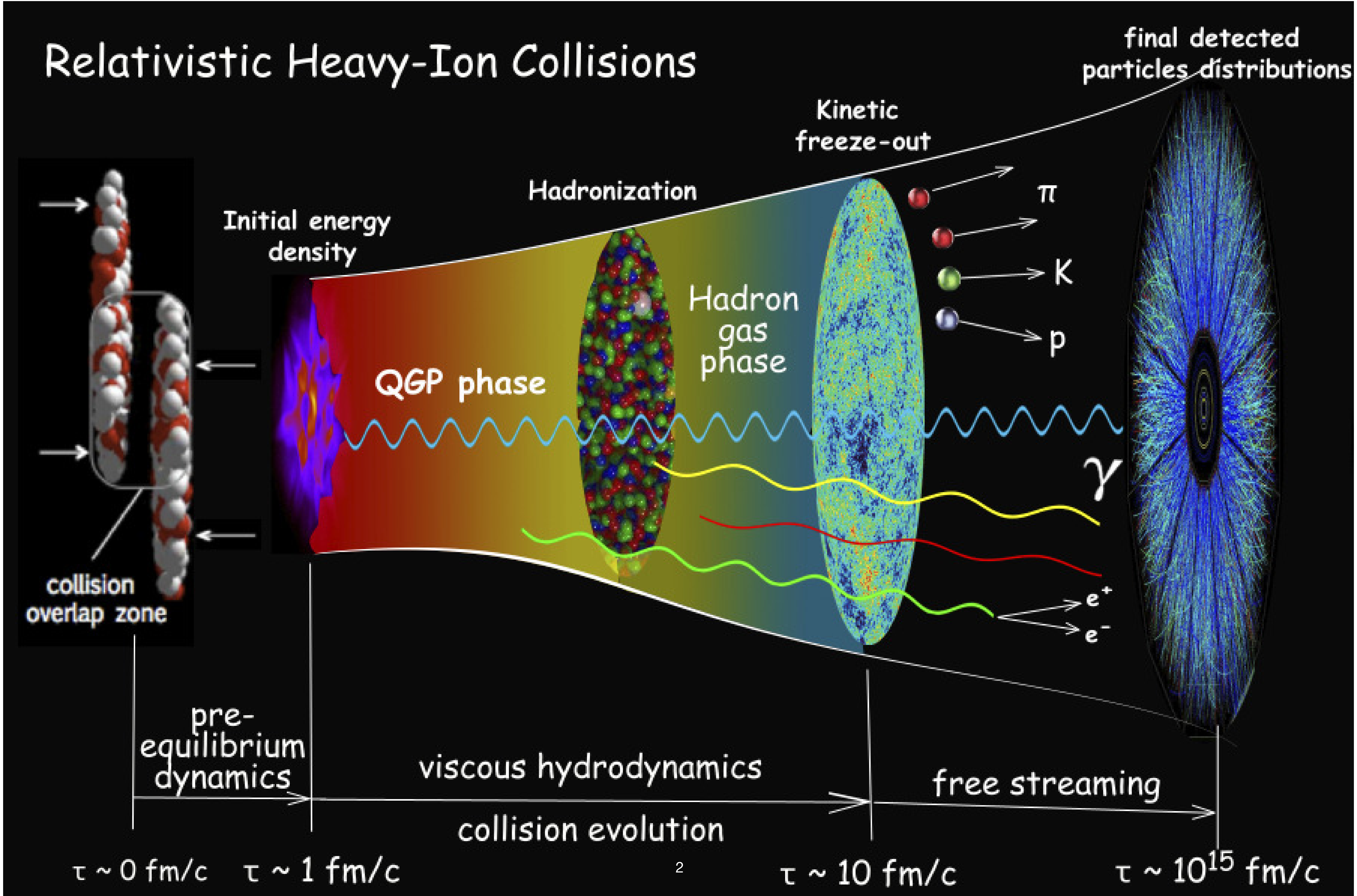
Quarkonium transport in weakly and strongly coupled plasmas

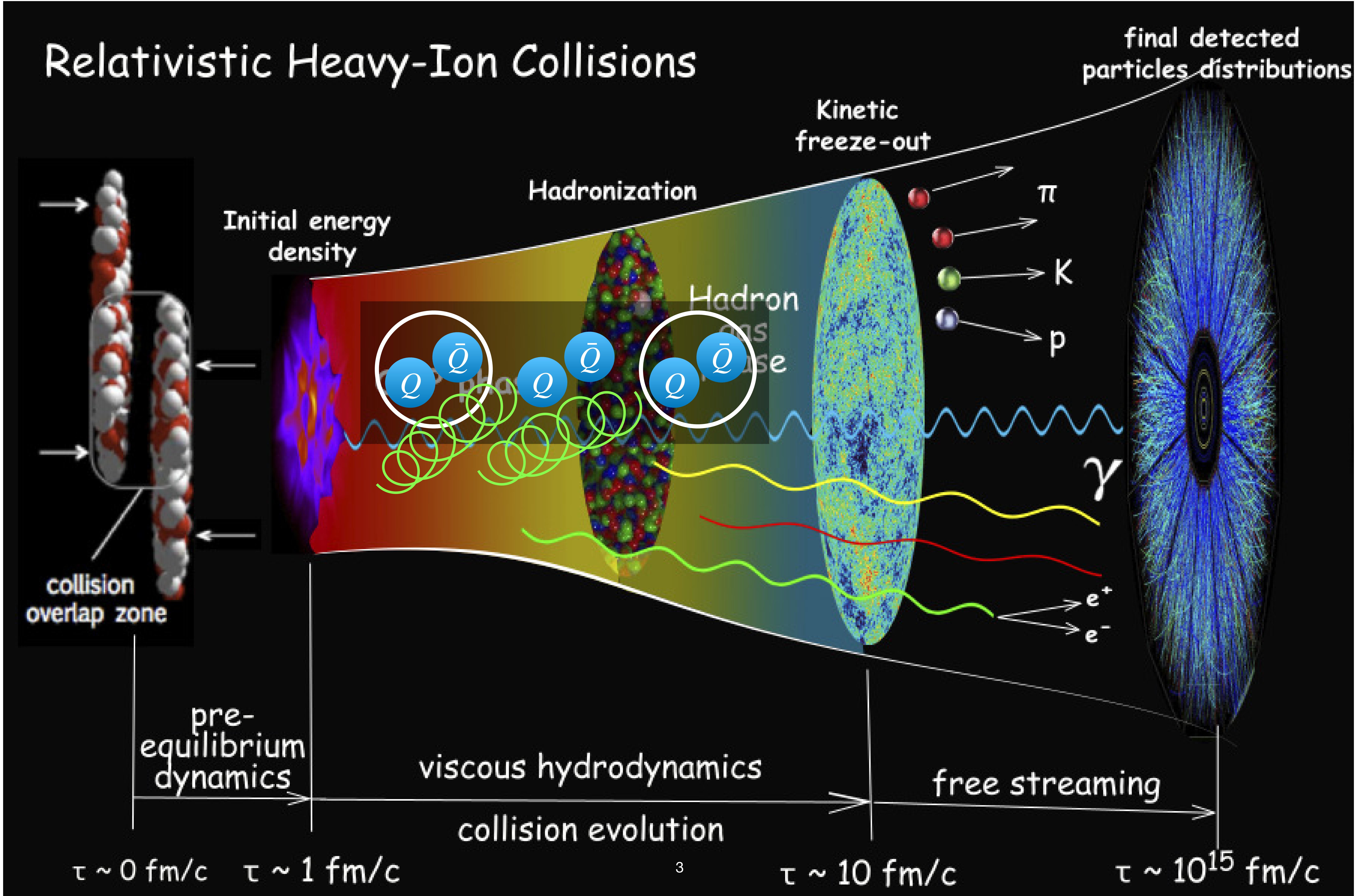
Heavy Ion Physics in the EIC Era
Institute for Nuclear Theory, University of Washington
August 6, 2024

Bruno Scheihing Hitschfeld

based on 2107.03945, 2205.04477, 2304.03298, 2306.13127, 2310.09325, 2312.12307



credit: Paul Sorensen and Chun Shen



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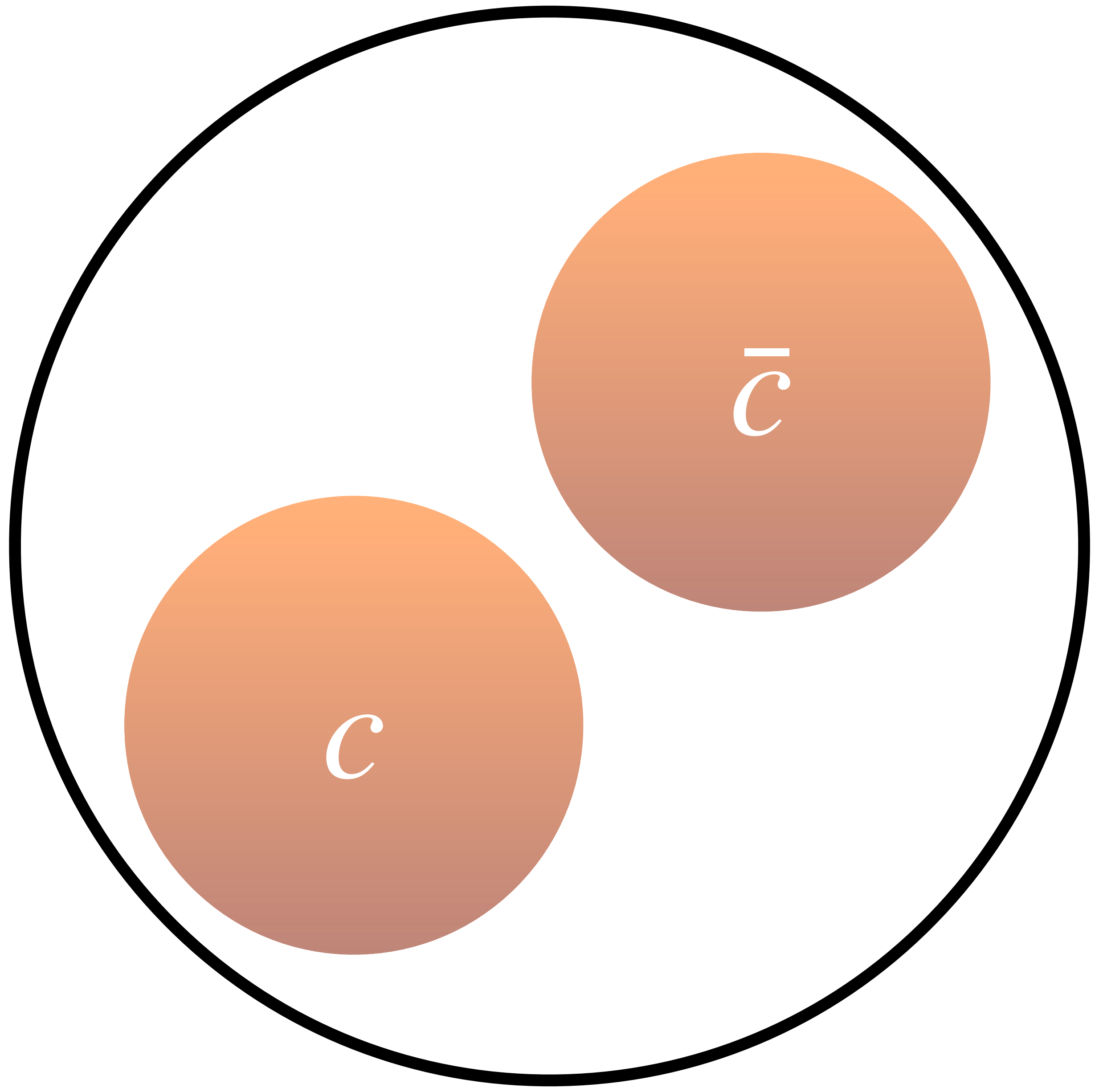
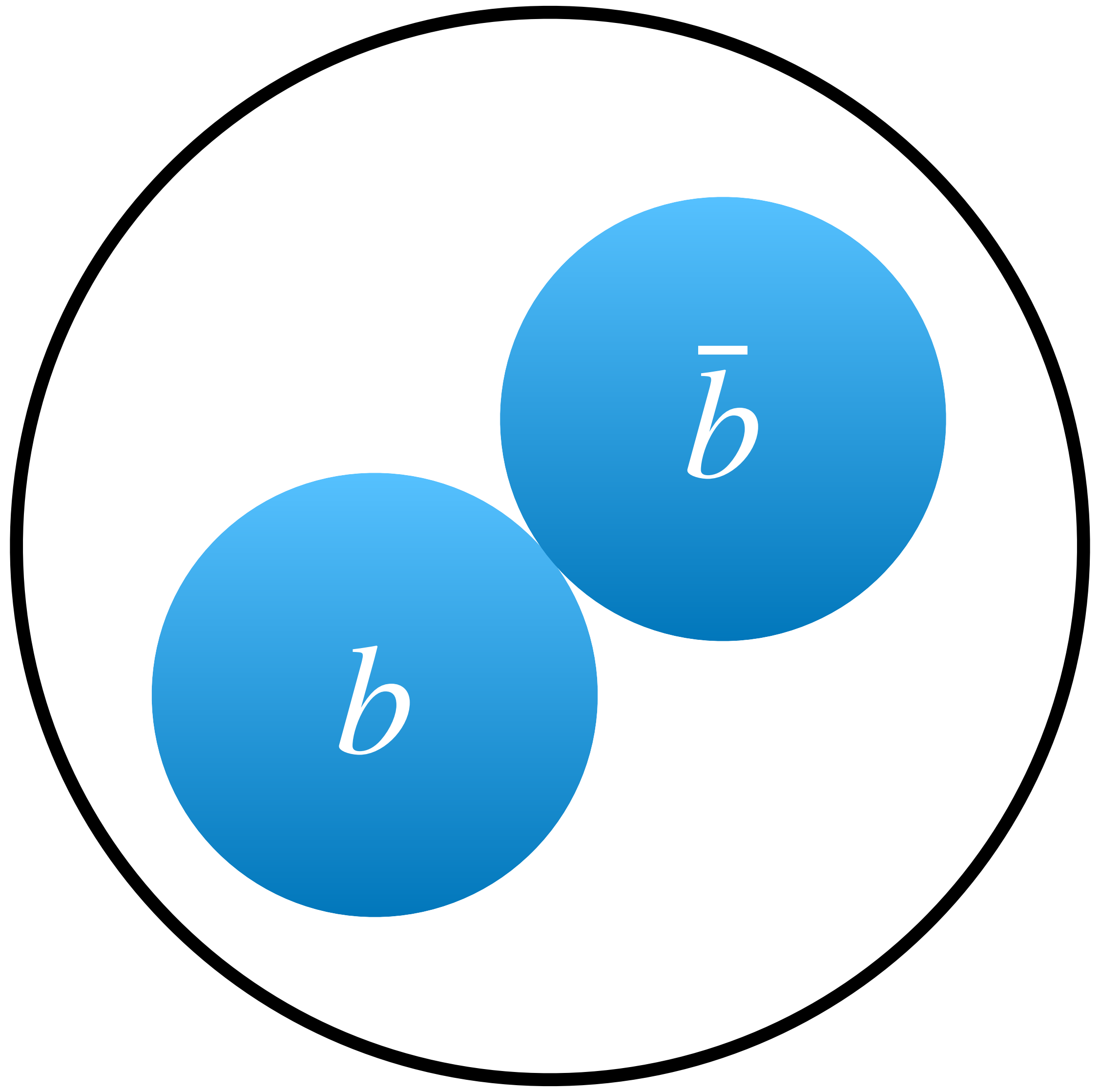
Quarkonium: charmonium and bottomonium





$$m(J/\psi(1S)) = 3.097\,\text{GeV}$$
$$m(\psi(2S)) = 3.686\,\text{GeV}$$

$$m(\Upsilon(1S)) = 9.460\,\text{GeV}$$
$$m(\Upsilon(2S)) = 10.023\,\text{GeV}$$
$$m(\Upsilon(3S)) = 10.355\,\text{GeV}$$

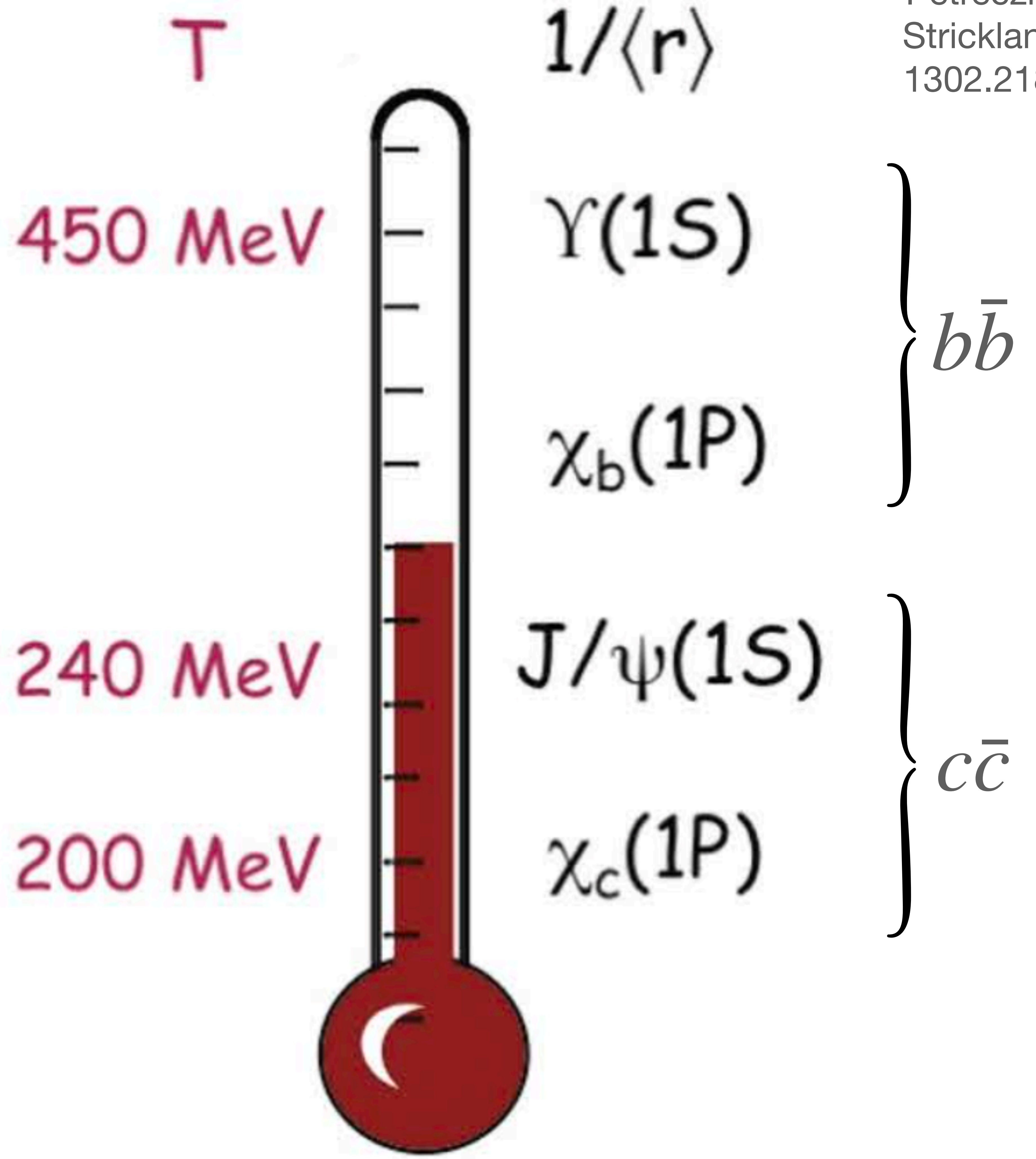
The (vacuum) decay widths of these quarkonium states satisfy

$$\Gamma((Q\bar{Q})_b \to X) \ll \tau_{\text{QGP}}^{-1}.$$

credit: Mocsy, Petreczky, Strickland, 1302.2180

Quarkonium in Heavy-Ion Collisions

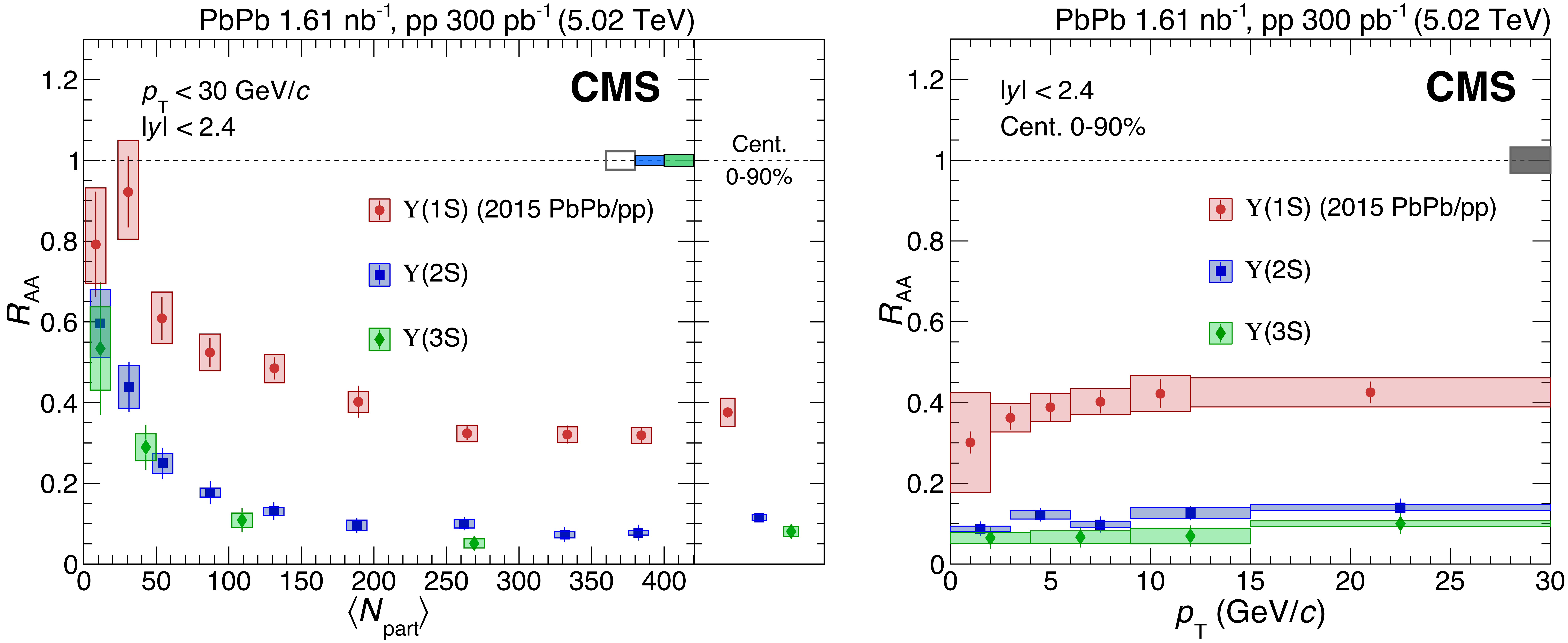
- Heavy quarks (HQ) and quarkonia are amongst the most informative probes of QGP.
- They are produced in the initial hard scattering, and never fully thermalize due to their large masses.
- To interpret the full wealth of data stemming from b and c quarks in heavy-ion collisions, we need a precise theoretical understanding of heavy quarks in a thermal medium.



Note that the decay rate of states with a single b, c quark will be much smaller than $\tau_{\rm QGP}^{-1} \sim 20\,{\rm MeV}$, as they proceed through electroweak interactions.

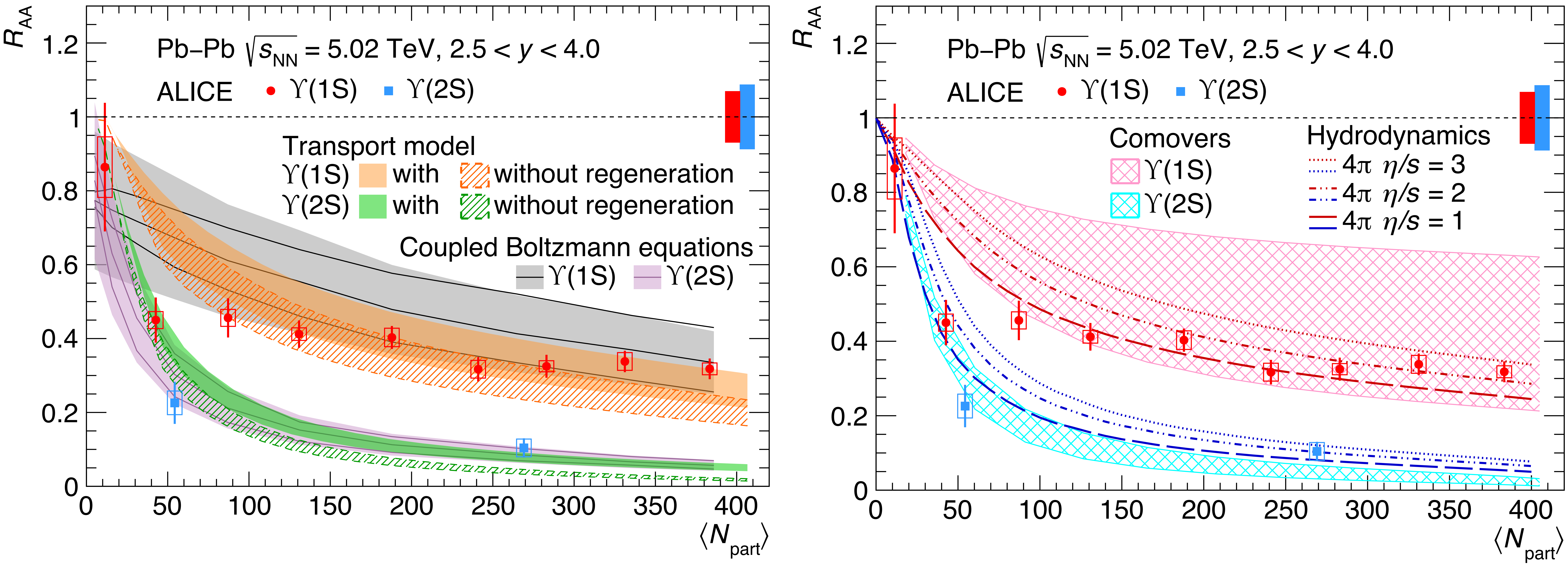
Quarkonium suppression

results from the CMS collaboration (2303.17026)



Quarkonium suppression

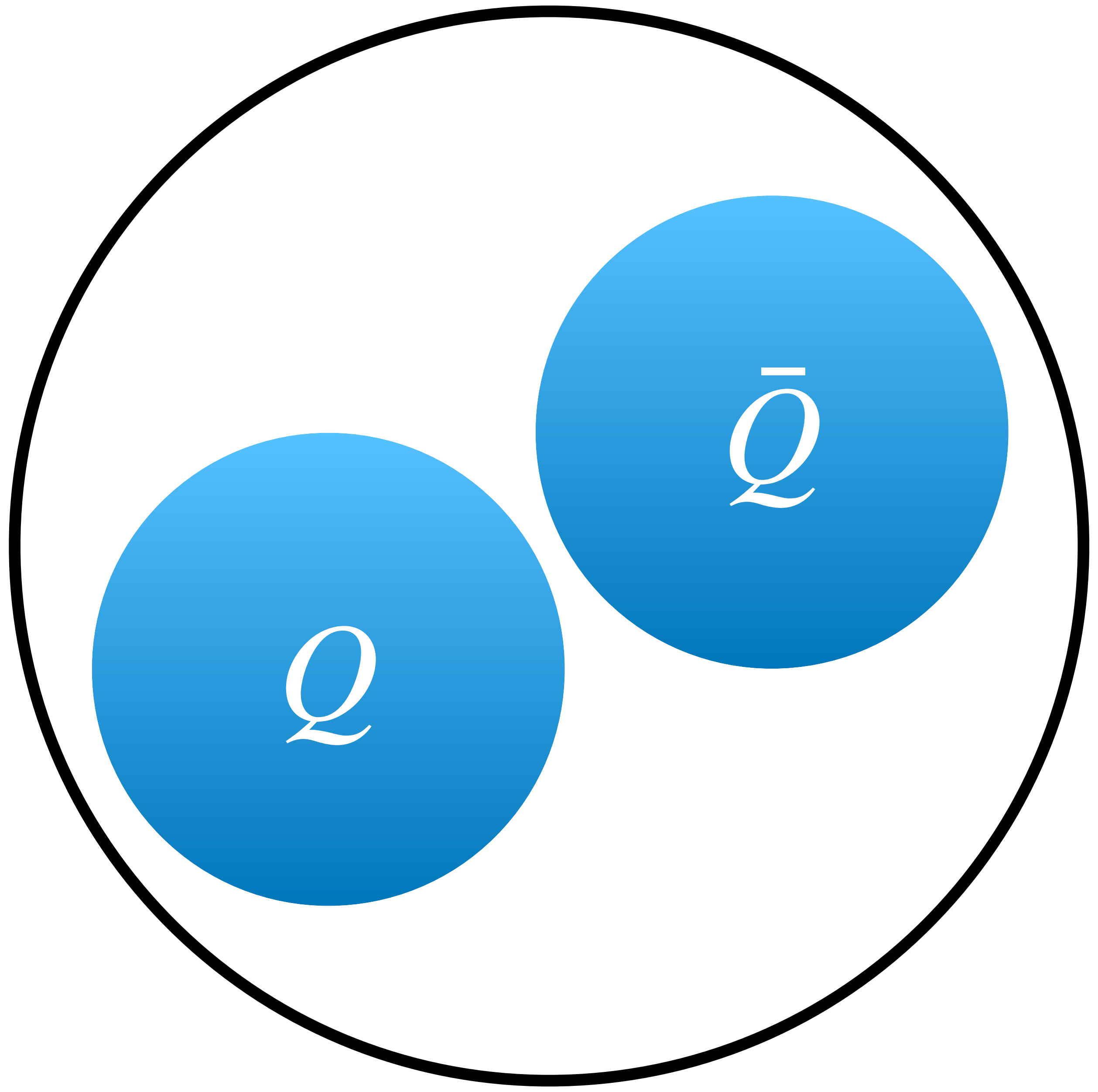
comparison to some models (2011.05758)



What are the energy scales of quarkonia?

M: heavy quark mass

v: typical relative velocity



$r \sim (Mv)^{-1}$

$\Delta E \approx 500\,\text{MeV}$

$\sim Mv^2$

$Mv(c\bar{c}) \approx 1\,\text{GeV}$

$Mv(b\bar{b}) \approx 1.5\,\text{GeV}$

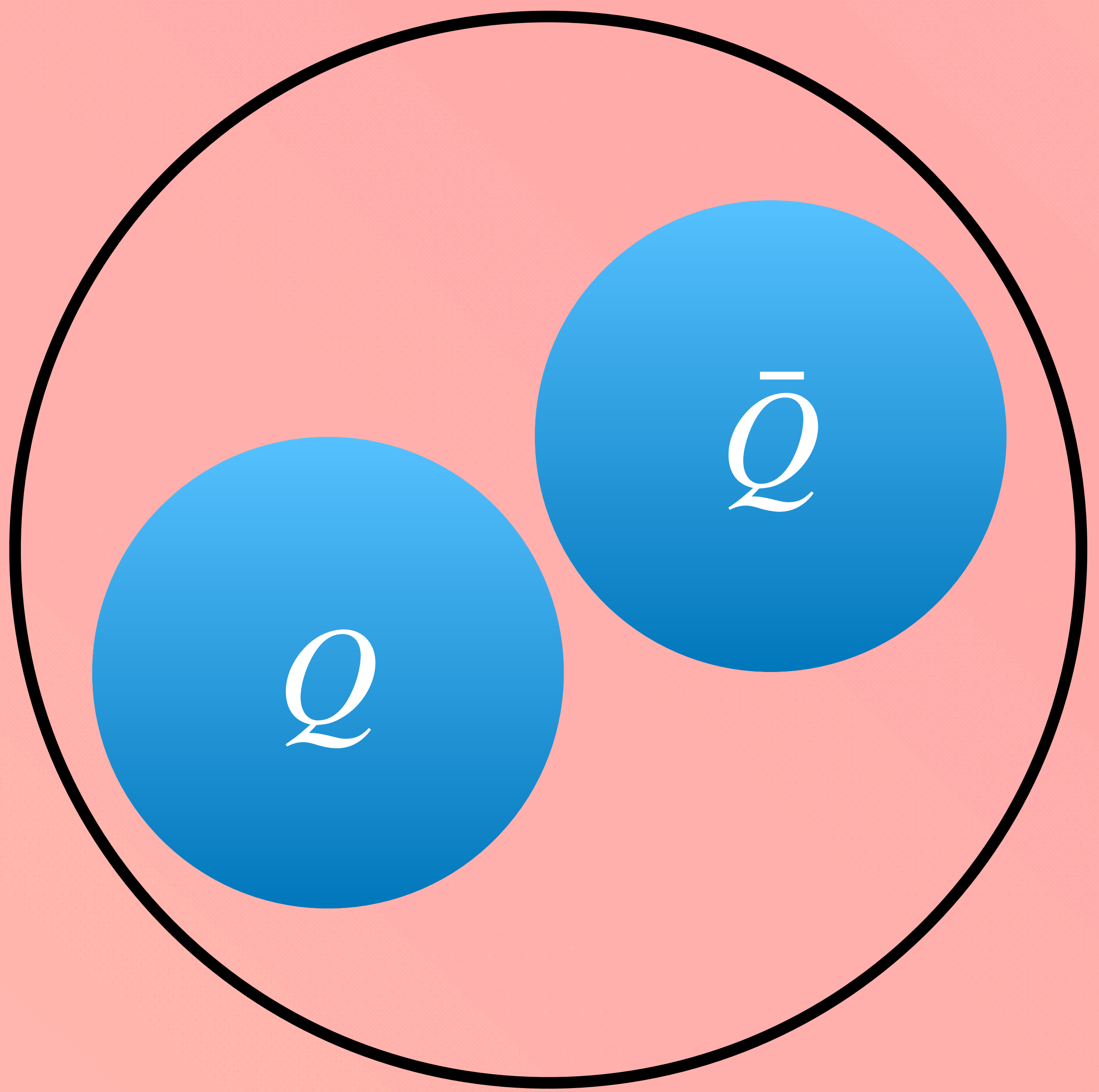
$M(c) \approx 1.5\,\text{GeV}$

$M(b) \approx 4.5\,\text{GeV}$

M: heavy quark mass
v: typical relative velocity

$T \sim 200\,\mathrm{MeV}$
$T^{-1} \sim 1\,\mathrm{fm}$

$\Delta E \approx 500\,\mathrm{MeV}$
$\sim Mv^2$



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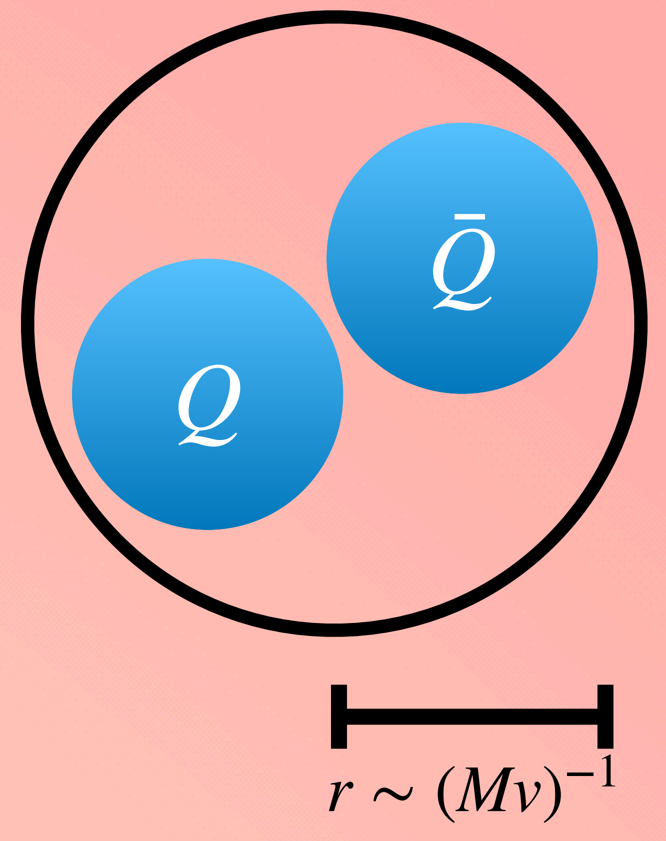

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$L \sim 5$ fm ~ nuclear radius of Au, Pb

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$\Longrightarrow$ For $b\bar{b}$, there is a clear hierarchy of scales

$$M \gg Mv \gg T$$

Let’s write an EFT for this system

(The longer way to go through this would be QCD —> NRQCD —> pNRQCD)

N. Brambilla, A. Pineda, J. Soto, and A. Vairo, hep-ph/9707481, hep-ph/9907240, hep-ph/0410047

The zeroth order Lagrangian

- At zeroth order on $rT \sim T/(Mv)$, the Lagrangian density is

$$\mathscr{L}^{(0)}_{\text{pNRQCD}}(\mathbf{x}, t) = \mathscr{L}_{\text{light QCD}}(\mathbf{x}, t) + \mathscr{L}_{Q\bar{Q}}(\mathbf{x}, t) \ ,$$

where

$$\mathscr{L}_{\text{light QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \sum_{I \in \{u,d,s\}} \bar{\psi}_I \left(i \not{D} - m_I \right) \psi_I \ ,$$

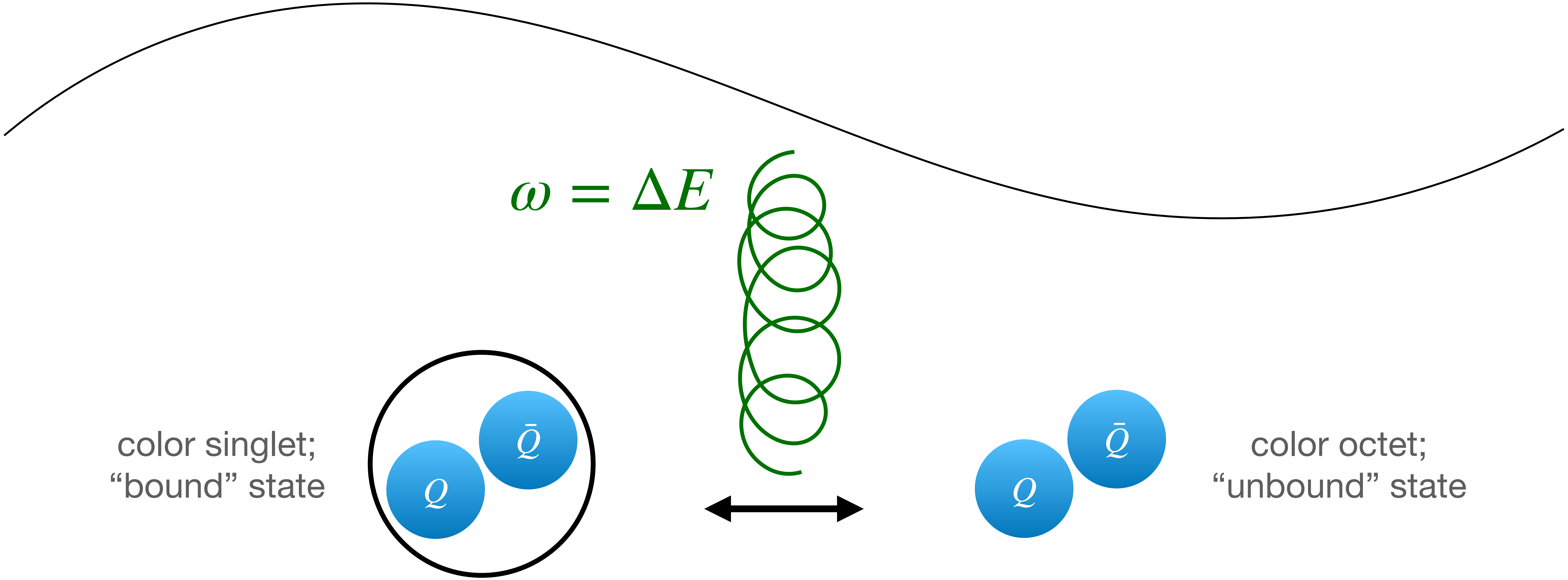
$$\mathscr{L}_{Q\bar{Q}} = \int d^3 r \, \text{Tr}_{\text{c}} \left[S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O \right].$$

- The operators $S = S(\mathbf{x}, \mathbf{r}, t)$, $O = T^a O^a(\mathbf{x}, \mathbf{r}, t)$ are annihilation operators for singlet and octet $Q\bar{Q}$ configurations, respectively.

