Mass from the vacuum energy density

Paul Hoyer
University of Helsinki

Unraveling the Proton Mass (13-17 June) INT Seattle (remotely)

PH 2101.06721

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  Yes: Poincaré invariance allows a single parameter Λ

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- Can we input Λ_{QCD} in a boundary condition on the gluon field?

  Yes: Poincaré invariance allows a single parameter Λ
- Implies confinement, and a universal vacuum energy density $\propto \Lambda^4$

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The QCD scale from a boundary condition

The QCD equations of motion do not involve the $\Lambda_{\rm QCD}$ scale.
This is illustrated by Gauss' law for an electric charge:

$$-\boldsymbol{\nabla}^2 A^0(t,\boldsymbol{x}) = e\,\delta(\boldsymbol{x}) \qquad \Longrightarrow \qquad A^0(t,\boldsymbol{x}) = \frac{e}{4\pi|\boldsymbol{x}|}$$

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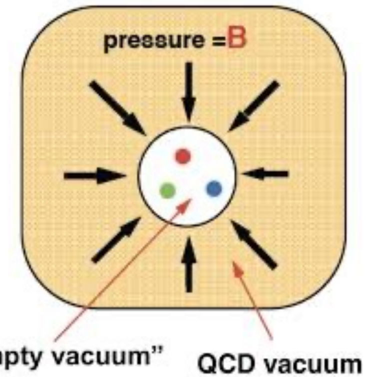
In QCD there is a boundary condition giving a universal energy density of the vacuum $\sim V'^{\,2}$

The result reminds of the "bag model", but without a "bag"



The perturbative S-matrix

$$S_{fi} = {}_{out}\langle f,\, t \to \infty| \left\{ \mathrm{T}\exp\left[-\, i \int_{-\infty}^{\infty} dt\, H_I(t) \right] \right\} |i,\, t \to -\infty\rangle_{in}$$

This defines the perturbative expansion around free *in* and *out* states.

The free gauge propagators have no scale. For photons in Coulomb gauge,

$$D^{00}(t, \boldsymbol{q}) = \frac{1}{\boldsymbol{q}^2} \quad \Longrightarrow \quad D^{00}(t, \boldsymbol{x}) = \frac{1}{4\pi\, |\boldsymbol{x}|}$$

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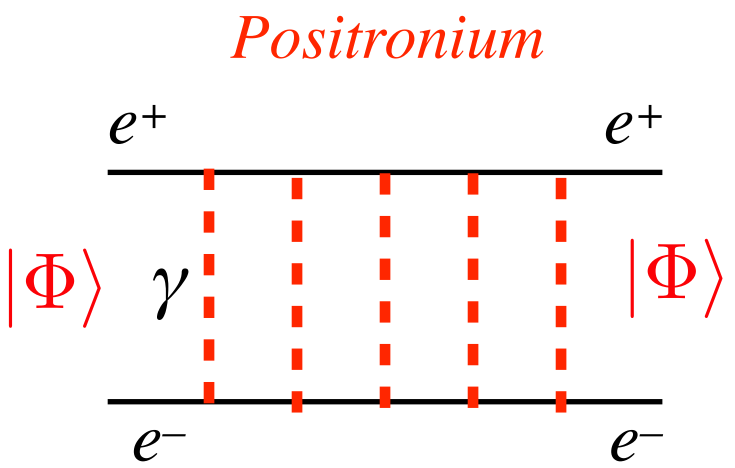
Bound states have a size (scale), and are orthogonal to free states:

No Feynman diagram has a bound state pole

Consider perturbative expansions for bound states

QED bound states: Atoms



Atoms are expanded around an initial bound state $|\Phi\rangle$

The initial state is usually chosen to be a solution of the Schrödinger equation.

