

Opening up baryon-number-violating operators

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INT Workshop on *Baryon Number Violation*

1/16/2025



Standard Model Effective Field Theory

- EFT (with Majorana neutrinos): [Weinberg, '79 & '80]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{\text{LLHH}}{\Lambda} + \sum_j \frac{\mathcal{O}_j}{\Lambda^2} + \sum_j \frac{\mathcal{O}'_j}{\Lambda^3} + \sum_j \frac{\mathcal{O}''_j}{\Lambda^4} + \dots$$

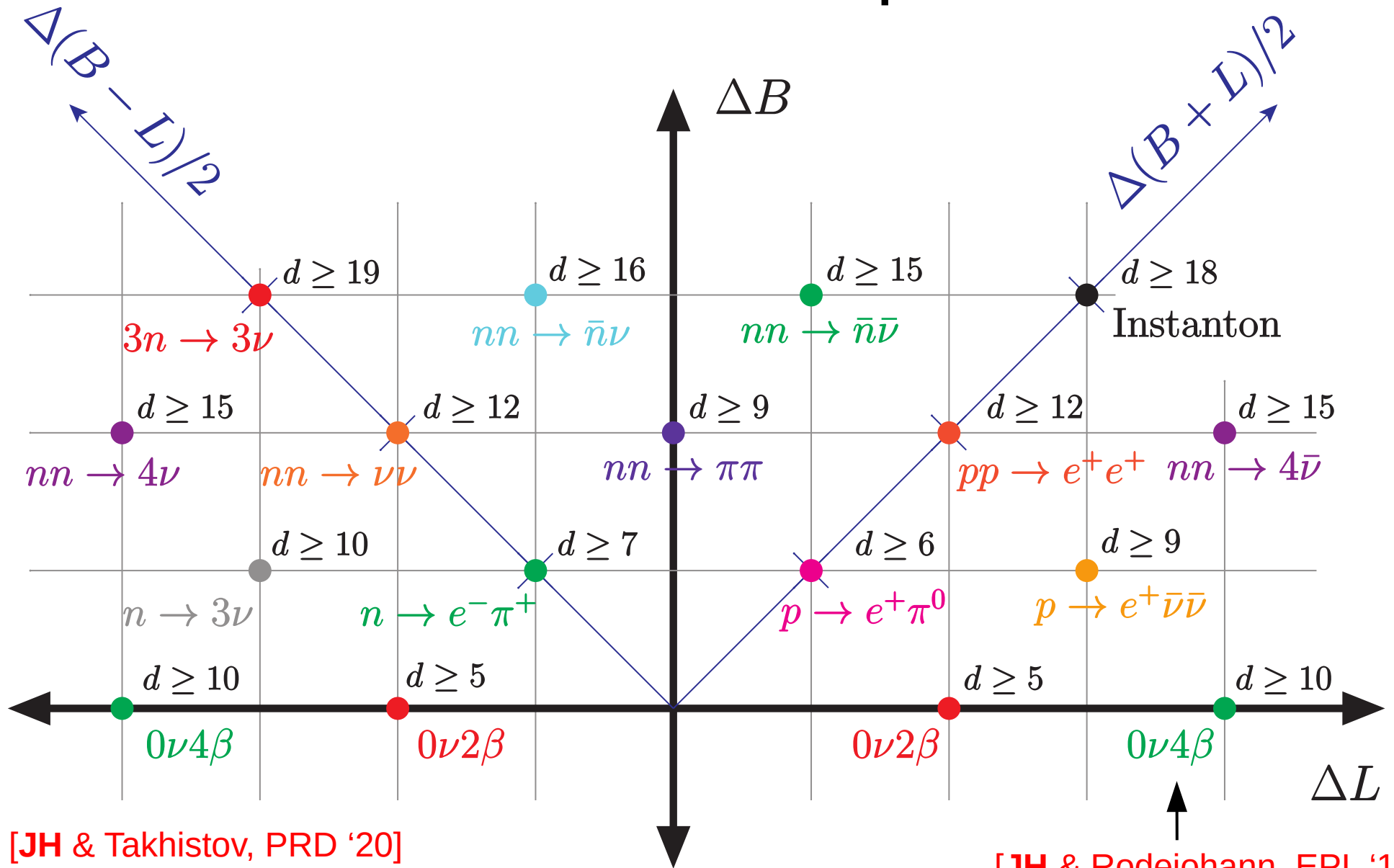
$\Delta L = 2$ $\Delta B = \Delta L = 1$ $\Delta B = -\Delta L = 1$

- Some symmetry/hierarchy **has to exist**, otherwise

$$\Lambda \sim \langle H \rangle^2 / M_\nu \sim 10^{14} \text{ GeV} \longrightarrow \text{Fast proton decay!}$$

- BNV sensitive to $d \gg 6$, unlike any other experiment.
- ΔB dominated by $d = 6$, unless forbidden by **symmetry!**
[Weinberg, '80]

BNV landscape



[JH & Takhistov, PRD '20]

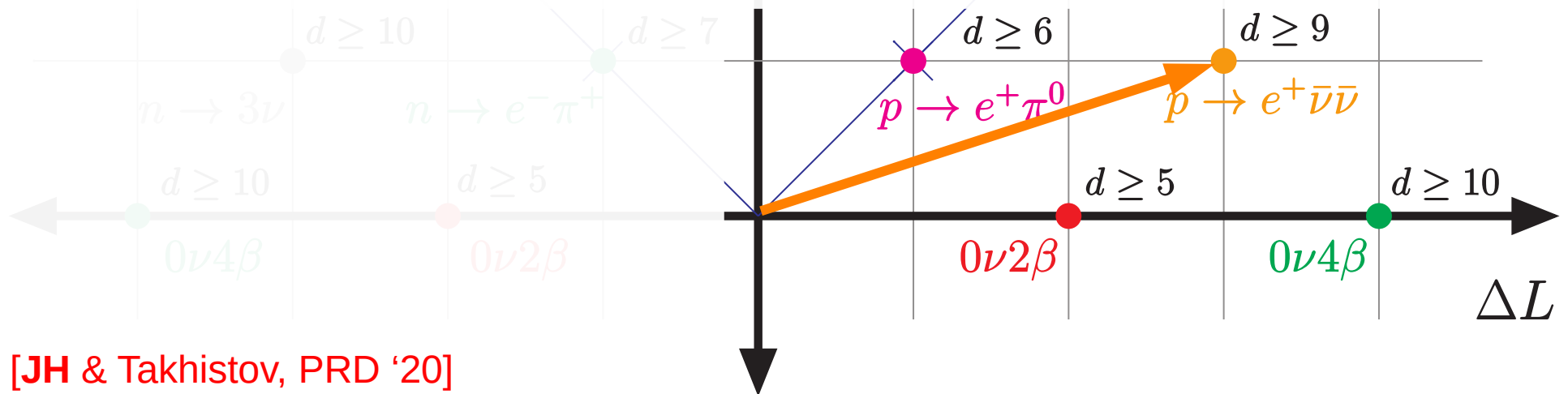
[JH & Rodejohann, EPL '13]
[NEMO-3, PRL '17]

Example for Weinberg's selection rules:

Impose $U(1)_{3B-L}$ on SMEFT,
 then the lowest BNV operators have $\Delta B = \Delta L/3 = 1$ and
 arise at $d = 9$.

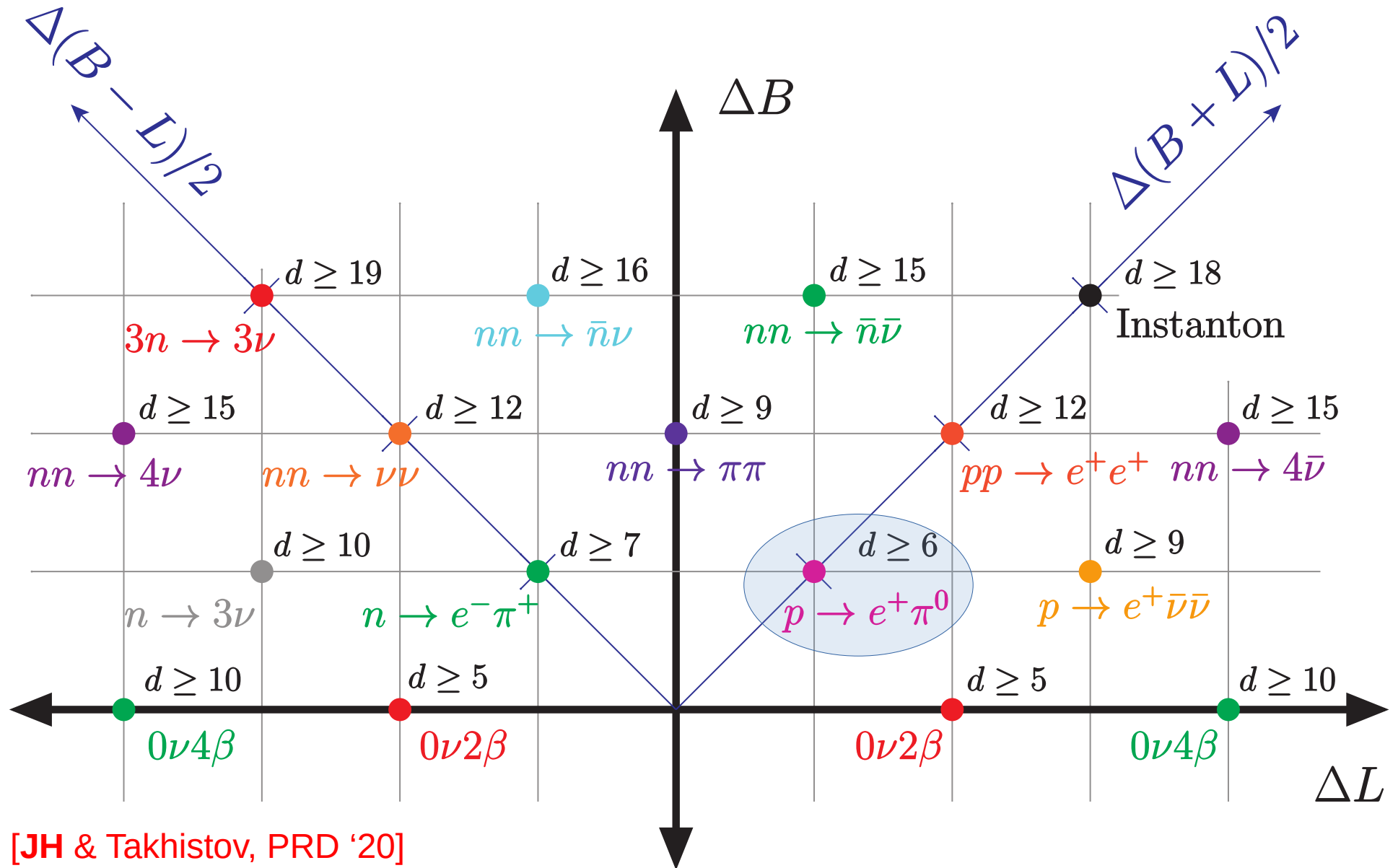
$$\tau(p \rightarrow e^+ \bar{\nu} \bar{\nu}) \simeq \underbrace{2 \times 10^{32} \text{yr}}_{\text{Super-K limit '14}} \left(\frac{\Lambda}{300 \text{TeV}} \right)^{10}.$$

BNV sensitive to $d \gg 6$ and multi-body final states!



[JH & Takhistov, PRD '20]

Probing the landscape point by point?



$$\Delta B = \Delta L = 1$$

- 546 **d=6** operators:

$$y_{abcd}^1 \epsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C u_{b,\beta}) (\bar{Q}_{i,c,\gamma}^C \epsilon_{ij} L_{j,d})$$

$$+ y_{abcd}^2 \epsilon^{\alpha\beta\gamma} \epsilon_{il} \epsilon_{jk} (\bar{Q}_{i,a,\alpha}^C Q_{j,b,\beta}) (\bar{Q}_{k,c,\gamma}^C L_{l,d})$$

$$+ y_{abcd}^3 \epsilon^{\alpha\beta\gamma} (\bar{Q}_{i,a,\alpha}^C \epsilon_{ij} Q_{j,b,\beta}) (\bar{u}_{c,\gamma}^C l_d)$$

$$+ y_{abcd}^4 \epsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C u_{b,\beta}) (\bar{u}_{c,\gamma}^C l_d) + \text{h.c.}$$

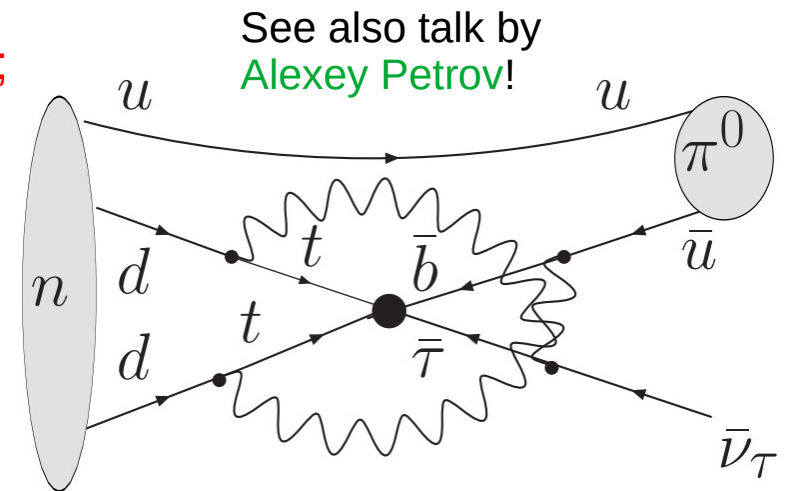
[Weinberg '79 & '80;
Wilczek & Zee '79]

- **All** induce 2-body nucleon decays, even those with **c,b,t, τ** .

[Marciano, NPB '95; Hou++, hep-ph/0509006;
Dong++, 1107.3805; Gargalionis++, 2401.04768;
Beneke++, 2404.09642;
JH & Watkins, 2405.18478;
Gisbert++, 2409.00218]

- d=6 BNV “covered” via simple two-body searches.

[**JH** & Takhistov, PRD '20]



Not necessarily the fastest mode!

