Weak Interactions in Neutron Star Mergers

Let's get rid of modified Urca

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Alford, Haber, Zhang arXiv:2406.13717
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Topic of this Talk

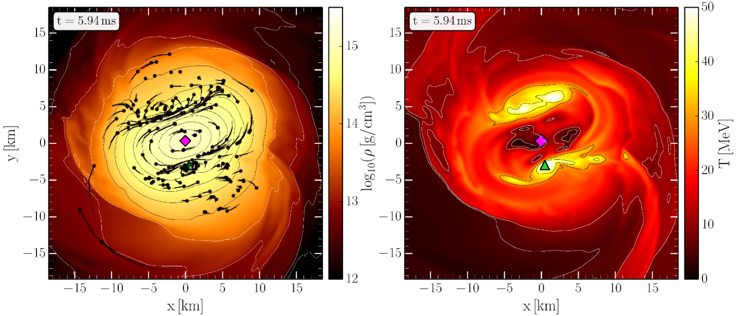
Question of the day:

Is there a consistent, systematically improvable way to calculate weak (nuclear) decay rates in dense matter?

Answer: Yes, we should:

- Forget about "modified" and "direct" Urca
- Take in-medium collisonal broadening for **all** participating nucleons into account

Neutron Star Mergers



Hanauske, M.; Steinheimer, J. et al. Particles 2019

- Mergers test properties of dense matter at high densities (up to $\approx 4 - 7\ n_{\mathrm{sat}}$) and high temperatures (up to $T \approx 60 - 80$ MeV)
- If we want to use mergers to learn about nuclear matter, we need to include all the relevant physics in our simulations.

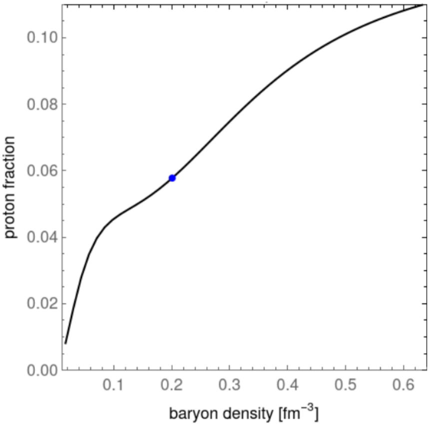
What does the weak interaction do for us?

Chemical Equilibrium

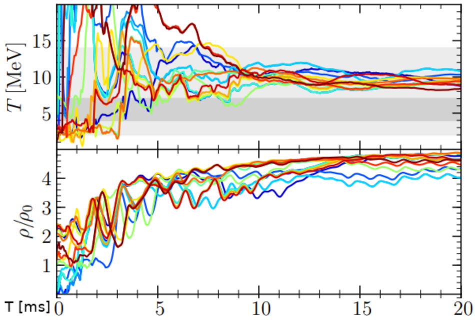
Chemical equilibrium: neutron decay and electron capture balance

$$n + \cdots \rightarrow p + e^- + \ldots \qquad p + e^- + \cdots \rightarrow n + \cdots$$

- Only weak interactions can change particle content
- In equilibrium: rates balance
- Cold equilibrium: $\mu_n = \mu_p + \mu_e$
- finite T correction:
  Alford, Harris: 1803.00662,
  Zhang et al: 2108.03324 , 2306.06180



Neutron Star Mergers



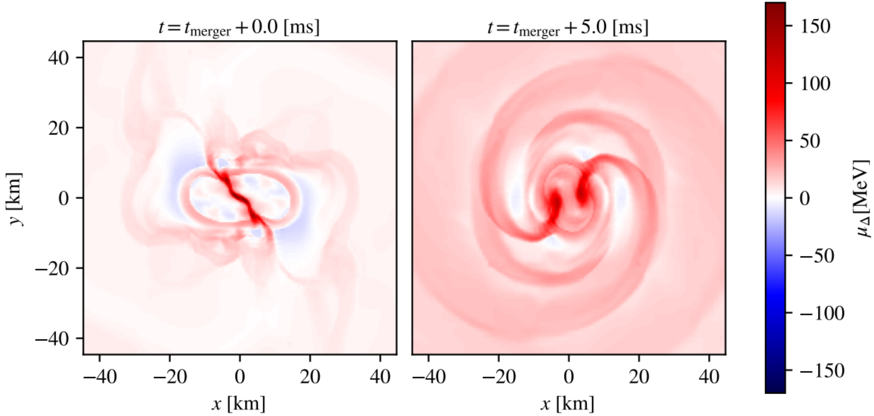
Alford, Bovard et.al., PRL 120 (2018)

- Oscillations drive matter out of equilibrium
- Weak interactions try to drive matter back to equilibrium
- impact on mergers depends on timescale of oscillations and equilibration times

Ignore Equilibration in Simulations: Frozen Composition

Hammond, Hawke, Andersson: 2108.08649

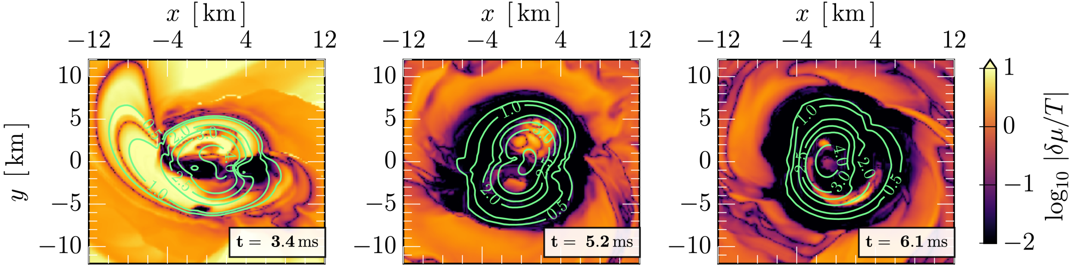
$$\mu_\Delta = \mu_n - (\mu_p + \mu_e)$$