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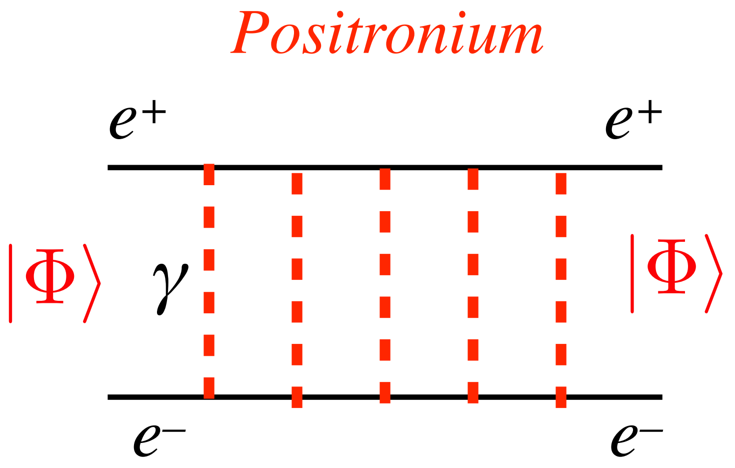

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Their higher order corrections $\Phi(\alpha)(1 + c_1\alpha + c_2\alpha^2 \ldots)$ depend on $\Phi(\alpha)$.

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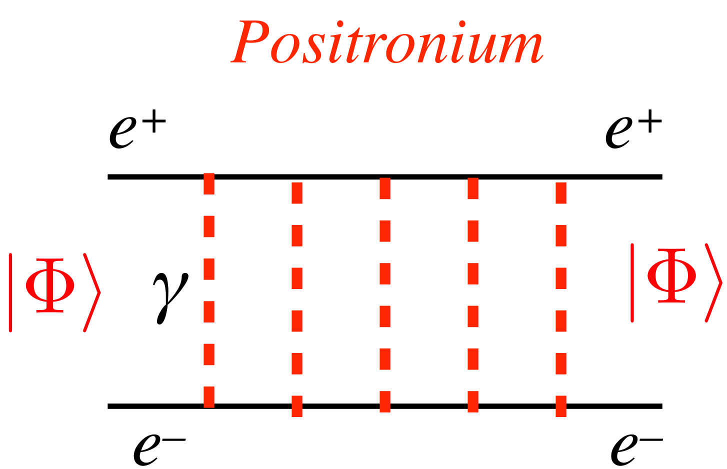

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The perturbative expansion for bound states is not unique, it depends on the choice of initial state.

Caswell & Lepage (1975)

Hadrons with light quarks: π, ρ, N,…

Valence quark Fock states govern quantum numbers,
even for highly relativistic constituents.

Particle Data Group

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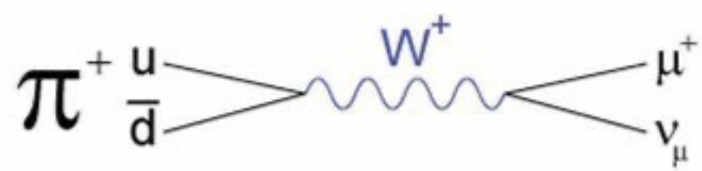

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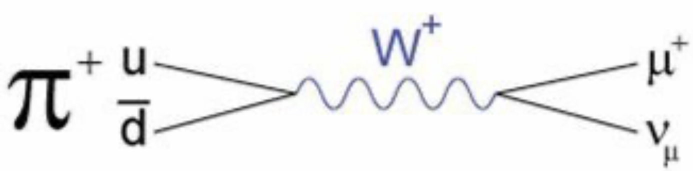

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Assume:
Current quark Fock states

A Bound Fock expansion

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Is there an instantaneous potential for the relativistic quarks of QCD?

Theories with a local action generally do not have instantaneous potentials:

Constituent velocities are bounded by the speed of light (causality)

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Due to the absence of $\partial_0 A^0$ in $\mathcal{L}_{QED}$ A^0 has no conjugate field

⇒ Canonical quantisation in Coulomb gauge requires Dirac constraints

Temporal gauge

No Dirac constraints in temporal gauge: $A^0 = \partial_0 A^0 = 0$

Canonical commutation relations, with $E^i = -\partial_0 A^i$, include E_L

$$\left[E^i(t,\boldsymbol{x}), A^j(t,\boldsymbol{y})\right] = i\delta^{ij}\delta(\boldsymbol{x}-\boldsymbol{y})$$

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$A^0(t,\boldsymbol{x}) = 0$ is preserved under time-independent gauge transformations, which are generated by the operator of “Gauss’ law”: Willemsen (1978)

