

# Three fluids for BES and resonance widths for LHC

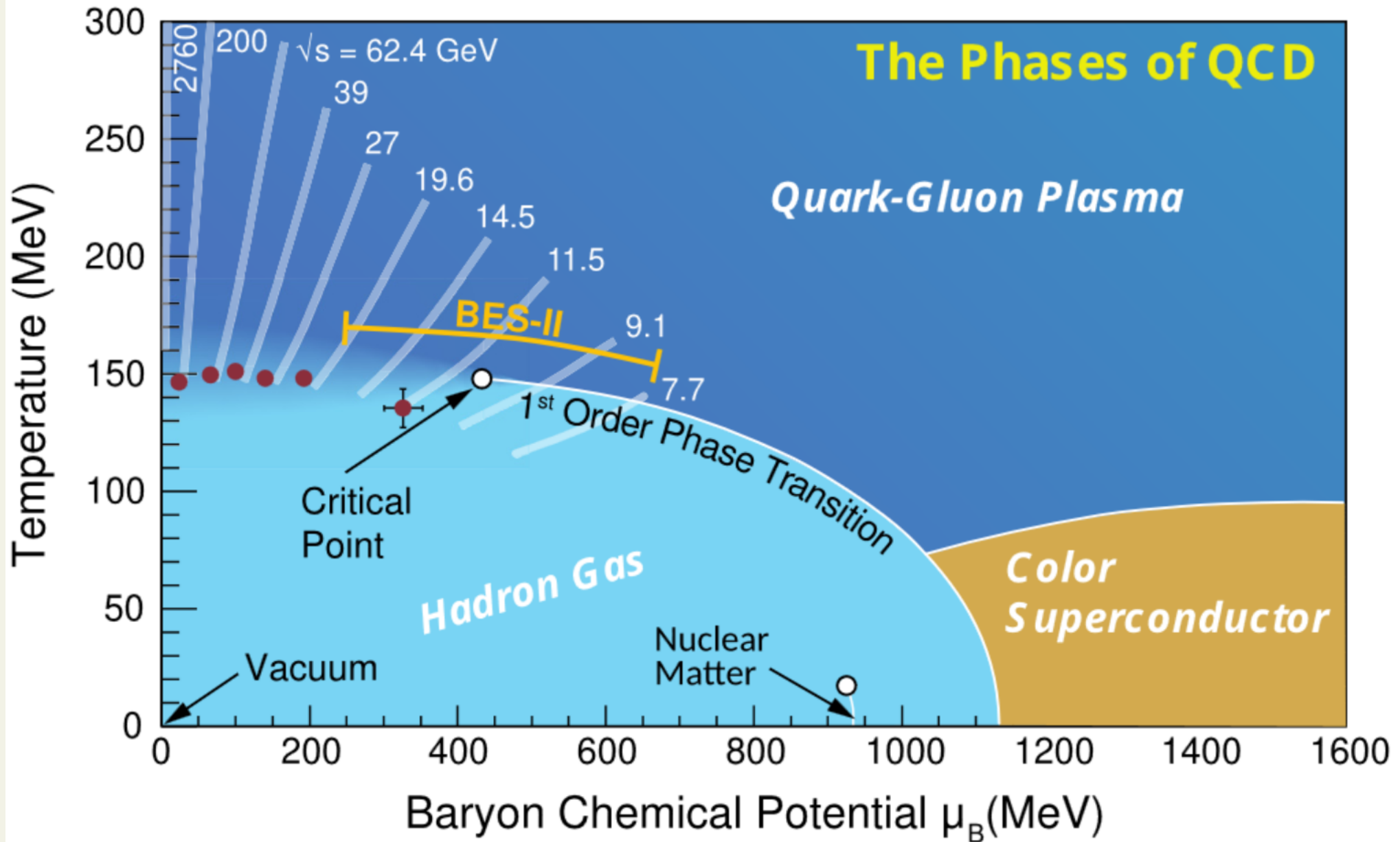
**Pasi Huovinen**

**Incubator of Scientific Excellence—Centre for Simulations of Superdense Fluids  
University of Wrocław**

**Heavy Ion Physics in the EIC Era**

**July 30, 2024, Institute for Nuclear Theory**

**work done by Jakub Cimerman, Iurii Karpenko, Boris Tomasik,  
Clemens Werthmann, Bithika Karmakar,  
Pok Man Lo and Michał Marczenko**



# Challenges

1. lower multiplicity  $\implies$  smaller system  
 $\implies$  **larger deviations from equilibrium?**

# Challenges

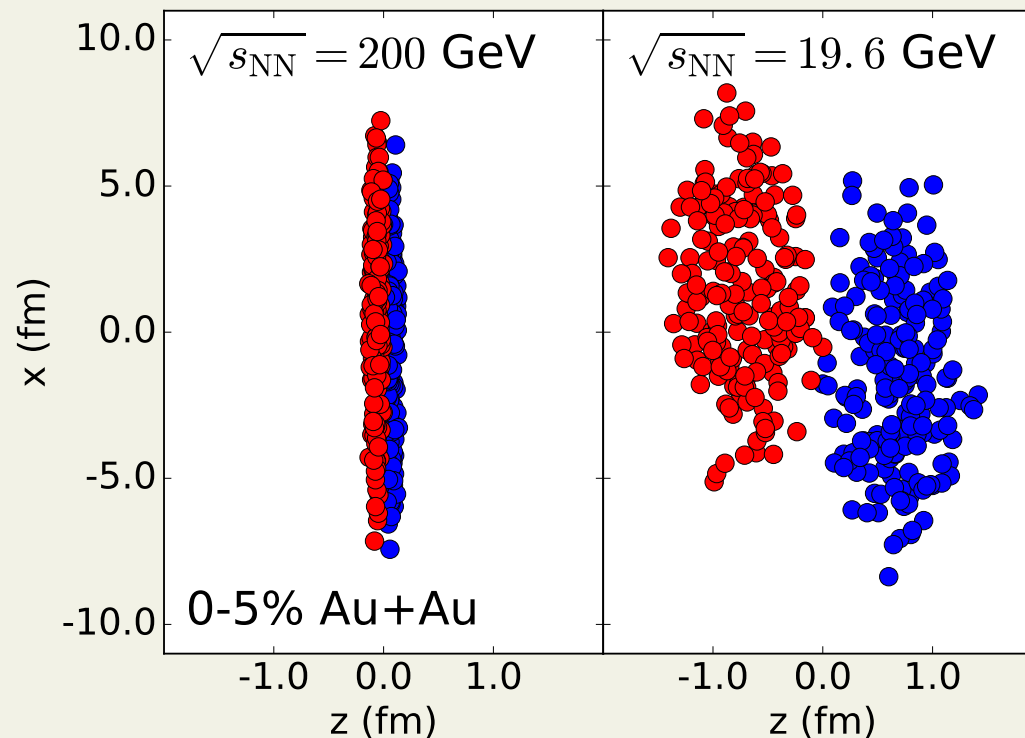
1. lower multiplicity  $\implies$  smaller system  
 $\implies$  **larger deviations from equilibrium?**
2. model for primary processes?

# Challenges

1. lower multiplicity  $\implies$  smaller system  
 $\implies$  **larger deviations from equilibrium?**
2. model for primary processes?
3. primary collisions overlap with secondary collisions

# Challenges

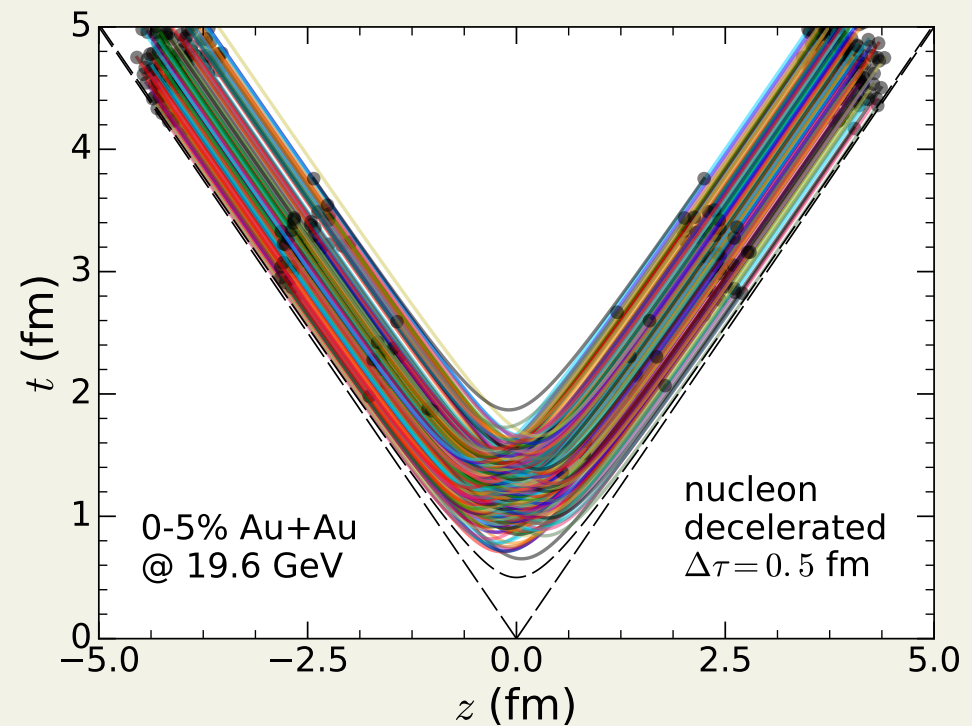
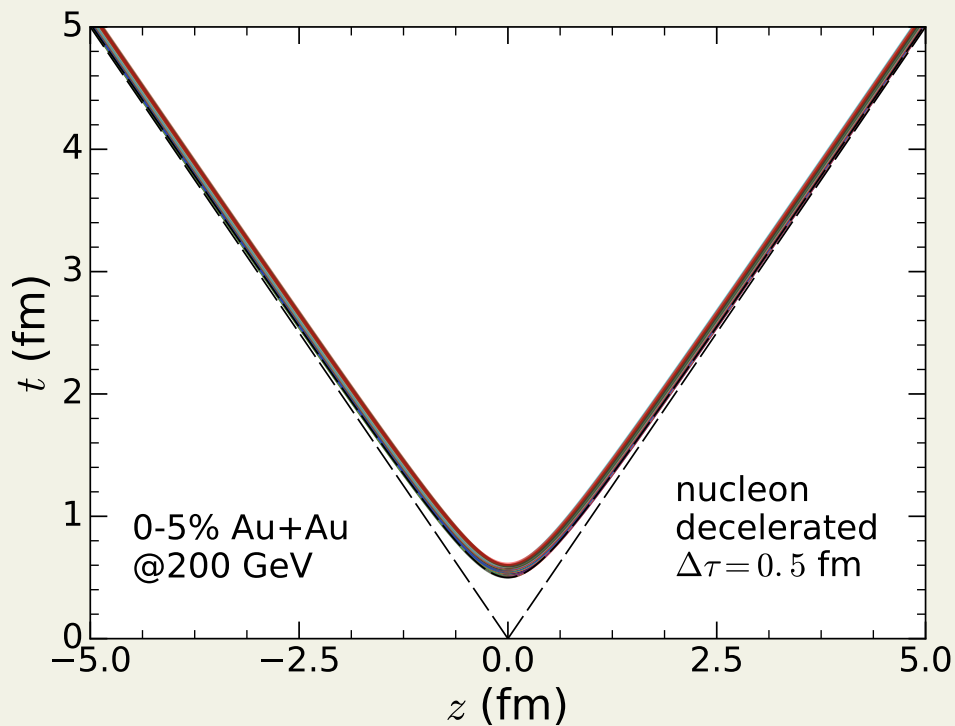
1. lower multiplicity  $\implies$  smaller system  
 $\implies$  **larger deviations from equilibrium?**
2. model for primary processes?
3. primary collisions overlap with secondary collisions



Shen & Schenke, PRC97, 024907 (2018)

# Challenges

1. lower multiplicity  $\implies$  smaller system  
 $\implies$  **larger deviations from equilibrium?**
2. model for primary processes?
3. primary collisions overlap with secondary collisions

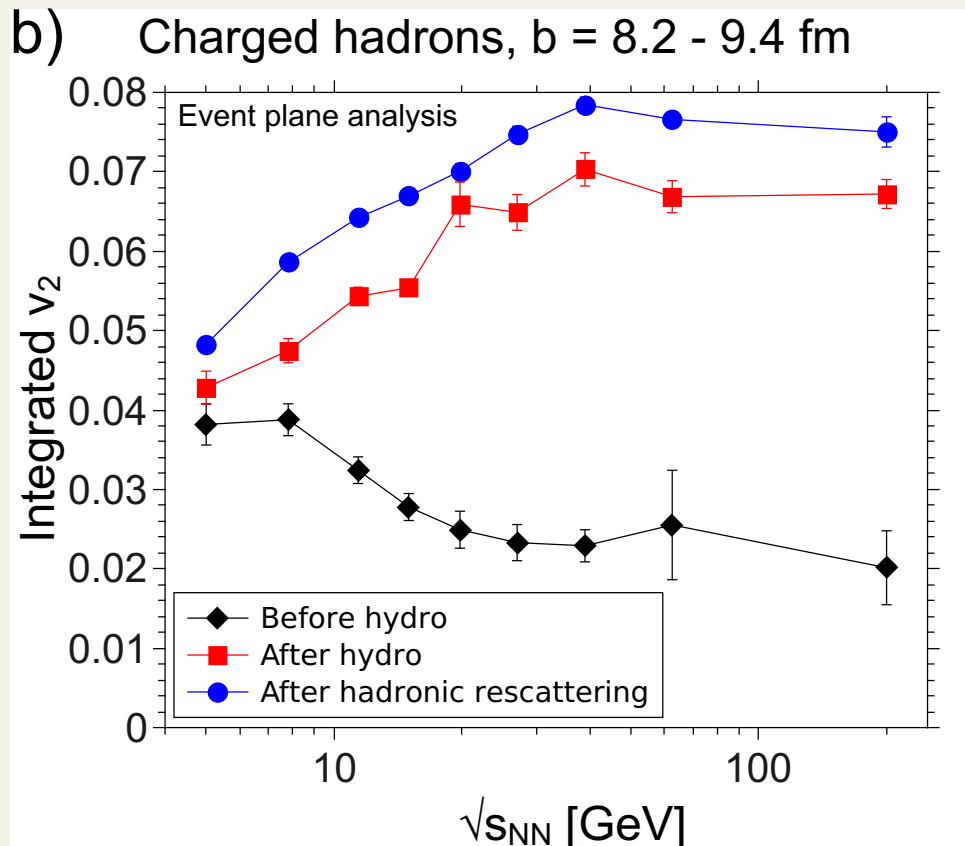


Shen & Schenke, PRC97, 024907 (2018)

# Solutions

- “Sandwich hybrid”

- cascade until the nuclei have passed each other
- fluid until hadronisation
- cascade until freeze out



- at  $\sqrt{s_{NN}} < 10$  GeV not much happens during the hydro stage

- sensitivity to EoS?

Auvinen & Petersen, PRC88, 064908 (2013)



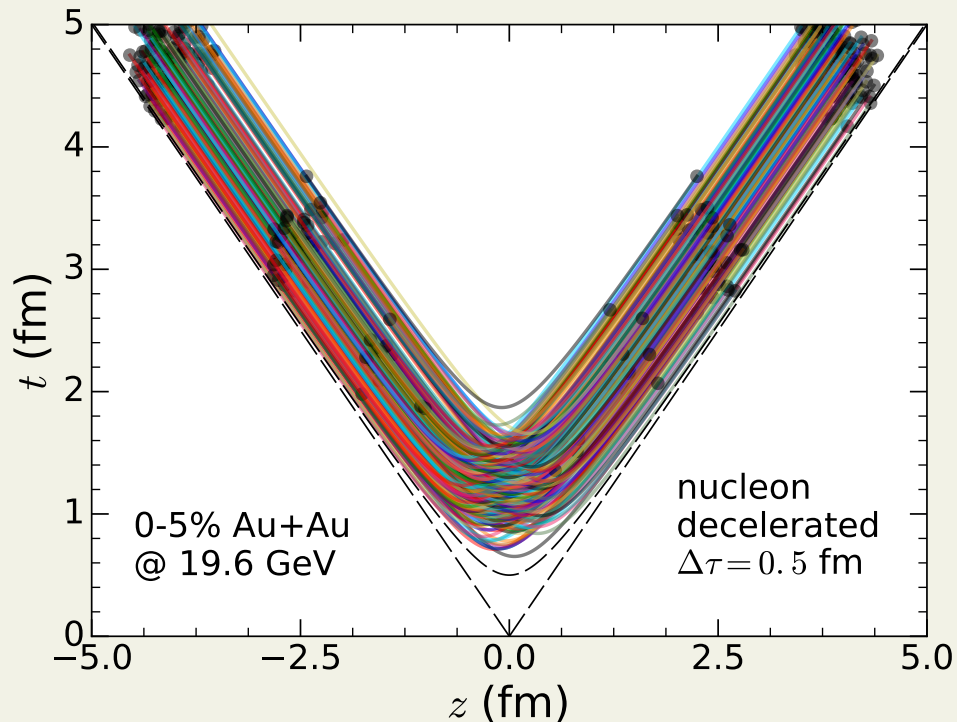
# Solutions

- **Dynamical initialisation**

- each primary collision a source term for fluid

- $\partial_\mu T^{\mu\nu} = J^\nu$

- $\partial_\mu N_B^\mu = \rho_B$



- **no interaction between incoming nucleons and produced particles**

**Shen & Schenke, PRC97, 024907 (2018)**

# 3-fluid dynamics

$$0 = \partial_{\mu} T^{\mu\nu}$$

# 3-fluid dynamics

$$\begin{aligned} 0 &= \partial_\mu T^{\mu\nu} \\ &= \partial_\mu T_t^{\mu\nu} \end{aligned}$$

$$T_t^{\mu\nu} = \text{target fluid}$$

# 3-fluid dynamics

$$\begin{aligned} 0 &= \partial_\mu T^{\mu\nu} \\ &= \partial_\mu T_t^{\mu\nu} + \partial_\mu T_p^{\mu\nu} \end{aligned}$$

$T_t^{\mu\nu}$  = target fluid

$T_p^{\mu\nu}$  = projectile fluid

# 3-fluid dynamics

$$\begin{aligned} 0 &= \partial_\mu T^{\mu\nu} \\ &= \partial_\mu T_t^{\mu\nu} + \partial_\mu T_p^{\mu\nu} + \partial_\mu T_{\text{fb}}^{\mu\nu} \end{aligned}$$

$T_t^{\mu\nu}$  = target fluid

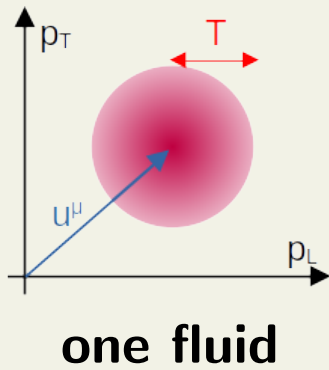
$T_p^{\mu\nu}$  = projectile fluid

$T_{\text{fb}}^{\mu\nu}$  = fireball fluid

- target and projectile represent colliding nucleons
- fireball (loosely) represents produced particles
- three fluids, each with temperature and flow velocity of its own

