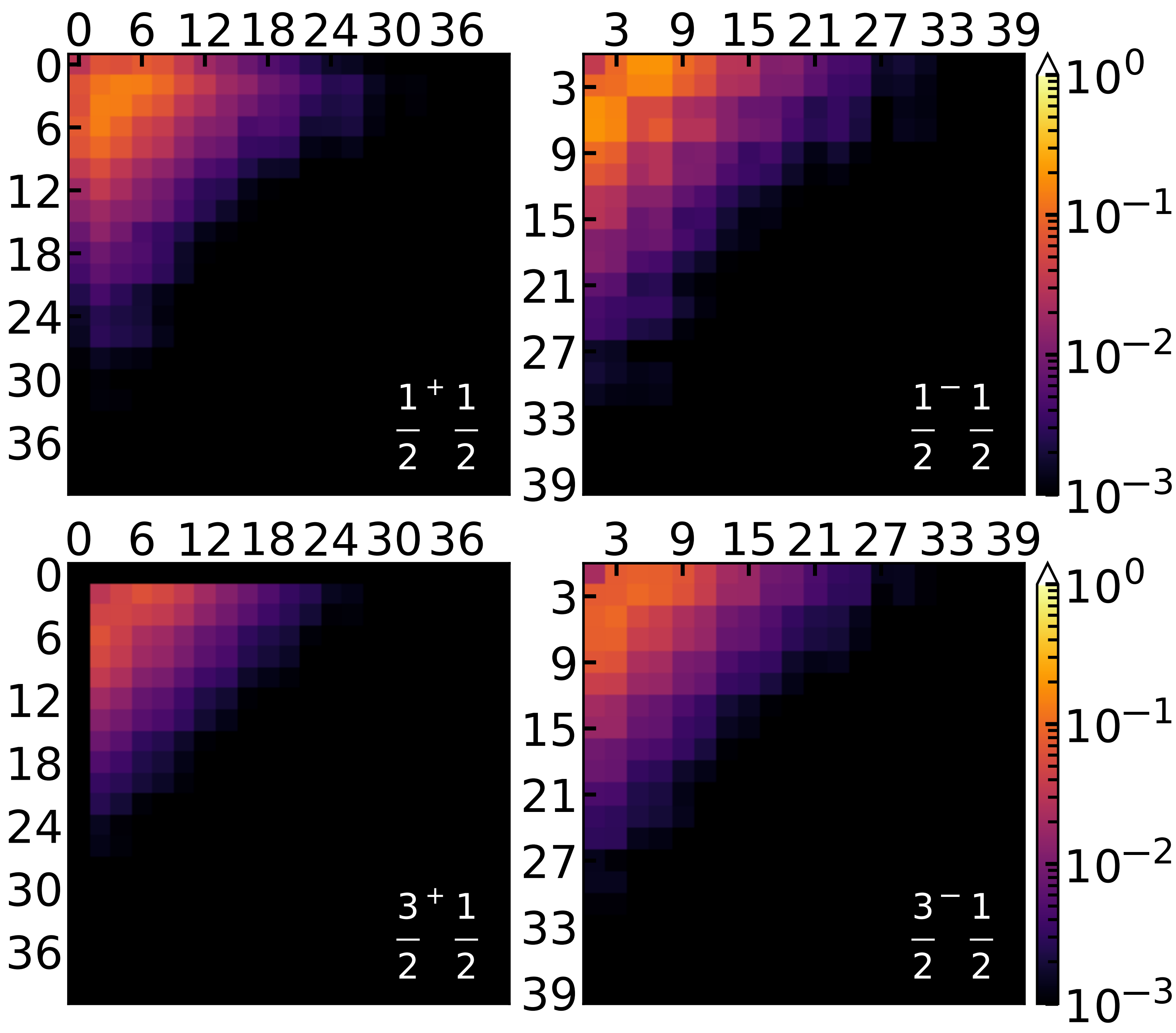
Projection (c_D, c_E):

$\Delta c_1 = \Delta c_3 = \Delta c_4 = 0$

$\Delta c_D = 0.16,\ \Delta c_E = 0.01$

- contributions from **all energies** (up to ...del space ...n)
- ...**ant diagonal**
- **different from N²LO topologies**

Evolved 3N Interaction



$$|\langle EJ^\pi T|V_{3\to3}|E'J^\pi T\rangle|$$

$$\hbar\omega = 36\,\mathrm{MeV}$$

$$\lambda = 1.8\,\mathrm{fm}^{-1}$$

- shape is similar to initial 3N force
- **weak** compared to $V_{2\to3}$

Projection of Evolved Interaction

LEC	2.0/2.0	2.0/2.0 →1.8		1.8/2.0
		Full	c_D, c_E	
c_1	-0.81	-0.67	-0.81	-0.81
c_3	-3.20	-2.92	-3.20	-3.20
c_4	5.40	5.14	5.40	5.40
c_D	1.26	1.38	1.45	1.27
c_E	-0.12	-0.13	-0.11	-0.13

- **Full:** 10% correction to c_is, 2PE suppressed, contacts enhanced
- c_D, c_E **only:** D term enhanced, E term (slightly) suppressed
- **Final values quite different from EM1.8/2.0**

Conclusions (?)

- **“Magic”** of EM1.8/2.0 seems to be an **accidental cancellation:** induced 3N terms and **excluded** higher order NN, 3N, 4N, … cancel, except for contact terms
- c_D, c_E have the **right size** to fit few-body observables and provide **correct shift** in E/A
- **Use this protocol to analyze Δ-full interactions, impact of new 3N forces**

SRG in Many-Body Systems

SRG Scales

- **split the Hamiltonian:** $H(s) = H_d(s) + H_{od}(s)$
- assume that

$$H_d(s)\,|\,n\rangle = E_n(s)\,|\,n\rangle\,, \qquad \langle n\,|\,H_{od}(s)\,|\,n\rangle = 0$$

- **generator** - e.g., Wegner:

$$\begin{aligned}\langle i\,|\,\eta\,|\,j\rangle &= \sum_k \big(\langle i\,|\,H_d\,|\,k\rangle\langle k\,|\,H_{od}\,|\,j\rangle - \langle i\,|\,H_{od}\,|\,k\rangle\langle k\,|\,H_d\,|\,j\rangle\big)\\ &= -\,(E_i - E_j)\langle i\,|\,H_{od}\,|\,j\rangle\end{aligned}$$

- **flow equation:**

$$\begin{aligned}\frac{d}{ds}\langle i\,|\,H\,|\,j\rangle = -(E_i - E_j)^2\langle i\,|\,H_{od}\,|\,j\rangle \\ + \sum_k (E - i + E_j - 2E_k)\langle i\,|\,H_{od}k\rangle\langle k\,|\,H_{od}\,|\,j\rangle\end{aligned}$$

SRG Scales

- assume $H_{od}(s)$ is small - should be a **good assumption** for some $s > s_0$ if the SRG flow is working as intended (or if there are perturbative arguments)

$$\frac{d}{ds}E_i = \frac{d}{ds}\langle i|H_d|i\rangle = 2\sum_k (E_i - E_k)|\langle i|H_{od}|k\rangle|^2 \approx 0$$

$$\frac{d}{ds}\langle i|H|j\rangle = \frac{d}{ds}\langle i|H_{od}|j\rangle \approx -(E_i - E_j)^2\langle i|H_{od}|j\rangle$$

- **integrate:**

$$\langle i|H_{od}(s)|j\rangle = \langle i|H_{od}(s_0)|j\rangle\, e^{-(E_i-E_j)^2(s-s_0)}$$

- **White generator:** e^{-s}
- **imaginary time / Brillouin:** $e^{-|E_i-E_j|s}$

Takeaways

- s characterizes **decoupling of energy scales** in the many-body system
  - $s \sim f\left(\Delta E^{-1}\right)$
  - concrete interpretation depends on **choice of generator**
- carries forward from **many-body states to operator formulation in IMSRG** - applies in the same way to 0B, 1B, 2B, … operators
- **Can this be used (more) ?**

In-Medium SRG

Operator Bases for the IMSRG

- choose a **basis of operators** to represent the flow (might involve an educated guess about physics):

$$H(s) = \sum_i c_i(s) O_i\,, \qquad \eta(s) = \sum_i f_i(\{c(s)\}) O_i$$

- **close algebra by truncation,** if necessary:

$$\left[O_i, O_j\right] = \sum_{ijk} g_{ijk} O_k$$

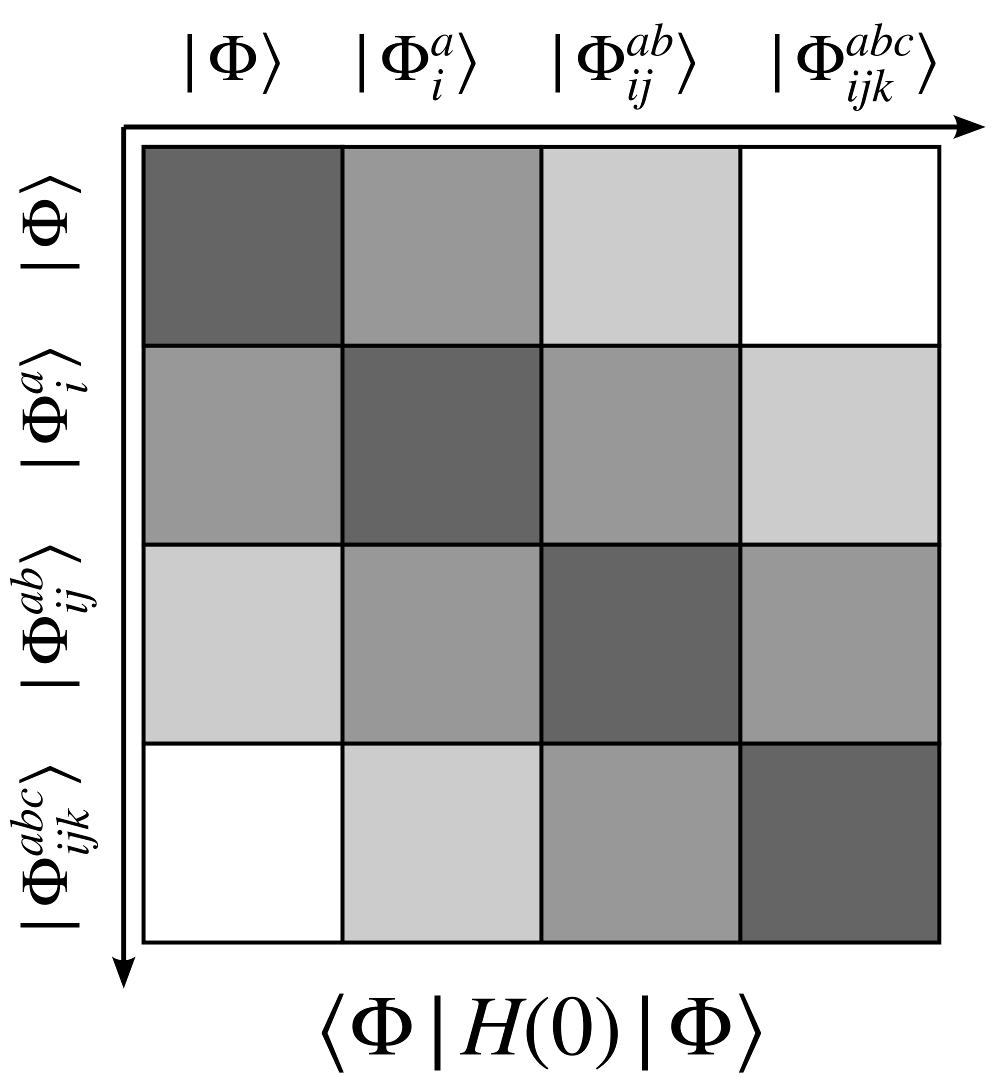
- **flow equations** for the coefficient (**coupling constants**):

$$\frac{d}{ds} c_k = \sum_{ij} g_{ijk} f_i(c) c_j$$

- "obvious" choice for many-body problems

$$\{O_{pq}, O_{pqrs}, \ldots\} = \{a^\dagger_p a_q,\ a^\dagger_p a^\dagger_q a_s a_r,\ \ldots\}$$