Two-body nucleon decays (38)

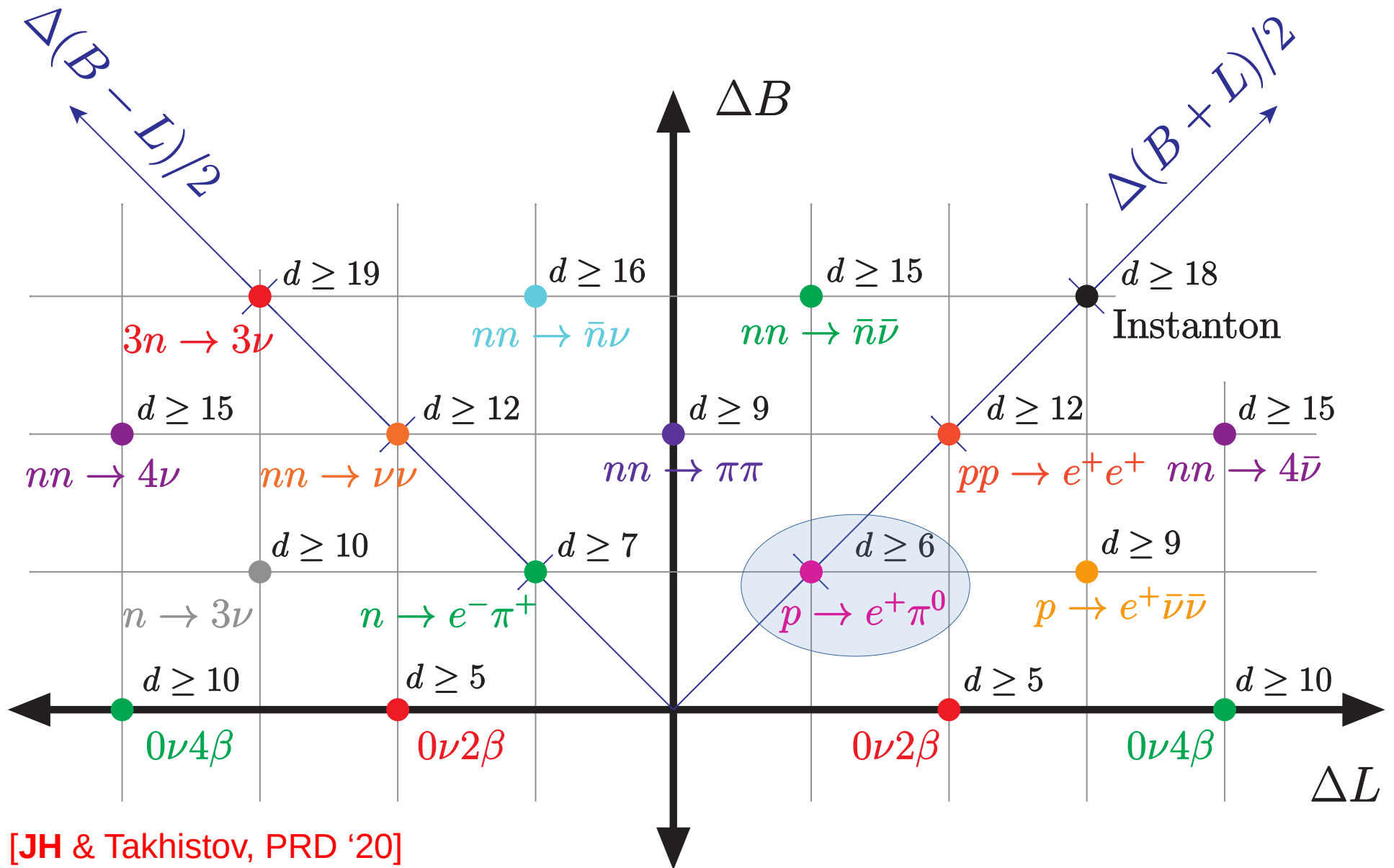
Channel	$\Gamma^{-1}/10^{30}$ yr	Year
$p \rightarrow e^+ + \gamma$	41000	2018
$p \rightarrow e^+ + \pi^0$	16000	2016
$p \rightarrow e^+ + \eta$	10000	2017
$p \rightarrow e^+ + \rho^0$	720	2017
$p \rightarrow e^+ + \omega$	1600	2017
$p \rightarrow e^+ + K^0$	1000	2005
$p \rightarrow e^+ + K^{*,0}$	84	1999
$p \rightarrow \mu^+ + \gamma$	21000	2018
$p \rightarrow \mu^+ + \pi^0$	7700	2016
$p \rightarrow \mu^+ + \eta$	4700	2017
$p \rightarrow \mu^+ + \rho^0$	570	2017
$p \rightarrow \mu^+ + \omega$	2800	2017
$p \rightarrow \mu^+ + K^0$	1600	2012
$p \rightarrow \nu + \pi^+$	390	2013
$p \rightarrow \nu + \rho^+$	162	1999
$p \rightarrow \nu + K^+$	5900	2014
$p \rightarrow \nu + K^{*,+}$	130	2007

$n \rightarrow e^- + \pi^+$	65	1988
$n \rightarrow e^- + \rho^+$	62	1988
$n \rightarrow e^- + K^+$	32	1991
$n \rightarrow e^- + K^{*,+}$		
$n \rightarrow e^+ + \pi^-$	5300	2017
$n \rightarrow e^+ + \rho^-$	217	1999
$n \rightarrow e^+ + K^-$	17	1999
$n \rightarrow e^+ + K^{*,-}$		
$n \rightarrow \mu^- + \pi^+$	49	1988
$n \rightarrow \mu^- + \rho^+$	7	1988
$n \rightarrow \mu^- + K^+$	57	1991
$n \rightarrow \mu^+ + \pi^-$	3500	2017
$n \rightarrow \mu^+ + \rho^-$	228	1999
$n \rightarrow \mu^+ + K^-$	26	1999
$n \rightarrow \nu + \gamma$	550	2015
$n \rightarrow \nu + \pi^0$	1100	2013
$n \rightarrow \nu + \eta$	158	1999
$n \rightarrow \nu + \rho^0$	19	1988
$n \rightarrow \nu + \omega$	108	1999
$n \rightarrow \nu + K^0$	130	2005
$n \rightarrow \nu + K^{*,0}$	78	1999

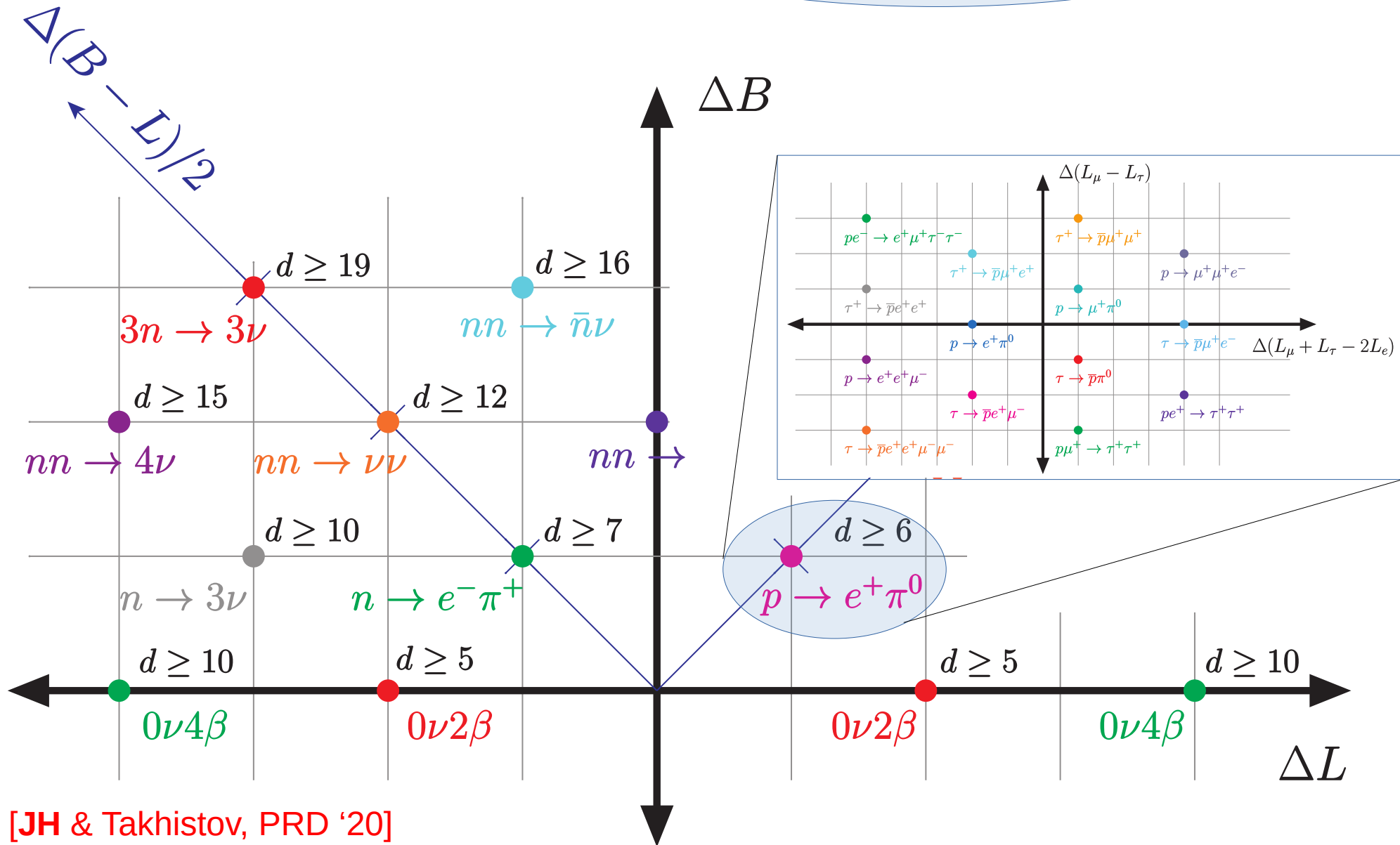
Many of these limits are
decades old.

[JH & Takhistov, PRD '20]

$\Delta B = \Delta L = 1$ covered?



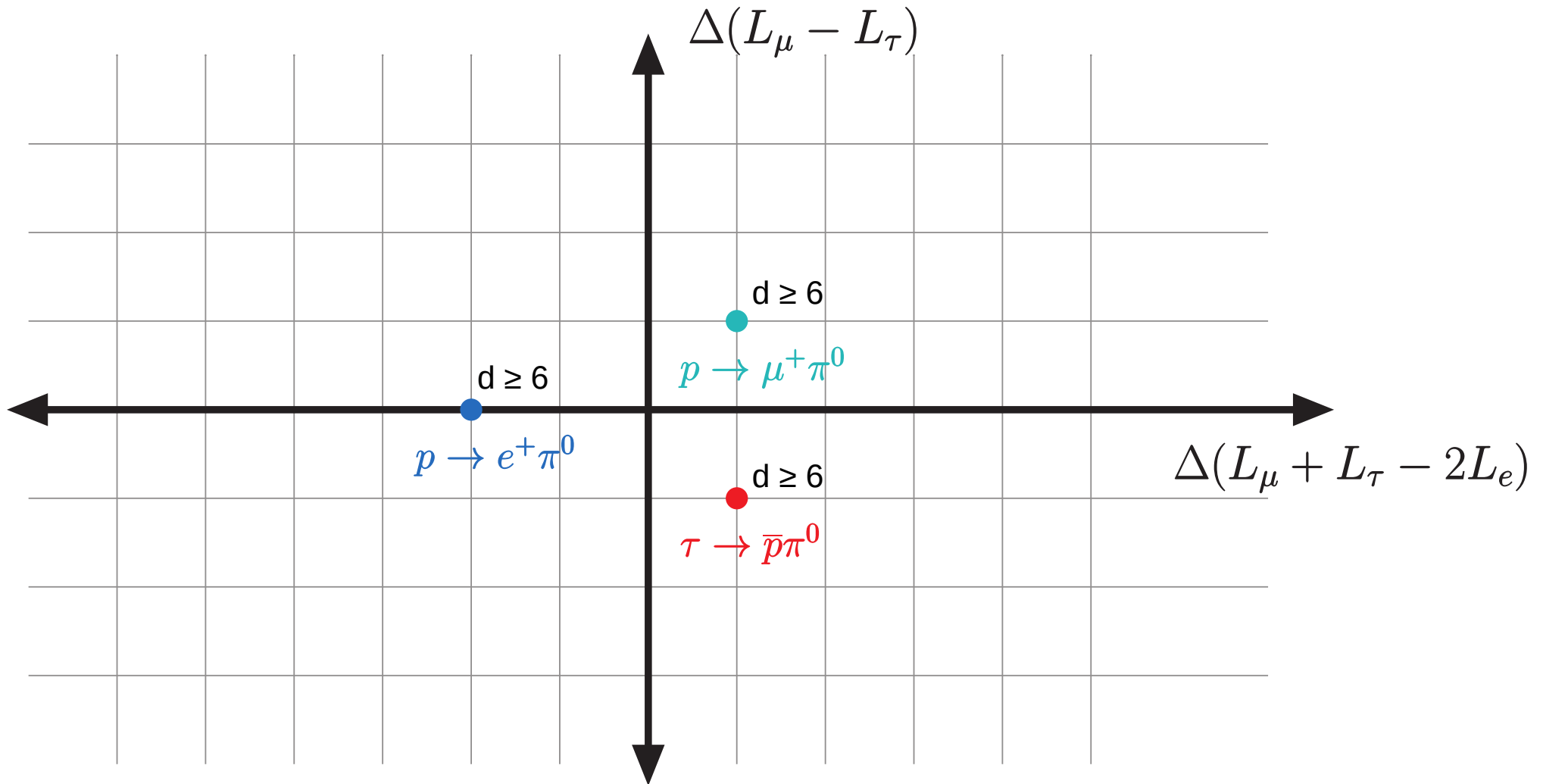
$$U(1)_B \times U(1)_L \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}.$$



[JH & Takhistov, PRD '20]

