Cooling, Masses and Emulators

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INT, Washington

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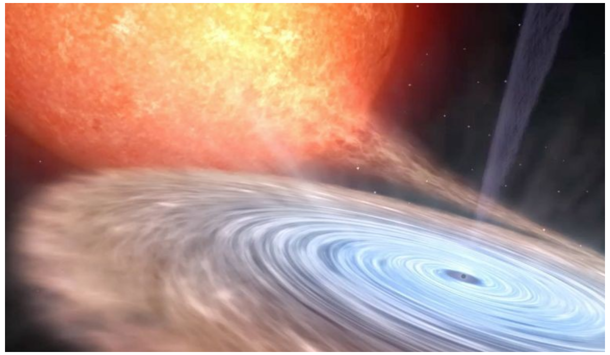
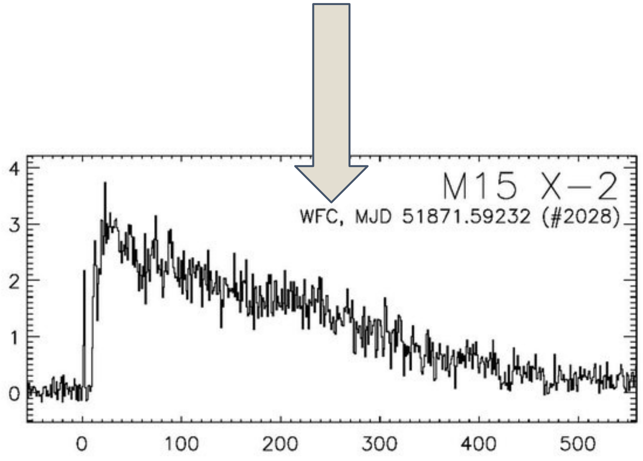
Accreting Neutron Stars Observables

Accretion Outburst





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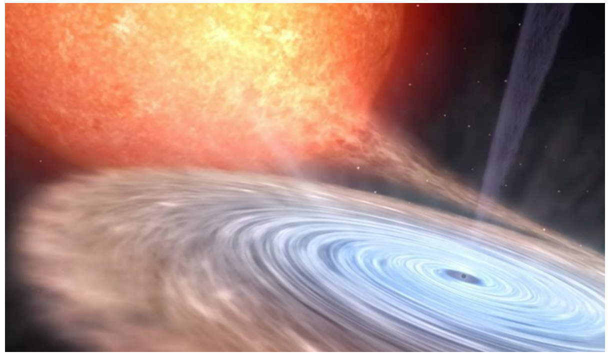
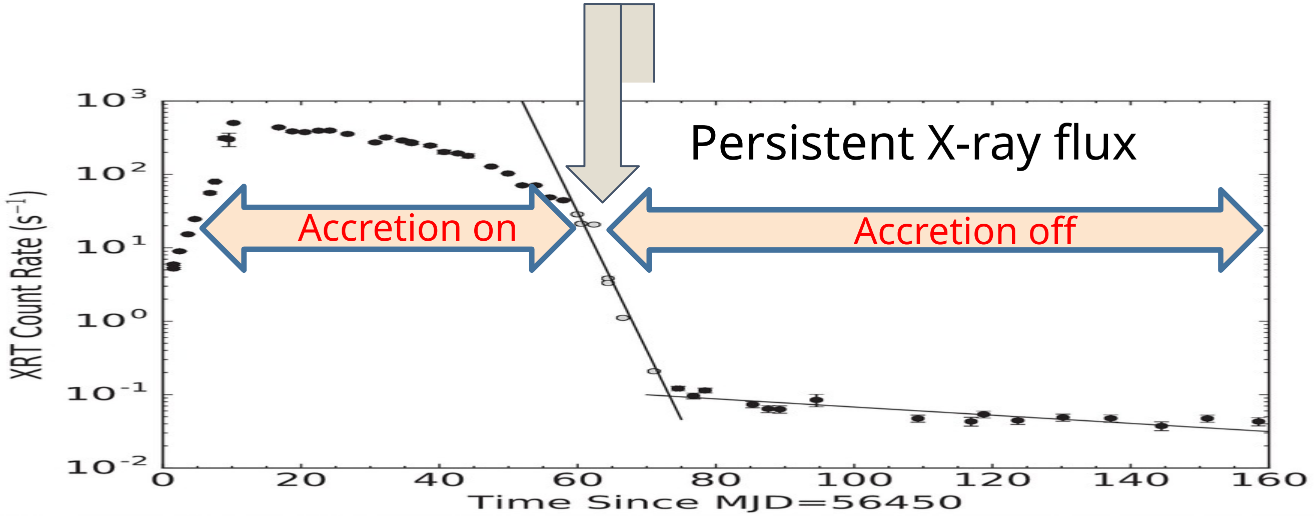
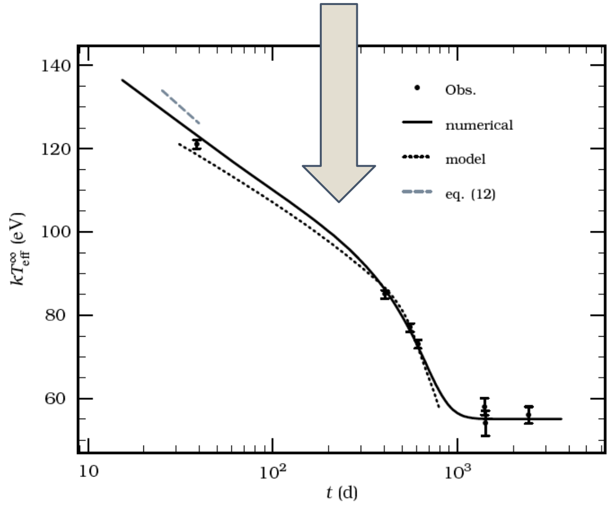




Quiescence



Brown and Cumming, ApJ (2009)

Accreting Neutron Stars Observables

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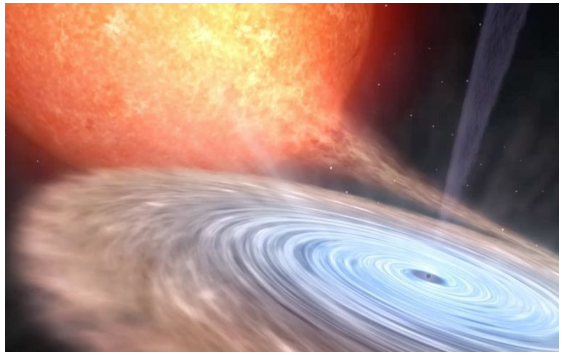

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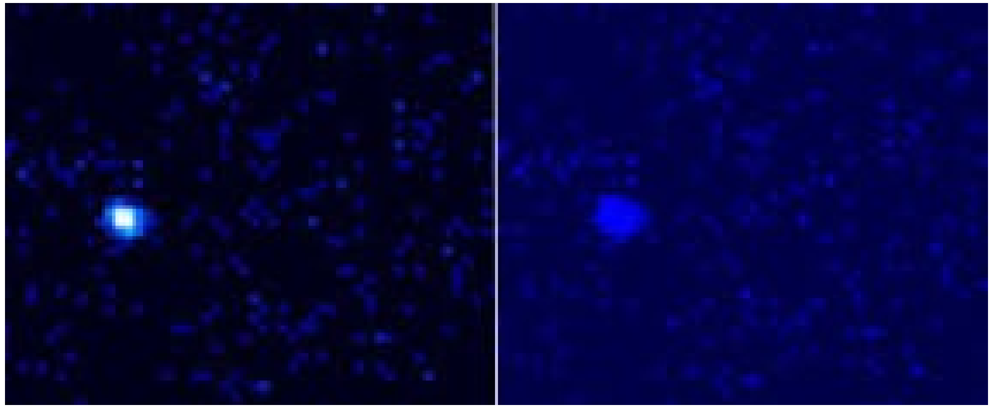