Include Equilibration in Simulations

Most, A.H., Harris, Zhang, Alford, Noronha; arXiv:2207.00442

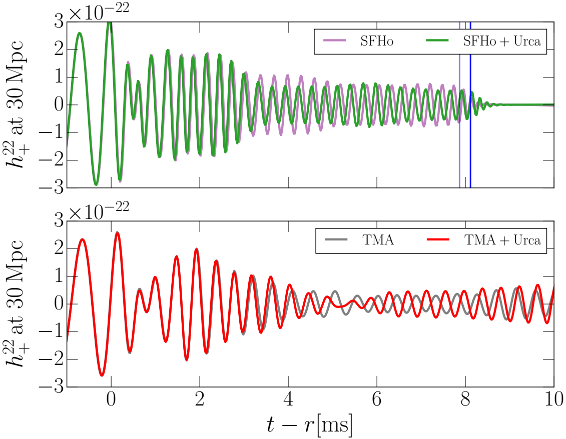
$$\delta\mu = \mu_n - (\mu_p + \mu_e)$$



Include Equilibration in Simulations

Most, A.H., Harris, Zhang, Alford, Noronha; arXiv:2207.00442

Gravitational Wave Signal



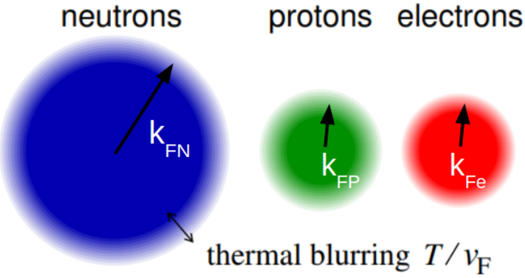
Difference same order as finite T, resolution effects, **uncertainty in EOS**, ...

Urca processes

direct Urca (dU)

neutron decay: $n \to p + e^- + \bar{\nu}_e$ electron capture: $p + e^- \to n + \nu_e$

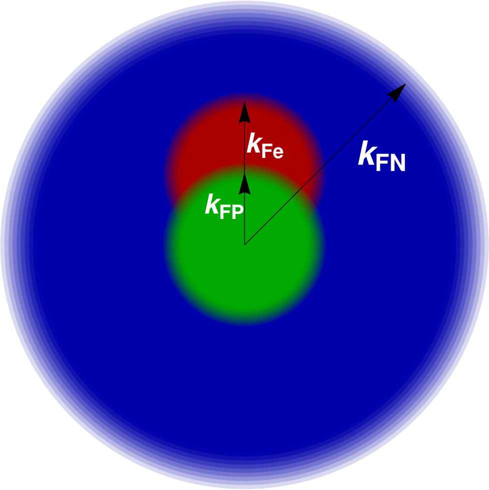
- Strongly degenerate npe-matter:
  dominated by particles on their Fermi surface (FS)



$$\Gamma_{dU,ND} \propto \prod_{i=1}^{4} \int \frac{d^3 p_i}{2E_i} \sum_{\text{spins}} |M|^2 \delta^4(E-p) \times f_n(1-f_p)(1-f_e)$$

Direct Urca Threshold

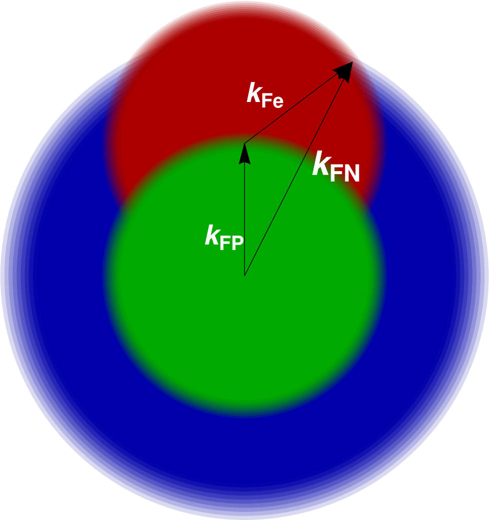
Below threshold: proton fraction too low



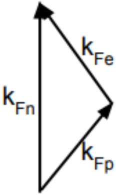
- Momentum conservation on FS demands $\vec{k}_{Fn} \leq \vec{k}_{Fp} + \vec{k}_{Fe}$
- If momentum cons. on FS not possible: rate heavily suppressed

Direct Urca Threshold

Above threshold: proton fraction $\geq 11\%$

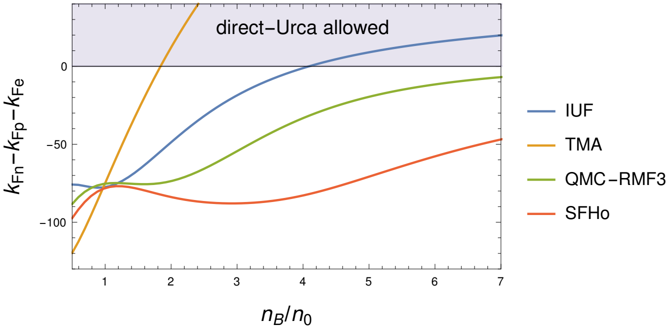


- Momentum conservation on FS demands $\vec{k}_{Fn} \leq \vec{k}_{Fp} + \vec{k}_{Fe}$
- If momentum cons. on FS possible: rate dominated by direct Urca



Direct Urca Threshold

strongly EOS depended



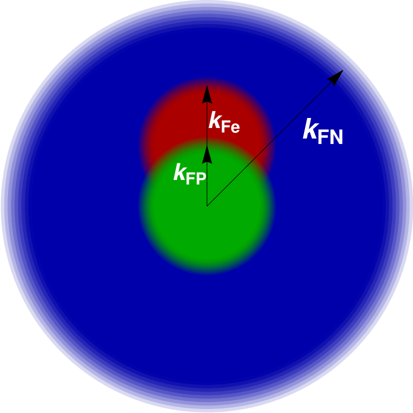
- Momentum conservation on FS demands $\vec{k}_{Fn} \leq \vec{k}_{Fp} + \vec{k}_{Fe}$
- Proton fraction x_p is monotonic with density
- Need $x_p \approx 11\%$ for $k_{Fn} = k_{Fp} + k_{Fe}$
- Threshold density = direct Urca threshold
- Impact on cooling, bulk viscosity, ...