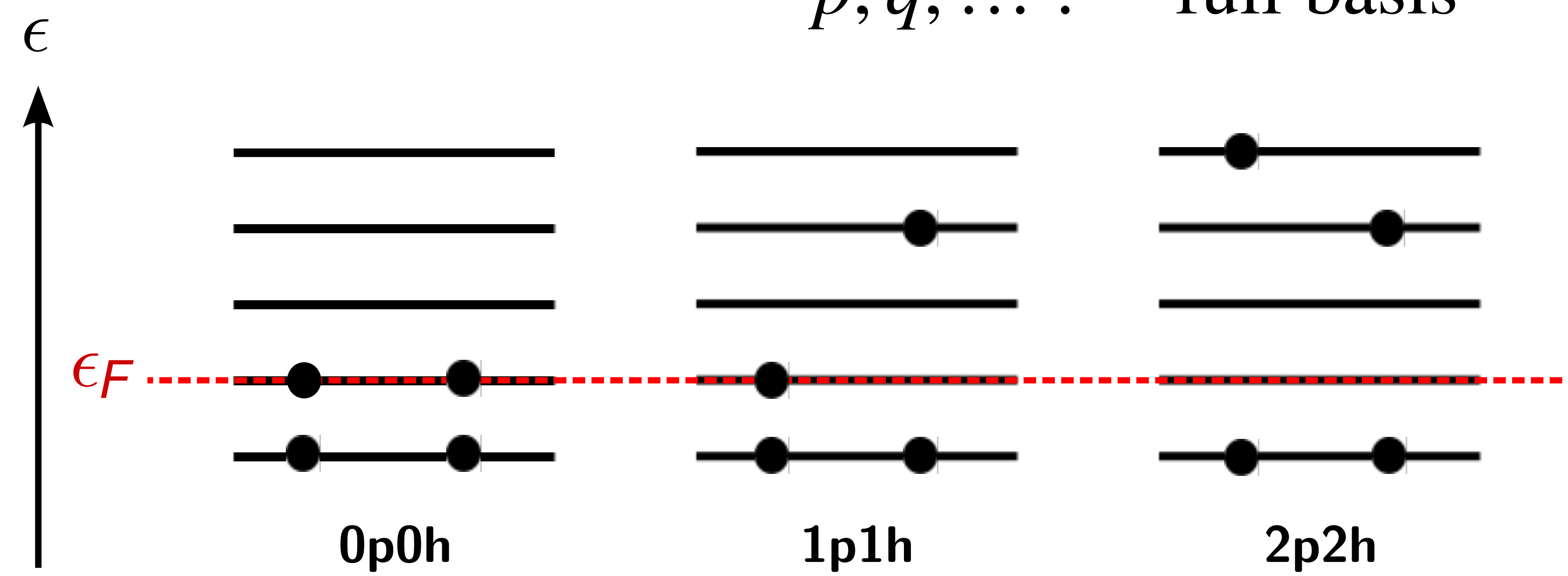
Transforming the Hamiltonian



$a, b, \ldots :$ $\epsilon > \epsilon_F$

$i, j, \ldots :$ $\epsilon \leq \epsilon_F$

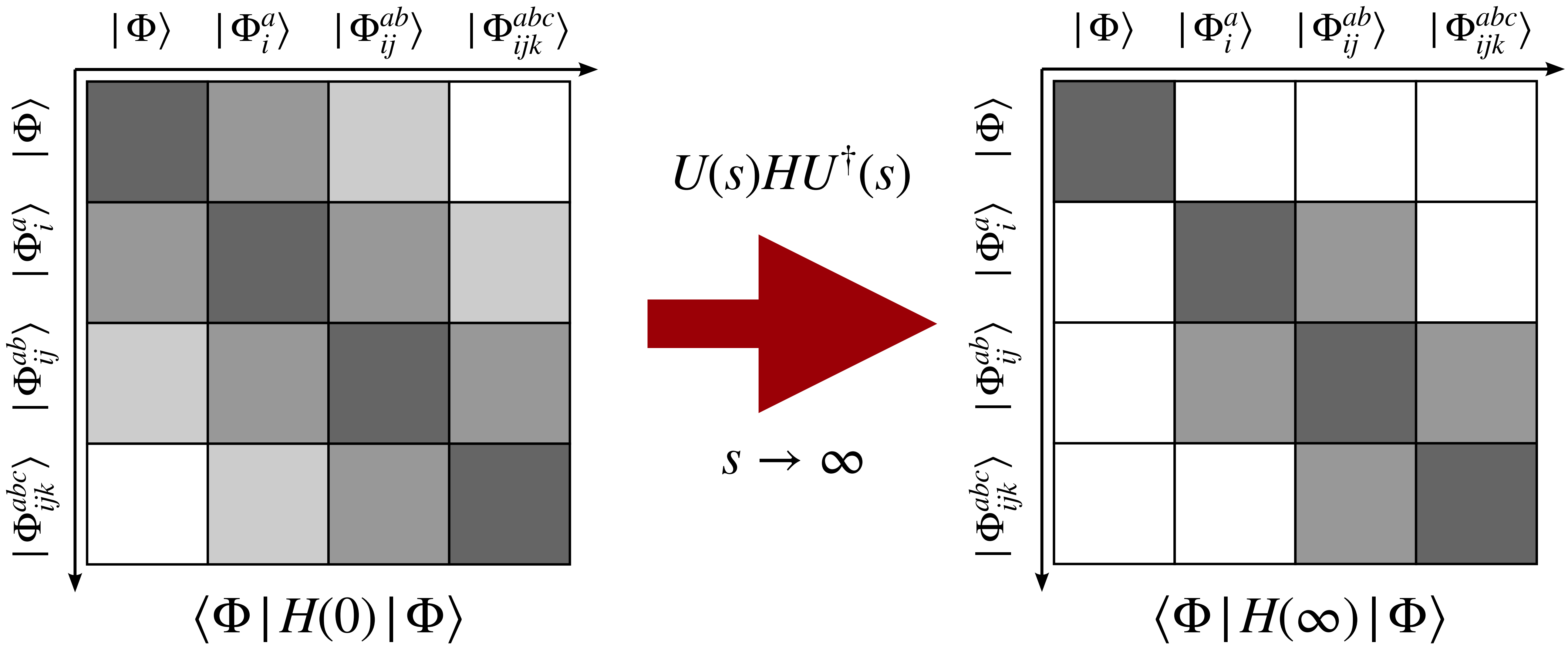
$p, q, \ldots :$ full basis



excitations relative to reference state: **normal-ordering**

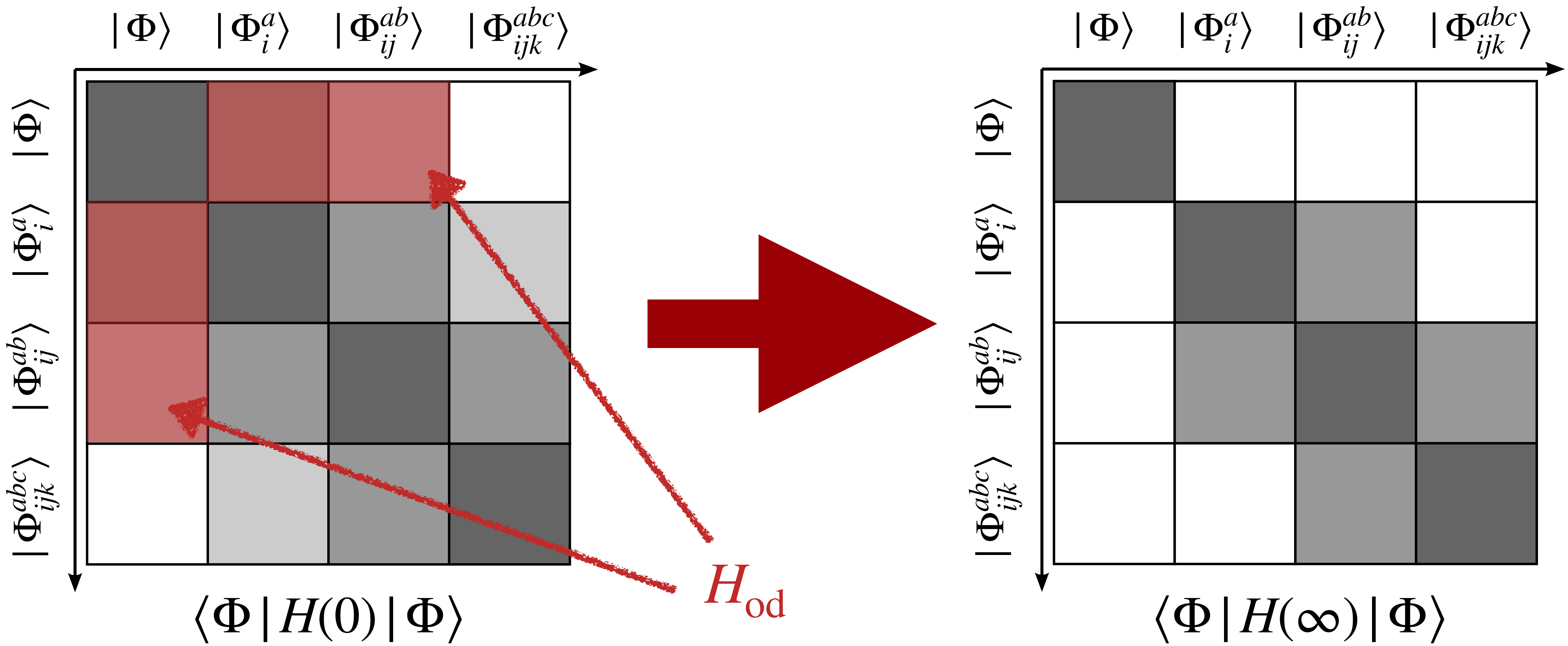
- reference state: **single Slater determinant**