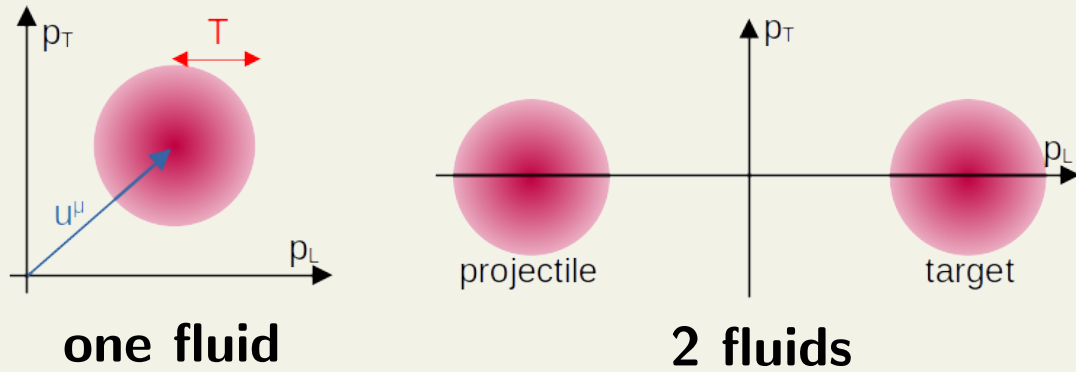
# 3-fluid dynamics

- distributions in momentum space



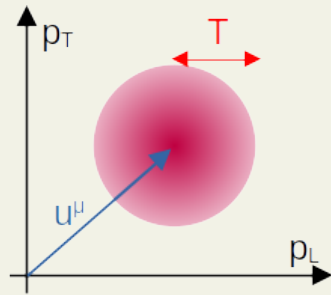
# 3-fluid dynamics

- distributions in momentum space

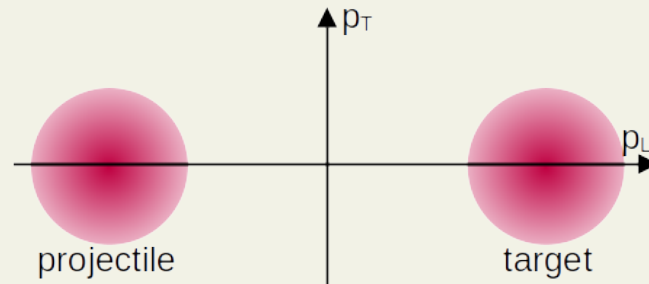


# 3-fluid dynamics

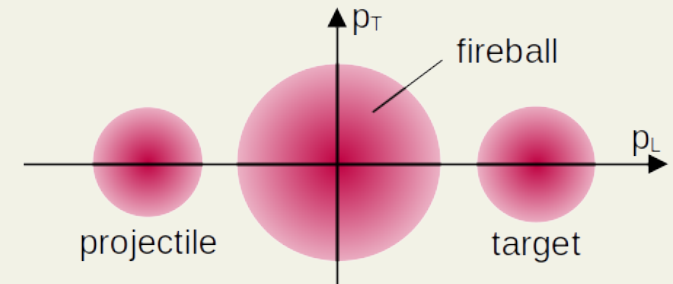
- distributions in momentum space



one fluid



2 fluids

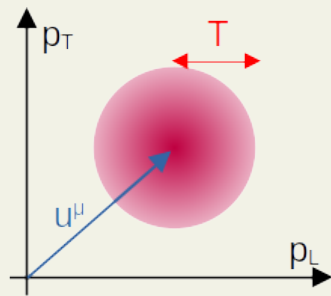


3 fluids

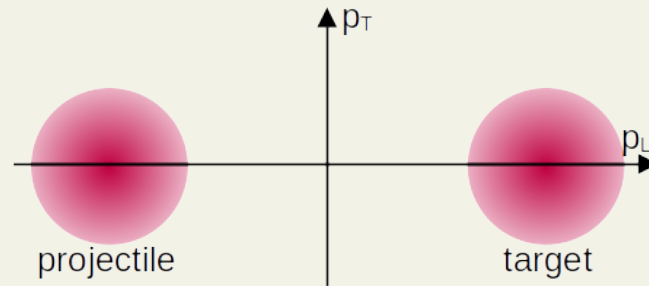


# 3-fluid dynamics

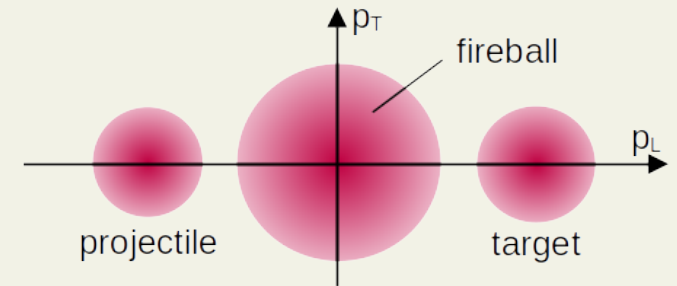
- distributions in momentum space



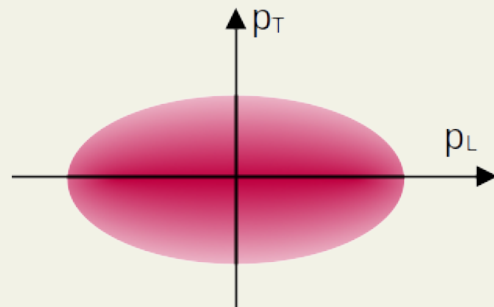
one fluid



2 fluids



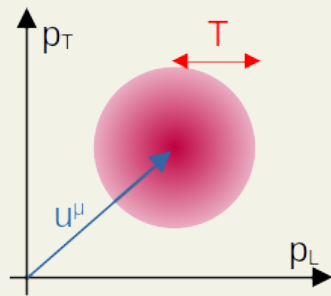
3 fluids



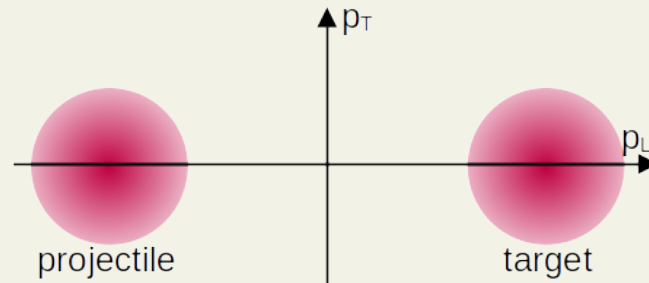
anisotropic hydro

# 3-fluid dynamics

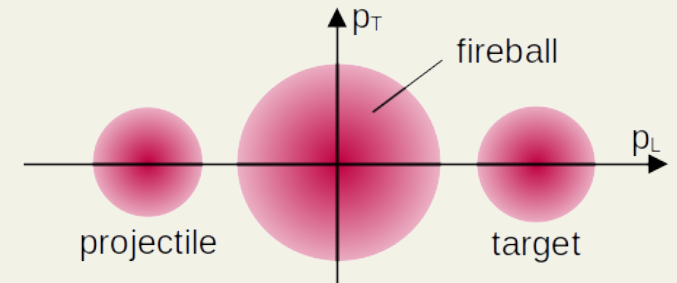
- distributions in momentum space



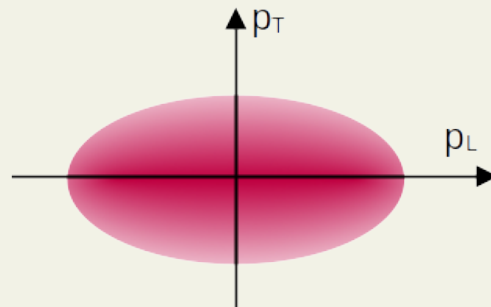
one fluid



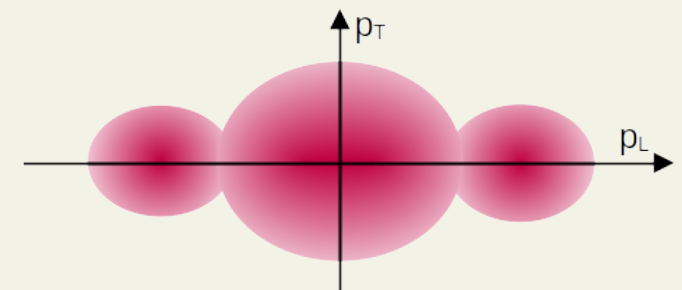
2 fluids



3 fluids



anisotropic hydro



somewhat realistic distribution

# 3-fluid dynamics

$$\partial_{\mu} T_{\text{t}}^{\mu\nu}(x) = -F_{\text{t}}^{\nu}(x) + F_{\text{ft}}^{\nu}(x)$$

$$\partial_{\mu} T_{\text{p}}^{\mu\nu}(x) = -F_{\text{p}}^{\nu}(x) + F_{\text{fp}}^{\nu}(x)$$

$$\partial_{\mu} T_{\text{fb}}^{\mu\nu}(x) = F_{\text{p}}^{\nu}(x) + F_{\text{t}}^{\nu}(x) - F_{\text{fp}}^{\nu}(x) - F_{\text{ft}}^{\nu}(x)$$

- interaction between **target** and **projectile**:  
friction terms  $-F_{\text{t}}^{\nu}(x)$  and  $-F_{\text{p}}^{\nu}(x)$
- interaction between fireball and **target/projectile**:  
friction terms  $F_{\text{fp}}^{\nu}(x)$  and  $F_{\text{ft}}^{\nu}(x)$

# Friction from kinetic theory

**Boltzmann equation for three fluids**

$$p^\mu \partial_\mu f_i = C_i[f_p, f_t, f_f] = \sum_{j,k} C_i^{jk}[f_j, f_k], \quad i, j, k \in \{p, t, f\}$$

$C_i^{jk}$ : change in distribution/fluid  $i$  due to interactions of particles in  $j$  and  $k$   
for given  $C_i^{jk}$ , friction obtained as

$$\partial_\mu T_i^{\mu\nu} = \int \frac{d^3p}{p^0} p^\nu C_i = F_i^\nu, \quad \partial_\mu J_{B,i}^\mu = B_i \int \frac{d^3p}{p^0} C_i = R_{B,i}$$

# Friction from kinetic theory

collision integrals in terms of scattering cross sections

$$C_i^{ij}[f_i, f_j](p_i) = \int d^3p_j p_i^0 \left[ \underbrace{-f_i(p_i) f_j(p_j) v_{\text{rel}} \sigma_{ij \rightarrow X}}_{\text{loss}} + \underbrace{\int d^3q_i f_i(q_i) f_j(p_j) v_{\text{rel}} \frac{d\sigma_{ij \rightarrow iX}}{d^3p_i}}_{\text{gain}} \right]$$

from these, approximative friction formulae are derived

## problems:

- cross sections may not be fully measured in experiment
- what stays in a fluid, what's moved to another?
- d.o.f. change in deconfinement transition

# Csernai approach

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26, 149 (1982)

- all that scatters goes to the fireball
- projectile and target stay cold
- no baryon transparency!

# Csernai approach

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26, 149 (1982)

- all that scatters goes to the fireball
- projectile and target stay cold
- no baryon transparency!

## Note:

- dynamical initialization is analogous to this approach!
- finite formation time & spatial distribution  $\Rightarrow$  baryon transparency

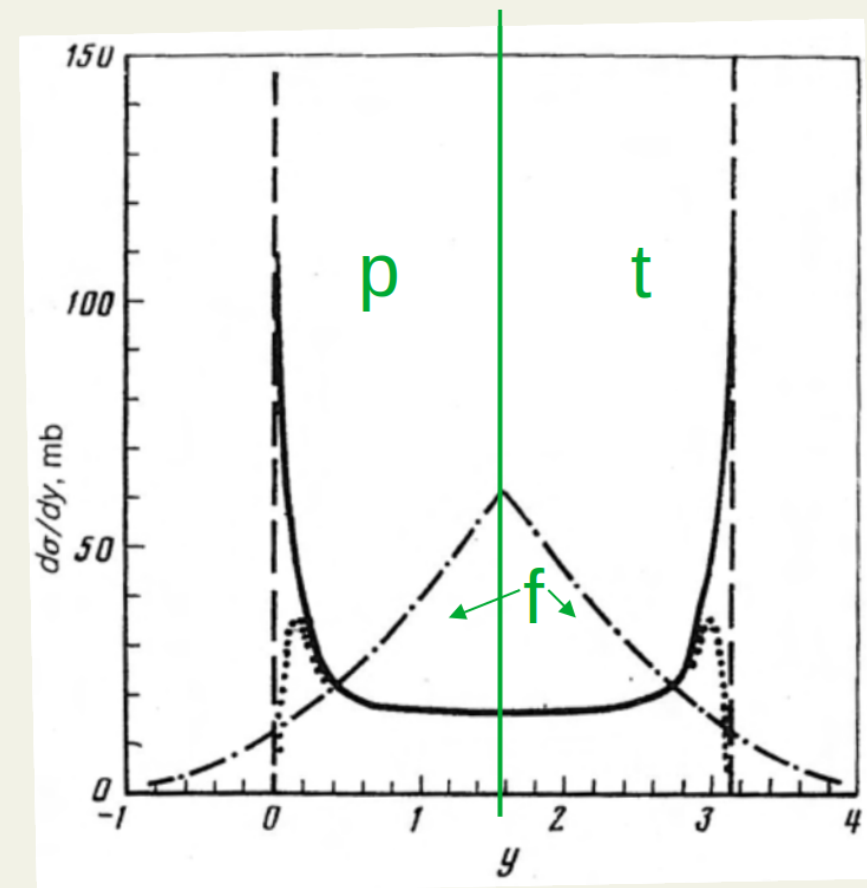
# Satarov/Ivanov approach

Ivanov, Russkikh, Toneev PRC 73, 044904 (2006)

- **N+N scattering: N strongly peaked at ingoing rapidities,  $\pi$  at midrapidity**  
 $\Rightarrow$  in p-t friction: N stay in p/t,  $\pi$  go to f
- **$\pi + N$  mostly resonance formation**  
 $\Rightarrow$  all outgoing particles from p-f friction go to p
- **uncertainty in deconfined phase: densities multiplied with  $\sqrt{s}$ -dependent prefactor**

pros: only need total crosssections.  
can describe the double peak in baryon distributions!

cons:  $\mu_B = 0$  in fireball

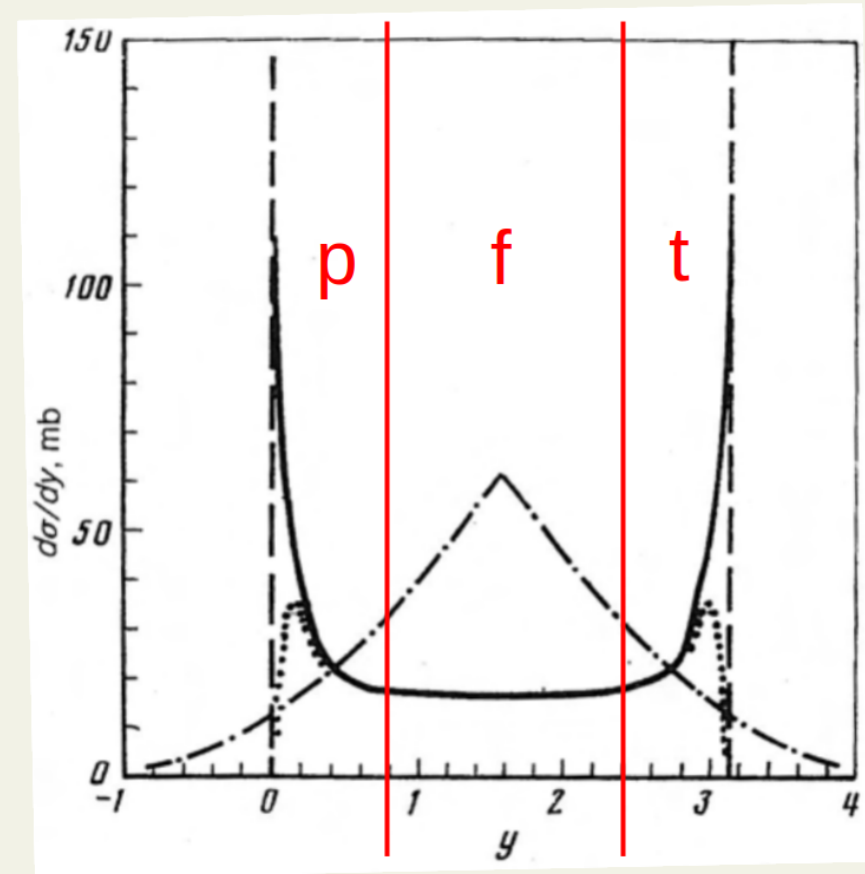




# modified Satarov/Ivanov approach

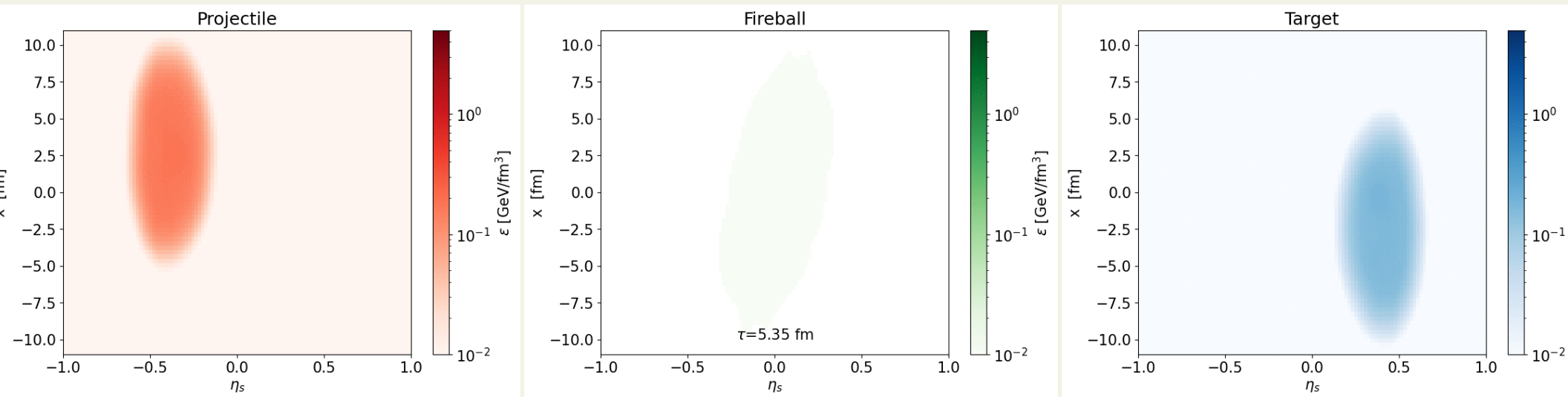
- for our purposes:  
need high  $\mu_B$  also in fireball!
- idea: divide outgoing N from N+N into **3** regions  
⇒ modified p+t friction moves B to fireball

but: need doubly differential cross sections!  $(y, E)$



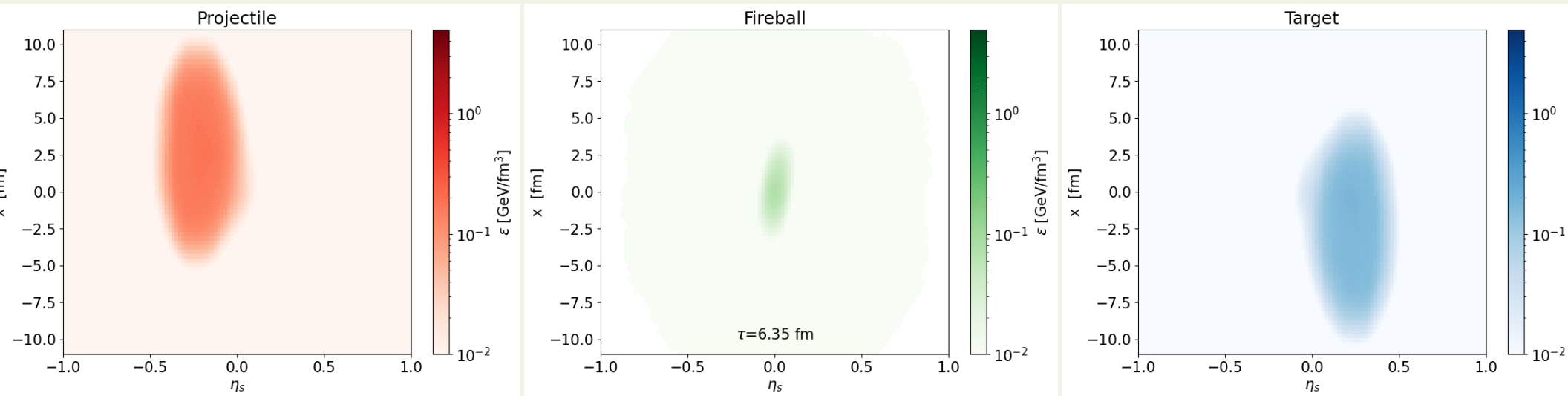
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{NN}} = 7.7$  GeV



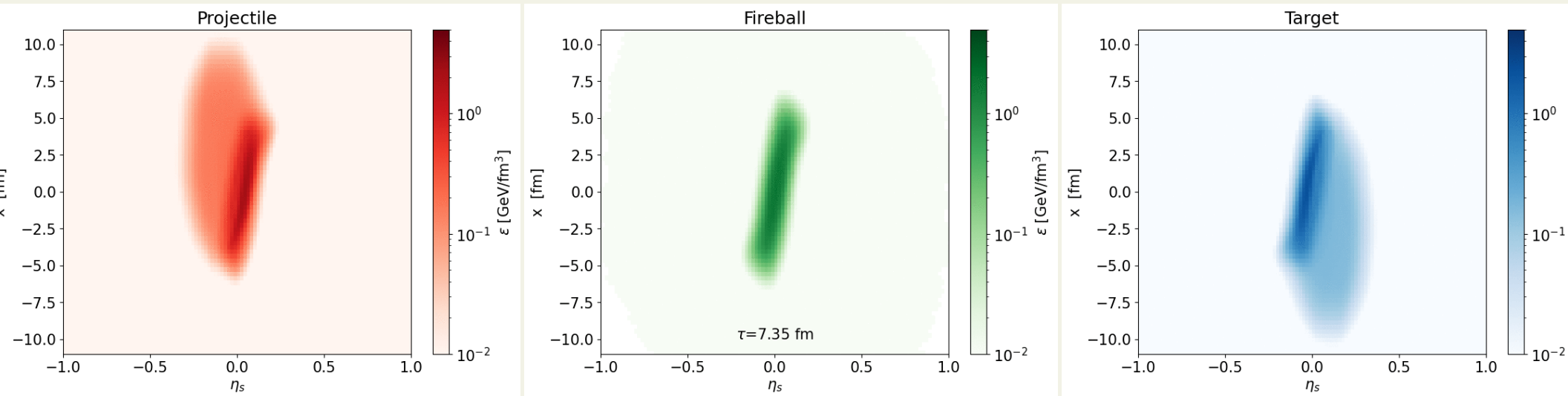
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$



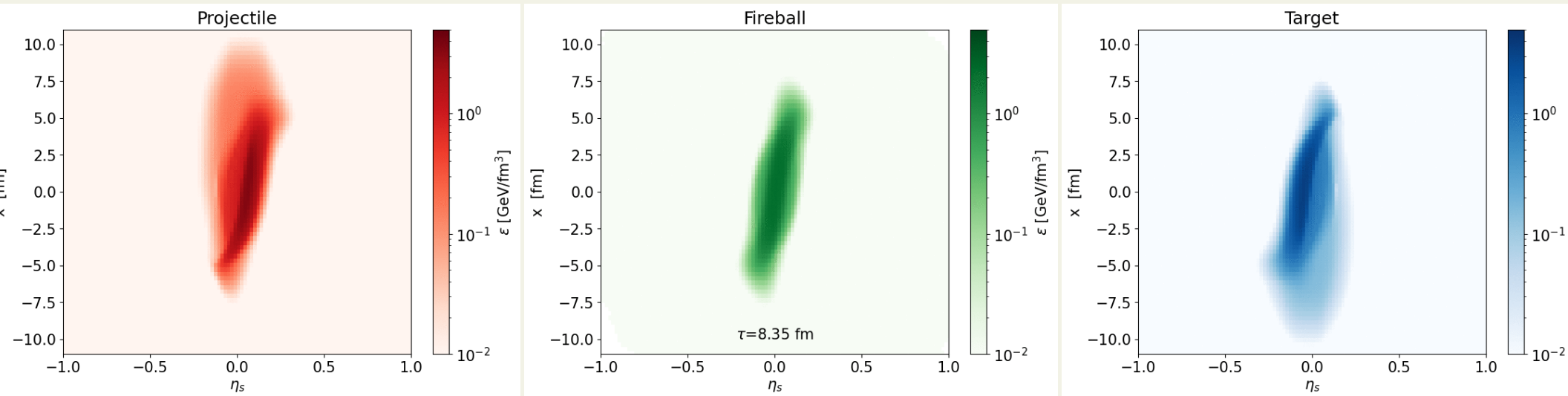
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$