KS 1731-260
observed with
Chandra X-ray Satellite

Impact of Mass Models

Mass Model	Urca Pair
BCPM	$^{59}Ca \leftrightarrow {}^{59}Sc$

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FRDM-2012	$^{59}Sc \leftrightarrow {}^{59}Ti$
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HFB-24	^{57}Sc ↔ ^{57}Ti
UNEDF1	^{57}Sc ↔ ^{57}Ti
UNEDF2	^{57}Sc ↔ ^{57}Ti

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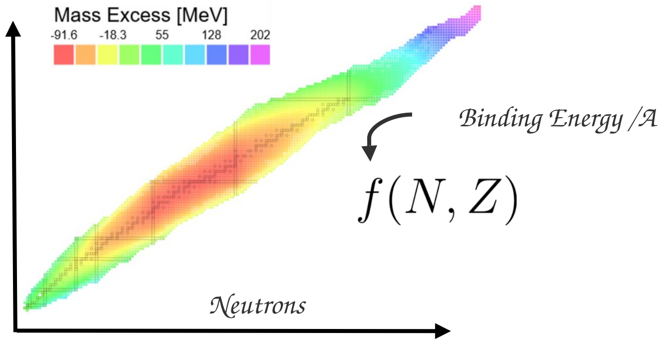
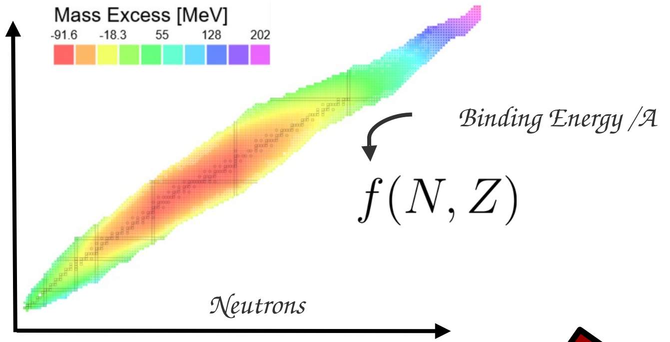
SkM*	$^{57}Ca \leftrightarrow {}^{57}Sc$
SkP	$^{57}Ca \leftrightarrow {}^{57}Sc$
SLy4	$^{57}Ca \leftrightarrow {}^{57}Sc$
SV-Min	$^{57}Ca \leftrightarrow {}^{57}Sc$
UNEDF0	$^{57}Ca \leftrightarrow {}^{57}Sc$

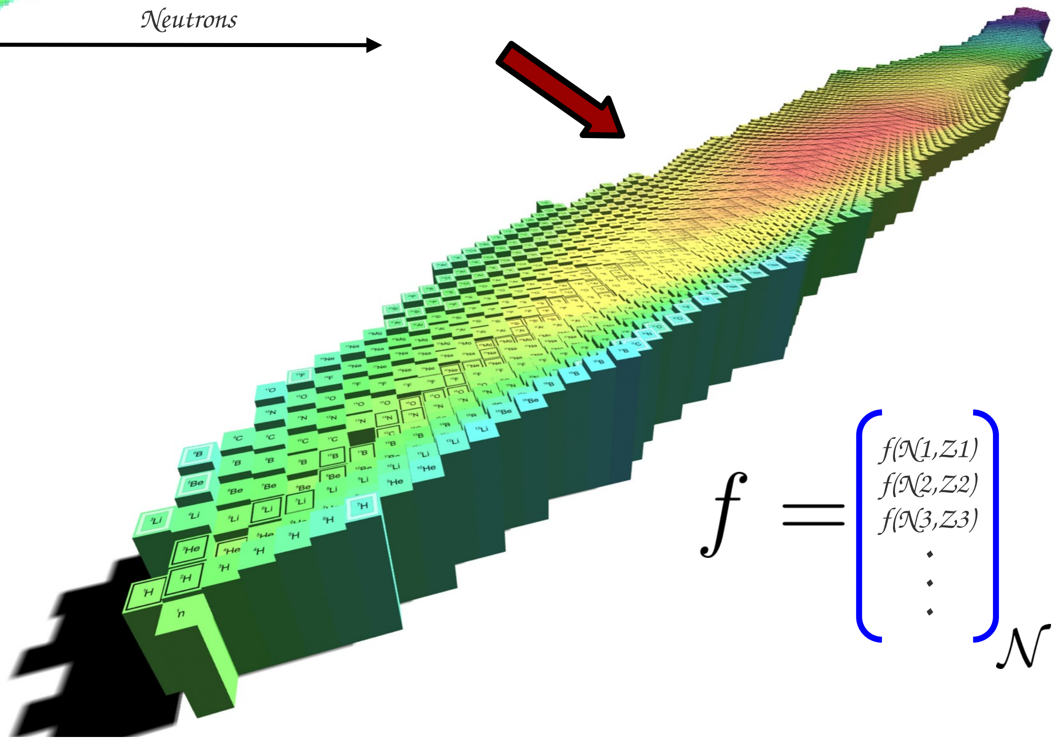
Bayesian Model Averaging – A Brief Introduction



Courtesy: P. Guiliani

Bayesian Model Averaging





$$f = \begin{pmatrix} f(N1,Z1) \\ f(N2,Z2) \\ f(N3,Z3) \\ \vdots \end{pmatrix}_{\mathcal{N}}$$

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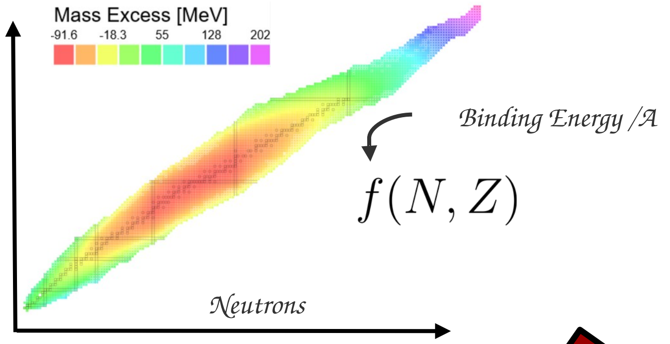