Below threshold: in-medium corrections important

modified Urca

modified Urca (mU): dU with spectator

neutron decay: $n + N \rightarrow p + e^- + \bar{\nu}_e + N$ electron capture $p + e^- + N \rightarrow n + \nu_e + N$

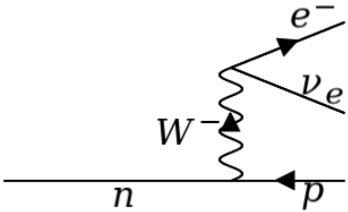


$$\Gamma_{mU,ND} \propto \prod_{i=1}^{6} \int \frac{d^3 p_i}{2E_i} \sum_{\text{spins}} |M|^2_{mU} \delta^4(E-p) \times$$
$$f_n f_N (1-f_p)(1-f_e)(1-f_N)$$

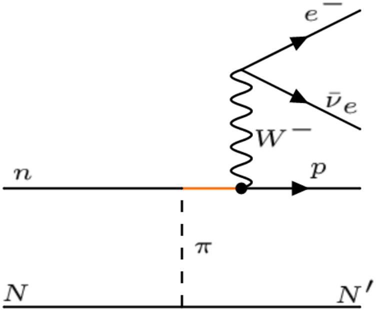
Direct Urca and Modified Urca Matrix Element

Approximations for internal propagator

direct Urca



modified Urca

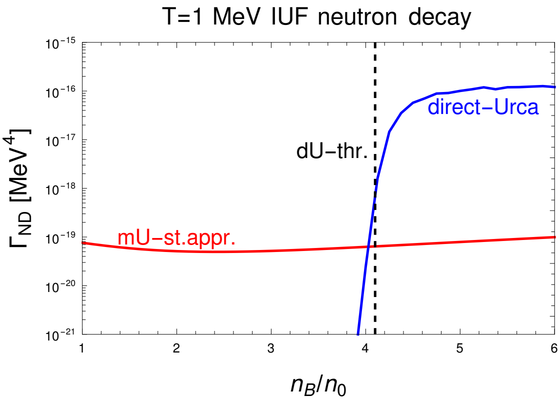


Propgator for off-shell nucleon

How to deal with propagator G_n for internal, off-shell nucleon?

Direct Urca and Modified Urca

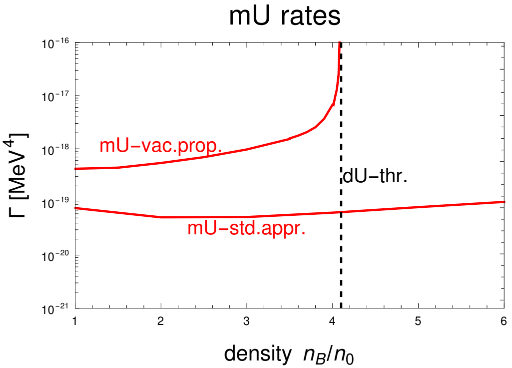
$T = 1$ MeV - neutrino transparent, IUF-EOS



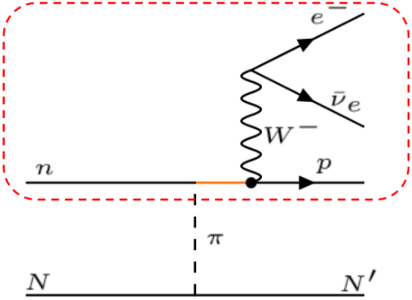
- standard approximation for mU: $G_n = 1/\mu_e$
- Full phase space calculation for direct Urca: **arXiv:2306.06180, arXiv:2108.03324**

Improved modified Urca is worse?

Divergent rate in Shternin et al. 2018



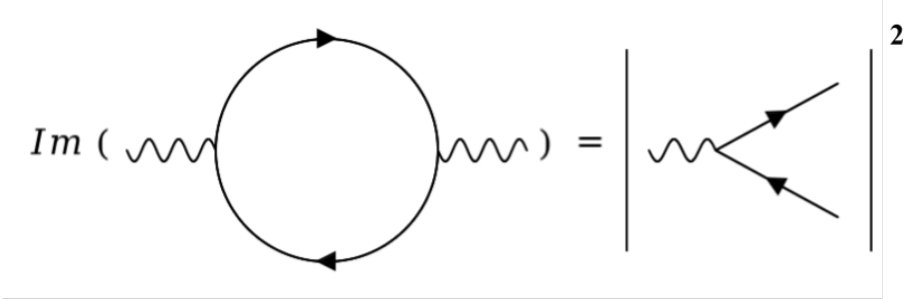
- ✓ Improved treatment: $G_n^{-1} \propto \left(E^2 - \varepsilon_N^2(k)\right)$
- ✗ Divergence at dU threshold
- ✗ Internal nucleon goes on shell!