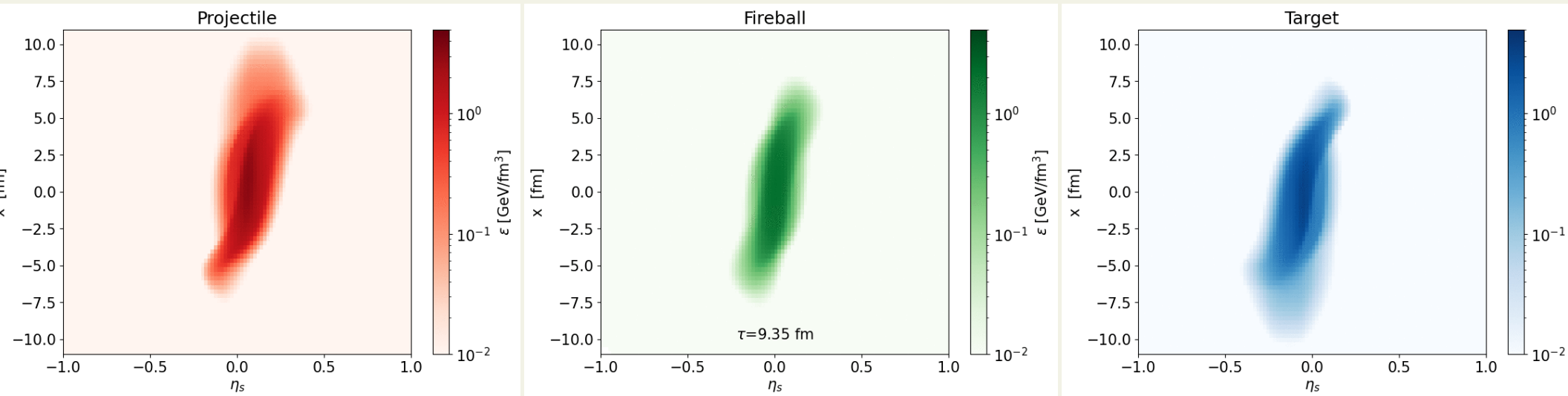
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$



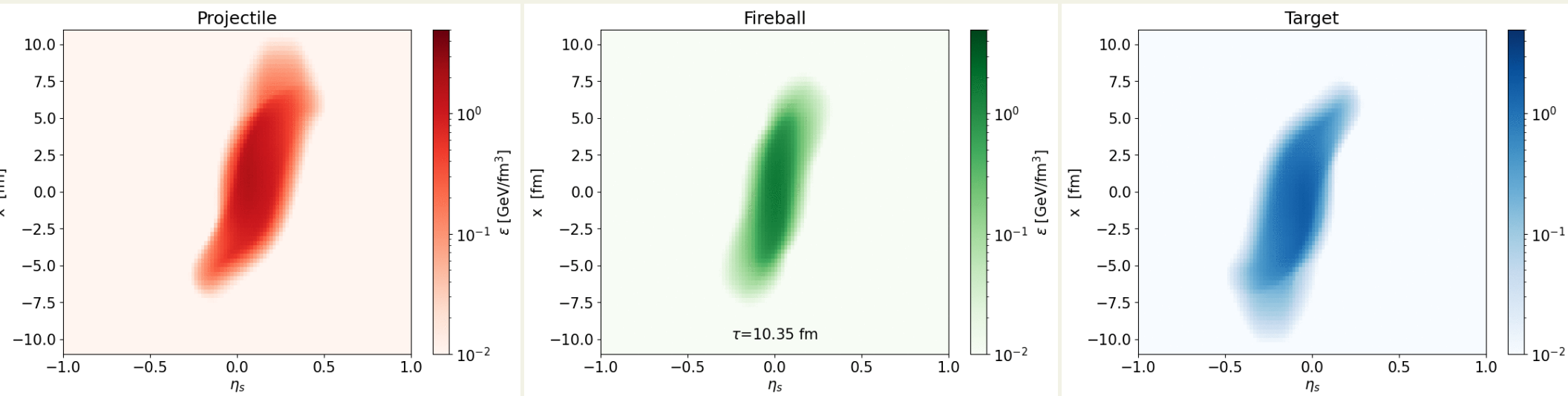
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$



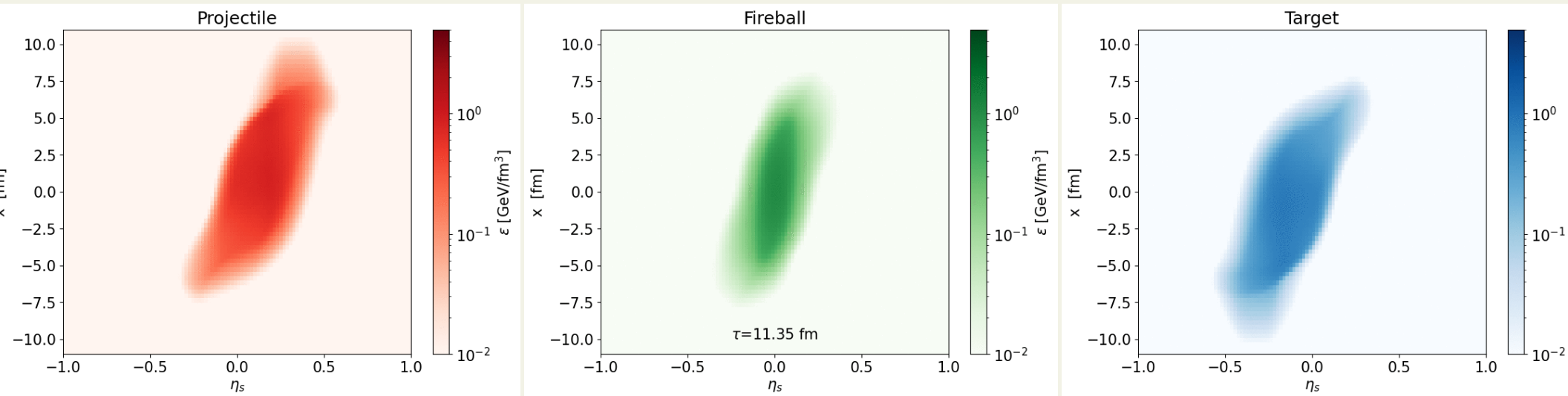
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{NN}} = 7.7$  GeV



# Evolution of energy density

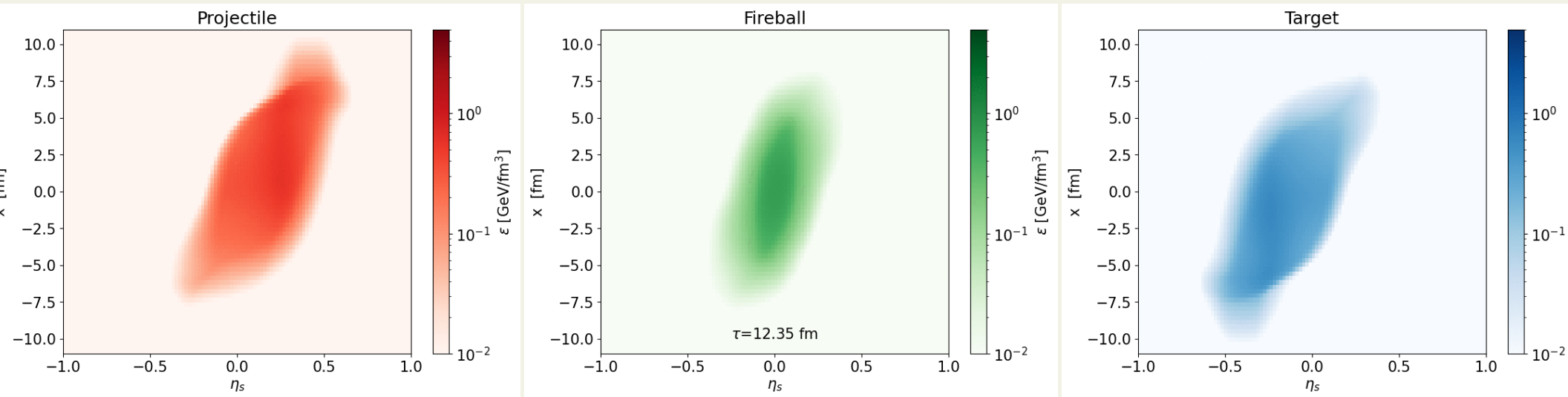
- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$





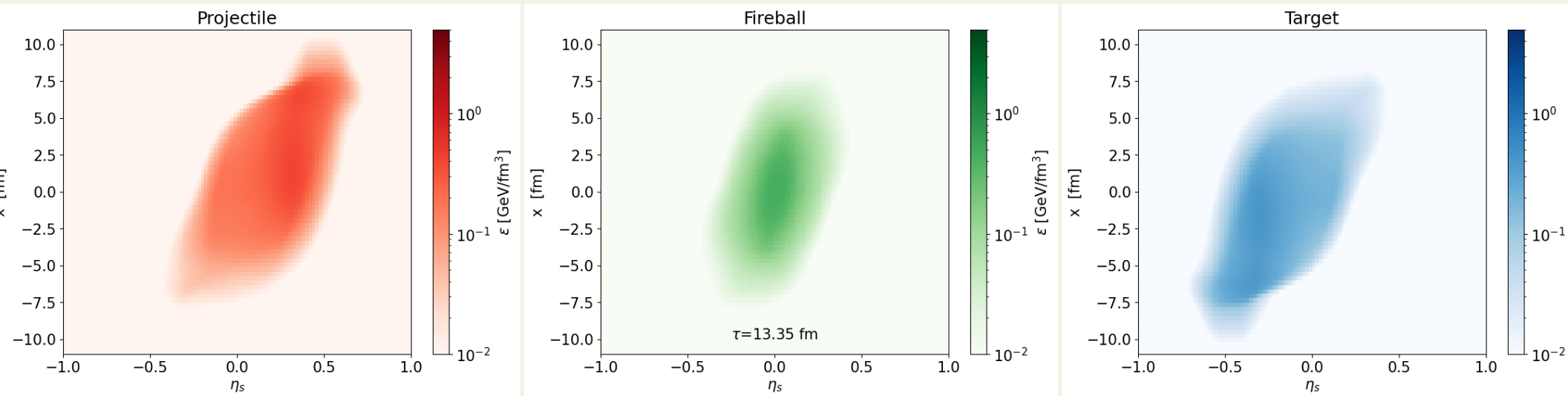
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$



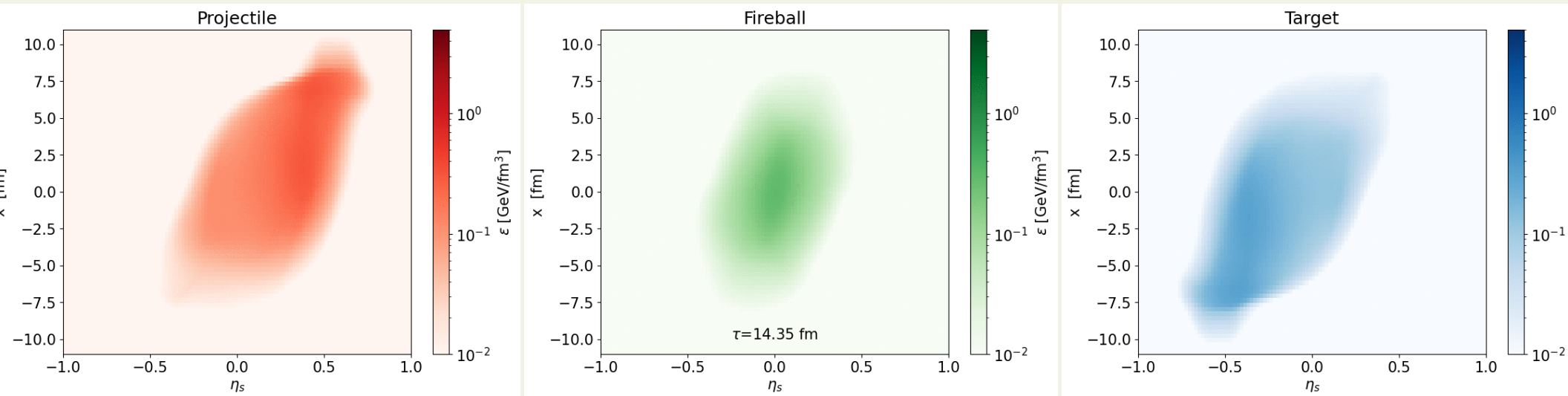
# Evolution of energy density

- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$

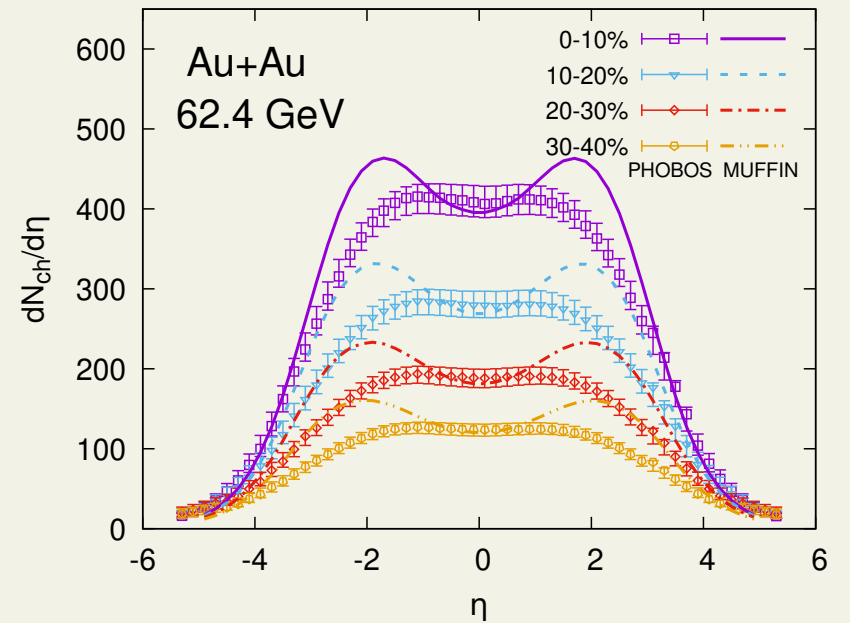
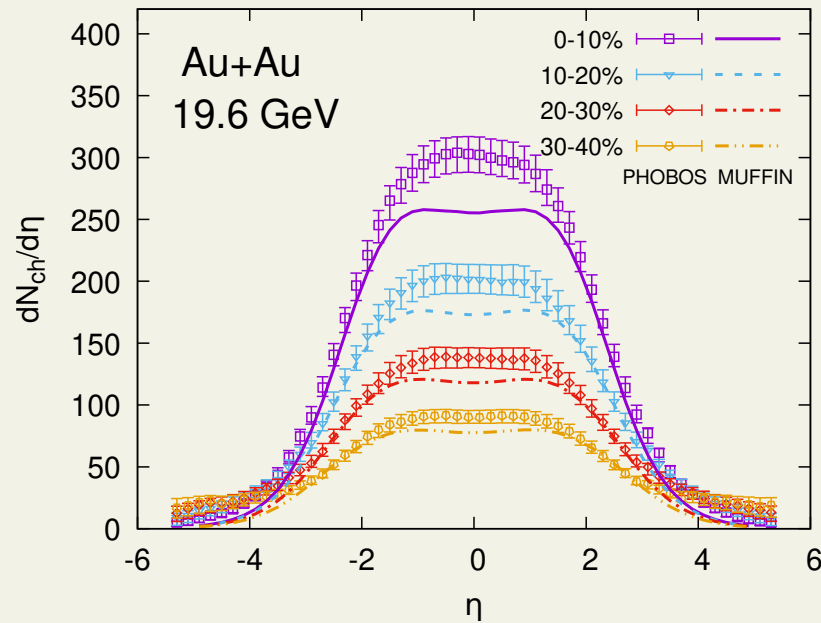
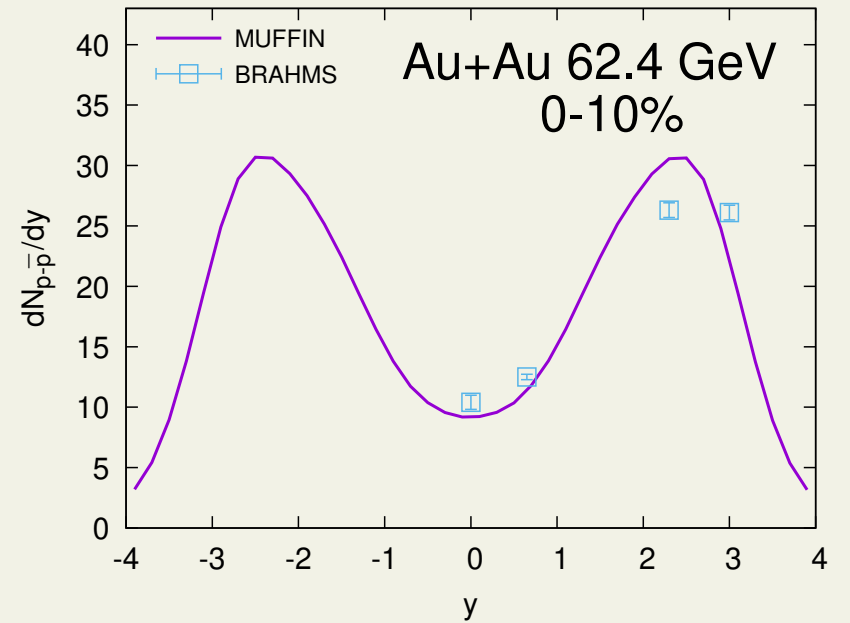
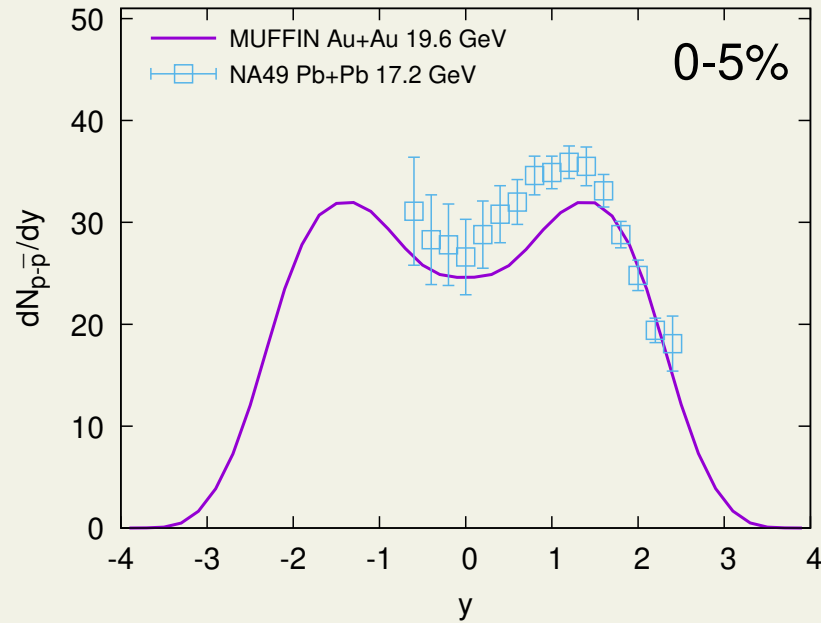


# Evolution of energy density

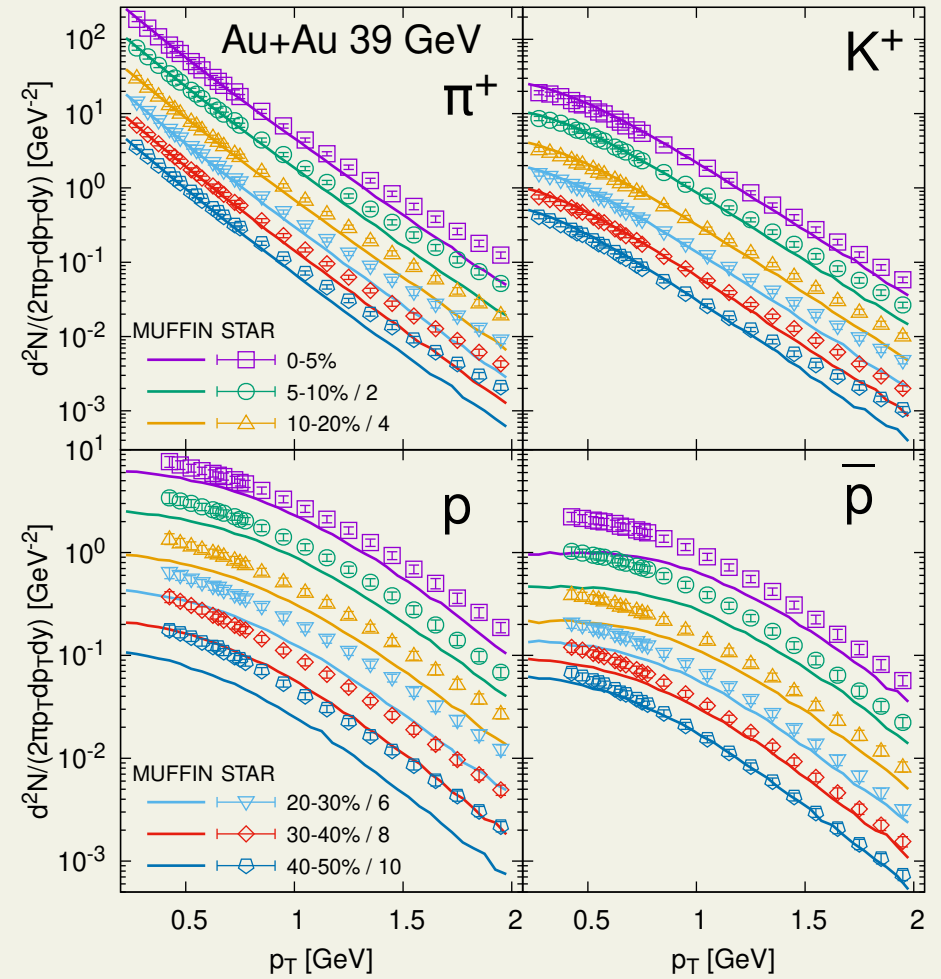
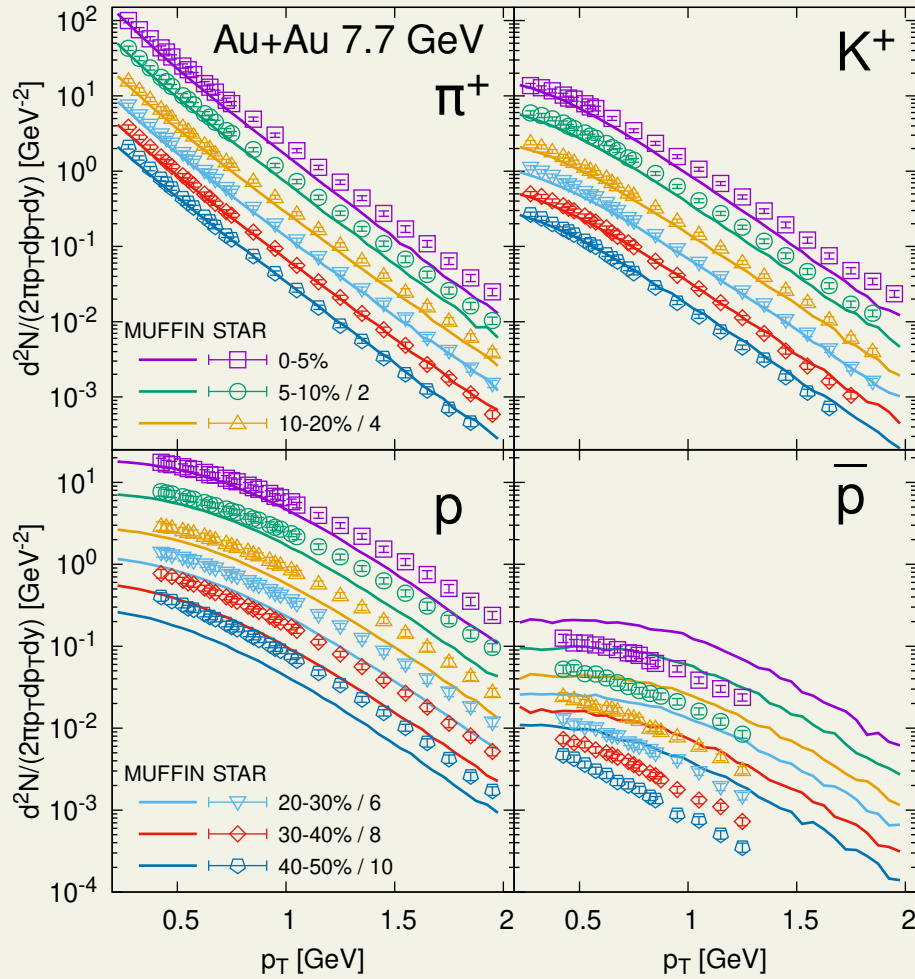
- Au+Au collision at  $\sqrt{s_{\text{NN}}} = 7.7 \text{ GeV}$