Decoupling in A-Body Space



goal: decouple reference state $|\Phi\rangle$ from excitations

Flow Equation



$$\frac{d}{ds}H(s) = \left[\eta(s), H(s)\right] \quad , \quad \text{e.g.,} \quad \eta(s) \equiv \left[H_d(s), H_{od}(s)\right]$$

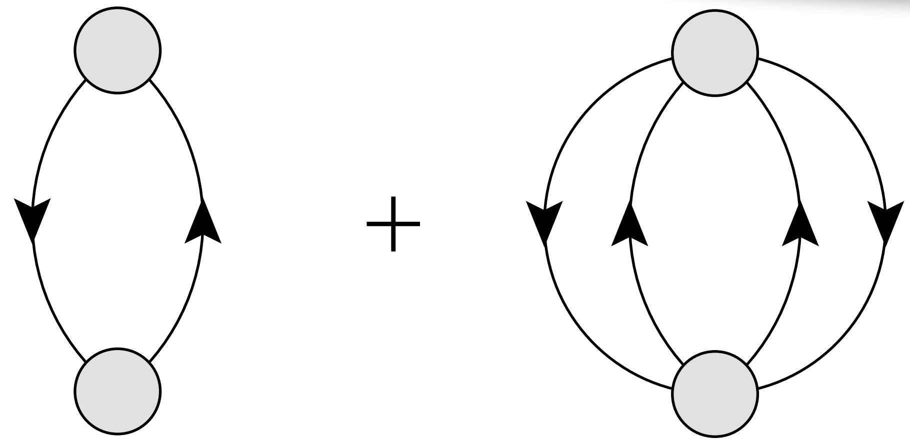
Operators truncated at **two-body level** - **matrix is never constructed explicitly!**