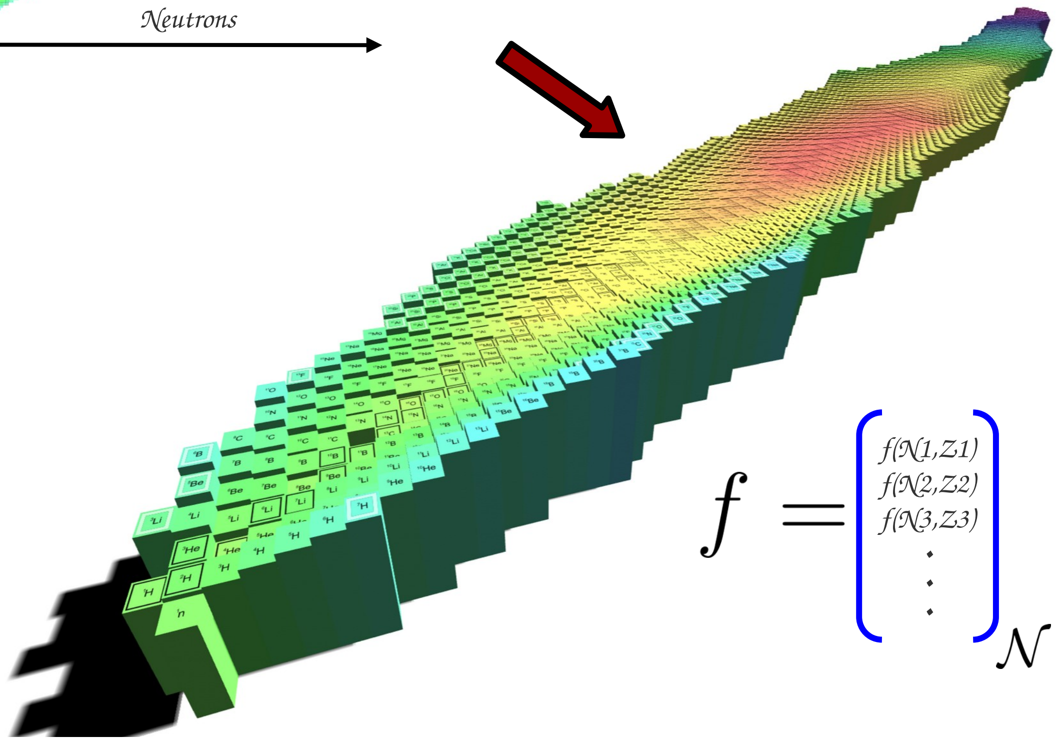

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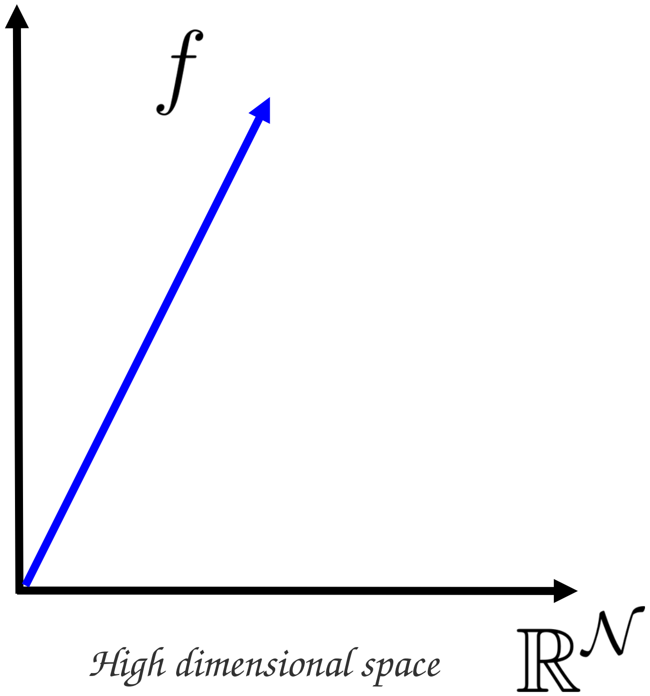

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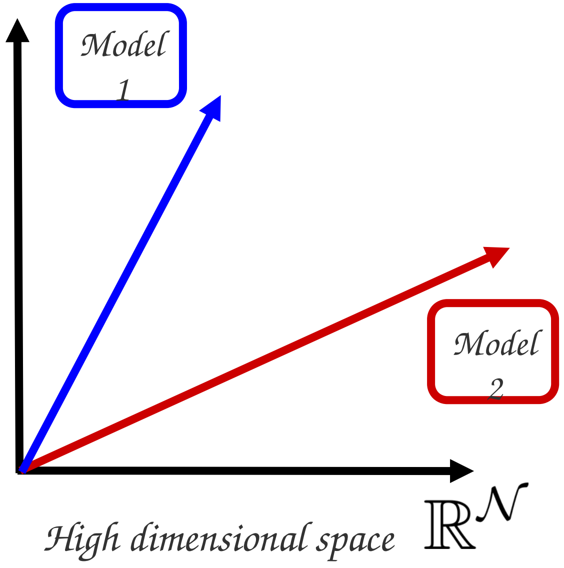

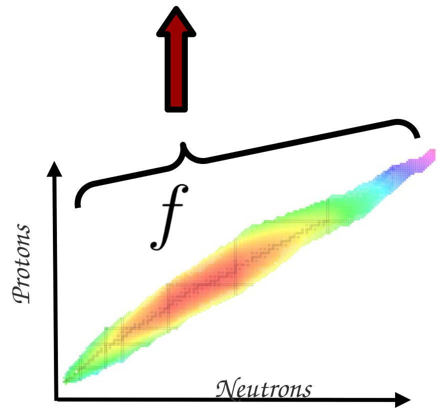

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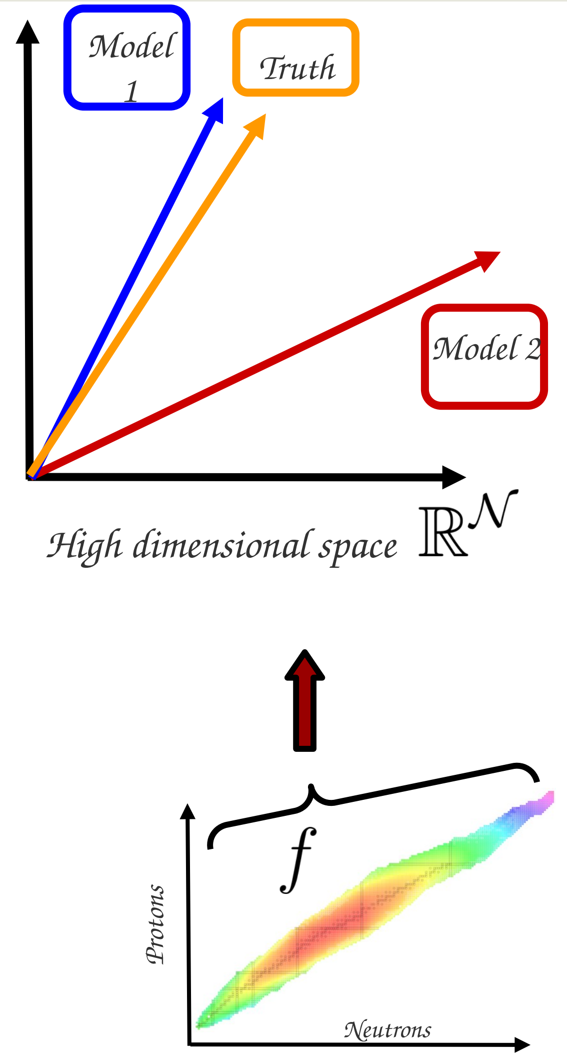
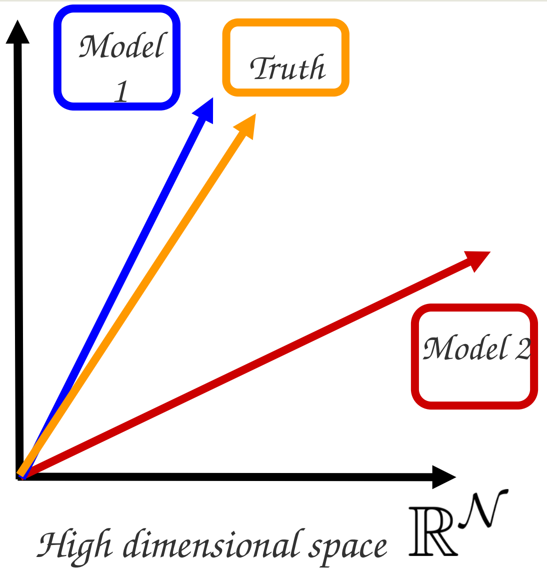
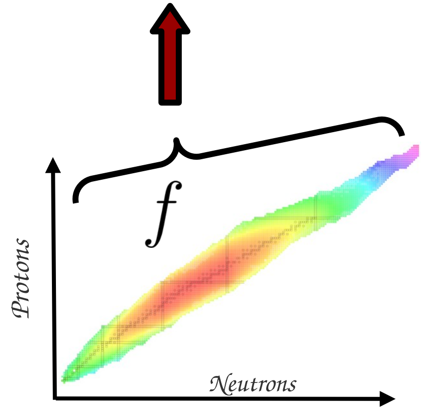