[*] N. Brambilla, A. Pineda, J. Soto, A. Vairo hep-ph/9707481, hep-ph/9907240, hep-ph/0410047

Interactions from a multipole expansion

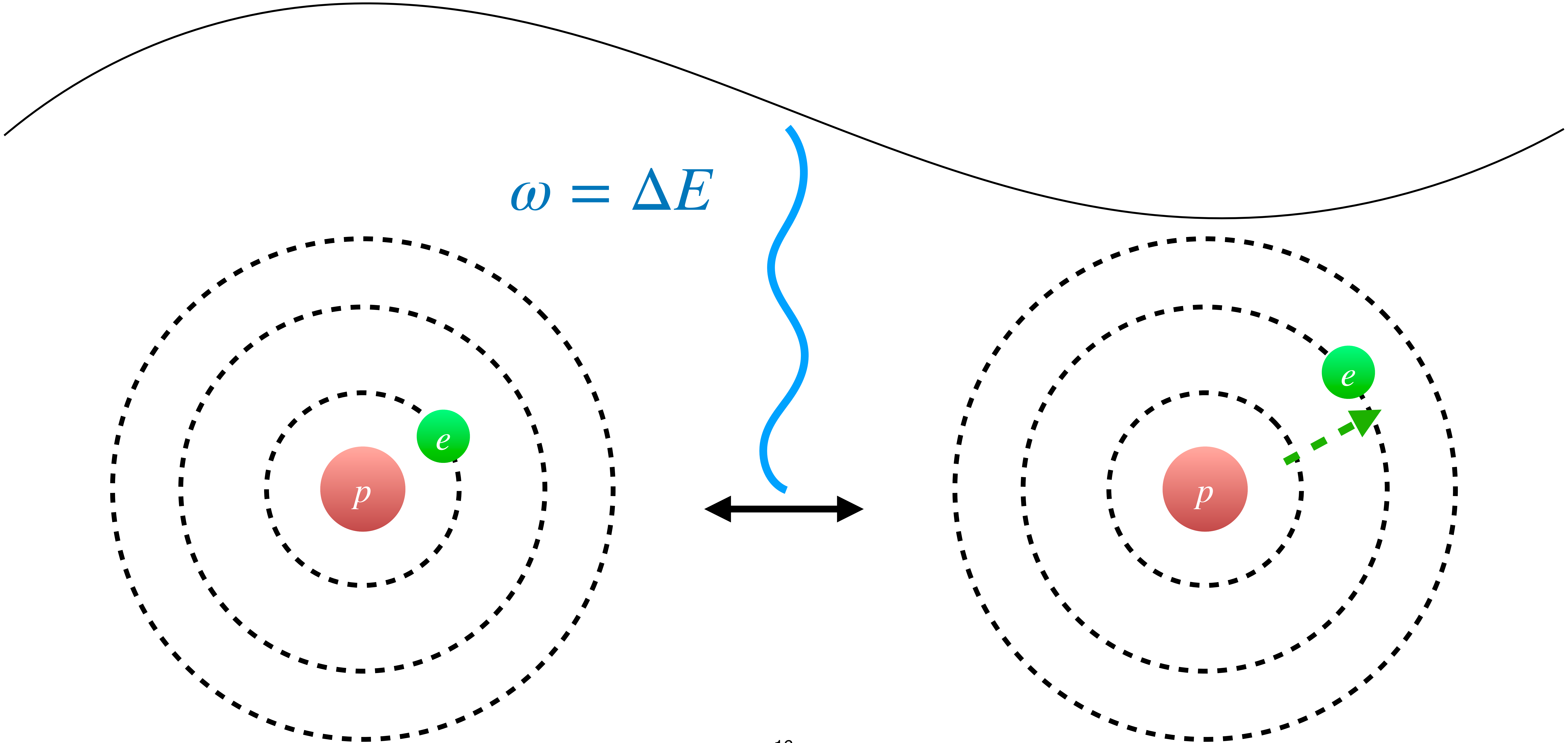
and the EFT that describes them (pNRQCD [*])



$$(Q\bar{Q})_s + g \longleftrightarrow (Q\bar{Q})_o \implies H_{\mathrm{int}} = S^\dagger r_i g E_i^a O^a + \mathrm{h.c.}$$

Analog situation

transitions in Hydrogen



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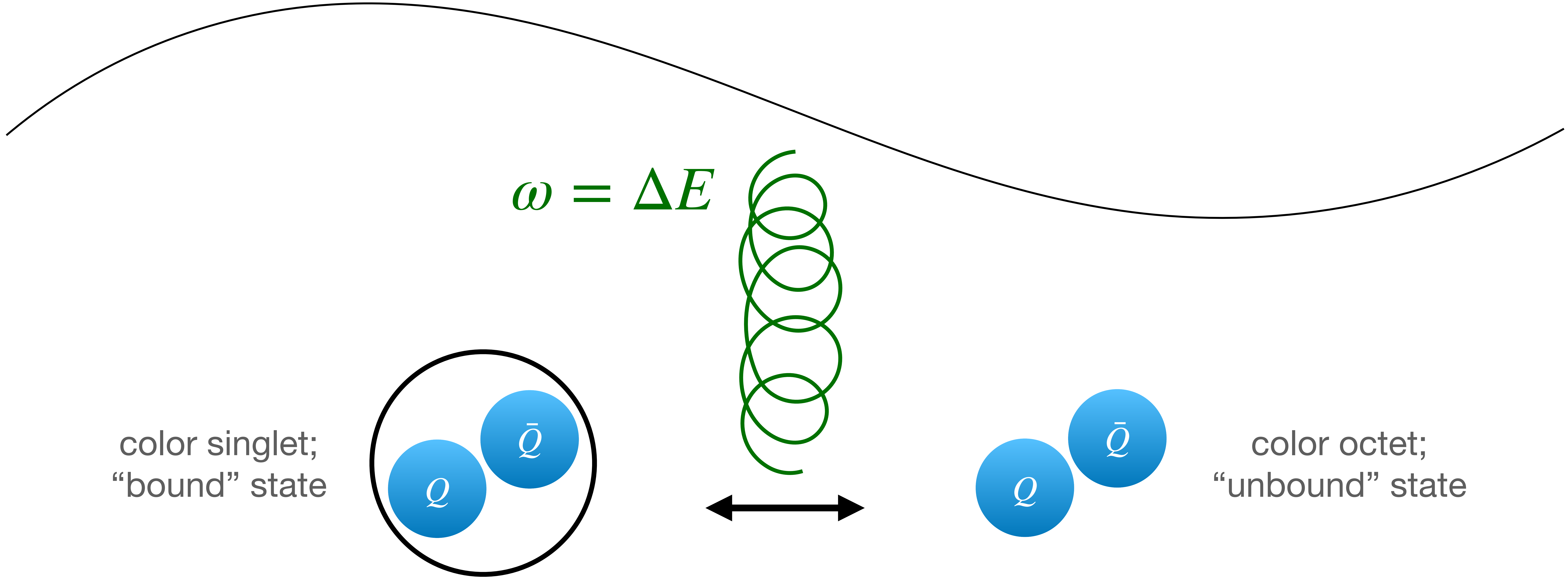

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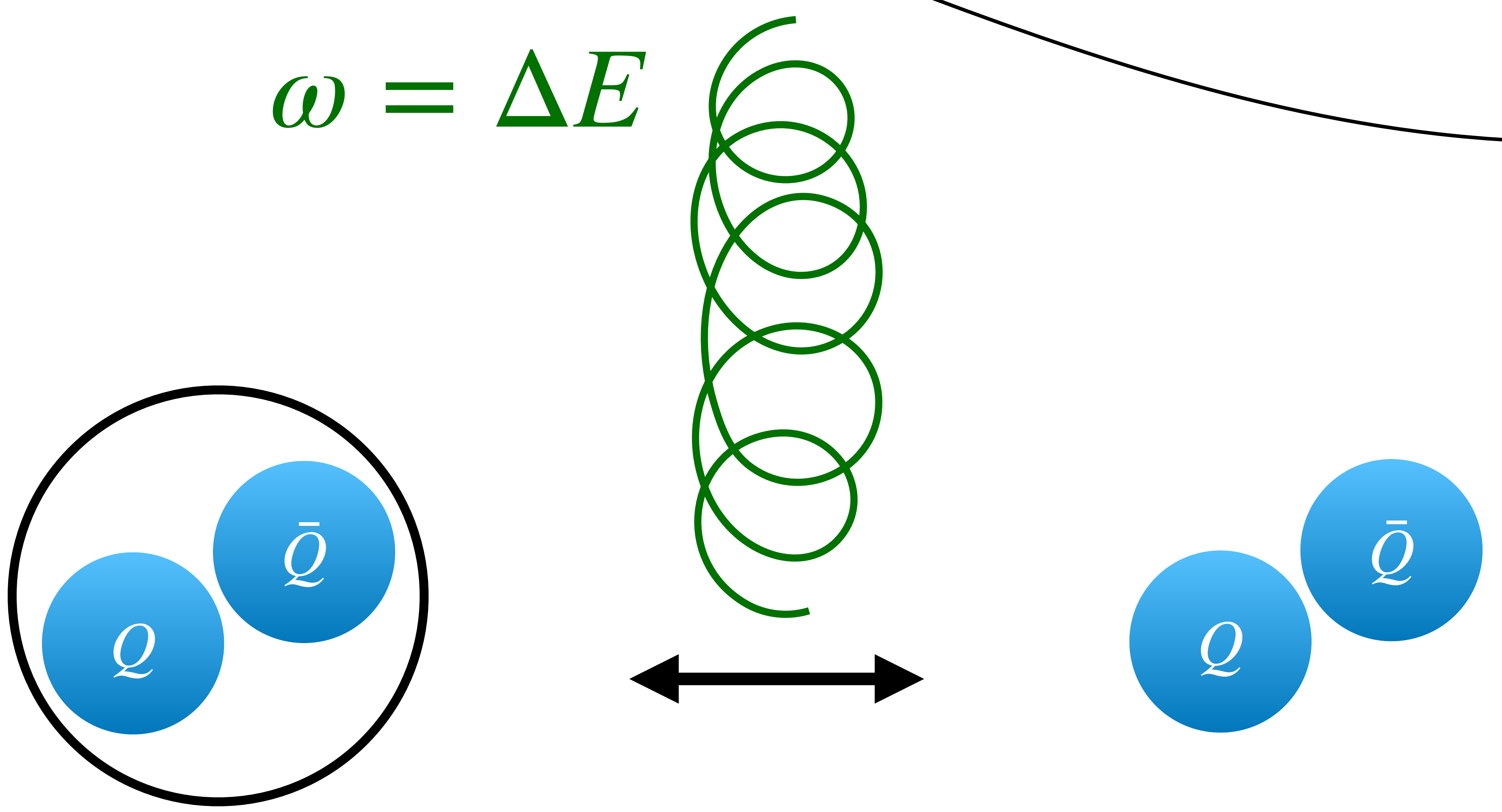


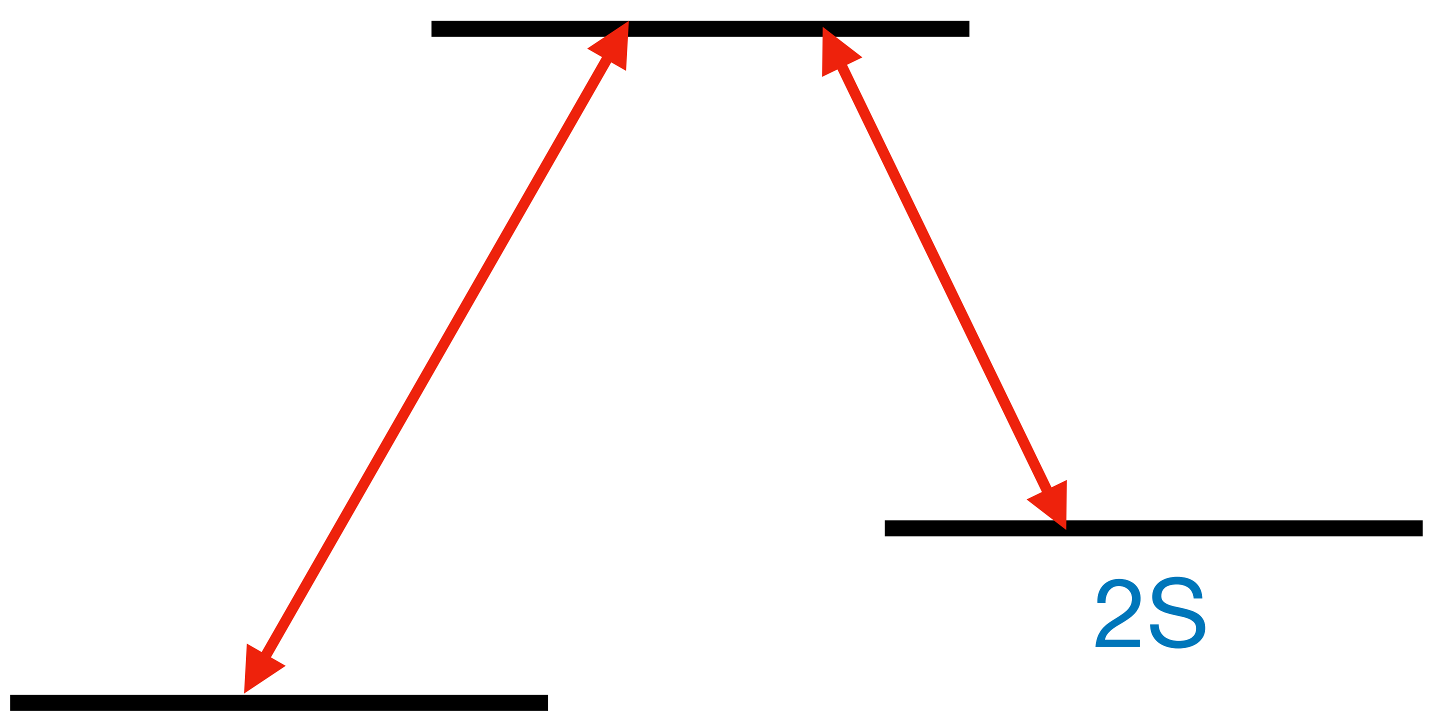
- These interactions can (re)combine and dissociate quarkonium as it propagates in the QGP
  - Crucial to predict final abundances in HICs!

(one also needs $E_{nl}(T)$)

$$(Q\bar{Q})_s + g \longleftrightarrow (Q\bar{Q})_o \implies H_{\text{int}} = S^\dagger r_i g E_i^a O^a + \text{h.c.}$$

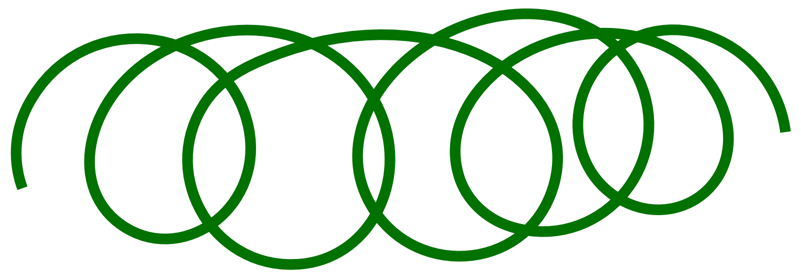
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim M v^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T \frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,QCD} + \int d^3 r {\rm Tr}_{\rm color} \Big[S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O$$

$$+ V_A \big(O^\dagger \mathbf{r} \cdot g\mathbf{E} S + {\rm h.c.} \big) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \cdots \Big]$$

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X. Yao, hep-ph/2102.01736

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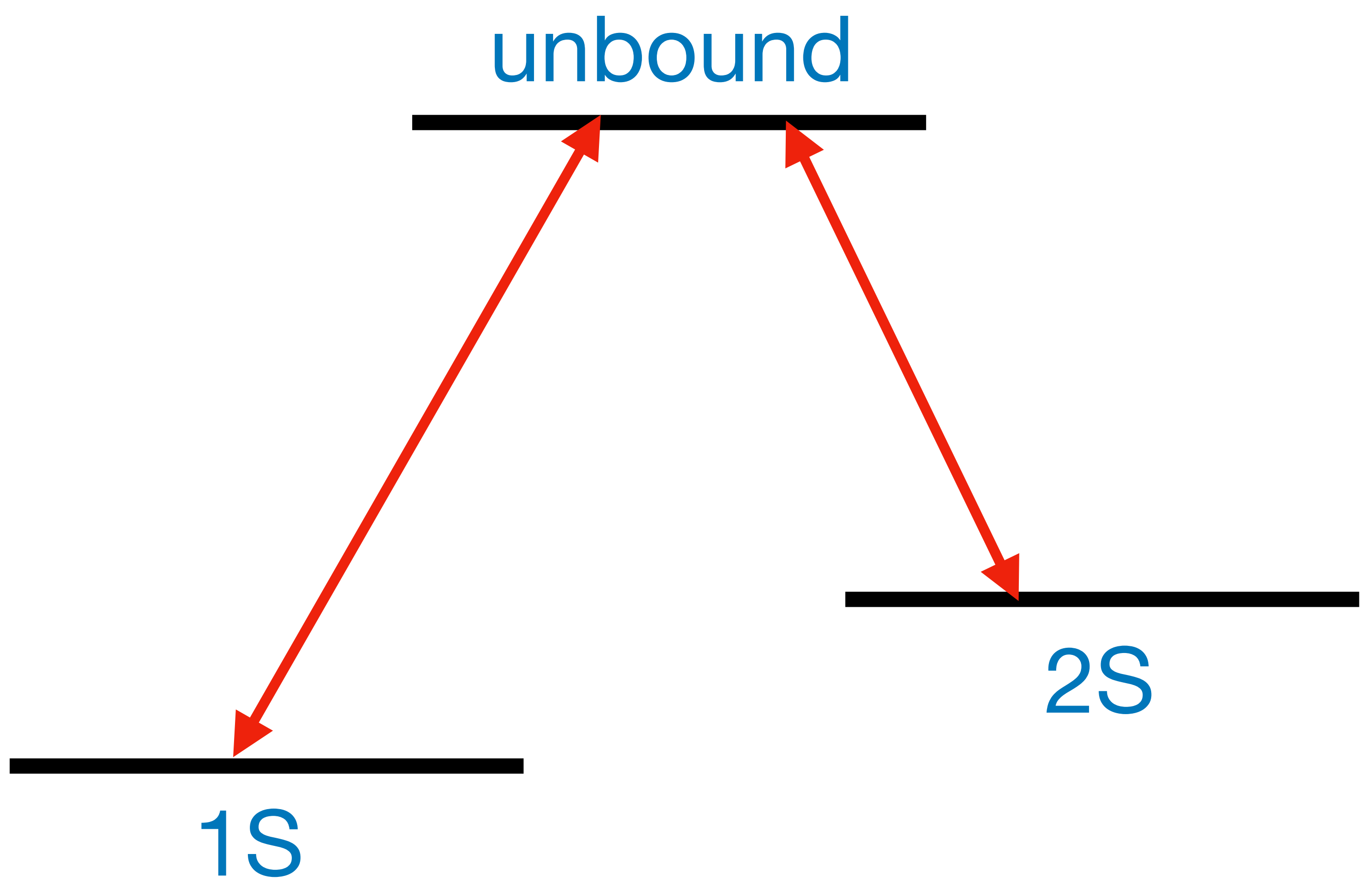

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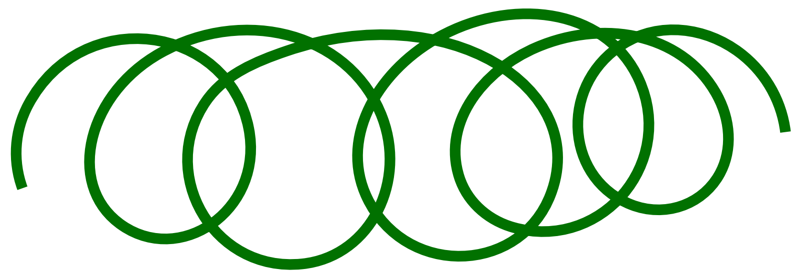

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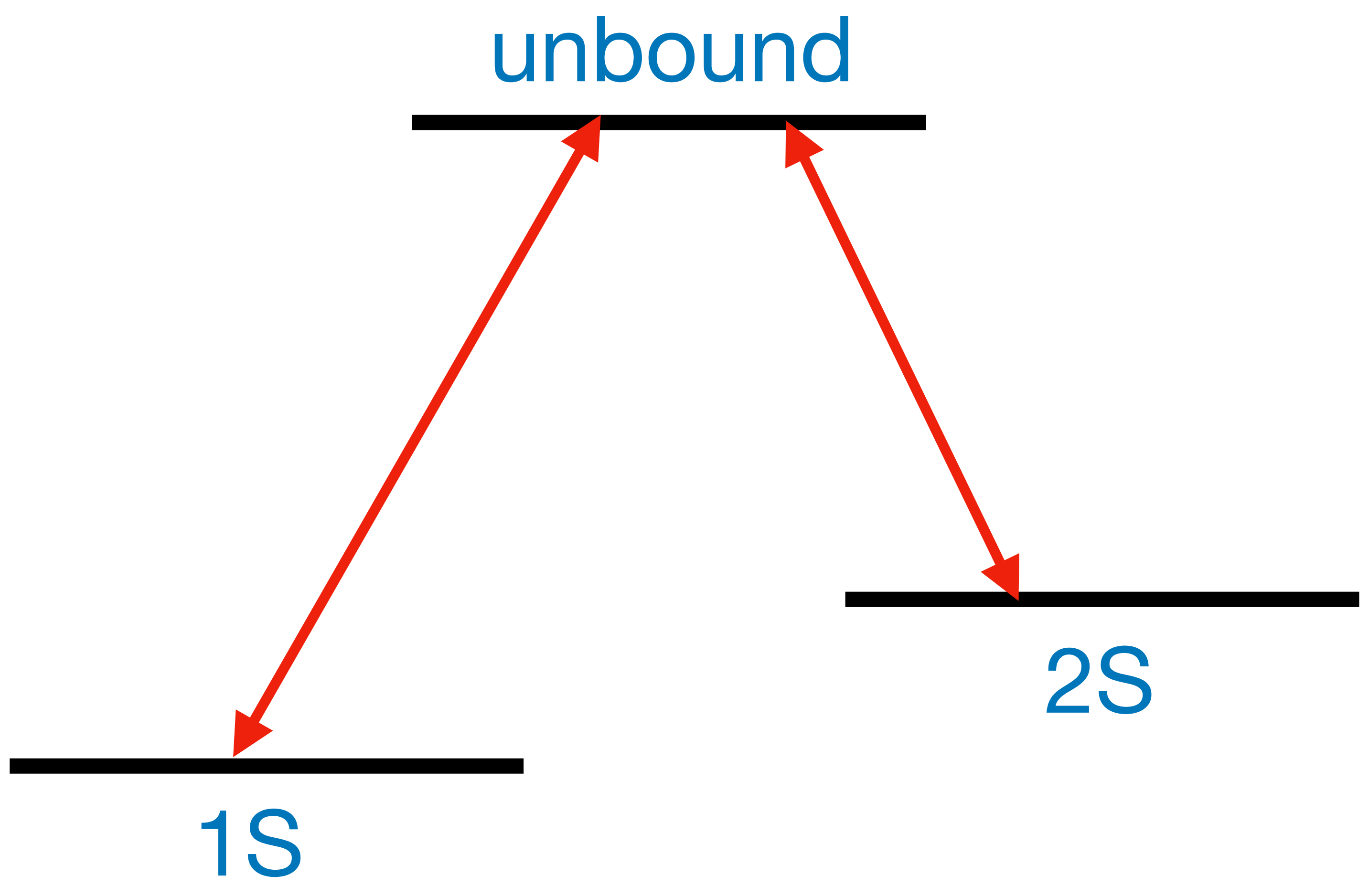

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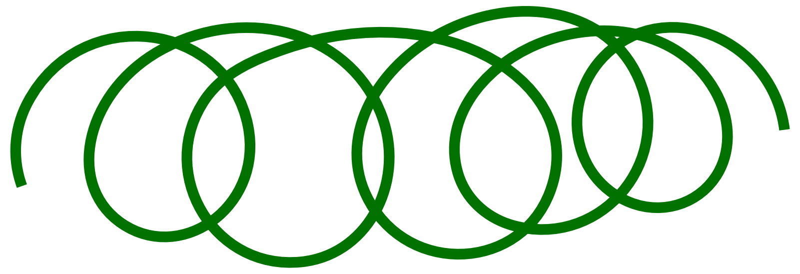

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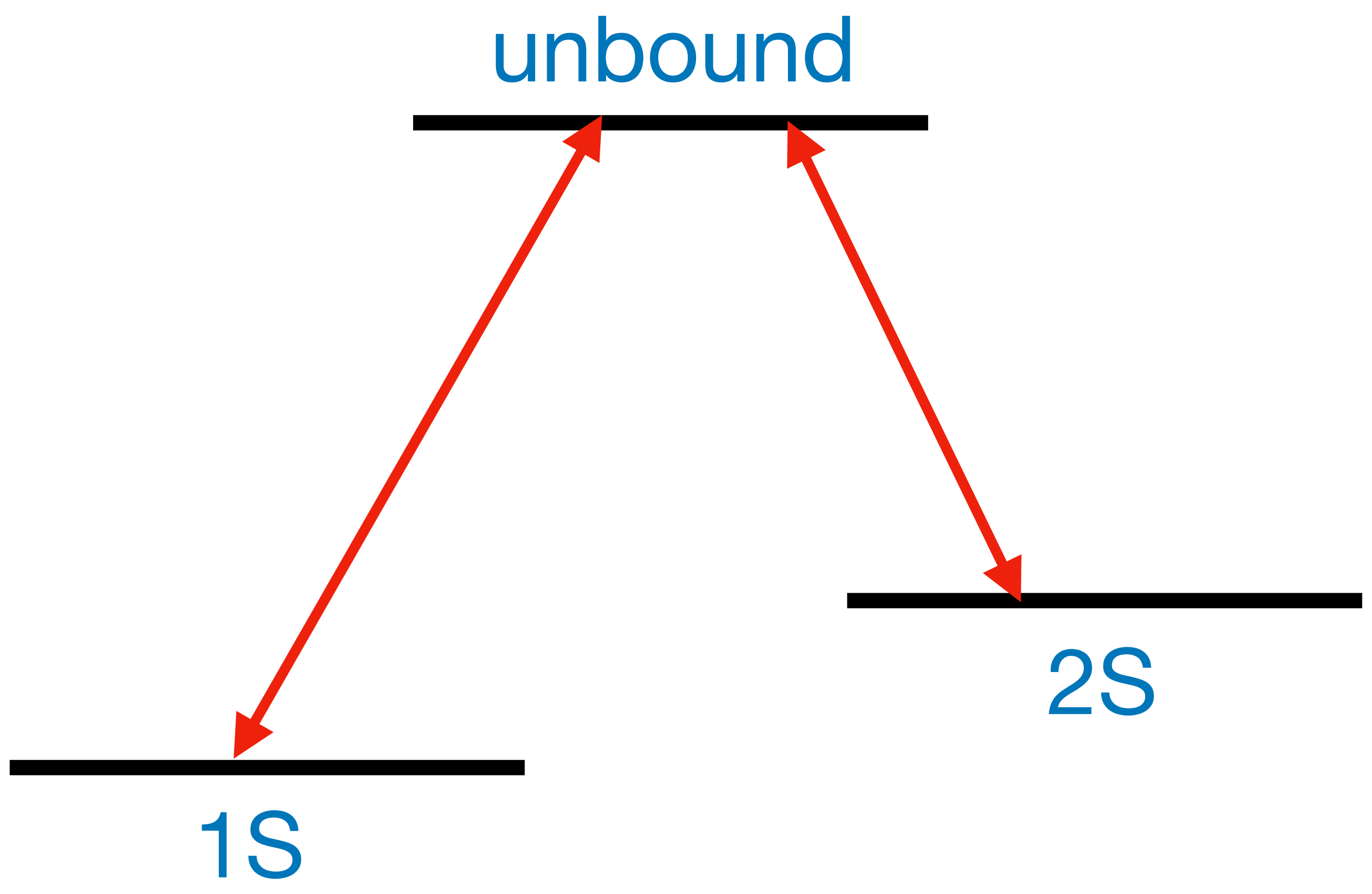

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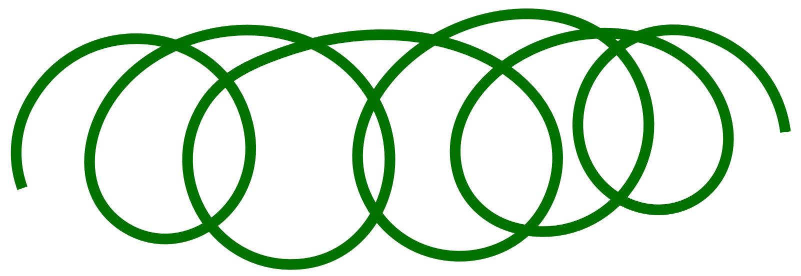

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Quarkonium as an open quantum system

isolating the observables of interest

- Given an initial density matrix $\rho_{\text{tot}}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\text{tot}}(t) = U(t)\rho_{\text{tot}}(t=0)U^{\dagger}(t).$$

- We will only be interested in describing the evolution of quarkonium and its final state abundances

$$\implies \rho_{Q\bar{Q}}(t) = \text{Tr}_{\text{QGP}}\left[U(t)\rho_{\text{tot}}(t=0)U^{\dagger}(t)\right].$$

- Then, one derives an evolution equation for $\rho_{Q\bar{Q}}(t)$, assuming that at the initial time we have $\rho_{\text{tot}}(t=0) = \rho_{Q\bar{Q}}(t=0) \otimes e^{-H_{\text{QGP}}/T}/\mathcal{Z}_{\text{QGP}}$.

What do we need to calculate?

What do we need to calculate from QFT?

Non-perturbative generalization of Peskin-Bhanot process

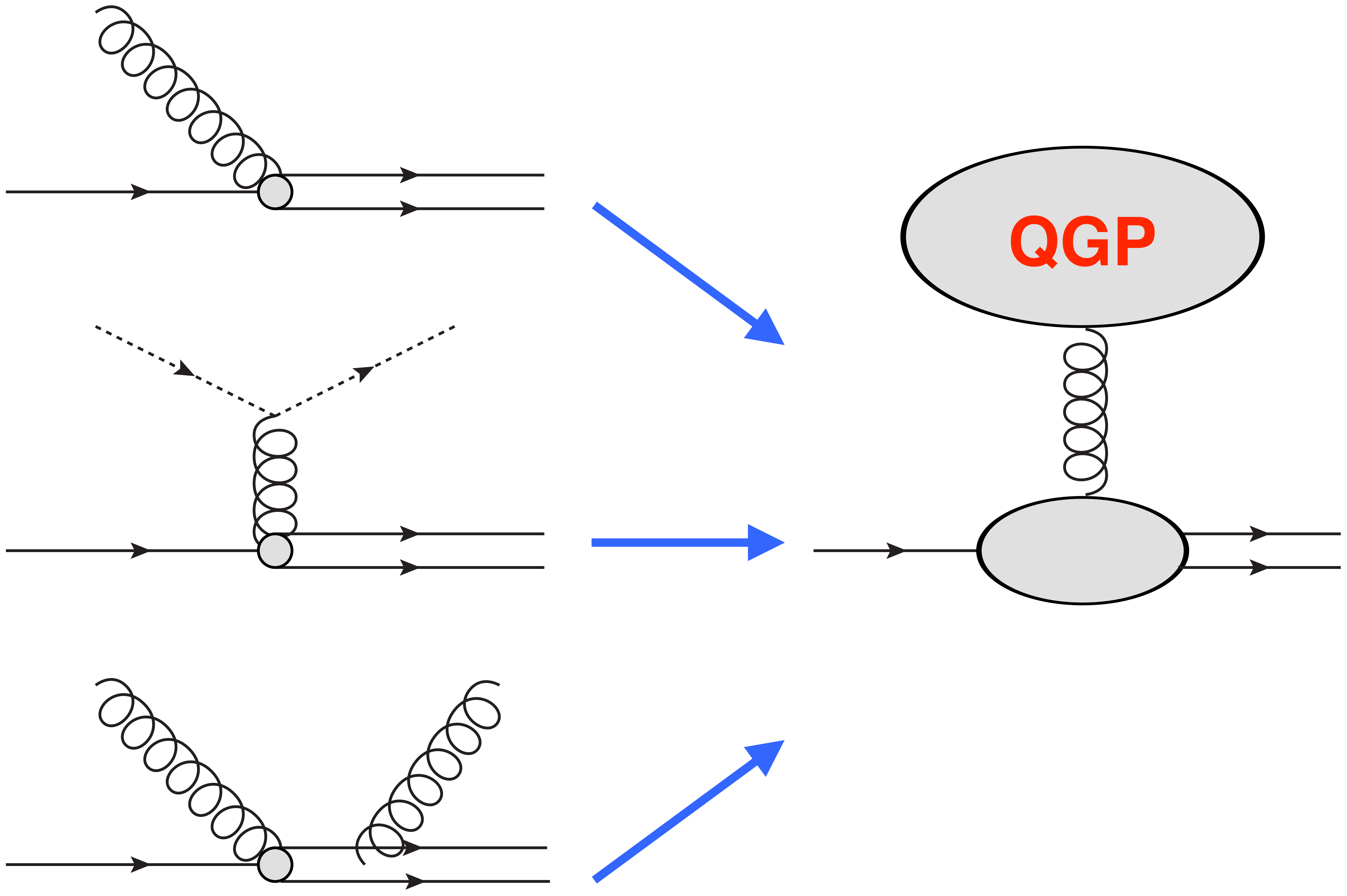
The singlet-octet transitions are governed by two generalized gluon distributions (GGDs):

- Dissociation:

$$[g_{\mathrm{adj}}^{++}]^{>}(\omega)$$

- (Re)combination:

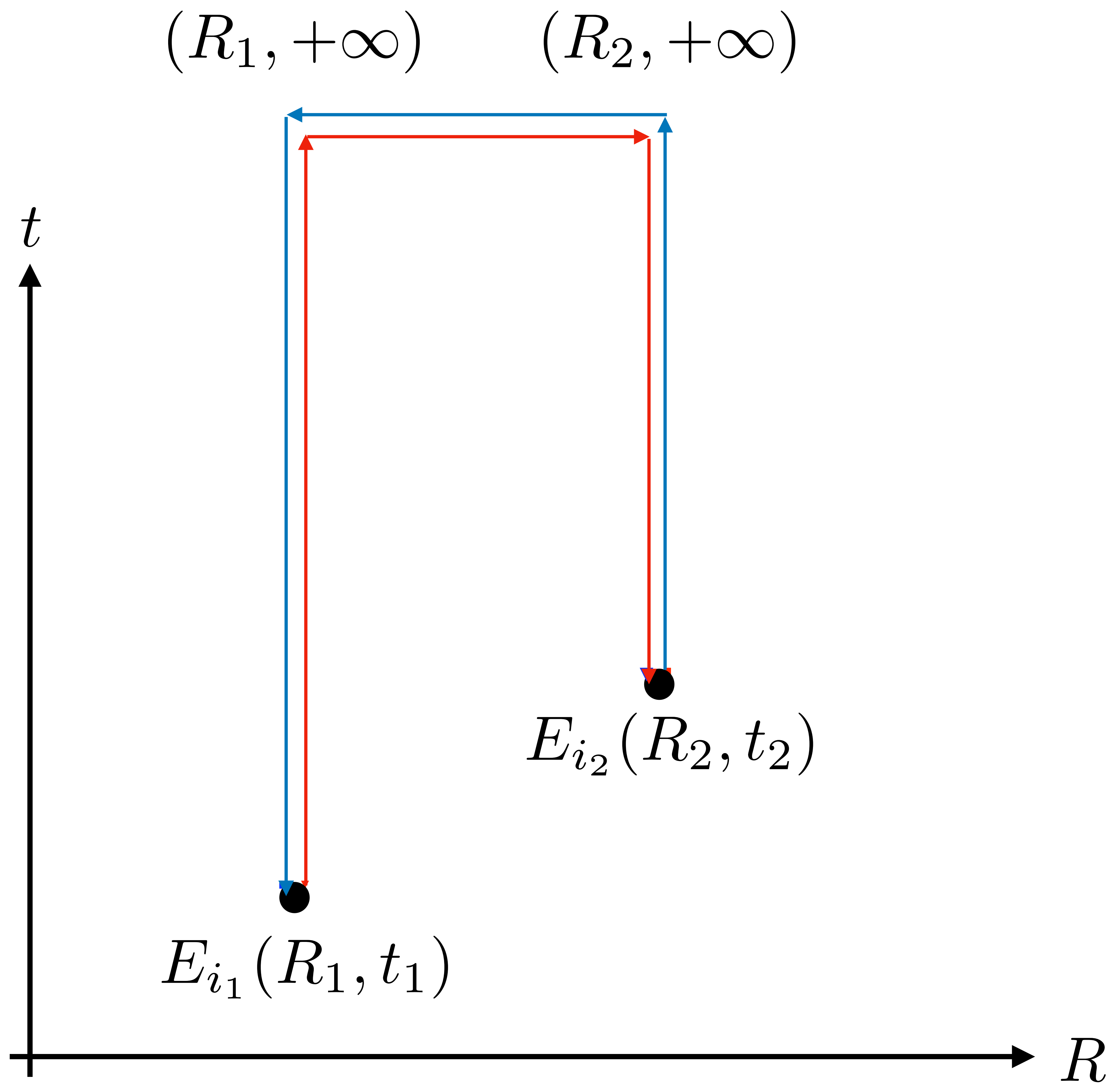
$$[g_{\mathrm{adj}}^{--}]^{>}(\omega)$$

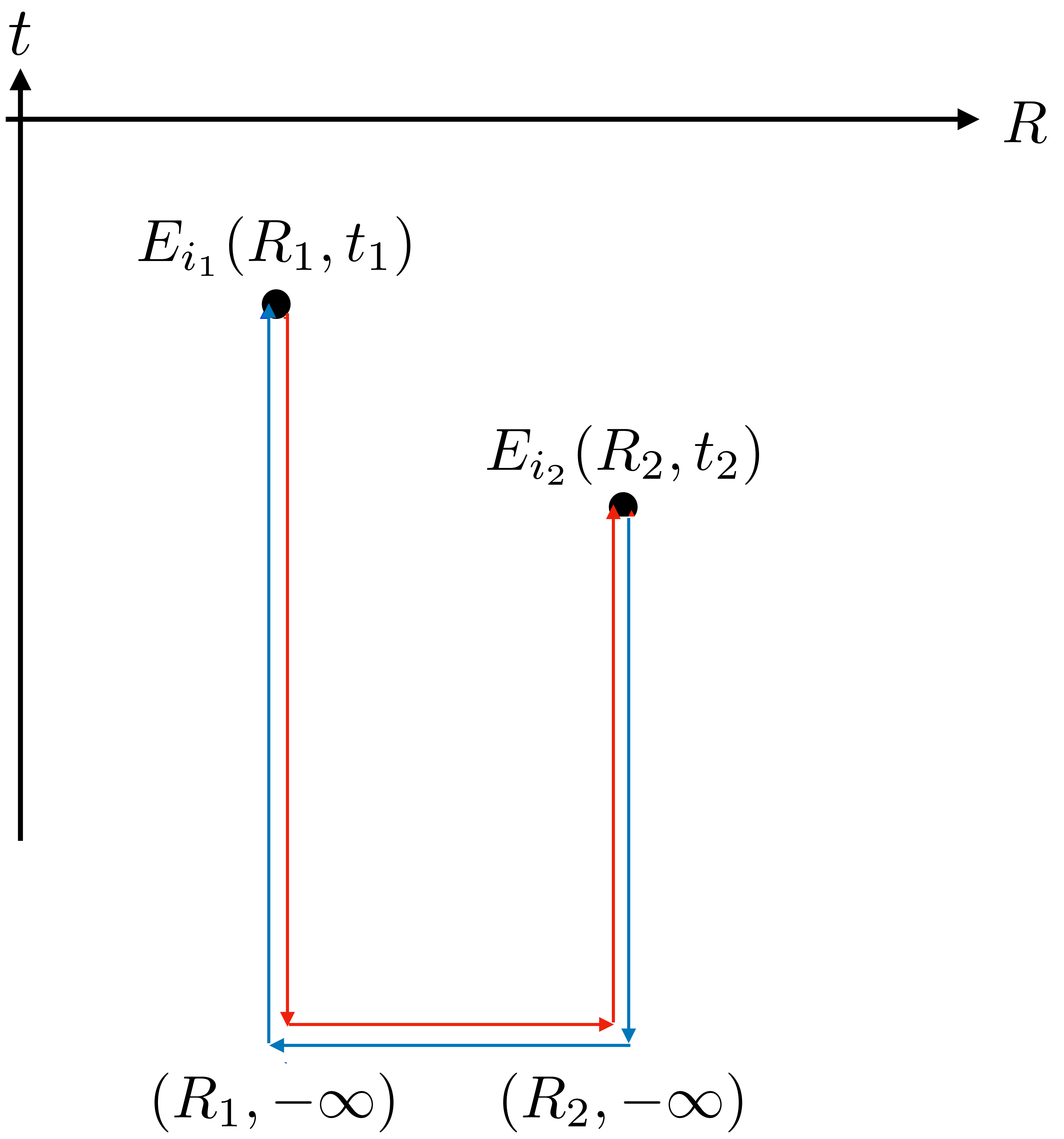


Generalized Gluon Distributions

for quarkonia transport

$$[g_{\rm adj}^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$





$$[g_{\rm adj}^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_{2}\right)^a \left(\mathscr{W}_{1} E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

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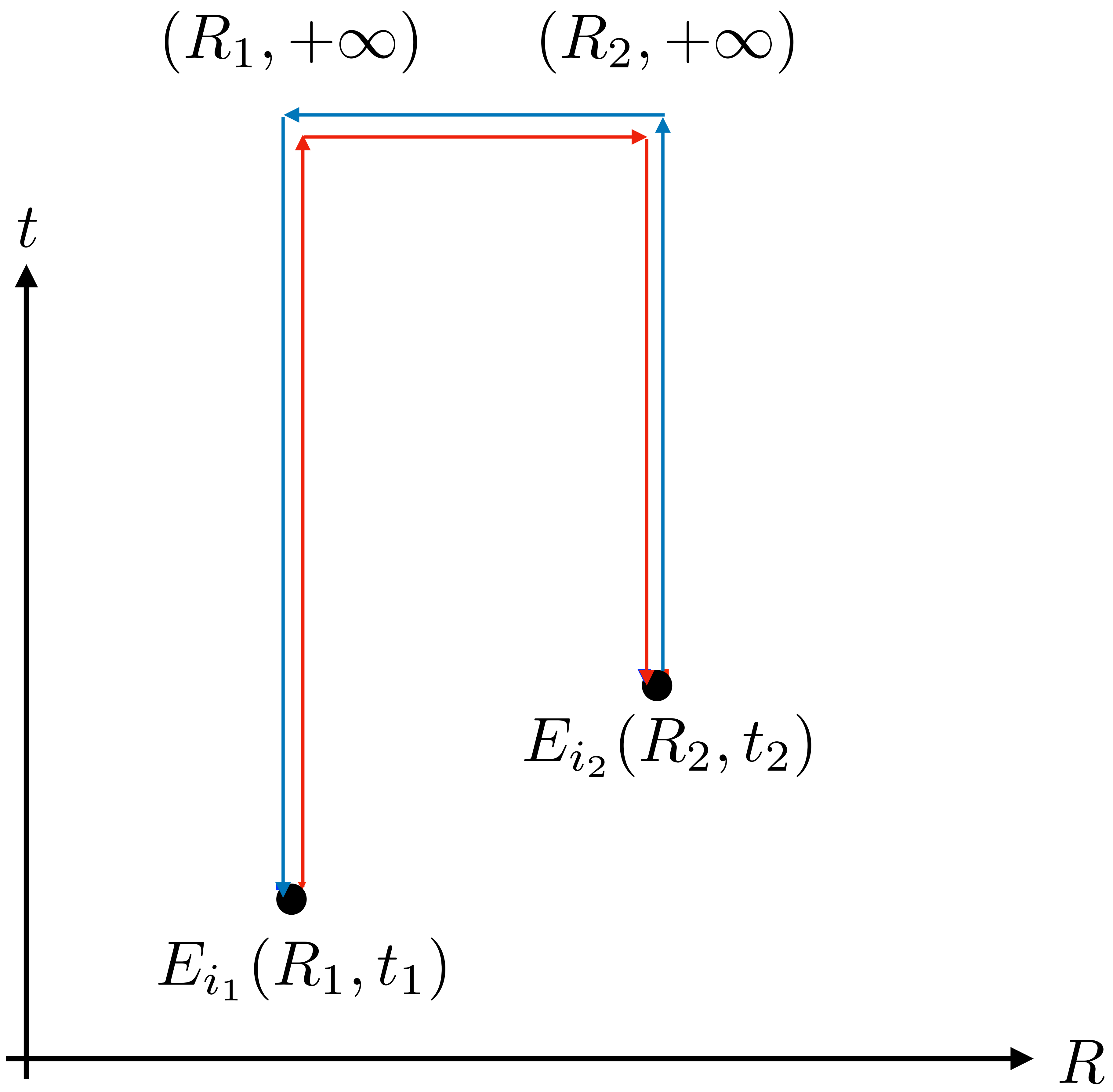

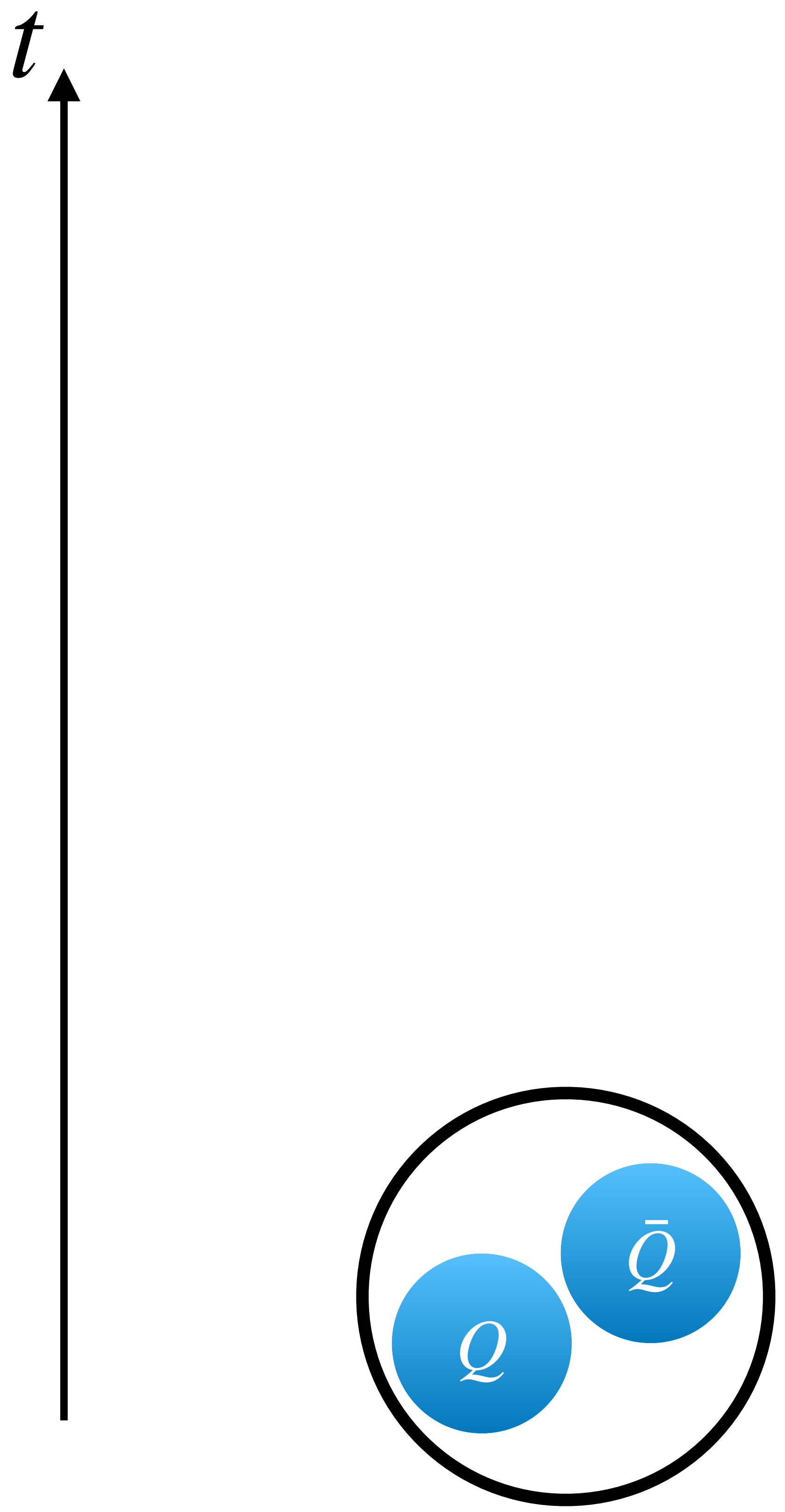

"bound" state:
color singlet

$$[g_{\rm adj}^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408

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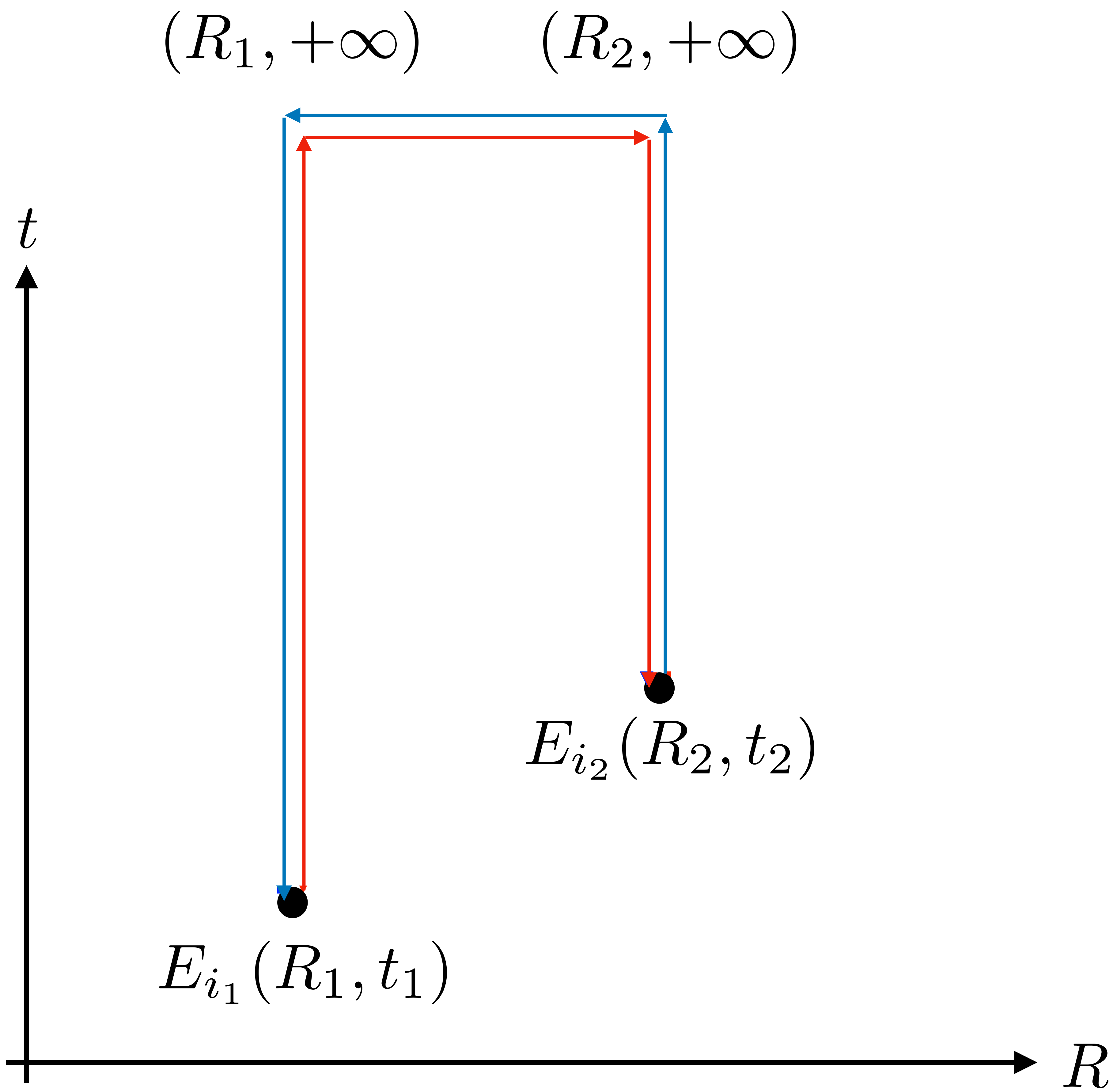

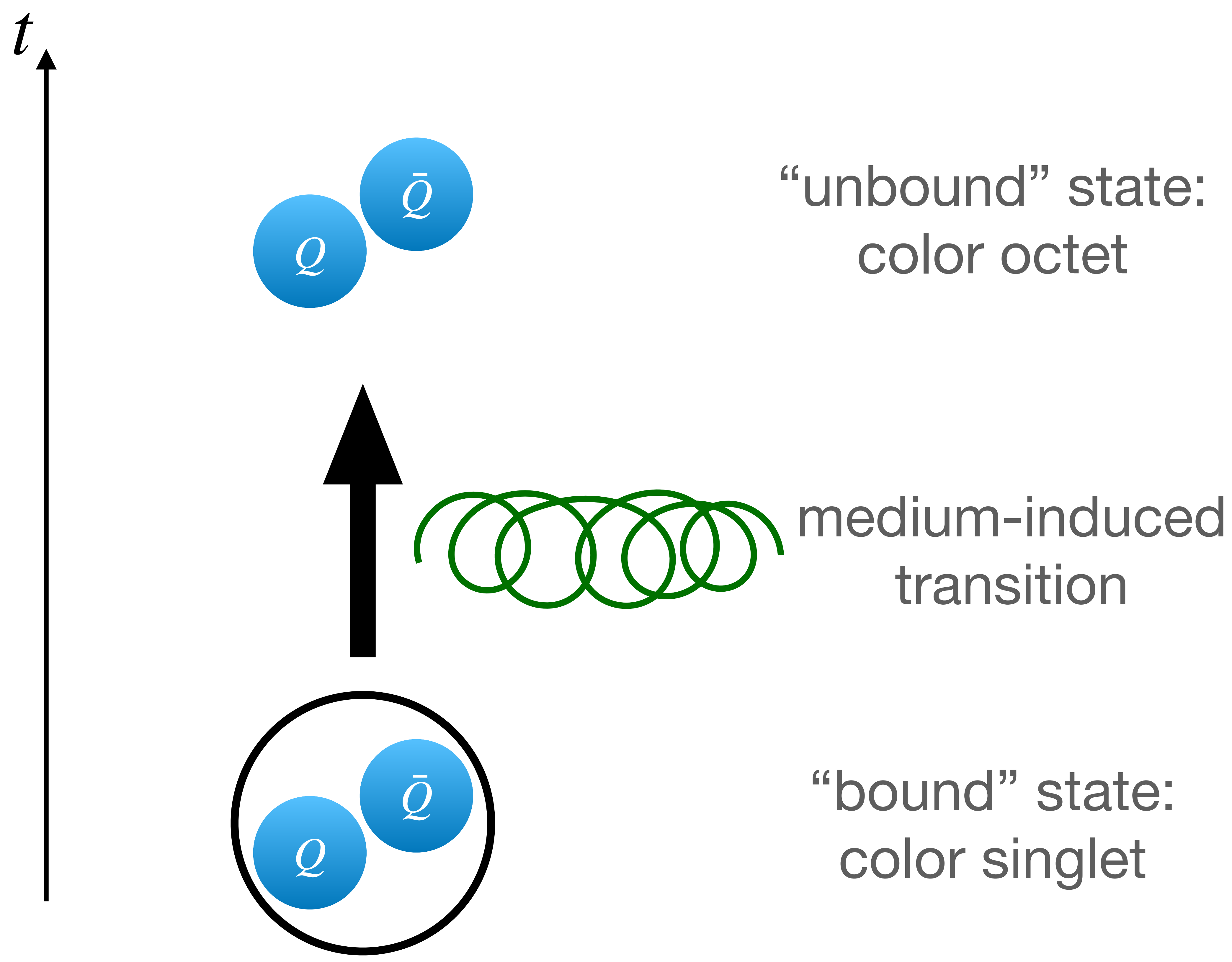

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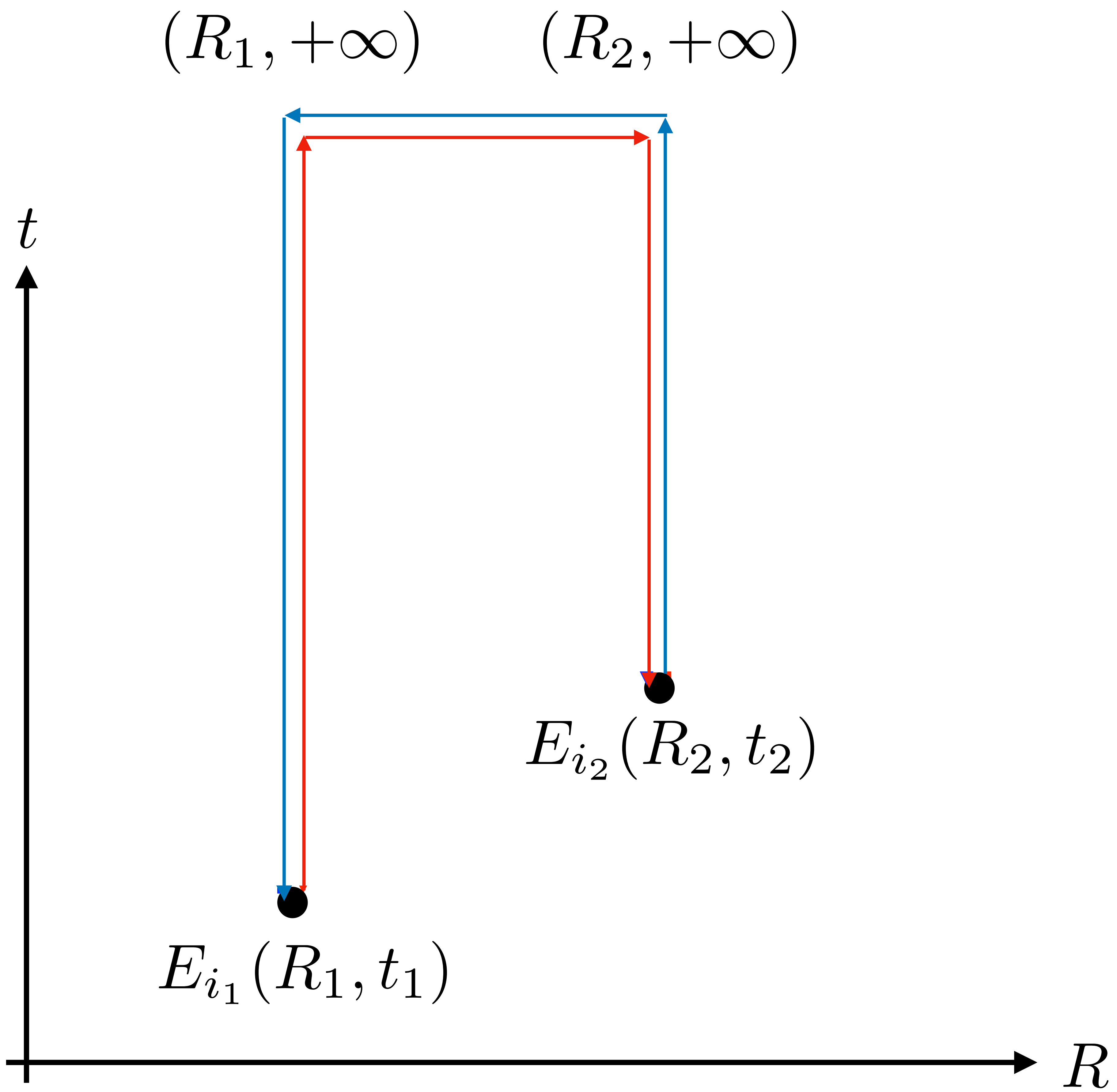

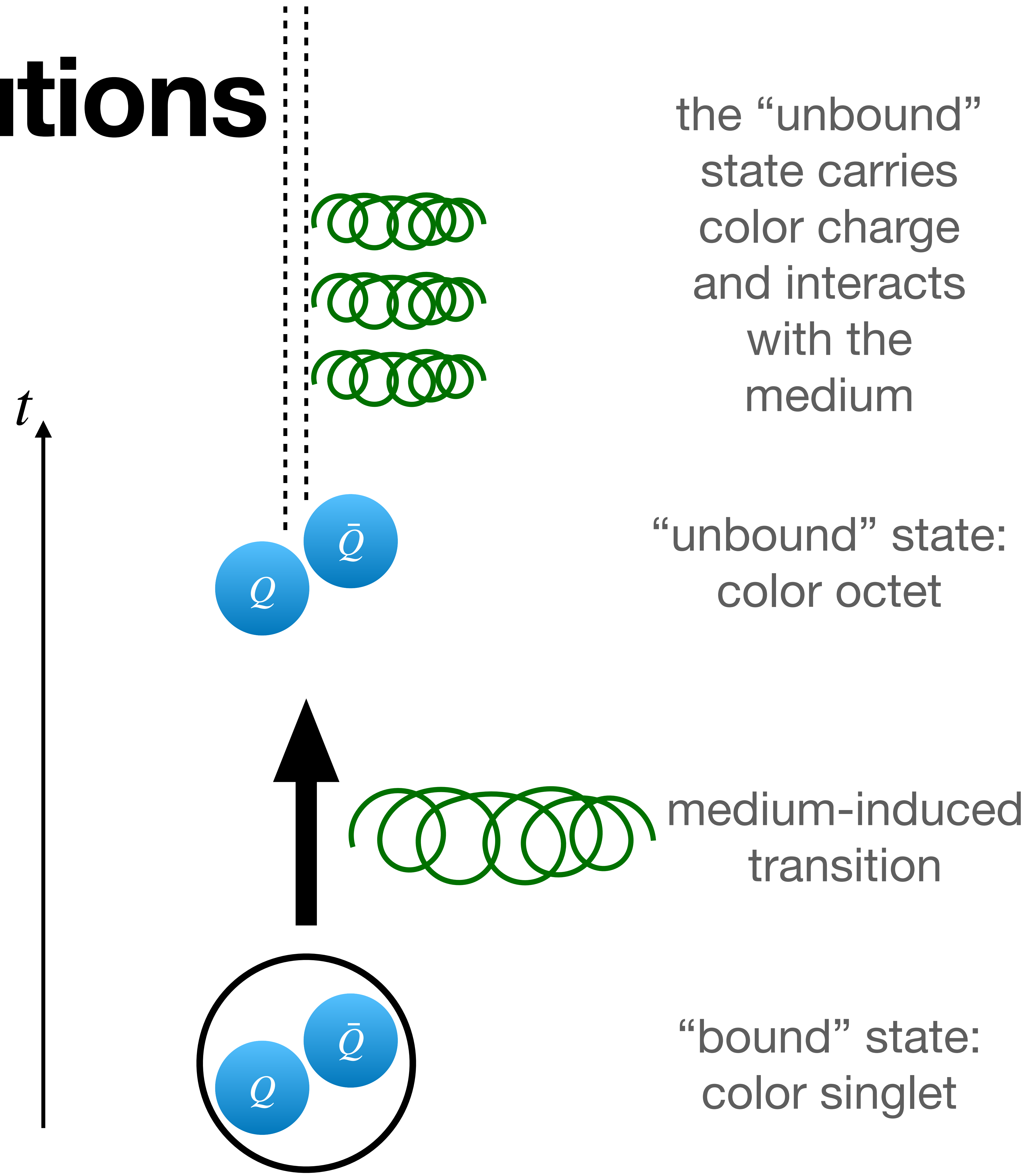

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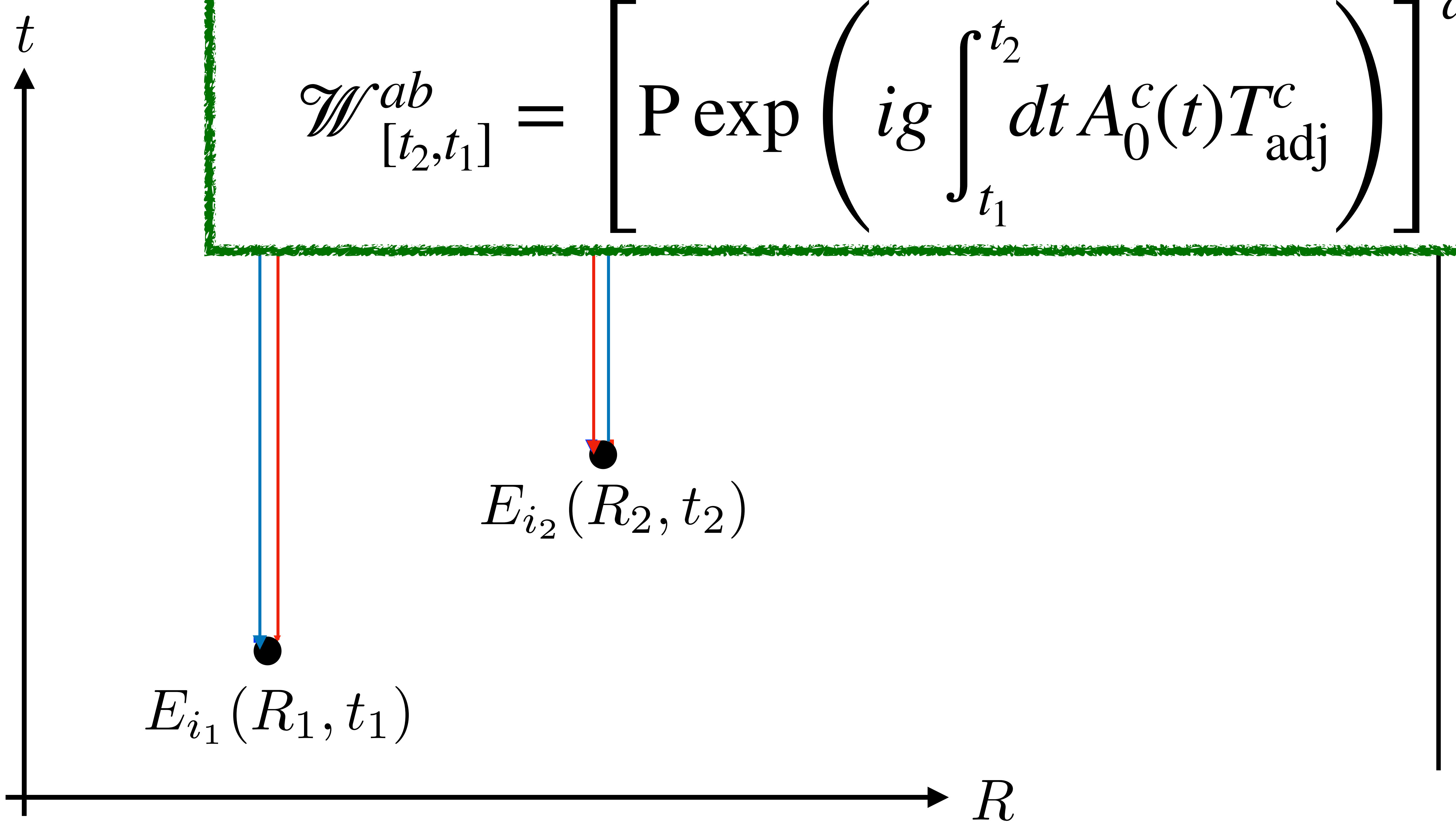
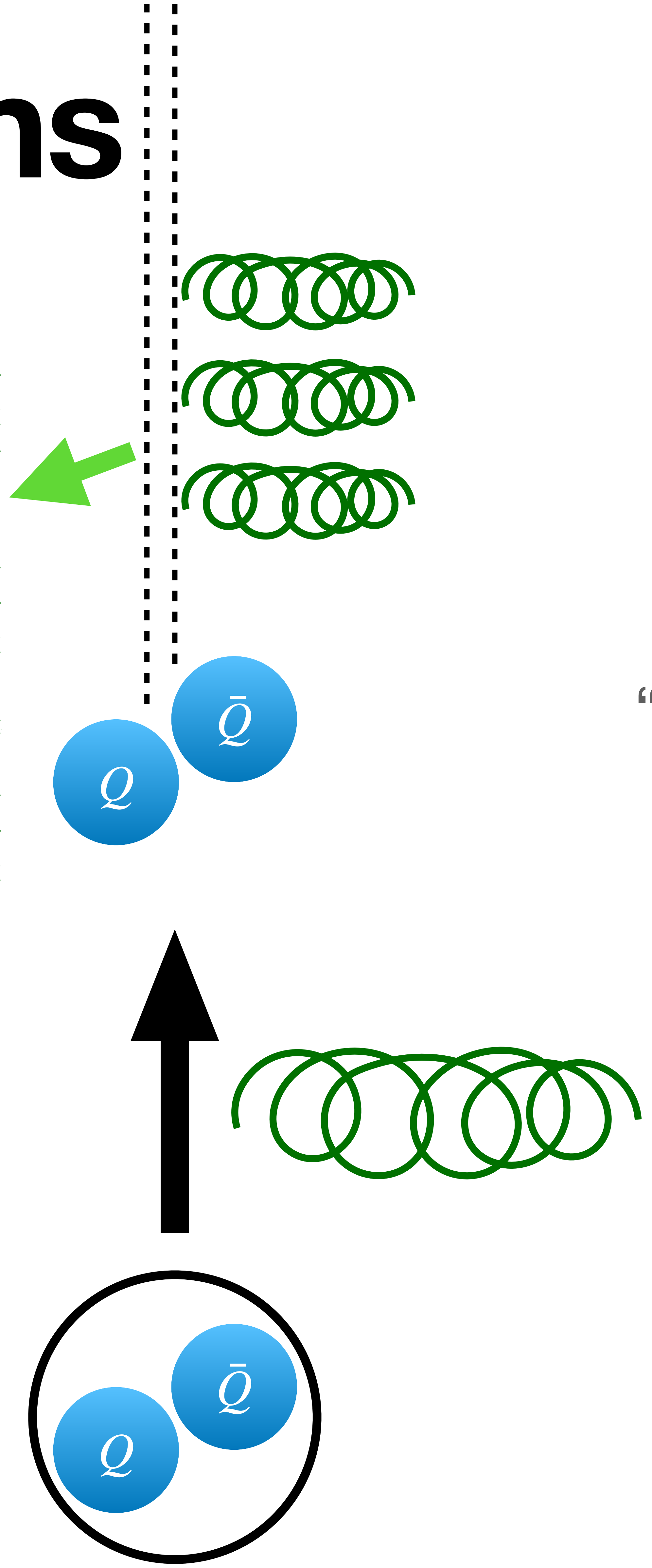
Generalized Gluon Distributions

for quarkonia transport

Summing the one-gluon insertions along the octet $Q\bar{Q}$ path generates a Wilson line:

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\rm adj} \right) \right]^{ab}$$