IMSRG(2) Flow Equations

0-body Flow

~ 2nd order MBPT for $H(s)$

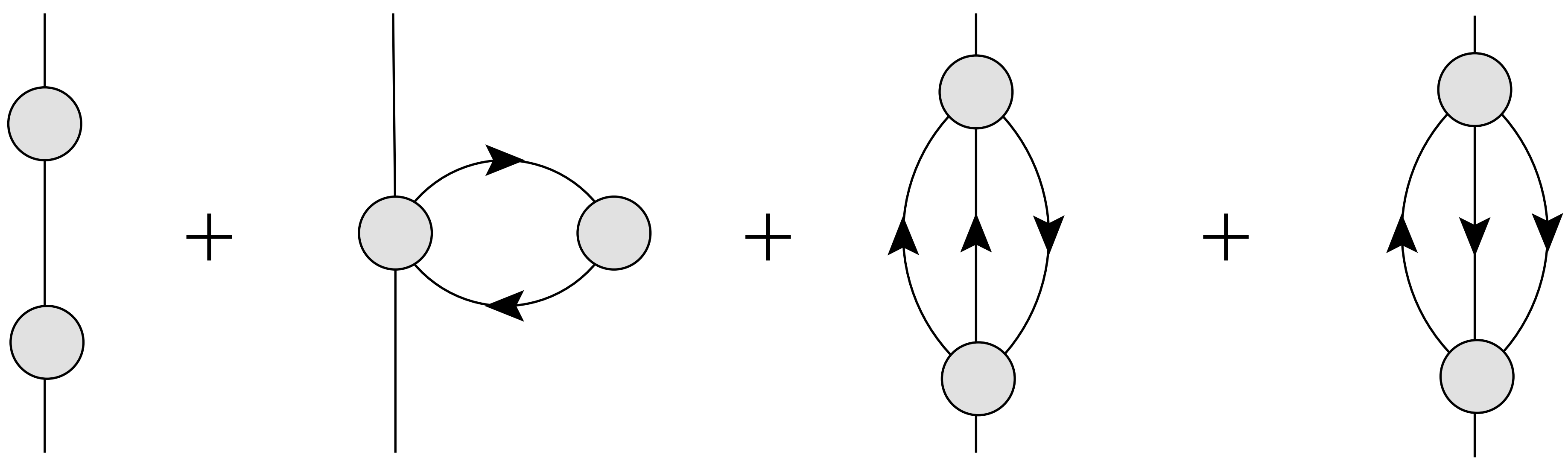
$$\frac{dE}{ds} =$$



1-body Flow

coefficients (couplings) of $H(s)$

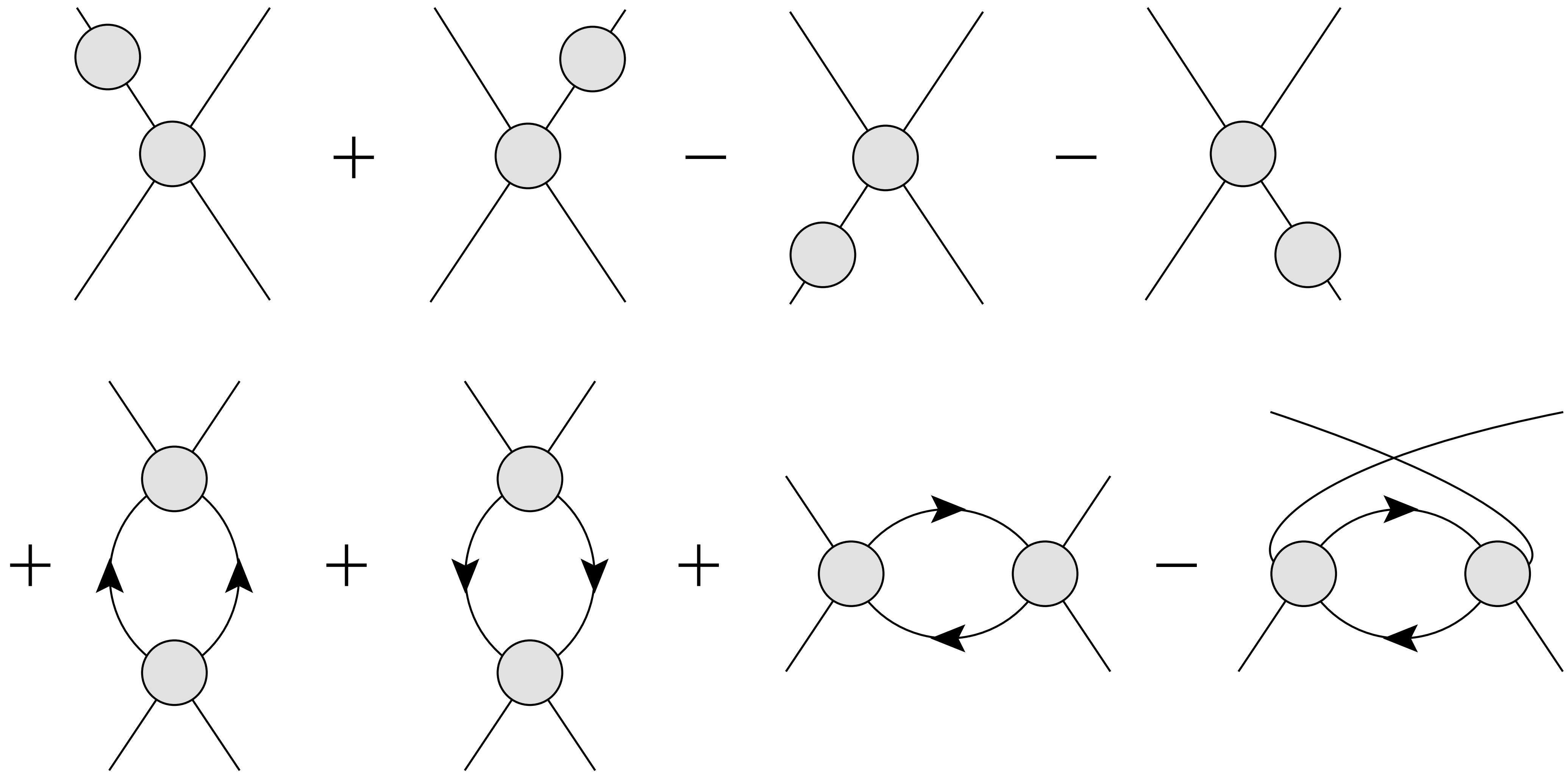
$$\frac{df}{ds} =$$



IMSRG(2) Flow Equations

2-body Flow

$$\frac{d\Gamma}{ds} =$$

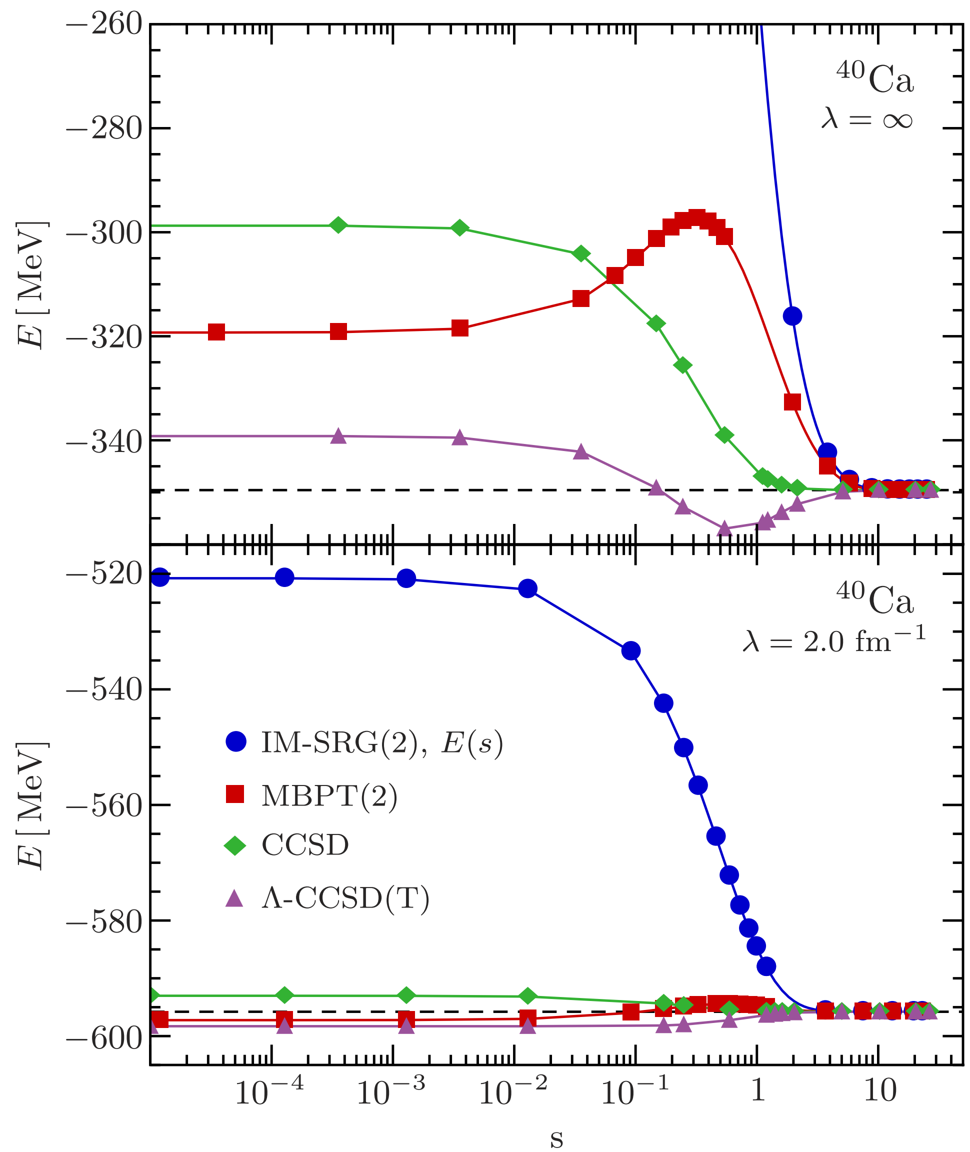


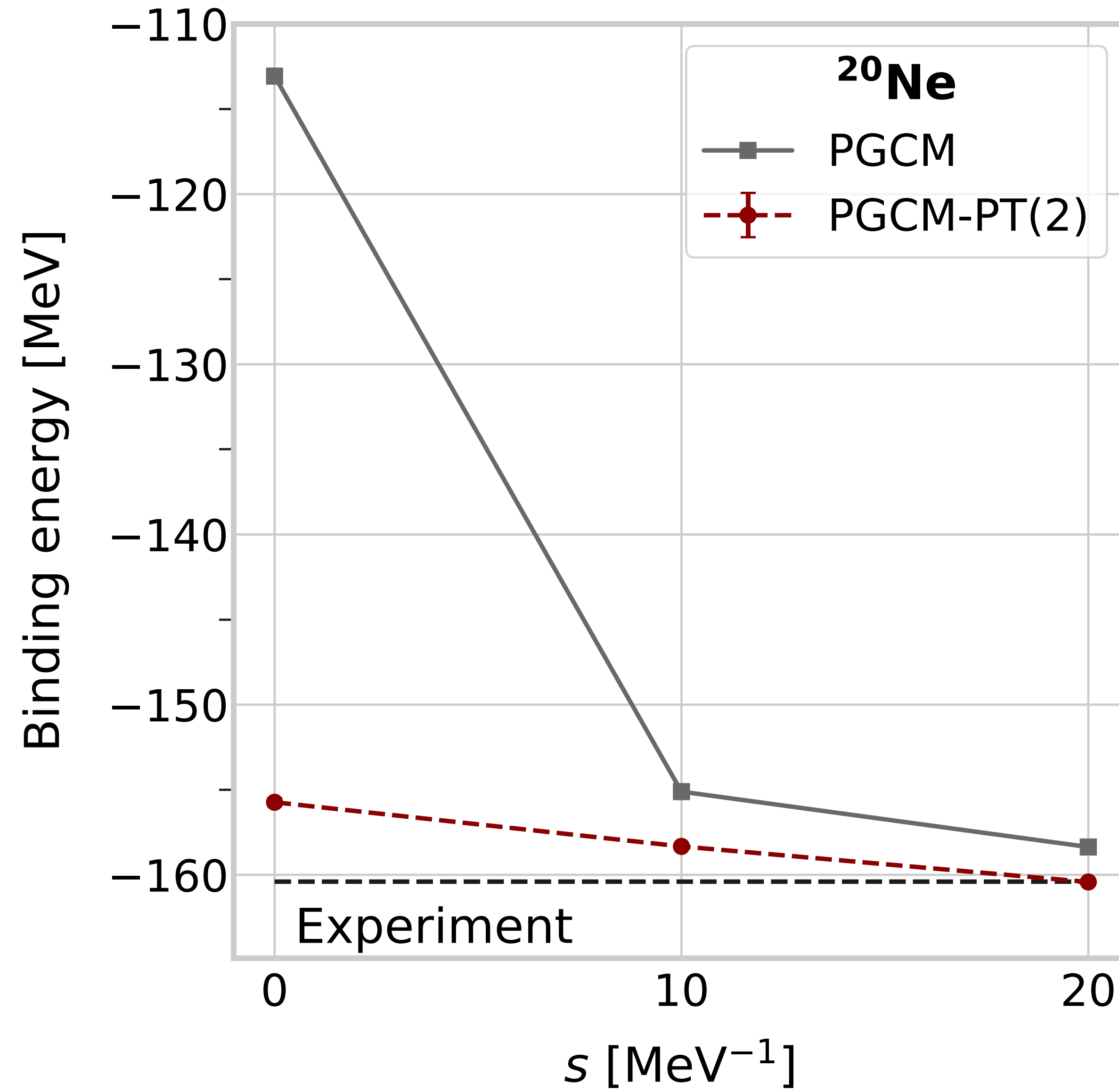
s channel | *t* channel | *u* channel

ladders | rings

Examples of (IM)SRG Invariance

HH et al., Phys. Rept. **621**, 165
M. Frosini et al., EPJA **58**, 64

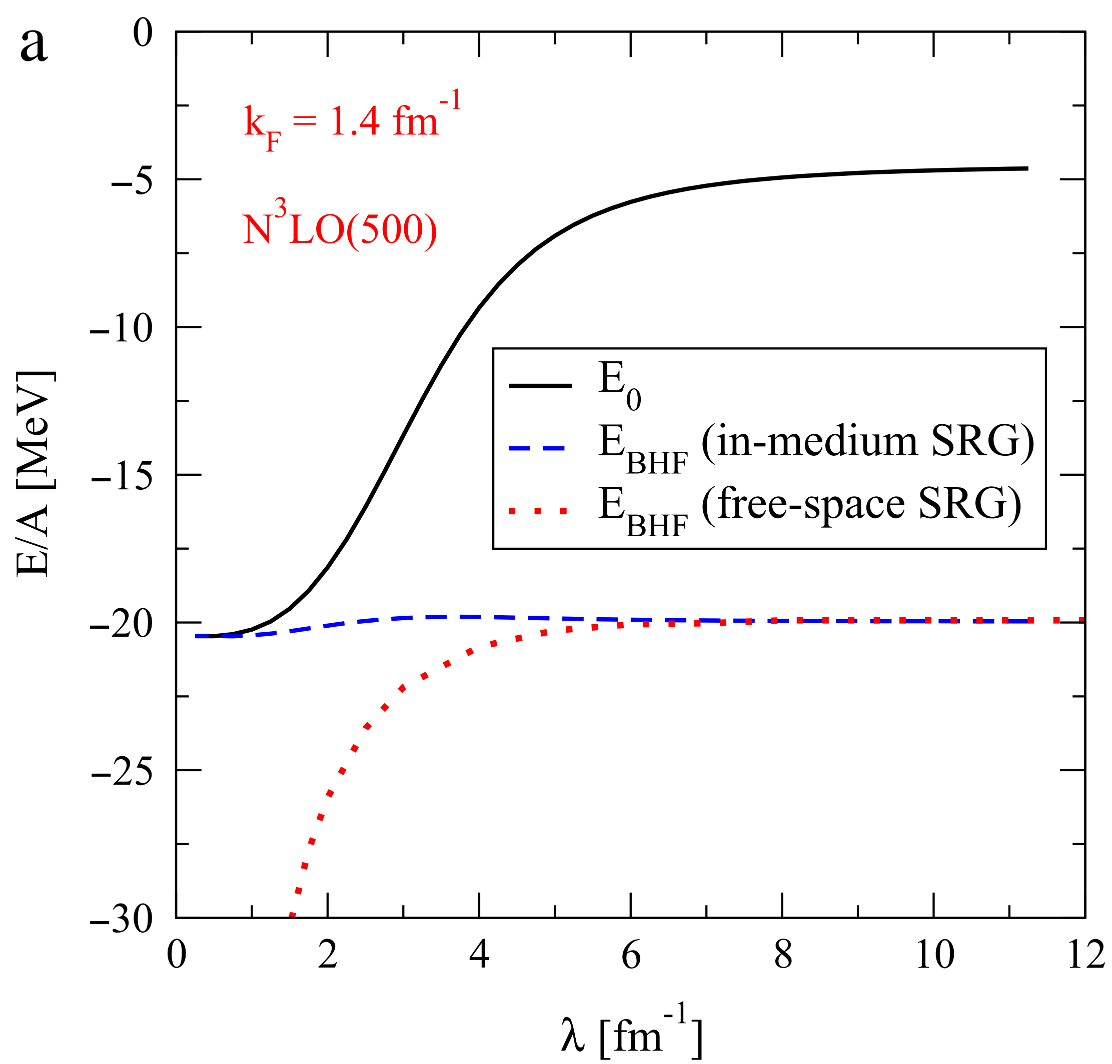


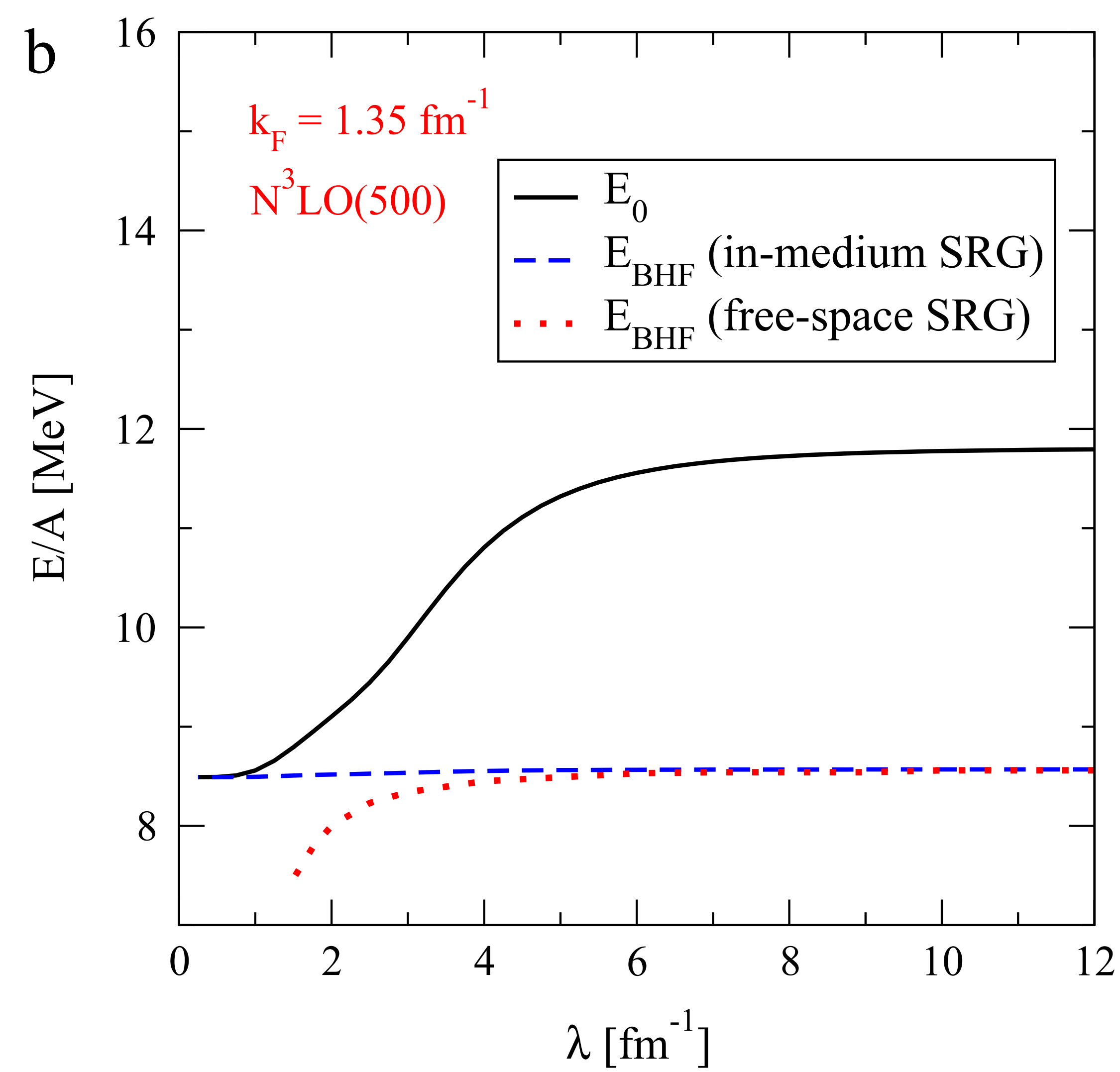


- observables should be invariant under unitary evolution
  - (tunable) combination of relevant operators in basis + degrees of freedom of many-body Hilbert space

Examples of (IM)SRG Invariance

S. K. Bogner et al., PPNP **65**, 94





Magnus Series Formulation