Bayesian Model Averaging

$$p(y(x^*)|\mathbf{y}) = \sum_{k=1}^{p} p(y(x^*)|\mathbf{y}, \mathcal{M}_k)\, p(\mathcal{M}_k|\mathbf{y})$$

$$p(\mathcal{M}_k|\mathbf{y}) = \frac{p(\mathbf{y}|\mathcal{M}_k)\pi(\mathcal{M}_k)}{\sum_{\ell=1}^{p} p(\mathbf{y}|\mathcal{M}_\ell)\pi(\mathcal{M}_\ell)}$$





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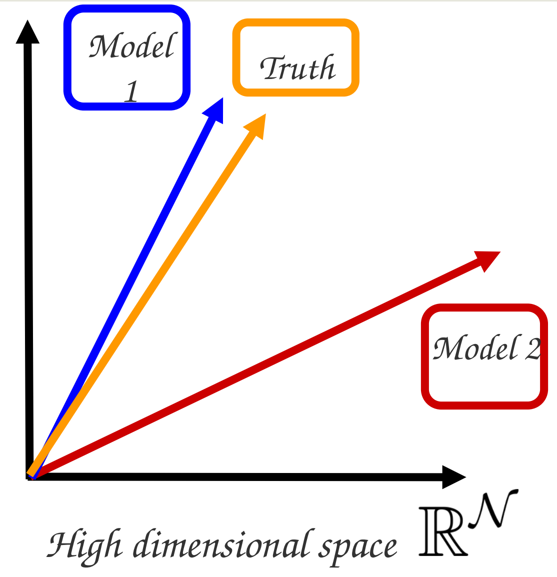
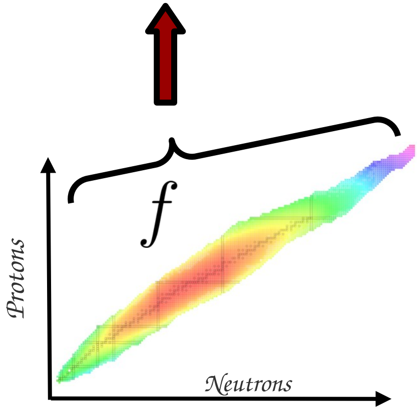


Great job!

*



*Warning: Collinearity of models affects the BMA

Modeling Residuals

Modeling Residuals

$$y_i \;=\; y^{exp}(x_i) - y^{th}(x_i)$$

$$y_i = f(x_i, \theta) + \sigma\epsilon_i$$

Formulation of a GP

$$f(x, \theta) \sim \mathcal{GP}(\mu, k_{\eta,\rho}(x, x'))$$

$$k_{\eta,\rho}(x, x') \;=\; \eta^2 e^{\sum_i -\frac{(x_i - x_i')^2}{\rho_{x_i}}}$$

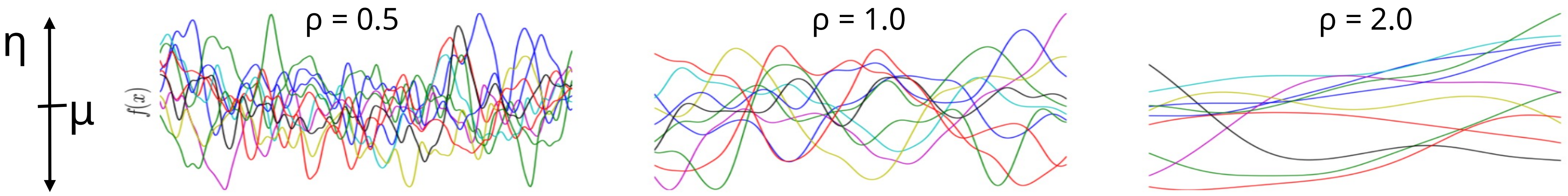
Input features:

$x_i := \{Z, N, p, \boldsymbol{\delta}\}$

p is the promiscuity (shell effects) and $\boldsymbol{\delta}$ is the pairing factor

Model Parameters:

$\theta := \{\mu, \eta, \rho_i\}$



R. Jain et al., (to be submitted to PRC)

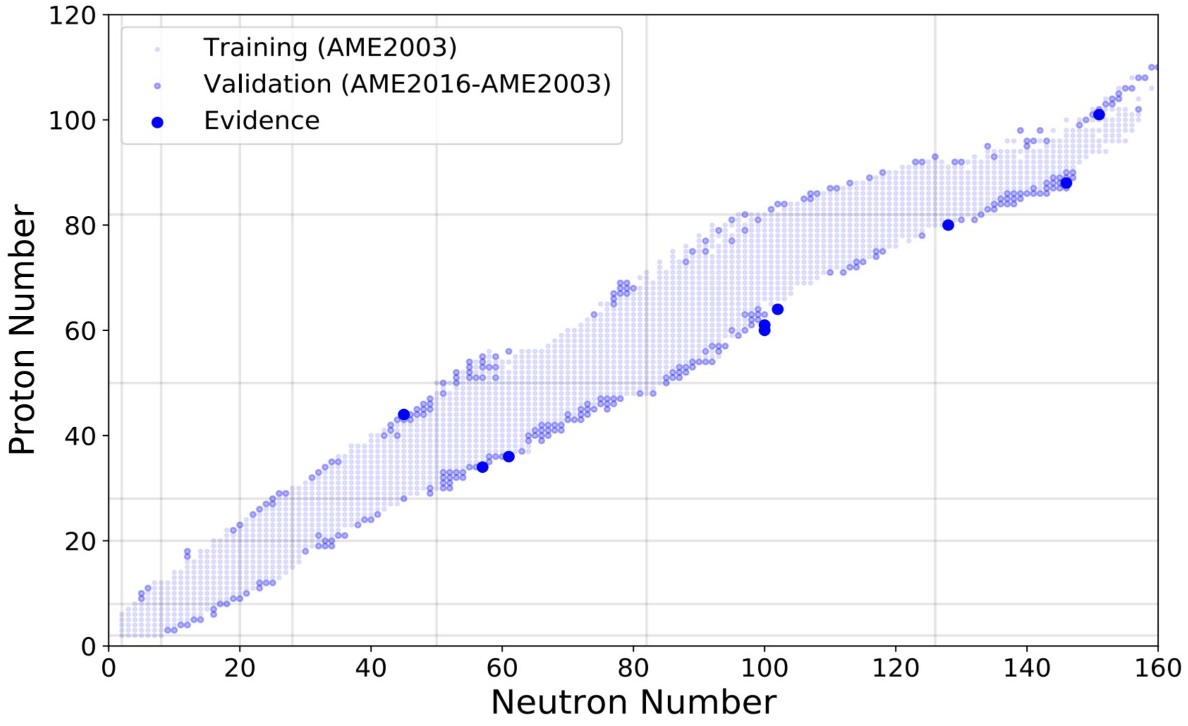
Bayesian Model Averaging

Normalized Weights

$$w_k = p(\mathcal{M}_k|y^*) = \frac{p(y^*|\mathcal{M}_k)\pi(\mathcal{M}_k)}{\sum_{\ell=1}^{11} p(y|\mathcal{M}_\ell)\pi(\mathcal{M}_\ell)}$$

Evidence Integrals

$$p(y|\mathcal{M}_k) = \int p(y|\theta_k, \mathcal{M}_k)\pi(\theta_k, \mathcal{M}_k)d\theta_k$$



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Final Predictions

$$y(x) = \sum_k w_k y^{(k)}(x)$$

$$\sigma_y^2(x) = \sum_k w_k(y^{(k)}(x) - y(x))^2 + \sum_k w_k \sigma_{y_k}^2(x)$$

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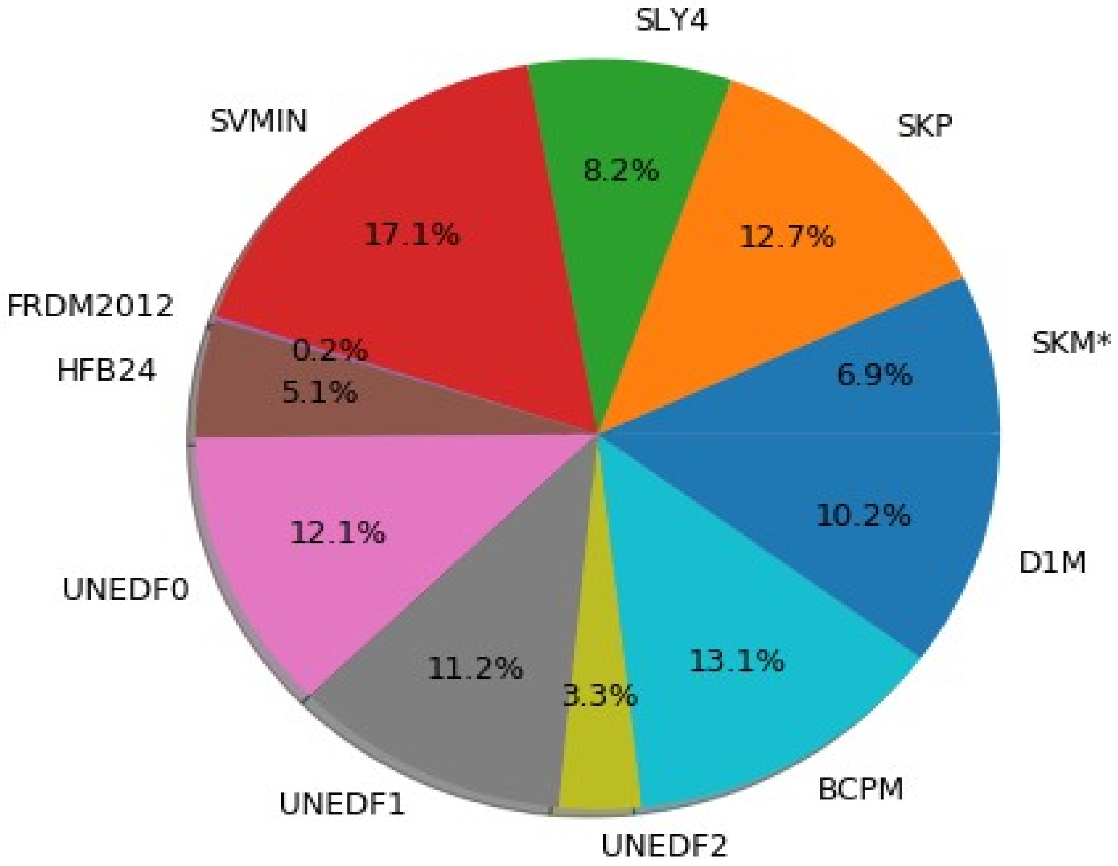

Impact on Urca Cooling

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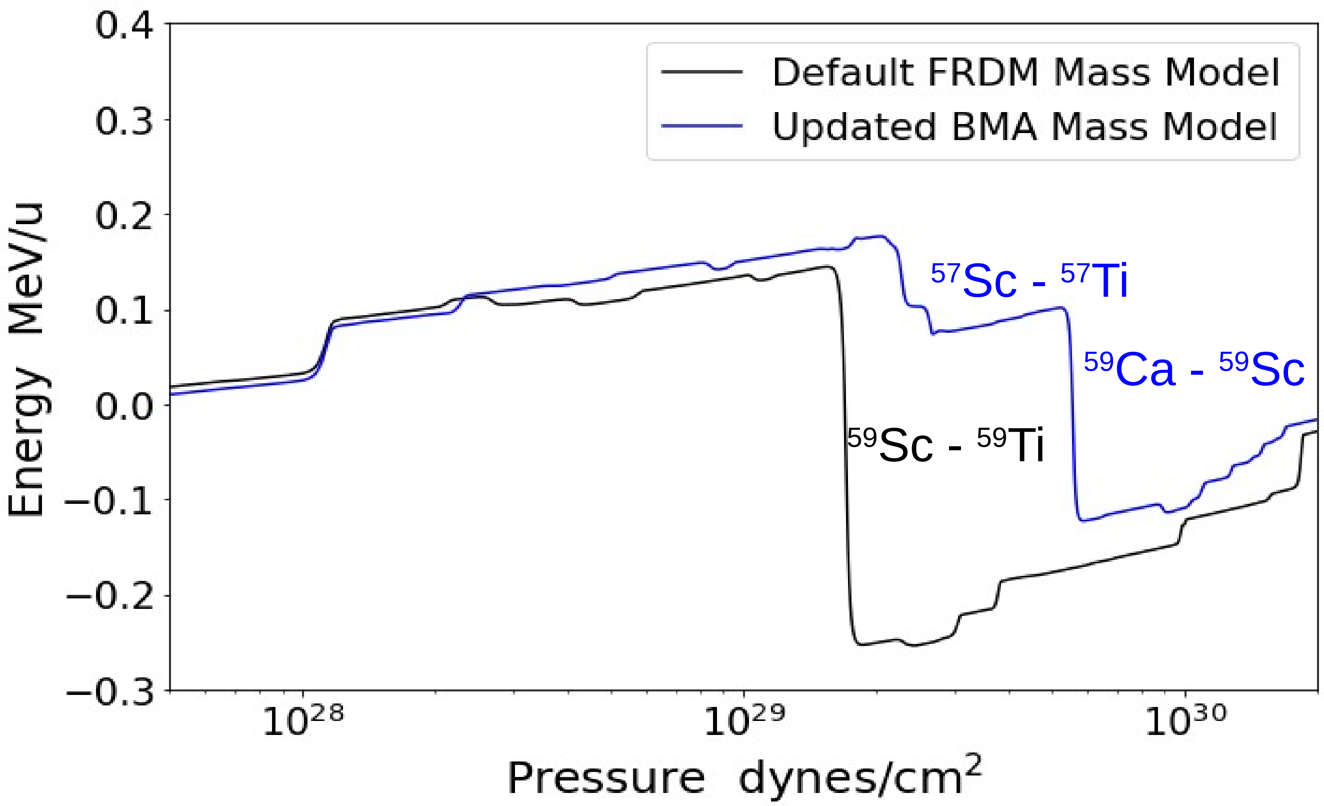