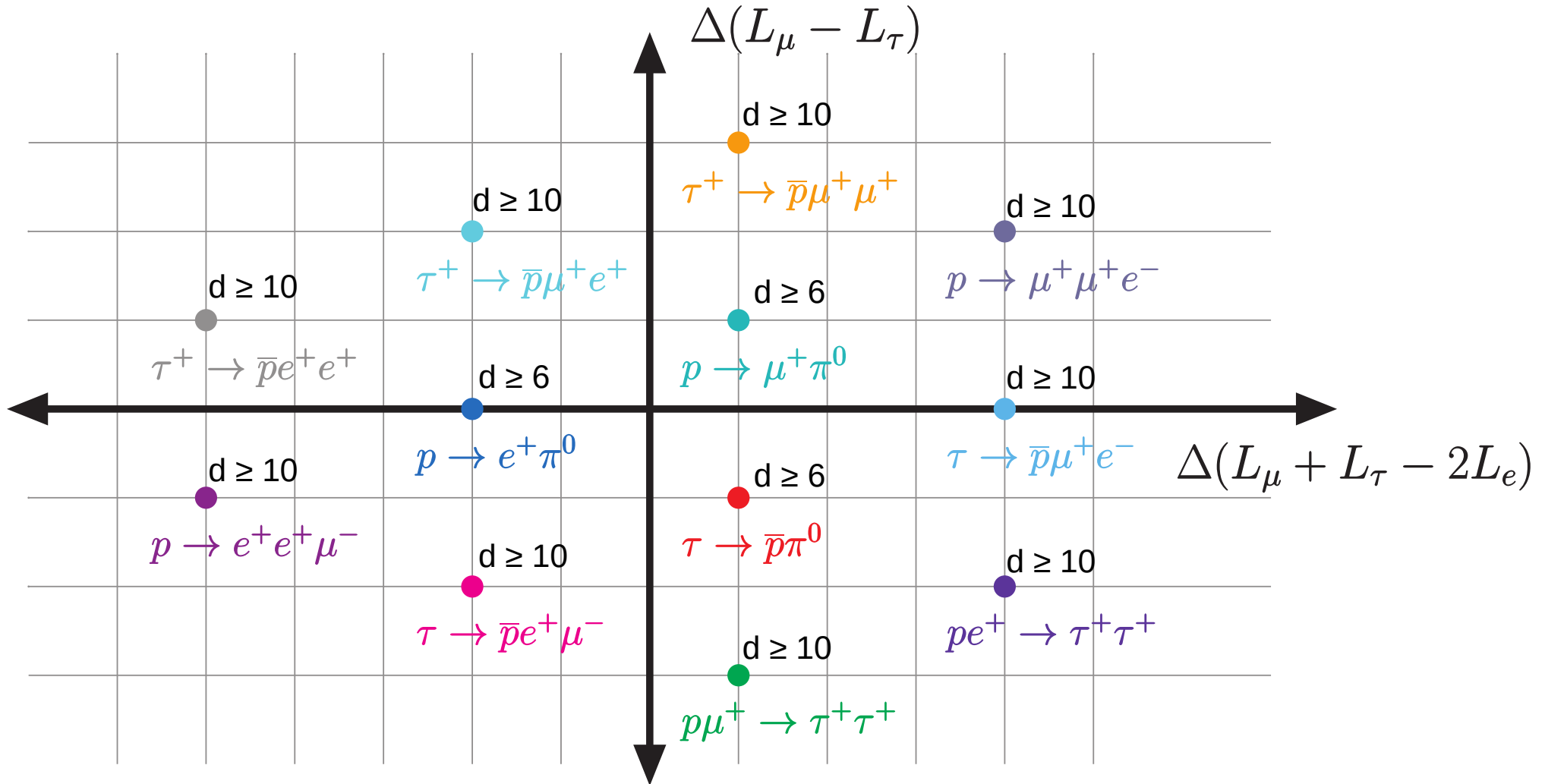
Proton decay = lepton flavor violation

$$\Delta B = \Delta L = 1$$



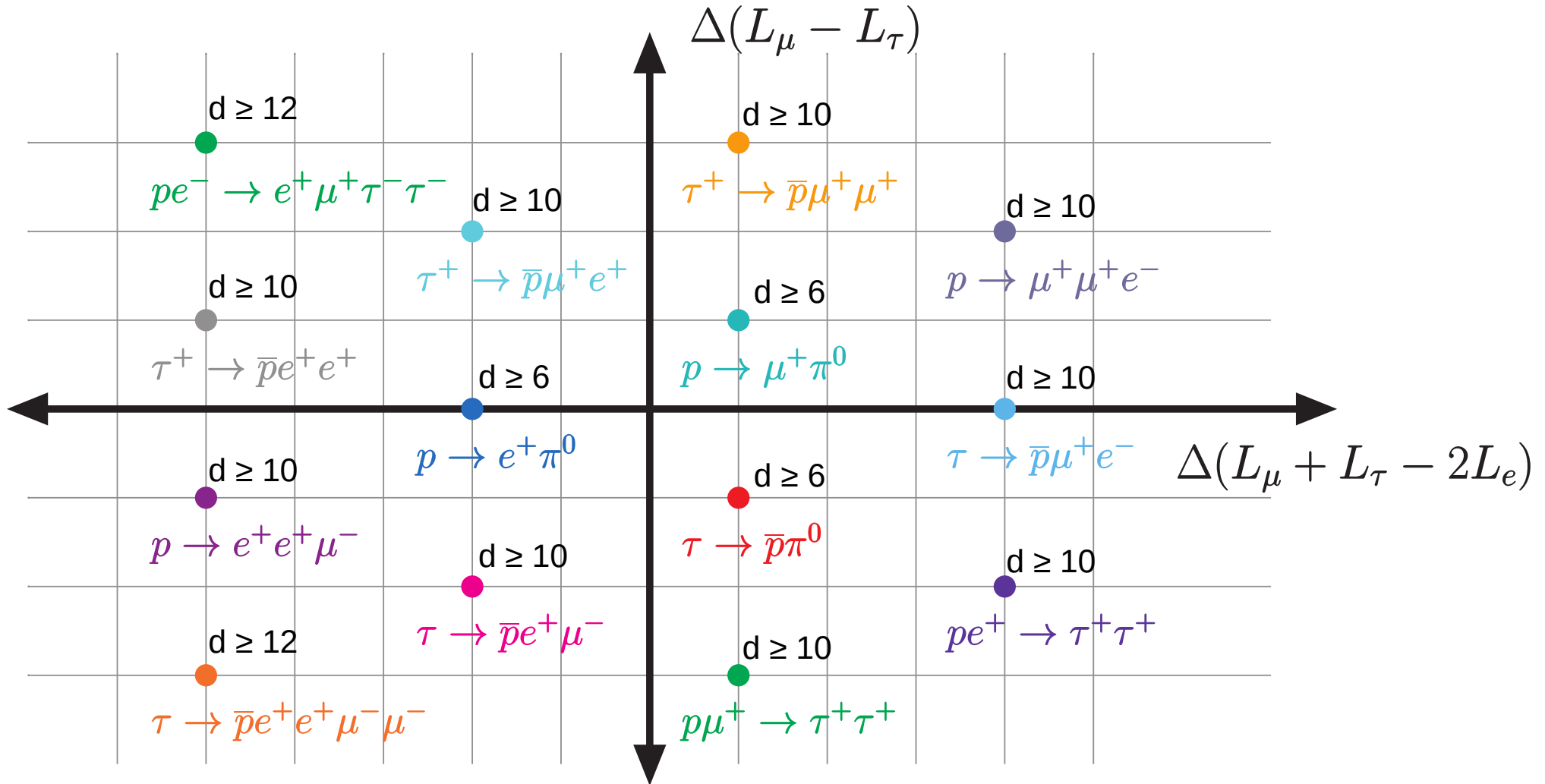
Proton decay = lepton flavor violation

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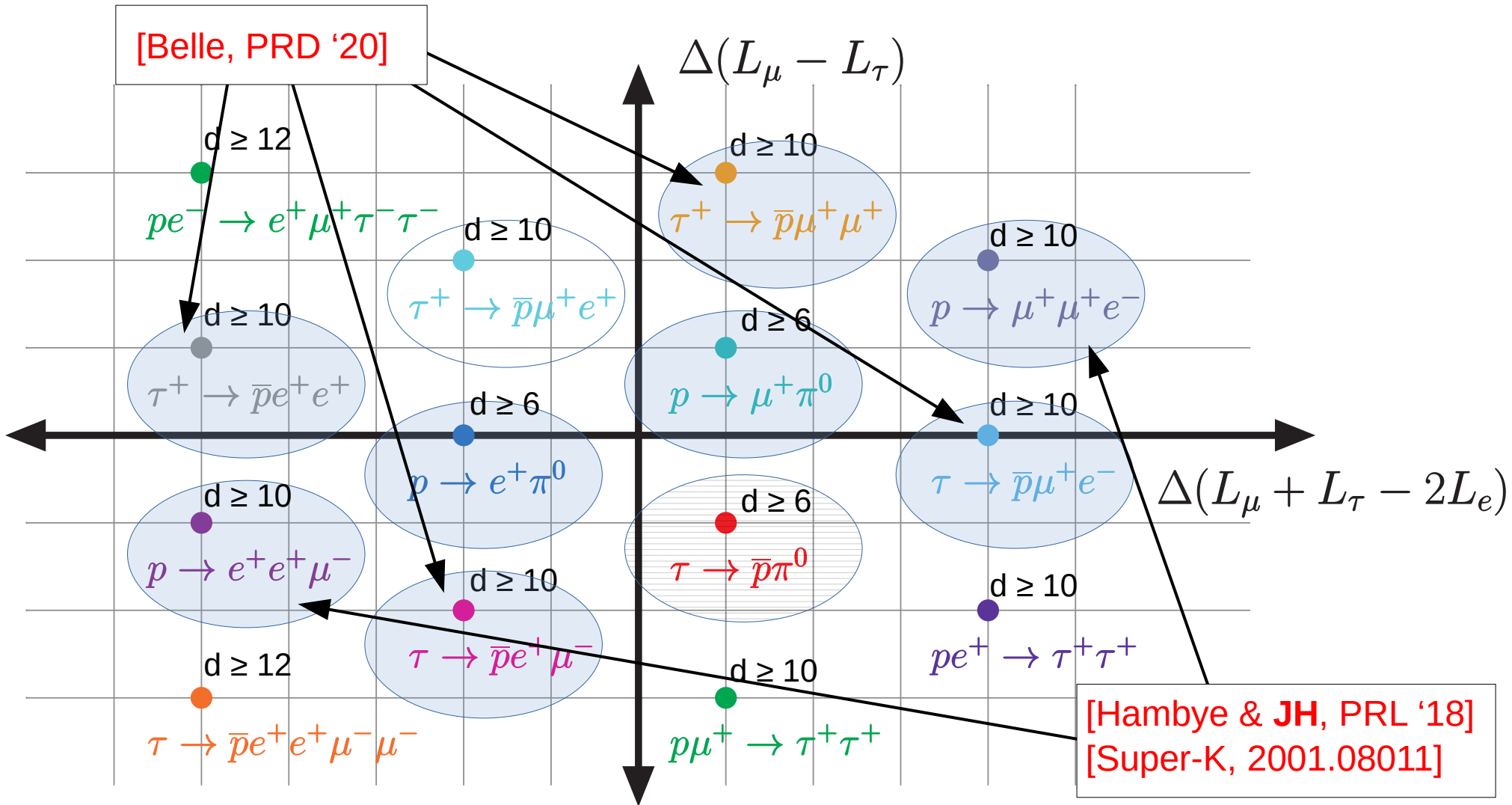
Proton decay = lepton flavor violation

$$\Delta B = \Delta L = 1$$



Currently being probed: Old results: Doable:

$\Delta B = \Delta L = 1$

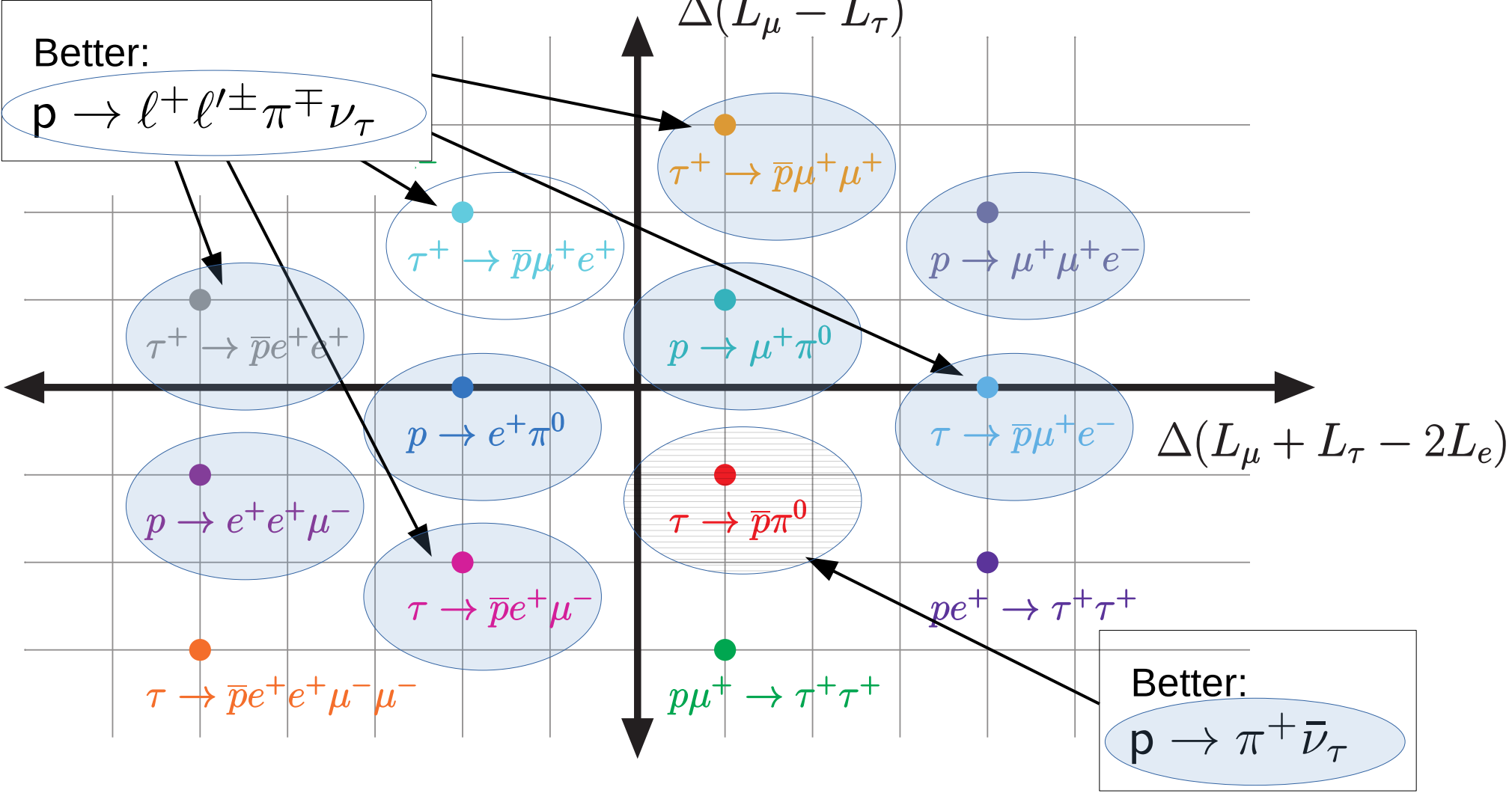


$$\tau(p \rightarrow \mu^+ \mu^+ e^-) \simeq 10^{33} \text{yr} \left(\frac{\Lambda}{100 \text{TeV}} \right)^{12}$$

Currently being probed: Old results: Doable:

$\Delta B = \Delta L = 1$

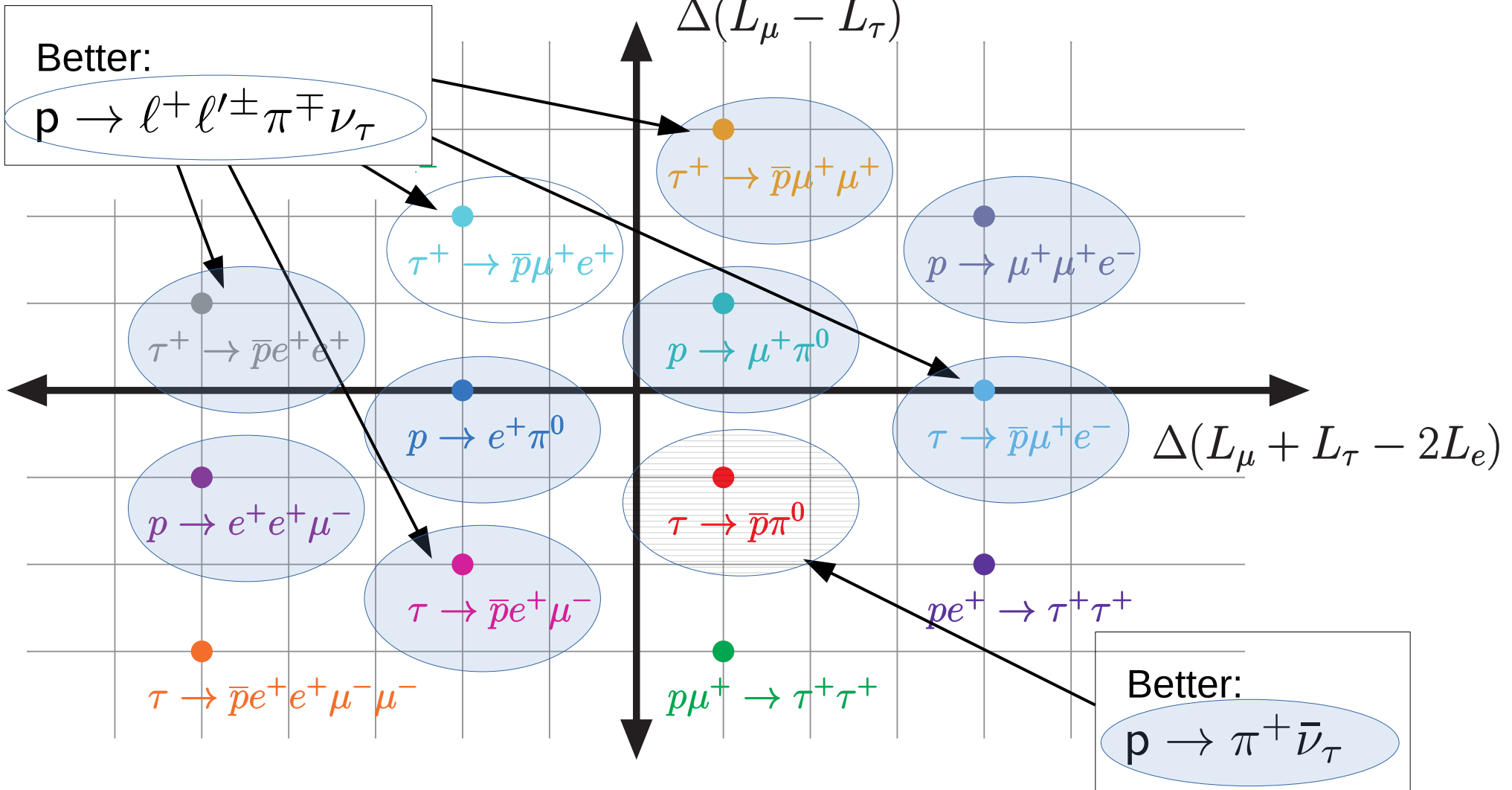
[JH & Watkins, 2405.18478]



[Marciano, NPB '95]
 [JH & Watkins, 2405.18478]

$\Delta B = \Delta L = 1$ covered now?

[JH & Watkins, 2405.18478]



[Marciano, NPB '95]
 [JH & Watkins, 2405.18478]

Beyond SMEFT

- So far: SMEFT + “ $U(1)_B \times U(1)_L \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$ ” to identify **potentially dominant BNV**.
- Now, find **UV completions** for BNV operators:
 - Generates a *physically motivated* operator basis;
 - Could have interesting *accidental symmetries*;
 - Useful to have in case of a *BNV observation*.
- Analogous to UV completions of **$\Delta L=2$ Weinberg operator**.
[too many to cite; exhaustive up to $d=11$: Gargalionis & Volkas, 2009.13537]
- UV completions for all SMEFT operators exist up to $d = 8$.
[Li++, 2309.15933]

Opening up $d = 6$ operators

Leptoquark	spin	representation	Leptoquark	spin	representation
\mathcal{S}_1	0	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	\mathcal{S}_3	0	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$
$\tilde{\mathcal{S}}_1$	0	$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	\mathcal{V}_2	1	$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$
$\bar{\mathcal{S}}_1$	0	$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	$\tilde{\mathcal{V}}_2$	1	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$

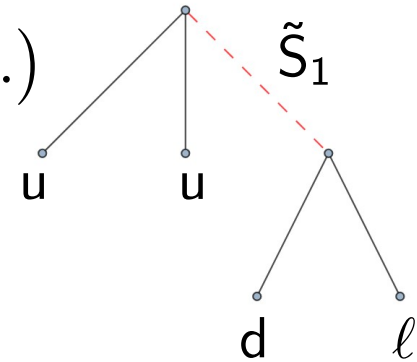
$duQL$	$\mathcal{S}_1, \mathcal{V}_2, \tilde{\mathcal{V}}_2$
$QQul$	$\mathcal{S}_1, \mathcal{V}_2$
$QQQL$	$\mathcal{S}_1, \mathcal{S}_3$
$duul$	$\mathcal{S}_1, \tilde{\mathcal{S}}_1$

[Buchmuller, Ruckl, Weyler, '87; Dorsner++, 1603.04993]

- For example:

$$L_{\tilde{\mathcal{S}}_1} \supset -m_{\tilde{\mathcal{S}}_1}^2 |\tilde{\mathcal{S}}_1|^2 + (\tilde{y}_{1ab}^{RR} \bar{d}_{Ra}^C \tilde{\mathcal{S}}_1 e_{Rb} + \tilde{z}_{1ab}^{RR} \bar{u}_{Ra}^C \tilde{\mathcal{S}}_1^* u_{Rb} + \text{h.c.})$$

$$\rightarrow \frac{2 \tilde{y}_{1ad}^{RR} \tilde{z}_{1bc}^{RR}}{m_{\tilde{\mathcal{S}}_1}^2} \epsilon^{\alpha\beta\gamma} (\bar{d}_{a,\alpha}^C u_{b,\beta}) (\bar{u}_{c,\gamma}^C \ell_d) + \text{h.c.}$$



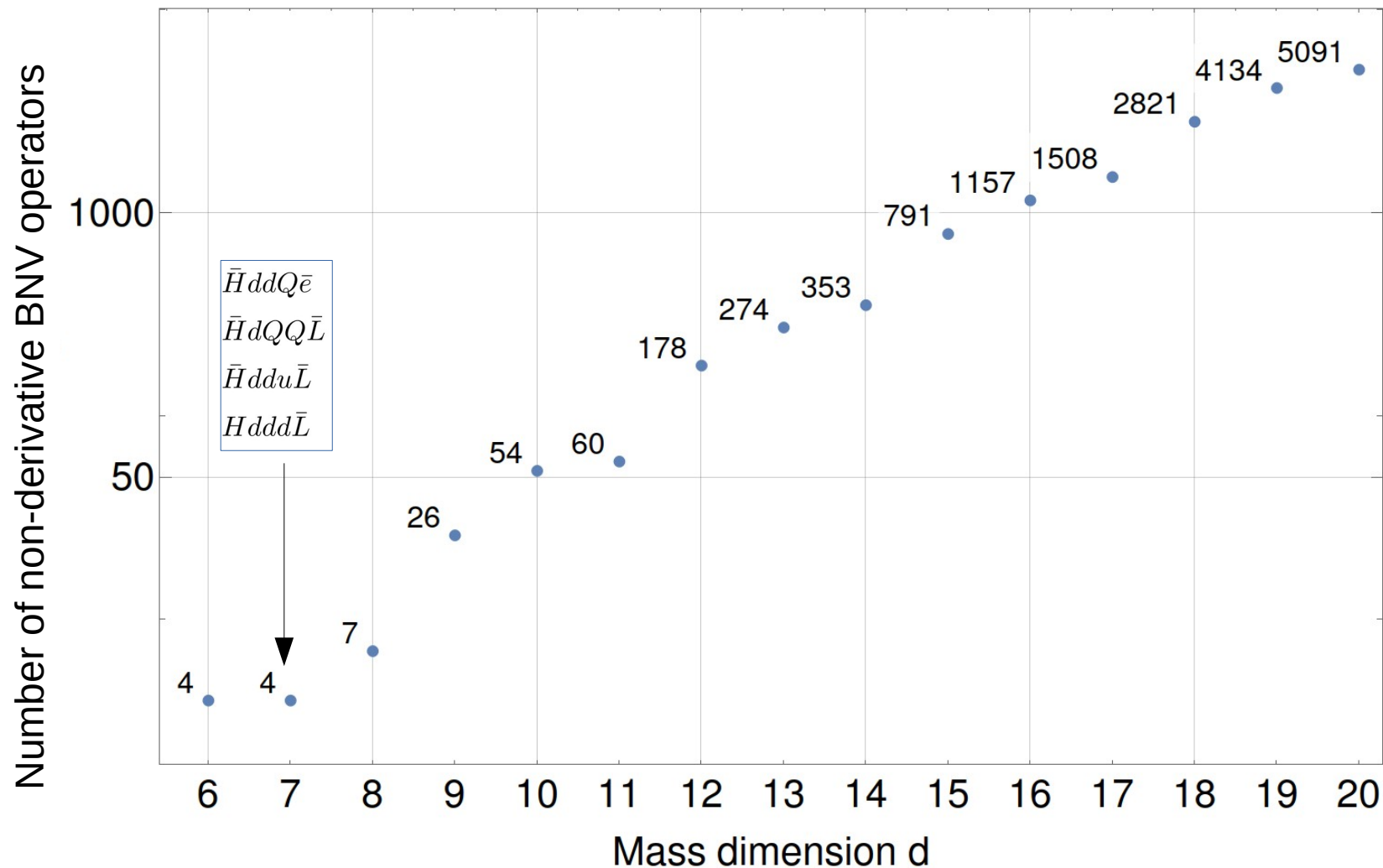
- \tilde{z}_{1ab}^{RR} is **antisymmetric** \rightarrow charm or top quark BNV!

[Dong++, 1107.3805; Dorsner, Fajfer, Kosnik, 1204.0674]

UV completions shed different light on BNV

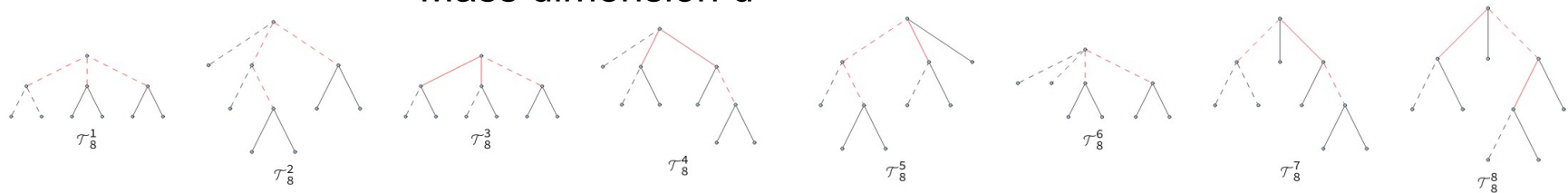
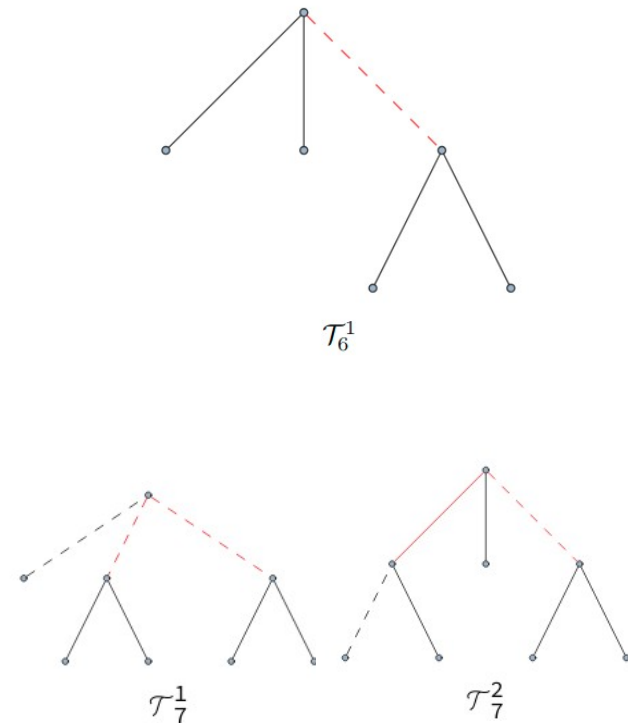
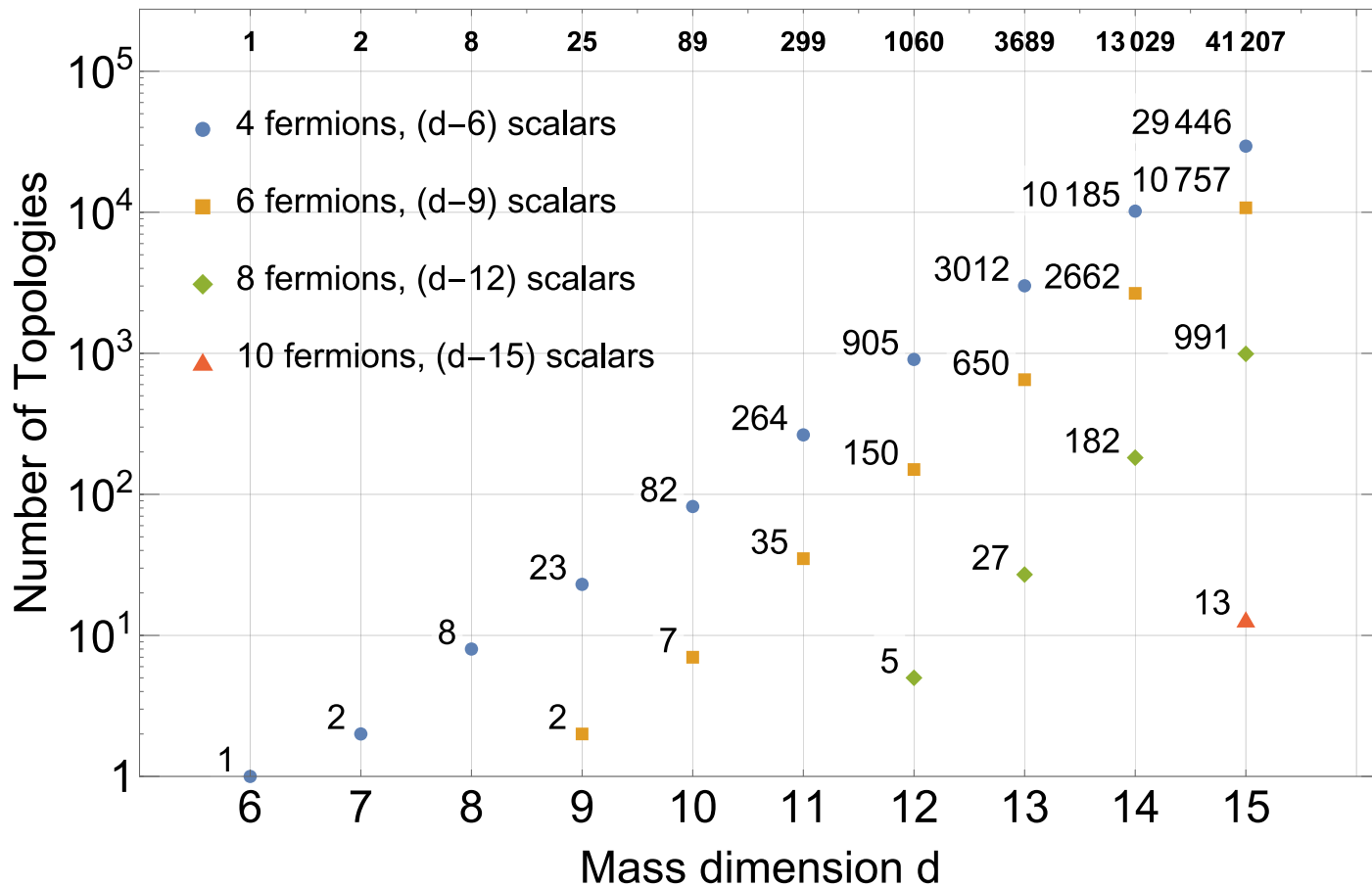
Opening up *all* BNV operators