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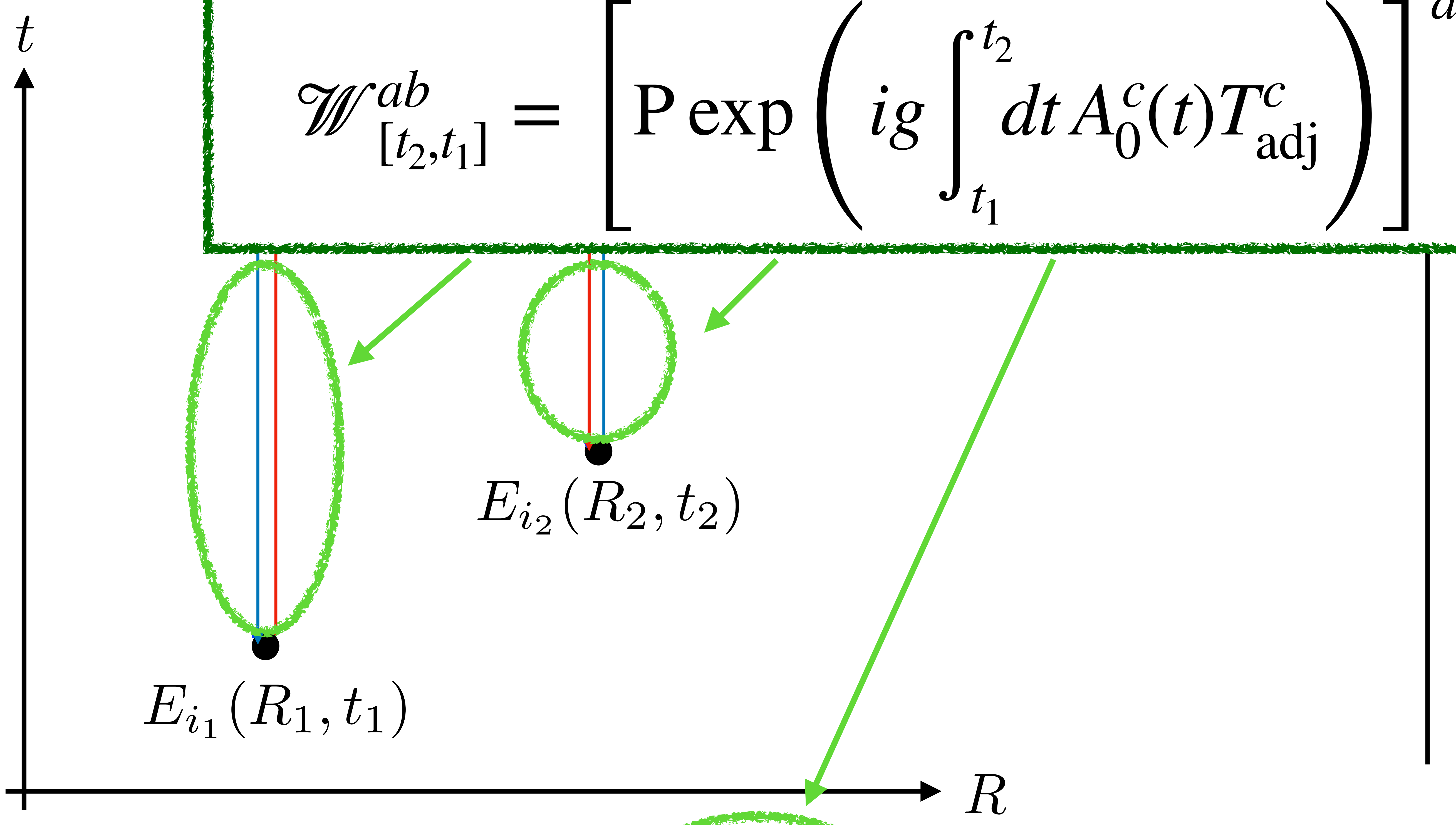
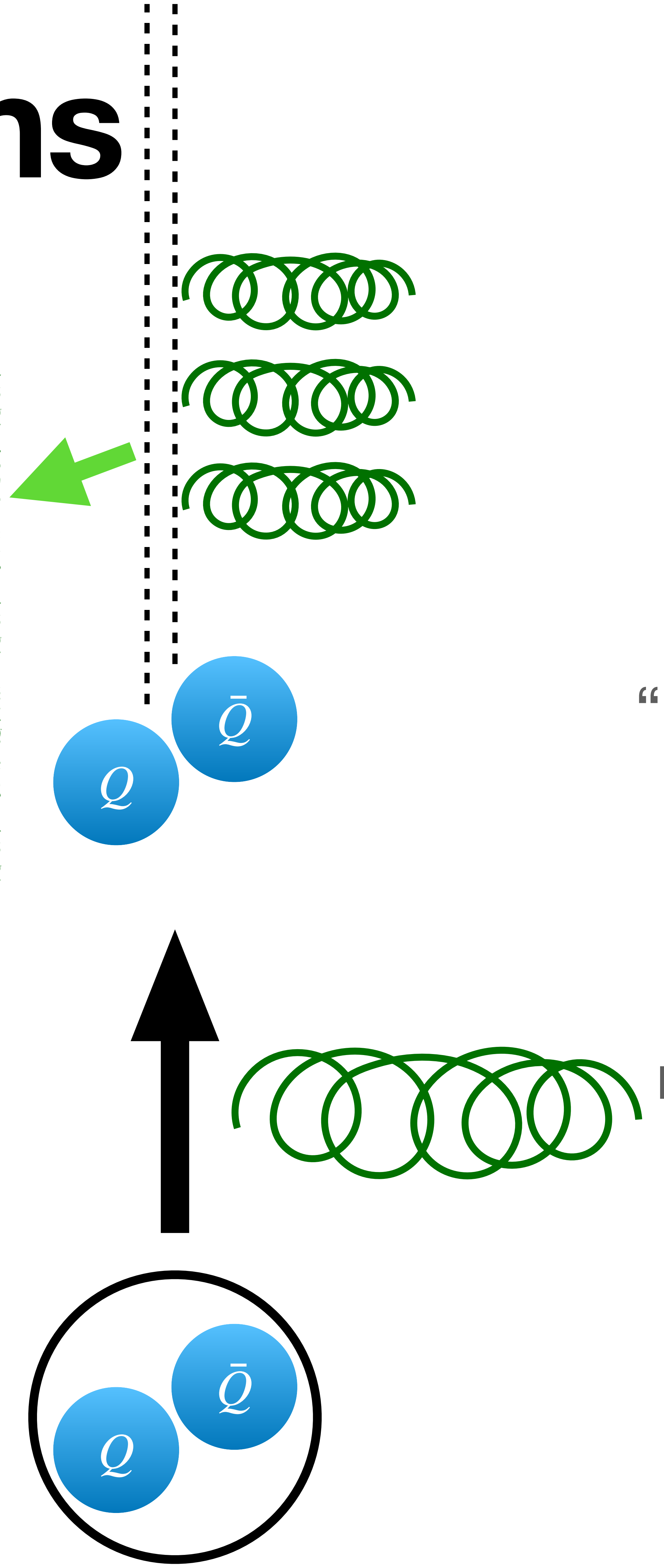
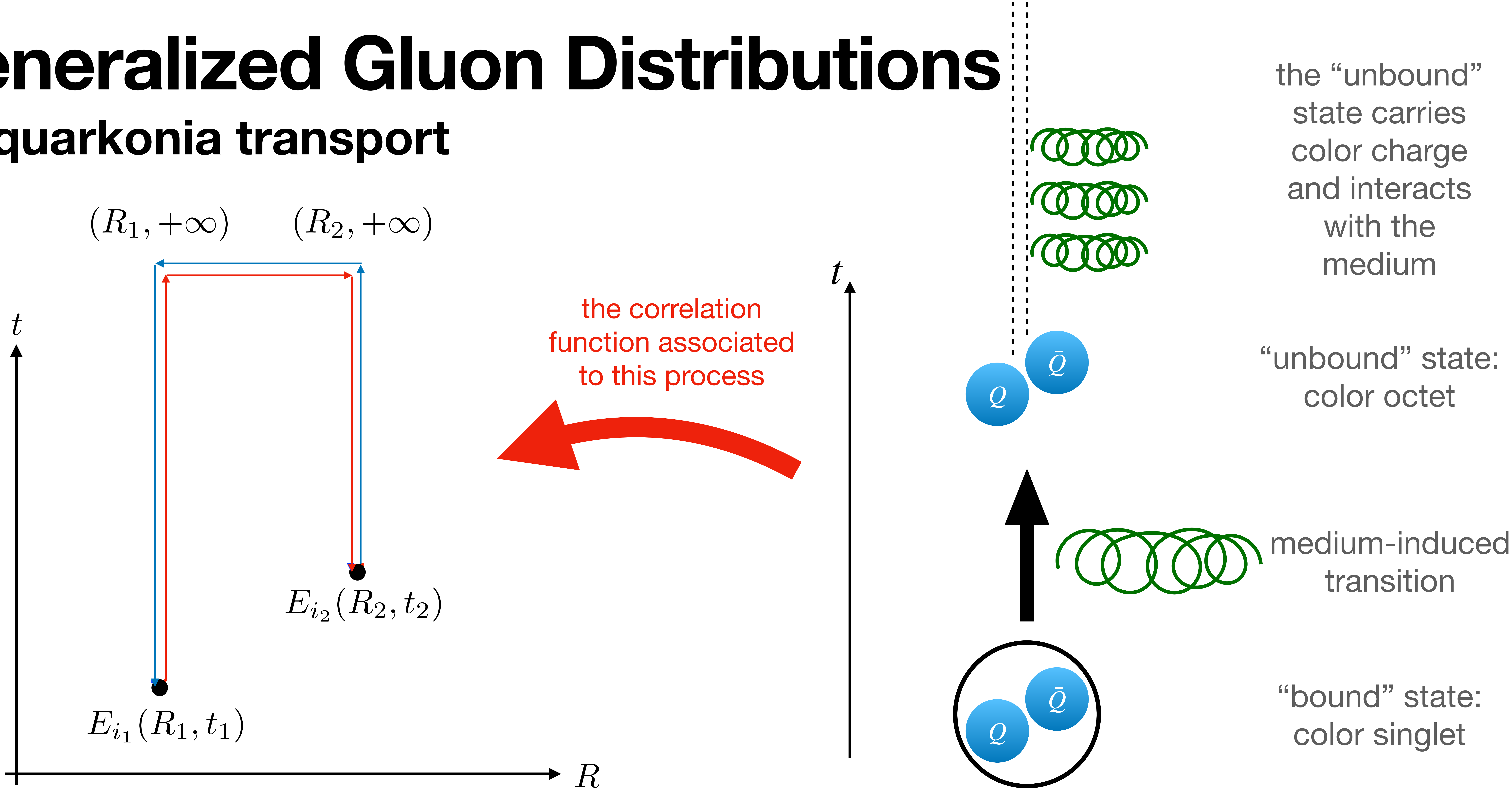




$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

Generalized Gluon Distributions

for quarkonia transport



$$[g_{\rm adj}^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408

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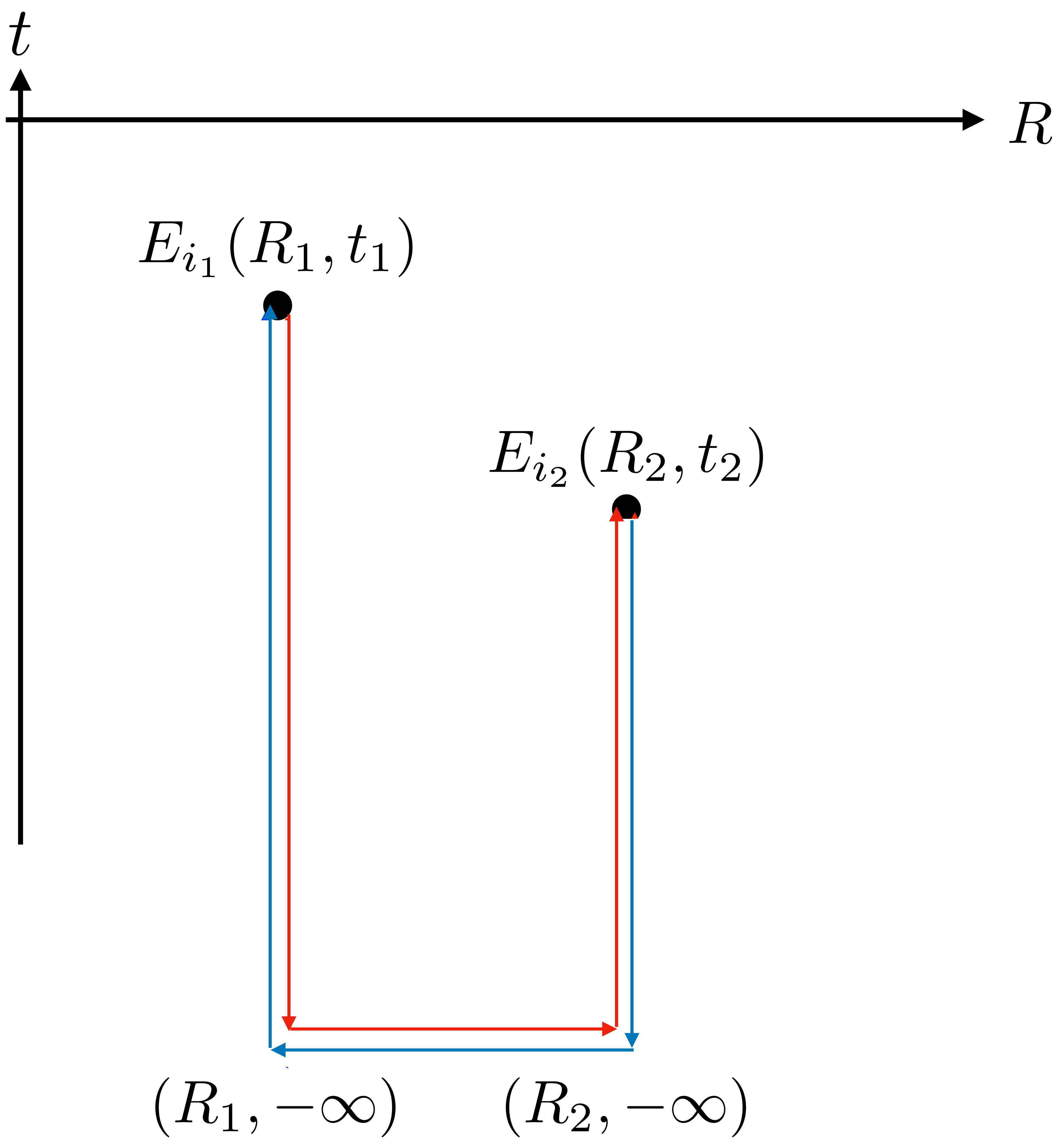

“unbound” state:
color octet

the “unbound”
state carries
color charge
and interacts
with the
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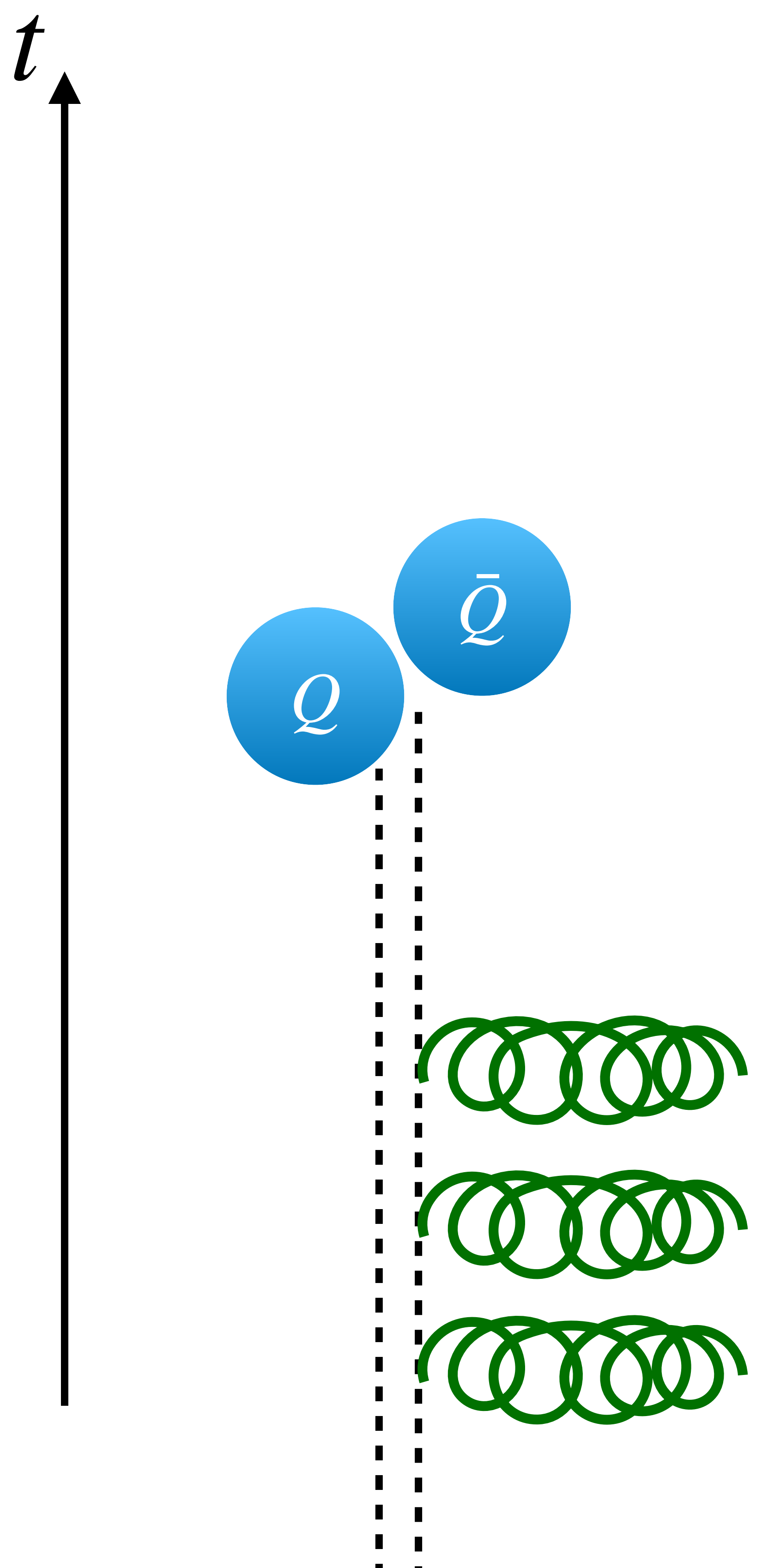

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medium-induced transition

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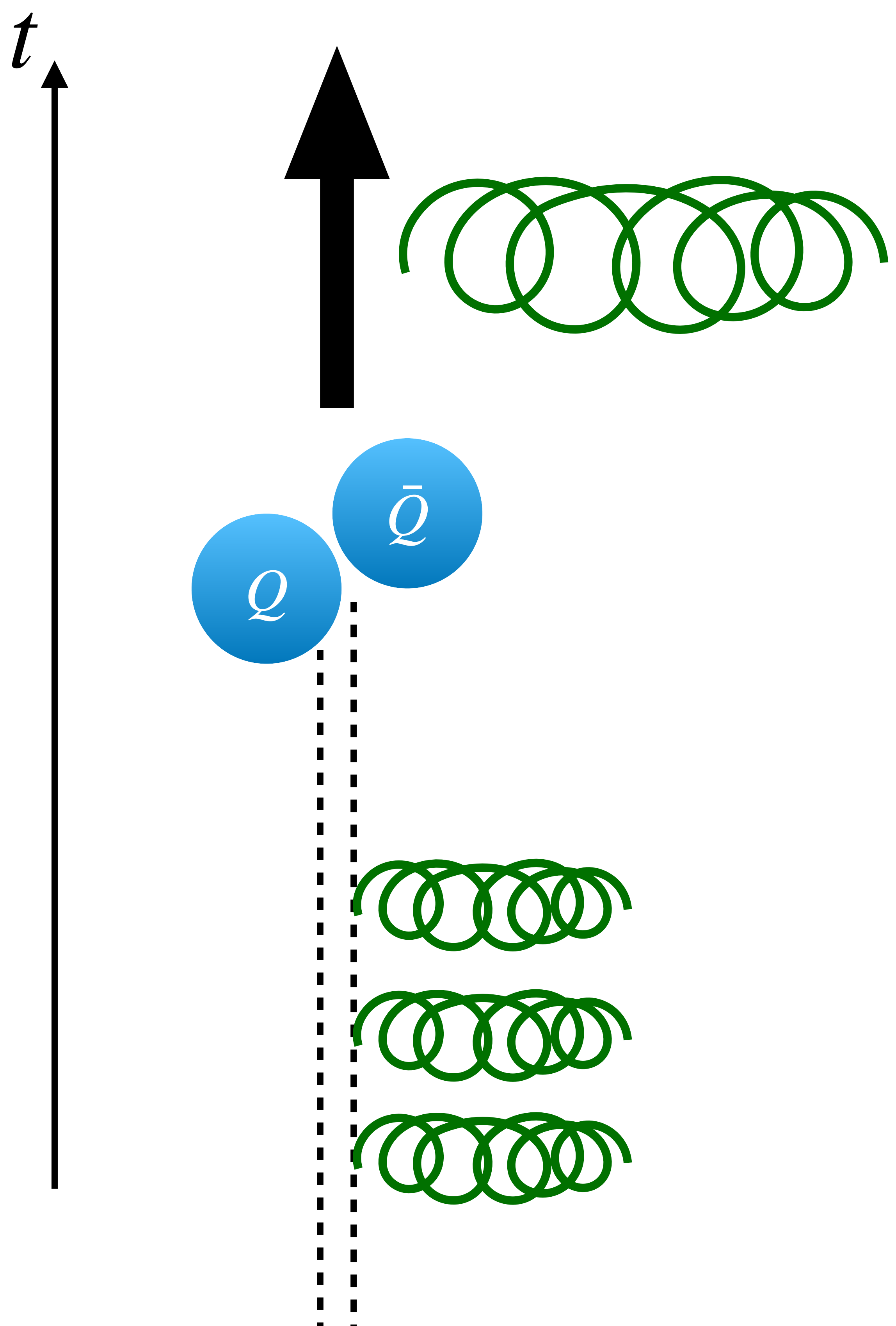

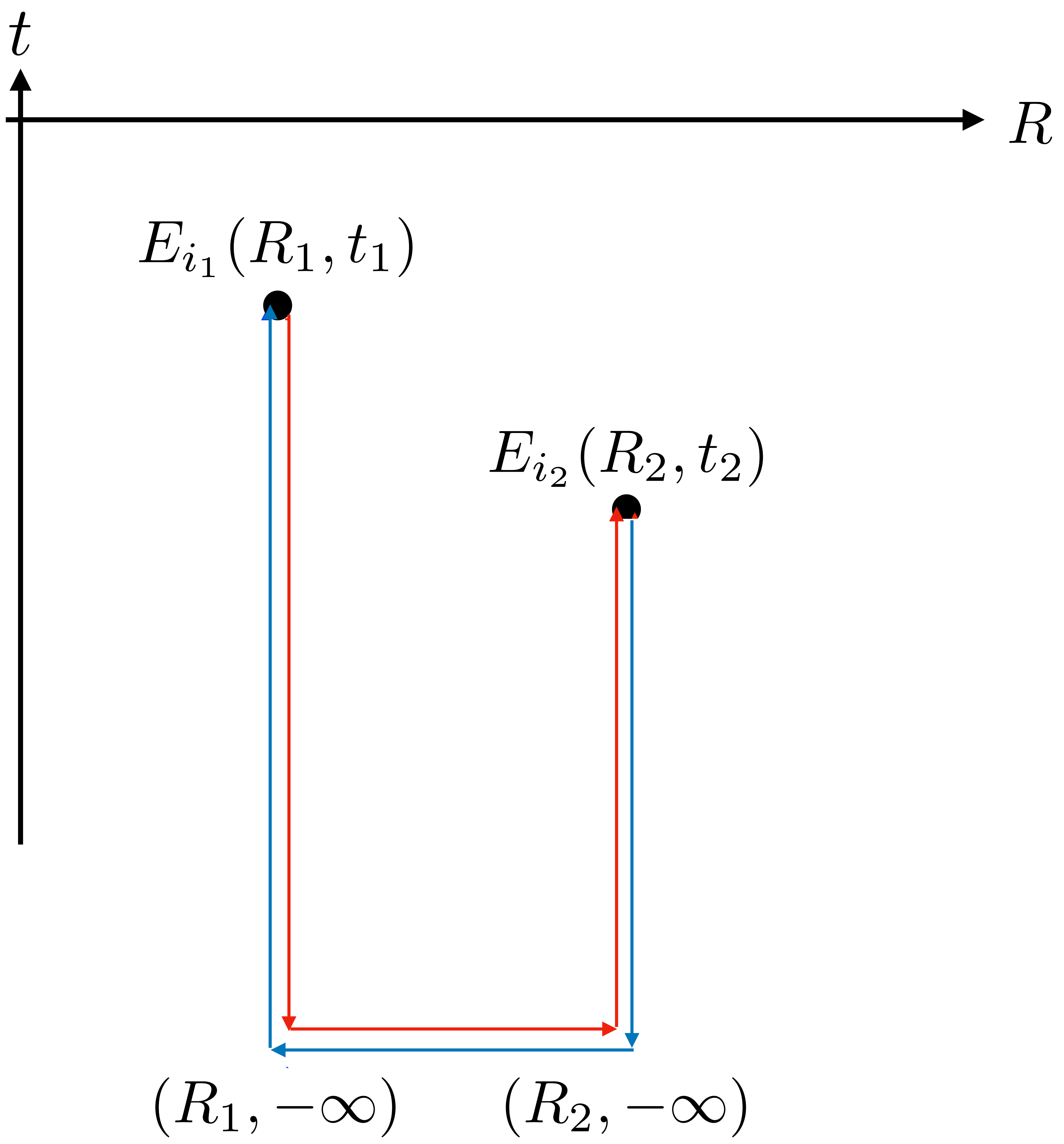

Generalized Gluon Distributions

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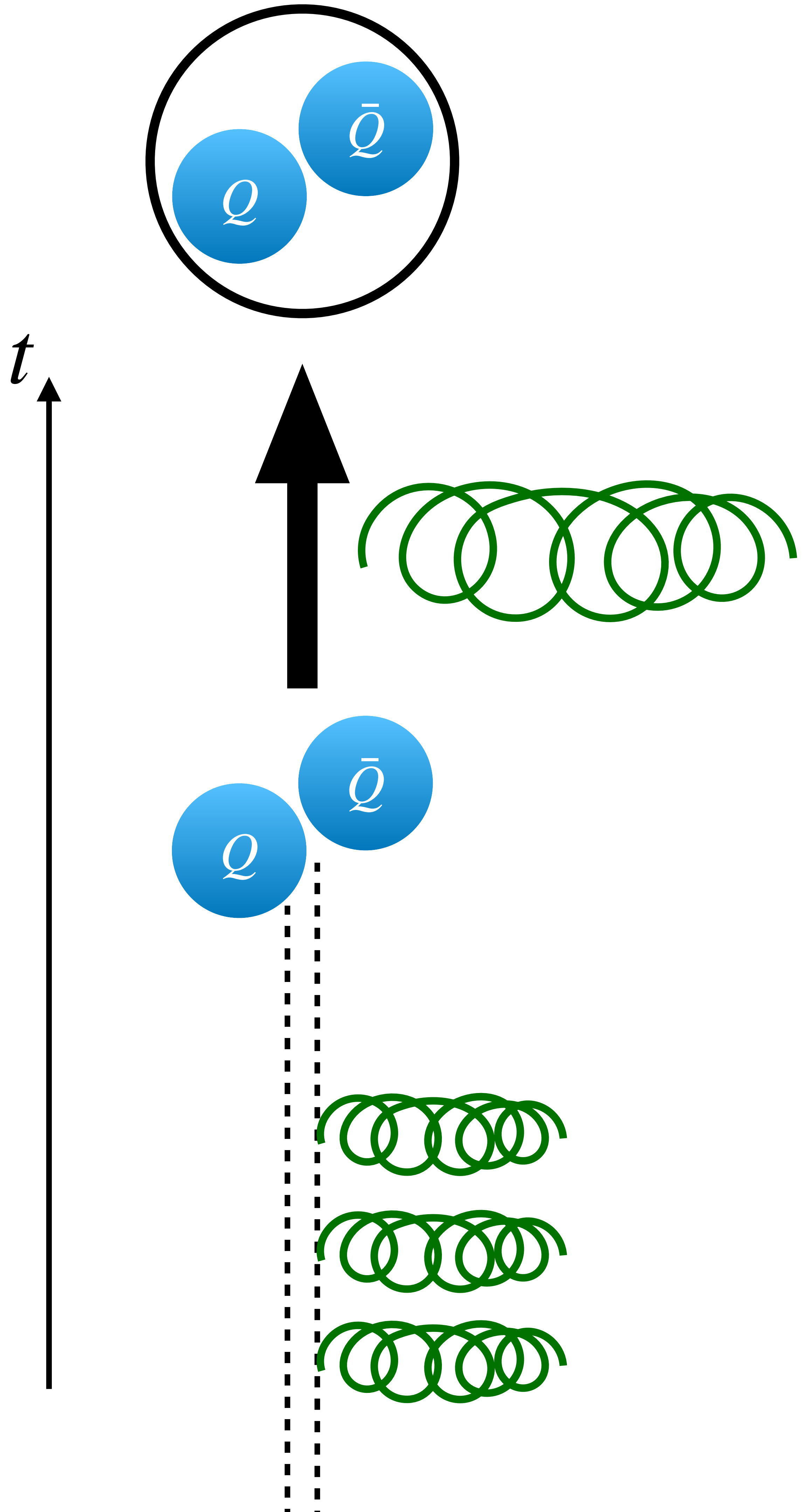

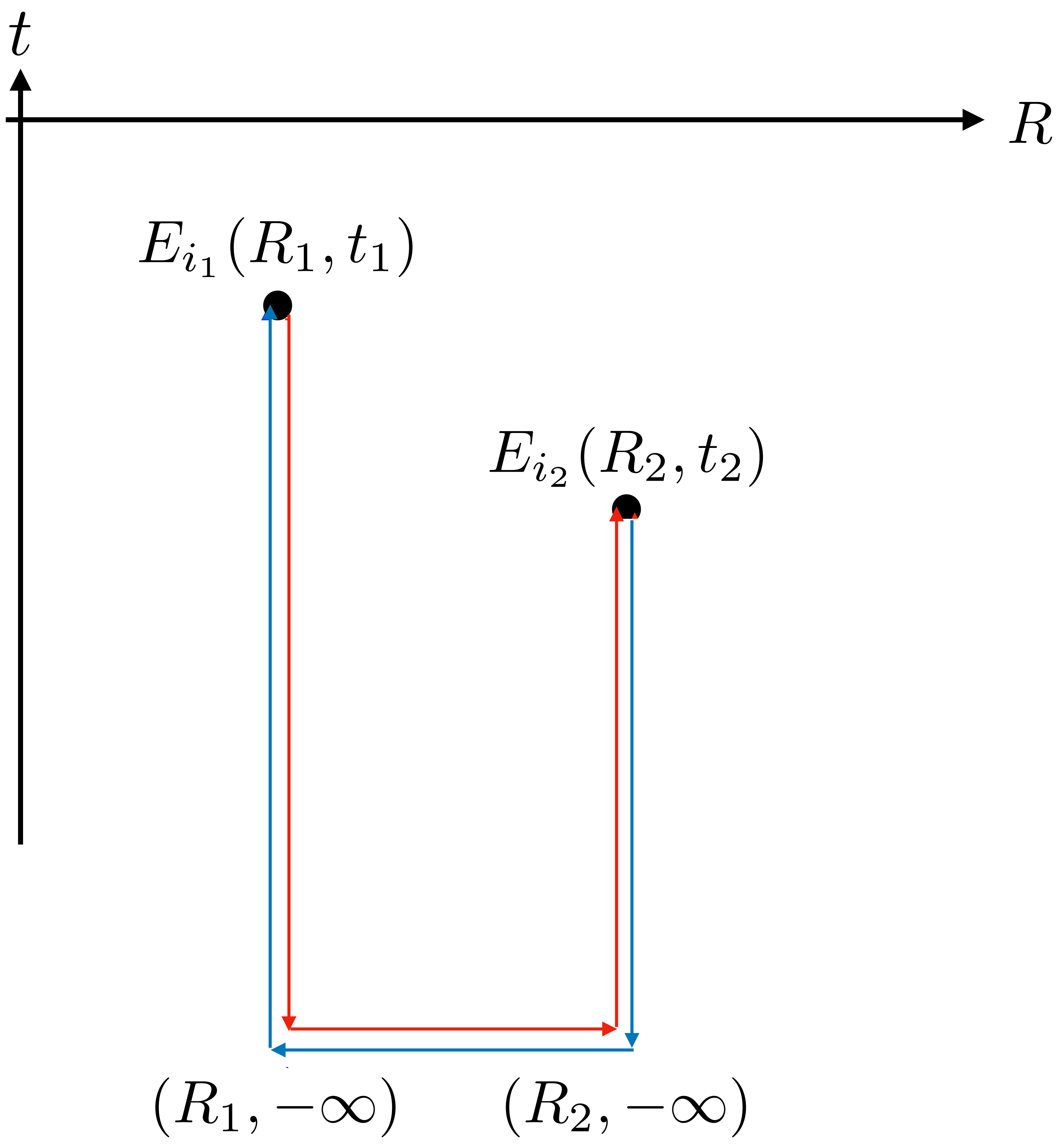

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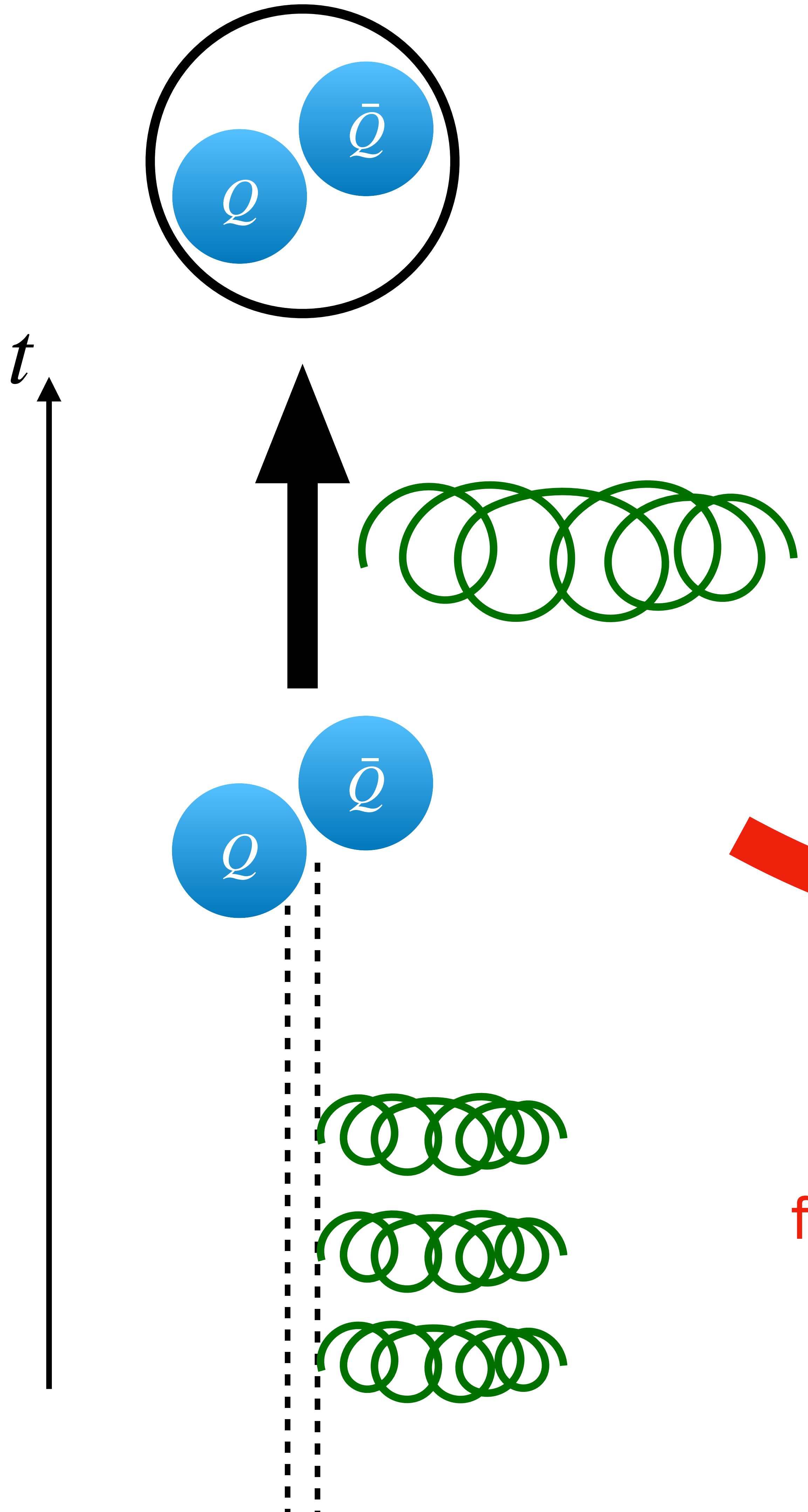
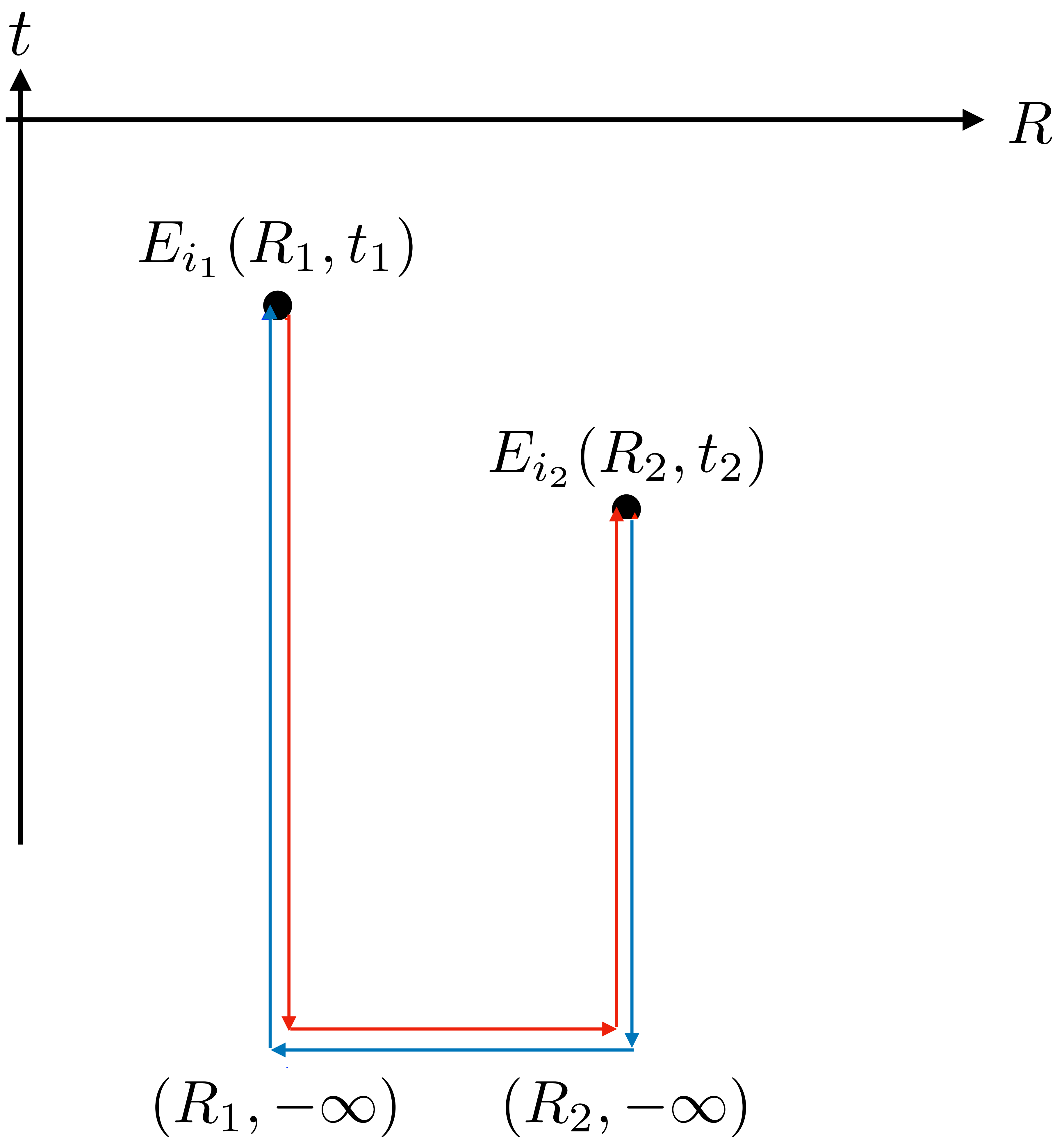


the correlation function associated to this process



Generalized Gluon Distributions

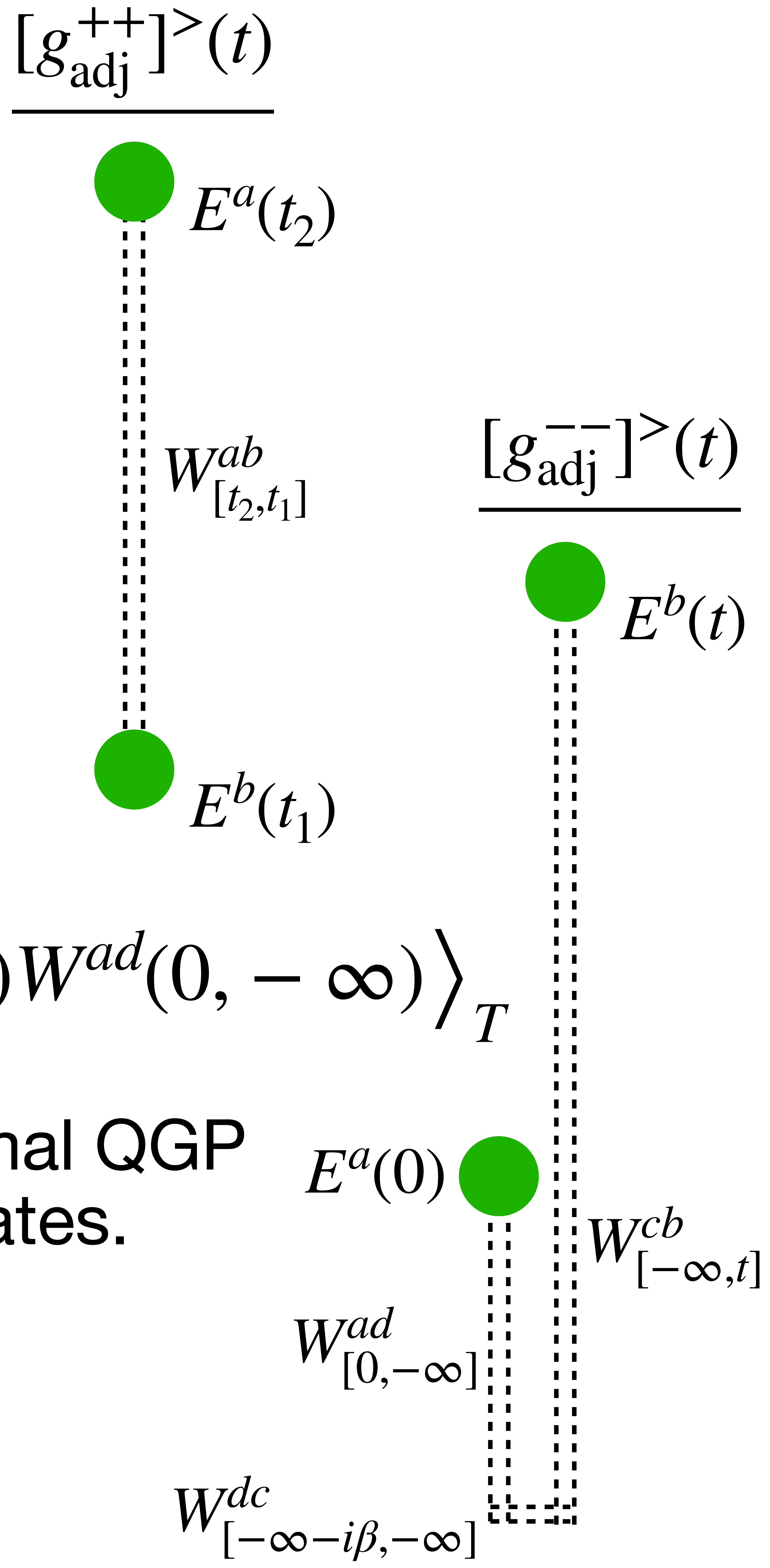
as they appear in transport equations

The GGDs that we will discuss from now on have $\mathbf{R}_1 = \mathbf{R}_2$:

$$[g_{\rm adj}^{++}]^{>}(t) \equiv \frac{g^2 T_F}{3N_c} \left\langle E_i^a(t) W^{ab}(t,0) E_i^b(0) \right\rangle_T$$

$$[g_{\rm adj}^{--}]^{>}(t) \equiv \frac{g^2 T_F}{3N_c} \left\langle W^{dc}(-i\beta-\infty,-\infty) W^{cb}(-\infty,t) E_i^b(t) E_i^a(0) W^{ad}(0,-\infty) \right\rangle_T$$

They encode *non-perturbative* information about how the thermal QGP environment mediates transitions between singlet and octet states.



