Trace Anomaly in Dense Matter with Strong Phase Transitions

[arXiv: 2408.11614]

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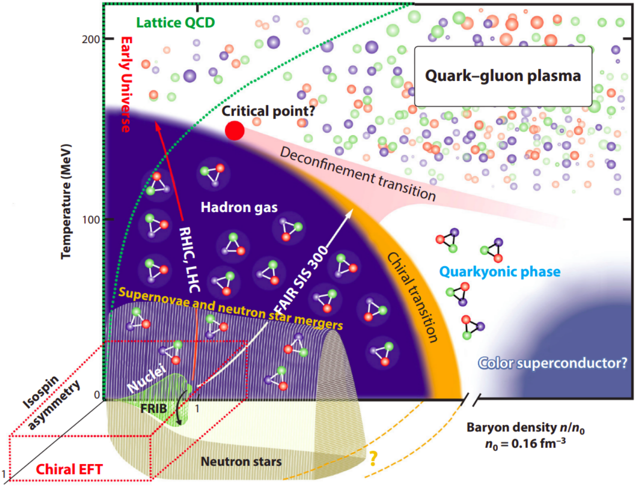
EoS Measurements with Next-Generation Gravitational-Wave Detectors (INT-24-89W, Seattle, Washington, USA 2024)

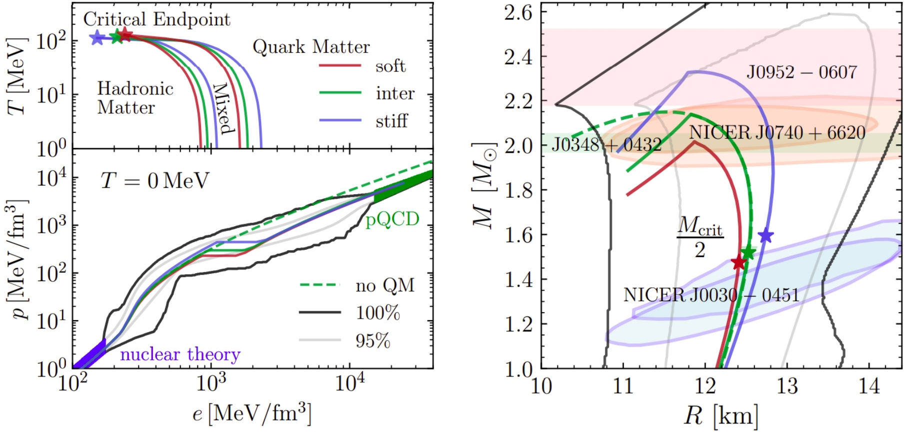
Outline

1. Motivation



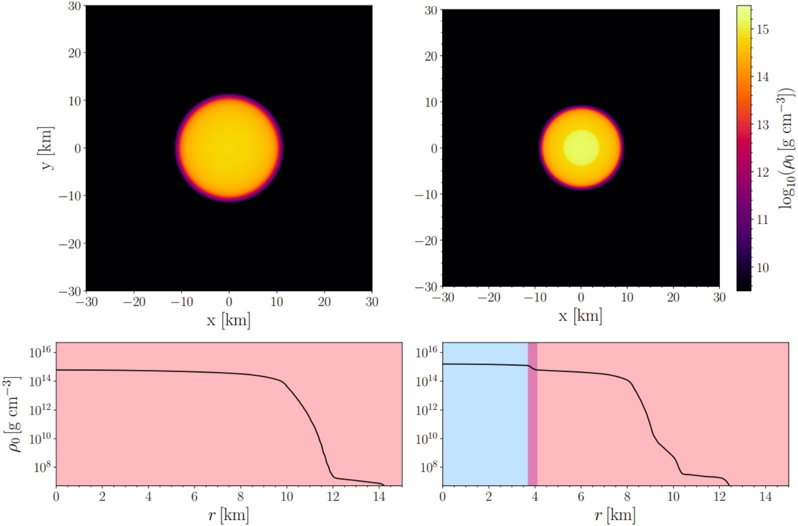
Cartoon of the QCD phase diagram
[Drischler *et al.*, 2021]