- explicit exponential ansatz for unitary transformation:

$$U(s) = S\exp\int_0^s ds'\,\eta(s') = e^{\Omega(s)}$$

- flow equation for **Magnus** operator:

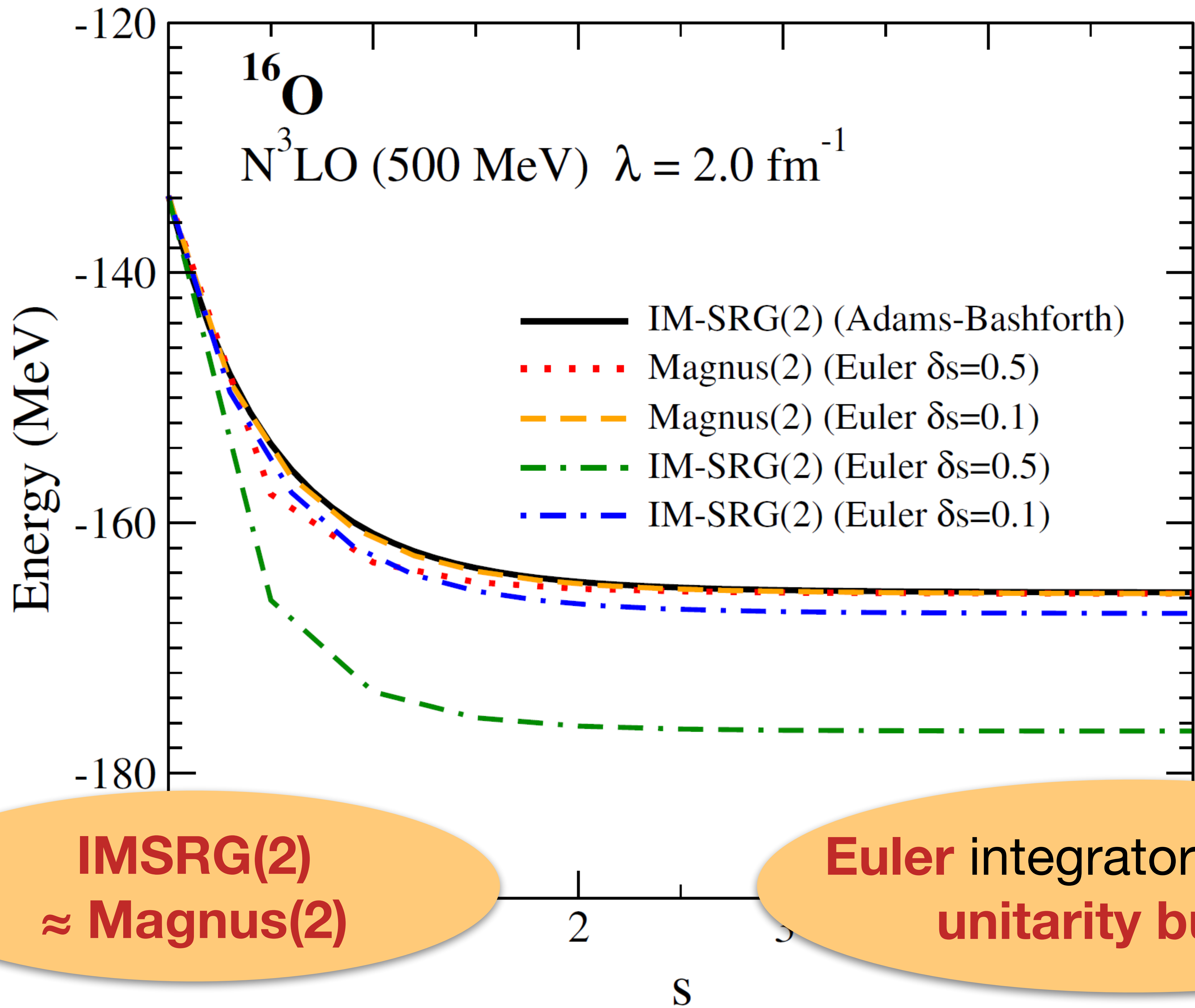
$$\frac{d}{ds}\Omega = \sum_{k=0}^{\infty}\frac{B_k}{k!}\mathrm{ad}_{\Omega}^{k}(\eta)\,,\quad \mathrm{ad}_{\Omega}(O) = \left[\Omega, O\right]$$

(B_k: Bernoulli numbers)

- construct $O(s) = U(s)OU^{\dagger}(s)$ using Baker-Campbell-Hausdorff expansion (**Hamiltonian + effective operators**)

- **Magnus(2): two-body truncation** (as in NO2B, IMSRG(2))

Magnus vs. Direct Integration



IMSRG(2) ≈ Magnus(2)

Euler integrator sufficient, **unitarity built in**

Perturbative Treatment of Subleading Forces

- Magnus formulation is convenient for treating **specific interactions perturbatively or non-perturbatively**
  - possible in MR-IMSRG formalism as well, but much more cumbersome - can be used for cross-checks in the future