Summary: Cooling + Masses

- Quantifying the strength of Urca Cooling in accreting neutron star crusts is essential to improve crust models.
- Urca cooling rates are sensitive to nuclear masses.
- A quantified global nuclear mass model developed in Bayesian Framework predicts a lower cooling at a higher depth.

Reduced Order Models

- Surrogate models that are simplified versions of high-dimensional models to reduce computational effort while preserving accuracy
- Have been used in fluid dynamics, structural mechanics, control systems, etc.
- Methods include:
  - Proper Orthogonalization Decomposition: Reduces dimensions by identifying dominant modes in data
  - Reduced Basis Methods: Builds reduced models from snapshots of high fidelity solutions
  - Balanced Truncation: Focuses on preserving system dynamics while reducing state space
- Advantages:
  - Reduces computational costs and times
  - Enables real-time analysis and control of systems
  - Facilitates understanding and decision-making in complex systems

Dynamic Mode Decomposition

- Dynamic Mode Decomposition (DMD) is a data-driven technique used to analyze the dynamics of complex systems by decomposing them into spatiotemporal modes. It bridges numerical simulation and experimental data, revealing key features of system evolution.

- Key Features:
  - Extracts dominant dynamic structures from time-series data.
  - Provides insight into the system's temporal behavior, including growth, decay, and oscillations.
  - Works with both linear and nonlinear systems (via linear approximations).

- How it Works:
  - Collect snapshots of the system's state at different time steps.
  - Represent the data as a matrix and approximate it using low-rank matrices.
  - Identify eigenvalues and eigenvectors to derive modes and their dynamics.

Dynamic Mode Decomposition

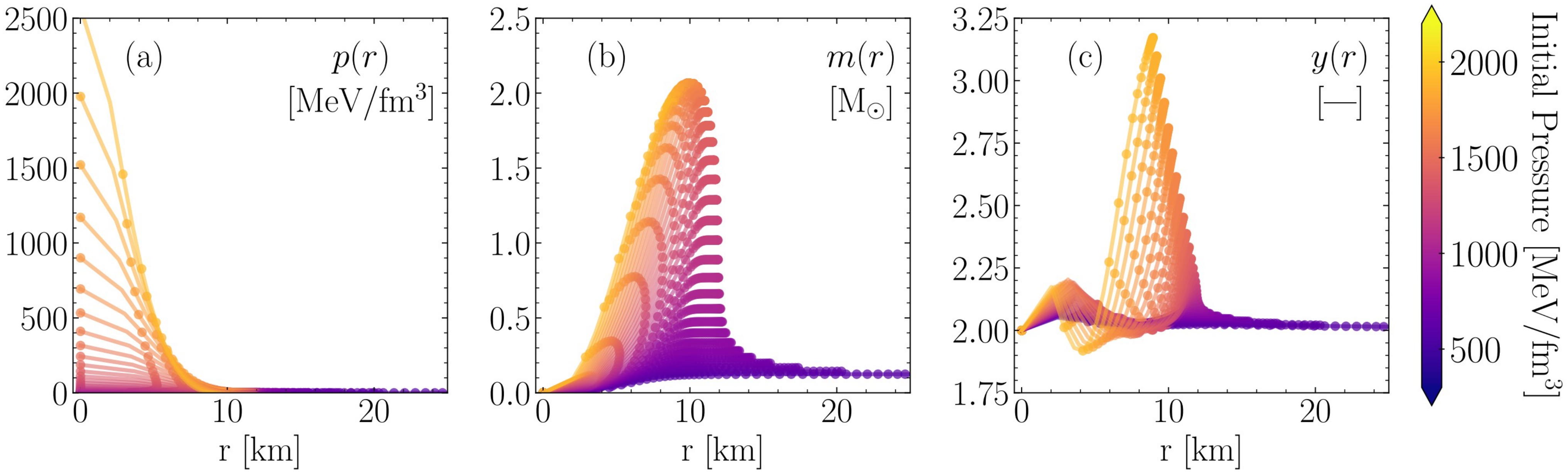
- Advantages:
  - Requires no prior knowledge of the governing equations.
  - Captures dominant dynamics with reduced complexity.
- Limitations:
  - Assumes linear dynamics between snapshots.
  - May require modifications (e.g., Extended DMD) for highly nonlinear systems.

Star Log-extended eMulator (SLM)

- Logarithm-based emulator that overcomes the limitations of DMDs
  - Does not assume linear dynamics between snapshots
  - Includes Extended DMD formalism for nonlinear systems
- Advantages:
  - Understands the underlying dynamics of system
  - High accuracy, low computational cost
  - Uses greedy algorithms to make predictions

FRIB

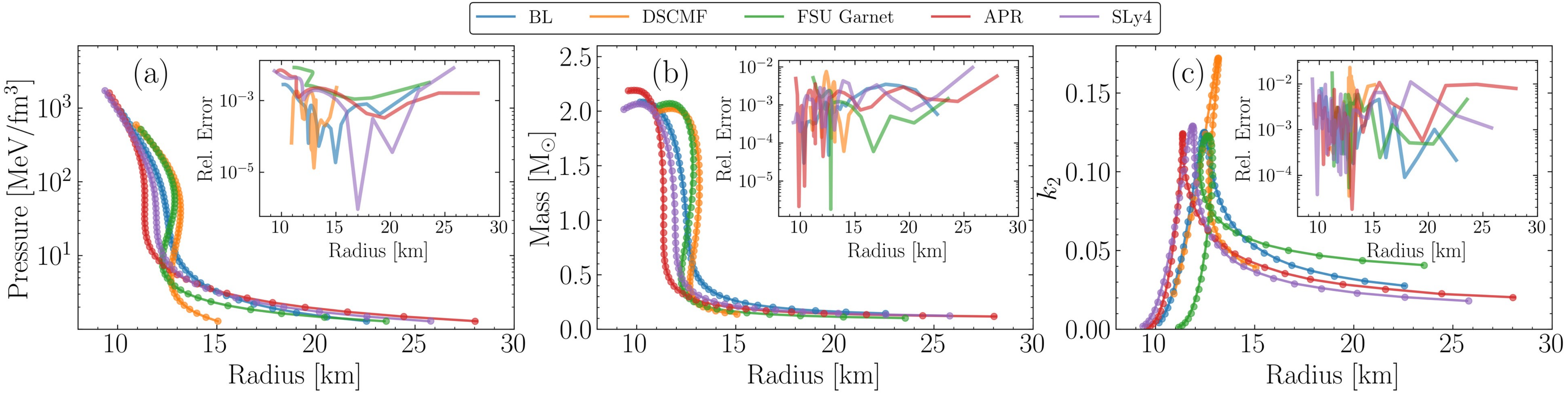
Applications: Tolmann-Oppenheimer-Volkoff Equations