Motivation



Phase boundaries and EoS (left)
and corresponding *M*–*R* diagram (right) [Ecker et al., 2402.11013]

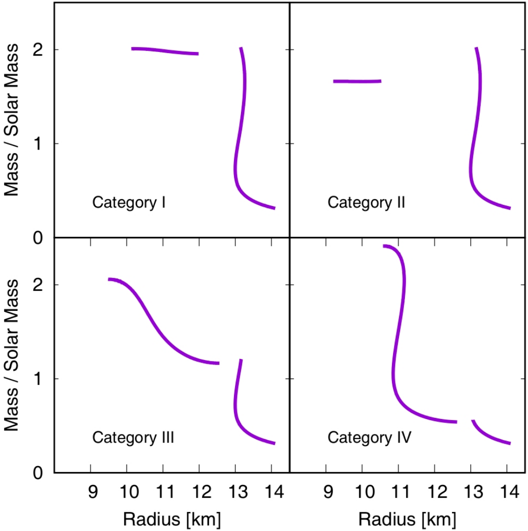
Motivation



Dynamical general-relativistic twin-star formation

[Naseri *et al.*, 2406.15544]

2. Twin-star matter essentials



Categories of twin stars according to their *M*'s
[J-E Christian *et al.*, Eur. Phys. J. A (2018) 54:28]

3. Trace anomaly in dense matter

- QCD trace anomaly as measure of breaking conformal invariance:

$$\eta_{\mu\nu} T^{\mu\nu}_{\mathrm{QCD}} \equiv T^{\mu}_{\mu} = \frac{\beta_{\mathrm{QCD}}}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + (1+\gamma_m) \sum_f m_f \overline{q}_f q_f .$$

- Thermal/dense case:

$$\left\langle T^{\mu}_{\mu} \right\rangle_{\mu_B, T} = \epsilon - 3P.$$

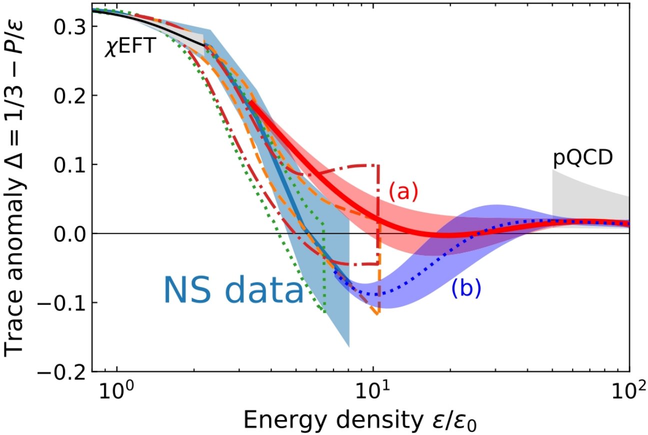
- Normalized thermal/dense case:

$$\Delta \equiv \frac{\left\langle T^{\mu}_{\mu} \right\rangle_{\mu_B, T}}{3\epsilon} = \frac{1}{3} - \frac{P}{\epsilon}.$$