- generate $\Omega_0(s)$ **nonperturbatively**
- construct $H_1(s) = e^{\Omega_0(s)} H_1(0) e^{-\Omega_0(s)}$ and treat in finite-order MBPT
  - first tests with interactions from **pionless EFT** - **see Matthias Heinz' talk**
- **challenge:** implementation of 4N force at NLO
  - will explore **normal-ordered approximations**

Epilogue

Summary

- **Projection methods** to help **analyze 3N** (or other) nuclear forces / operators
- Opportunities from **embracing the RG** in IMS**RG** ?
- IMSRG offers convenient pathways for treating **specific components of the Hamiltonian perturbatively instead of non-perturbatively** - see Matthias Heinz' talk

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Thank you for your attention!

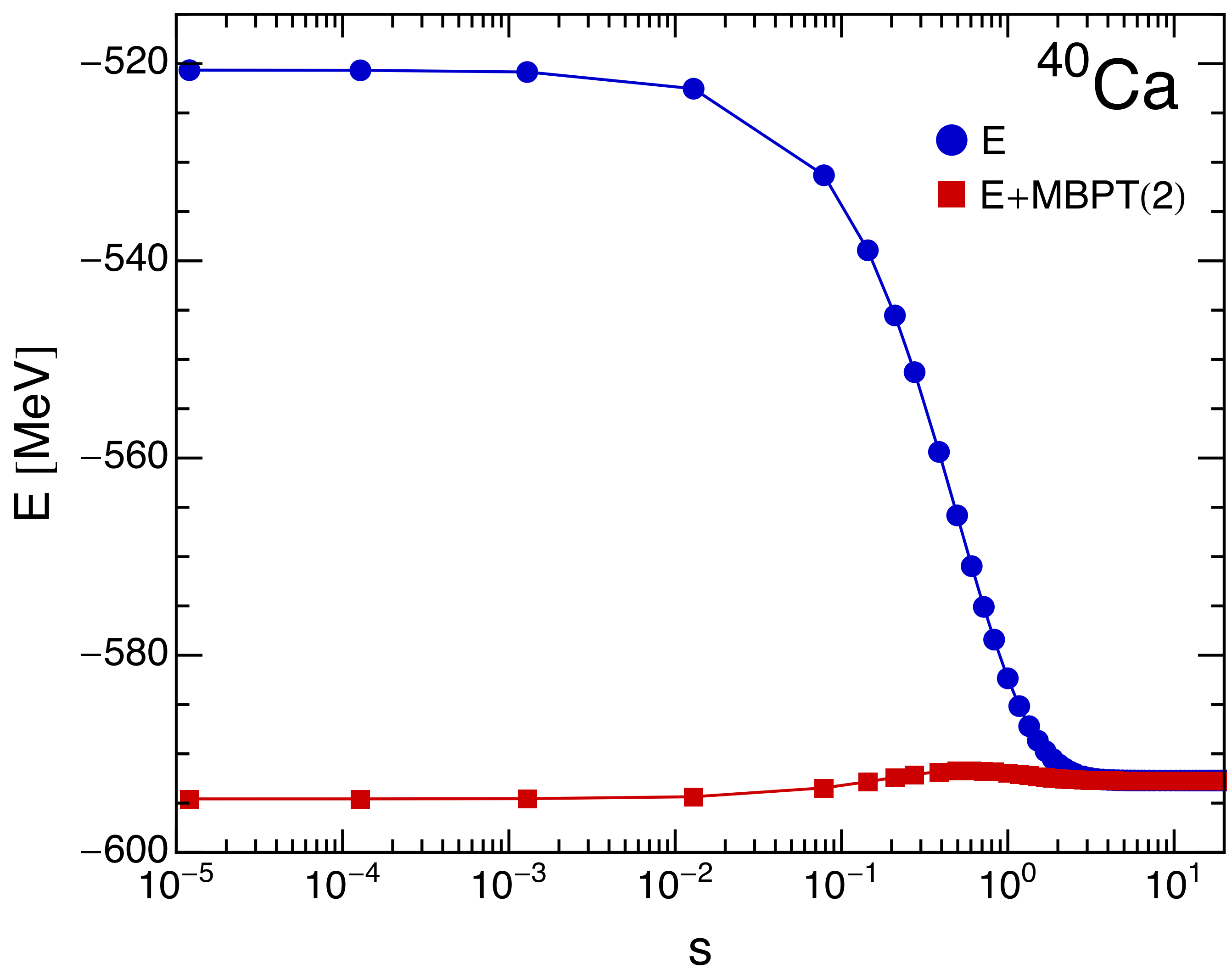
NUCLEI
Nuclear Computational Low-Energy Initiative