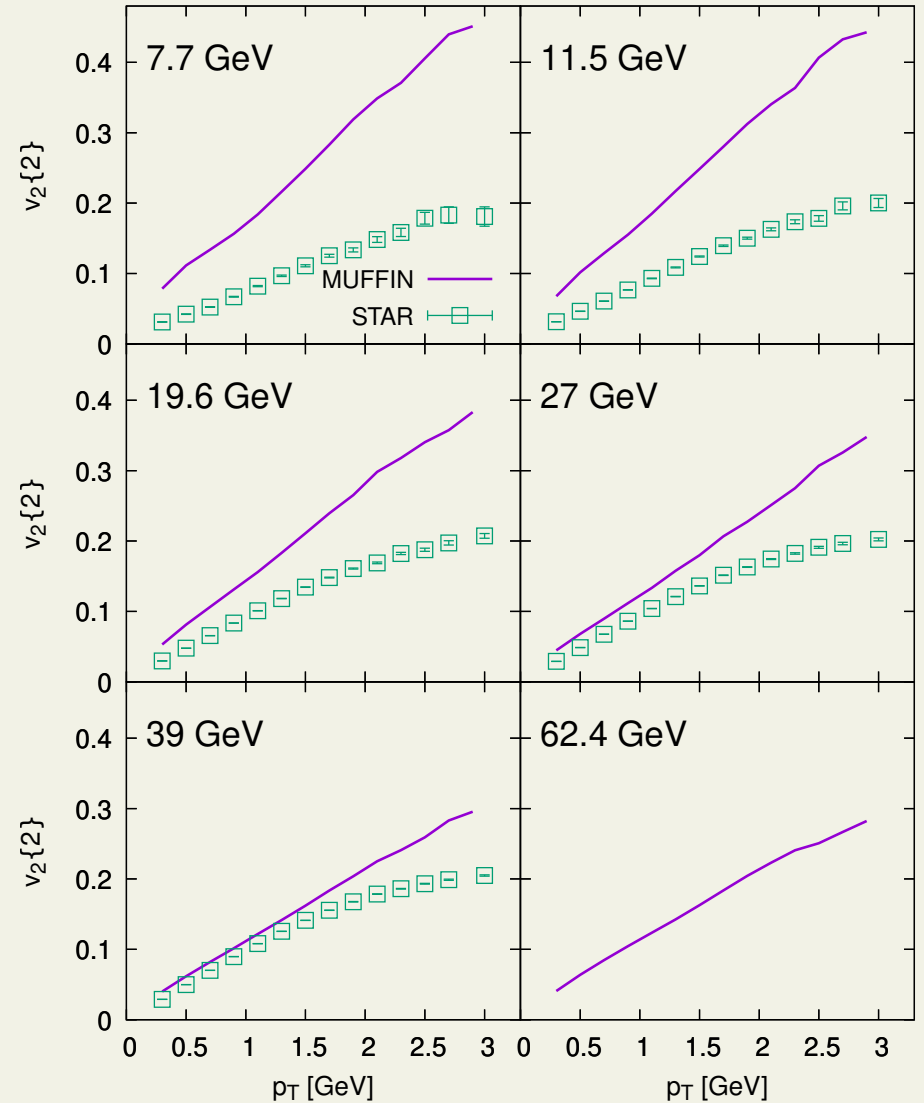
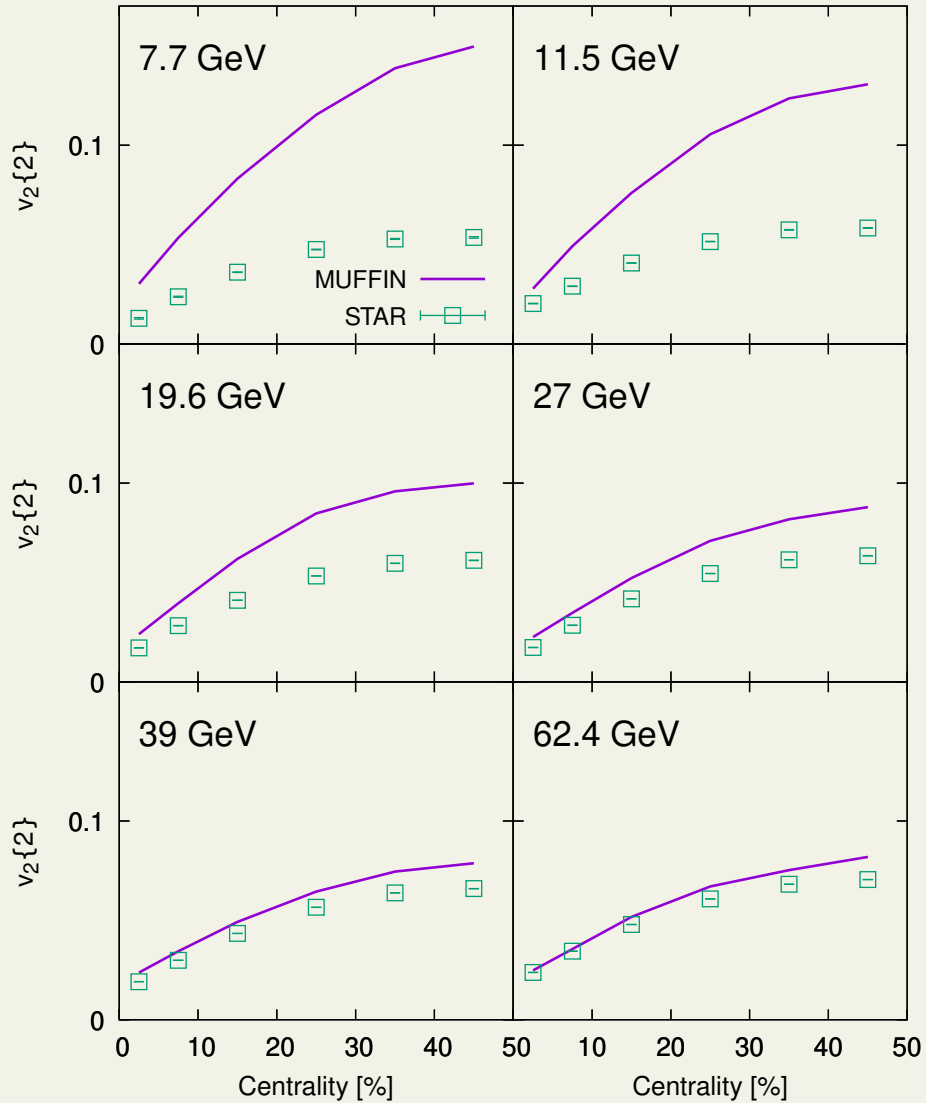
# Results: (pseudo)rapidity distributions



# Results: transverse momentum distributions



# Results: elliptic flow



**Viscosity not yet included!**

# Dissipation

$$T_i^{\mu\nu} = \epsilon_i u_i^\mu u_i^\nu + P_i \Delta_i^{\mu\nu} + \pi_i^{\mu\nu}, \quad i \in \{t, p, f\}$$

$$\partial_\mu T_i^{\mu\nu} = \partial_\mu (\epsilon_i u_i^\mu u_i^\nu) + \partial_\mu (P_i \Delta_i^{\mu\nu}) + \partial_\mu \pi_i^{\mu\nu} = F_i^\nu$$

where  $\pi_i^{\mu\nu}$  obeys

$$u^\alpha \partial_\alpha \pi_i^{\mu\nu} = -\frac{1}{\tau_\pi} \left( \pi_i^{\mu\nu} - 2\eta \nabla^{\langle\mu} u_i^{\nu\rangle} \right) + \dots$$

independent of  $F_i^\mu$ ?

⇒ corrections to the evolution equations needed

- rederive DMNR—work in progress

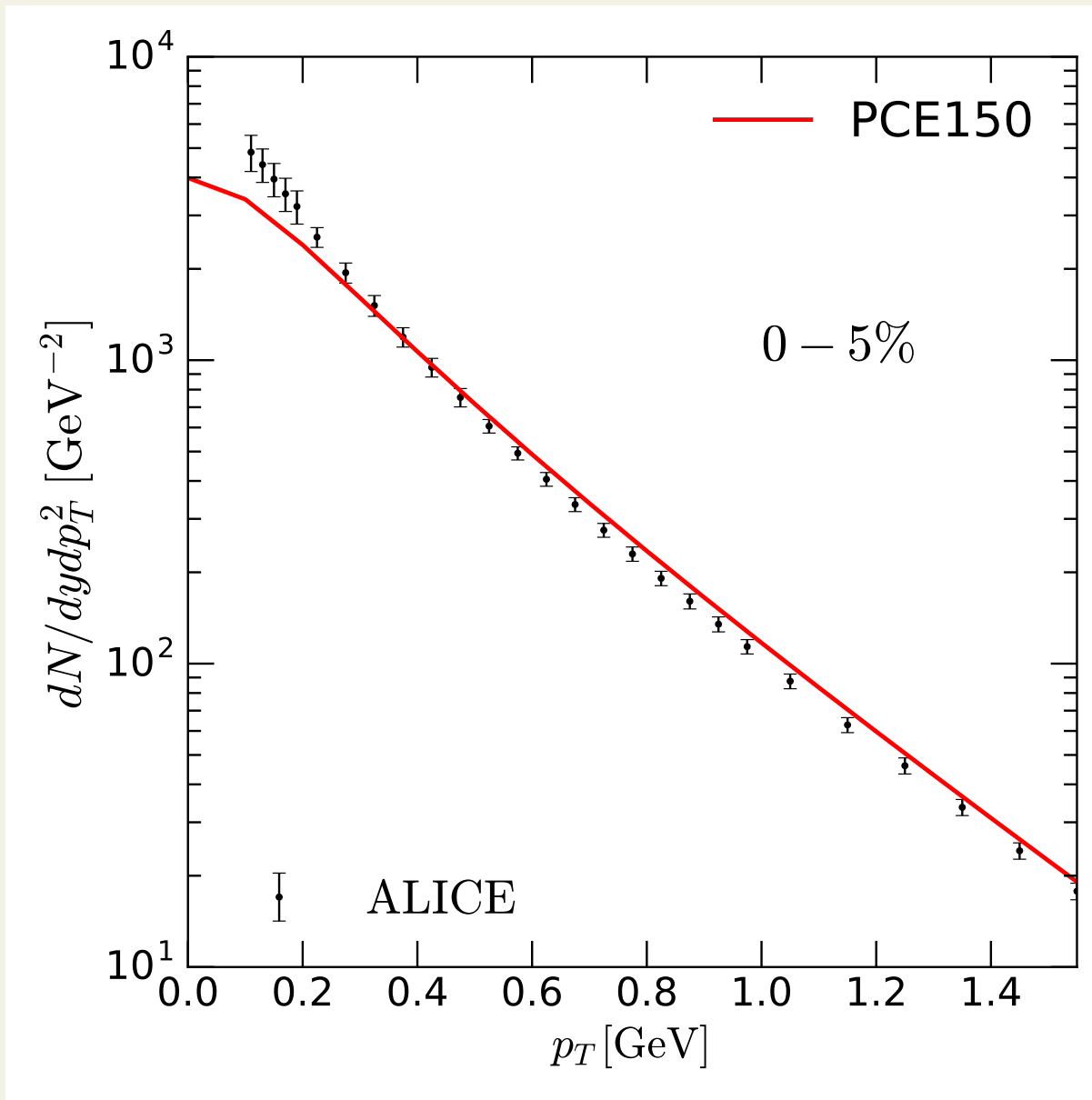
# End of part I

- **3-fluid approach to collisions at BES energies**
  - projectile, target, produced particles described as separate fluids
- **rough reproduction of rapidity and  $p_T$  distributions**
- **overshoots anisotropies—no viscosity**
- **work in progress—stay tuned!**



# Effects of resonance widths on EoS and particle distributions

# Pion $p_T$ spectrum at LHC (Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV)

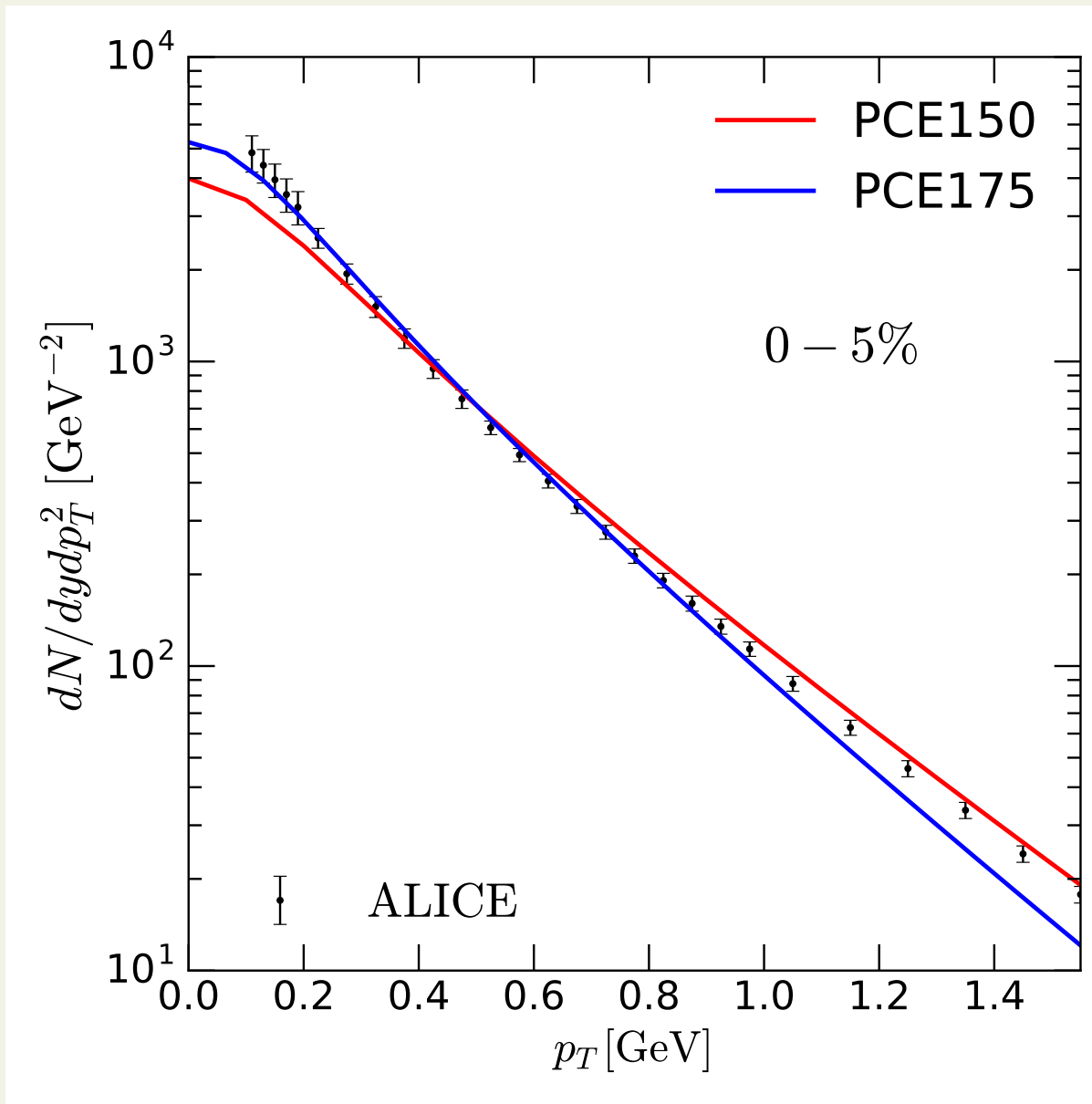


- viscous hydro
- initial state:  
pQCD+saturation
- $\tau_0 \approx 0.2\text{fm}/c$

**PCE150:**  
fit to  $\pi$ ,  $K$ ,  $p$  yields  
no fit to spectrum

© H. Niemi

# Pion $p_T$ spectrum at LHC (Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV)



- viscous hydro
- initial state:  
pQCD+saturation
- $\tau_0 \approx 0.2\text{fm}/c$

**PCE150:**  
fit to  $\pi$ ,  $K$ ,  $p$  yields  
no fit to spectrum

**PCE175:**  
no fit to yields  
fits the spectrum

© H. Niemi

- **need more resonances**
- **yield proportional to Boltzmann factor**

$$N \propto \exp\left(-\frac{m}{T}\right)$$

- **need more resonances**
- **yield proportional to Boltzmann factor**

$$N \propto \exp\left(-\frac{m}{T}\right)$$

- **resonance mass?**

- **need more resonances**
- **yield proportional to Boltzmann factor**

$$N \propto \exp\left(-\frac{m}{T}\right)$$

- **resonance mass?**
- **usually no width, i.e. resonances have their pole mass**

## Dashen-Ma-Bernstein:

If interactions mediated by *narrow* resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas

⇒ **Hadron resonance gas model**

## Dashen-Ma-Bernstein:

If interactions mediated by *narrow* resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas

⇒ **Hadron resonance gas model**

## Dashen-Ma-Berstein: S-matrix formulation of statistical mechanics:

⇒ **Second virial coefficient can be evaluated in terms of scattering phase shift (as far as interaction is manifested in elastic scattering)**

⇒ **relativistic Beth-Uhlenbeck form**

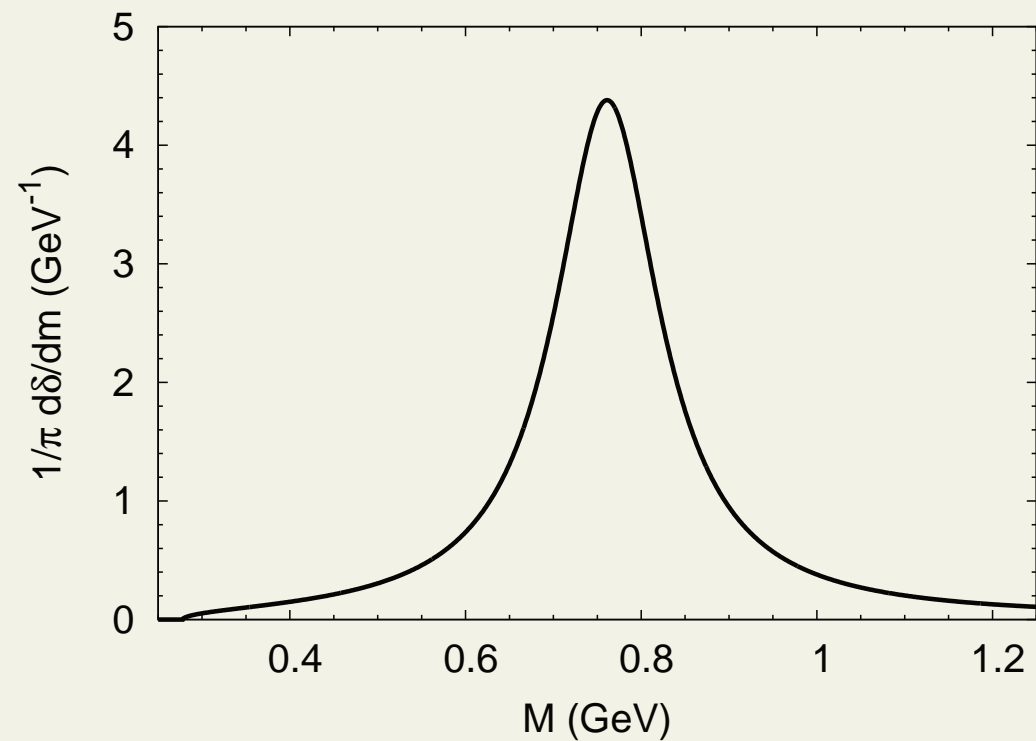
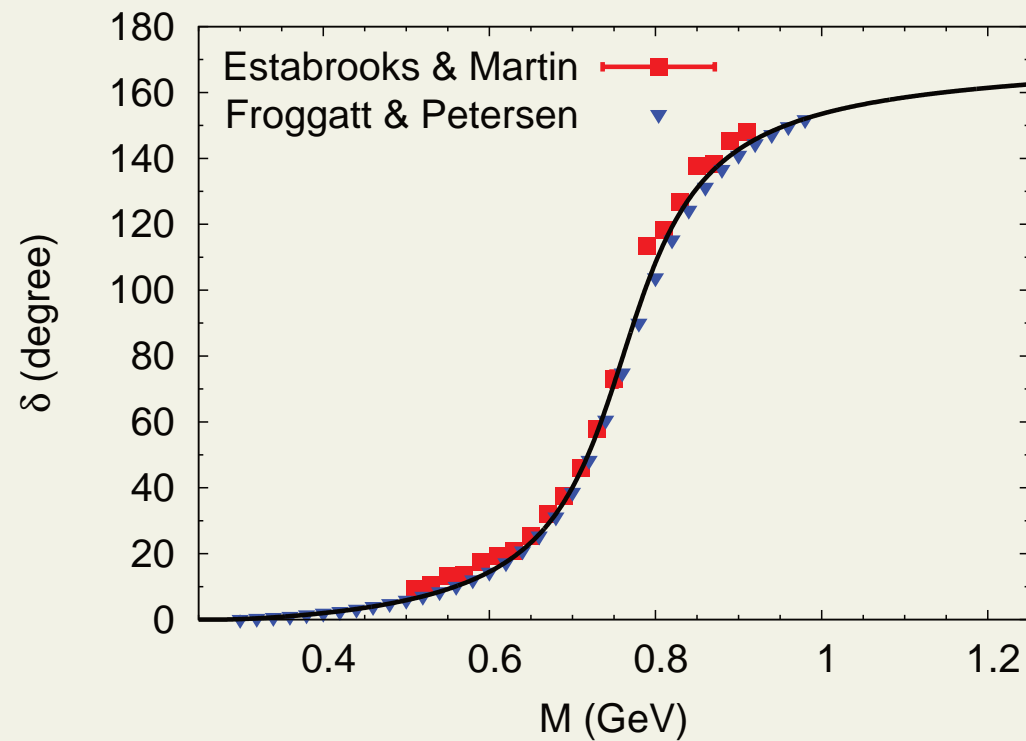