- Causality and non-relativistic limits

$$-\frac{2}{3}(\approx -0.667) \leq \Delta < \frac{1}{3}(\approx 0.333).$$

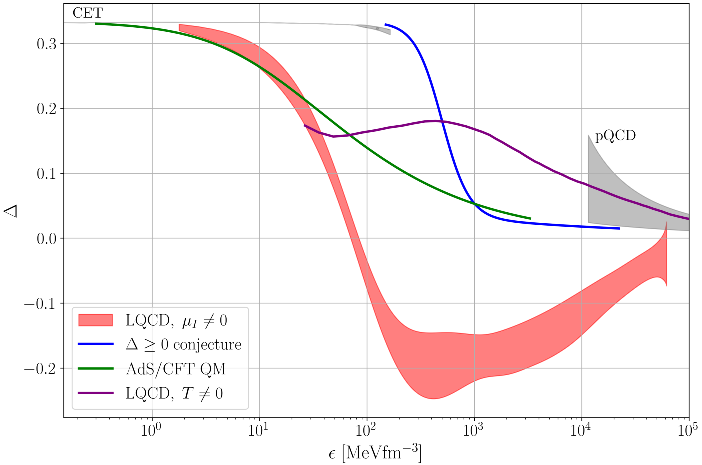
Trace anomaly in neutron-star interiors



Trace anomaly behavior with different NS data

[Y. Fujimoto *et al.*, PRL 129, 252702 (2022)]

4. Trace Anomaly and 1st-order Phase Transitions



Behavior of Δ for different kinds of extreme matter

[J. C. J. *et al.*, 2408.11614]

Twin-star Matter and Seidov's Criterium

- Constant-speed-of-sound parametrization

$$\epsilon(P) = \begin{cases} \epsilon_{\mathrm{H}}(P) & P < P_t\,, \\ \epsilon_{\mathrm{H}}(P_t) + \Delta\epsilon + s^{-1}(P - P_t) & P > P_t\,. \end{cases}$$

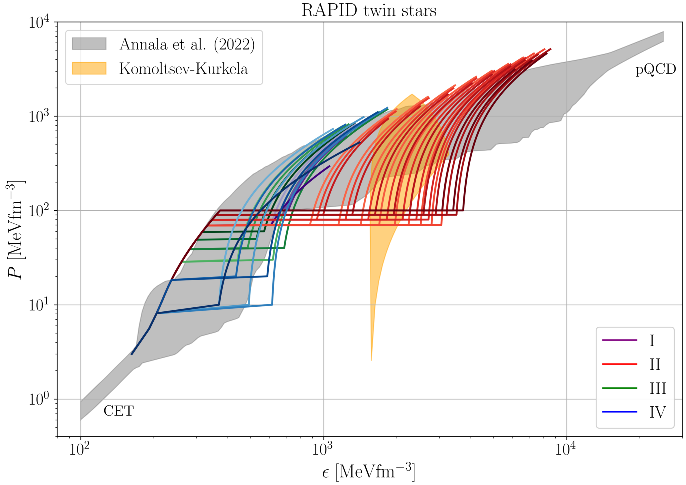
- Seidov's criterium

$$\Delta\epsilon \geq \Delta\epsilon_{\mathrm{crit}} \equiv \frac{1}{2}\epsilon_t + \frac{3}{2}P_t.$$

- Particular set of parameters (in units of MeV fm^{-3})

Category	$\epsilon_H^{\mathrm{max}} = \epsilon_t$	$\epsilon_Q^{\mathrm{min}}$	P_t	$\Delta\epsilon$	c_s^2
I	333.08	607.34	70	274	1
II	333.08	878.88	70	545	1
III	263.73	441.62	30	178	1
IV	212.91	370.85	10	157	1

Studied Twin-Star Equations of State



Family of EoSs for Category I-IV stable twin stars with rapid conversions.

Trace Anomaly (Δ) for the 3 sectors of the EoS

- Behavior at the mixed phase (Maxwell construction)

$$c_s^2 = c^2_{\mathrm{s,\ deriv}} + c^2_{\mathrm{s,\ nonderiv}} = \frac{dP}{d\epsilon}.$$