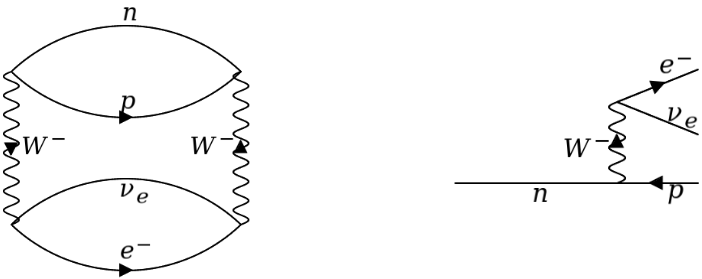
Cutkosky Cutting Rules

QFT version of optical theorem

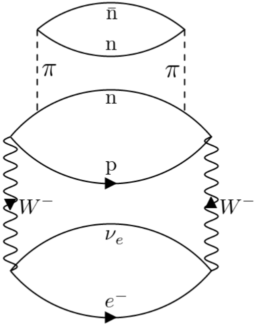
$$\mathrm{Im}(\text{diagram}) \propto \text{decay rate}$$

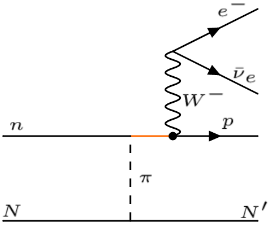


direct Urca from bubble diagram



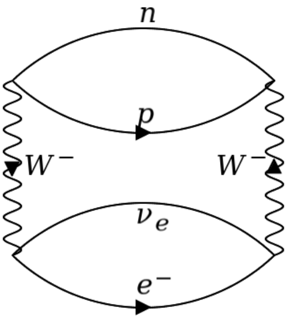
modified Urca from bubble diagram



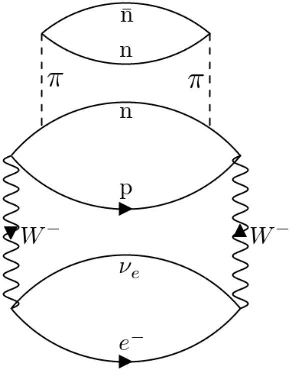


Total Rate

$\Gamma_\nu =$



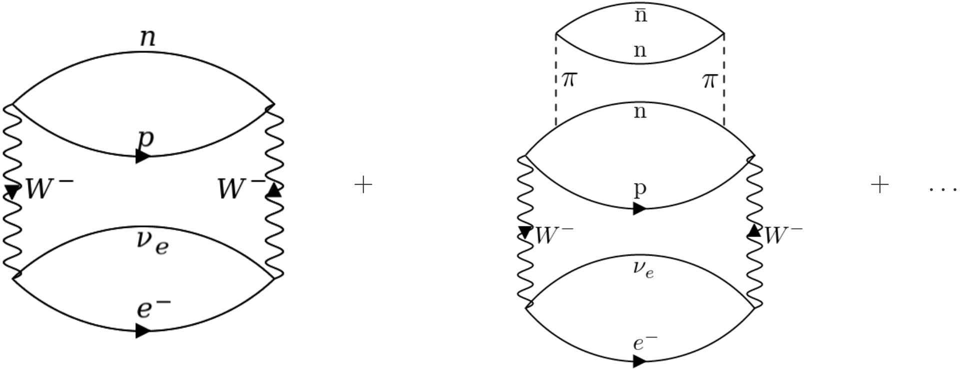
$+$



$+ \cdots$

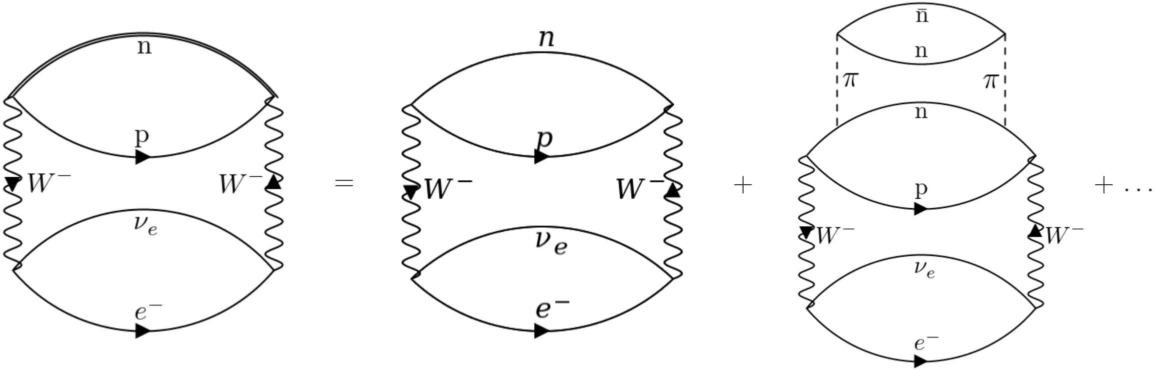
Dyson Schwinger Equation

$\Gamma_\nu =$



Nucleon Width Approximation NWA

Alford, Haber, Zhang, 2406.13717



Cure of the Divergence

- Fixed the IR divergence by including **nucleon width** $W \propto \Sigma$
- Corresponds to summing over an arbitrary number of collisions with the medium
- Collisional broadening taken into account for for all baryons