- Collect all SMEFT BNV operators, trivial with [Sym2Int](#).
[Renato M. Fonseca, 1703.05221 & 1907.12584]
- Exponential growth of “operators” (~field strings):



Opening up *all* BNV operators

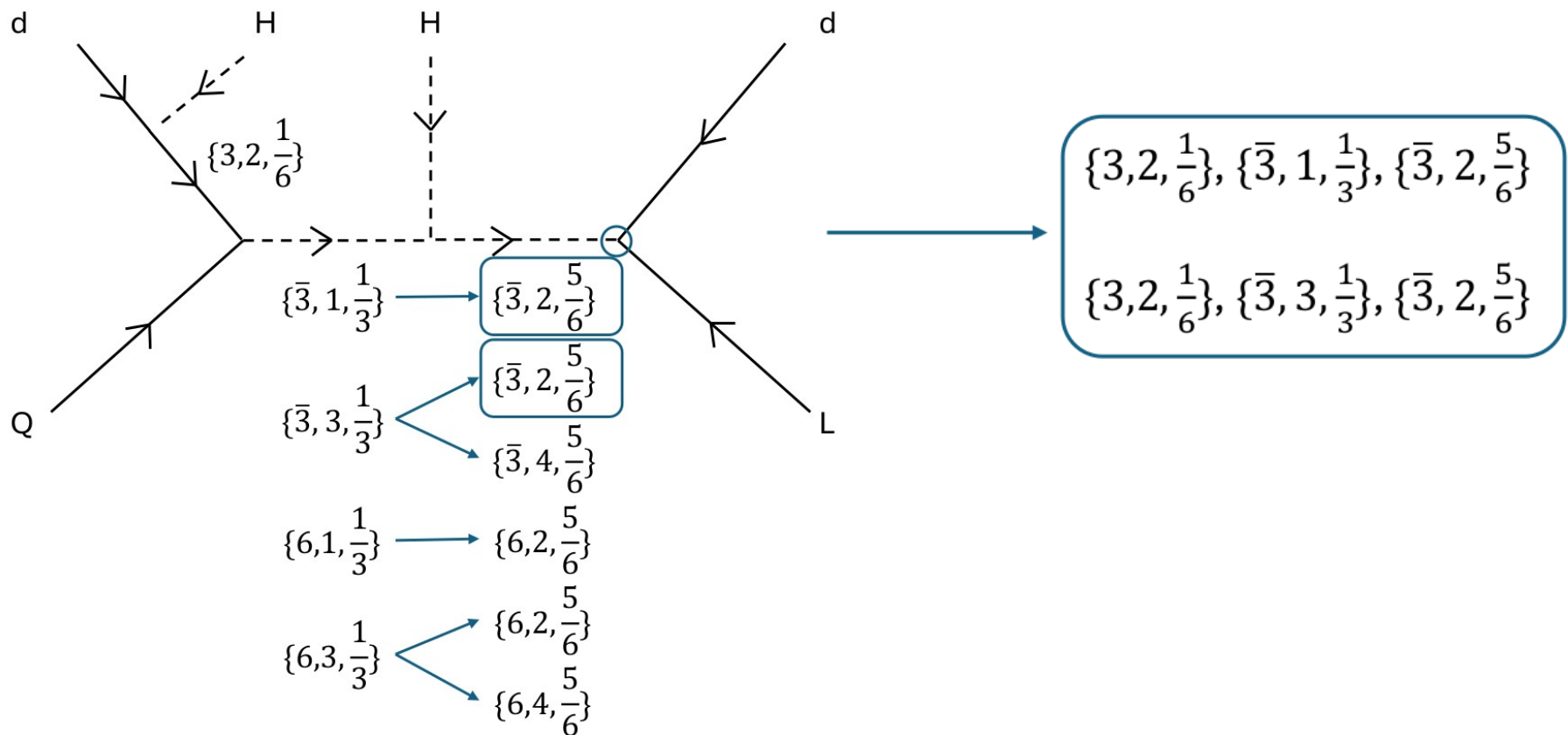
- Generate all irreducible tree-level **topologies**.
- Exponential growth:



Opening up *all* BNV operators

- For each operator, pick topology and distribute fields.
- Multiply group representations using [GroupMath](#).
- E.g. **ddLQHH**:

[Renato M. Fonseca, 2011.01764]



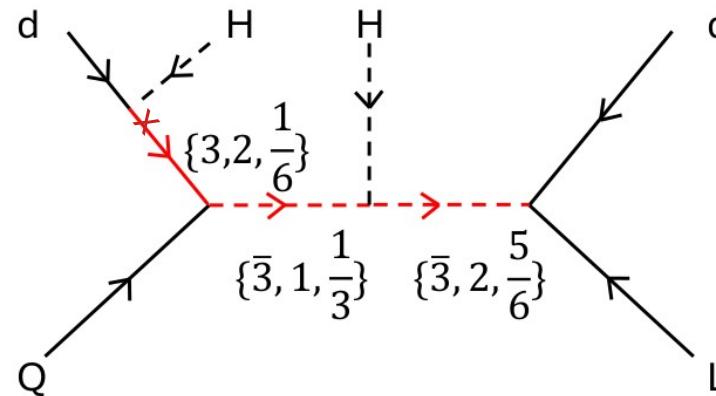
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- E.g. **ddLQHH**:

[Renato M. Fonseca, 2011.01764]



$$\left\{ 3, 2, \frac{1}{6} \right\}_F, \left\{ \bar{3}, 1, \frac{1}{3} \right\}_V, \left\{ \bar{3}, 2, \frac{5}{6} \right\}_V$$

[JH, D. Sokhashvili, Thapa, to appear]

- Also include global Lorentz $SU(2)_{\text{left}} \times SU(2)_{\text{right}}$ for spin.
- Then permute external particles over topology and repeat...

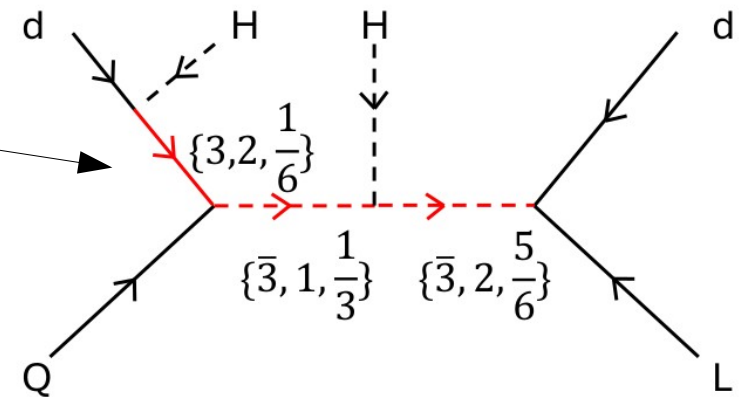
Derivative operators

- So far only BNV operators without derivatives. Why?
- More complicated, not always tree-level completion.
- Generically **sub-dominant** at tree-level:

$$\frac{1}{\not{p} - M} \simeq -\frac{1}{M} - \frac{\not{p}}{M^2}$$

Non-derivative operator

derivative operator



- Same UV completions, dominant only through **finetuning**.
- **Exception**: operators with $HD_\mu H$ that vanish without D_μ .

[Gargalionis & Volkas, 2009.13537]

Opening up *all* BNV operators

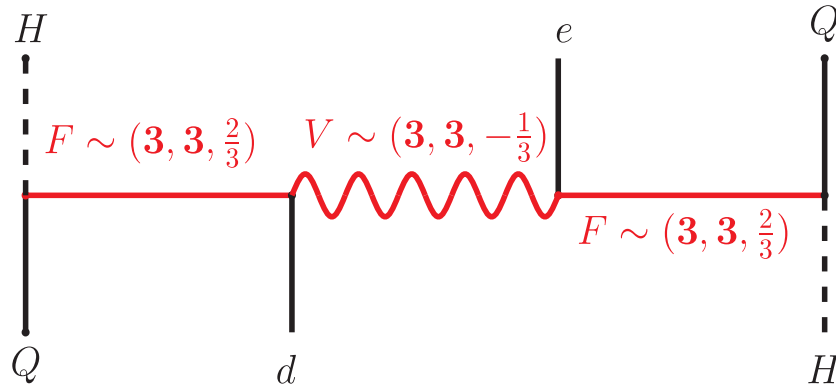
- Code fast enough to reach $d \sim 15$.
- Similar code developed for $\Delta L=2$ operators. [Gargalionis & Volkas, '21]
- Already revealed some mistakes in literature.
- Can be used to open up **any** (non-derivative) EFT operator.

Opening up *all* BNV operators

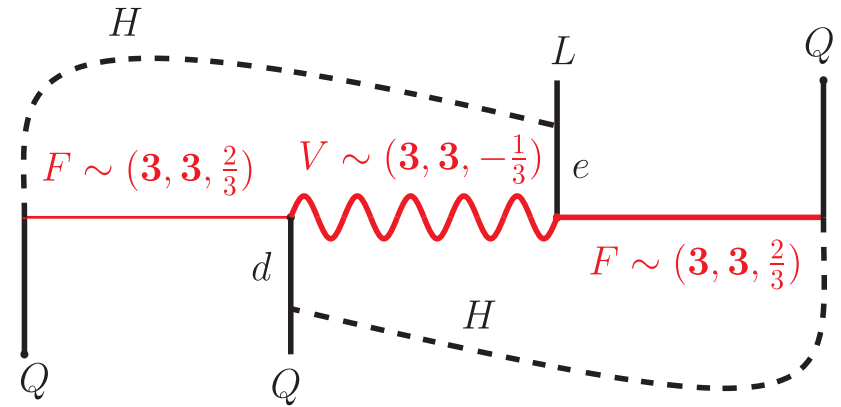
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- Similar code developed for $\Delta L=2$ operators. [Gargalionis & Volkas, '21]
- Already revealed some mistakes in literature.
- Can be used to open up **any** (non-derivative) EFT operator.
- Find **accidentally protected operators**. [Weinberg, '80]
- E.g. add **Dirac fermion $(\mathbf{3}, \mathbf{3}, 2/3)$** and **vector LQ $(\mathbf{3}, \mathbf{3}, -1/3)$** .
 - Only generates BNV operator **QQdeHH** ($d=8$, $B=L=1$).
 - Could be dominant but *not* protected by symmetry.
 - Gives genuine **loop realization** of $d=6$ BNV operator.
[also discussed in Gargalionis, Herrero-Garcia, Schmidt, 2401.04768]
See also talk by **Svjetlana Fajfer** about BNV loops!

Opening up *all* BNV operators

a)



b)



- E.g. add Dirac fermion $(\mathbf{3}, \mathbf{3}, 2/3)$ and vector LQ $(\mathbf{3}, \mathbf{3}, -1/3)$.
 - Only generates BNV operator $QQdeHH$ ($d=8$, $B=L=1$).
 - Could be dominant but *not* protected by symmetry.
 - Gives genuine loop realization of $d=6$ BNV operator.

[also discussed in Gargalionis, Herrero-Garcia, Schmidt, 2401.04768]

See also talk by Svjetlana Fajfer about BNV loops!

Protected operators

- $d > 6$ operators could dominate either because we impose a **B/L symmetry** à la Weinberg or due to **UV structure**.
- Can make *any* $d=7$ or $d=8$ operator dominant through UV:

$HHddQL, \bar{H}\bar{H}uuQL, H\bar{H}duQL, H\bar{H}duue, HHdQQe, H\bar{H}QQQL, H\bar{H}uQQe$

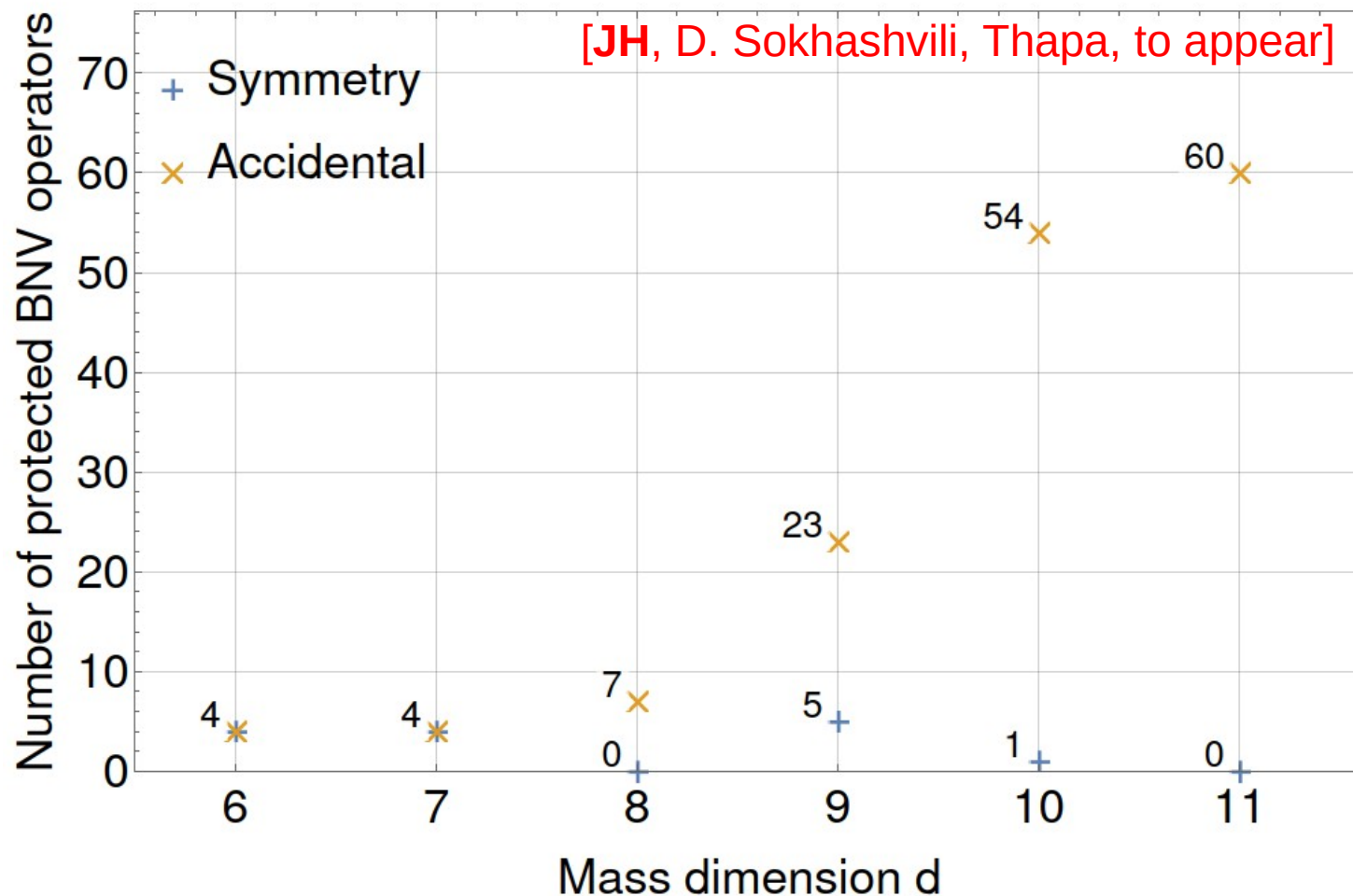
- Can make *most* $d=9$ operators (23/26), e.g.