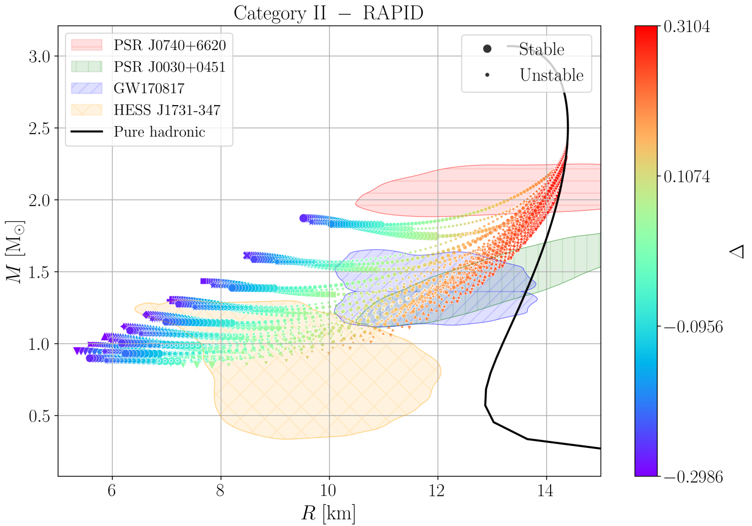
$$c^2_{\mathrm{s,\ deriv}} \equiv -\epsilon\frac{d\Delta}{d\epsilon}, \qquad c^2_{\mathrm{s,\ nonderiv}} \equiv \frac{1}{3} - \Delta.$$

$$\Delta_{\mathrm{mix}}(\epsilon) = \frac{1}{3}\left(1 - \frac{\epsilon_H^{\mathrm{max}}}{\epsilon}\right) + \frac{\epsilon_H^{\mathrm{max}}}{\epsilon}\Delta_H^{\mathrm{max}}.$$

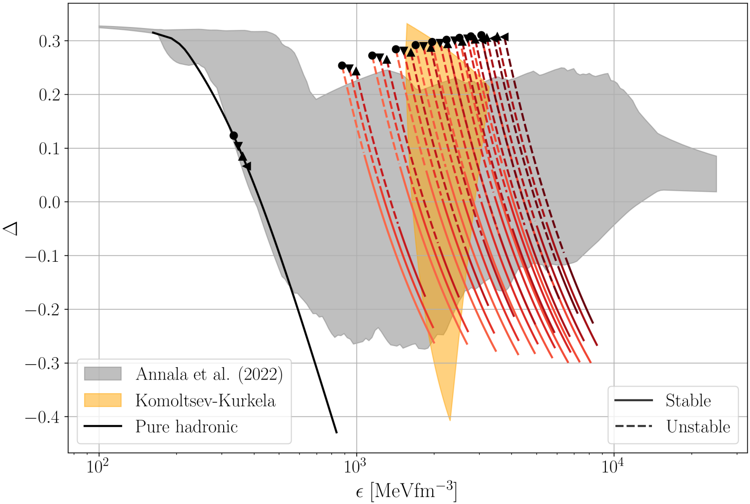
- Behavior in the quark phase in the CSS parametrization

$$\Delta_Q(\epsilon) = \left(\frac{1}{3} - s\right)\left(1 - \frac{\epsilon_Q^{\mathrm{min}}}{\epsilon}\right) + \frac{\epsilon_Q^{\mathrm{min}}}{\epsilon}\Delta_Q^{\mathrm{min}}.$$