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Physical states need to be invariant under all gauge transformations:

$$\frac{\delta \mathcal{S}_{QED}}{\delta A^0(x)}\,|phys\rangle = 0$$

Determines $\nabla\cdot \boldsymbol{E}_L$ from the fermion distribution

The classical, instantaneous field E_L

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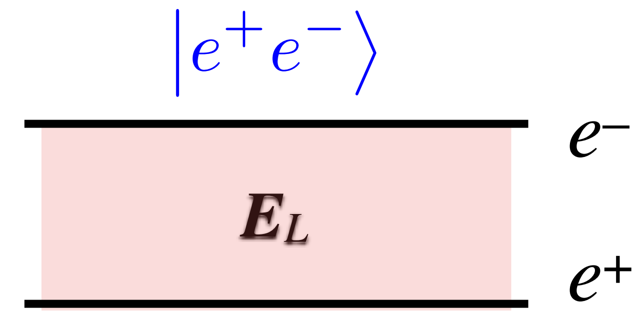
$\boldsymbol{E}_L$ can bind e^+e^- Fock states strongly, without pair creation.

Temporal gauge allows valence dominance even of relativistic states.

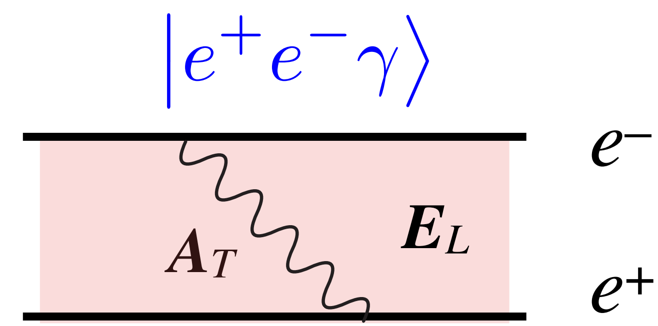
Contrast: In Coulomb gauge A^0 is a quantum field, which creates particles.

Bound Fock expansion for Positronium in $A^0=0$ gauge

The perturbative expansion in α is chosen to start from the $|e^+e^-\rangle$ Fock state, which is bound by its classical field $\boldsymbol{E}_L$:



Higher order corrections include states with transverse photons and e^+e^- pairs, as determined by $H_{QED}\,|e^+e^-\rangle$



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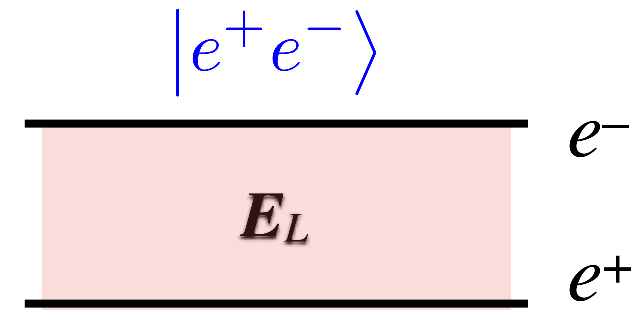

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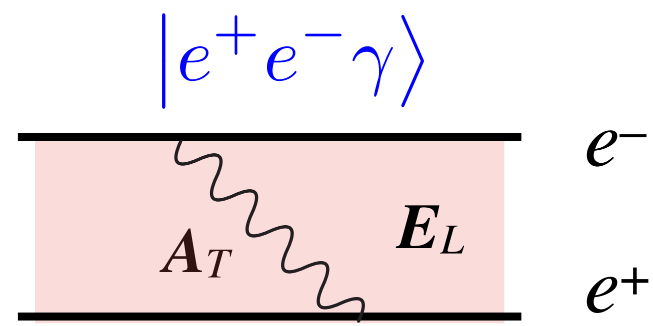

Each Fock component of the bound state includes its particular instantaneous $\boldsymbol{E}_L$ field.

This Fock expansion is valid in any frame, and is formally exact at $O(\alpha^\infty)$.