Simplicity

mass-spectral decomposition

$$G_a^{\mathrm{NWA}}(k, M_a^*, W_a) = \int_{-\infty}^{\infty} dm\, G_a^{\mathrm{mf}}(k, m)\, R_a(m)\ ,$$

with the Breit-Wigner spectral function

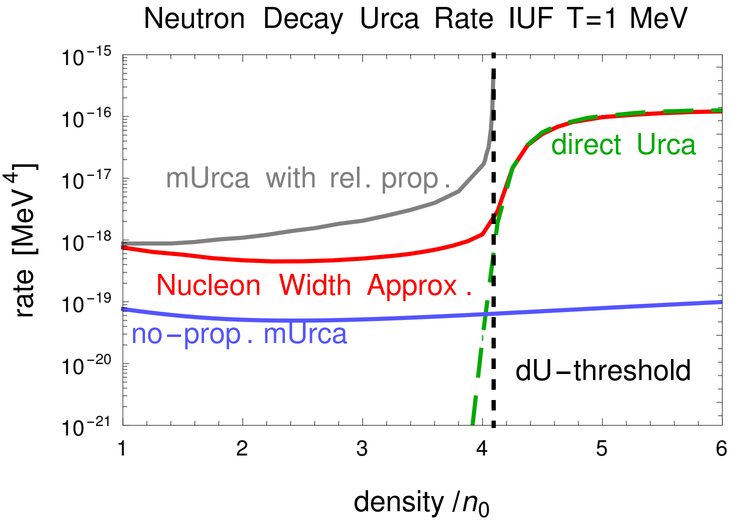
$$R_a(m) \equiv \frac{1}{\pi} \frac{W_a/2}{(m - M_a^*)^2 + W_a^2/4}\ .$$

Kuksa, 1408.6994

Nucleon Width Approximation - NWA

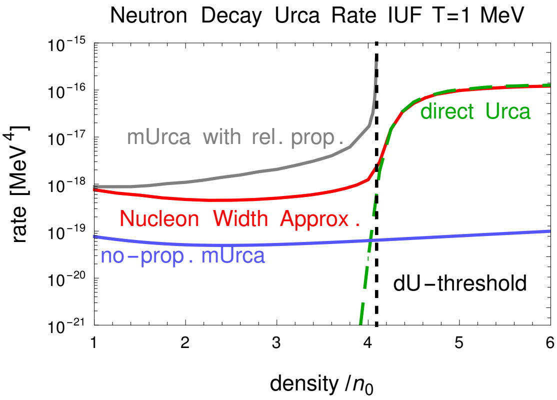
$$\Gamma^{\mathrm{NWA}} = \int_{-\infty}^{\infty} dm_{\mathrm{n}} dm_{\mathrm{p}} \Gamma^{\mathrm{dUrca}}(m_{\mathrm{n}}, m_{\mathrm{p}})\, R_{\mathrm{n}}(m_{\mathrm{n}}) R_{\mathrm{p}}(m_{\mathrm{p}})\ .$$

NWA rates


Neutron Decay Urca Rate IUF T=1 MeV
rate [MeV^4]
density /n_0
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
1
2
3
4
5
6
mUrca with rel. prop.
direct Urca
Nucleon Width Approx.
no-prop. mUrca
dU-threshold

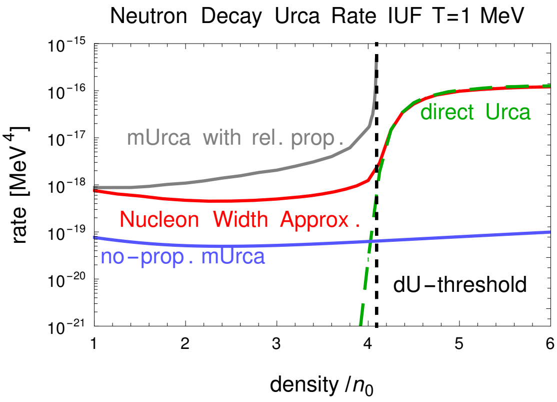
NWA - results

- smooth transition from "mU-regime" to "dU-regime"
- constant nucleon width ($W_n = W_p = T^2/(5\,\mathrm{MeV})$ obtained from Brueckner theory calculation for pure neutron matter using the Paris NN potential by Sedrakian astro-ph/0002228
- Allows us to go beyond Fermi-surface for mU
- Enhancement of low density rate by order of magnitude (see Shternin et al.)
- No divergence
- Matches dU calculation above threshold



NWA - advantages

- Consistent, simple approach to Urca
- Systematically improvable scheme
- Allows us to go beyond Fermi-surface for mU
- NWA can be applied in any context where dUrca rates can be calculated
    - finite temperature
    - matter with non-equilibrium neutrino distributions
    - strong magnetic fields
    - decays in some models of dark matter, hyperonic matter, quark matter . . .



Summary & Outlook

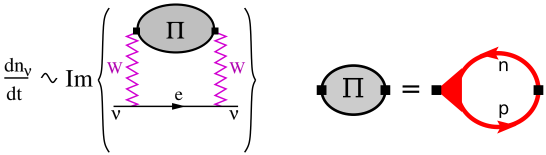
- There is room for improvement concerning the microphsyics in merger simulations
- Missing or inaccurate microphysics can hamper our aspirations to measure the EOS
- mU is crude, inconsistent, hard to improve, and wrong
- NWA is easy to implement alternative that gives more accurate results

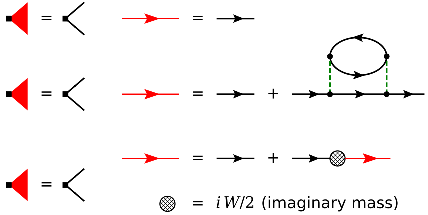
N.W.A

Outlook

- Calculate neutron width using different models
- Include vertex corrections
- Implement NWA rates/opacities in merger simulations