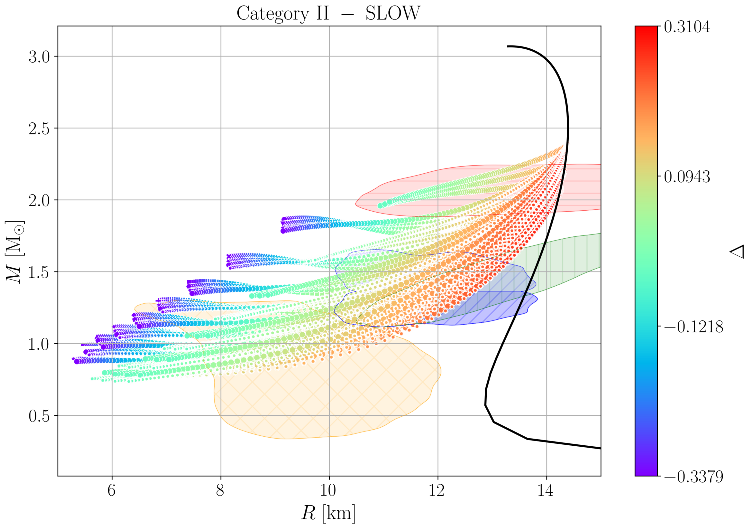
M–R for *rapid* Category II twin stars



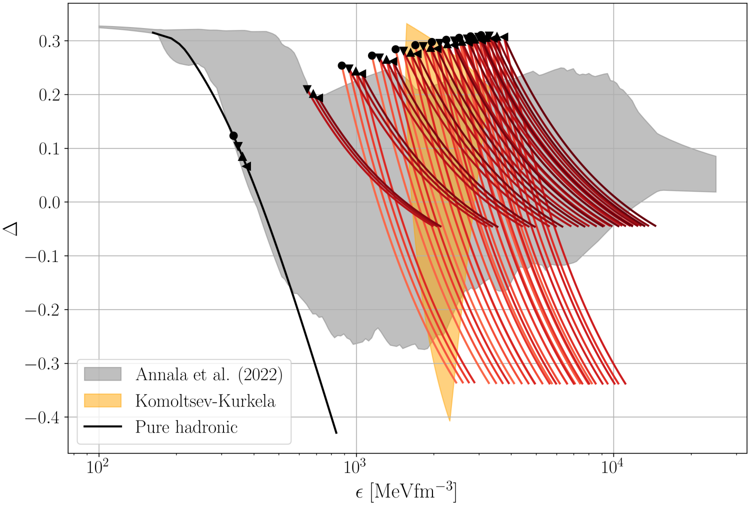
Δ for *rapid* Category II twin stars



M–R for *slow* Category II twin stars



Δ for *slow* Category II twin stars



5. Some insights for dense QCD

- Conjecture of $\Delta > 0$ (Fujimoto et al., 2022) through

$$\frac{\epsilon - 3P}{P_{\mathrm{ideal}}} = \mu_B \frac{dN_{\mathrm{eff}}}{d\mu_B} > 0,$$

where $N_{\mathrm{eff}} \equiv P/P_{\mathrm{ideal}}$ and $P_{\mathrm{ideal}} \equiv N_c N_f \frac{\mu_B^4}{12\pi^2}$.

- In our case, a finite latent heat, Q, is present:

$$Q = \mu_c \Delta n_B = \left[T^{\mu}_{\mu}(\mu_B^+ \to \mu_c)\right]_{\mathrm{Q}} - \left[T^{\mu}_{\mu}(\mu_B^- \to \mu_c)\right]_{\mathrm{H}},$$

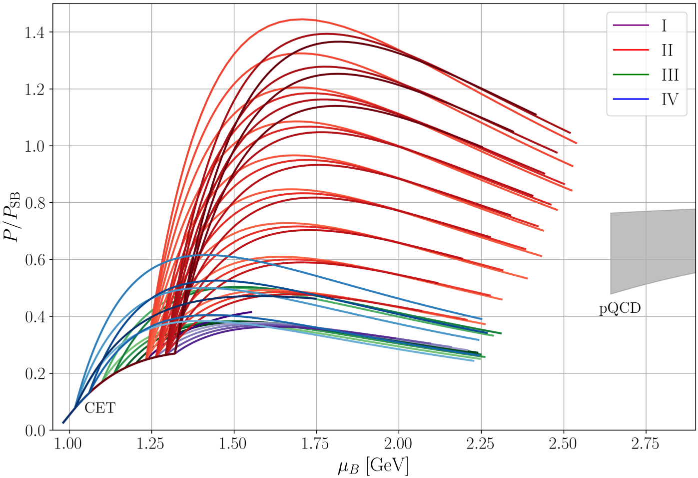
or equivalently

$$\frac{Q}{\mu_c^4} = \mu_c \left[\left(\frac{dN^Q_{\mathrm{eff}}}{d\mu_B^+}\right) - \left(\frac{dN^H_{\mathrm{eff}}}{d\mu_B^-}\right)\right]_{\mu_B^{\pm} \to \mu_c}.$$

- Now, the quark phase in CSS parametrization (1st-order transition)

$$P_Q(\mu_B) = N\mu_B^{1+\gamma} - B.$$

Some insights for dense QCD



Corresponding $P/P_{\rm SB}$ vs μ_B for twin stars.