Application to QCD

The classical fields of QED and QCD differ

Global gauge invariance allows a classical gauge field for neutral atoms…

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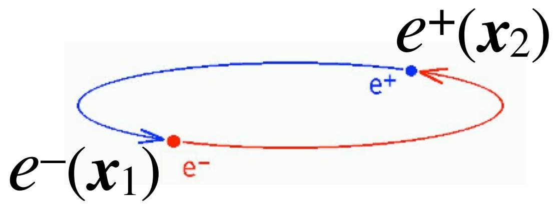
Positronium (QED)



External observers see a dipole field:

$$\boldsymbol{E}_L(\boldsymbol{x}) = -\frac{e}{4\pi}\boldsymbol{\nabla}_x\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right)$$

The electron is bound by a monopole field:

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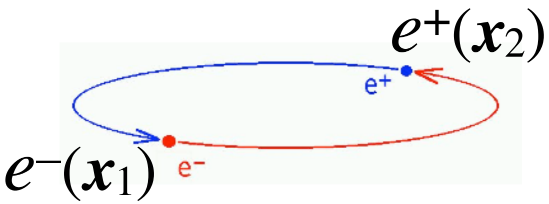

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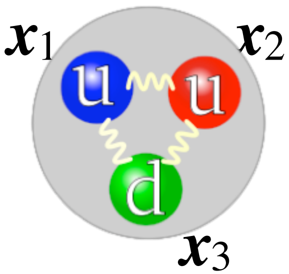
Proton (QCD) is a color singlet



External observers see no field:

$$\boldsymbol{E}_L^a(\boldsymbol{x}) = 0 \quad \text{for all } \boldsymbol{x}$$

The blue quark sees the field of the red and green quarks.

$$\boldsymbol{E}_L^a[\boldsymbol{x}_1;\, q_r(\boldsymbol{x}_2), q_g(\boldsymbol{x}_3)] \neq 0$$

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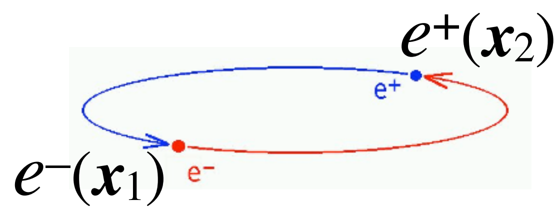

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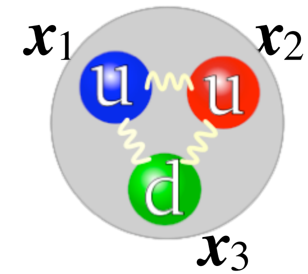

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An external observer sees no color field due to the sum over quark colors.

Each color component of the Fock state does have $\boldsymbol{E}_L^a \neq 0$

Temporal gauge in QCD: $A_a^0 = 0$

The temporal gauge constraint determines $\nabla\cdot \boldsymbol{E}_{L,a}$ for each state:

$$\partial_i E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = g\big[-f_{abc}A^i_b E^i_c + \psi^\dagger T^a \psi(\boldsymbol{x})\big]\,|phys\rangle$$

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In QED we impose the boundary condition: $\boldsymbol{E}_L(\boldsymbol{x}) \to 0$ for $|\boldsymbol{x}| \to \infty$

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The color electric field $\boldsymbol{E}_{L,a}(\boldsymbol{x}) \neq 0$ for each quark color component

Include a homogeneous solution, $\nabla\cdot \boldsymbol{E}_{L,a}(\boldsymbol{x}) = 0$ with $\boldsymbol{E}_{L,a}(\boldsymbol{x}) \neq 0$.
$\boldsymbol{E}_{L,a}(\boldsymbol{x})$ binds each quark color component of a hadron.
The field cancels in the sum over quark colors for singlet states.

Including a homogeneous solution for $E^i_{L,a}$

$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial_i^x \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A^i_b E^i_c(\boldsymbol{y}) + \psi^\dagger T^a\psi(\boldsymbol{y})$ and $\mathcal{E}_a(\boldsymbol{y})\,|0\rangle = 0$

The contribution $\propto$ g gives the gluon exchange potential: $V(r) = -\frac{4}{3}\frac{\alpha_s}{r}$

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The homogeneous solution $\propto$ $\varkappa$ of the gauge constraint is the only one that gives invariance under translations and rotations

$\boldsymbol{E}_L \propto \varkappa$ is independent of $\boldsymbol{x}$, as required by translation invariance:
The gluon field energy density is spatially constant.