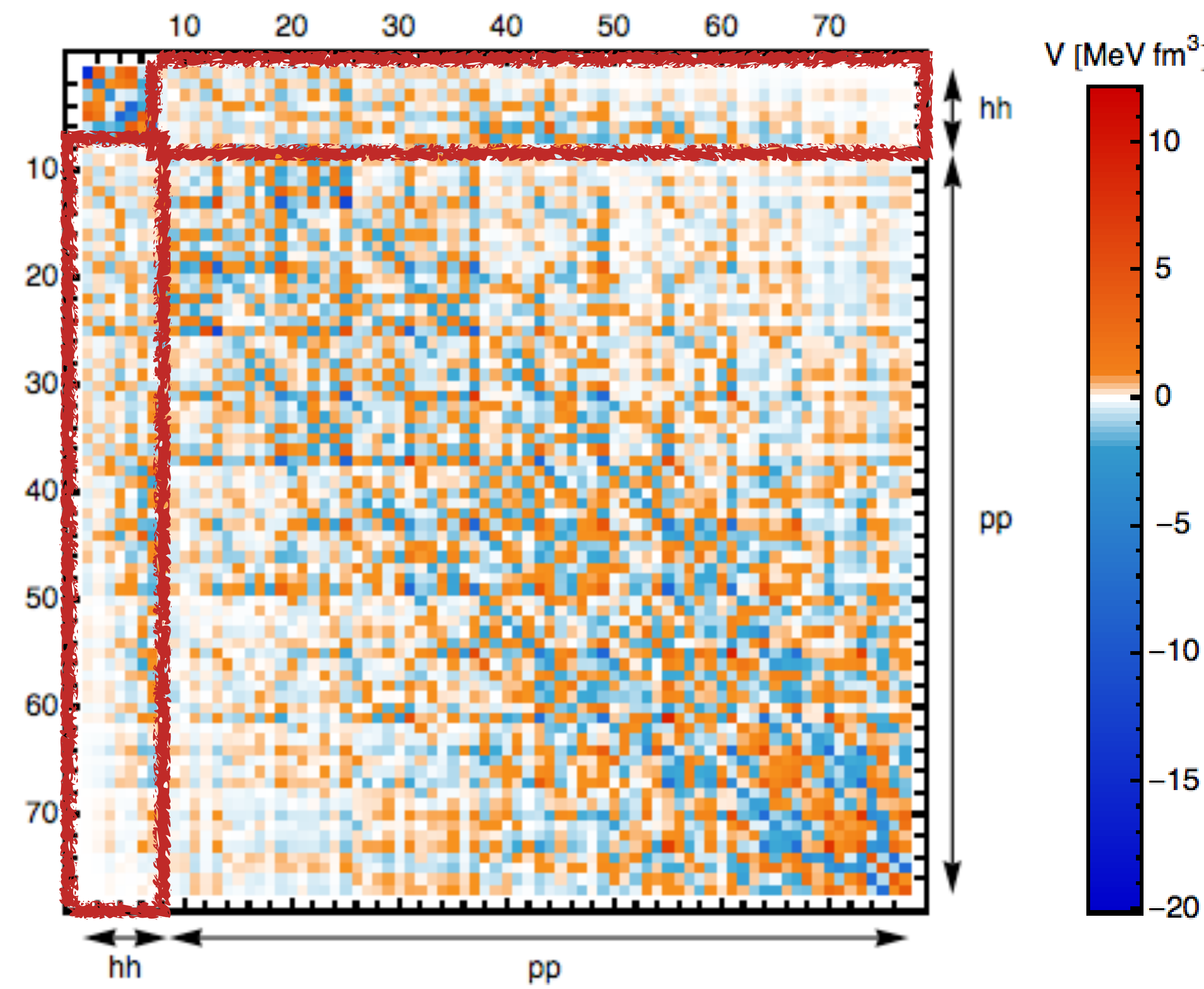
NERSC

ICER

Supplements

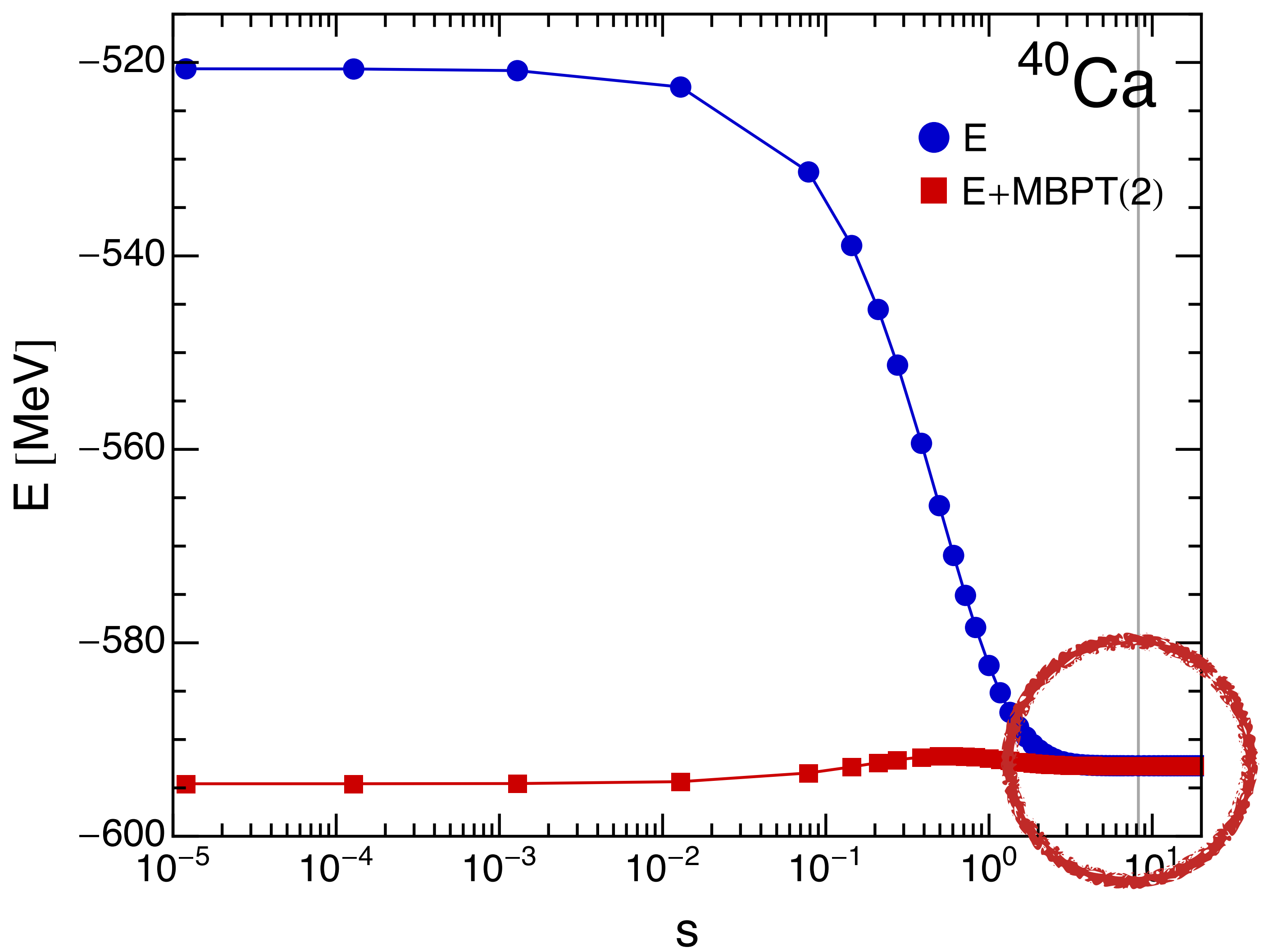
Decoupling

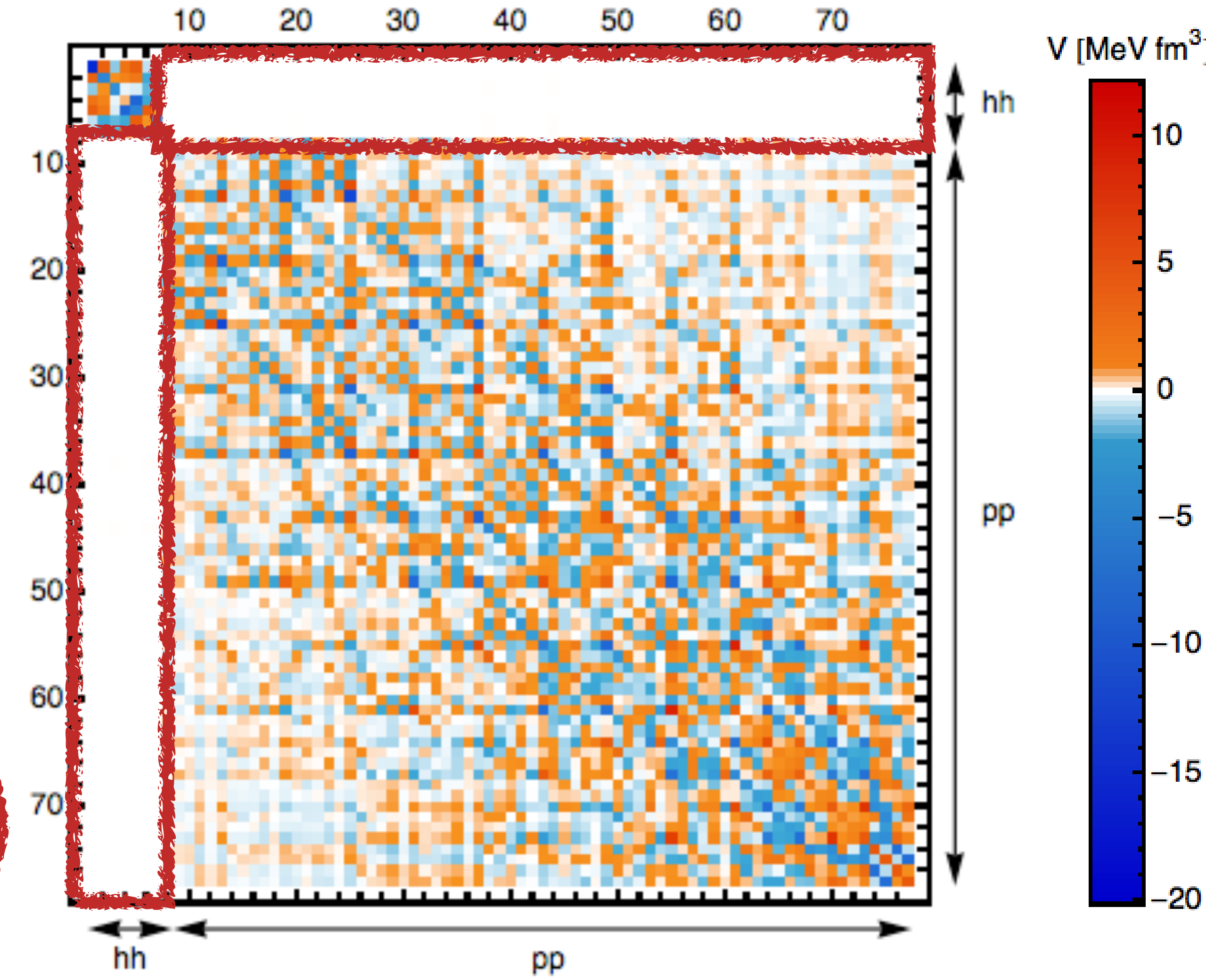




N3LO, $\lambda = 2.0\ \text{fm}^{-1}, e_{\text{Max}} = 8$

Decoupling



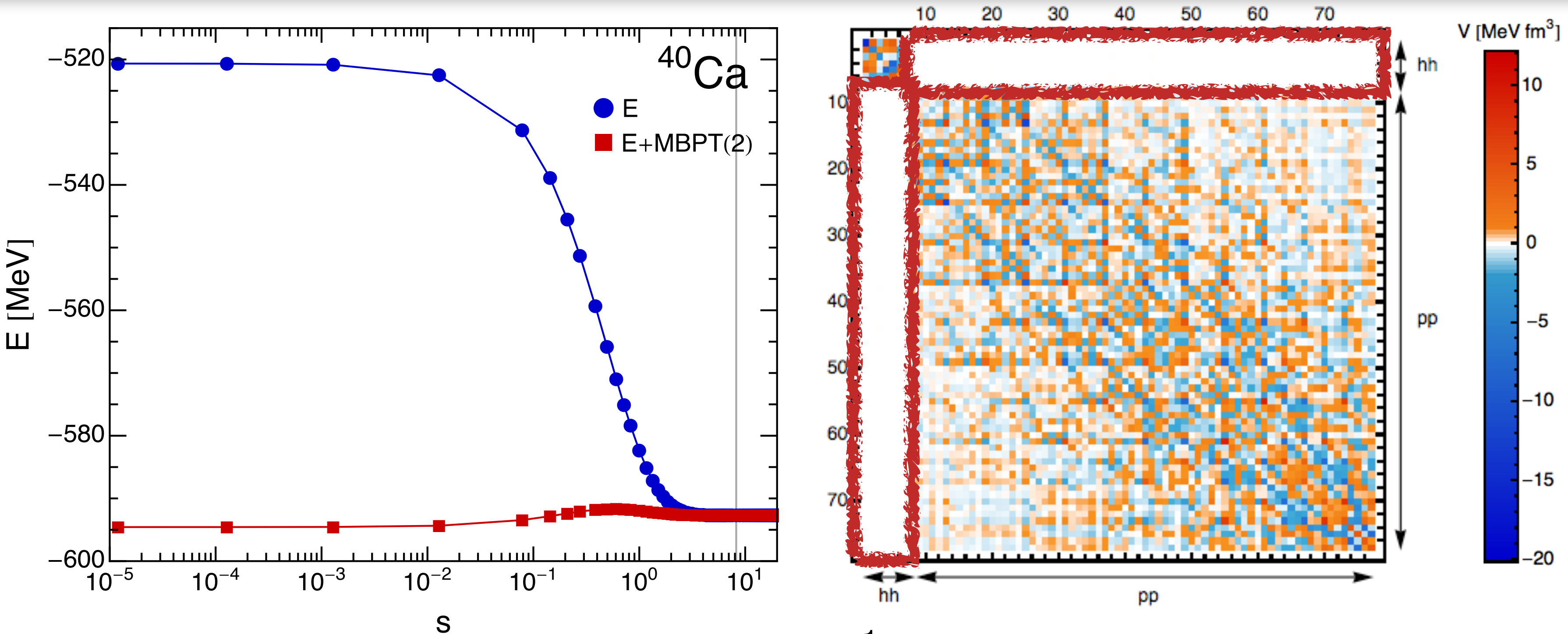


N3LO, $\lambda = 2.0\ \text{fm}^{-1}, e_{\text{Max}} = 8$

non-perturbative resummation of MBPT series **(correlations)**

off-diagonal couplings are rapidly driven to zero

Decoupling



N3LO, $\lambda = 2.0\ \mathrm{fm}^{-1}, e_{\mathrm{Max}} = 8$

- absorb correlations into **RG-improved Hamiltonian**

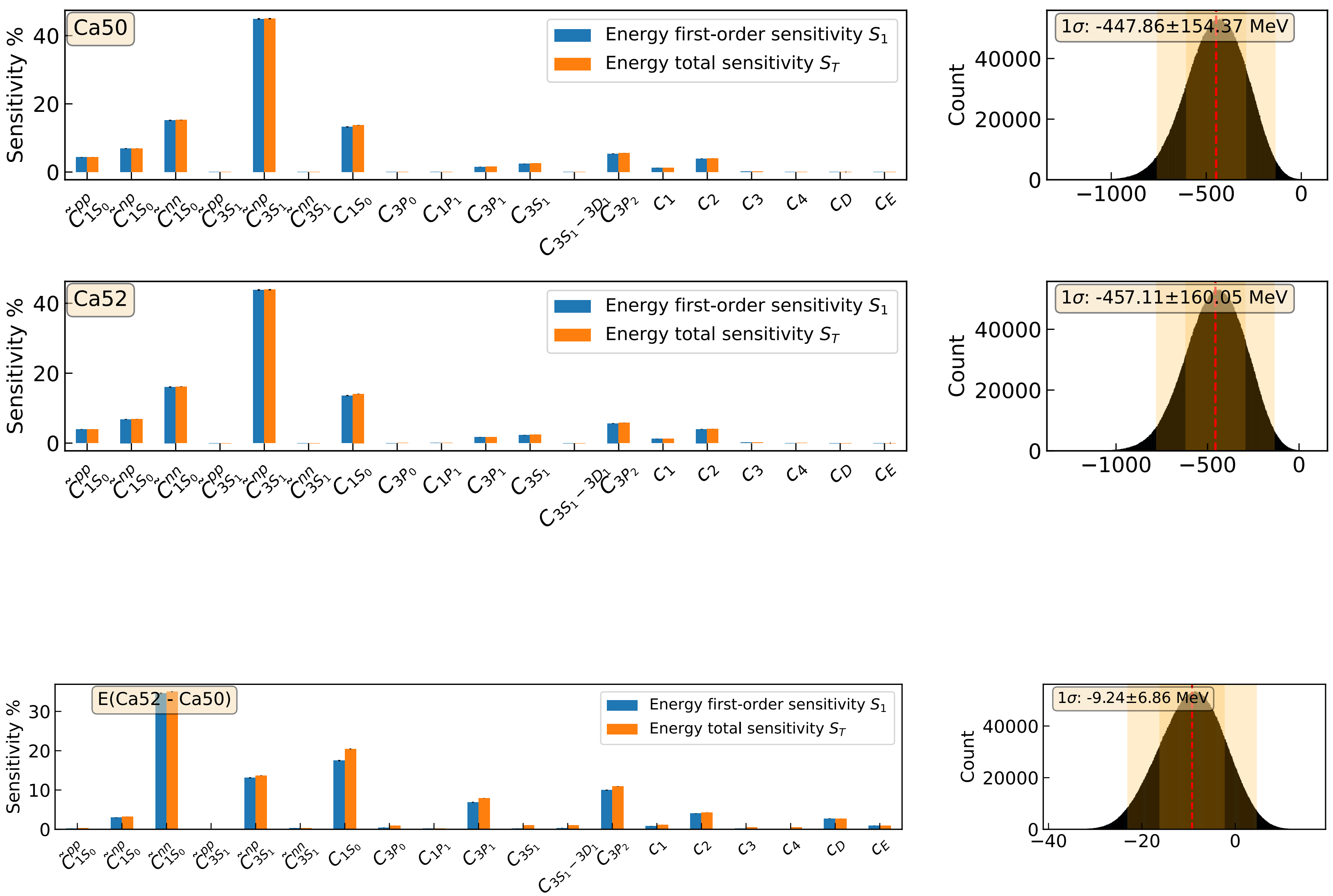
$$U(s) H U^\dagger(s) U(s) \left| \Psi_n \right\rangle = E_n U(s) \left| \Psi_n \right\rangle$$