Corresponding 'Δ' vs 'μ_B' for twin stars

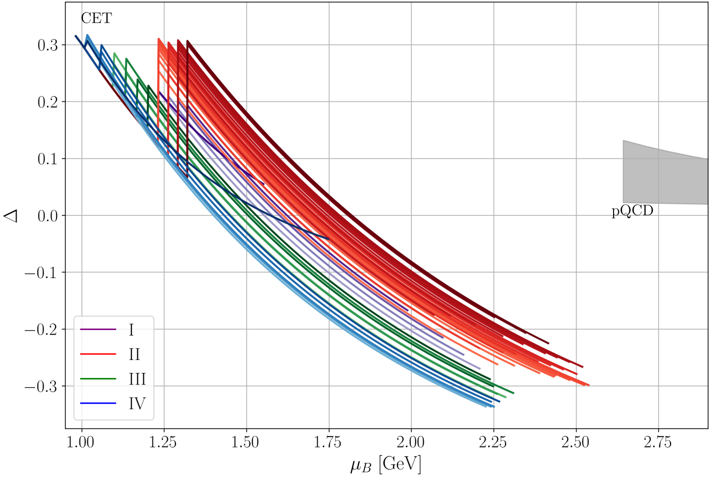
- It can be written at all densities as

$$\Delta(\mu_B) = \begin{cases} \Delta_H(\mu_B) & \mu_B < \mu_c \,, \\ \Delta_Q(\mu_B) & \mu_B > \mu_c \end{cases}$$

- In this case, the normalize trace anomaly for the quark phase is

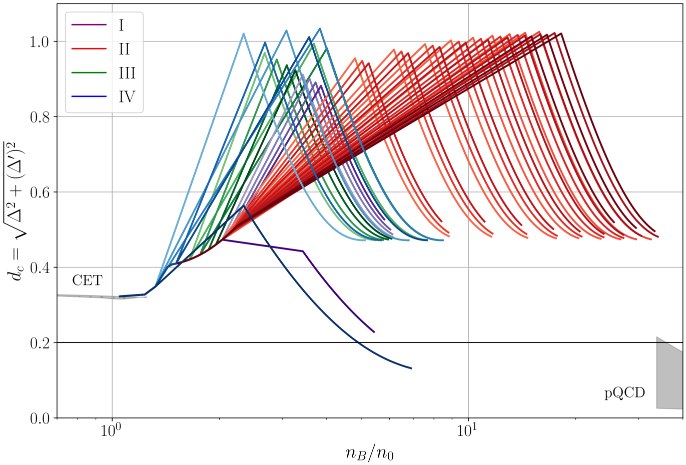
$$\Delta_Q = \frac{4B - 3N\gamma\mu_B^{1+\gamma}\left(c_{s,Q}^2 - 1/3\right)}{B + N\gamma\mu_B^{1+\gamma}},$$

Corresponding 'Δ' vs 'μ_B' for twin stars



Corresponding 'Δ' vs 'μ_B' for twin stars.

Conformality, $d_c = \sqrt{\Delta^2 + (\Delta')^2}$, vs n_B/n_0



Conformality factor 'd_c'

defined in [E. Annala et al., Nat Commun 14, 8451 (2023)].

6. Summary

- We performed a comprehensive analysis of Δ in the presence of strong first-order phase transitions realized at the cores of twin NS.
- We found that these Δ's show an abrupt decreasing trend to negative values at intermediate densities after the onset of stiff quark matter.
- If fully stable twin stars exist in nature (through large $c^2_{s,Q} \sim 1$), the $\Delta < 0$, in particular, reaching $\Delta \simeq -0.35$.
- Future (agnostic) Bayesian studies constraining the NS EoS should take into account these non-trivial possibilities.

Acknowledgements

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