This solution is excluded by the free field BC of Feynman diagrams.

The instantaneous potential from the Hamiltonian

$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial^x_i \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

$$\mathcal{H}_V \equiv \tfrac{1}{2}\int d\boldsymbol{x}\sum_a \boldsymbol{E}^a_L\cdot\boldsymbol{E}^a_L$$

$$= \int d\boldsymbol{y}d\boldsymbol{z}\Big\{\boldsymbol{y}\cdot\boldsymbol{z}\Big[\tfrac{1}{2}\kappa^2\int d\boldsymbol{x} + g\kappa\Big] + \tfrac{1}{2}\frac{\alpha_s}{|\boldsymbol{y}-\boldsymbol{z}|}\Big\}\mathcal{E}_a(\boldsymbol{y})\mathcal{E}_a(\boldsymbol{z})$$

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where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A^i_b E^i_c(\boldsymbol{y}) + \psi^\dagger T^a\psi(\boldsymbol{y})$

The field energy ∝ volume of space is irrelevant only if it is universal.

This relates the normalisation $\varkappa$ of all Fock components,

leaving an overall scale Λ as the single parameter.

Meson $q\bar{q}$ Fock state potential

$$|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_A \bar{\psi}^A(\boldsymbol{x}_1)\,\psi^A(\boldsymbol{x}_2)\,|0\rangle$$ globally color singlet

$$\mathcal{H}_V \equiv \tfrac{1}{2}\int d\boldsymbol{x}\sum_a \boldsymbol{E}_L^a\cdot\boldsymbol{E}_L^a$$ does not create particles

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$$\mathcal{H}_V\,|q\bar{q}\rangle = V_{q\bar{q}}\,|q\bar{q}\rangle$$

$$V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$ Cornell potential

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This potential is valid also for relativistic $q\bar{q}$ Fock states, in any frame

The universal vacuum energy density is $E_\Lambda = \dfrac{\Lambda^4}{2g^2C_F}$

Baryon Fock state potential

Baryon: $$|q(\boldsymbol{x}_1)q(\boldsymbol{x}_2)q(\boldsymbol{x}_3)\rangle \equiv \sum_{A,B,C} \epsilon_{ABC}\psi^\dagger_A(\boldsymbol{x}_1)\,\psi^\dagger_B(\boldsymbol{x}_2)\,\psi^\dagger_C(\boldsymbol{x}_3)\,|0\rangle$$

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \Lambda^2 d_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) - \frac{2}{3}\,\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_3|}+\frac{1}{|\boldsymbol{x}_3-\boldsymbol{x}_1|}\Big)$$

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When two of the quarks coincide the potential reduces to the $q\bar{q}$ potential:

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - \frac{4}{3}\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} = V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2)$$

Analogous potentials are obtained for any globally color singlet quark and gluon Fock state, such as $q\bar{q}g$ and gg.

$\mathcal{O}\left(\alpha_s^0\right)$ q$\bar{\text{q}}$ bound states

An $\mathcal{O}\left(\alpha_s^0\right)$ meson state with $\boldsymbol{P} = 0$ and wave function Φ:

$$|M\rangle = \sum_{A,B;\alpha,\beta} \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t=0,\boldsymbol{x}_1)\delta^{AB}\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_\beta^B(t=0,\boldsymbol{x}_2)\,|0\rangle$$

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The (rest frame) bound state condition $H\,|M\rangle = M\,|M\rangle$ gives

$$\left[i\gamma^0\boldsymbol{\gamma}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0\right]\Phi(\boldsymbol{x}) + \Phi(\boldsymbol{x})\left[i\gamma^0\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}} - m\gamma^0\right] = \left[M - V(|\boldsymbol{x}|)\right]\Phi(\boldsymbol{x})$$

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In the non-relativistic limit ($m \gg \Lambda$) this reduces to the Schrödinger equation.

$\Longrightarrow$ The quarkonium phenomenology with the Cornell potential.