# S-matrix

- effects of interactions expressed in terms of scattering phase shifts

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p, m) \quad \text{with} \quad \frac{d\rho}{dm} = \frac{1}{\pi} \frac{d\delta}{dm}$$

- $\pi\pi$  scattering, P-wave, i.e.  $\rho$  resonance

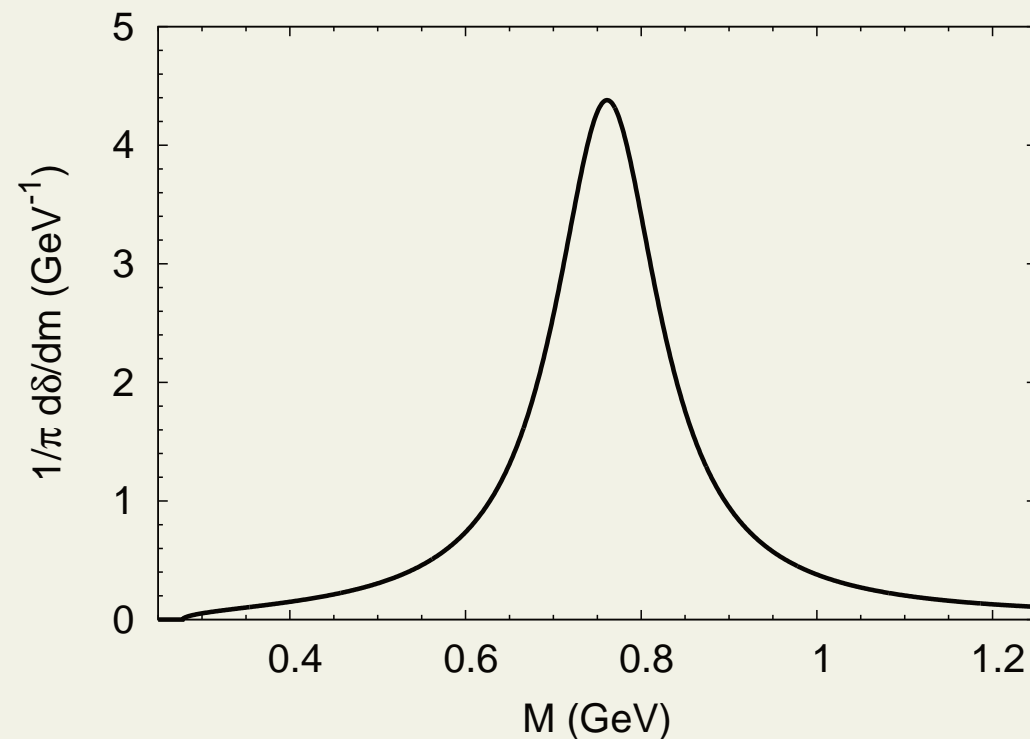
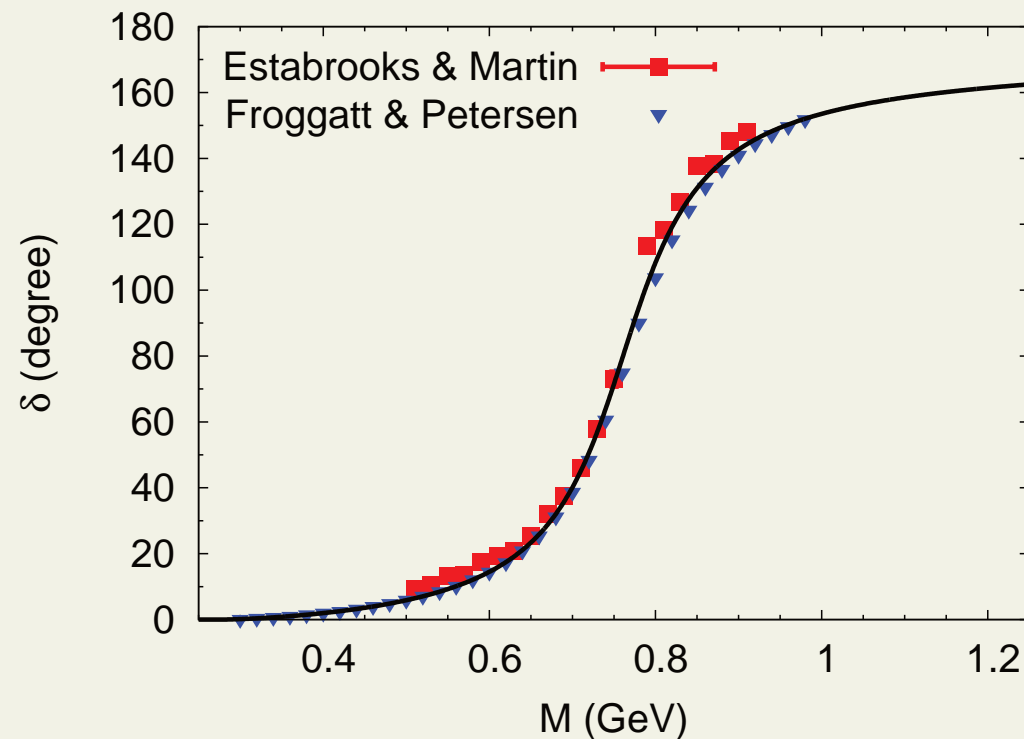


# S-matrix

- effects of interactions expressed in terms of scattering phase shifts

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p, m) \quad \text{with} \quad \frac{d\rho}{dm} = \frac{1}{\pi} \frac{d\delta}{dm}$$

- $\pi\pi$  scattering, P-wave, i.e.  $\rho$  resonance

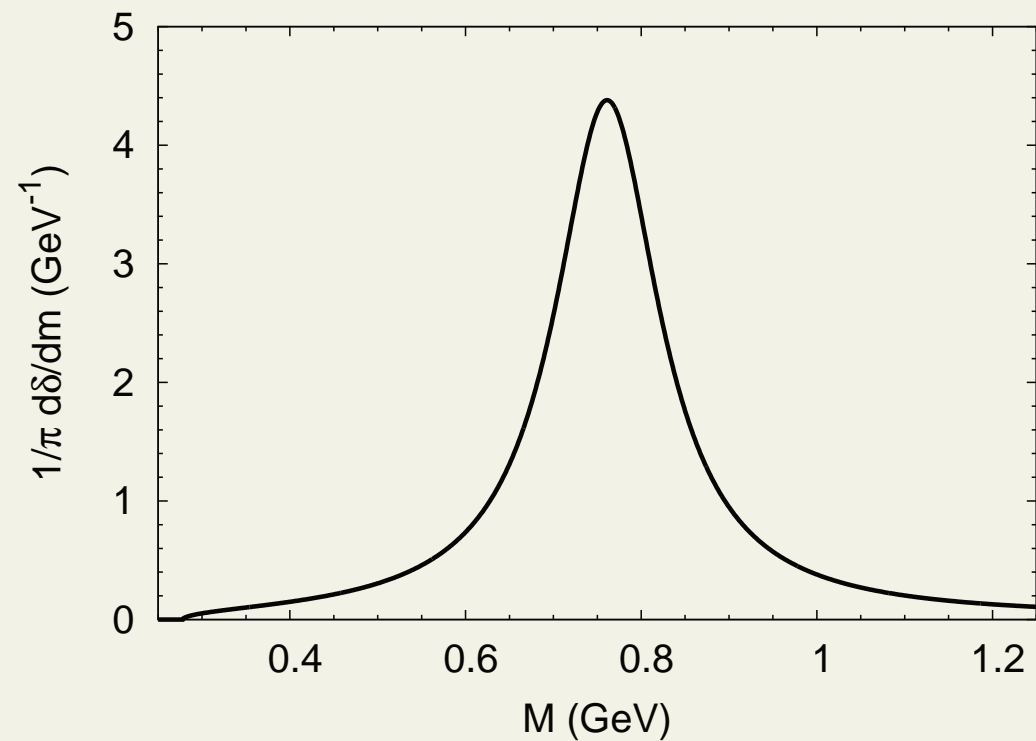
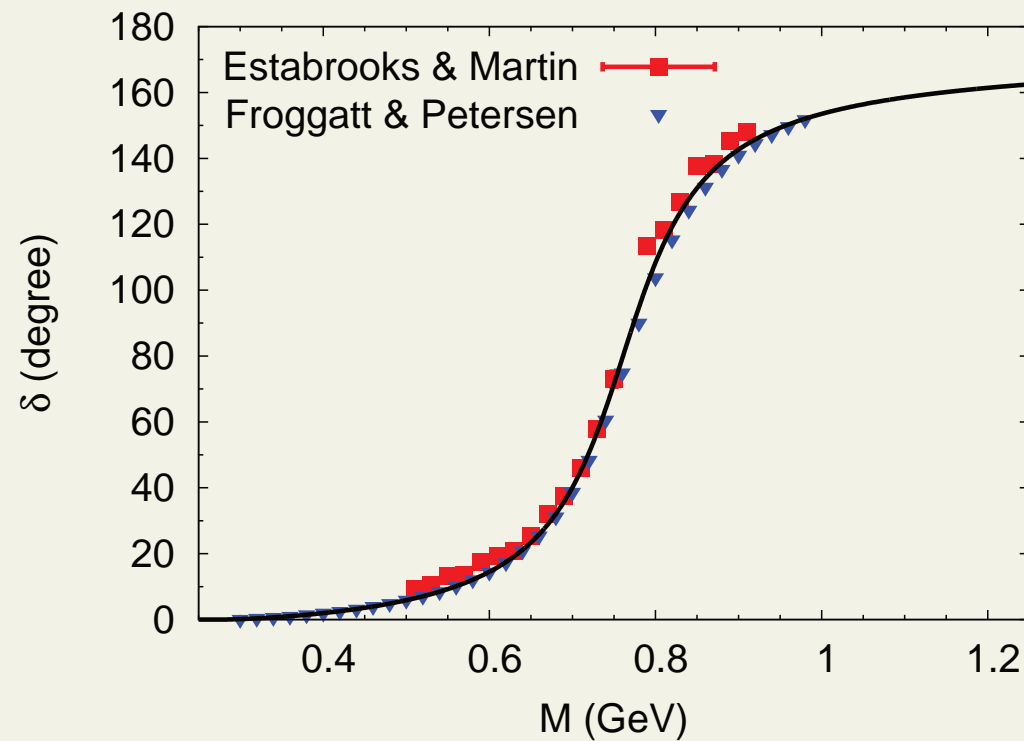


# S-matrix

- effects of interactions expressed in terms of scattering phase shifts

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p, m) \quad \text{with} \quad \frac{d\rho}{dm} = \frac{1}{\pi} \frac{d\delta}{dm}$$

- $\pi\pi$  scattering, P-wave, i.e.  $\rho$  resonance

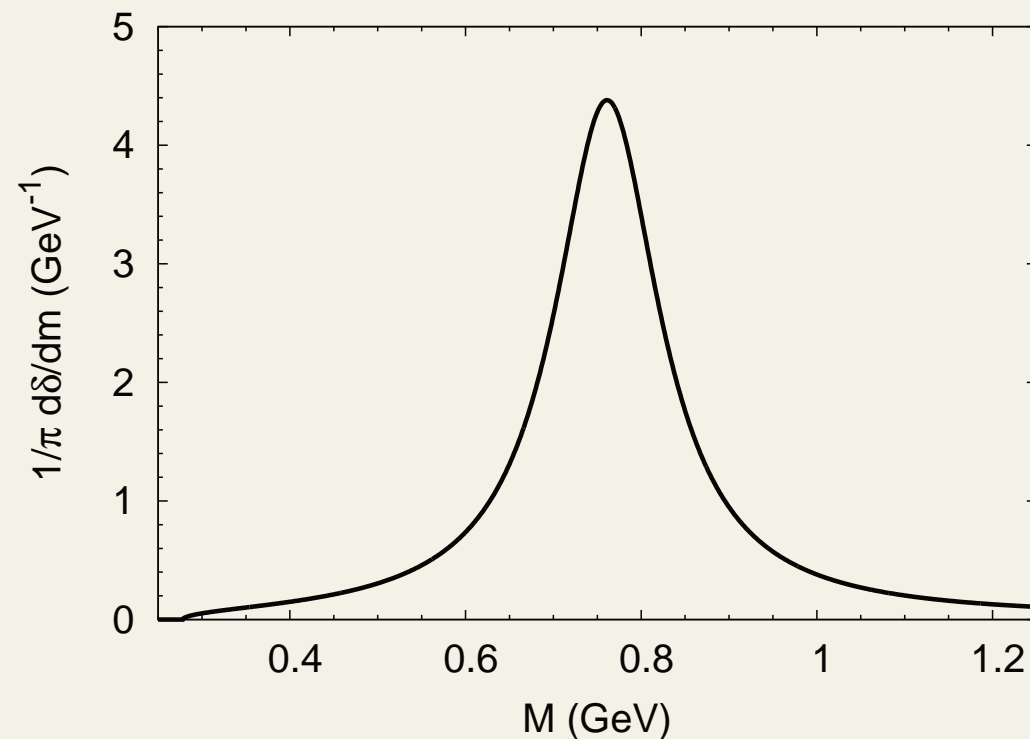
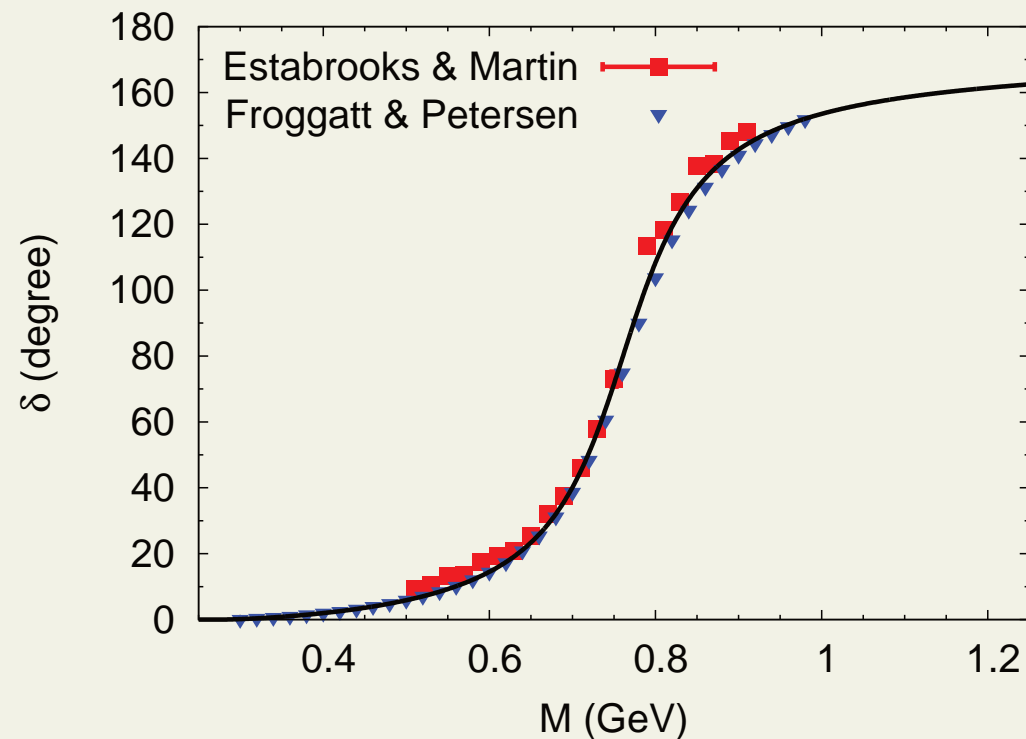


# S-matrix

- effects of interactions expressed in terms of scattering phase shifts

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p, m) \quad \text{with} \quad \frac{d\rho}{dm} = \frac{1}{\pi} \frac{d\delta}{dm}$$

- $\pi\pi$  scattering, P-wave, i.e.  $\rho$  resonance

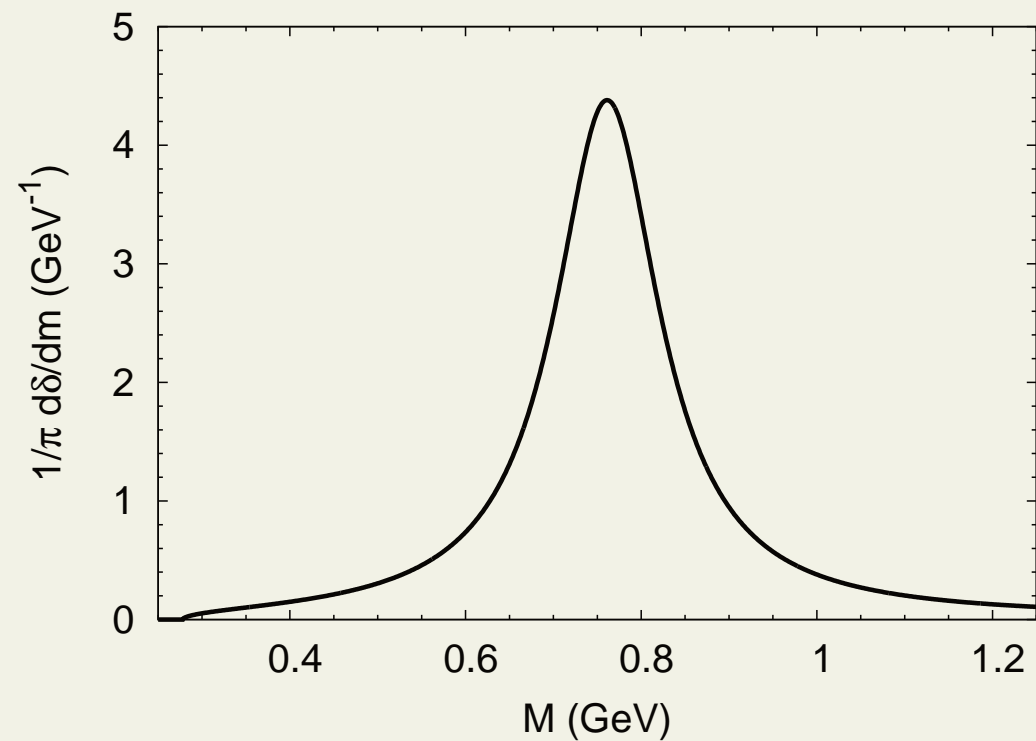
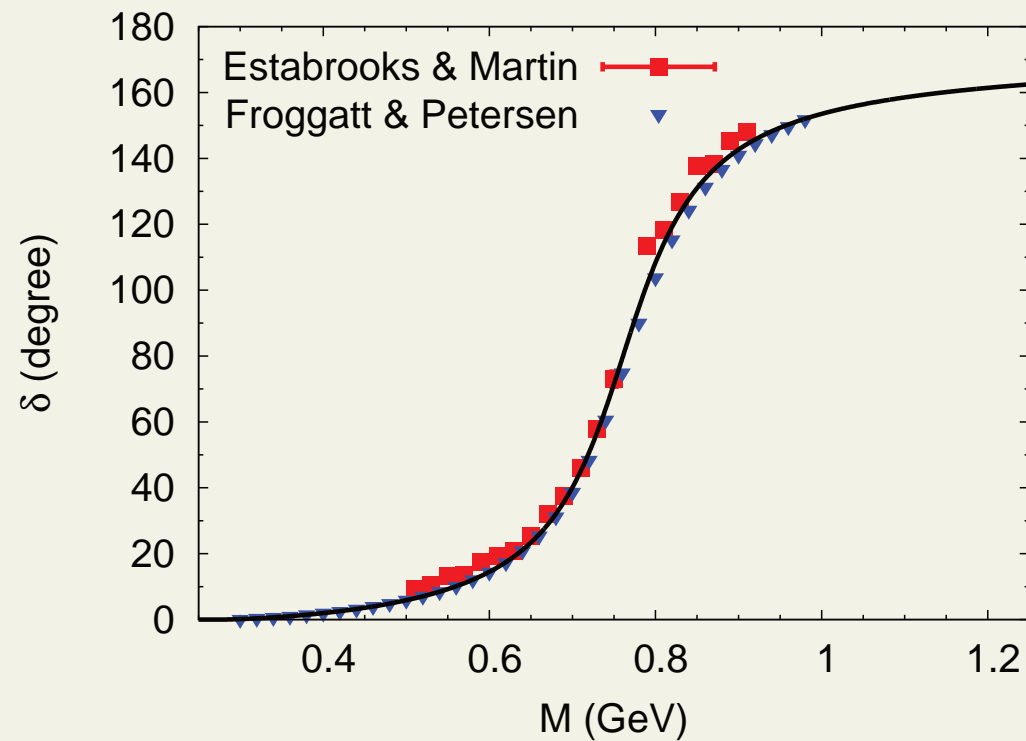


# S-matrix

- effects of interactions expressed in terms of scattering phase shifts

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p, m) \quad \text{with} \quad \frac{d\rho}{dm} = \frac{1}{\pi} \frac{d\delta}{dm}$$