$dddd\bar{d}\bar{e}, ddde\bar{e}\bar{e}, ddue\bar{L}\bar{L}, ddu\bar{u}Q\bar{L}, H\bar{H}\bar{H}ddQ\bar{e}, \dots$

- Can make *all* 54 $d=10$ operators, *all* 60 $d=11$ operators, ...
- More abundant than symmetry-protected operators!
- These either have **accidental B/L symmetry** or give **loop realization** of lower-dim operators.

[**JH**, D. Sokhashvili, Thapa, to appear]

Protected operators



...so many operators, many with multi-particle BNV final states.
Can we test all of them?

Full BNV coverage possible?

- Cannot to go through all $\Delta B > 0$ decays:
 - 38 two-body $\Delta B = 1$ modes: $N \rightarrow AB$. 36 limits.
 - 76 three-body $\Delta B = 1$ modes: $N \rightarrow ABC$. 33 limits.
 - 300 four-body $\Delta B = 1$ modes: $N \rightarrow ABCD$. 0 limits.
 - 118 two-body $\Delta B = 2$ modes: $NN \rightarrow AB$. 18 limits.
 - 500 three-body $\Delta B = 2$ modes: $NN \rightarrow ABC$. 0 limits.
 - ...
- *Exclusive* searches can reach $t \sim 10^{34}$ yr in Super-K.

Inclusive searches to the rescue!

Inclusive searches

- Current limits from PDG:

$$\Gamma^{-1}(\text{N} \rightarrow \text{e} + \text{anything}) > 0.6 \times 10^{30} \text{ yr}, \quad [\text{Learned, Reines, Soni, '79}]$$

$$\Gamma^{-1}(\text{N} \rightarrow \mu + \text{anything}) > 12 \times 10^{30} \text{ yr}. \quad [\text{Cherry, Deakyne, Lande, Lee, Steinberg, Cleveland, '81}]$$

- **45 years old**, improve with new tech!
- $\text{p} \rightarrow \text{e}^+ + \text{anything}$ in SK could reach **10^{32} yr**, judging by

$$\Gamma^{-1}(\text{p} \rightarrow \text{e}^+ \nu \nu) > 1.7 \times 10^{32} \text{ yr}. \quad [\text{Super-K, PRL '14}]$$

- Do inclusive searches for $\text{N} \rightarrow \ell/\text{meson} + \text{anything}$.
- Also probes $\Delta B > 1$, light new physics, and dark matter!

[**JH** & Takhistov, PRD '20]

Invisible neutron decay

- Special case of inclusive searches:

$$\Gamma^{-1}(n \rightarrow \text{neutrinos}) > 0.58 \times 10^{30} \text{ yr},$$

$$\Gamma^{-1}(nn \rightarrow \text{neutrinos}) > 1.4 \times 10^{30} \text{ yr},$$

$$\Gamma^{-1}(nnn \rightarrow \text{neutrinos}) > 1.8 \times 10^{23} \text{ yr},$$

$$\Gamma^{-1}(nnnn \rightarrow \text{neutrinos}) > 1.4 \times 10^{23} \text{ yr}.$$

[KamLAND, PRL '06; see also SNO+, PRD '19]

[Hazama, Ejiri, Fushimi, Ohsumi, PRC '94]

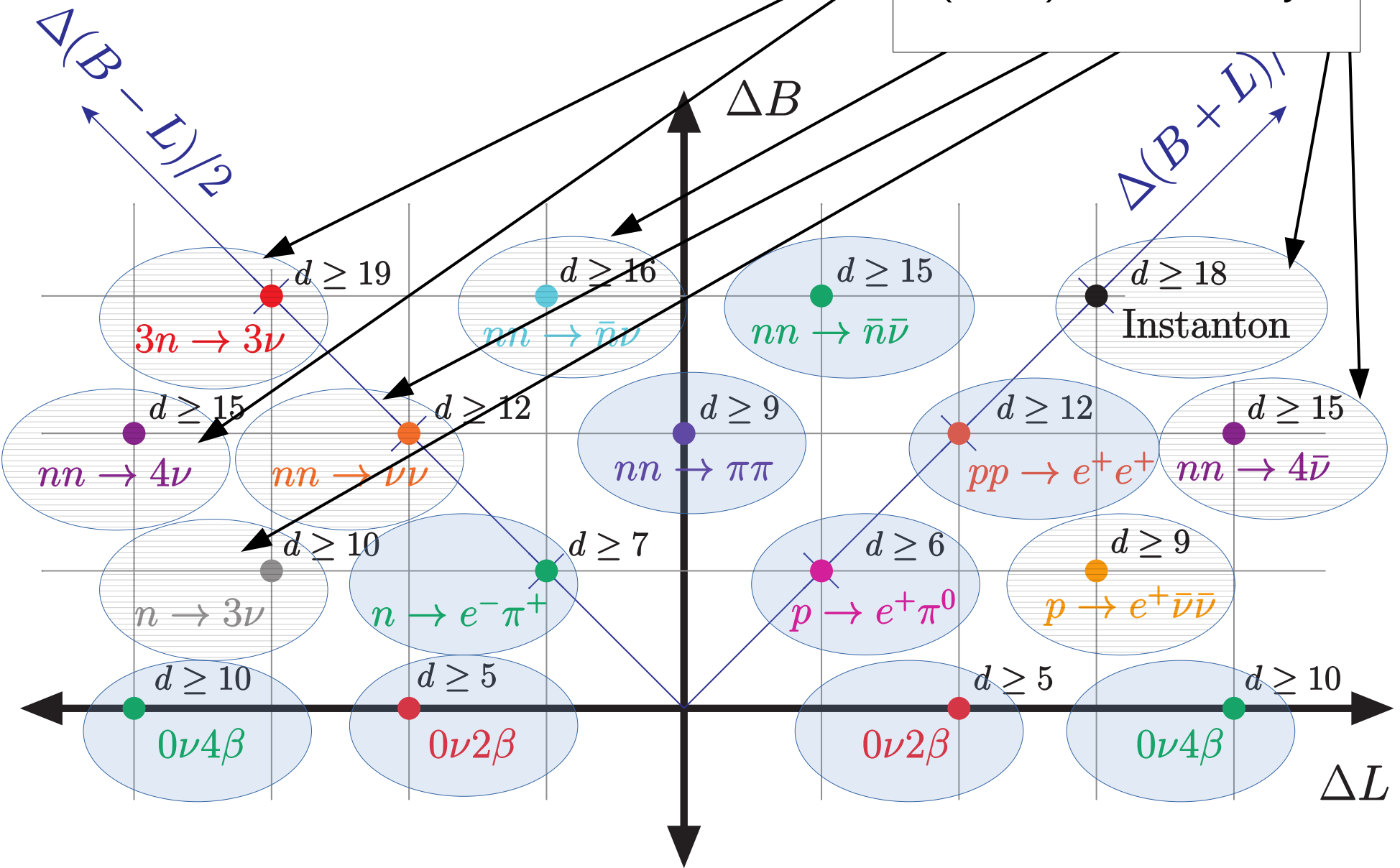
- Only signature is **de-excitation** of daughter nucleus. [Ejiri, '93]
- **Every** $\Delta B = k$ operator gives rise to **k neutrons** \rightarrow **neutrinos**.
- Neutrinos carry away arbitrary lepton number & flavor!
- Also probes light new physics and dark matter.
- JUNO can improve KamLAND limit. [JUNO, 2405.17792]

See also talk by [Cailian Jiang](#)!

[**JH** & Takhistov, PRD '20]

Recent limits:  Older than 5 yr: 

Limits from invisible (multi-)neutron decay!



Summary

- BNV nuclear decays probe
 - high *scales* (10^{15} GeV) *or*
 - high *multiplicities* ($N \rightarrow 15$ particles) *or*
 - high operator *dimensions* ($d \sim 15$)!
- Nearly **every $d \geq 6$ BNV operator** could be the starting point, either because of B/L/ L_α symmetry or UV completion!
- Embarrassment of riches, BNV landscape much more **difficult to map** than e.g. $\Delta L = 2$ operators.
 - Inclusive searches + few theory-motivated exclusives?
- Still more: light new physics, dark matter induced ΔB ...

SK/HK,
DUNE,
JUNO,
 $0\nu\beta\beta$ exp.?

Time to cast a wider net!

Backup

$$p \rightarrow \mu^+ \mu^+ e^-$$

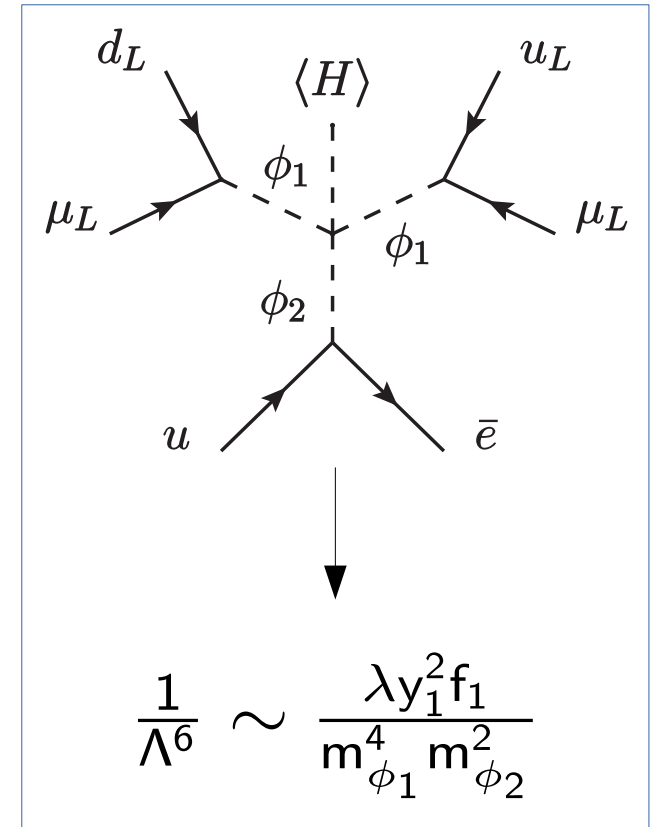
- Minimal **leptoquark** example:

$$\phi_1 \sim (\mathbf{3}, \mathbf{3}, -2/3), \phi_2 \sim (\mathbf{3}, \mathbf{2}, 7/3).$$

- $L_\mu + 2L_e - 3L_\tau$ ensures simple structure

$$y_j \bar{L}_\mu \phi_1 Q_j^c + f_j \bar{u}_j \phi_2 L_e + \lambda \phi_1^2 \phi_2 H.$$

- Final **$\Delta B=1$** operator: $\frac{1}{\Lambda^6} Q Q u L_\mu L_\mu \bar{L}_e H.$
- Lattice QCD input: $\langle 0 | u u d | p \rangle.$



[Hambye, JH, PRL '18]

$$\Gamma(p \rightarrow \mu^+ \mu^+ e^-) \simeq \frac{\langle H \rangle^2 \beta^2 m_p^5}{6144 \pi^3 \Lambda^{12}} \simeq \frac{(100 \text{ TeV} / \Lambda)^{12}}{10^{33} \text{ yr}}$$

Two-body nucleon decays

Channel	$ \Delta(B - L) $	$\frac{\Gamma^{-1}}{10^{30} \text{ yr}}$	
$p \rightarrow e^+ + \gamma$	0	41000	[72]
$p \rightarrow e^+ + \pi^0$	0	16000	[24]
$p \rightarrow e^+ + \eta$	0	10000	[73]
$p \rightarrow e^+ + \rho^0$	0	720	[73]
$p \rightarrow e^+ + \omega$	0	1600	[73]
$p \rightarrow e^+ + K^0$	0	1000	[74]
$p \rightarrow e^+ + K^{*,0}$	0	84	[65]
$p \rightarrow \mu^+ + \gamma$	0	21000	[72]
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$p \rightarrow \mu^+ + \eta$	0	4700	[73]
$p \rightarrow \mu^+ + \rho^0$	0	570	[73]
$p \rightarrow \mu^+ + \omega$	0	2800	[73]
$p \rightarrow \mu^+ + K^0$	0	1600	[75]
$p \rightarrow \nu + \pi^+$	0,2	390	[76]
$p \rightarrow \nu + \rho^+$	0,2	162	[65]
$p \rightarrow \nu + K^+$	0,2	5900	[77]
$p \rightarrow \nu + K^{*,+}$	0,2	130	[78]
$n \rightarrow e^- + \pi^+$	2	65	[79] (5300* [73])
$n \rightarrow e^- + \rho^+$	2	62	[79] (217* [65])
$n \rightarrow e^- + K^+$	2		32 [62]
$n \rightarrow e^- + K^{*,+}$	2		
$n \rightarrow e^+ + \pi^-$	0		5300 [73]
$n \rightarrow e^+ + \rho^-$	0		217 [65]
$n \rightarrow e^+ + K^-$	0		17 [65]
$n \rightarrow e^+ + K^{*,-}$	0		
$n \rightarrow \mu^- + \pi^+$	2	49	[79] (3500* [73])
$n \rightarrow \mu^- + \rho^+$	2	7	[79] (228* [65])
$n \rightarrow \mu^- + K^+$	2		57 [62]
$n \rightarrow \mu^+ + \pi^-$	0		3500 [73]
$n \rightarrow \mu^+ + \rho^-$	0		228 [65]
$n \rightarrow \mu^+ + K^-$	0		26 [65]
$n \rightarrow \nu + \gamma$	0,2		550 [28]
$n \rightarrow \nu + \pi^0$	0,2		1100 [76]
$n \rightarrow \nu + \eta$	0,2		158 [65]
$n \rightarrow \nu + \rho^0$	0,2		19 [79]
$n \rightarrow \nu + \omega$	0,2		108 [65]
$n \rightarrow \nu + K^0$	0,2		130 [74]
$n \rightarrow \nu + K^{*,0}$	0,2		78 [65]

[JH, Takhistov, PRD '20]