Separation of radial and angular variables

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\}+m\left[\gamma^0,\Phi(\boldsymbol{x})\right]=\left[M-V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

Expanding the 4×4 wave function in a basis of 16 Dirac structures $\Gamma_i(\boldsymbol{x})$

$$\Phi(\boldsymbol{x})=\sum_i \Gamma_i(\boldsymbol{x})F_i(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

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We may use rotational, parity and charge conjugation invariance to determine which $\Gamma_i(\boldsymbol{x})$ may occur for a state of given j^{PC}:

0^{-+} trajectory $[s=0,\ \ell=j]:$ $\quad -\eta_P=\eta_C=(-1)^j\quad \gamma_5,\ \gamma^0\gamma_5,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L}$

0^{--} trajectory $[s=1,\ \ell=j]:$ $\quad \eta_P=\eta_C=-(-1)^j\quad \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \boldsymbol{\alpha}\cdot\boldsymbol{L},\ \gamma^0\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{++} trajectory $[s=1,\ \ell=j\pm 1]:$ $\quad \eta_P=\eta_C=+(-1)^j\quad 1,\ \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{+-} trajectory [exotic] $:$ $\quad \eta_P=-\eta_C=(-1)^j\quad \gamma^0,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

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0^{++} trajectory	$[s=1,\ \ell=j\pm1]:$	$\eta_P=\eta_C=+(-1)^j$	$1,\ \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$
0^{+-} trajectory	[exotic] :	$\eta_P=-\eta_C=(-1)^j$	$\gamma^0,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

$\Longrightarrow$ There are no solutions for quantum numbers that would be exotic in the NR quark model (despite the relativistic dynamics)

The BSE gives the radial equations for the $F_i(r)$

Example: $-\eta_P = \eta_C = (-1)^j$ states at O(α_s^0)

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\vec{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\tfrac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

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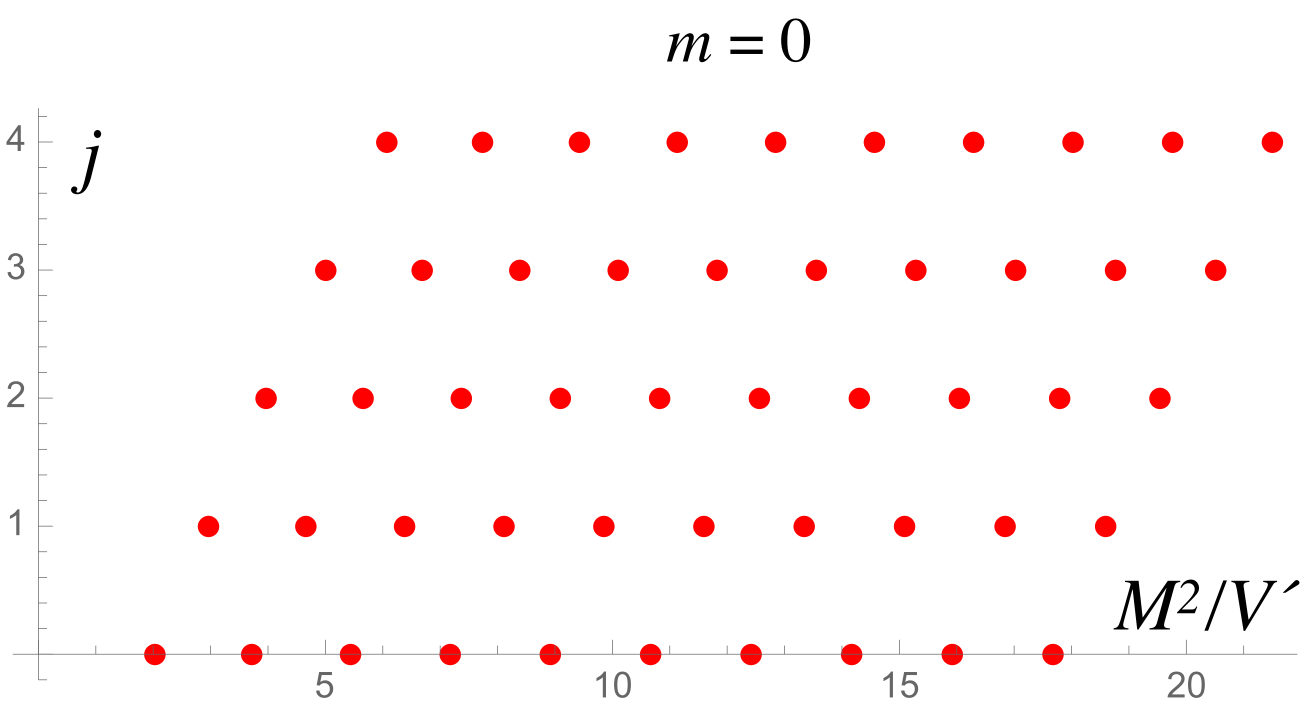
Regularity at $r = 0$ and at $V(r) = M$ determines the bound state masses M