- $\pi\pi$  scattering, P-wave, i.e.  $\rho$  resonance

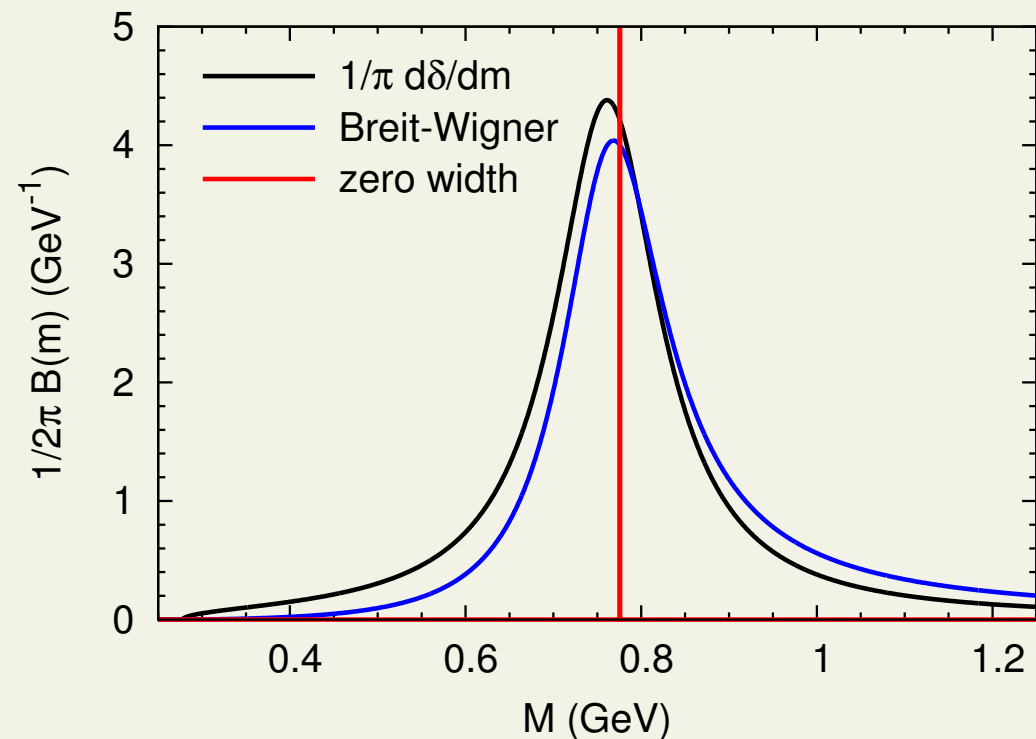
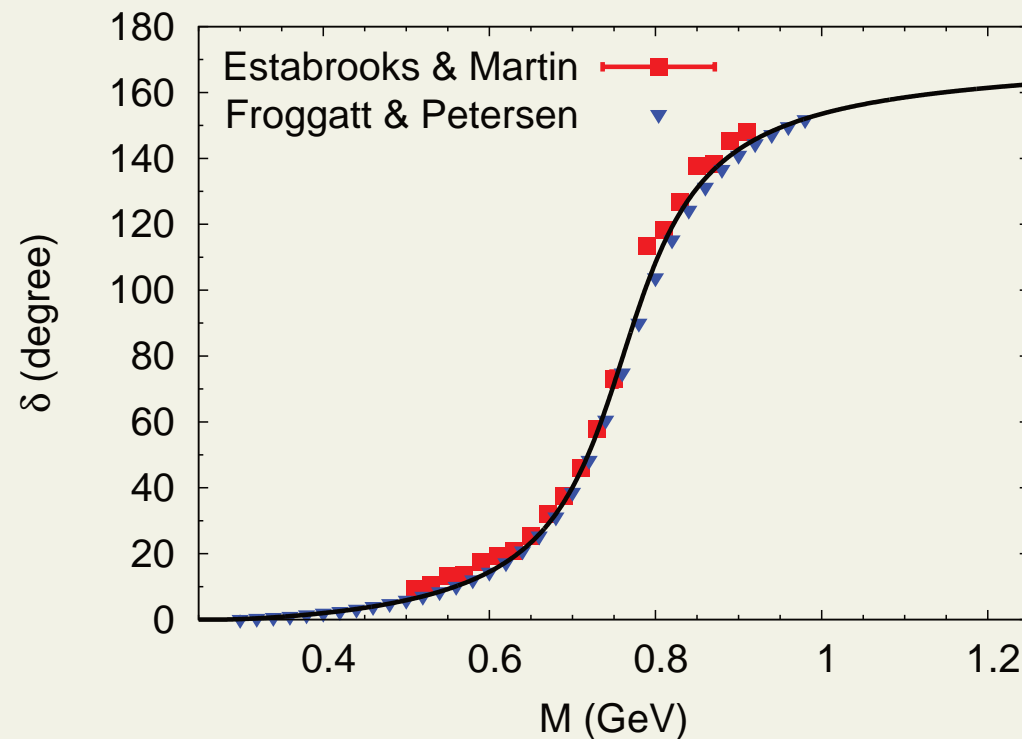


# S-matrix

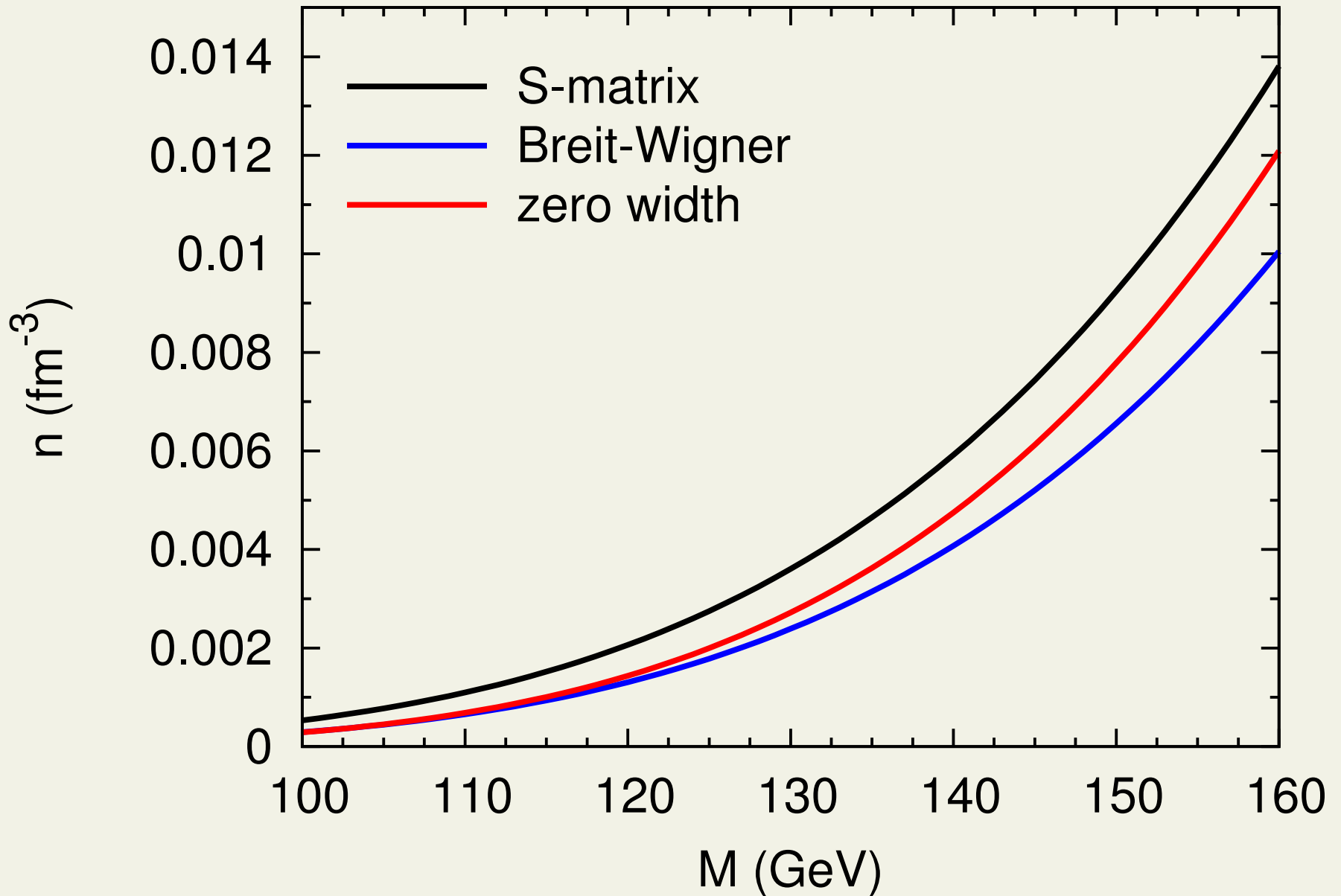
- effects of interactions expressed in terms of scattering phase shifts

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p, m) \quad \text{with} \quad \frac{d\rho}{dm} = \frac{1}{\pi} \frac{d\delta}{dm}$$

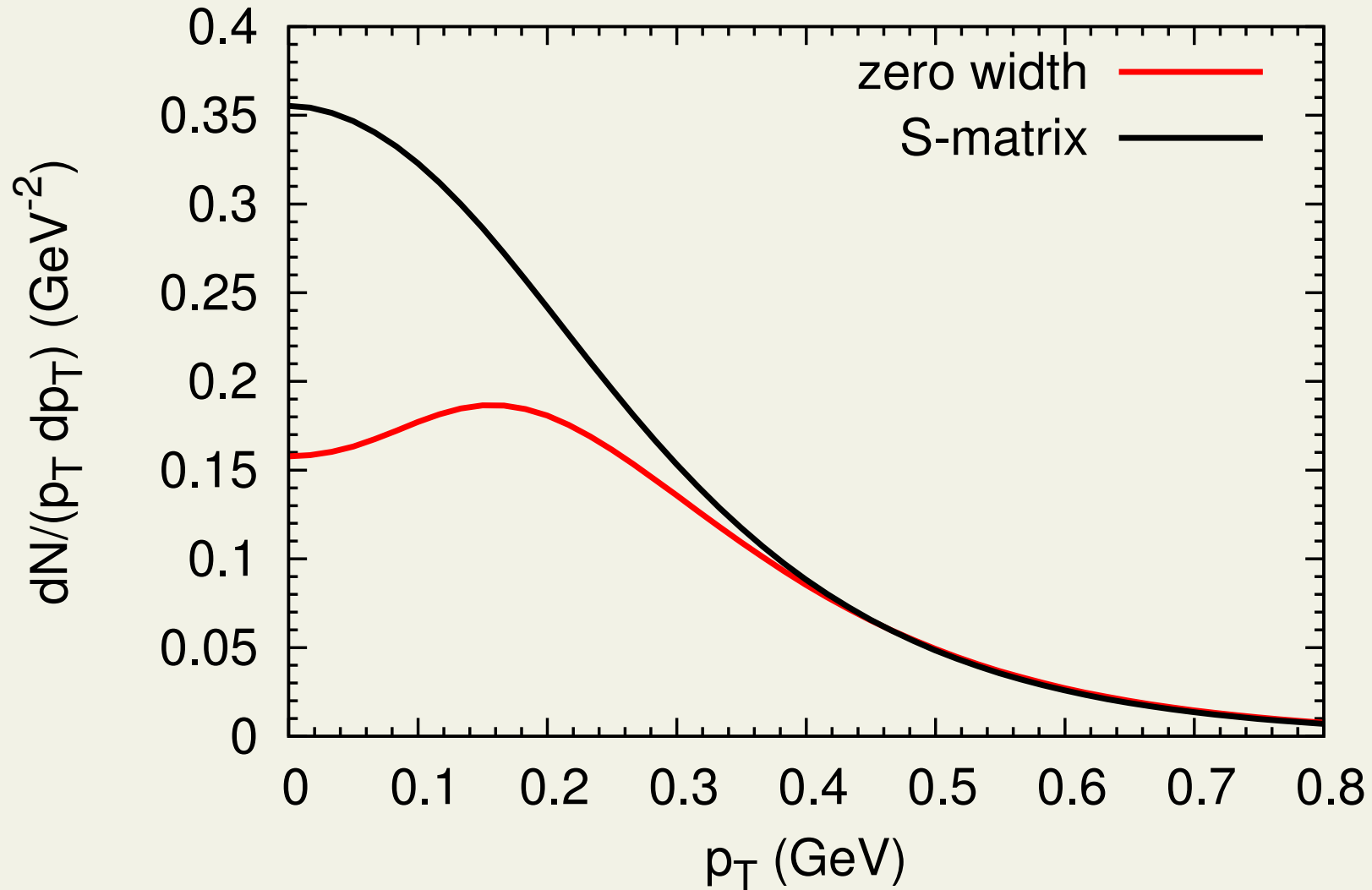
- $\pi\pi$  scattering, P-wave, i.e.  $\rho$  resonance



# $\rho$ -density



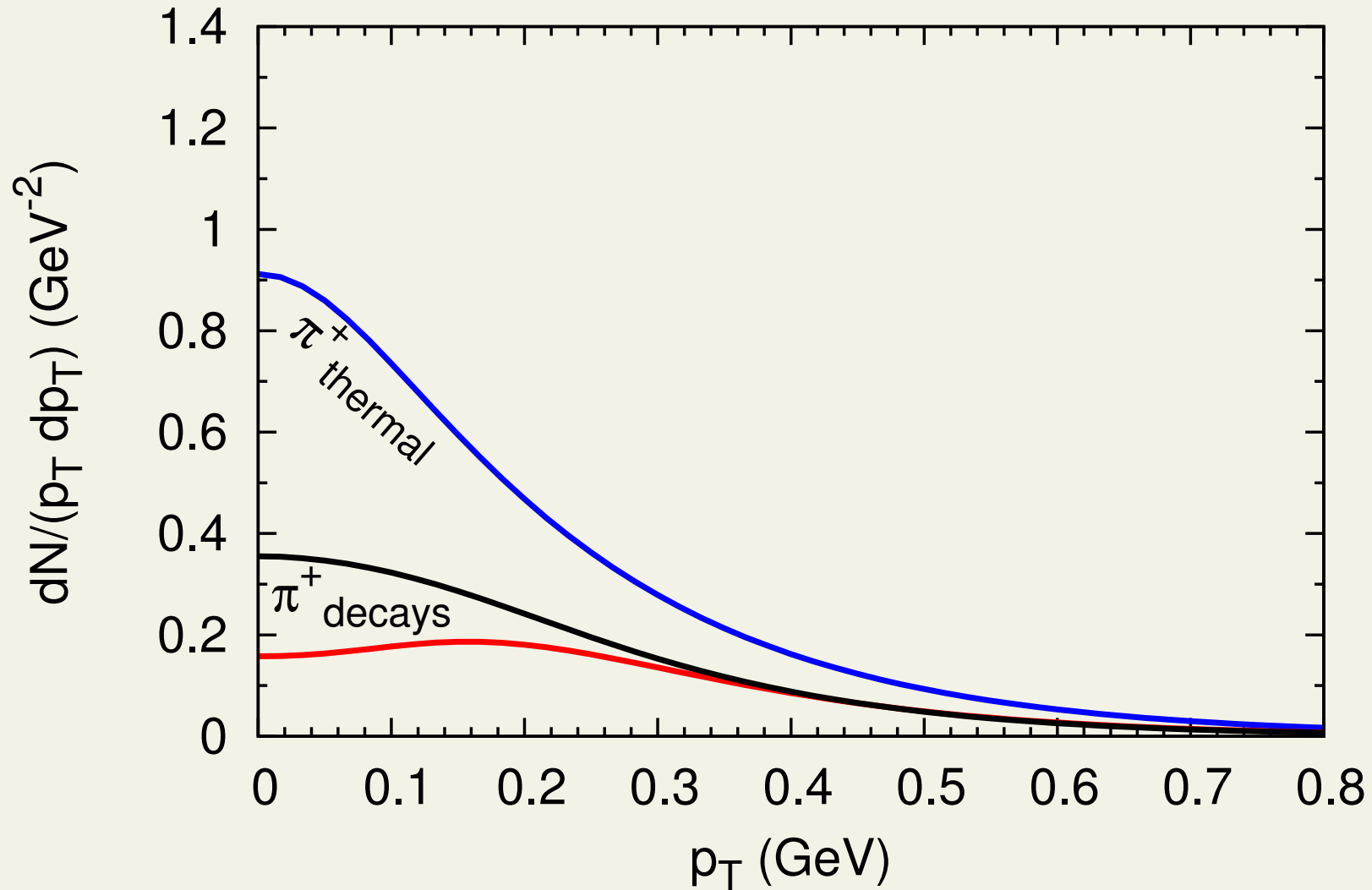
# Pions from $\rho$ decays



- static source,  $T = 155$  MeV

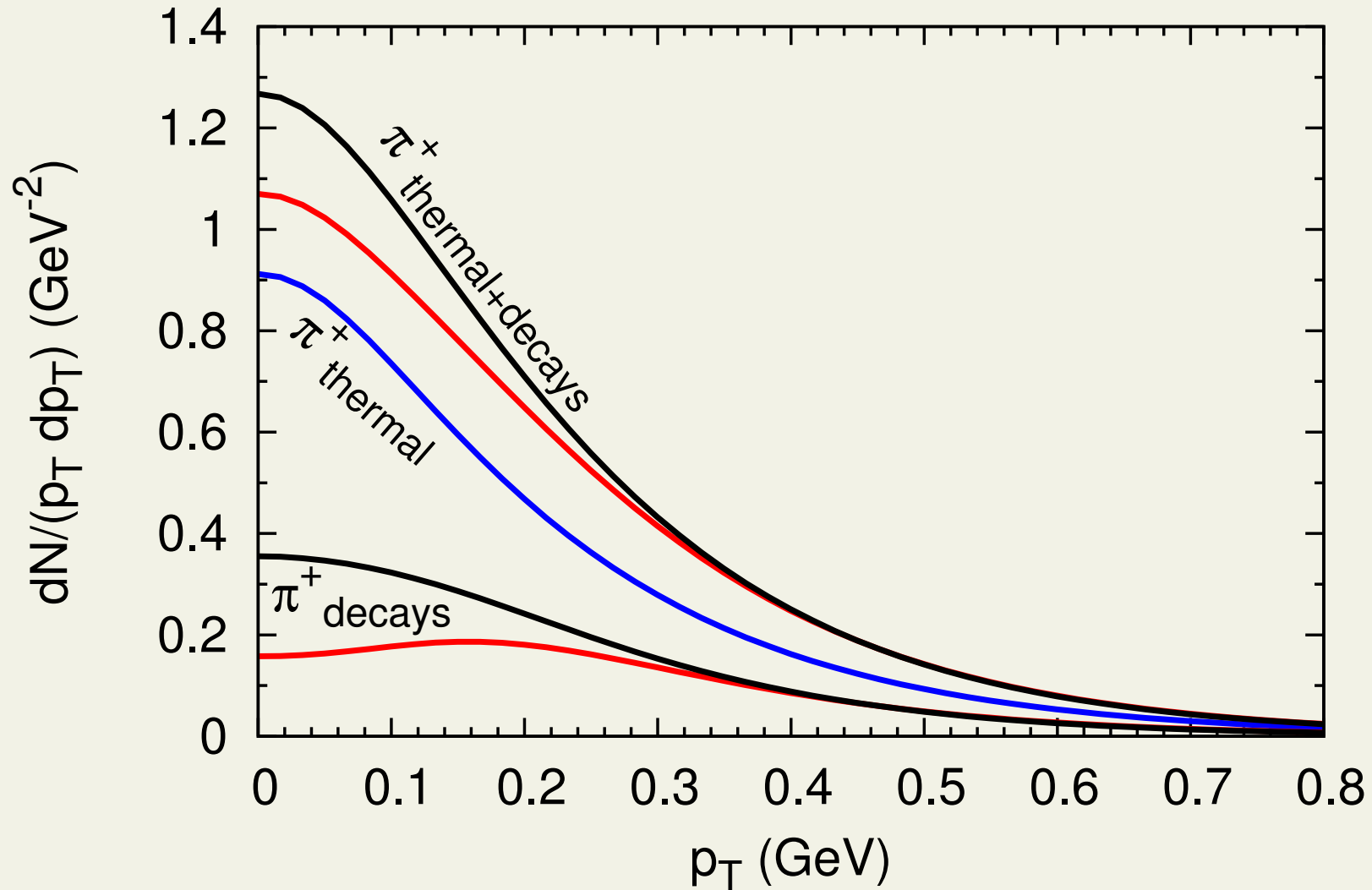


# Thermal pions + pions from $\rho$ decays



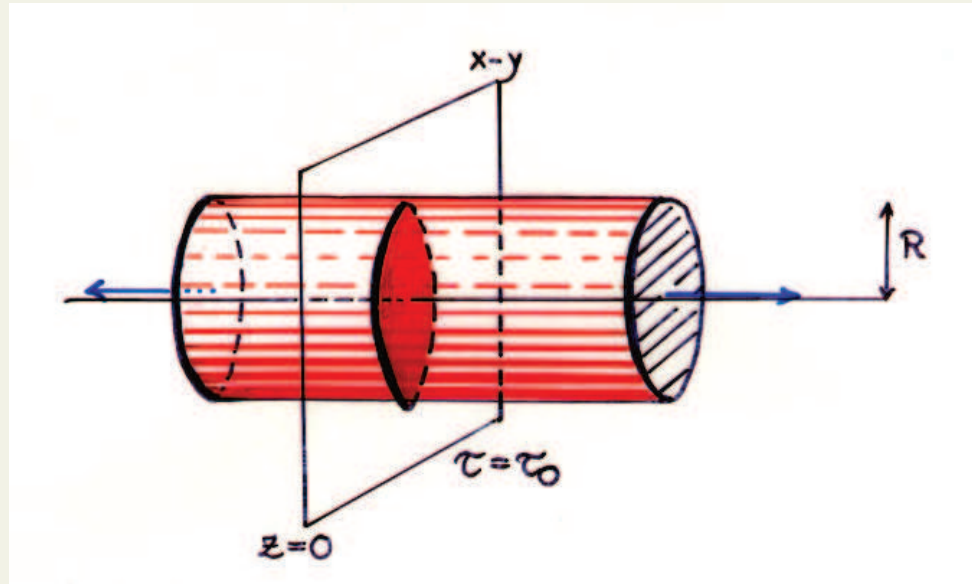
- **static source**,  $T = 155 \text{ MeV}$