Three-body nucleon decays

Channel	$ \Delta(B-L) $	$\frac{\Gamma^{-1}}{10^{30} \text{ yr}}$
$p \rightarrow e^- + e^+ + e^+$	0	793 [65]
$p \rightarrow e^- + e^+ + \mu^+$	0	529 [65]
$p \rightarrow e^+ + e^+ + \mu^-$	0	529* [65]
$p \rightarrow e^- + \mu^+ + \mu^+$	0	6 [64] (359* [65])
$p \rightarrow e^+ + \mu^- + \mu^+$	0	359 [65]
$p \rightarrow \mu^- + \mu^+ + \mu^+$	0	675 [65]
$p \rightarrow e^+ + 2\nu$	0,2	170 [81]
$p \rightarrow \mu^+ + 2\nu$	0,2	220 [81]
$p \rightarrow e^- + 2\pi^+$	2	30 [62] (82* [65])
$p \rightarrow e^- + \pi^+ + \rho^+$	2	
$p \rightarrow e^- + K^+ + \pi^+$	2	75 [65]
$p \rightarrow e^+ + 2\gamma$	0	100 [82] (793* [65])
$p \rightarrow e^+ + \pi^- + \pi^+$	0	82 [65]
$p \rightarrow e^+ + \rho^- + \pi^+$	0	
$p \rightarrow e^+ + K^- + \pi^+$	0	75* [65]
$p \rightarrow e^+ + \pi^- + \rho^+$	0	
$p \rightarrow e^+ + \pi^- + K^+$	0	75* [65]
$p \rightarrow e^+ + 2\pi^0$	0	147 [65]
$p \rightarrow e^+ + \pi^0 + \eta$	0	
$p \rightarrow e^+ + \pi^0 + \rho^0$	0	
$p \rightarrow e^+ + \pi^0 + \omega$	0	
$p \rightarrow e^+ + \pi^0 + K^0$	0	
$p \rightarrow \mu^- + 2\pi^+$	2	17 [62] (133* [65])
$p \rightarrow \mu^- + K^+ + \pi^+$	2	245 [65]
$p \rightarrow \mu^+ + 2\gamma$	0	529* [65]
$p \rightarrow \mu^+ + \pi^- + \pi^+$	0	133 [65]
$p \rightarrow \mu^+ + K^- + \pi^+$	0	245* [65]
$p \rightarrow \mu^+ + \pi^- + K^+$	0	245* [65]
$p \rightarrow \mu^+ + 2\pi^0$	0	101 [65]
$p \rightarrow \mu^+ + \pi^0 + \eta$	0	
$p \rightarrow \mu^+ + \pi^0 + K^0$	0	
$p \rightarrow \nu + \pi^+ + \pi^0$	0,2	
$p \rightarrow \nu + \pi^+ + \eta$	0,2	
$p \rightarrow \nu + \pi^+ + \rho^0$	0,2	
$p \rightarrow \nu + \pi^+ + \omega$	0,2	
$p \rightarrow \nu + \pi^+ + K^0$	0,2	
$p \rightarrow \nu + \rho^+ + \pi^0$	0,2	
$p \rightarrow \nu + K^+ + \pi^0$	0,2	

Channel	$ \Delta(B-L) $	$\frac{\Gamma^{-1}}{10^{30} \text{ yr}}$
$n \rightarrow \nu + e^- + e^+$	0,2	257 [65]
$n \rightarrow \nu + e^- + \mu^+$	0,2	83 [65]
$n \rightarrow \nu + e^+ + \mu^-$	0,2	83* [65]
$n \rightarrow \nu + \mu^- + \mu^+$	0,2	79 [65]
$n \rightarrow 3\nu$	0,2,4	0.58 [83]
$n \rightarrow e^- + \pi^+ + \pi^0$	2	29 [62] (52* [65])
$n \rightarrow e^- + \pi^+ + \eta$	2	
$n \rightarrow e^- + \pi^+ + \rho^0$	2	
$n \rightarrow e^- + \pi^+ + \omega$	2	
$n \rightarrow e^- + \pi^+ + K^0$	2	
$n \rightarrow e^- + \rho^+ + \pi^0$	2	
$n \rightarrow e^- + K^+ + \pi^0$	2	
$n \rightarrow e^+ + \pi^- + \pi^0$	0	52 [65]
$n \rightarrow e^+ + \pi^- + \eta$	0	
$n \rightarrow e^+ + \pi^- + \rho^0$	0	
$n \rightarrow e^+ + \pi^- + \omega$	0	
$n \rightarrow e^+ + \pi^- + K^0$	0	18 [82]
$n \rightarrow e^+ + \rho^- + \pi^0$	0	
$n \rightarrow e^+ + K^- + \pi^0$	0	
$n \rightarrow \mu^- + \pi^+ + \pi^0$	2	34 [62] (74* [65])
$n \rightarrow \mu^- + \pi^+ + \eta$	2	
$n \rightarrow \mu^- + \pi^+ + K^0$	2	
$n \rightarrow \mu^- + K^+ + \pi^0$	2	
$n \rightarrow \mu^+ + \pi^- + \pi^0$	0	74 [65]
$n \rightarrow \mu^+ + \pi^- + \eta$	0	
$n \rightarrow \mu^+ + \pi^- + K^0$	0	
$n \rightarrow \mu^+ + K^- + \pi^0$	0	
$n \rightarrow \nu + 2\gamma$	0,2	219 [65]
$n \rightarrow \nu + \pi^- + \pi^+$	0,2	
$n \rightarrow \nu + \rho^- + \pi^+$	0,2	
$n \rightarrow \nu + K^- + \pi^+$	0,2	
$n \rightarrow \nu + \pi^- + \rho^+$	0,2	
$n \rightarrow \nu + \pi^- + K^+$	0,2	
$n \rightarrow \nu + 2\pi^0$	0,2	
$n \rightarrow \nu + \pi^0 + \eta$	0,2	
$n \rightarrow \nu + \pi^0 + \rho^0$	0,2	
$n \rightarrow \nu + \pi^0 + \omega$	0,2	
$n \rightarrow \nu + \pi^0 + K^0$	0,2	

[JH, Takhistov, PRD '20]
Does not include SK's 2020
limits on $p \rightarrow \ell\ell\ell$.

Two-body di-nucleon decays

Channel	$ \Delta(B-L) $	$\frac{\Gamma^{-1}}{10^{30} \text{ yr}}$
$pp \rightarrow e^+ + e^+$	0	4200 [72]
$pp \rightarrow \mu^+ + \mu^+$	0	4400 [72]
$pp \rightarrow e^+ + \mu^+$	0	4400 [72]
$pp \rightarrow e^+ + \tau^+$	0	
$pp \rightarrow \pi^+ + \pi^+$	2	72 [115]
$pp \rightarrow \pi^+ + \rho^+$	2	
$pp \rightarrow \pi^+ + K^+$	2	
$pp \rightarrow \pi^+ + K^{*,+}$	2	
$pp \rightarrow \rho^+ + \rho^+$	2	
$pp \rightarrow \rho^+ + K^+$	2	
$pp \rightarrow \rho^+ + K^{*,+}$	2	
$pp \rightarrow K^+ + K^+$	2	170 [116]
$pp \rightarrow K^+ + K^{*,+}$	2	
$pp \rightarrow K^{*,+} + K^{*,+}$	2	

$nn \rightarrow e^+ + e^-$	2	4200 [72]
$nn \rightarrow e^+ + \mu^-$	2	4400 [72]
$nn \rightarrow \mu^+ + e^-$	2	4400 [72]
$nn \rightarrow \mu^+ + \mu^-$	2	4400 [72]
$nn \rightarrow e^+ + \tau^-$	2	
$nn \rightarrow \tau^+ + e^-$	2	
$nn \rightarrow 2\nu$	0,2,4	1.4 [83]
$nn \rightarrow 2\gamma$	2	4100 [72]
$nn \rightarrow \gamma + \pi^0$	2	
$nn \rightarrow \gamma + \eta$	2	
$nn \rightarrow \gamma + \rho^0$	2	
$nn \rightarrow \gamma + \omega$	2	
$nn \rightarrow \gamma + \eta'$	2	
$nn \rightarrow \gamma + K^0$	2	
$nn \rightarrow \gamma + K^{*,0}$	2	
$nn \rightarrow \gamma + D^0$	2	
$nn \rightarrow \gamma + \phi$	2	
$nn \rightarrow \pi^- + \pi^+$	2	0.7 [62] (72* [115])
$nn \rightarrow \pi^+ + \rho^-$	2	
$nn \rightarrow K^- + \pi^+$	2	
$nn \rightarrow K^{*,-} + \pi^+$	2	
$nn \rightarrow \pi^- + \rho^+$	2	
$nn \rightarrow K^+ + \pi^-$	2	
$nn \rightarrow K^{*,+} + \pi^-$	2	
$nn \rightarrow 2\pi^0$	2	404 [115]
$nn \rightarrow \eta + \pi^0$	2	
$nn \rightarrow \pi^0 + \rho^0$	2	
$nn \rightarrow \pi^0 + \omega$	2	
$nn \rightarrow \eta' + \pi^0$	2	
$nn \rightarrow K^0 + \pi^0$	2	
$nn \rightarrow K^{*,0} + \pi^0$	2	

Channel	$ \Delta(B-L) $	$\frac{\Gamma^{-1}}{10^{30} \text{ yr}}$
$nn \rightarrow \pi^0 + \phi$	2	
$nn \rightarrow 2\eta$	2	
$nn \rightarrow \eta + \rho^0$	2	
$nn \rightarrow \eta + \omega$	2	
$nn \rightarrow \eta + \eta'$	2	
$nn \rightarrow \eta + K^0$	2	
$nn \rightarrow \eta + K^{*,0}$	2	
$nn \rightarrow \eta + \phi$	2	
$nn \rightarrow 2\rho^0$	2	
$nn \rightarrow \rho^0 + \omega$	2	
$nn \rightarrow \eta' + \rho^0$	2	
$nn \rightarrow K^0 + \rho^0$	2	
$nn \rightarrow K^{*,0} + \rho^0$	2	
$nn \rightarrow \rho^0 + \phi$	2	
$nn \rightarrow \rho^- + \rho^+$	2	
$nn \rightarrow K^+ + \rho^-$	2	
$nn \rightarrow K^{*,+} + \rho^-$	2	
$nn \rightarrow K^- + \rho^+$	2	
$nn \rightarrow K^{*,-} + \rho^+$	2	
$nn \rightarrow 2\omega$	2	
$nn \rightarrow \eta' + \omega$	2	
$nn \rightarrow K^0 + \omega$	2	
$nn \rightarrow K^{*,0} + \omega$	2	
$nn \rightarrow \omega + \phi$	2	
$nn \rightarrow \eta' + K^0$	2	
$nn \rightarrow \eta' + K^{*,0}$	2	
$nn \rightarrow K^- + K^+$	2	170* [116]
$nn \rightarrow K^+ + K^{*,-}$	2	
$nn \rightarrow K^- + K^{*,+}$	2	
$nn \rightarrow 2K^0$	2	
$nn \rightarrow K^{*,0} + K^0$	2	
$nn \rightarrow K^0 + \phi$	2	
$nn \rightarrow 2K^{*,0}$	2	
$nn \rightarrow K^{*,-} + K^{*,+}$	2	

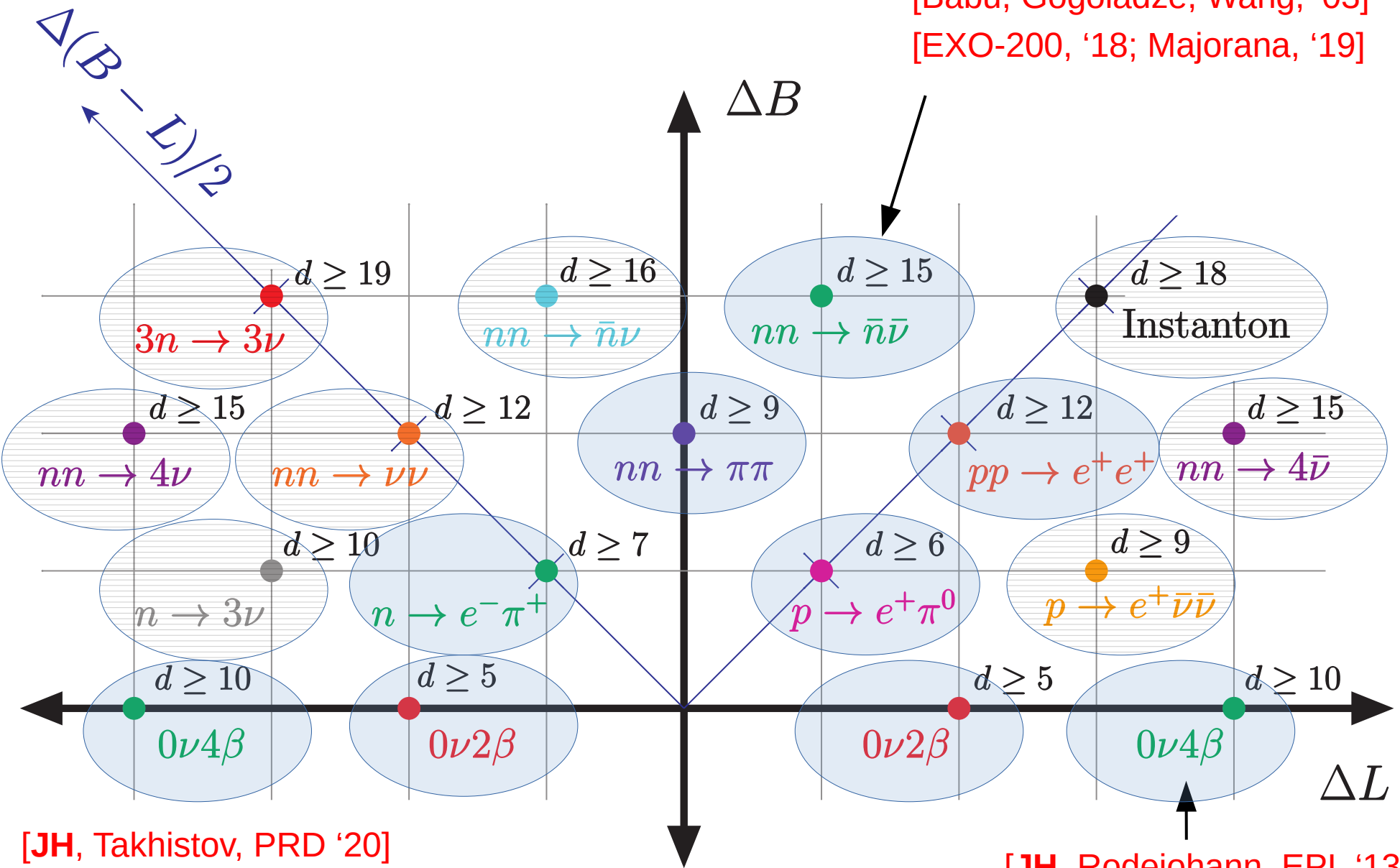
Channel	$ \Delta(B-L) $	$\frac{\Gamma^{-1}}{10^{30} \text{ yr}}$
$pn \rightarrow e^+ + \nu$	0,2	260 [28]
$pn \rightarrow \mu^+ + \nu$	0,2	200 [28]
$pn \rightarrow \tau^+ + \nu$	0,2	29 [28]
$pn \rightarrow \gamma + \pi^+$	2	
$pn \rightarrow \gamma + \rho^+$	2	
$pn \rightarrow \gamma + K^+$	2	
$pn \rightarrow \gamma + K^{*,+}$	2	
$pn \rightarrow \gamma + D^+$	2	
$pn \rightarrow \pi^+ + \pi^0$	2	170 [115]
$pn \rightarrow \eta + \pi^+$	2	
$pn \rightarrow \pi^+ + \rho^0$	2	
$pn \rightarrow \pi^+ + \omega$	2	
$pn \rightarrow \eta' + \pi^+$	2	
$pn \rightarrow K^0 + \pi^+$	2	
$pn \rightarrow K^{*,0} + \pi^+$	2	
$pn \rightarrow \pi^+ + \phi$	2	
$pn \rightarrow \pi^0 + \rho^+$	2	
$pn \rightarrow K^+ + \pi^0$	2	
$pn \rightarrow K^{*,+} + \pi^0$	2	
$pn \rightarrow \eta + \rho^+$	2	
$pn \rightarrow \eta + K^+$	2	
$pn \rightarrow \eta + K^{*,+}$	2	
$pn \rightarrow \rho^+ + \rho^0$	2	
$pn \rightarrow K^+ + \rho^0$	2	
$pn \rightarrow K^{*,+} + \rho^0$	2	
$pn \rightarrow \rho^+ + \omega$	2	
$pn \rightarrow \eta' + \rho^+$	2	
$pn \rightarrow K^0 + \rho^+$	2	
$pn \rightarrow K^{*,0} + \rho^+$	2	
$pn \rightarrow \rho^+ + \phi$	2	
$pn \rightarrow K^+ + \omega$	2	
$pn \rightarrow K^{*,+} + \omega$	2	
$pn \rightarrow \eta' + K^+$	2	
$pn \rightarrow \eta' + K^{*,+}$	2	
$pn \rightarrow K^+ + K^0$	2	
$pn \rightarrow K^+ + K^{*,0}$	2	
$pn \rightarrow K^+ + \phi$	2	
$pn \rightarrow K^{*,+} + K^0$	2	
$pn \rightarrow K^{*,+} + K^{*,0}$	2	