- reference state is ansatz for transformed, **less correlated** eigenstate:

$$U(s) \left| \Psi_n \right\rangle \stackrel{!}{=} \left| \Phi \right\rangle$$

Emulators for the IMSRG

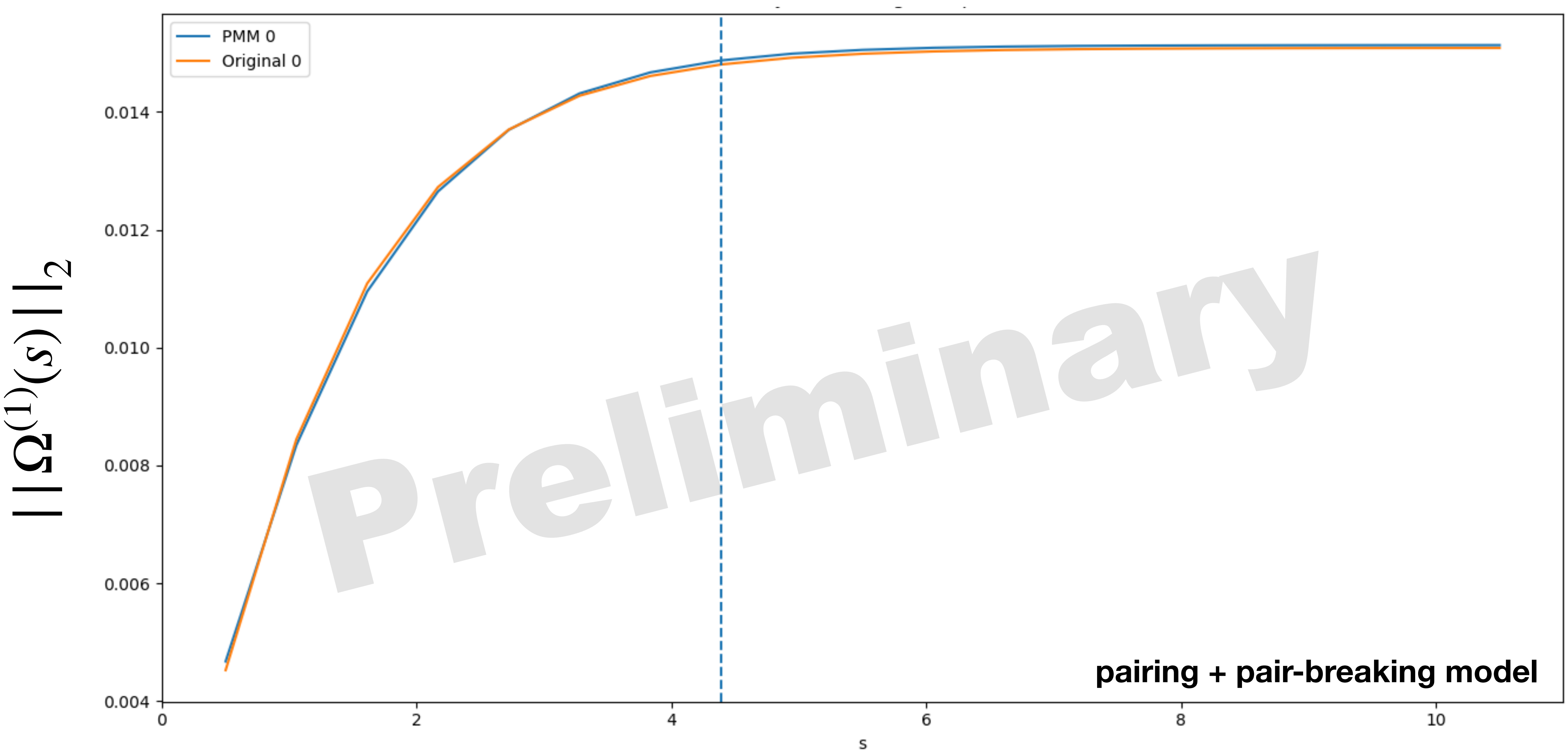
J. Davison, HH, J. Crawford, S. Bogner, arXiv:**2504**.xxxx



- non-invasive **ROM emulator** based on **Dynamic Mode Decomposition**
- ΔNNLO$_{GO}$, NN+3N, $e_{max} = 12, E_{3max} = 14$
- O(10M) samples - **computational effort reduced by 5+ orders of magnitude**

Parametric Matrix Model Emulators

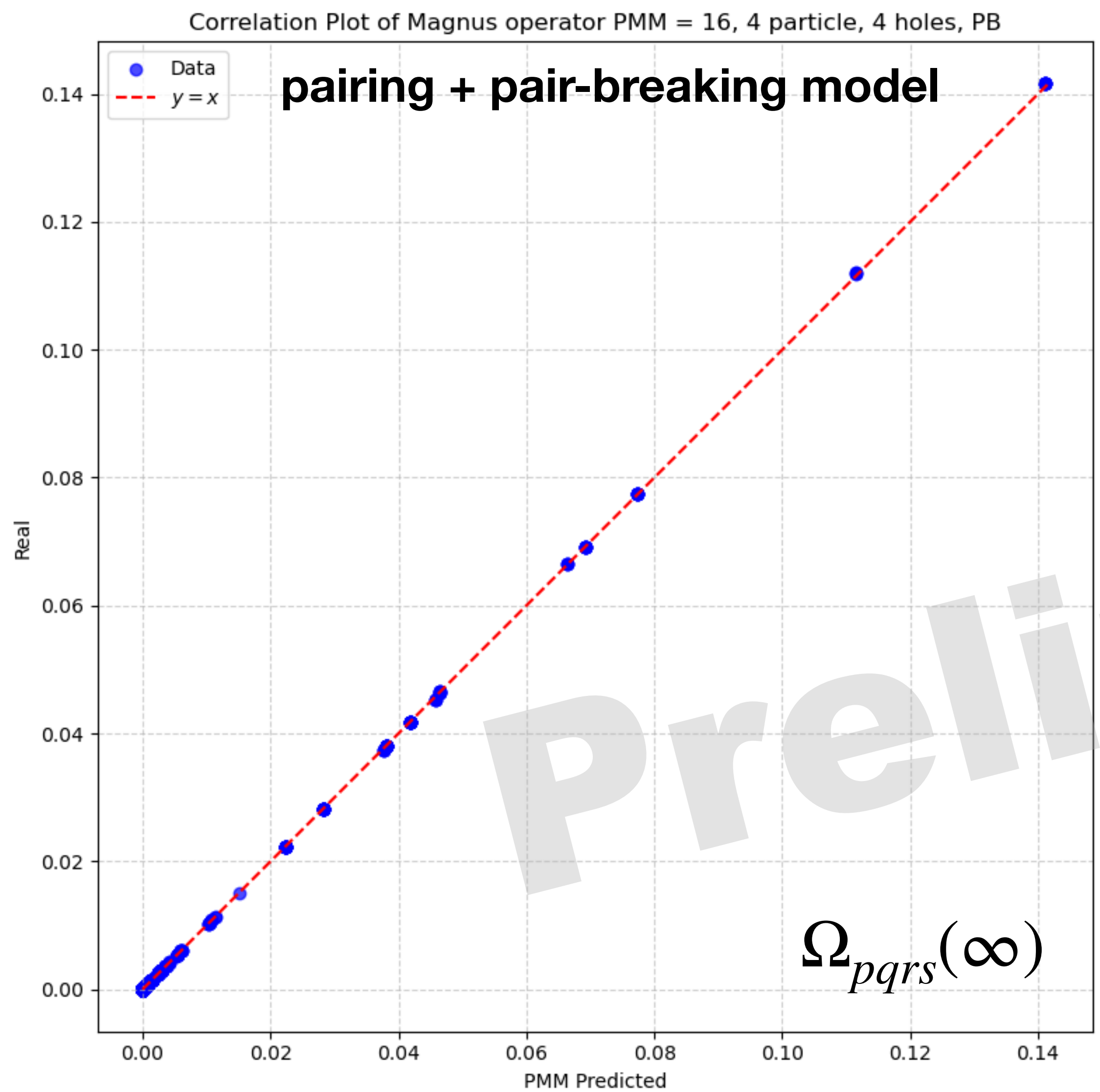
B. Clark, P. Cook, …
also see: S. Yoshida, Particles 2025, 8

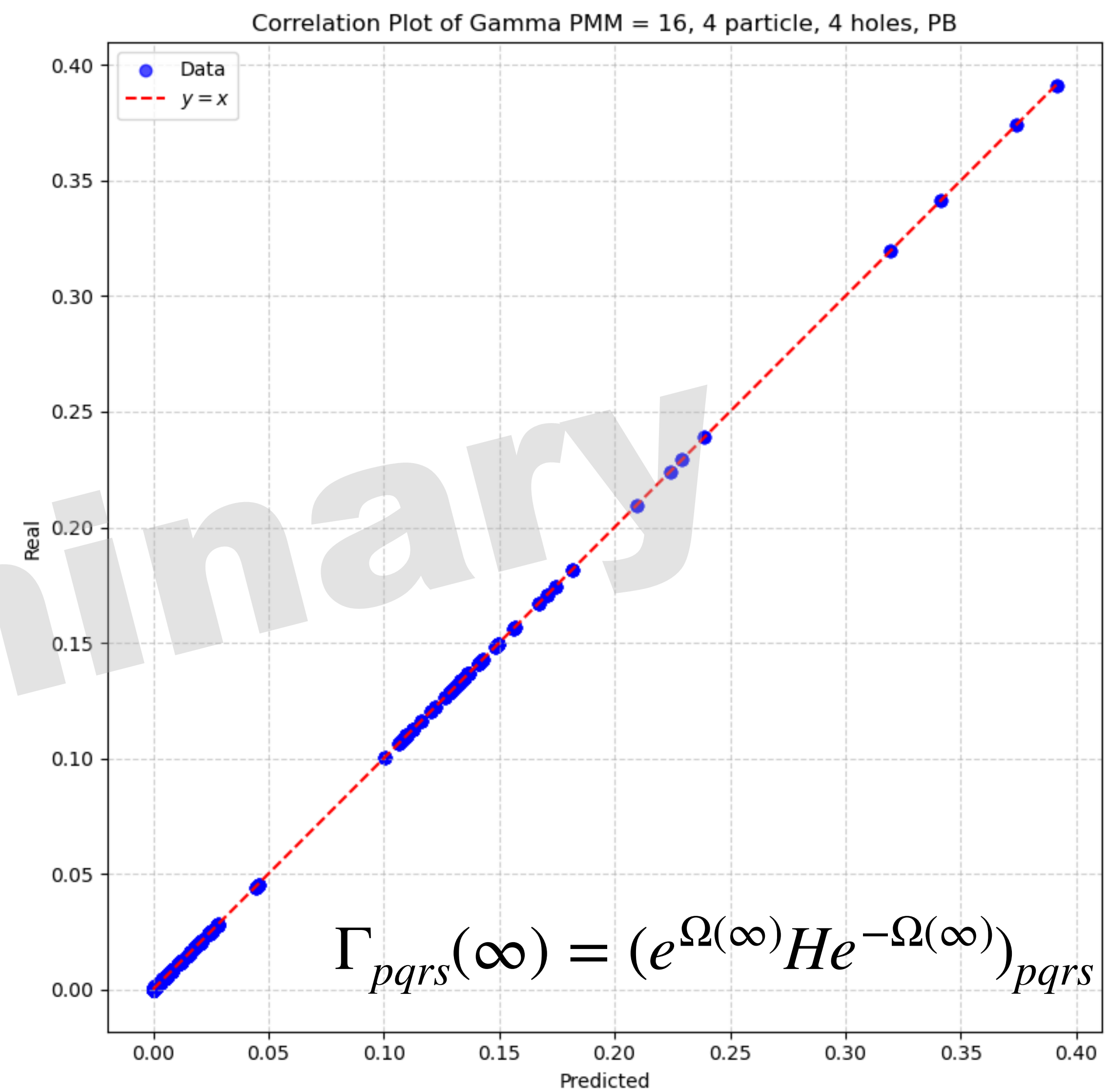


- DMD fails for Magnus operator if snapshots are taken during initial stages of flow…
- … but PMMs seem to work

Parametric Matrix Model Emulators

B. Clark, P. Cook, …
also see: S. Yoshida, Particles 2025, 8





- DMD fails for Magnus operator if snapshots are taken during initial stages of flow…
- … but PMMs seem to work