(with X. Yao) 2306.13127

Thermal equilibrium

KMS conditions and a spectral function

- The GGDs satisfy a Kubo-Martin-Schwinger relation

$$[g_{\rm adj}^{++}]^{>}(\omega) = e^{\omega/T}\,[g_{\rm adj}^{--}]^{>}(-\omega)$$

which means detailed balance (and thus thermalization) can be achieved between dissociation and (re)combination.

- This motivates the introduction of a spectral function

$$\rho_{\rm adj}^{++}(\omega) = \left(1 - e^{-\omega/T}\right)[g_{\rm adj}^{++}]^{>}(\omega) = [g_{\rm adj}^{++}]^{>}(\omega) - [g_{\rm adj}^{--}]^{>}(-\omega)$$

which encodes all of the information in the GGDs.

A comparison with heavy quark diffusion

Different physics with the same building blocks

Heavy quark diffusion

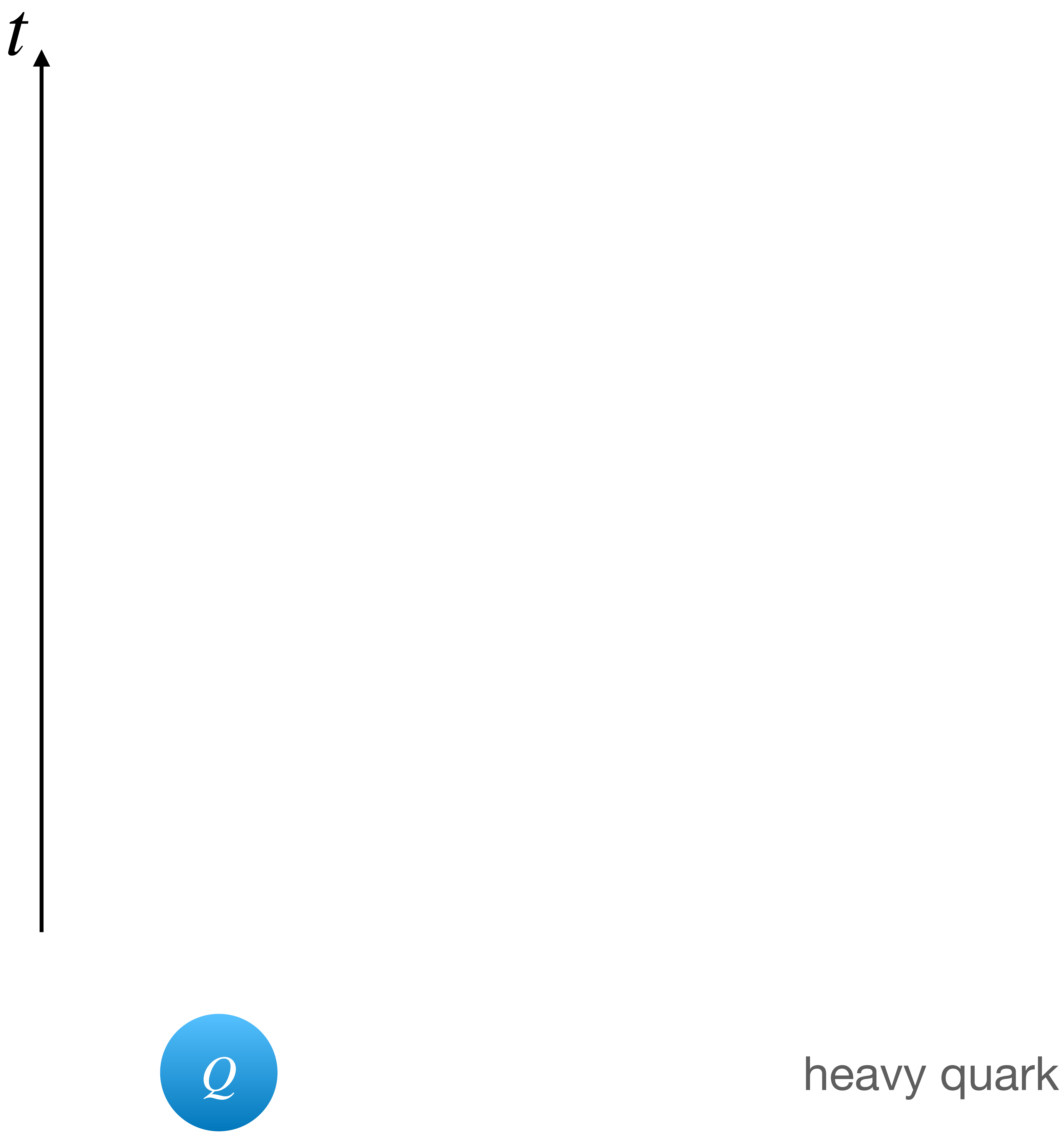
an analogous picture

J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from "kicks" from the medium.



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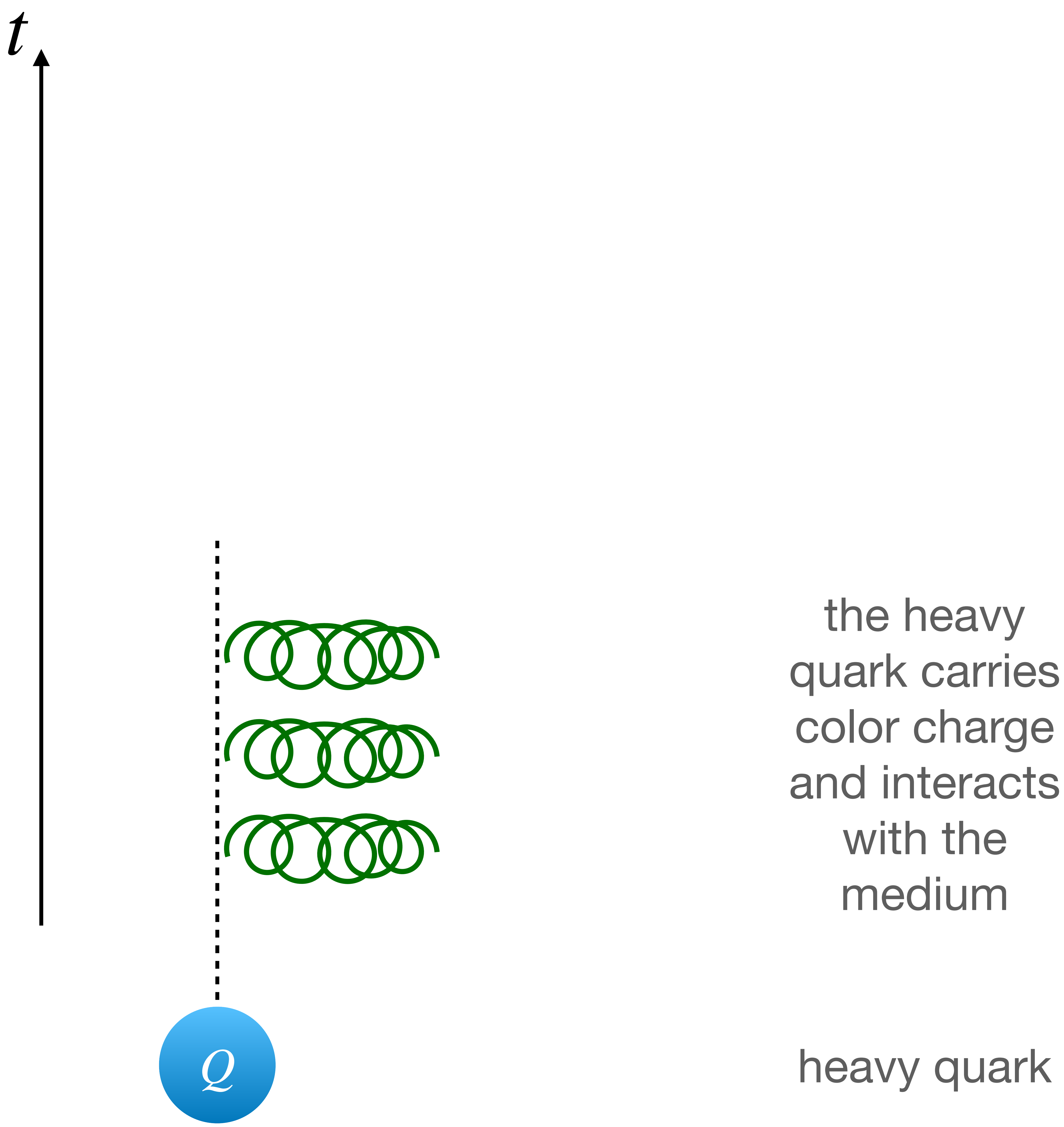

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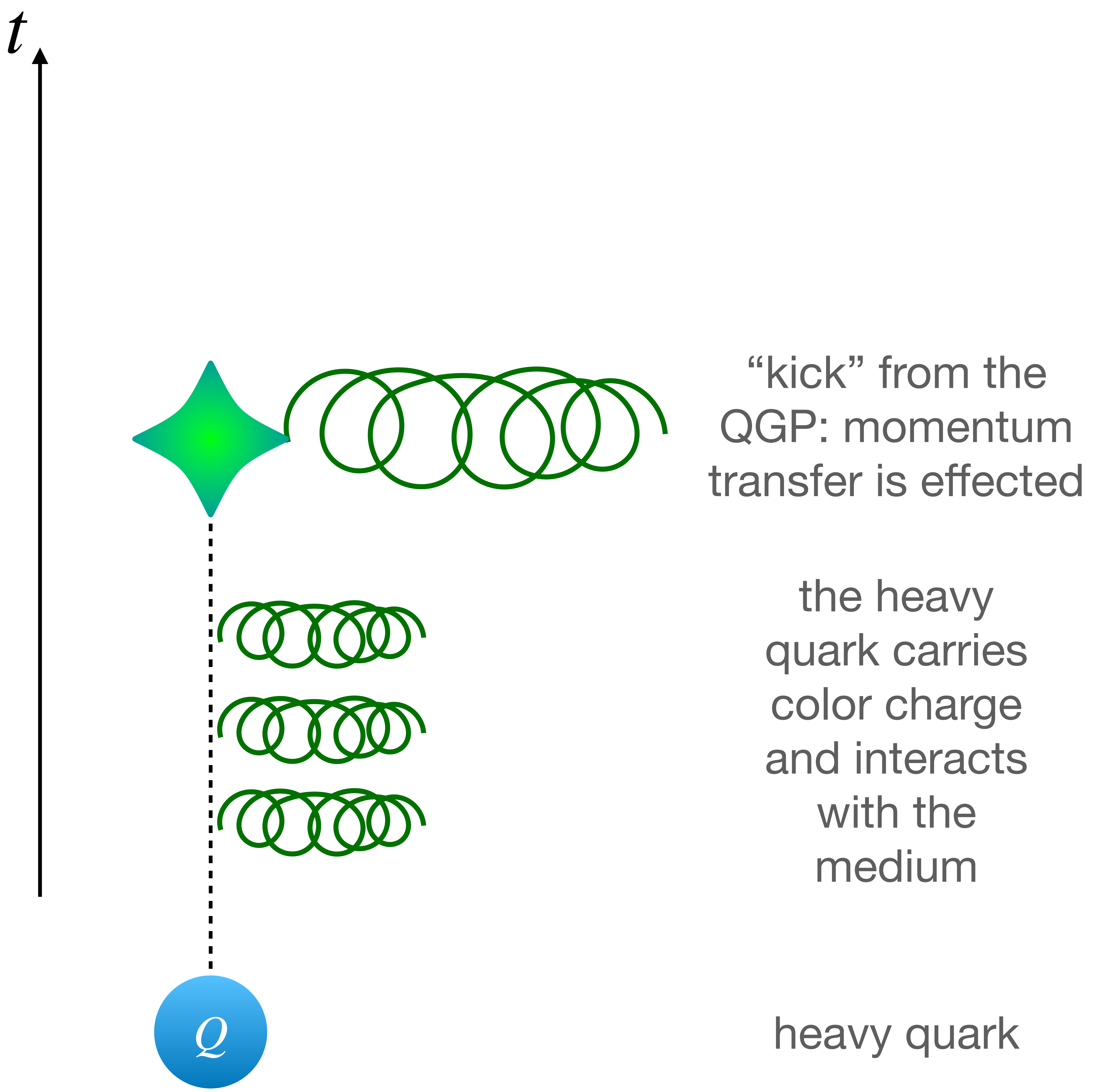

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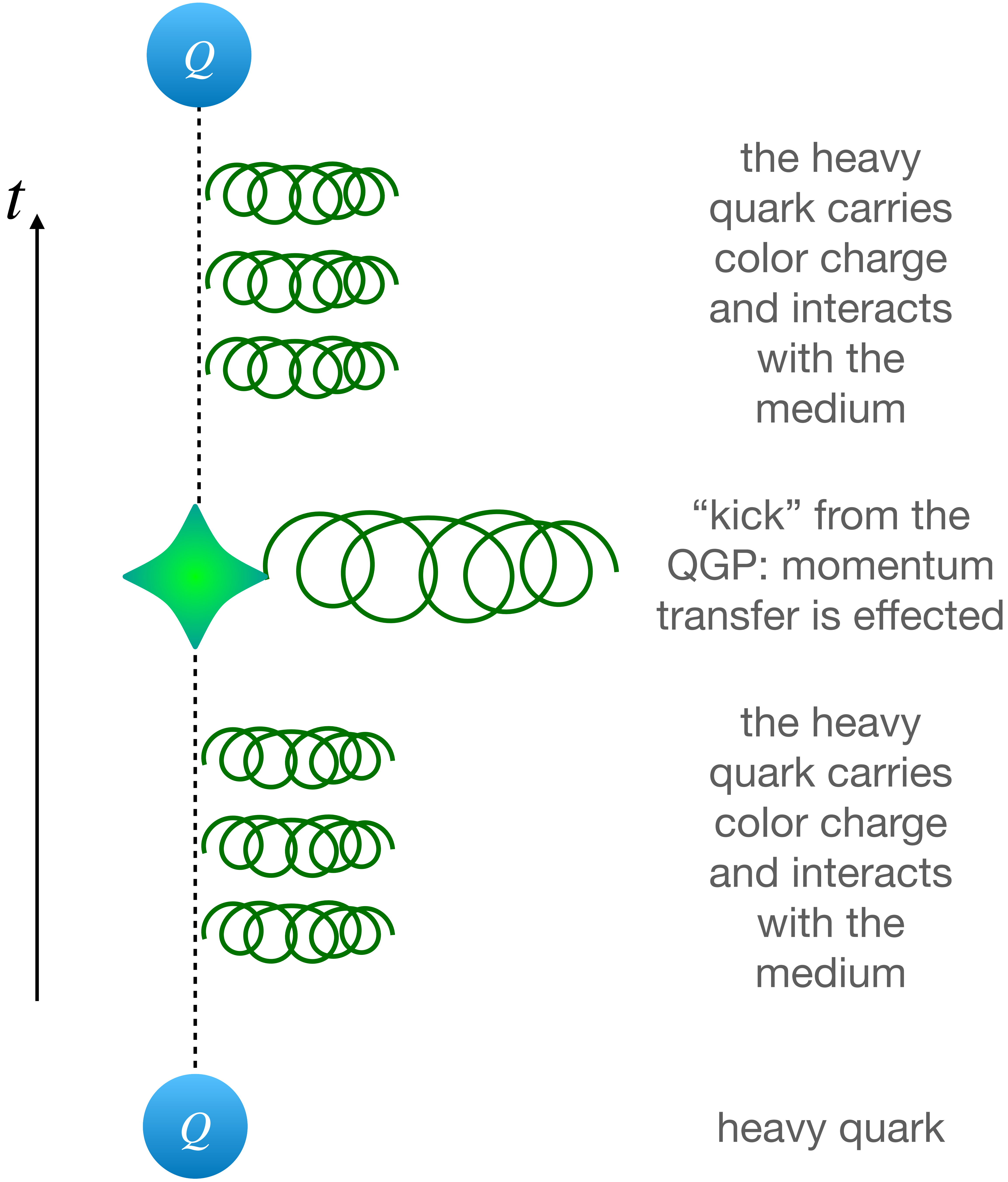
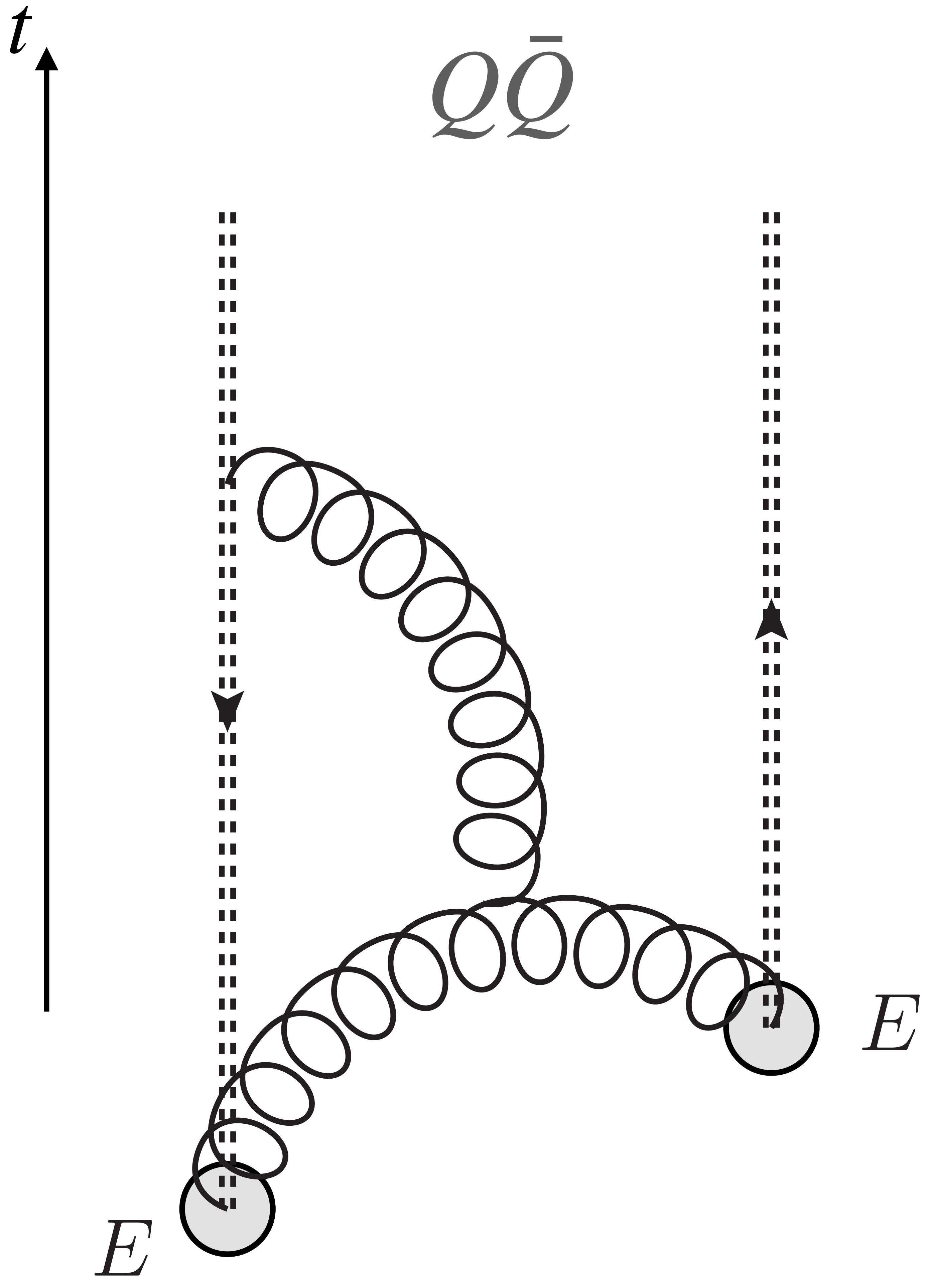


see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

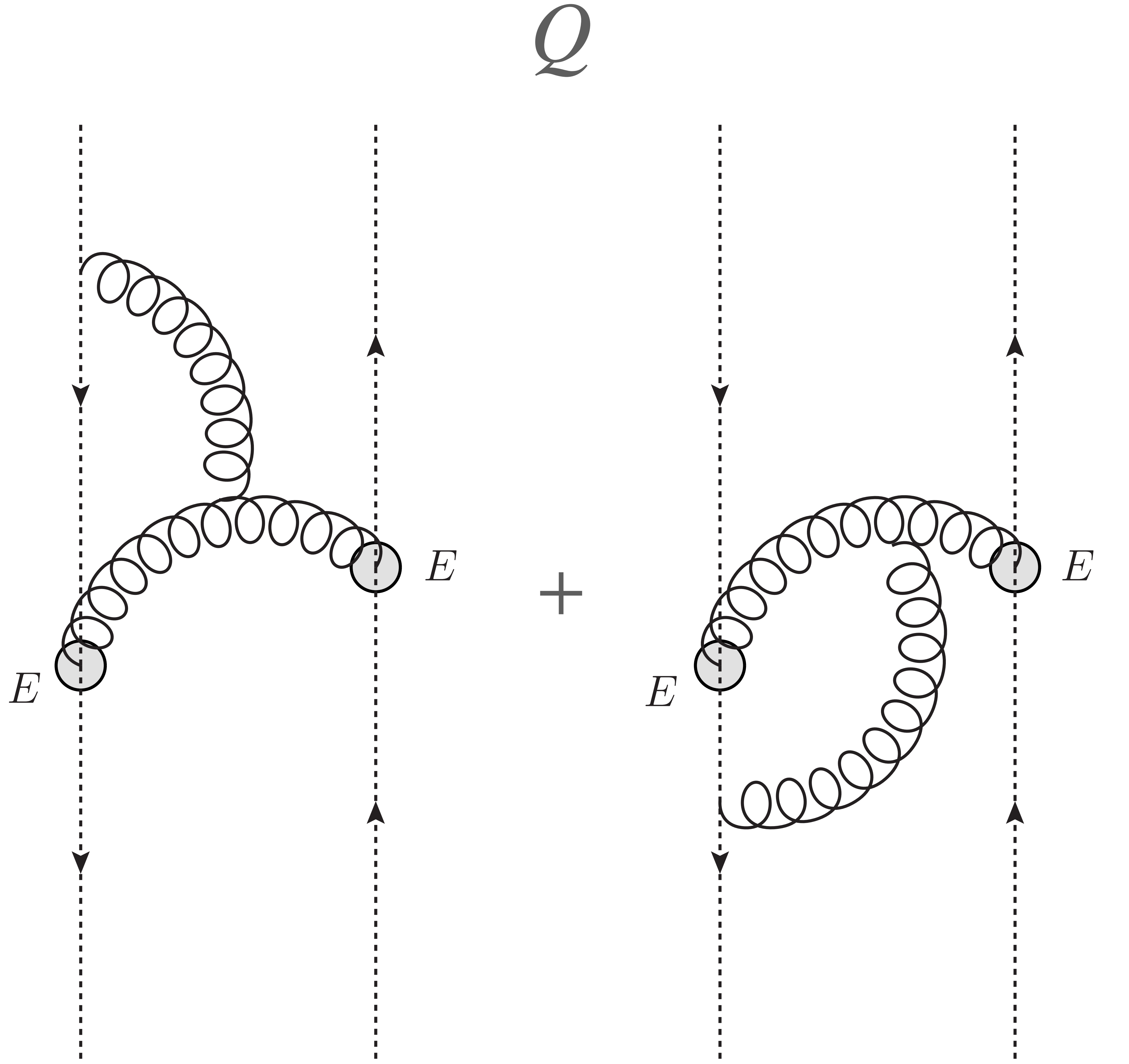
The difference in pQCD

operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi} |\,\omega\,|^3$$



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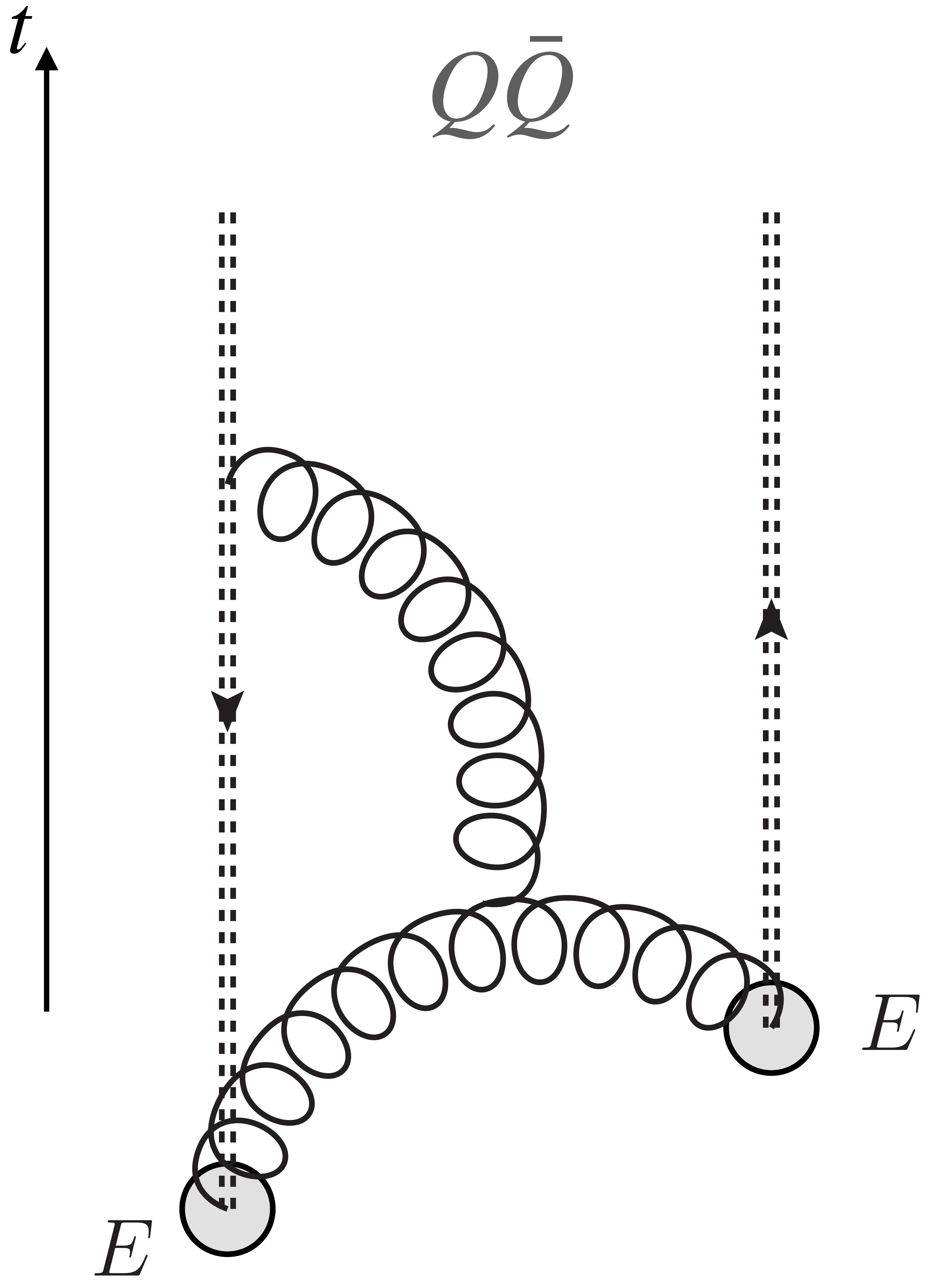

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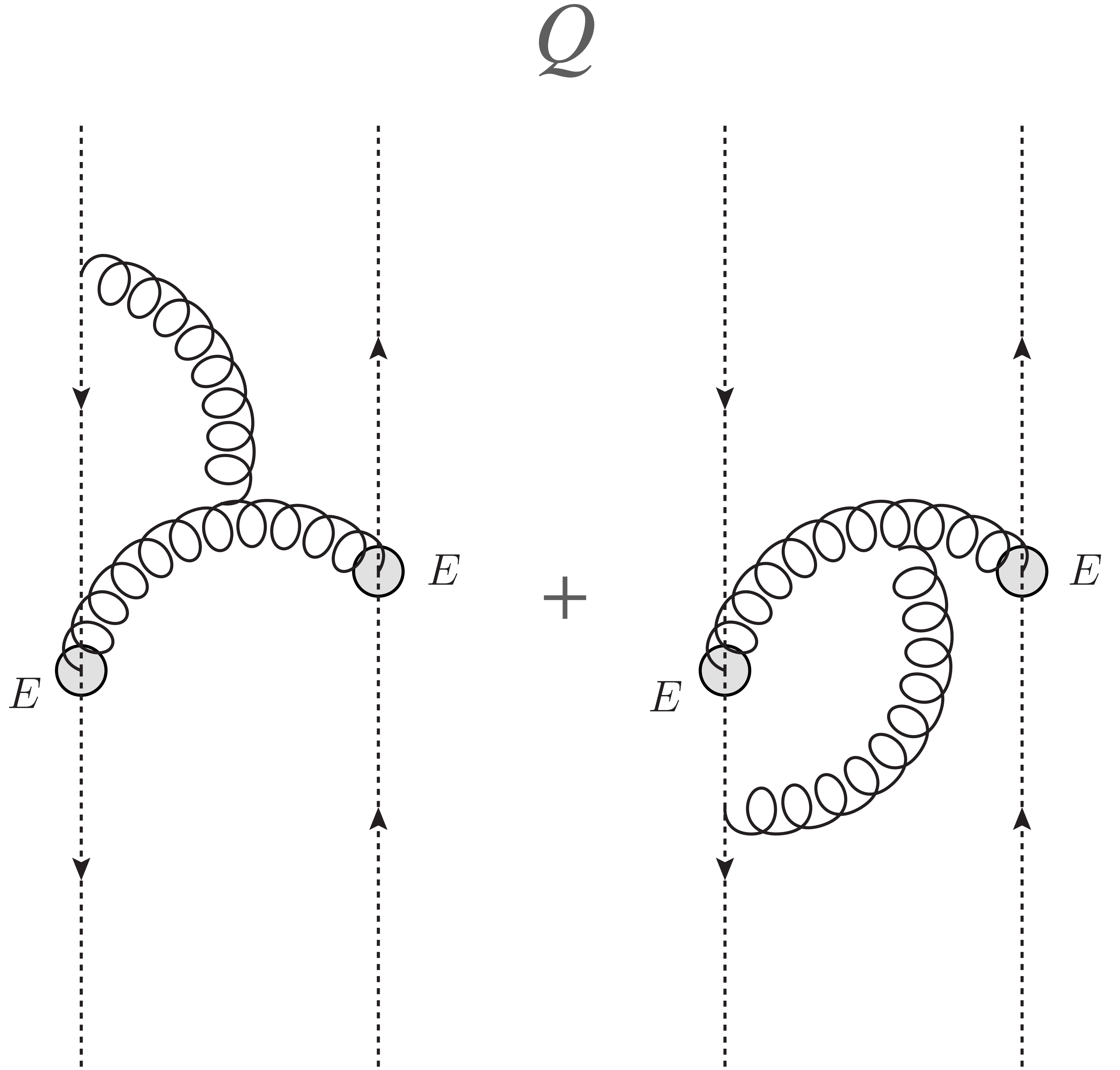
The difference is due to different operator orderings (different possible gluon insertions).