NWA - dU - mU framework



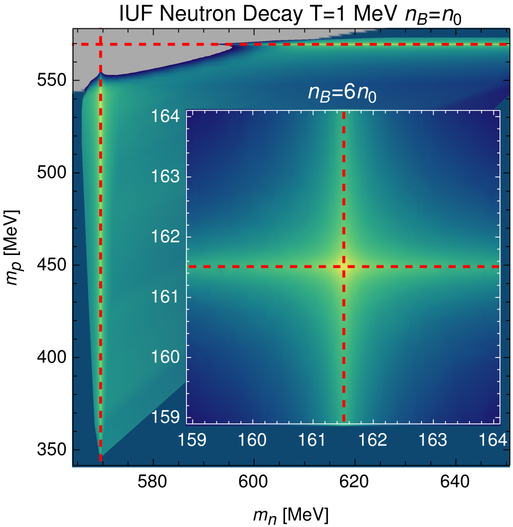

dUrca

dUrca+mUrca
(squared terms)

nucleon width

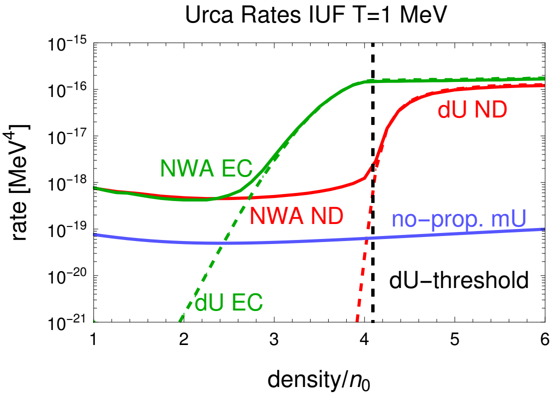
$iW/2$ (imaginary mass)

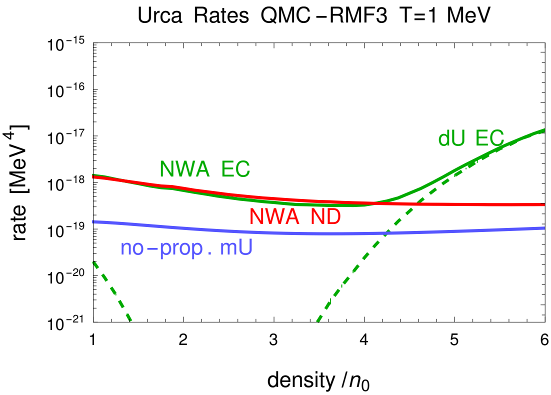
Mass Integral



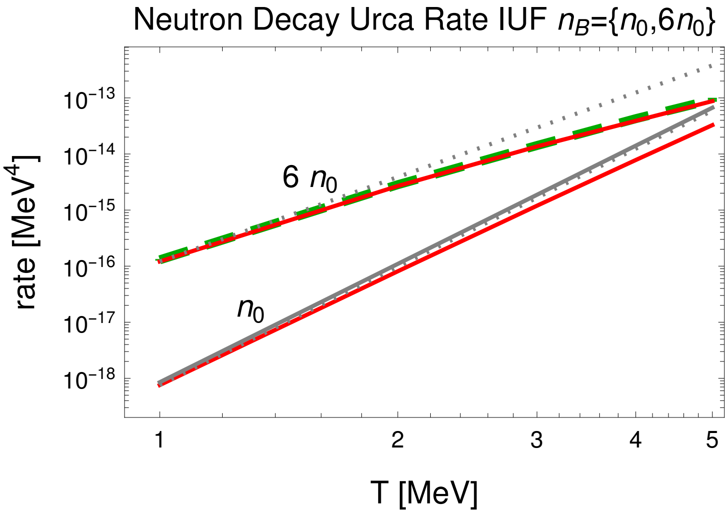
NWA results

QMC-RMF3 Alford, Brodie, Haber, Tews 2205.10283





NWA temperature dependence

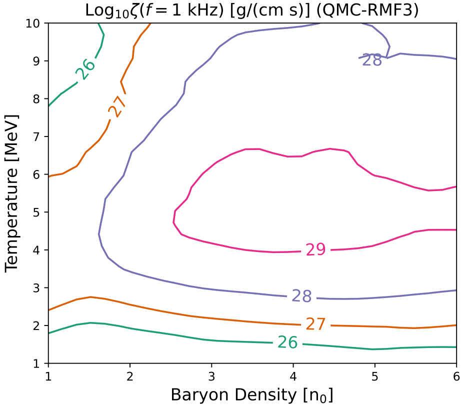


Bulk Viscosity: Resonant Behavior

Alford, Haber, Zhang, 2306.06180

$$\zeta \propto \gamma/(\omega^2 + \gamma^2)$$

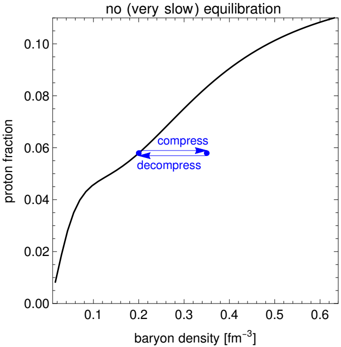
- $\gamma = 1/\tau \propto$ (ND-rates - EC-rates)
- γ strong T and n_B dependence
- Resonant behavior: $\gamma \approx \omega$
- QMC-RMF models: Alford, Brodie, Haber, Tews: 2205.10283 (Compose)



Bulk Viscosity from Beta Equilibration

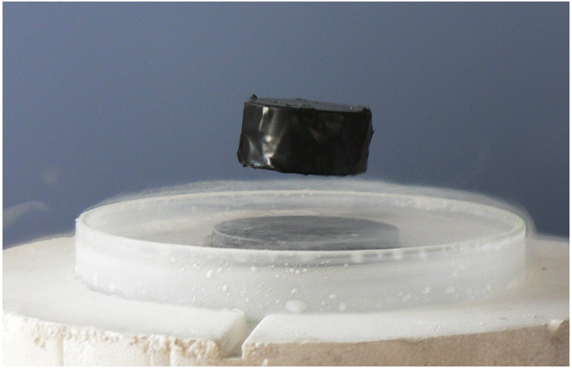
Path of fluid element as it is compressed and decompressed

Density Oscillations in merger drive matter out of equilibrium



Why Transport?