[JH, Takhistov, PRD '20]

Recent limits:  Older than 5 yr: 

$ppp \rightarrow e^+ \pi^+ \pi^+$

[Babu, Gogoladze, Wang, '03]
[EXO-200, '18; Majorana, '19]



[JH, Takhistov, PRD '20]

[JH, Rodejohann, EPL '13]
[NEMO-3, PRL '17]

ppp \rightarrow $e^+ \pi^+ \pi^+$

- Symmetry

	Q	u^c	d^c	ℓ	e^c	ν^c	H
Z_6	6	5	1	2	5	3	1

$$\mathbb{Z}_6 \subset U(1)_{2Y-B+3L}$$

[Babu, Gogoladze, Wang, '03]

allows for $d = 15$ $\Delta B = 3\Delta L = 3$ operators $\frac{1}{\Lambda^{11}} Q^5 d^4 \bar{\ell}, \dots$

- $ppp \rightarrow e^+ \pi^+ \pi^+$, $ppn \rightarrow e^+ \pi^+$, $pnn \rightarrow e^+ \pi^0$, $nn \rightarrow \bar{n} \bar{\nu}, \dots$

- $\tau(pnn \rightarrow e^+ \pi^0) \simeq 3 \times 10^{33} \text{ yr} \left(\frac{\Lambda}{100 \text{ GeV}} \right)^{22}$.

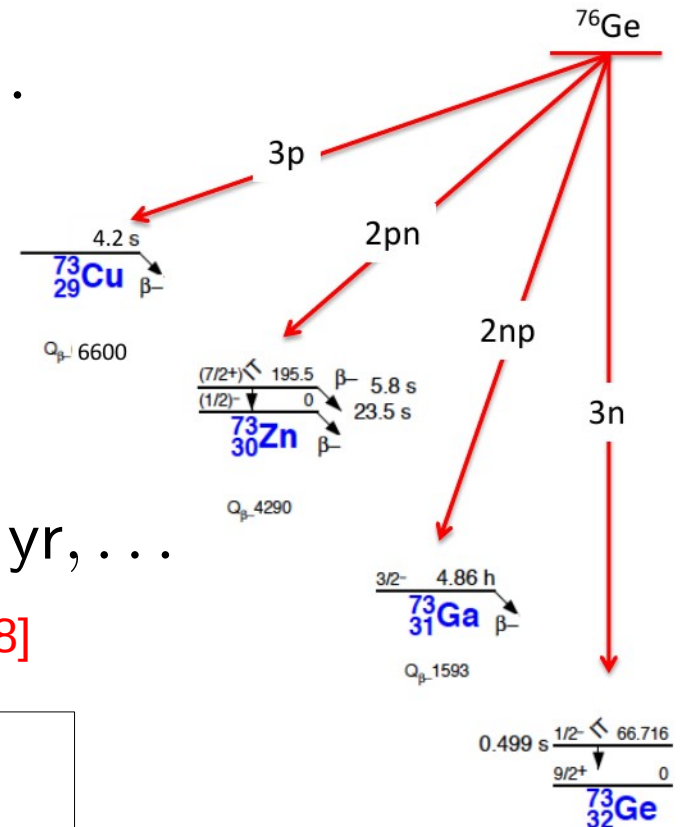
- Limits:

$$\tau(^{73}\text{Ge}(pnn) \rightarrow ^{70}\text{Ga} e^+ \pi^0) > 7 \times 10^{23} \text{ yr},$$

$$\tau(^{76}\text{Ge}(ppn) \rightarrow ^{73}\text{Zn} e^+ \pi^+) > 5 \times 10^{25} \text{ yr},$$

$$\tau(^{76}\text{Ge}(ppp) \rightarrow ^{73}\text{Cu} e^+ \pi^+ \pi^+) > 5 \times 10^{25} \text{ yr}, \dots$$

[Majorana Demonstrator, PRD '19; see also EXO-200, '18]



SK, JUNO, DUNE, HK?

$$\begin{aligned}
\mathcal{O}_{7,(1,-1)}^1 &\equiv \bar{H}ddQ\bar{e}, \\
\mathcal{O}_{7,(1,-1)}^2 &\equiv \bar{H}dQQ\bar{L}, \\
\mathcal{O}_{7,(1,-1)}^3 &\equiv \bar{H}ddu\bar{L}, \\
\mathcal{O}_{7,(1,-1)}^4 &\equiv Hddd\bar{L}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{8,(1,1)}^1 &\equiv HHddQL, \\
\mathcal{O}_{8,(1,1)}^2 &\equiv HHdQQe, \\
\mathcal{O}_{8,(1,1)}^3 &\equiv \bar{H}\bar{H}uuQL, \\
\mathcal{O}_{8,(1,1)}^4 &\equiv H\bar{H}QQQL, \\
\mathcal{O}_{8,(1,1)}^5 &\equiv H\bar{H}duQL, \\
\mathcal{O}_{8,(1,1)}^6 &\equiv H\bar{H}uQQe, \\
\mathcal{O}_{8,(1,1)}^7 &\equiv H\bar{H}duue,
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{9,(1,-1)}^1 &\equiv dddd\bar{e}, & \mathcal{O}_{9,(1,-1)}^{39} &\equiv H\bar{H}\bar{H}ddQ\bar{e}, \\
\mathcal{O}_{9,(1,-1)}^2 &\equiv ddde\bar{e}, & \mathcal{O}_{9,(1,-1)}^{40} &\equiv H\bar{H}\bar{H}dQQ\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^3 &\equiv dddQ\bar{Q}\bar{e}, & \mathcal{O}_{9,(1,-1)}^{41} &\equiv H\bar{H}\bar{H}ddu\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^4 &\equiv dd\bar{u}QQ\bar{e}, & \mathcal{O}_{9,(1,-1)}^{42} &\equiv HH\bar{H}ddd\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^5 &\equiv dddu\bar{u}\bar{e}, & \mathcal{O}_{9,(1,-1)}^{43} &\equiv \bar{H}\bar{H}\bar{H}QQQ\bar{e}, \\
\mathcal{O}_{9,(1,-1)}^6 &\equiv dddd\bar{Q}\bar{L}, & \mathcal{O}_{9,(1,-1)}^{44} &\equiv \bar{H}\bar{H}\bar{H}uQQ\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^7 &\equiv ddd\bar{e}L\bar{L}, & \mathcal{O}_{9,(1,3)}^1 &\equiv uuQLLL, \\
\mathcal{O}_{9,(1,-1)}^8 &\equiv ddQe\bar{e}\bar{L}, & \mathcal{O}_{9,(1,3)}^2 &\equiv uuueLL, \\
\mathcal{O}_{9,(1,-1)}^9 &\equiv ddQL\bar{L}\bar{L}, & \mathcal{O}_{9,(2,0)}^1 &\equiv ddQQQQ, \\
\mathcal{O}_{9,(1,-1)}^{10} &\equiv dQQe\bar{L}\bar{L}, & \mathcal{O}_{9,(2,0)}^2 &\equiv ddduQQ, \\
\mathcal{O}_{9,(1,-1)}^{11} &\equiv ddue\bar{L}\bar{L}, & \mathcal{O}_{9,(2,0)}^3 &\equiv dddduu \\
\mathcal{O}_{9,(1,-1)}^{12} &\equiv ddQQ\bar{Q}\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^{13} &\equiv dddu\bar{Q}\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^{14} &\equiv d\bar{u}QQQ\bar{L}, \\
\mathcal{O}_{9,(1,-1)}^{15} &\equiv ddu\bar{u}Q\bar{L},
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{10,(1,1)}^1 &\equiv Hdd\bar{d}QQL, & \mathcal{O}_{10,(1,1)}^{16} &\equiv \bar{H}duQ\bar{e}LL, & \mathcal{O}_{10,(1,1)}^{31} &\equiv HdQQLL\bar{L}, & \mathcal{O}_{10,(1,1)}^{46} &\equiv Hduu\bar{u}Qe, \\
\mathcal{O}_{10,(1,1)}^2 &\equiv Hdd\bar{d}QQQe, & \mathcal{O}_{10,(1,1)}^{17} &\equiv \bar{H}uQQe\bar{e}L, & \mathcal{O}_{10,(1,1)}^{32} &\equiv HQQQeL\bar{L}, & \mathcal{O}_{10,(1,1)}^{93} &\equiv HHH\bar{H}ddQL, \\
\mathcal{O}_{10,(1,1)}^3 &\equiv Hddd\bar{d}uL, & \mathcal{O}_{10,(1,1)}^{18} &\equiv \bar{H}duue\bar{e}L, & \mathcal{O}_{10,(1,1)}^{33} &\equiv HdduLL\bar{L}, & \mathcal{O}_{10,(1,1)}^{94} &\equiv HHH\bar{H}dQQe, \\
\mathcal{O}_{10,(1,1)}^4 &\equiv Hdd\bar{d}uQe, & \mathcal{O}_{10,(1,1)}^{19} &\equiv \bar{H}uuQee\bar{e}, & \mathcal{O}_{10,(1,1)}^{34} &\equiv HduQeL\bar{L}, & \mathcal{O}_{10,(1,1)}^{95} &\equiv H\bar{H}\bar{H}\bar{H}uuQL, \\
\mathcal{O}_{10,(1,1)}^5 &\equiv HddQ\bar{e}LL, & \mathcal{O}_{10,(1,1)}^{20} &\equiv \bar{H}uQQLL\bar{L}, & \mathcal{O}_{10,(1,1)}^{35} &\equiv HuQQee\bar{L}, & \mathcal{O}_{10,(1,1)}^{96} &\equiv HHH\bar{H}QQQL, \\
\mathcal{O}_{10,(1,1)}^6 &\equiv HdQQe\bar{e}L, & \mathcal{O}_{10,(1,1)}^{21} &\equiv \bar{H}duuLL\bar{L}, & \mathcal{O}_{10,(1,1)}^{36} &\equiv Hduue\bar{L}, & \mathcal{O}_{10,(1,1)}^{97} &\equiv HHH\bar{H}\bar{H}duQL, \\
\mathcal{O}_{10,(1,1)}^7 &\equiv HQQQee\bar{e}, & \mathcal{O}_{10,(1,1)}^{22} &\equiv \bar{H}uuQeL\bar{L}, & \mathcal{O}_{10,(1,1)}^{37} &\equiv HdQQQ\bar{Q}L, & \mathcal{O}_{10,(1,1)}^{98} &\equiv HHH\bar{H}\bar{H}uQQe, \\
\mathcal{O}_{10,(1,1)}^8 &\equiv Hddue\bar{e}L, & \mathcal{O}_{10,(1,1)}^{23} &\equiv \bar{H}uuue\bar{L}, & \mathcal{O}_{10,(1,1)}^{38} &\equiv HQQQQ\bar{Q}e, & \mathcal{O}_{10,(1,1)}^{99} &\equiv HHH\bar{H}\bar{H}duue, \\
\mathcal{O}_{10,(1,1)}^9 &\equiv HduQee\bar{e}, & \mathcal{O}_{10,(1,1)}^{24} &\equiv \bar{H}uQQQ\bar{Q}L, & \mathcal{O}_{10,(1,1)}^{39} &\equiv HdduQ\bar{Q}L, & & \\
\mathcal{O}_{10,(1,1)}^{10} &\equiv \bar{H}\bar{d}QQQQL, & \mathcal{O}_{10,(1,1)}^{25} &\equiv \bar{H}duuQ\bar{Q}L, & \mathcal{O}_{10,(1,1)}^{40} &\equiv HduQQ\bar{Q}e, & \mathcal{O}_{10,(1,-3)}^1 &\equiv \bar{H}ddd\bar{L}\bar{L}\bar{L}, \\
\mathcal{O}_{10,(1,1)}^{11} &\equiv \bar{H}d\bar{d}uQQL, & \mathcal{O}_{10,(1,1)}^{26} &\equiv \bar{H}uuQQ\bar{Q}e, & \mathcal{O}_{10,(1,1)}^{41} &\equiv Hdduu\bar{Q}e, & & \\
\mathcal{O}_{10,(1,1)}^{12} &\equiv \bar{H}\bar{d}uQQQe, & \mathcal{O}_{10,(1,1)}^{27} &\equiv \bar{H}duuu\bar{Q}e, & \mathcal{O}_{10,(1,1)}^{42} &\equiv H\bar{u}QQQQL, & & \\
\mathcal{O}_{10,(1,1)}^{13} &\equiv \bar{H}ddd\bar{d}uL, & \mathcal{O}_{10,(1,1)}^{28} &\equiv \bar{H}uu\bar{u}QQL, & \mathcal{O}_{10,(1,1)}^{43} &\equiv Hdu\bar{u}QQL, & & \\
\mathcal{O}_{10,(1,1)}^{14} &\equiv \bar{H}d\bar{d}uuQe, & \mathcal{O}_{10,(1,1)}^{29} &\equiv \bar{H}duuu\bar{u}L, & \mathcal{O}_{10,(1,1)}^{44} &\equiv Hu\bar{u}QQQe, & & \\
\mathcal{O}_{10,(1,1)}^{15} &\equiv \bar{H}QQQ\bar{e}LL, & \mathcal{O}_{10,(1,1)}^{30} &\equiv \bar{H}uuu\bar{u}Qe, & \mathcal{O}_{10,(1,1)}^{45} &\equiv Hdduu\bar{u}L, & &
\end{aligned}$$