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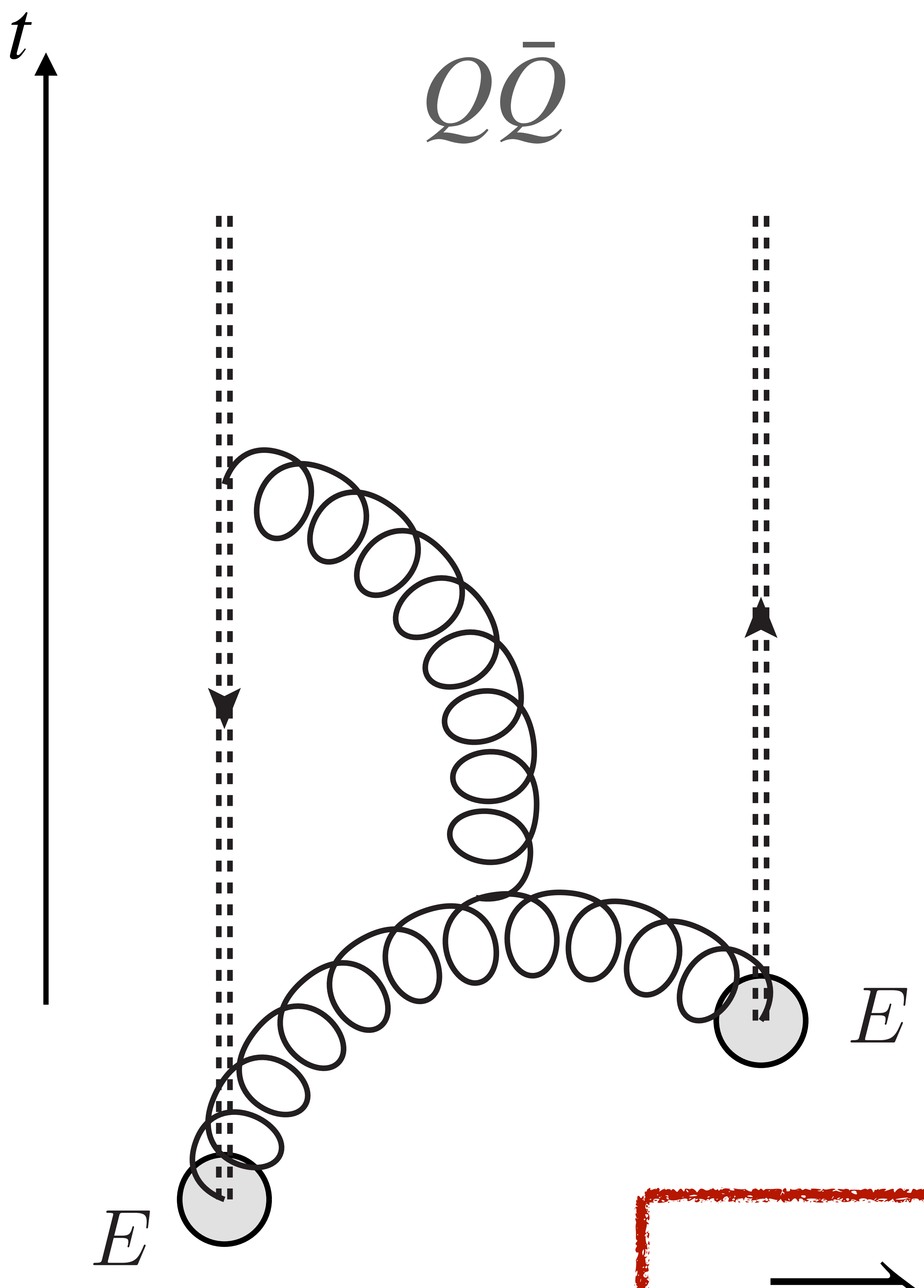

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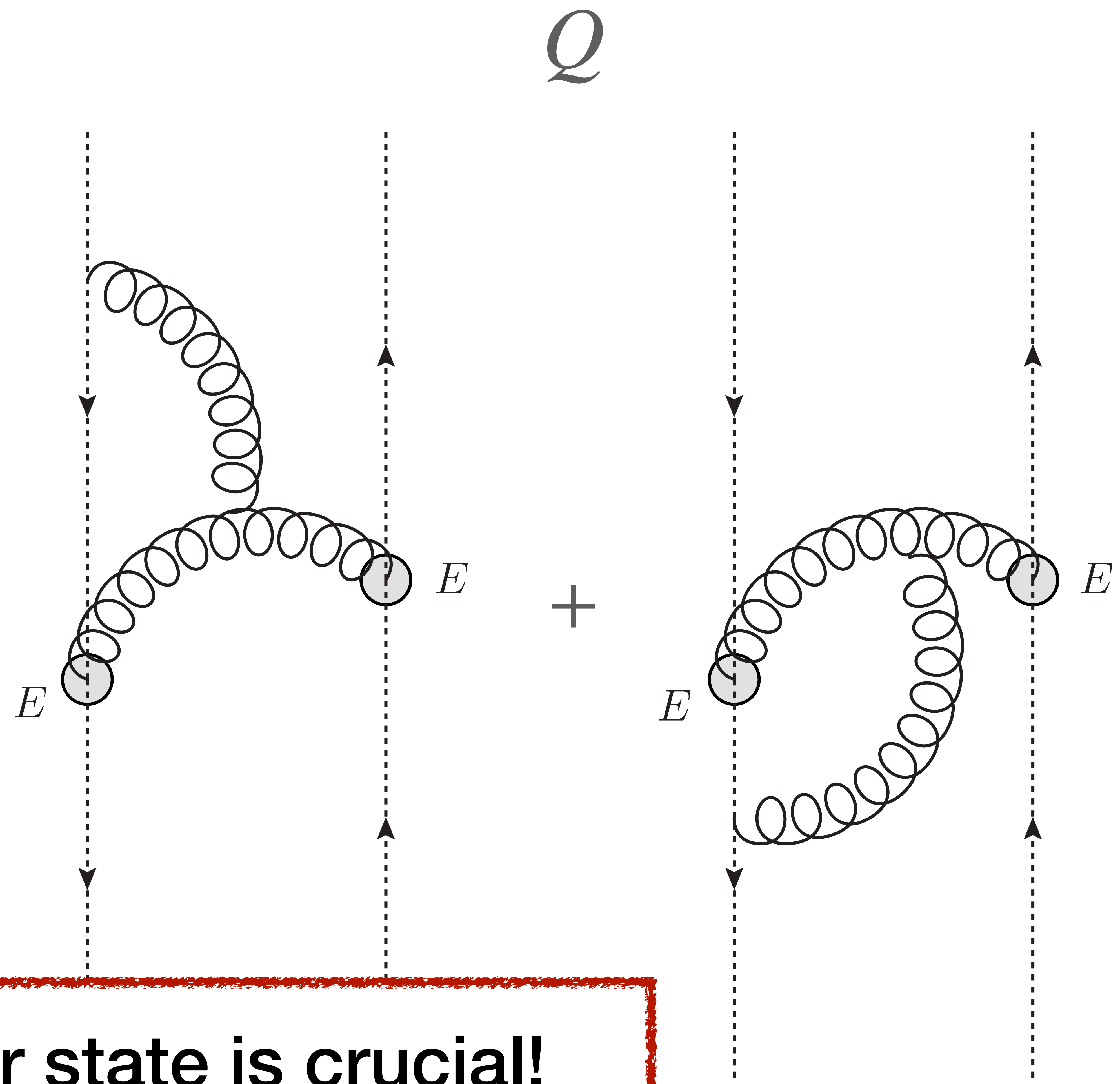
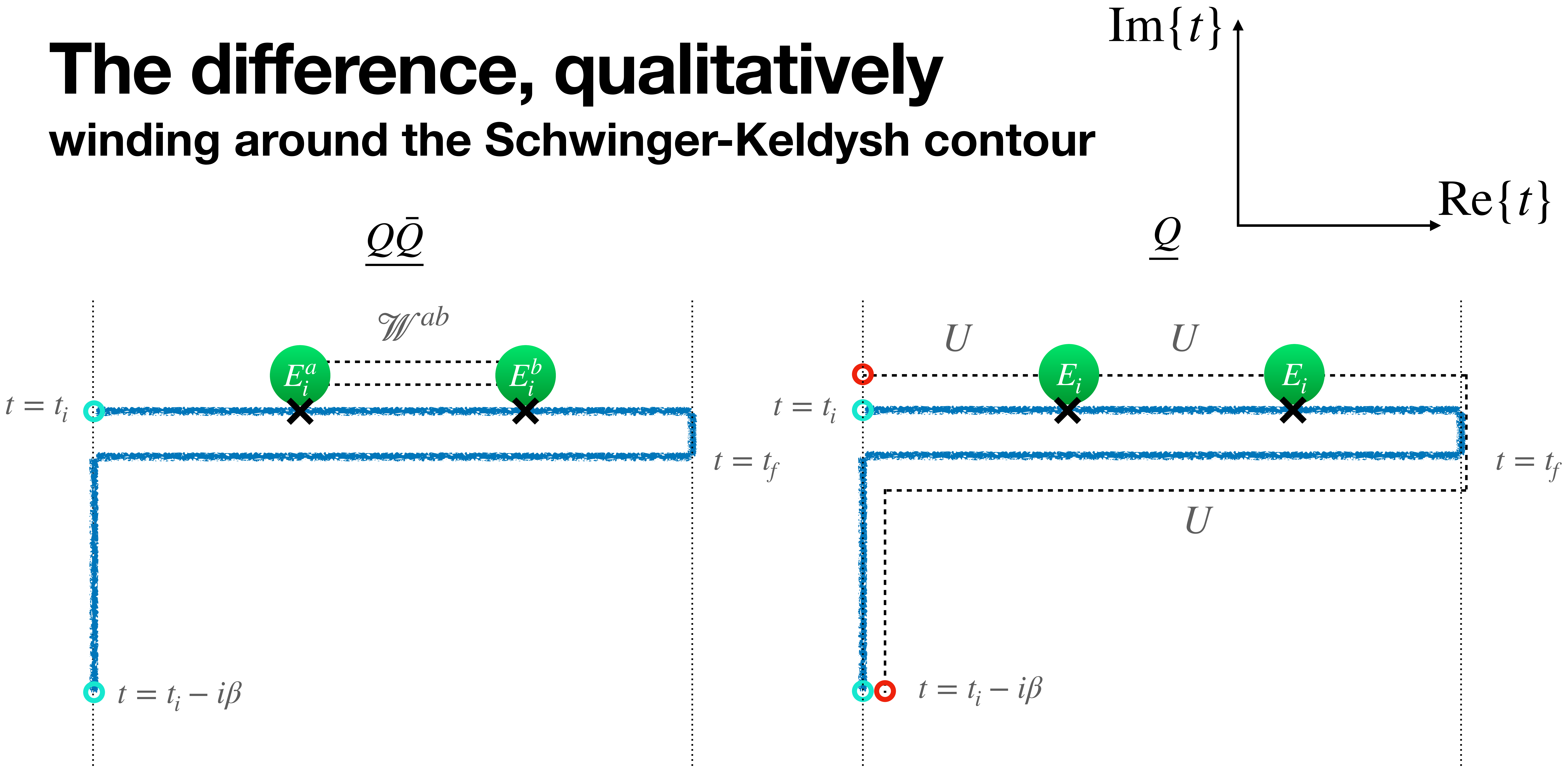


$\Longrightarrow$ **The dynamics of the color state is crucial!**

How does this difference show up non-perturbatively?

The difference, qualitatively

winding around the Schwinger-Keldysh contour



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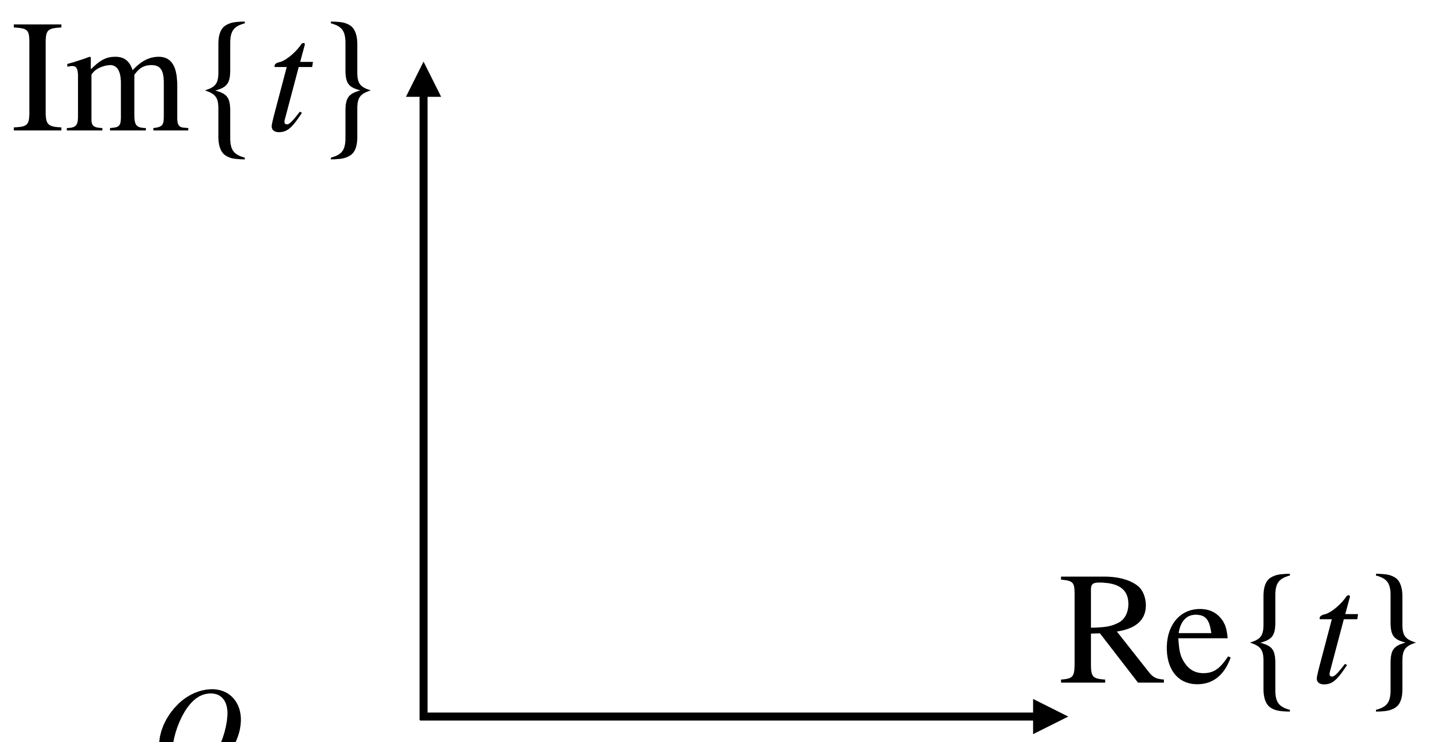
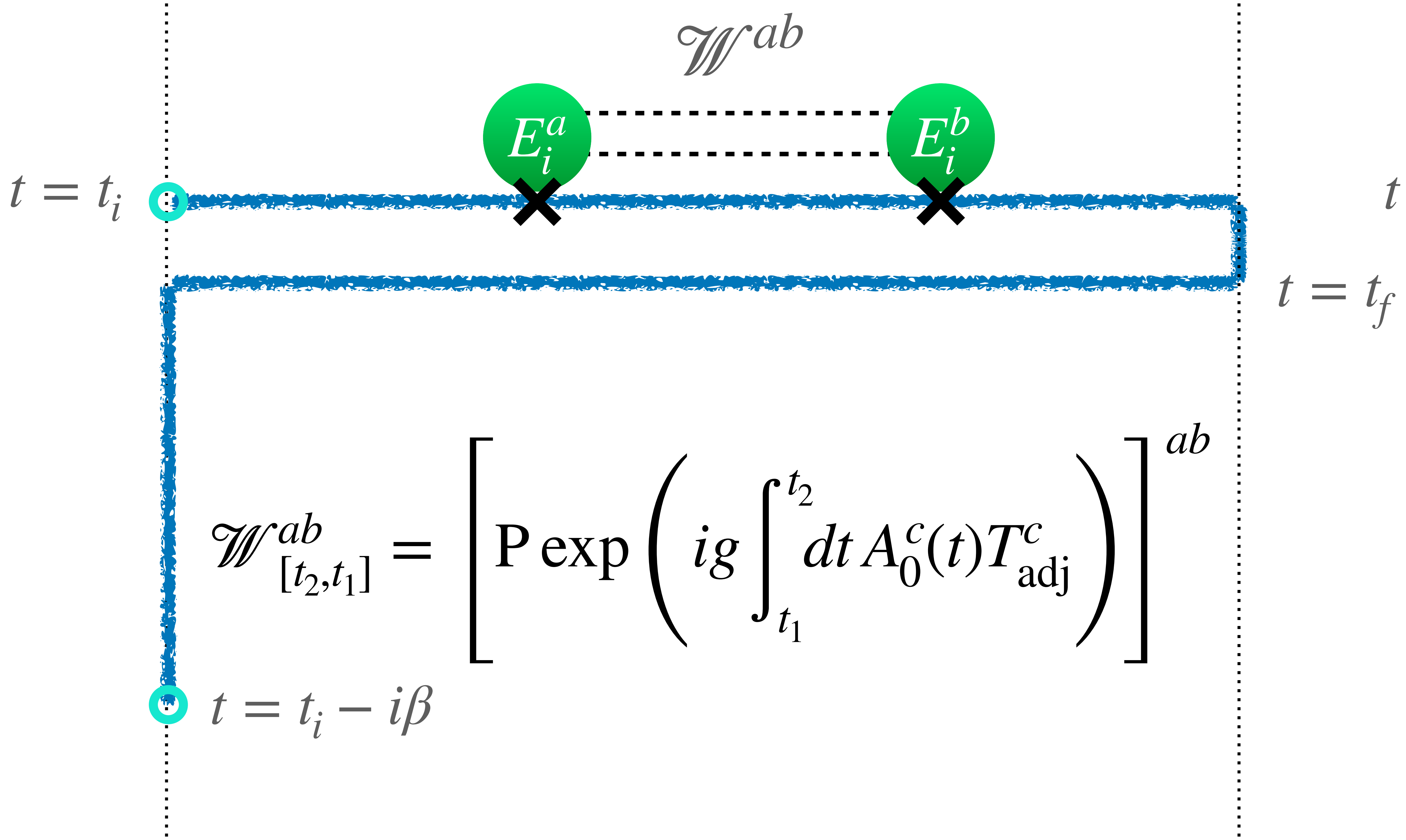
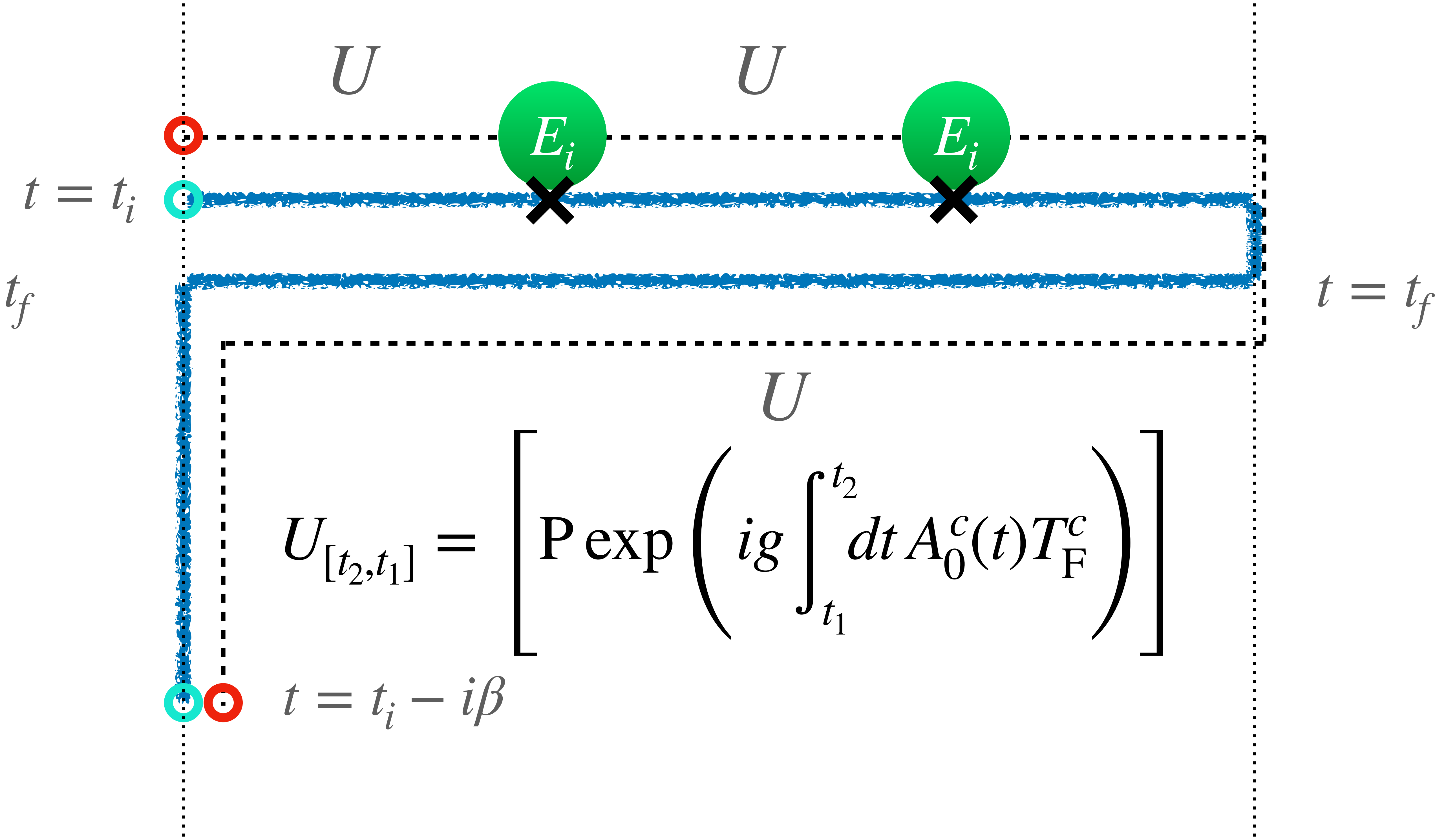


$Q\bar{Q}$



$$\mathscr{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$

Q

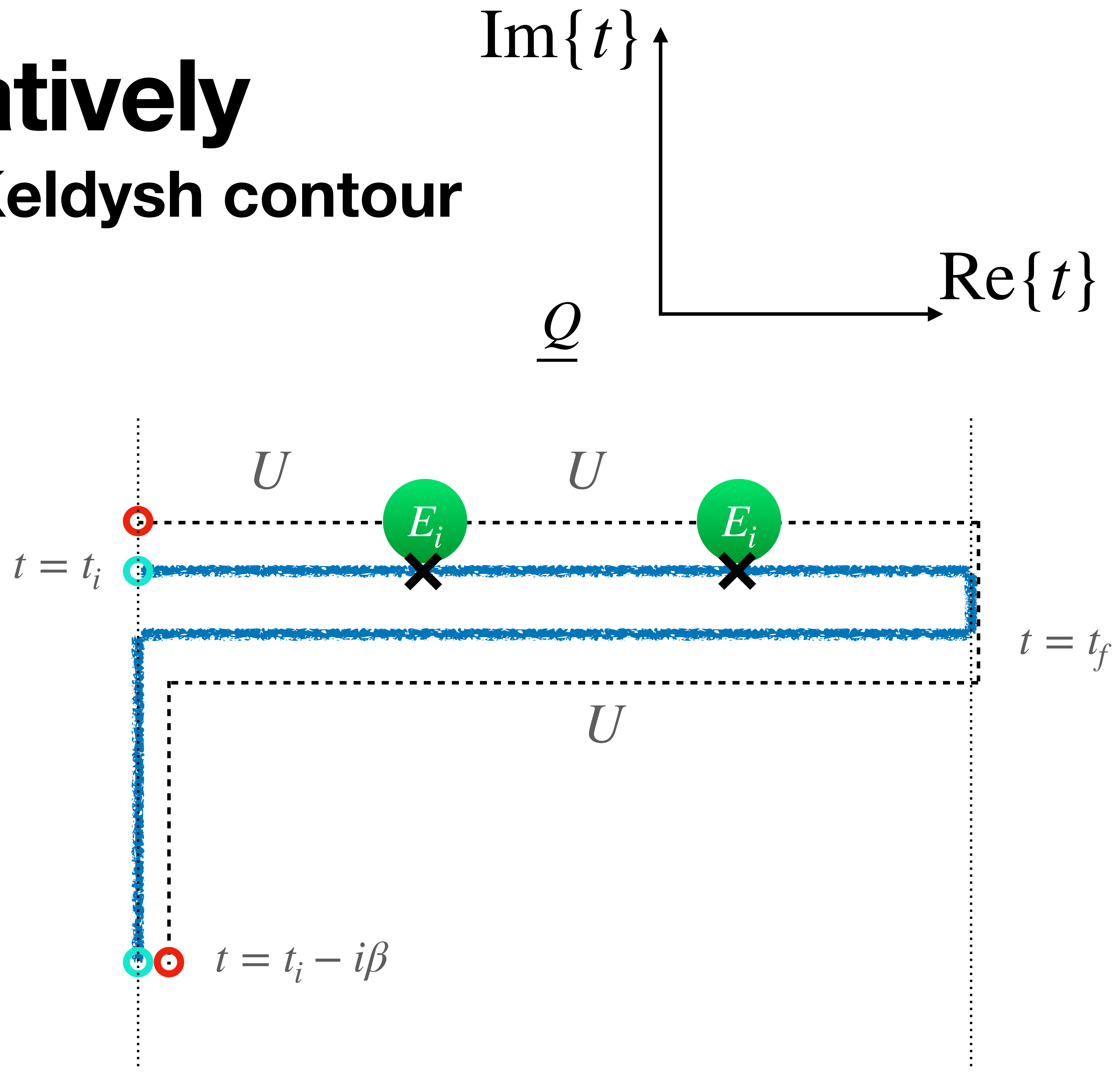


$$U_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{F}} \right) \right]$$

The difference, qualitatively

winding around the Schwinger-Keldysh contour

- The heavy quark is present at all times:
    - It is part of the construction of the thermal state of the QGP.
    - The Wilson line, which enforces the Gauss' law constraint due to the point charge, is also present on the Euclidean segment.



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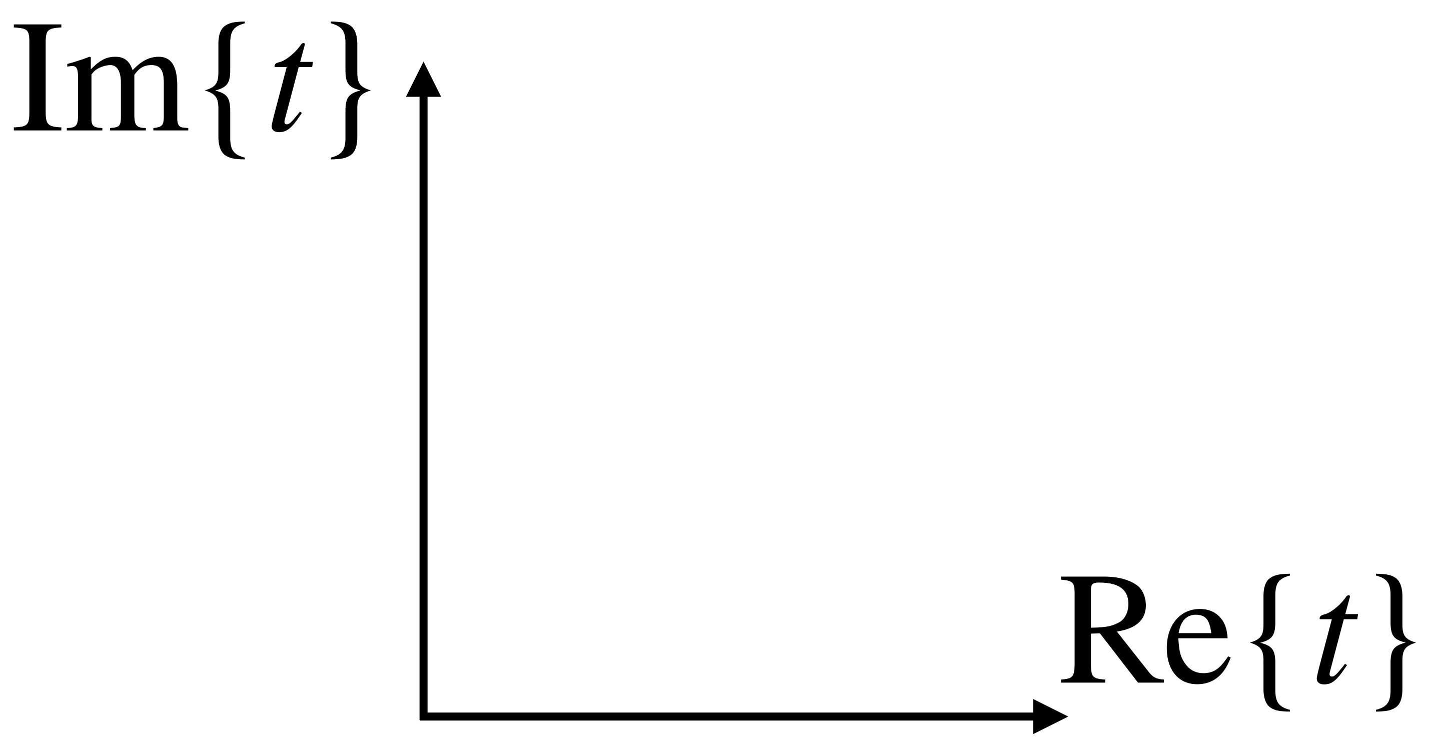

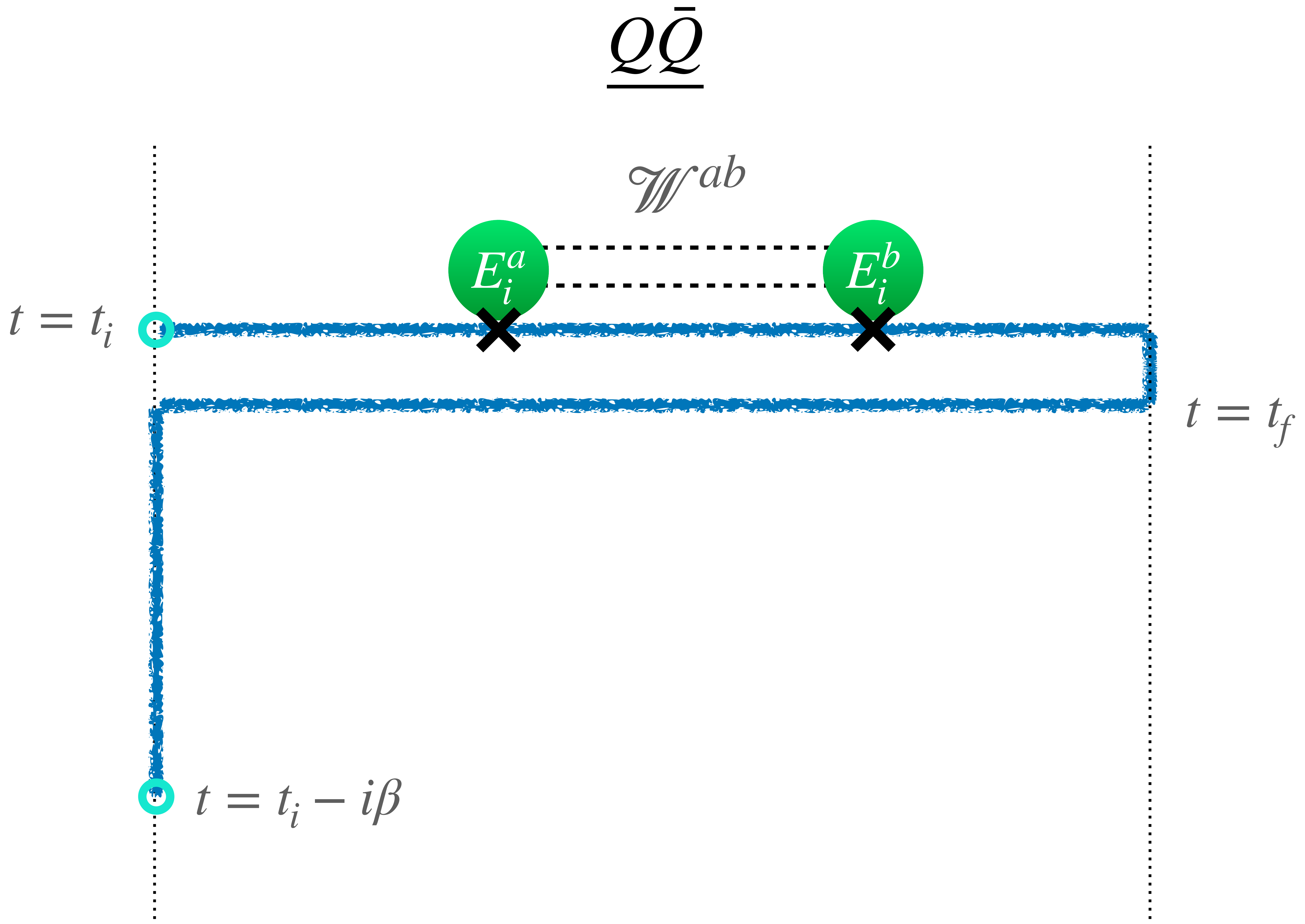

- In this correlator, the heavy quark pair is present at all times, but it is only color-charged for a finite time:
  - It is *not* part of the construction of the thermal state of the QGP.
  - The adjoint Wilson line, representing the propagation of unbound quarkonium (in the adjoint representation), is only present on the real-time segment.

A Lattice QCD perspective

heavy quark diffusion

- The heavy quark diffusion coefficient has been studied by evaluating the following correlation function (e.g., Altenkort et al. 2009.13553, 2302.08501; Leino et al. 2212.10941):

$$G_{\mathrm{fund}}(\tau) = -\frac{1}{3}\frac{\left\langle \mathrm{ReTr}_c[U(\beta,\tau)\,gE_i(\tau)\,U(\tau,0)\,gE_i(0)]\right\rangle}{\left\langle \mathrm{ReTr}_c[U(\beta,0)]\right\rangle}\,.$$

- The heavy quark diffusion coefficient is extracted by reconstructing the corresponding spectral function (Caron-Huot et al. 0901.1195):

$$G_{\mathrm{fund}}(\tau) = \int_0^{+\infty}\frac{d\omega}{2\pi}\frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}\rho_{\mathrm{fund}}(\omega)\,,\quad \kappa_{\mathrm{fund}} = \lim_{\omega\to 0}\frac{T}{\omega}\rho_{\mathrm{fund}}(\omega)\,.$$

A Lattice QCD perspective

quarkonium transport (2306.13127)

- The quarkonium correlator in imaginary time has received less attention:

$$G_{\rm adj}(\tau) = \frac{T_F g^2}{3N_c} \left\langle E_i^a(\tau) \mathscr{W}^{ab}(\tau,0) E_i^b(0) \right\rangle_T .$$

- The transport coefficients can also be extracted by spectral reconstruction:

$$G_{\rm adj}(\tau) = \int_{-\infty}^{+\infty} \frac{{\rm d}\omega}{2\pi} \frac{\exp\left(\omega(\frac{1}{2T} - \tau)\right)}{2\sinh\left(\frac{\omega}{2T}\right)} \rho_{\rm adj}^{++}(\omega) \, , \quad \kappa_{\rm adj} = \lim_{\omega \to 0} \frac{T}{2\omega} \left[\rho_{\rm adj}^{++}(\omega) - \rho_{\rm adj}^{++}(-\omega) \right] .$$

- Main new ingredient: the spectral function $\rho_{\rm adj}^{++}(\omega)$ is not odd under $\omega \to -\omega$, because $G_{\rm adj}(\tau)$ is not invariant under $\tau \to 1/T - \tau$.

How does one use the GGDs?

Transport equations for quarkonia

in the semiclassical limit (Mehen and Yao, 2009.02408)

- One can derive a Boltzmann equation for the bound state density, namely

$$\frac{dn_b(t,\mathbf{x})}{dt} = -\Gamma^{\text{diss}}\, n_b(t,\mathbf{x}) + \Gamma^{\text{form}}(t,\mathbf{x})\,.$$

- The transition rates are given by

$$\Gamma^{\text{diss}} = \int \frac{\mathrm{d}^3\mathbf{p}_{\text{rel}}}{(2\pi)^3} \,|\langle \psi_{\mathscr{B}}|\mathbf{r}|\Psi_{\mathbf{p}_{\text{rel}}}\rangle|^2 \frac{\rho^{++}_{\text{adj}}\big(-|E_{\mathscr{B}}| - \mathbf{p}^2_{\text{rel}}/M\big)}{1-\exp\big[(|E_{\mathscr{B}}| + \mathbf{p}^2_{\text{rel}}/M)/T\big]}\,,$$

$$\Gamma^{\text{form}}(t,\mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{p}_{\text{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_{\text{rel}}}{(2\pi)^3} \,|\langle \psi_{\mathscr{B}}|\mathbf{r}|\Psi_{\mathbf{p}_{\text{rel}}}\rangle|^2 \frac{\rho^{++}_{\text{adj}}\big(-|E_{\mathscr{B}}| - \mathbf{p}^2_{\text{rel}}/M\big)}{\exp\big[-(|E_{\mathscr{B}}| + \mathbf{p}^2_{\text{rel}}/M)/T\big]-1}\, f_{Q\bar{Q}}\,.$$

Transport equations for quarkonia

in the Brownian motion limit (Brambilla et al. 2302.11826 and previous work)

- Quantum Brownian motion limit: $Mv \gg T \gg Mv^2$

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S, \rho_S(t)\right] + \kappa_{\rm adj}\left(L_{\alpha i}\rho_S(t)L^\dagger_{\alpha i} - \frac{1}{2}\left\{L^\dagger_{\alpha i}L_{\alpha i}, \rho_S(t)\right\}\right)$$

where $\kappa_{\rm adj} + i\gamma_{\rm adj} = \dfrac{T_F g^2}{3N_c}\displaystyle\int_{-\infty}^{\infty} dt\, \langle \mathcal{T}\, E_i^a(t)\mathcal{W}^{ab}_{[t,0]}E_i^b(0)\rangle.$

- Note that

$$\frac{T_F g^2}{3N_c}\int_{-\infty}^{\infty} dt\, \langle \mathcal{T}\, E_i^a(t)\mathcal{W}^{ab}_{[t,0]}E_i^b(0)\rangle = 2\int_0^{\infty} dt\, [g^{++}_{\rm adj}]^{>}(t)\ .$$

Let’s calculate!

What is the result of an NLO calculation?