Emulating the TOV equations: dots – emulations, curves – HF solutions

ArXiv: 2411.10556

Applications: Tolmann-Oppenheimer-Volkoff Equations



Emulating the pressure, mass and k2 curves: dots – data, curves – emulations

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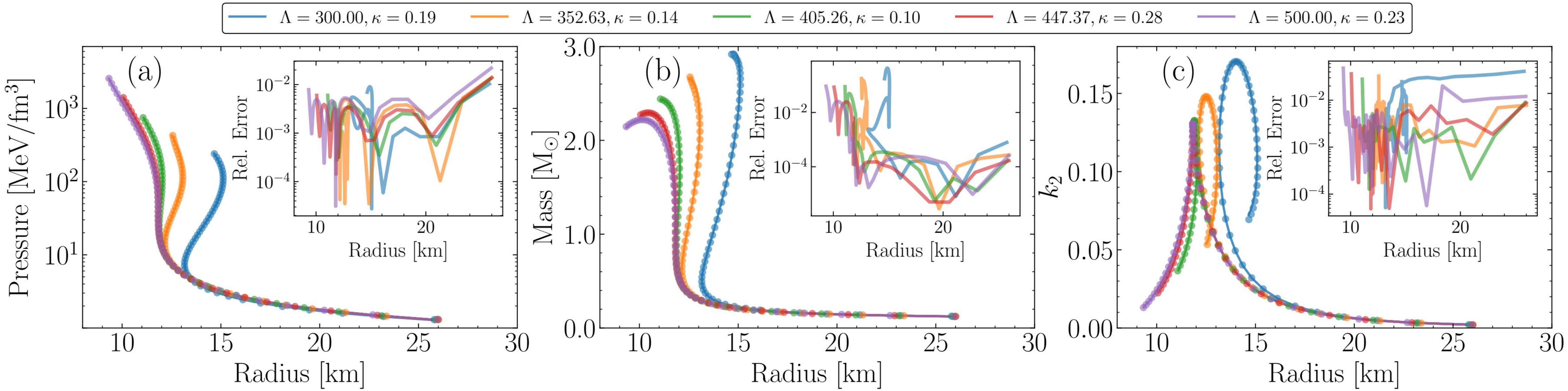
Applications: Tolmann-Oppenheimer-Volkoff Equations

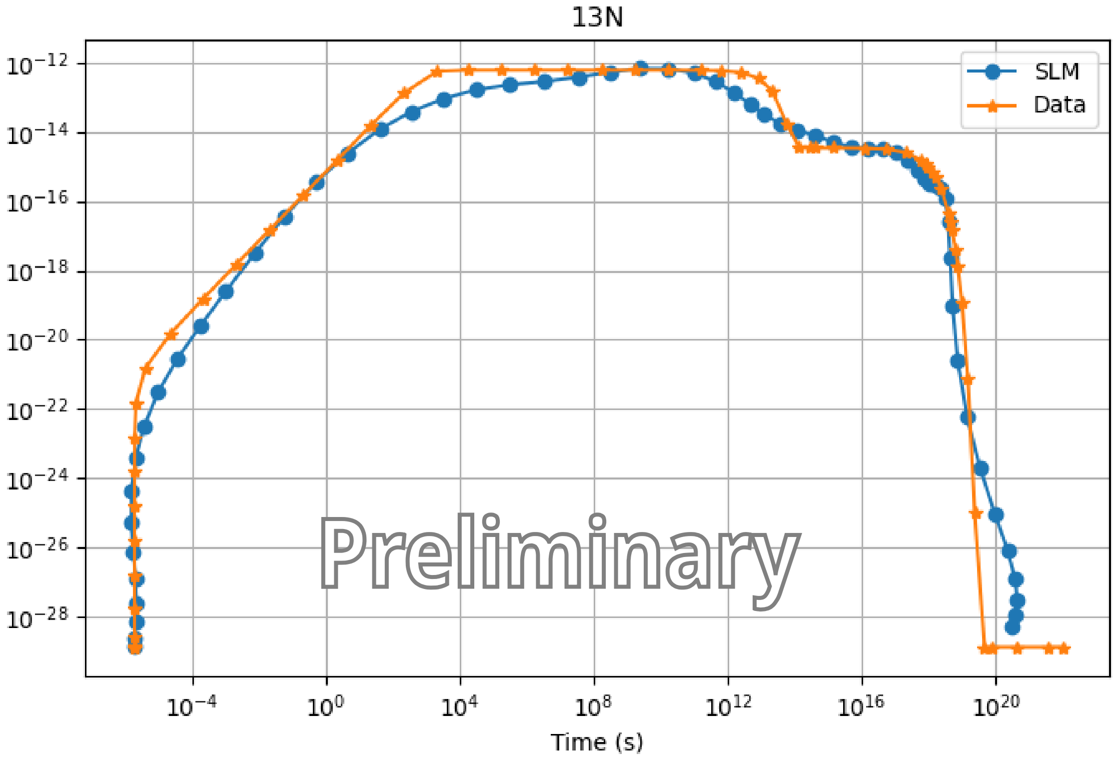


Parametric SLM (pSLM) predictions: the pressure, mass and k2 curves: dots – data, curves – emulations

-- Requires only 15 data sets for training
-- Speeds up the calculation by a factor of 30,000