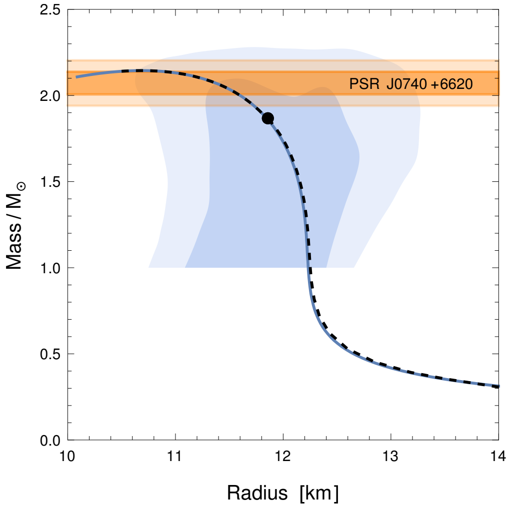
Better discriminator of different phases than EOS



What would you measure?

Masquerade Problem

We need to go **beyond the Equation of State**



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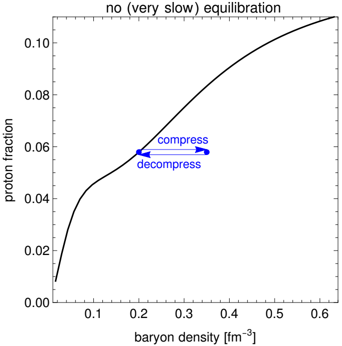

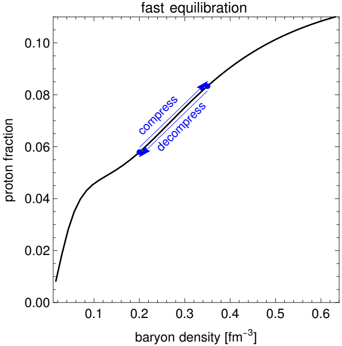

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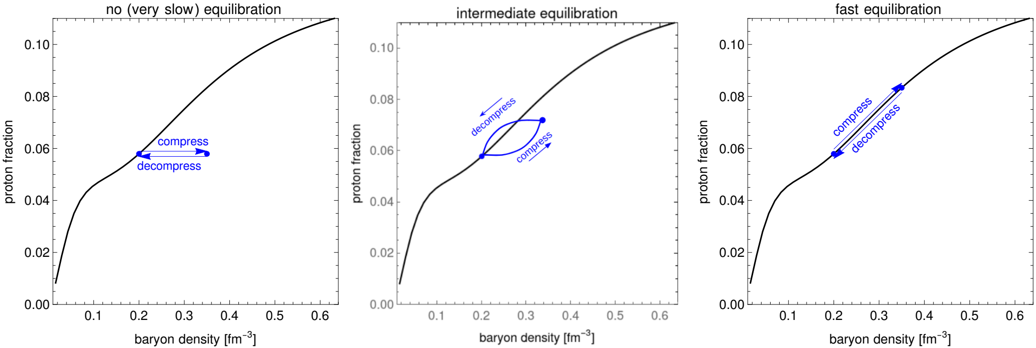
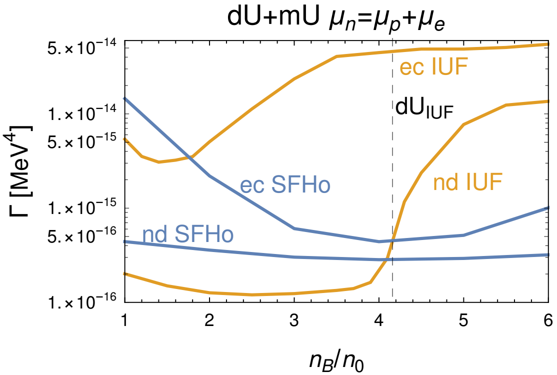
Density Oscillations in merger drive matter out of equilibrium



$\{x_P, n_B\}$ plane equiv. to $\{P, V\}$ plane → traversing a path in P-V plane leads to $\int PdV$ - **work**

Total Urca in Cold Beta-Equilibrium

$T = 3\,\mathrm{MeV}$ - neutrino transparent



- IUF-results show clear dU threshold
- Electron-capture and neutron-decay differ by 1 – 2 **orders of magnitude**
- Cold beta-equlibrium clearly violated

Reason:

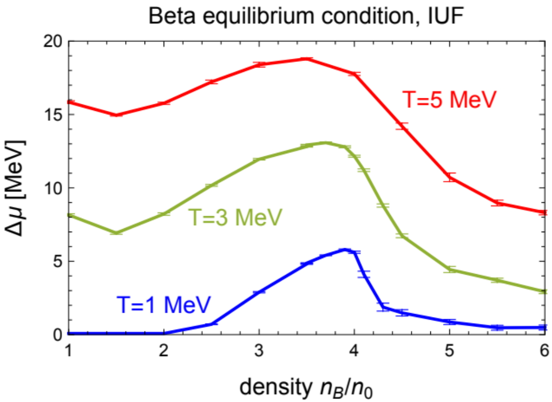
electron-capture and neutron-decay are **not** inverse processes: neutrino switches side