2107.03945, 2306.13127

- One finds, up to $\mathcal{O}(g^4)$

$$\rho_{\mathrm{adj}}^{++}(\omega) = \frac{T_F g^2 (N_c^2 - 1)\omega^3}{3\pi N_c}\left\{1 + \frac{g^2}{(2\pi)^2}\left[\left(\frac{11N_c}{12} - \frac{N_f}{6}\right)\ln\left(\frac{\mu^2}{4\omega^2}\right) + N_c\left(\frac{149}{36} - \frac{\pi^2}{6} + \frac{\pi^2}{2}\mathrm{sgn}(\omega)\right) - \frac{5N_f}{9} + F(\omega/T)\right]\right\}$$

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- For comparison, the spectral function for heavy quark diffusion is: cf. 1006.0867 [hep-ph]

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This difference has an effect in transport!

difference between γ_{adj} and γ_{fund}

The γ transport coefficients

$$\gamma_{\text{fund}} = \frac{g^2}{3N_c}\text{Im}\int_{-\infty}^{\infty} \mathrm{d}t \left\langle \text{Tr}_{\text{c}}[U(-\infty, t)E_i(t)U(t,0)E_i(0)U(0,-\infty)] \right\rangle_{T,Q},$$

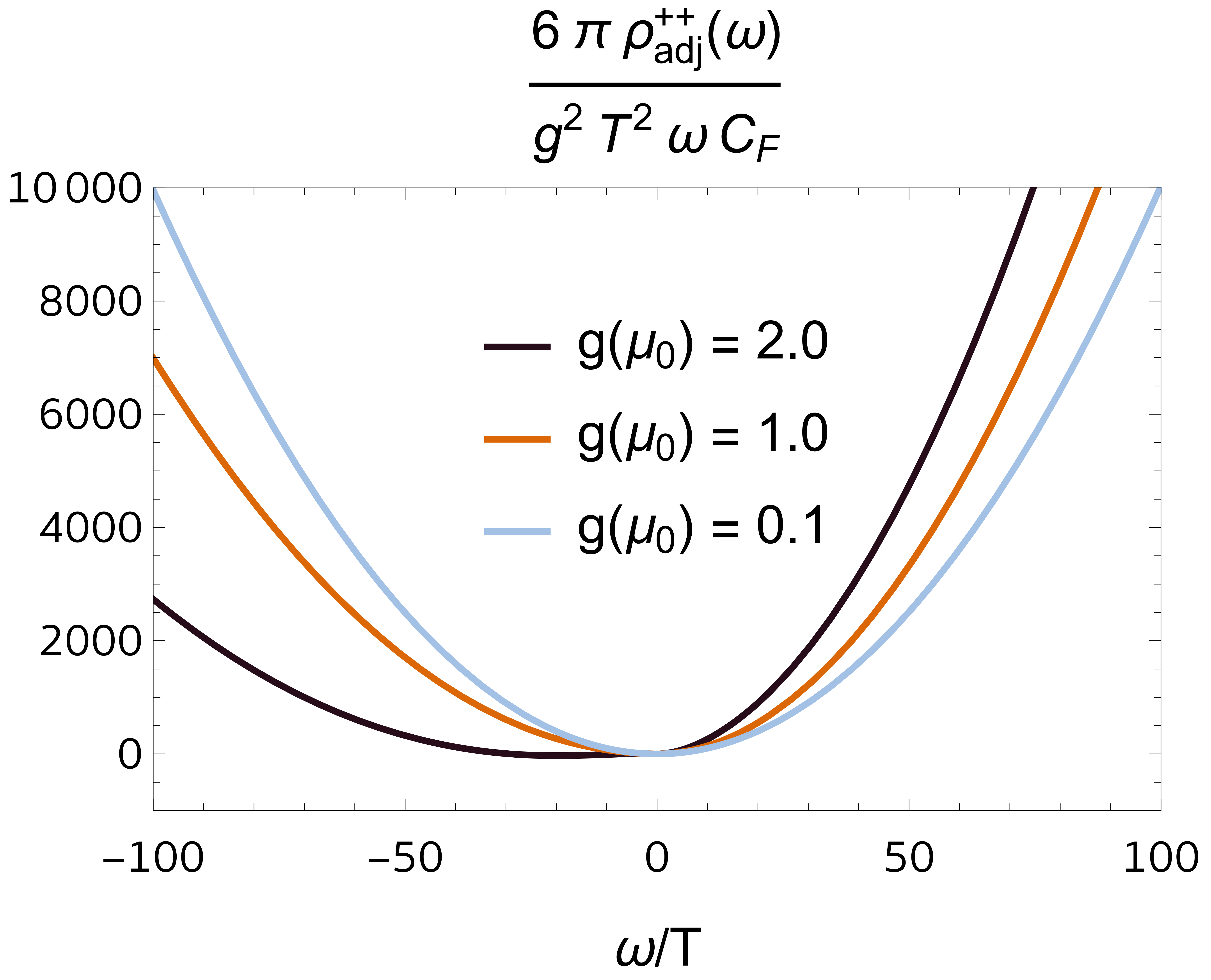
$$\gamma_{\text{adj}} = \frac{g^2 T_F}{3N_c}\text{Im}\int_{-\infty}^{\infty} \mathrm{d}t \left\langle \mathcal{T} E_i^a(t)W^{ab}(t,0)E_i^b(0) \right\rangle_{T},$$

differ at $\mathcal{O}(g^4)$: (first pointed out by Eller, Ghiglieri and Moore in 1903.08064)

$$\Delta\gamma = \gamma_{\text{fund}} - \gamma_{\text{adj}} = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{\pi|\omega|} n_B(|\omega|)\Delta\rho(\omega) = \frac{16\zeta(3)}{3} T_F C_F N_c \alpha_s^2 T^3$$

ρ_{adj}^{++} in perturbation theory
varying the coupling

- Let's look at $\rho_{\text{adj}}^{++}(\omega)$ in the regime where we trust perturbation theory: $|\omega| \gg T(\sim \Lambda_{\text{QCD}})$.
- We see that as we dial up the coupling from zero, an asymmetry develops between positive and negative frequencies.
- Why does this happen?



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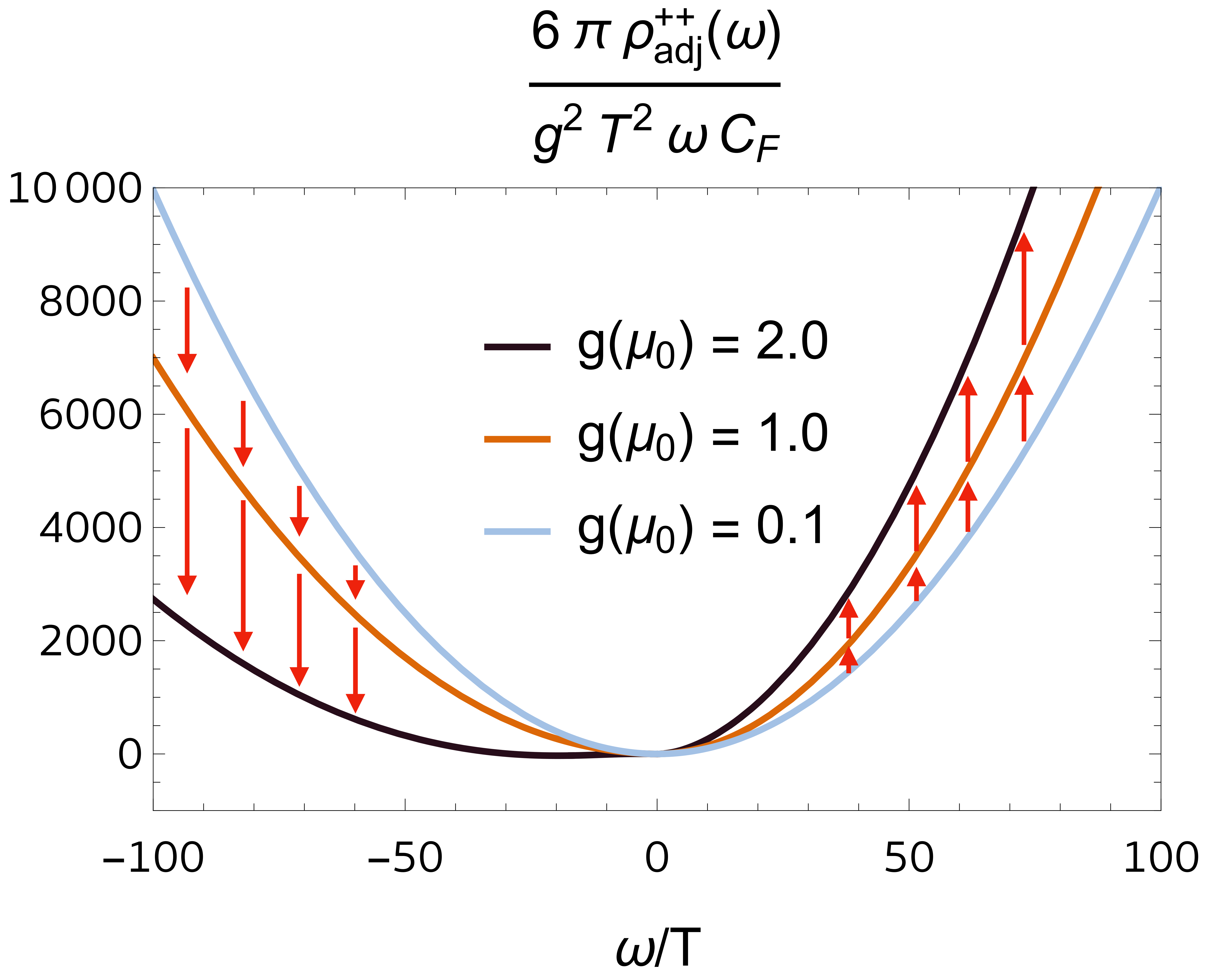

The spectral function $\rho_{\rm adj}^{++}$

a closer look

- Note that, contrary to usual thermal field theory correlators, $\rho_{\rm adj}^{++}(\omega)$ is not odd under $\omega \to -\omega$, because

$$[g_{\rm adj}^{++}]^{>}(\omega) \neq [g_{\rm adj}^{--}]^{>}(\omega)\,.$$

- This is a reflection of the fact that the initial state in a dissociation process is different than for (re)combination. Explicitly,

$$\rho_{\rm adj}^{++}(\omega) = \frac{g^2 T_F}{3N_c} \sum_{n,\tilde{n}} (2\pi)\delta(\omega + E_n - \tilde{E}_{\tilde{n}}) |\langle n | E_i^a(0) | \tilde{n}^a \rangle|^2 \left[e^{-\beta E_n} - e^{-\beta \tilde{E}_{\tilde{n}}} \right] .$$

$\{E_n, |n\rangle\}$ = eigenvalues/eigenstates of $H_{\rm QGP}$

$\{\tilde{E}_n, |\tilde{n}^a\rangle\}$ = eigenvalues/eigenstates of $H_{\rm QGP}\,\delta^{ab} - gA_0^c(0)[T_{\rm adj}^c]^{ab}$

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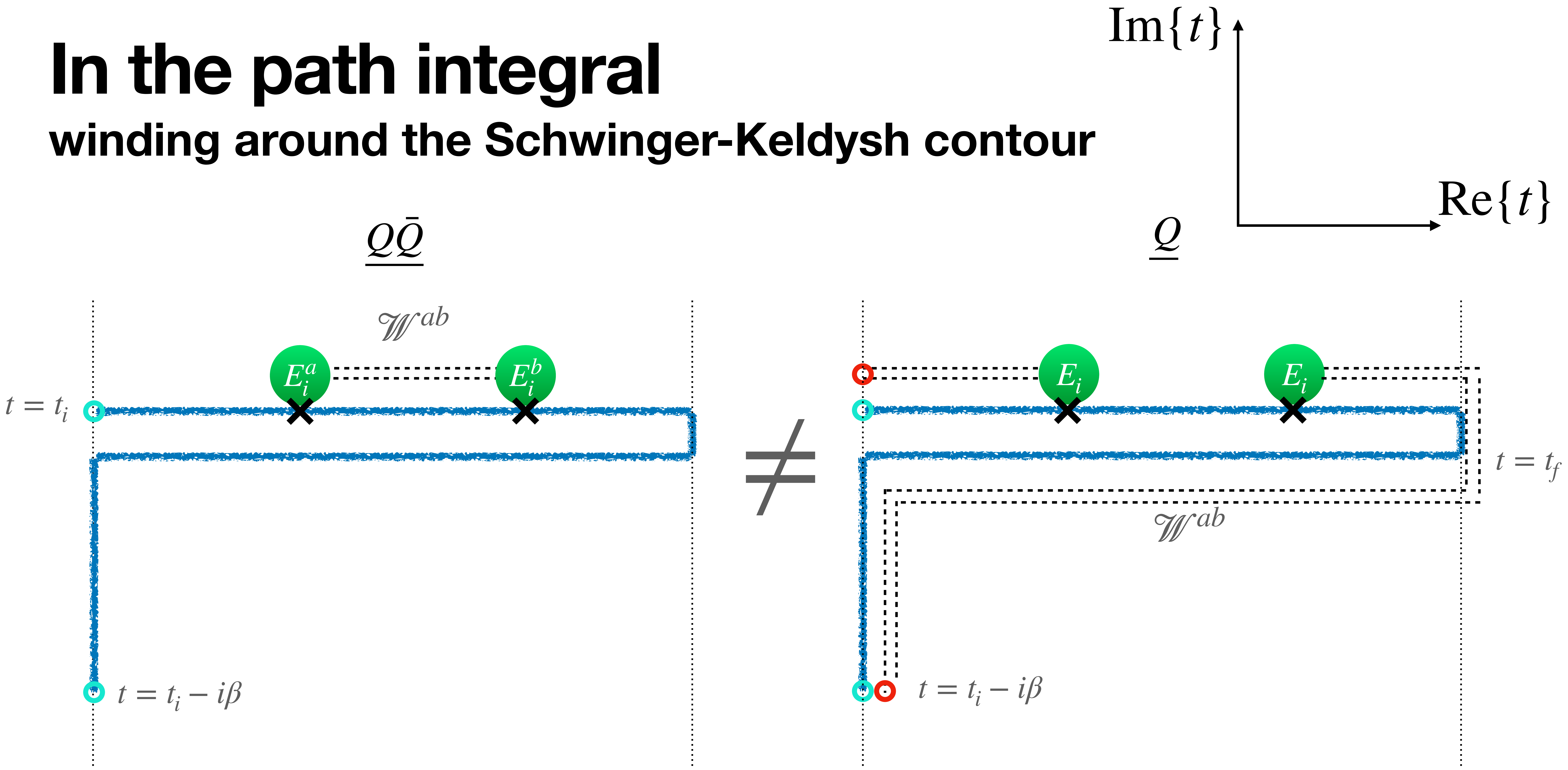
In the path integral

winding around the Schwinger-Keldysh contour



What should we expect for a strongly coupled plasma?

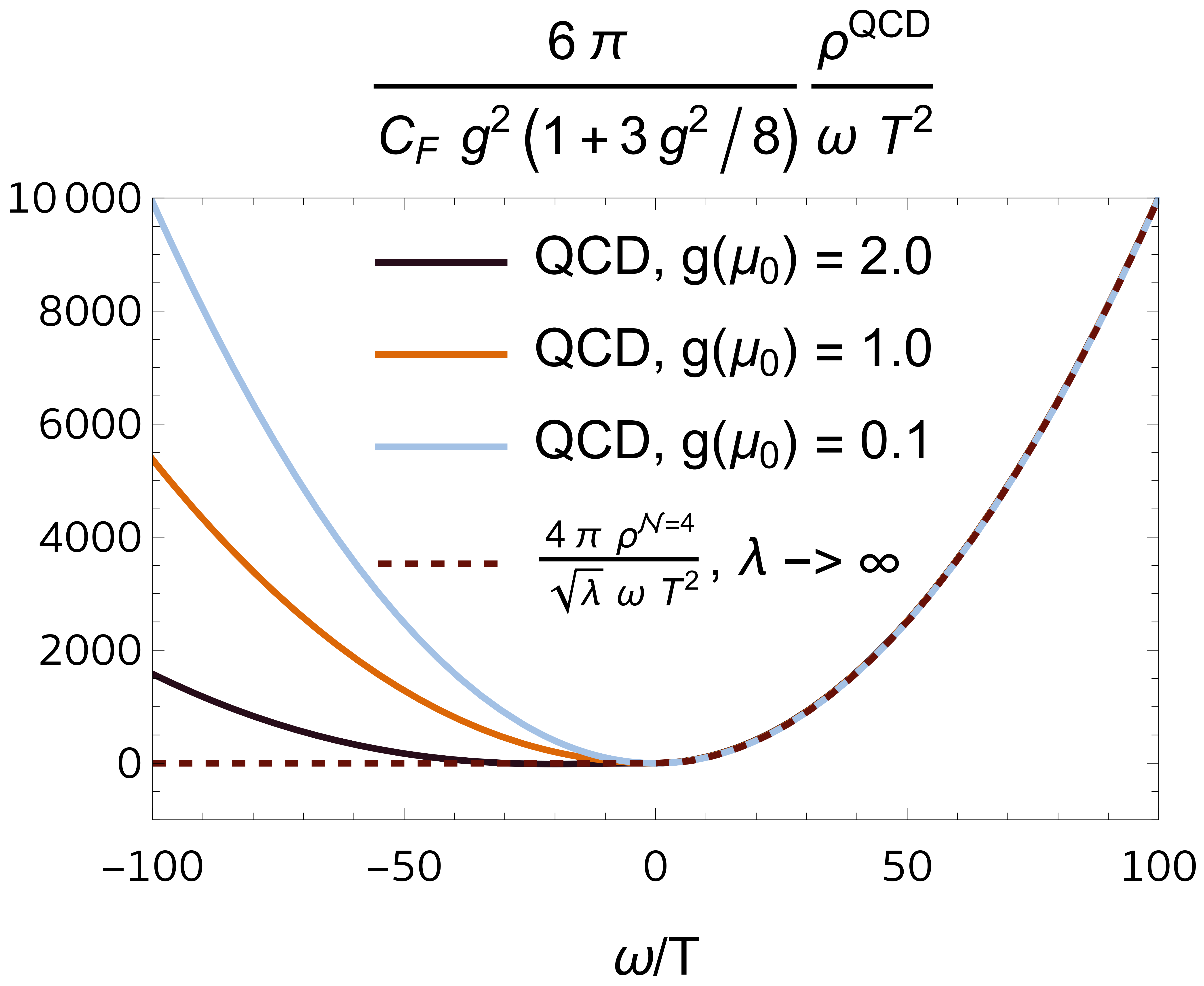
(with G. Nijs and X. Yao) 2310.09325

ρ_{adj}^{++} in perturbation theory

compared to $\mathcal{N} = 4$ SYM result at large N_c and strong coupling

- To clearly see the asymmetry between positive and negative ω, we normalize the different curves for $\rho_{\text{adj}}^{++}(\omega)$ so that their $\omega \to \infty$ limit agrees.

- The $\mathcal{N} = 4$ result at large N_c and strong coupling $\lambda = N_c g^2$ is compatible with the behavior of the weakly coupled limit as g is increased.



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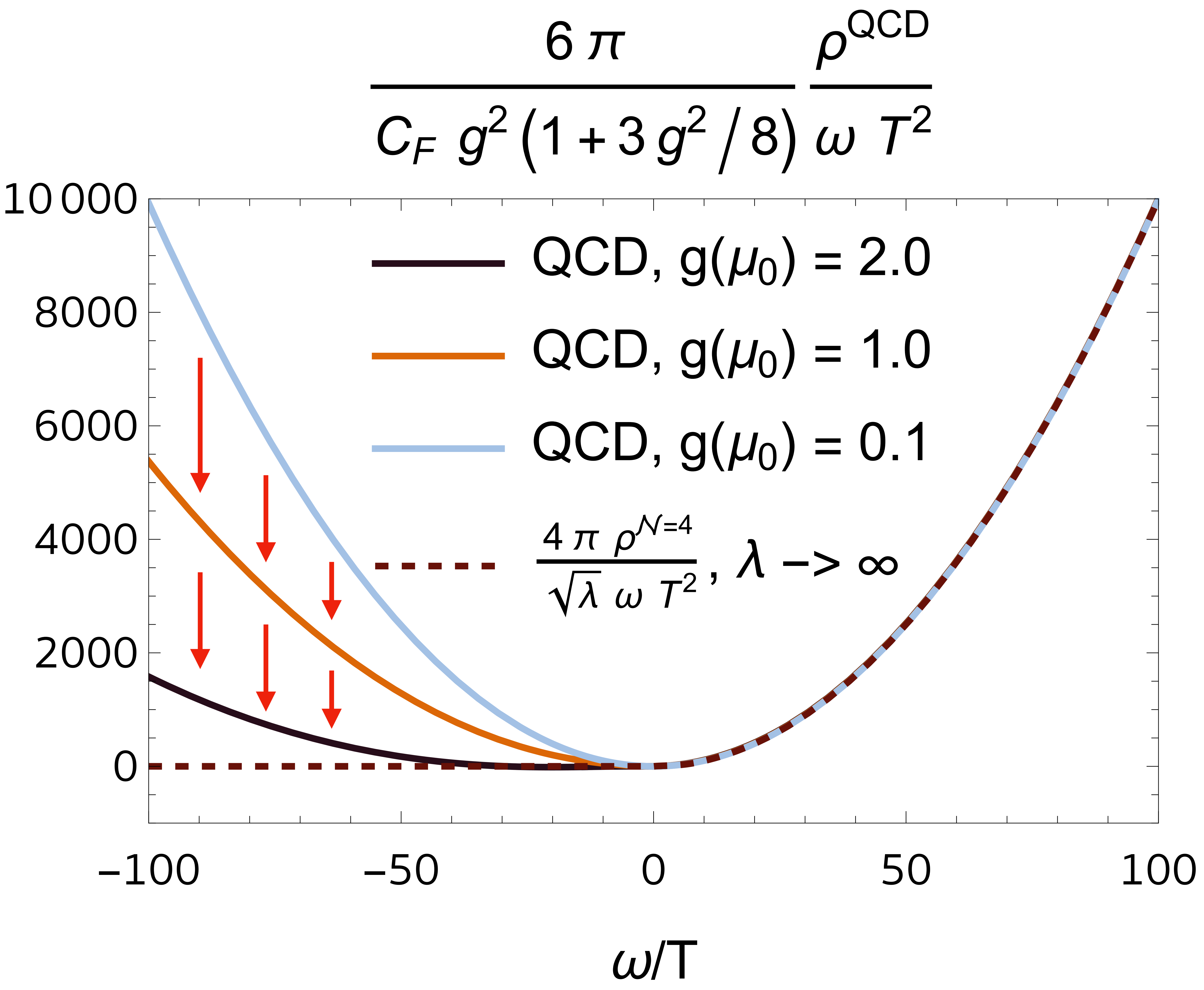

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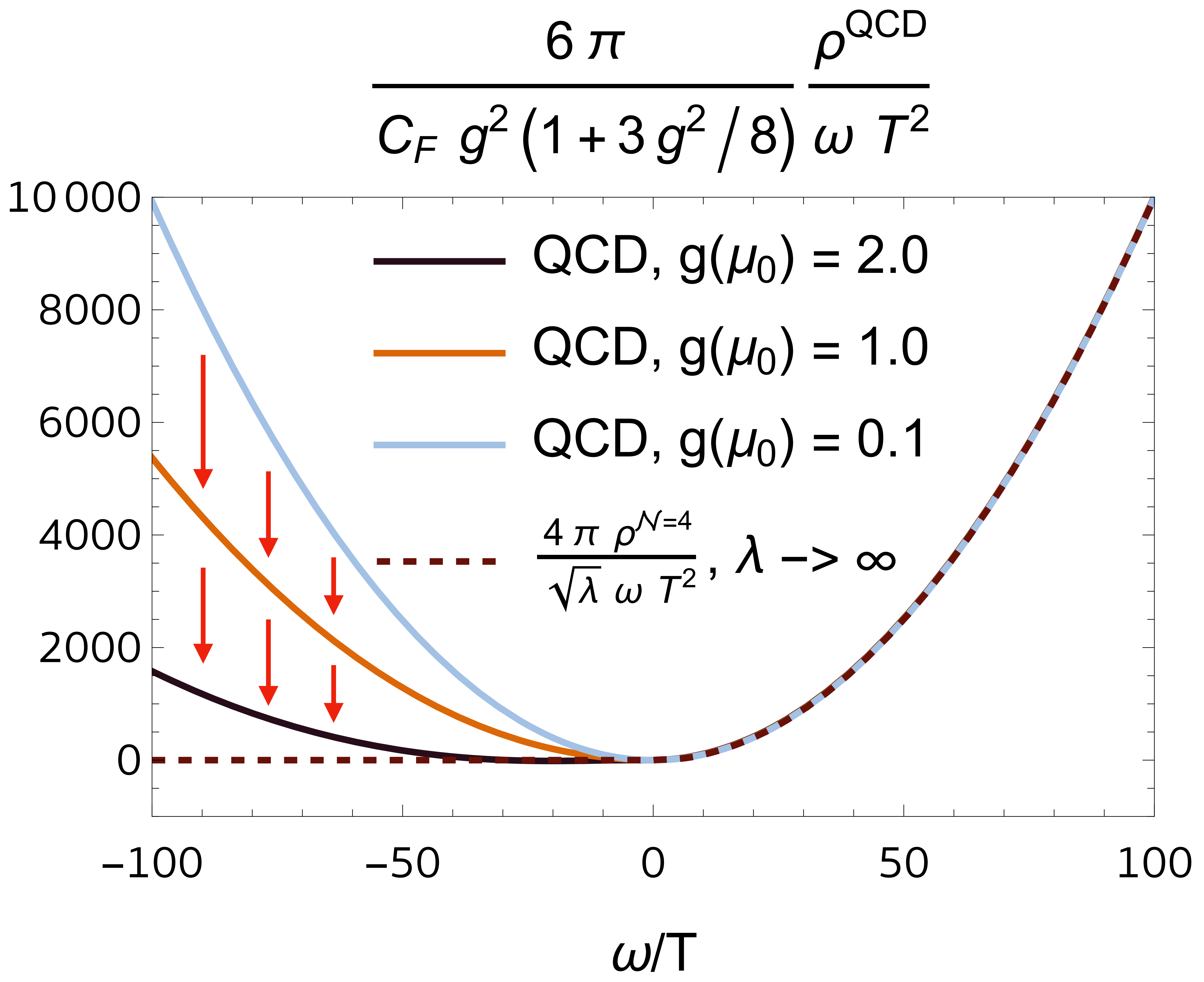

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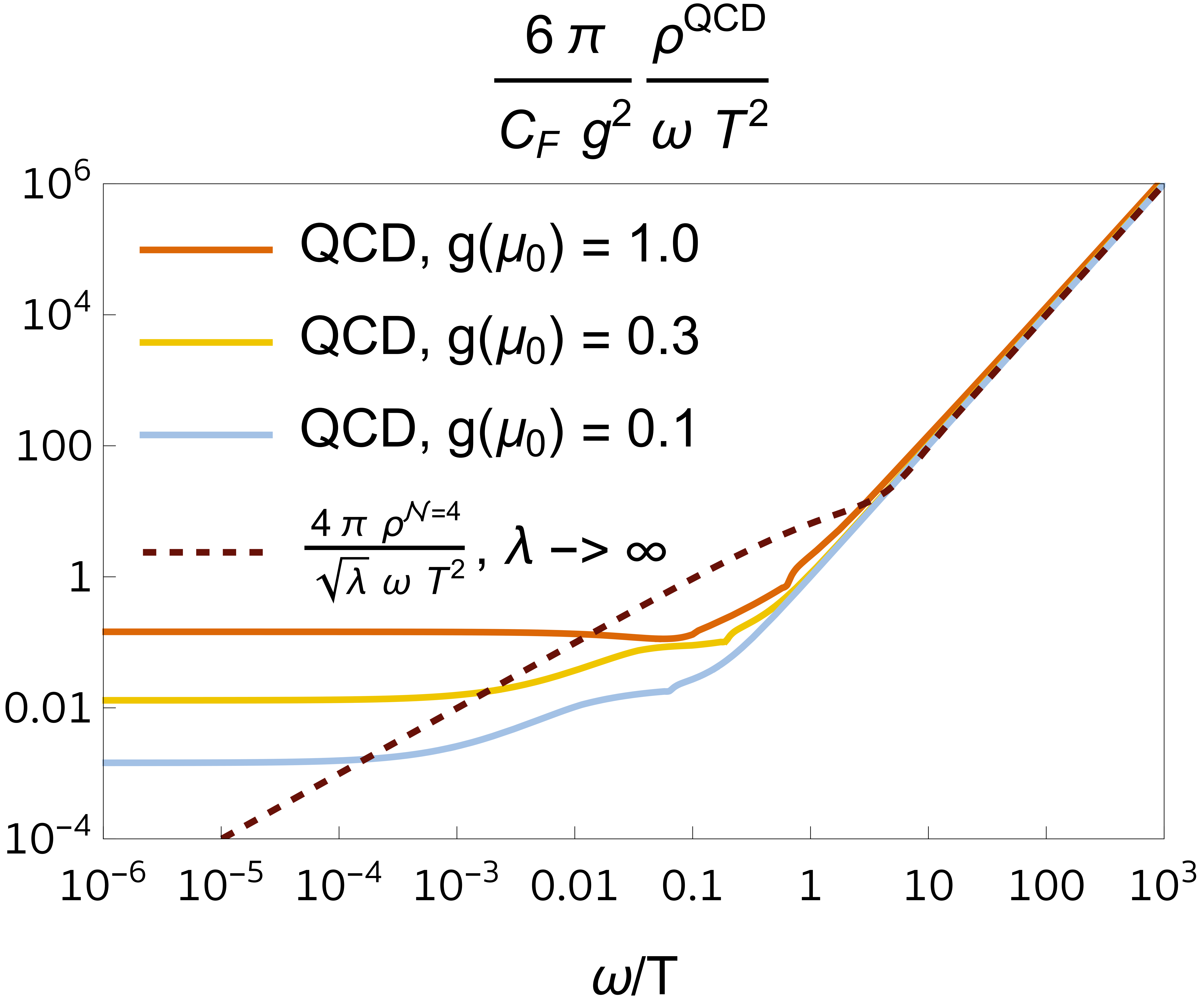


Consequences for transport

at strong coupling

- A naive application of the quantum optical limit gives trivial dynamics in the strongly coupled $\mathcal{N} = 4$ SYM plasma:

$$\rho_{\text{adj}}^{++}(-|\Delta E|) = 0 \,.$$

- Reason behind this: this transport description and every other EFT description currently on the market make assumptions that are intrinsically tied to weak coupling approximations.
    - Concretely, memory effects are neglected: this makes sense if every scattering can be approximated as independent, but not if the correlations of the medium are strong.
    - Since QGP at $T \sim 200\,\mathrm{MeV}$ is strongly coupled, we can't assume these effects are not present.

What to do then?

for a strongly coupled plasma (ongoing work)

- Back to open quantum systems basics:

$$\rho_{Q\bar{Q}}(t) = \mathrm{Tr}_{\mathrm{QGP}}\left[U(t)\rho_{\mathrm{tot}}(t=0)U^{\dagger}(t)\right] .$$

- From here, expanding up to second order in the interaction, one can derive a formula for the occupancies of the $Q\bar{Q}$ states after a proper time $t_f - t_i$ going through the plasma: e.g., for an octet $->$ nl transition

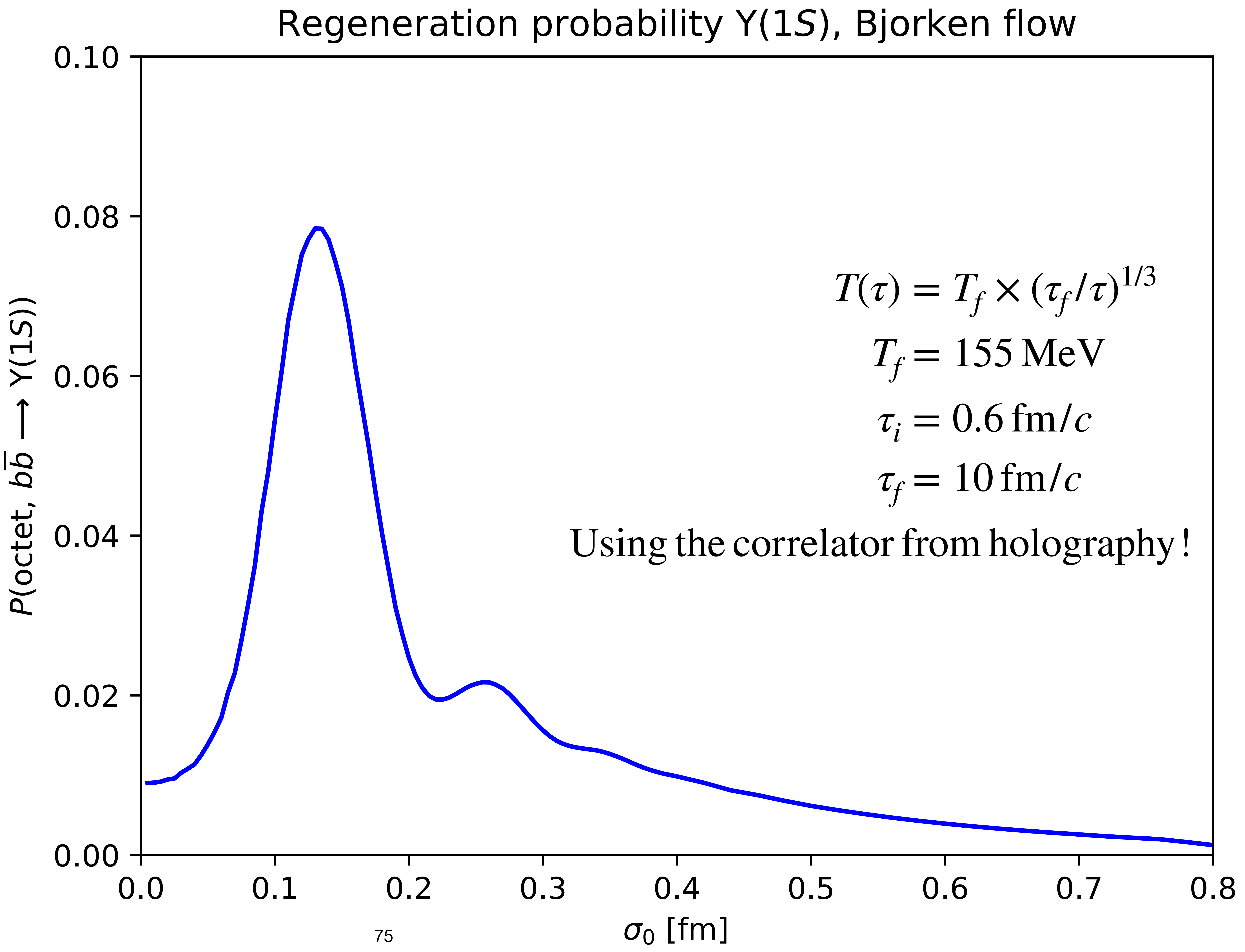
$$\langle nl \,|\, \rho_{Q\bar{Q}}(t_f) \,|\, nl \rangle = \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 \,[g_{\overline{\mathrm{adj}}}^{--}]^{>}(t_2, t_1)\,\langle nl \,|\, U^{\mathrm{singlet}}_{[t_f,t_1]} r_i U^{\mathrm{octet}}_{[t_1,t_i]} \,|\, \psi_0 \rangle \left(\langle nl \,|\, U^{\mathrm{singlet}}_{[t_f,t_2]} r_i U^{\mathrm{octet}}_{[t_2,t_i]} \,|\, \psi_0 \rangle \right)^{\dagger}$$

- Still to do: re-sum the Dyson series in different limits (e.g., large N_c).

(with G. Nijs and X. Yao) 2312.12307

We can now evolve a state of a heavy quark-antiquark pair, taking into account:

- ☑ Their wavefunction evolution using a potential model, allowing for different initial separations σ_0 between the pair.
- ☑ Their transition rates via the correlator we just discussed.



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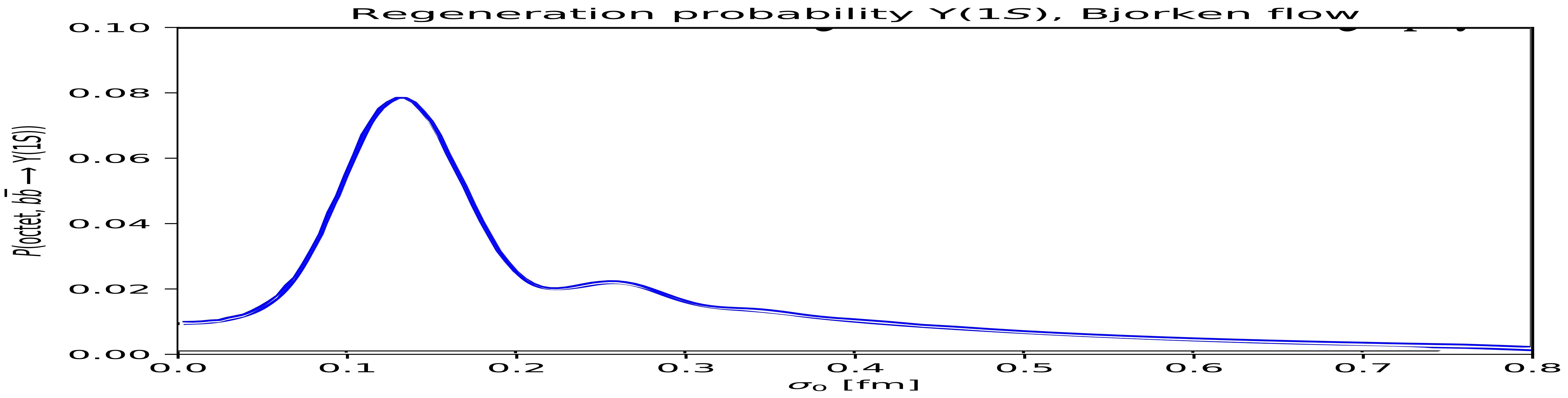
$$T(\tau) = T_f \times (\tau_f/\tau)^{1/3}$$

$$T_f = 155\,\mathrm{MeV}$$

$$\tau_i = 0.6\,\mathrm{fm}/c$$

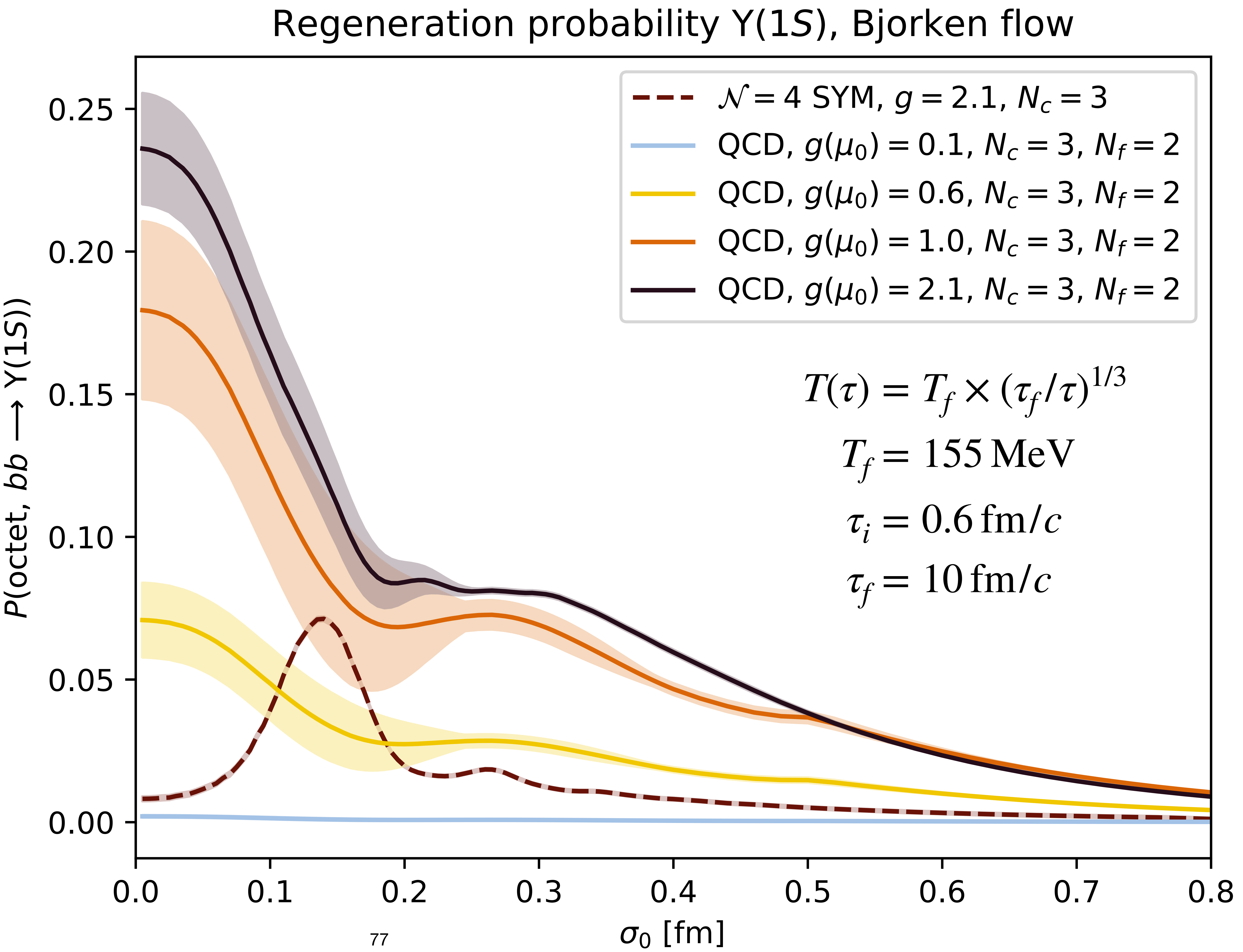
$$\tau_f = 10\,\mathrm{fm}/c$$



We can also compare weakly and strongly coupled plasmas:

- At weak coupling, it seems that the existence of quasiparticles increases the quarkonium transition rates relative to the strongly coupled case.