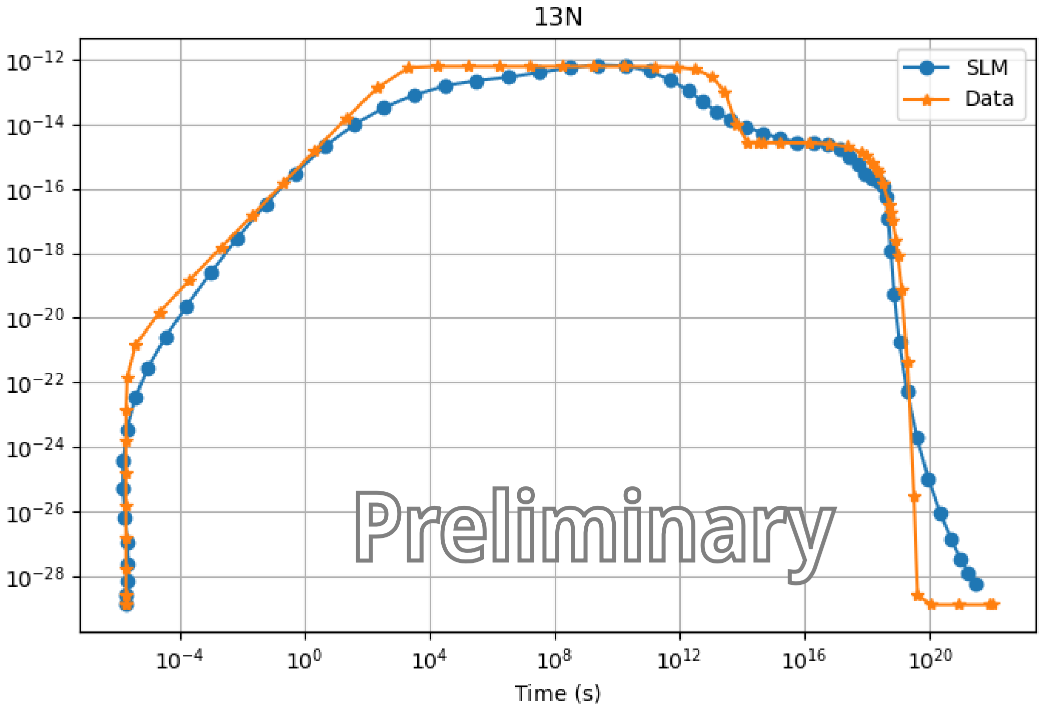
ArXiv: 2411.10556

Applications: Reaction Networks, CNO cycle



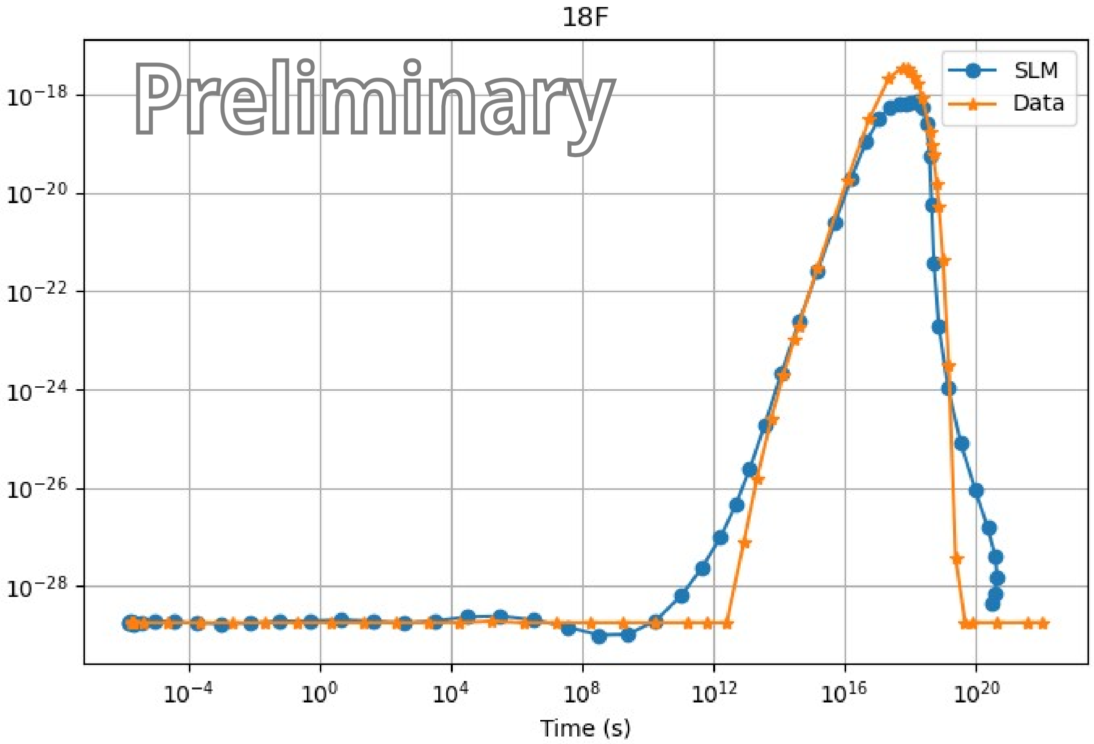
Training Data

Test Case



Abundances for various elements in the CNO cycle using pSLM

Applications: Reaction Networks, CNO cycle



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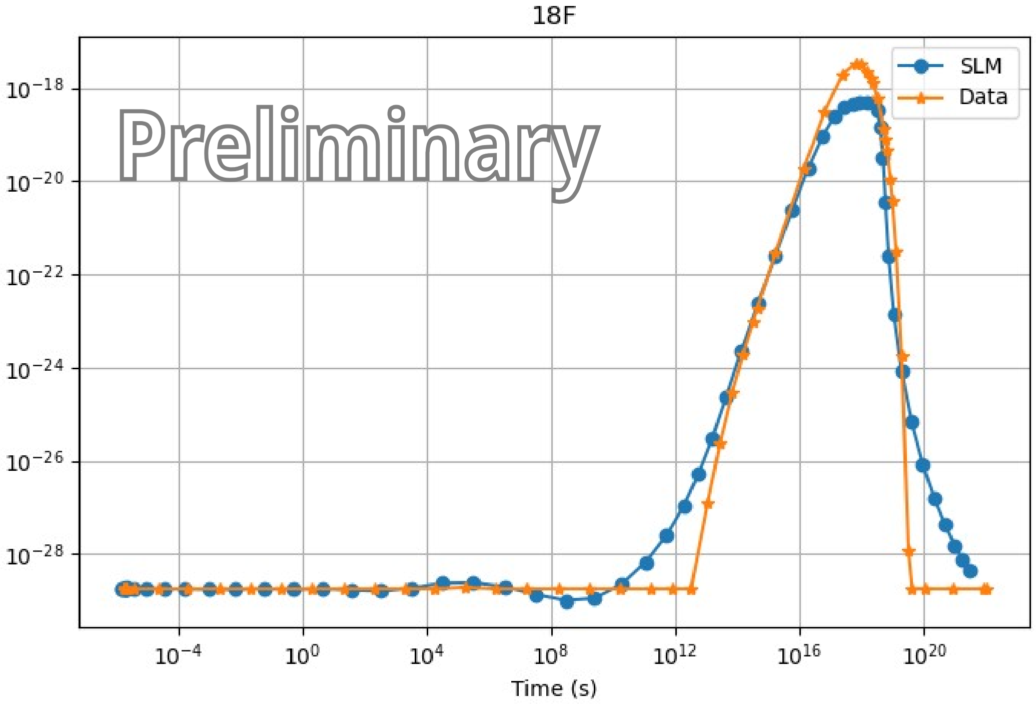


Abundances for various elements in the CNO cycle using pSLM

Summary: Emulators

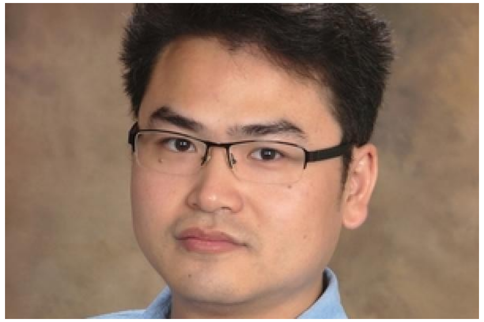
- Emulators are a useful alternative to high fidelity solvers for uncertainty quantification
- They are computationally inexpensive
- SLM accuracy seems to be independent of the number of parameters used

Thank you!

Hendrik Schatz

Witek Nazarewicz

Kyle Godbey



Xilin Zhang

Rahul Jain

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