Warm Beta Equilibrium

Alford, Harris PRC 98 (2018), **Alford, A.H., Harris, Zhang, arXiv:2108.03324**

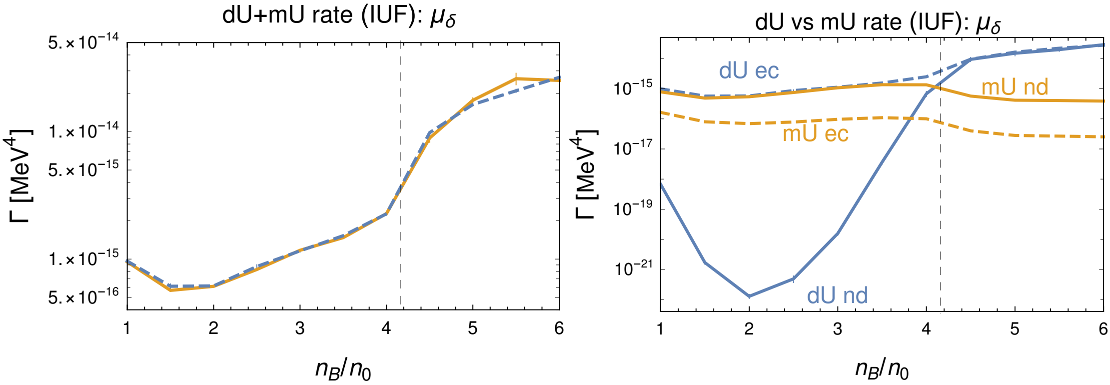
Warm Beta Equilibrium

$$\mu_n = \mu_p + \mu_e + \Delta\mu(n_B) \text{ where } \Delta\mu(n_B) \text{ is chosen s.t. } \Gamma_{nd} = \Gamma_{ec}$$



Corrected Rates

for IUF EOS at $T = 3\,\text{MeV}$

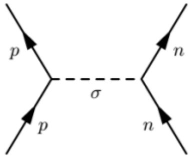


direct Urca electron capture dominates over modified Urca

New Model for Nuclear Matter

Alford, Brodie, A.H., Tews **Phys.Rev.C 106 (2022) arXiv:2205.10283**

Relativistic mean field theories:



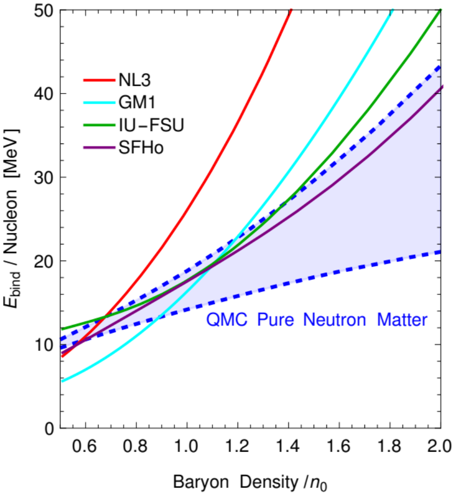
Based on meson-exchange Lagrangians:
nucleons interact via meson exchange

- ✓ Applicable to density/temperature range of NS mergers
- ✓ Fully relativistic model → always causal
- ✓ Provide microscopical model: dispersion relations, . . .
- ✓ Solvable on my laptop
- ✗ Mean field approx. not a controlled approximation (better at high densities)

Coupling constants: fit to saturation properties of (nearly) symmetric nuclear matter

Neutron stars are ≈ 90% neutrons!

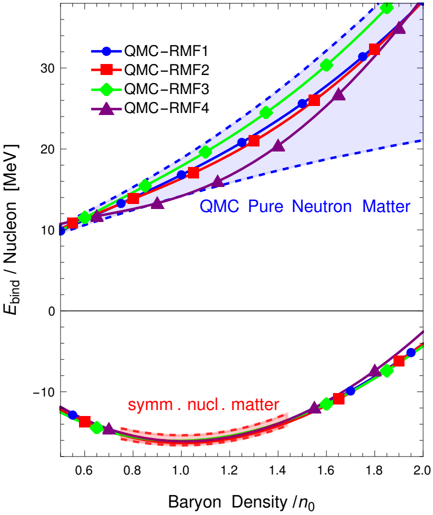
Common RMFs


50
40
30
20
10
0
E_{bind} / Nucleon [MeV]
NL3
GM1
IU−FSU
SFHo
QMC Pure Neutron Matter
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Baryon Density / n_0

QMC-RMFx EOS

Four Models

- Four different models: QMC-RMFx
- From soft to stiff
- Pressure = slope



QMC-RMFx EOS

Properties for Symmetric Nuclear Matter

Name	n_{sat} $[\mathrm{fm}^{-3}]$	$\mathcal{E}(n_{\mathrm{sat}})$ [MeV]	$\kappa(n_{\mathrm{sat}})$ [MeV]	$J(n_{\mathrm{sat}})$ [MeV]	$L(n_{\mathrm{sat}})$ [MeV]
Exp.				31.6 ± 3.2	58.7 ± 28.1
QMC-RMF1	0.159	-16.03	258	32.8	44.4
QMC-RMF2	0.160	-16.03	258	32.6	40.4
QMC-RMF3	0.158	-15.99	229	33.7	49.2
QMC-RMF4	0.162	-16.05	275	30.4	31.2

Adding a Crust

The best of two worlds

- Homogeneous, neutron-rich part: QMC-RMF
- Low-density crust and close to iso-spin symmetric: HS(IUF) Hempel, Schaffner-Bielich, Nuc.Phys. A 837 (2010), Fattoyev, Horowitz, Piekarewicz and Shen, PRC 82

combined in a thermodynamically consistent way to create **tabulated EOS**
as function of n_B, T, and x_P

Full 3D-table available on Compose now!
2304.07836

QMC-RMFx EOS II

Mass-Radius Curves

- Within 2σ of PSR J0740+6620: $M = 2.072 \pm 0.066\, M_\odot$
- consistent with NICER $R_{1.34} = 12.71 \pm 1.84$ km
- consistent with NICER+XMM+multi messanger constraints from P. T. H. Pang, I. Tews, M. W. Coughlin, M. Bulla, C. Van Den Broeck, and T. Dietrich, Astrophys. J. 922, 14 (2021)
- no direct Urca threshold