- In both cases, the relevant scale is the size of the bound state.



Outlook

the road ahead

- We have discussed how to calculate the generalized gluon distributions that govern quarkonium transport.
  - Interesting prospects for interpolating between weak & strong coupling, and describing non-perturbative QGP physics.
- Next steps:
  - Develop a transport formalism accounting for QGP memory effects.
  - Assess whether Markovian or non-Markovian effects are dominant in the QGP formed in heavy-ion collisions.
- Stay tuned!

Thank you!

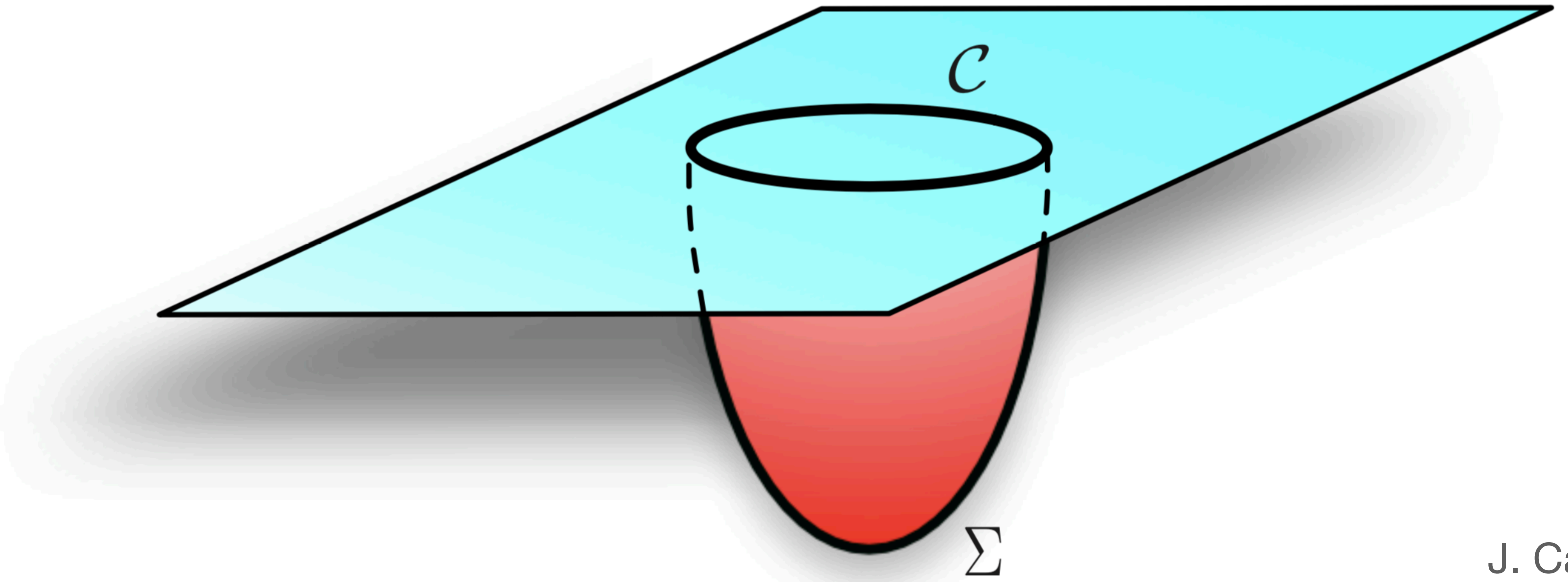
Extra Slides

Wilson loops in AdS/CFT

setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
  - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\rm NG}[\Sigma]}$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

How do Wilson loops help?

setup — pure gauge theory

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^\mu(s_2)}\frac{\delta}{\delta f^\nu(s_1)}W[\mathscr{C}_f]\bigg|_{f=0} = (ig)^2\mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^\rho(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^\sigma(s_1)U_{[s_1,0]}\Big]$$

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- Same as the lattice calculation of the heavy quark diffusion coefficient:

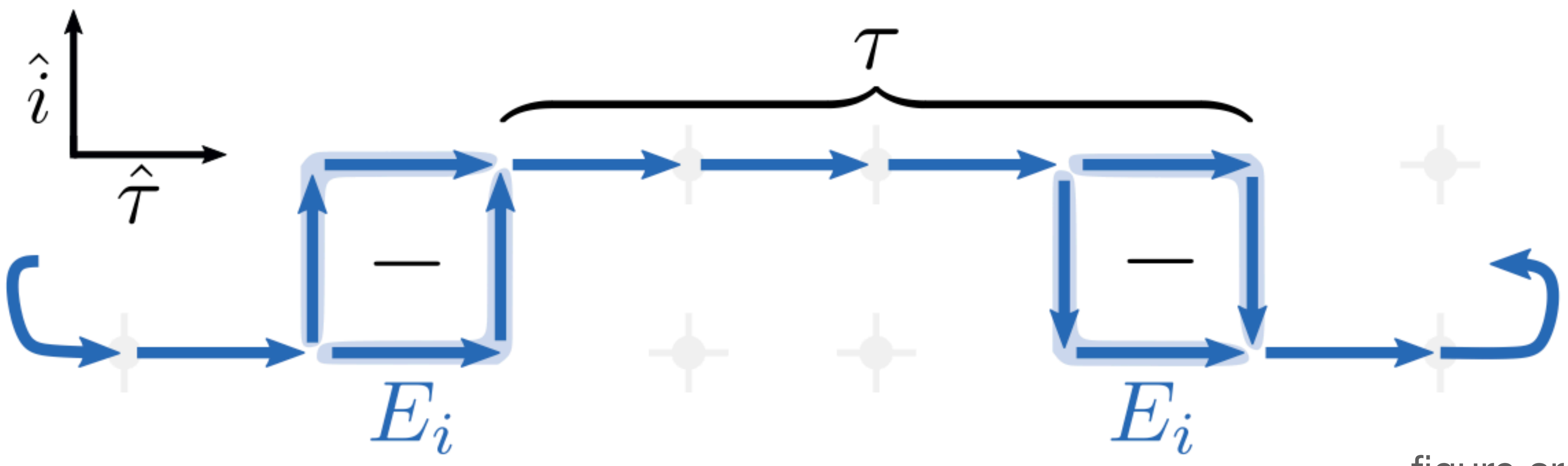

figure credit: L. Altenkort

[**] J. Maldacena, hep-th/9711200

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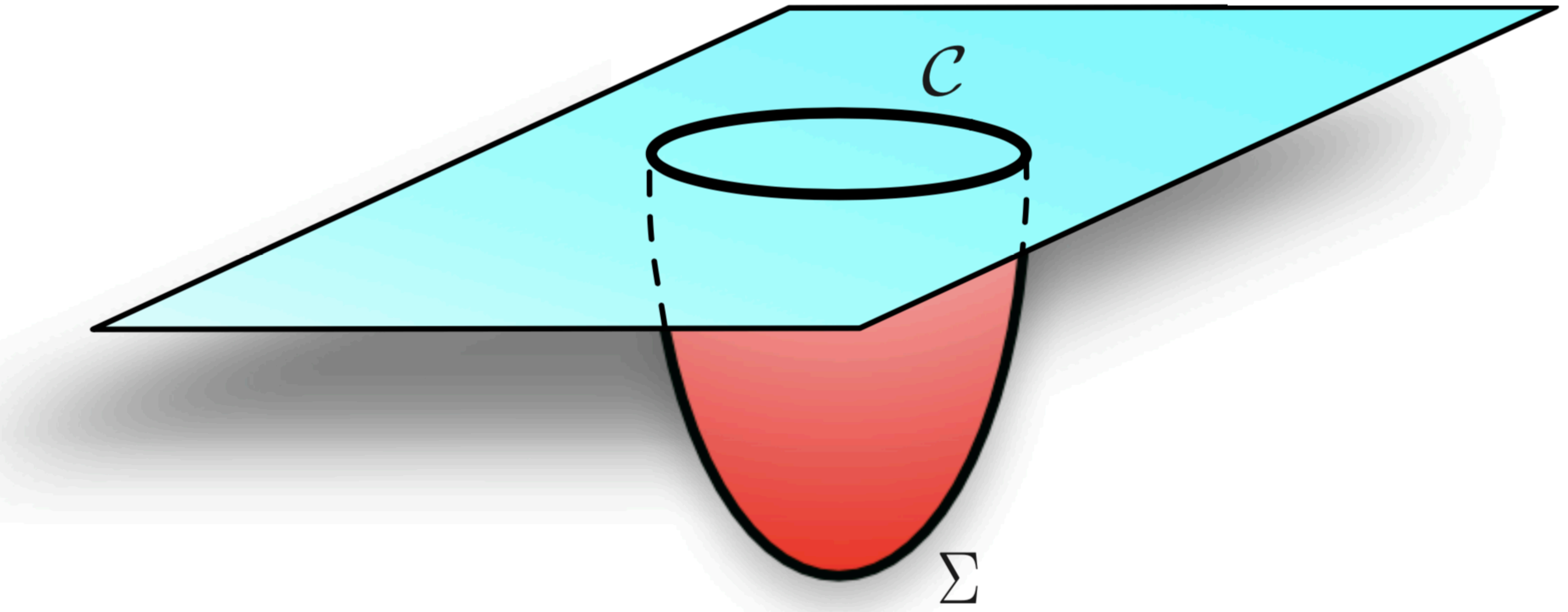
$$\langle W[\mathscr{C} = \partial\Sigma] \rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$

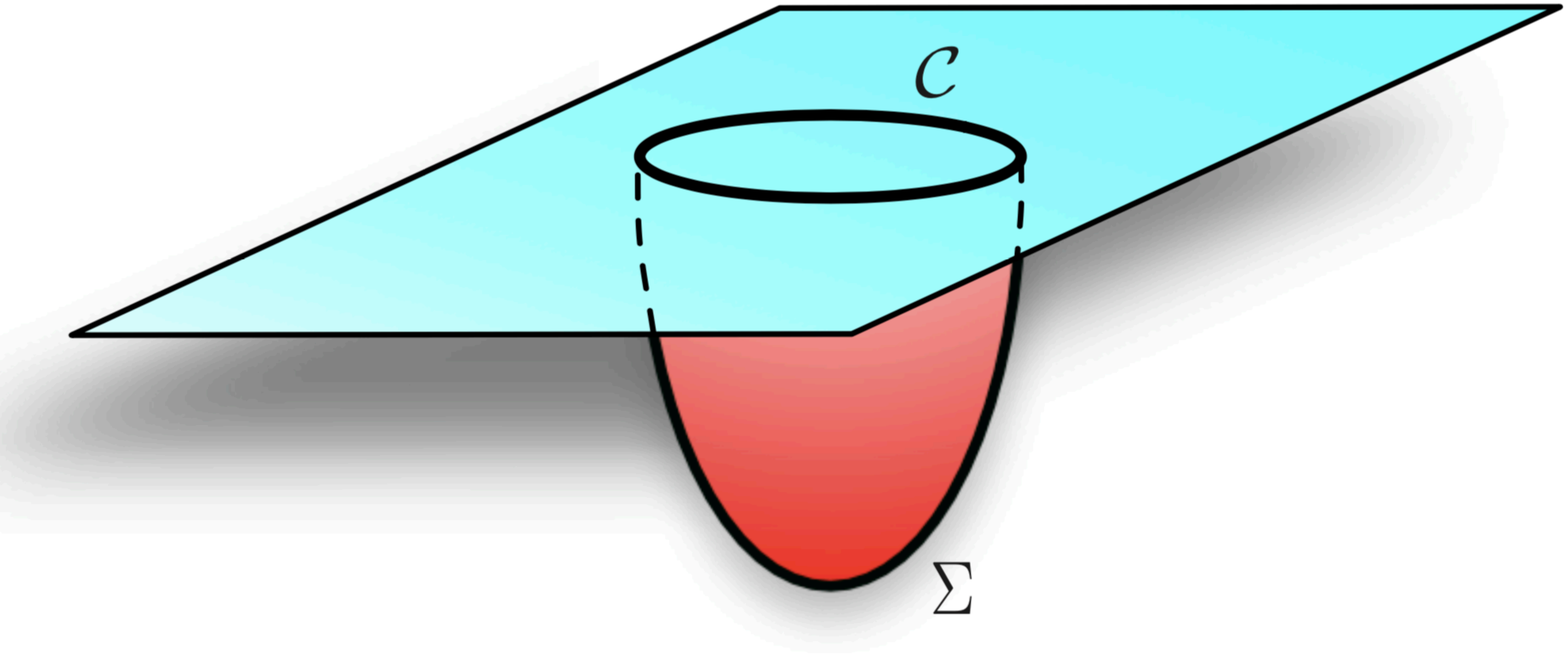
Metric of interest for finite T calculations:

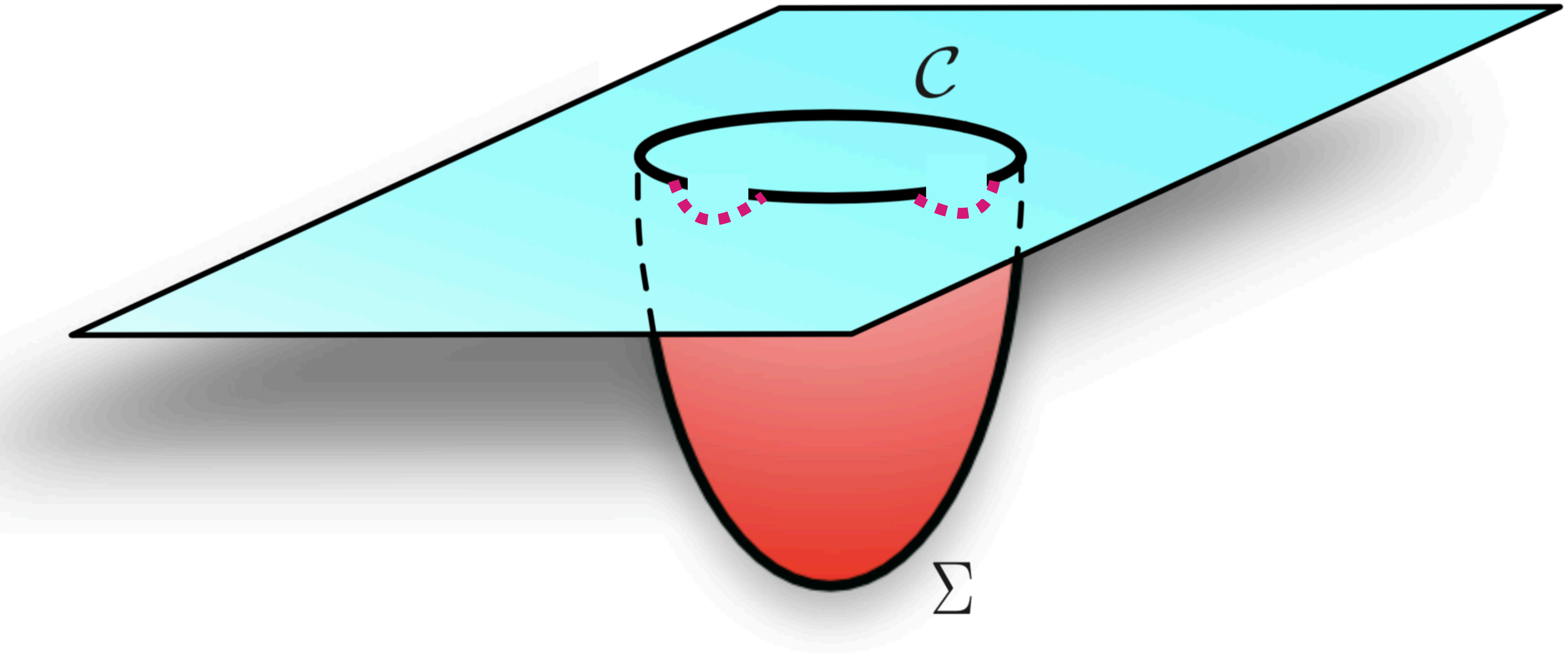
$$ds^2 = \frac{R^2}{z^2}\left[-f(z)\,dt^2 + d\mathbf{x}^2 + \frac{1}{f(z)}dz^2 + z^2 d\Omega_5^2\right]$$

$$f(z) = 1 - (\pi T z)^4$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618


$\mathcal{C}$
Σ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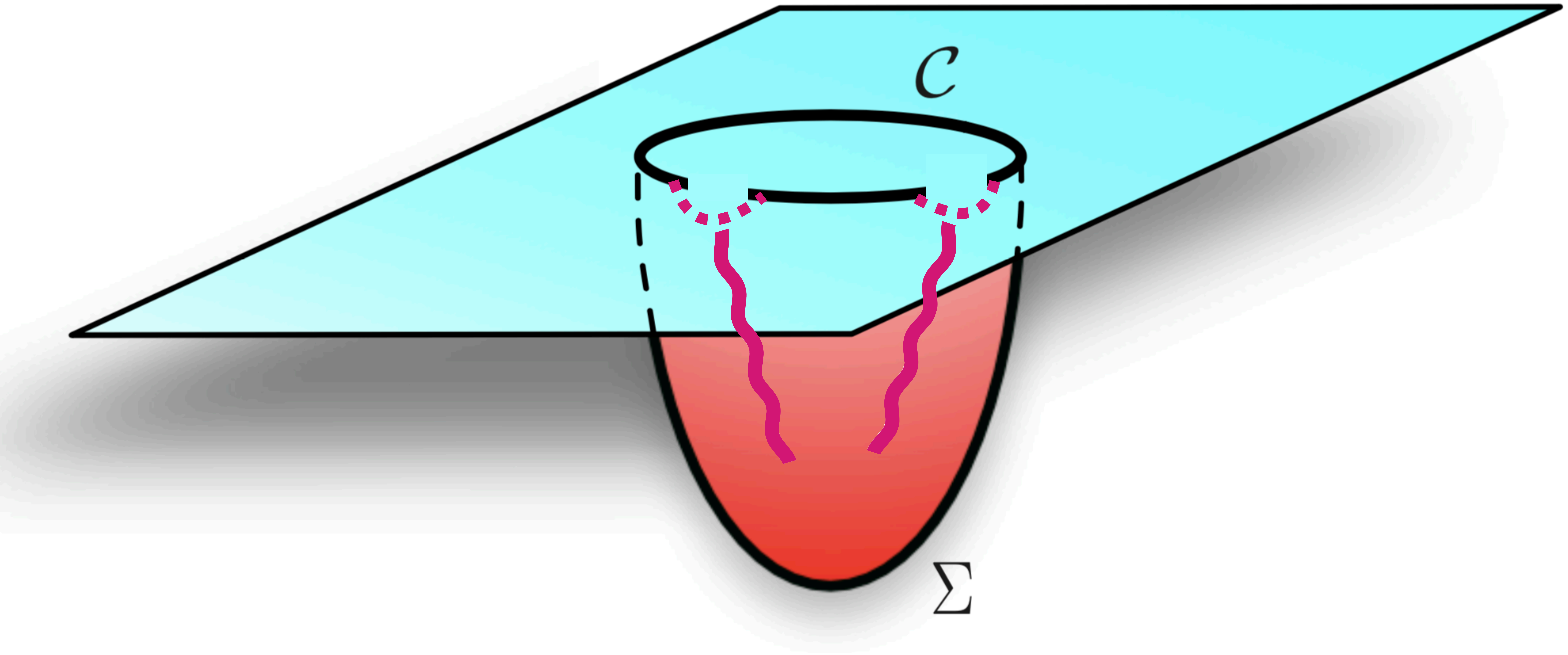
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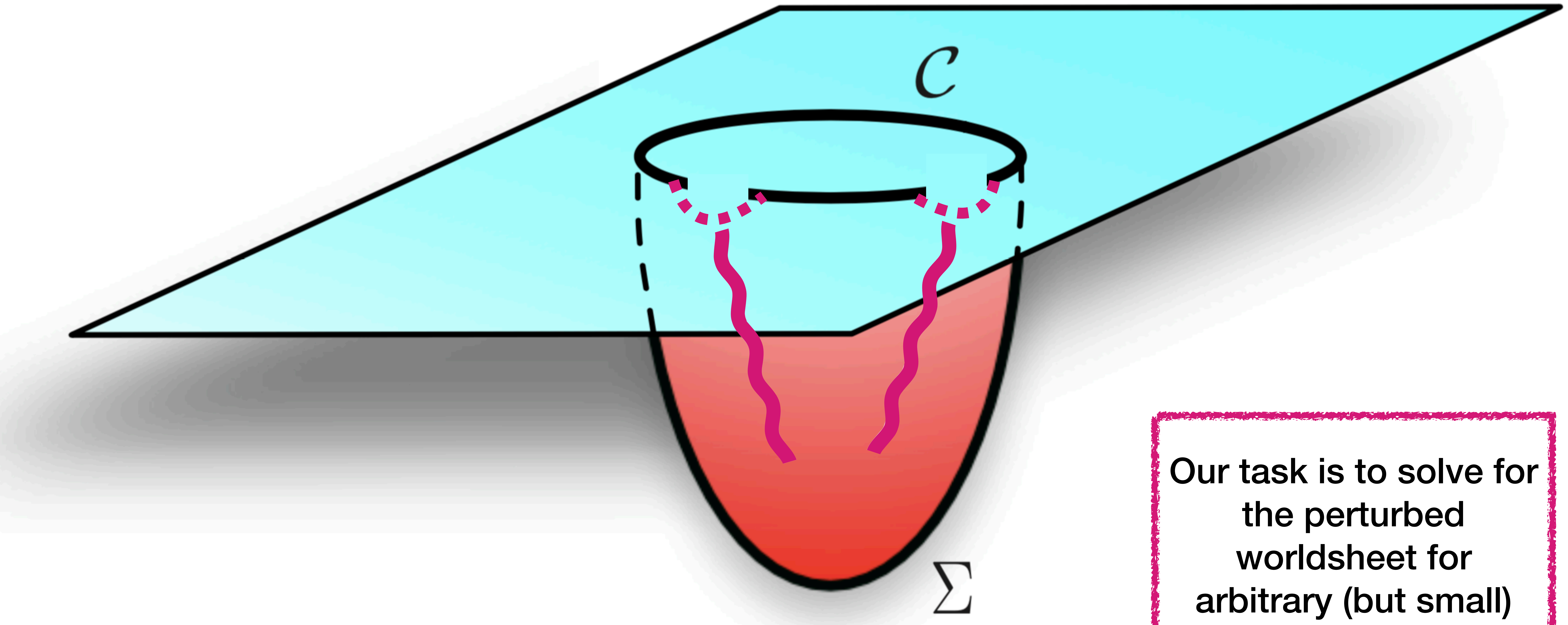
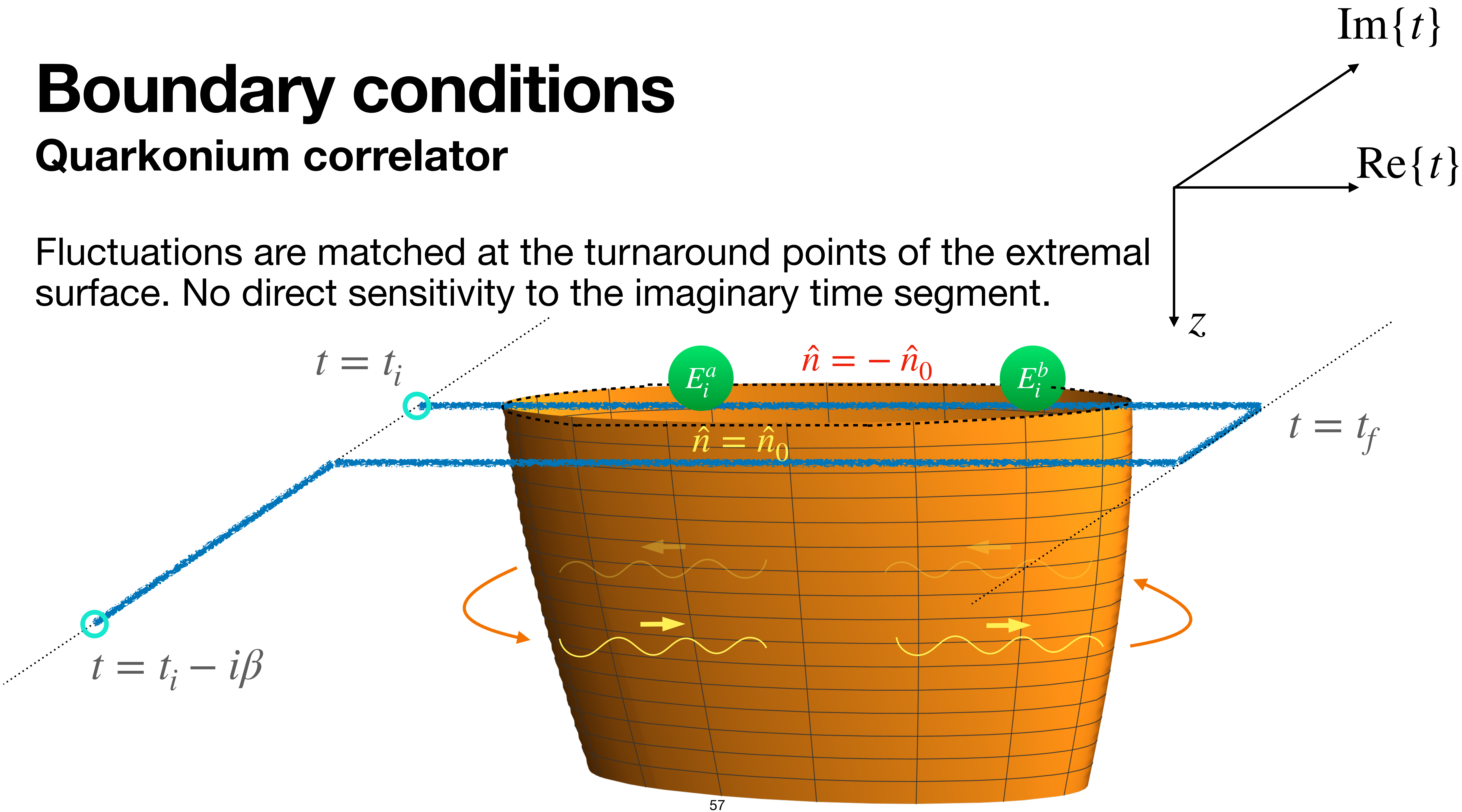


Our task is to solve for the perturbed worldsheet for arbitrary (but small) changes in the loop $\mathscr{C}$

Boundary conditions

Quarkonium correlator

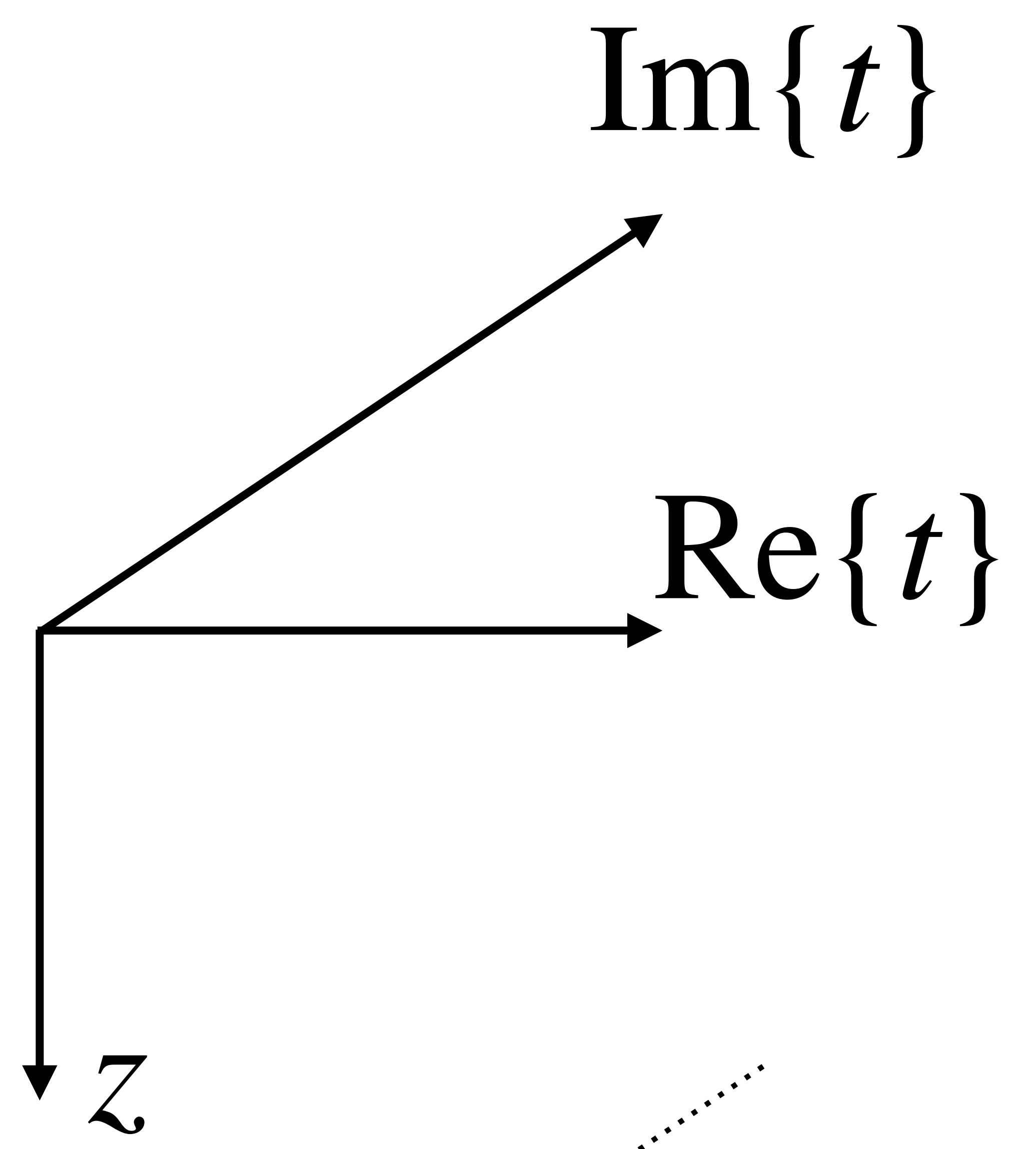
Fluctuations are matched at the turnaround points of the extremal surface. No direct sensitivity to the imaginary time segment.



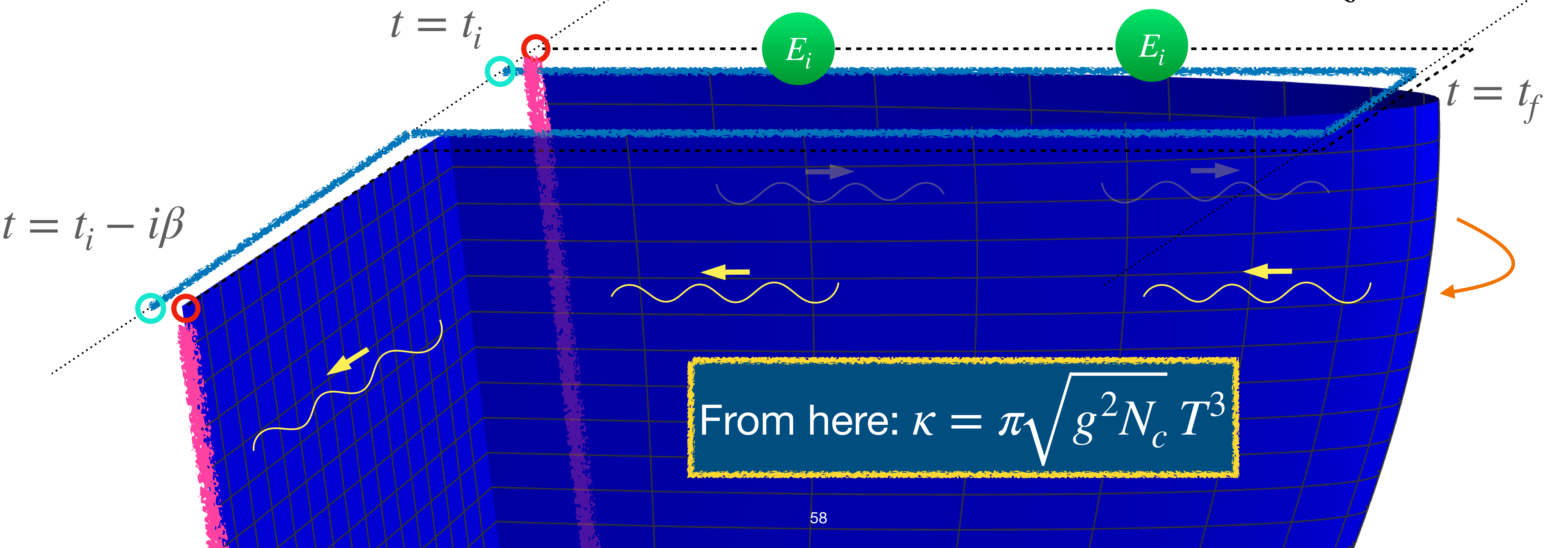
c.f. K. Skenderis and B. van Rees 0812.2909 [hep-th]

Boundary conditions

Heavy quark correlator



Fluctuations are matched through the imaginary time segment solving the equations of motion $\Longrightarrow$ factors of $e^{\beta\omega}$, KMS relations



From here: $\kappa = \pi\sqrt{g^2 N_c}\, T^3$

Wilson loops in $\mathcal{N} = 4$ SYM

a slightly different observable

- A holographic dual in terms of an extremal surface exists for

$$W_{\rm S}[\mathscr{C};\hat{n}] = \frac{1}{N_c}\mathrm{Tr}_{\rm color}\left[\mathscr{P}\exp\left(ig\oint_{\mathscr{C}} ds\, T^a \left[A^a_\mu \,\dot{x}^\mu \; + \; \hat{n}(s)\cdot \vec{\phi}^{\,a}\sqrt{\dot{x}^2}\,\right]\right)\right],$$

which is *not* the standard Wilson loop.

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- $\mathcal{N} = 4$ SYM has 6 scalar fields $\vec{\phi}^{\,a}$, which enter the above Wilson loop through a direction $\hat{n} \in \mathrm{S}_5$. Also, its dual gravitational description is $\mathrm{AdS}_5 \times \mathrm{S}_5$.

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- What to do with this extra parameter? For a single heavy quark, just set $\hat{n} = \hat{n}_0$.

Choosing $\hat{n}$

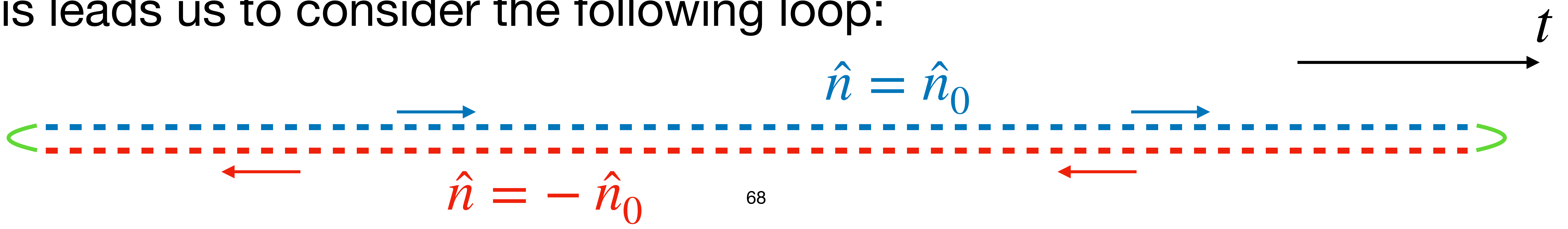
what is the best proxy for an adjoint Wilson line?

- A key property of the adjoint Wilson line is

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \frac{1}{T_F}\mathrm{Tr}\left[\mathscr{T}\left\{T^a U_{[t_2,t_1]} T^b U^\dagger_{[t_2,t_1]}\right\}\right],$$

which means that we can obtain the correlator we want by studying deformations of a Wilson loop of the form $W = \dfrac{1}{N_c}\mathrm{Tr}\left[UU^\dagger\right] = 1.$

- This leads us to consider the following loop:



[†] Alday and Maldacena, 2007
[‡] Polchinski and Sully, 2011

Another angle at the $\hat{n}$ configuration

the Neumann prescription for the pure gauge Wilson loop

- There exists a prescription to evaluate the pure gauge Wilson loop [†]. On the gravity side of the duality, it amounts to taking Neumann boundary conditions for $\hat{n}$. This is can be achieved by writing [‡]:

$$\langle W[\mathscr{C}]\rangle = N[\mathscr{C}] \int D\hat{n}\, \langle W_S[\mathscr{C};\hat{n}]\rangle$$

- If the path $\mathscr{C}$ is timelike and backtracks over itself, one can show that $|\langle W_S[\mathscr{C};\hat{n}]\rangle| \leq 1$, and moreover, that this bound is saturated when $\hat{n}$ takes antipodal positions on the S_5.

- Therefore, when $\sqrt{\lambda} \to \infty$, configurations where $\hat{n}(s) = \pm\,\hat{n}_0$ dominate.