# Thermal pions + pions from $\rho$ decays



- static source,  $T = 155$  MeV

# blast-wave parametrisation

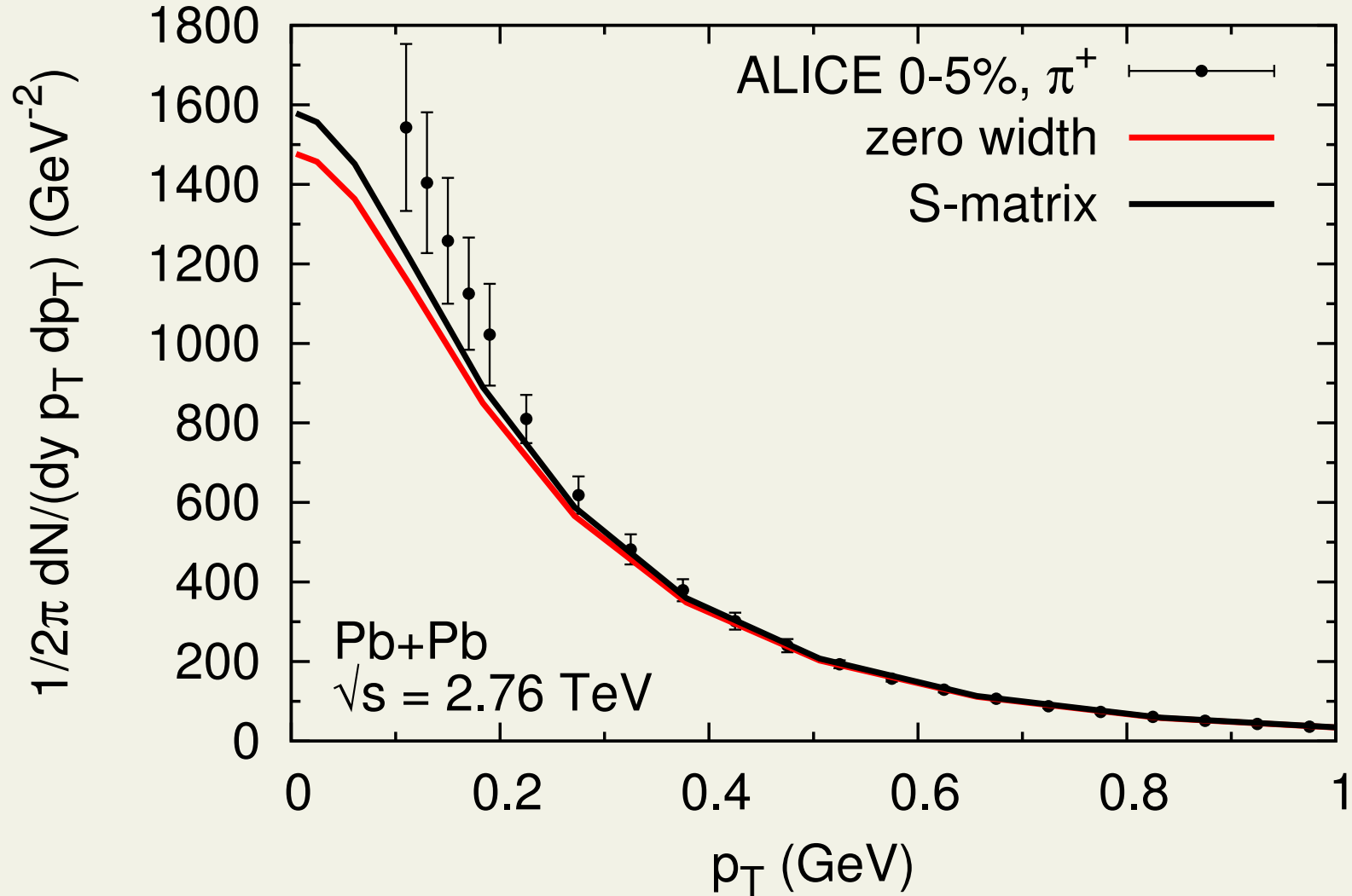


- boost invariant & cylindrically symmetric
- decoupling at constant  $\tau$ , i.e. volume emission
- transverse velocity  $v = v(r)$

$$E \frac{dN}{dp^3} = \frac{g\tau m_T}{2\pi^2} \int_0^R r dr \int_{m_{\text{th}}}^{\infty} dm \frac{d\rho}{dm} \sum_{n=1}^{\infty} (\mp 1)^{n+1} I_0 \left( n \frac{p_T \gamma_r(r) v_r(r)}{T} \right) K_1 \left( n \frac{m_T \gamma_r(r)}{T} \right)$$

$$\tau = 13.7 \text{ fm}, \quad R = 10 \text{ fm}, \quad v_{\text{max}} = 0.78$$

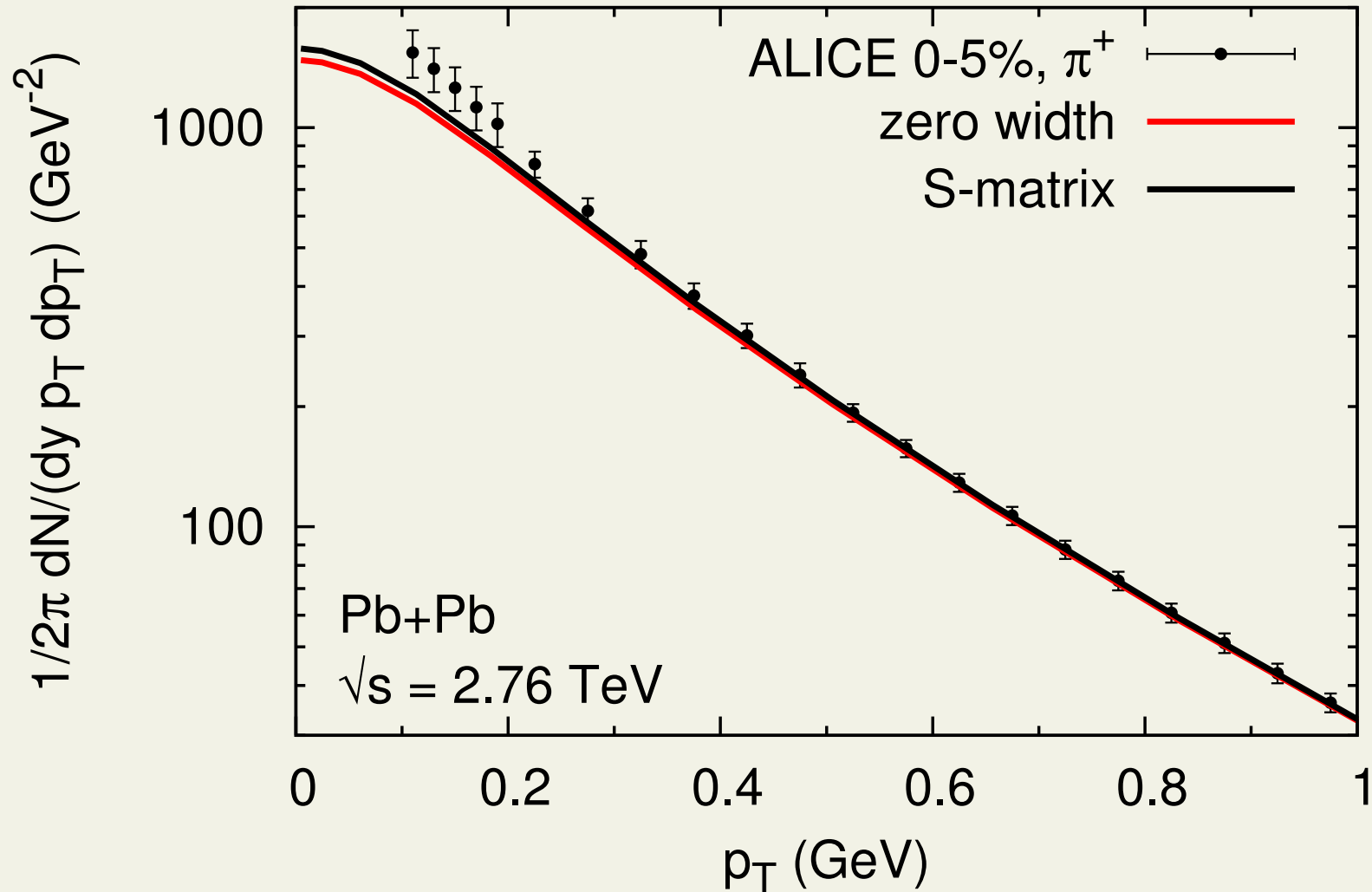
# Pions from blast wave



- $\tau = 14.1 \text{ fm}$
- $R = 10 \text{ fm}$
- $v_{max} = 0.8$

- all resonances up to 2 GeV
- Beth-Uhlenbeck for rhos
- zero width for everything else

# Pions from blast wave



- $\tau = 14.1 \text{ fm}$
- $R = 10 \text{ fm}$
- $v_{max} = 0.8$

- all resonances up to 2 GeV
- Beth-Uhlenbeck for rhos
- zero width for everything else

# caveats

- so far only rho mesons

# caveats

- so far only rho mesons
- Beth-Uhlenbeck applicable to **elastic scatterings only!**

# caveats

- so far only rho mesons
- Beth-Uhlenbeck applicable to **elastic scatterings only!**
- $\rho$ ,  $K^*(892)$ ,  $f_0(980)$ ,  $\Delta(1232)$ ,  $K_0^*(1430)$ 
  - data exists



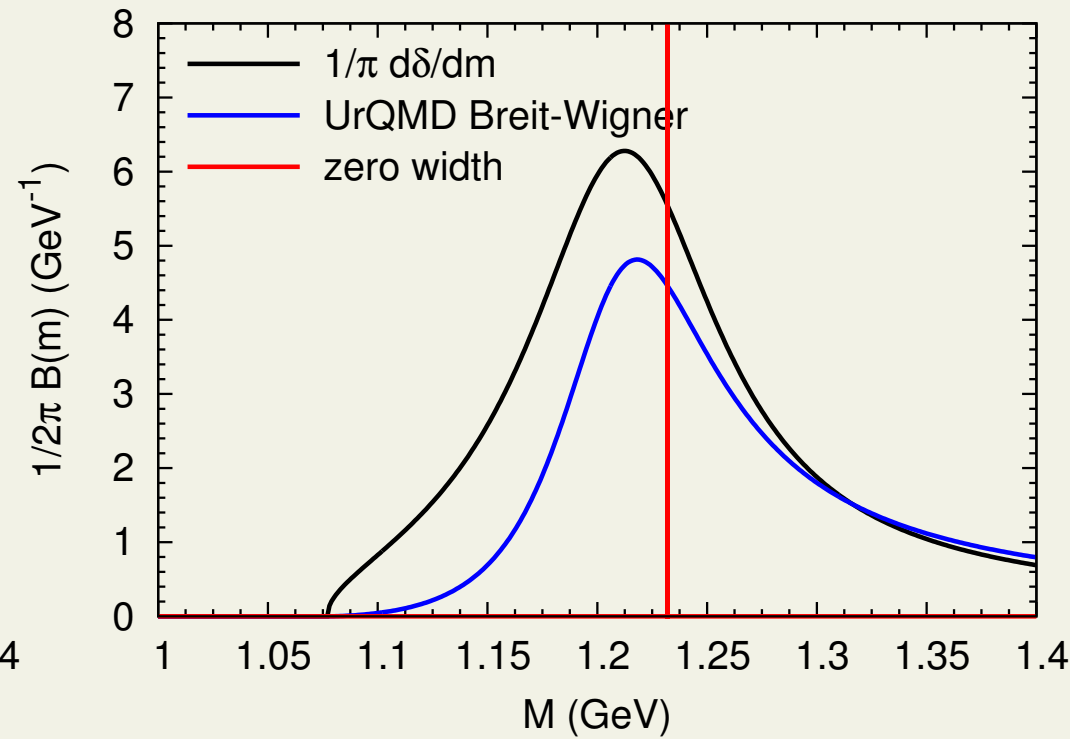
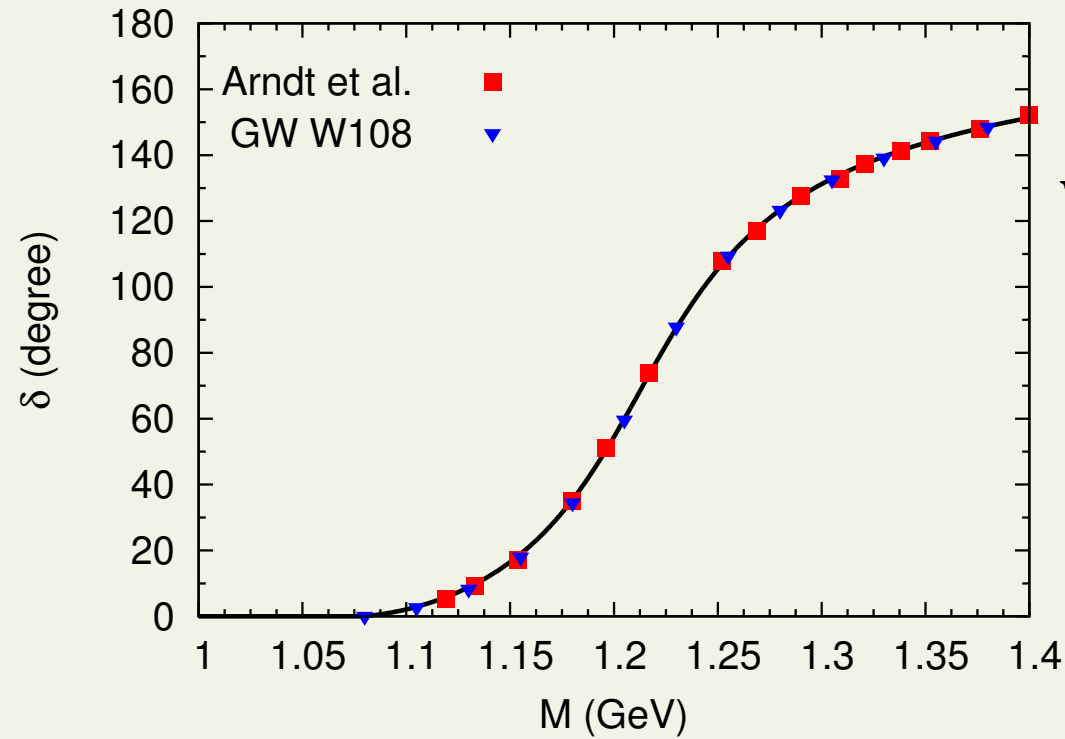
# caveats

- so far only rho mesons
- Beth-Uhlenbeck applicable to **elastic scatterings only!**
- $\rho$ ,  $K^*(892)$ ,  $f_0(980)$ ,  $\Delta(1232)$ ,  $K_0^*(1430)$ 
  - data exists
- $\Lambda(1405)$ ,  $\Xi(1530)$  applicable
  - no data

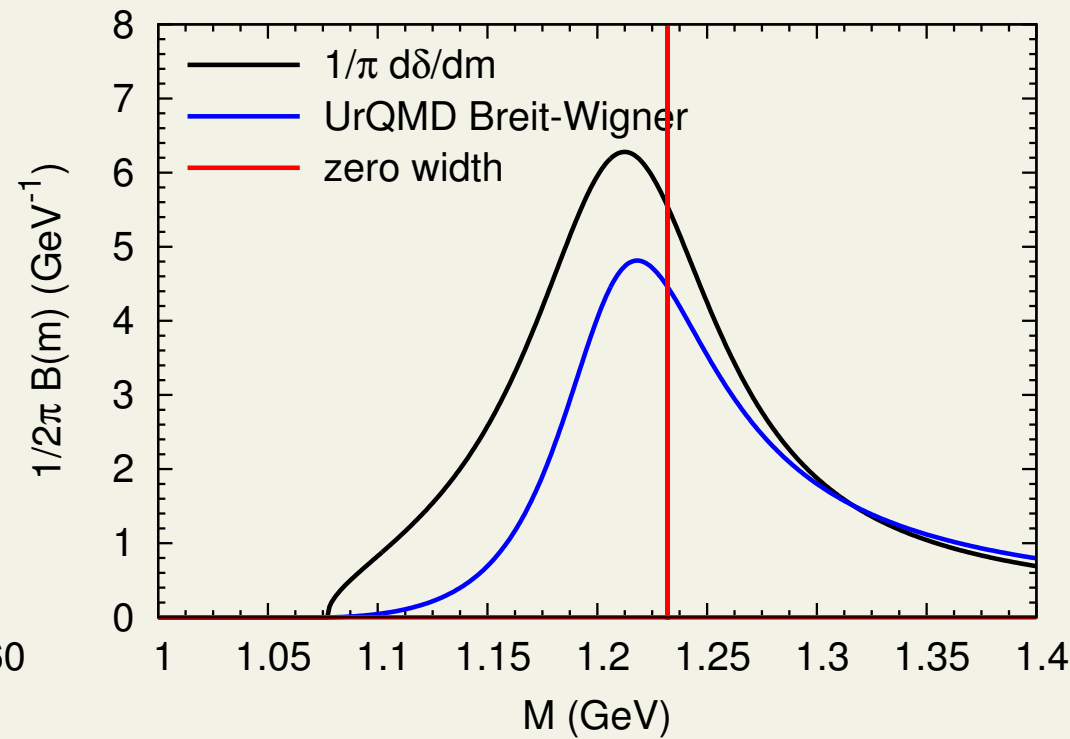
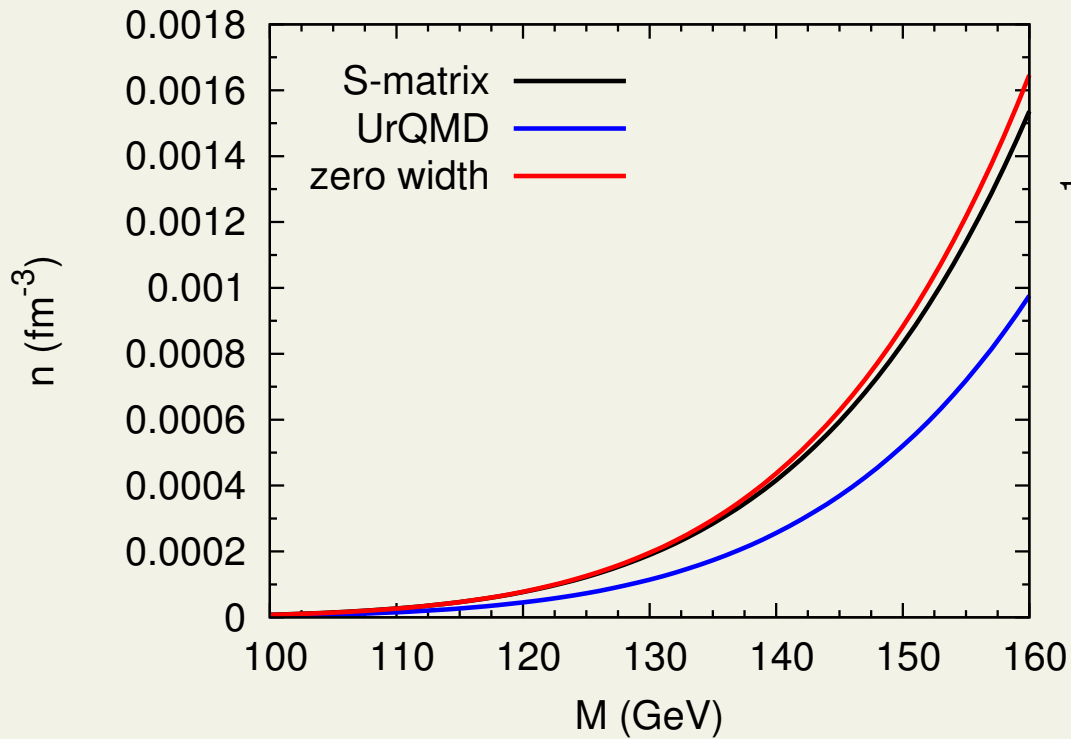
# caveats

- so far only rho mesons
- Beth-Uhlenbeck applicable to **elastic scatterings only!**
- $\rho$ ,  $K^*(892)$ ,  $f_0(980)$ ,  $\Delta(1232)$ ,  $K_0^*(1430)$ 
  - data exists
- $\Lambda(1405)$ ,  $\Xi(1530)$  applicable
  - no data
- **and everything else?**

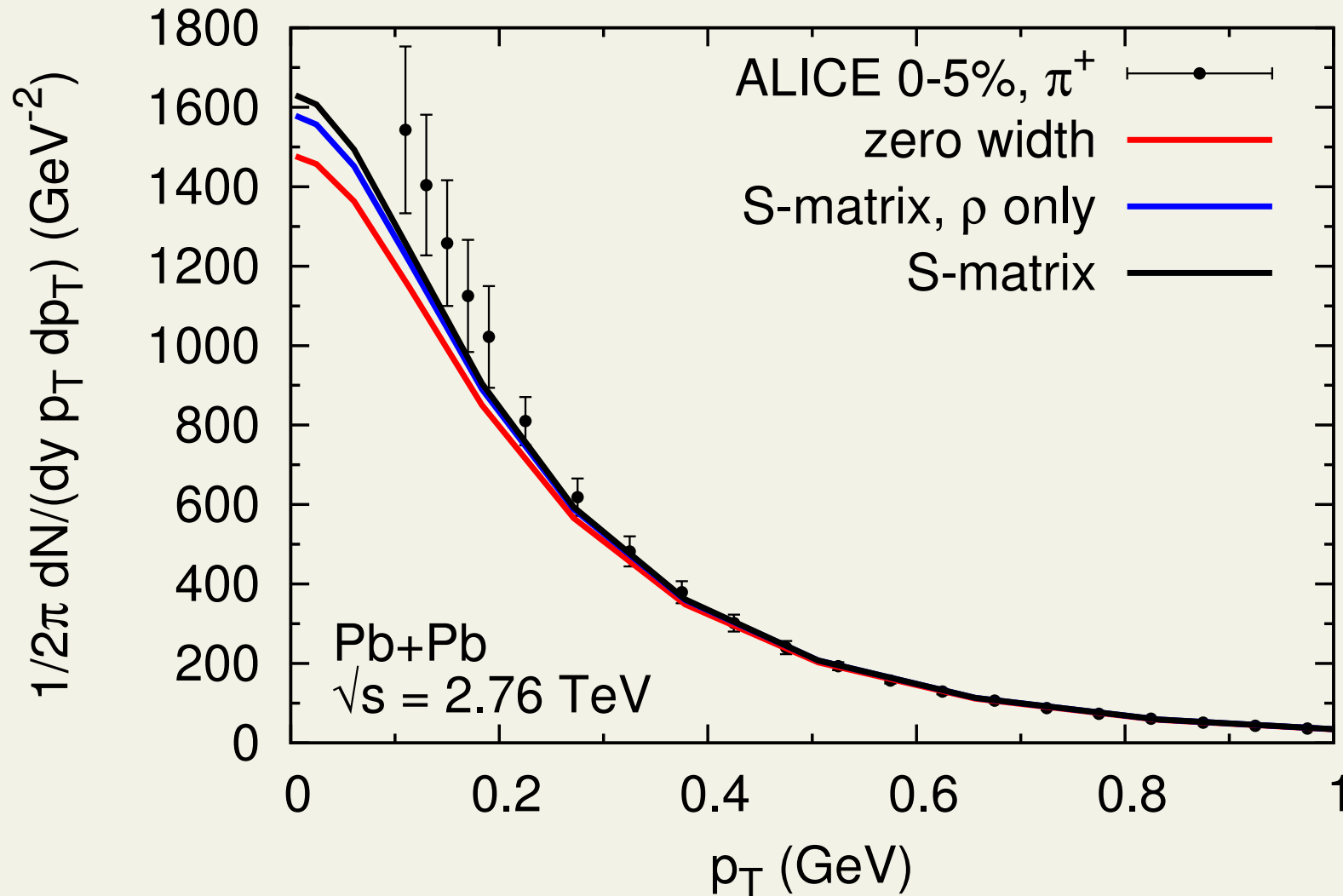
# $P_{33}$ $\pi N$ scattering, a.k.a. $\Delta$