Mass spectrum:

Linear Regge trajectories with daughters

Spectrum similar to dual models



Summary

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This allows a "Bound Fock expansion": The constituents are bound by $\boldsymbol{E}_L$.

Summary

The QCD scale Λ_{QCD} can be introduced via a boundary condition

Not violating the equations of motion is essential for Poincaré invariance

In temporal gauge ($A^0 = 0$) the charges instantaneously determine $\nabla\cdot\boldsymbol{E}_L$

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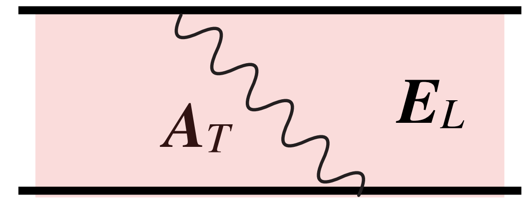
Poincaré invariance allows only a single parameter Λ

The features of hadrons thus obtained appear promising

Back-up slides

The qg$\bar{q}$ potential



A q$\bar{q}$ state, with the emission of a transverse gluon:

$$|q(\boldsymbol{x}_1)g(\boldsymbol{x}_g)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_{A,B,b} \bar{\psi}_A(\boldsymbol{x}_1)\, A_b^j(\boldsymbol{x}_g) T_{AB}^b \psi_B(\boldsymbol{x}_2)\, |0\rangle$$

$$V_{qgq}^{(0)}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) = \frac{\Lambda^2}{\sqrt{C_F}}\, d_{qgq}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) \qquad \text{(universal } \Lambda\text{)}$$

$$d_{qgq}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) \equiv \sqrt{\tfrac{1}{4}(N - 2/N)(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + N(\boldsymbol{x}_g - \tfrac{1}{2}\boldsymbol{x}_1 - \tfrac{1}{2}\boldsymbol{x}_2)^2}$$

$$V_{qgq}^{(1)}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) = \tfrac{1}{2}\,\alpha_s\Big[\frac{1}{N}\,\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} - N\Big(\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_g|} + \frac{1}{|\boldsymbol{x}_2 - \boldsymbol{x}_g|}\Big)\Big]$$

When q and g coincide:

$$V_{qgq}^{(0)}(\boldsymbol{x}_1 = \boldsymbol{x}_g, \boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1 - \boldsymbol{x}_2| = V_{q\bar{q}}^{(0)}$$

$$V_{qgq}^{(1)}(\boldsymbol{x}_1 = \boldsymbol{x}_g, \boldsymbol{x}_2) = V_{q\bar{q}}^{(1)}$$

The gg potential

A "glueball" component: $|g(\boldsymbol{x}_1)g(\boldsymbol{x}_2)\rangle \equiv \sum_a A_a^i(\boldsymbol{x}_1)\, A_a^j(\boldsymbol{x}_2)\,|0\rangle$

has the potential $V_{gg} = \sqrt{\dfrac{N}{C_F}}\,\Lambda^2\,|\boldsymbol{x}_1 - \boldsymbol{x}_2| - N\,\dfrac{\alpha_s}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$

This agrees with the qgq̄ potential where the quarks coincide:

$$V_{gg}(\boldsymbol{x},\boldsymbol{x}_g) = V_{qg\bar{q}}(\boldsymbol{x},\boldsymbol{x}_g,\boldsymbol{x})$$

It is straightforward to work out the instantaneous potential for any Fock state.