# $P_{33}$ $\pi N$ scattering, a.k.a. $\Delta$



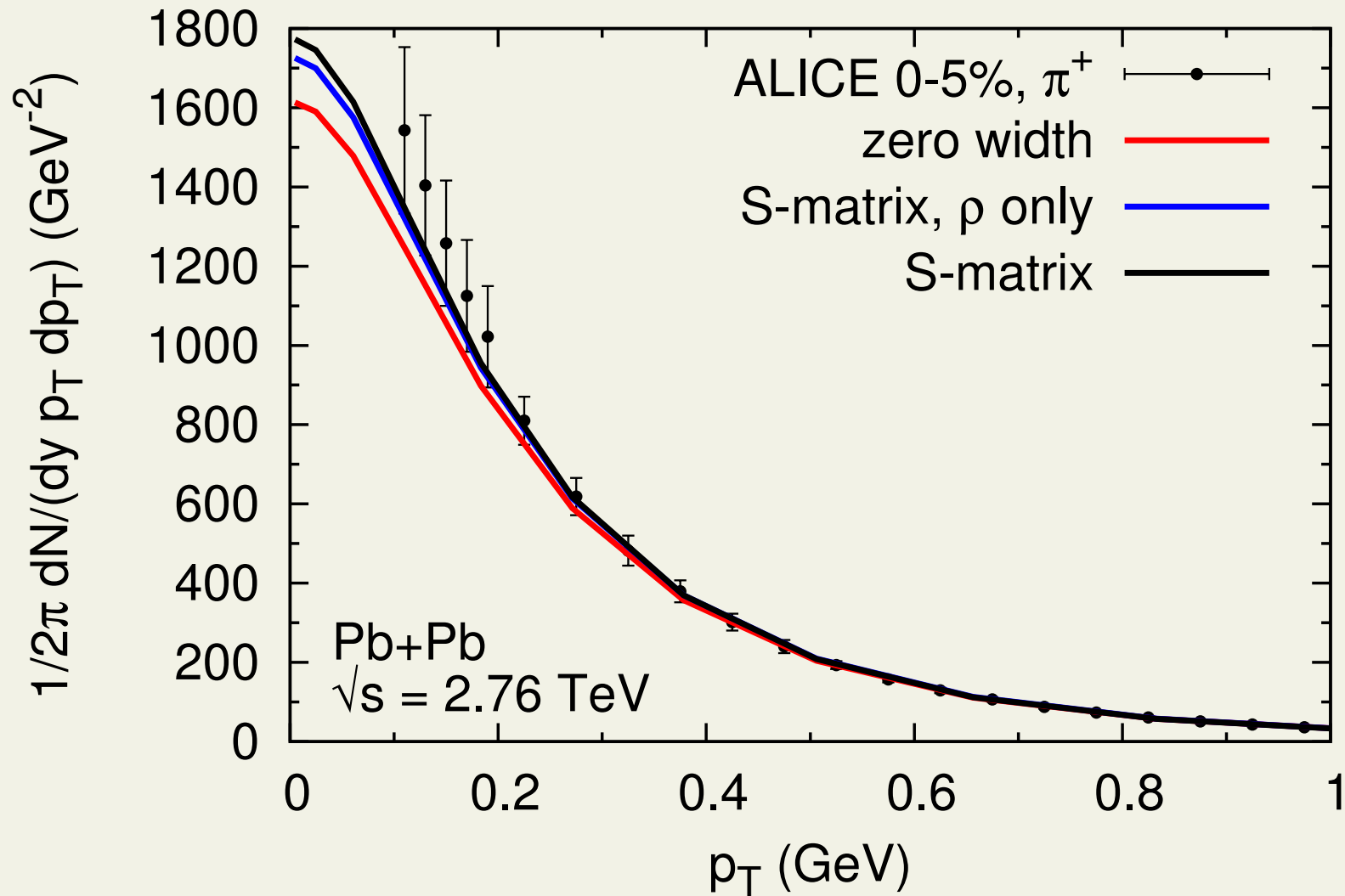
# Pions from blast wave, $T = 150$ MeV



- $\tau = 14.1$  fm
- $R = 10$  fm
- $v_{max} = 0.8$

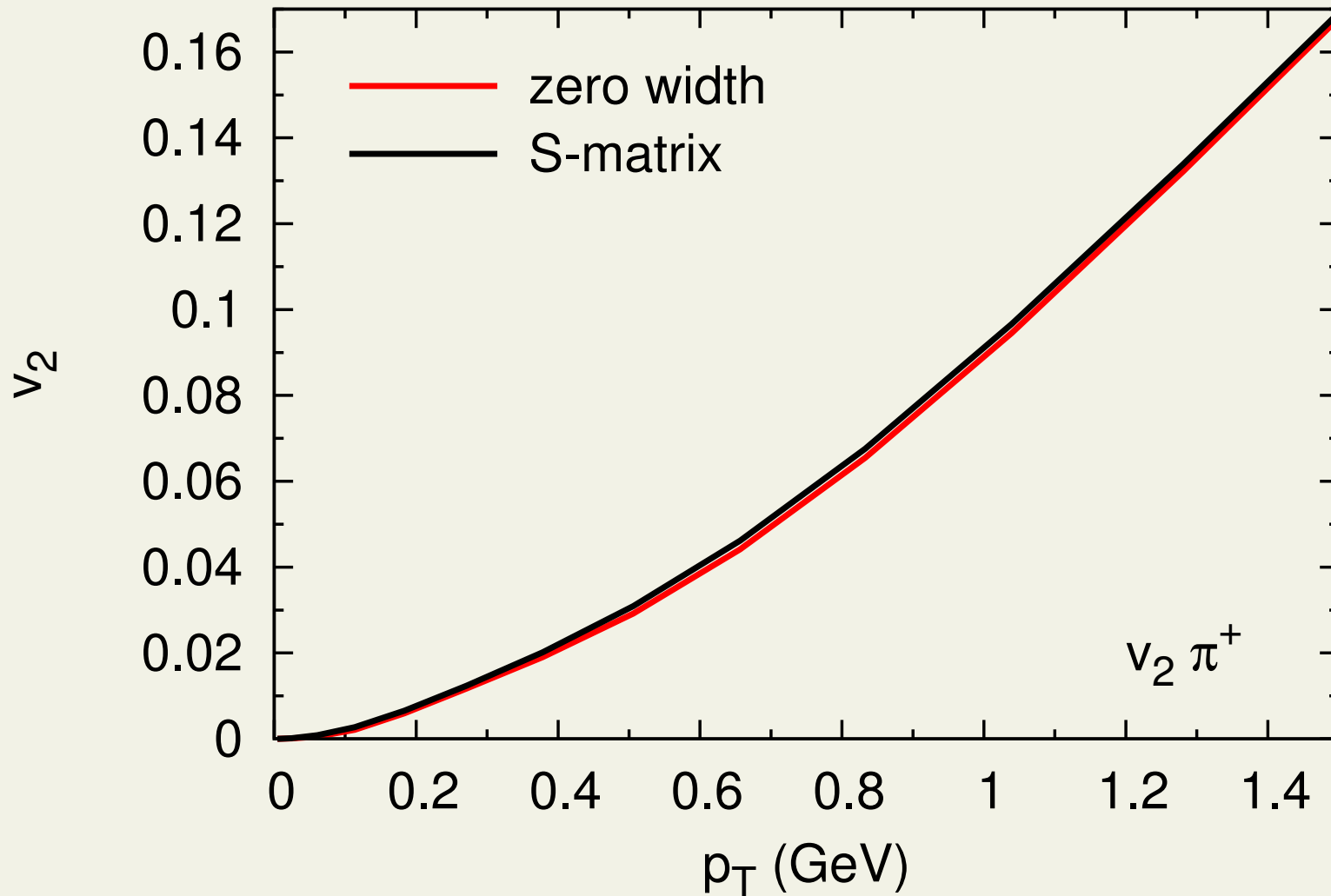
- all resonances up to 2 GeV
- Beth-Uhlenbeck for  $\rho$ ,  $\Delta$ ,  $f_0(980)$ ,  $K^*(892)$ ,  $K_0^*(1430)$
- zero width for everything else

# Pions from blast wave, $T = 120$ , $T_{\text{chem}} = 150$ MeV



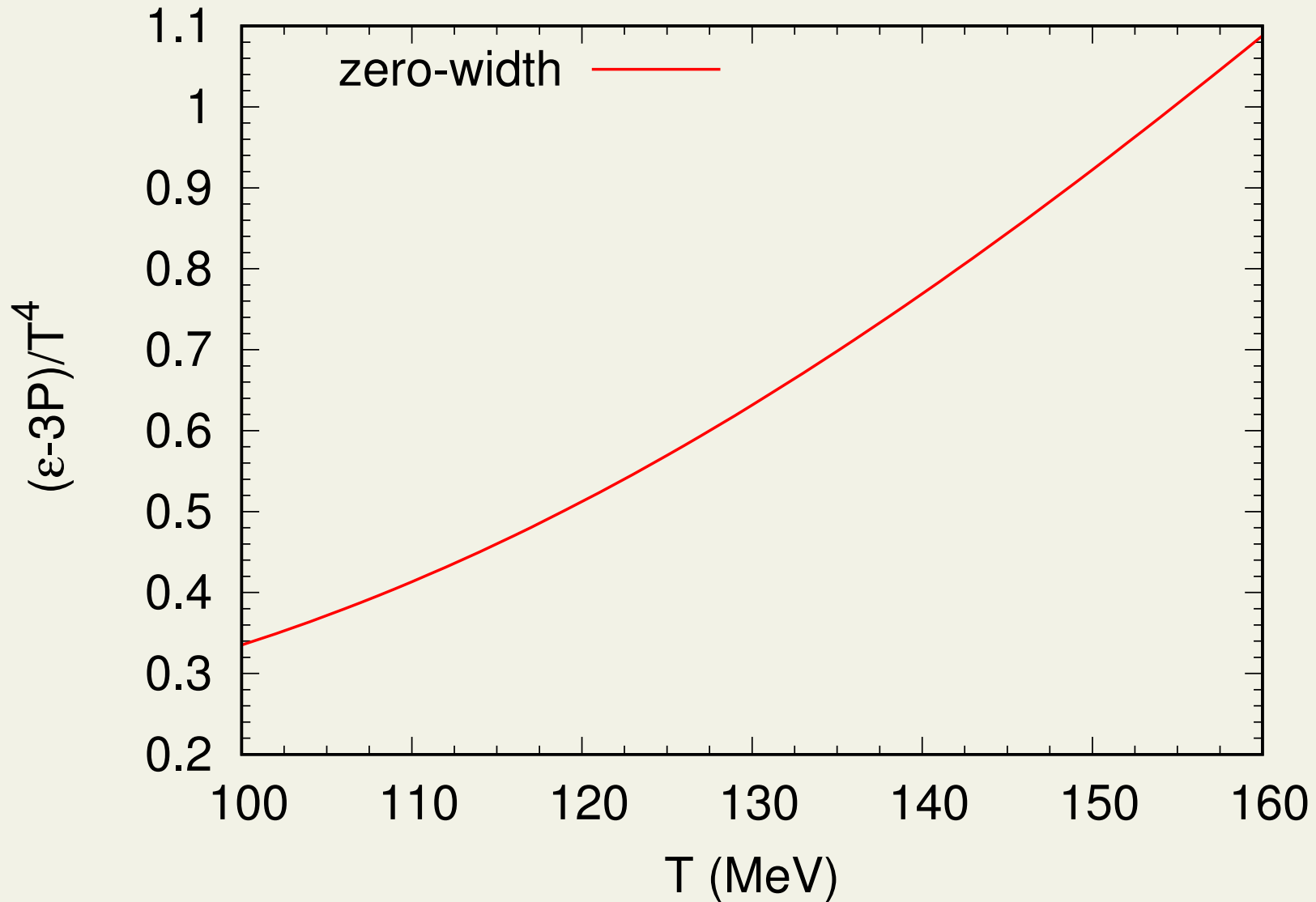
- $\tau = 31.0$  fm
- $R = 10$  fm
- $v_{max} = 0.87$
- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for  $\rho$ ,  $\Delta$ ,  $f_0(980)$ ,  $K^*(892)$ ,  $K_0^*(1430)$**
- **zero width for everything else**

# $v_2$ of pions from blast wave, $T = 120$ , $T_{\text{chem}} = 150$ MeV



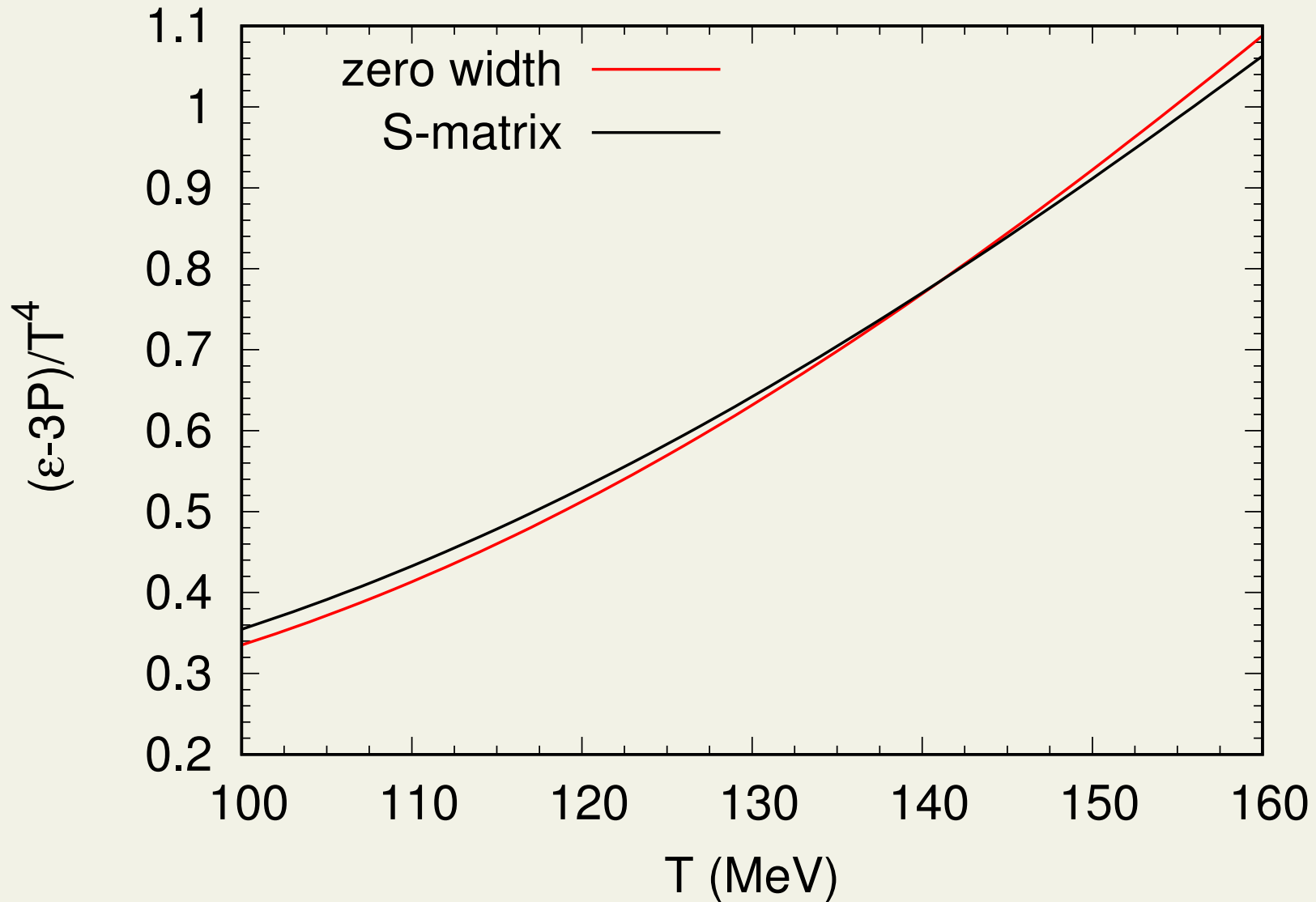
- $\tau = 31.0$  fm
- $R = 10$  fm
- $v_{max} = 0.87$
- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for  $\rho$ ,  $\Delta$ ,  $f_0(980)$ ,  $K^*(892)$ ,  $K_0^*(1430)$**
- **zero width for everything else**

$\pi, K, N, \rho, f_0(980), K^*, K_0(1430), \Delta$

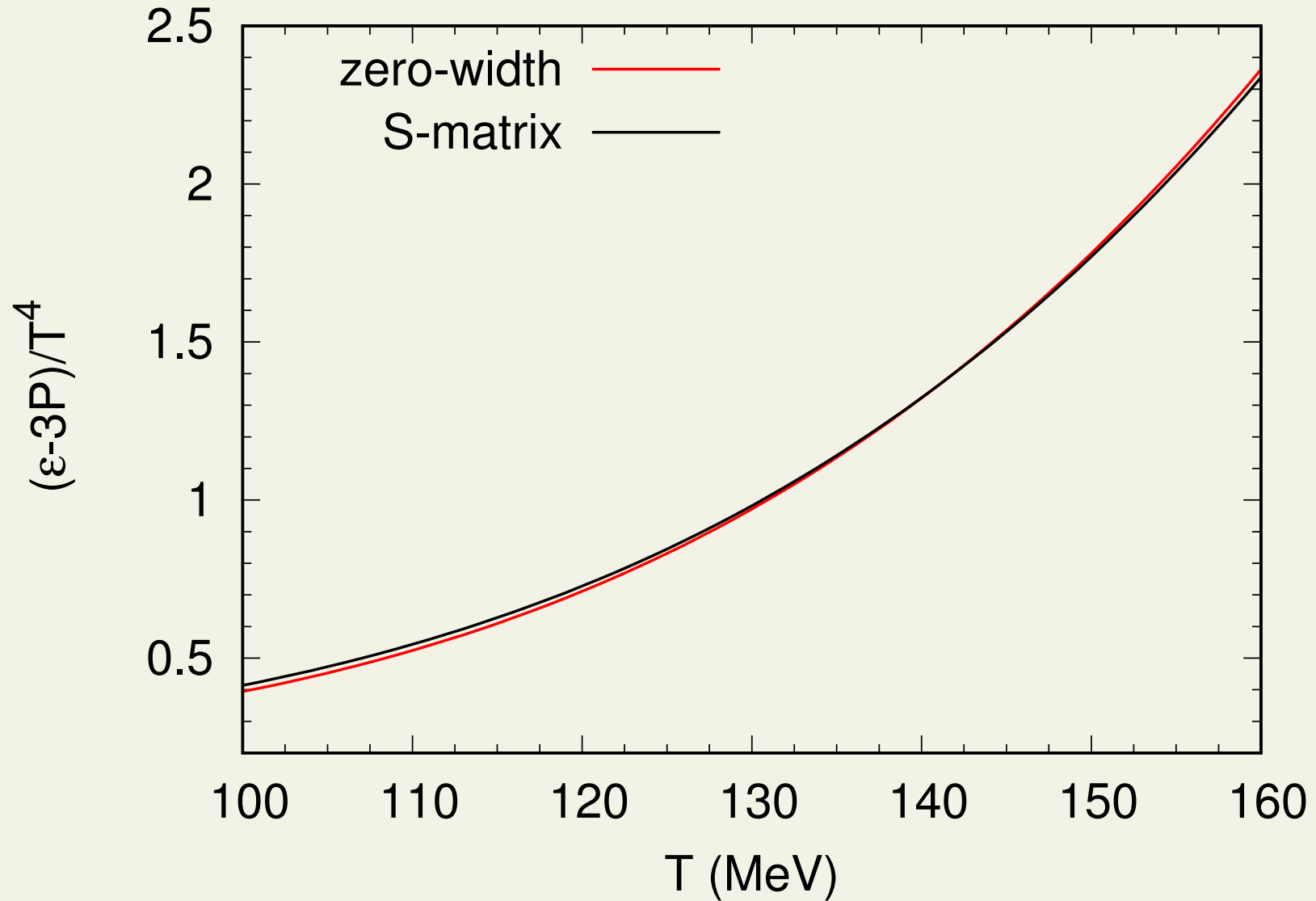




$\pi, K, N, \rho, f_0(980), K^*, K_0(1430), \Delta$



# the whole zoo



# Summary of part II

- Resonance widths change the low- $p_T$  distribution of pions

# Summary of part II


- Resonance widths change the low- $p_T$  distribution of pions
  - **Fortunately**  $v_2(p_T)$  is not affected

# Summary of part II

- Resonance widths change the low- $p_T$  distribution of pions
  - **Fortunately**  $v_2(p_T)$  is not affected
- Effect on EoS uncertain

# Summary of part II

- Resonance widths change the low- $p_T$  distribution of pions
  - **Fortunately**  $v_2(p_T)$  is not affected
- Effect on EoS uncertain
- Better treatment of resonances needed

 This talk consisted of 100% recycled electrons