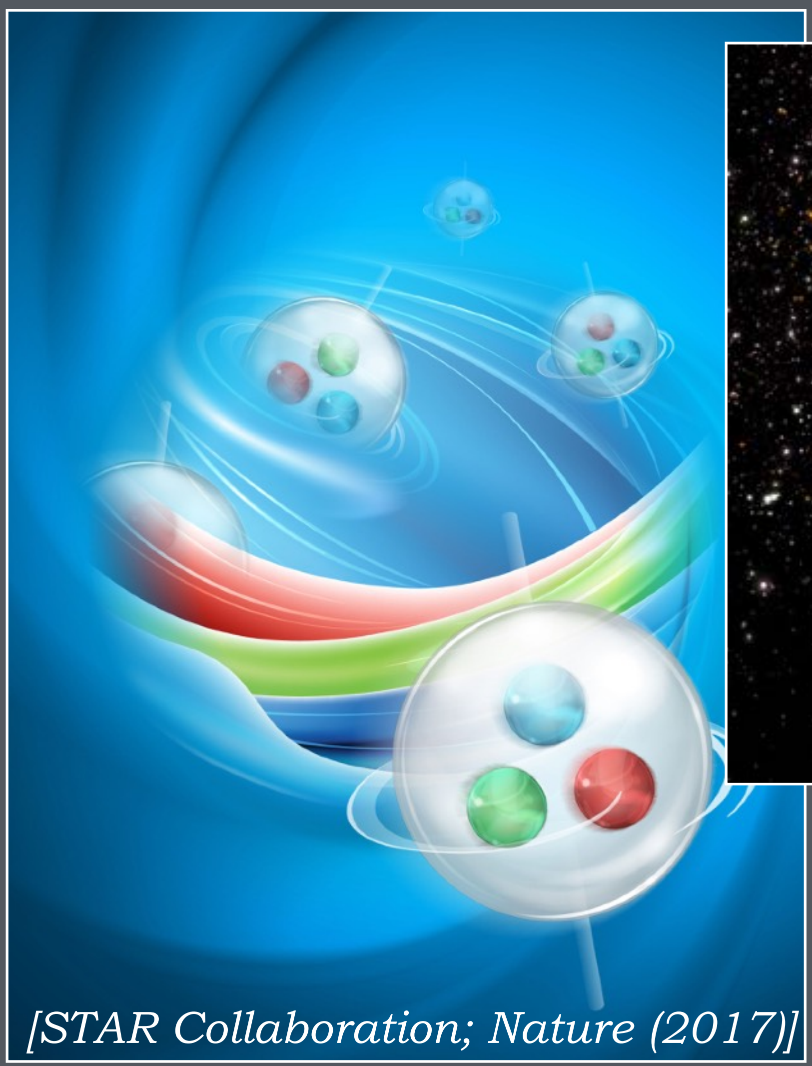
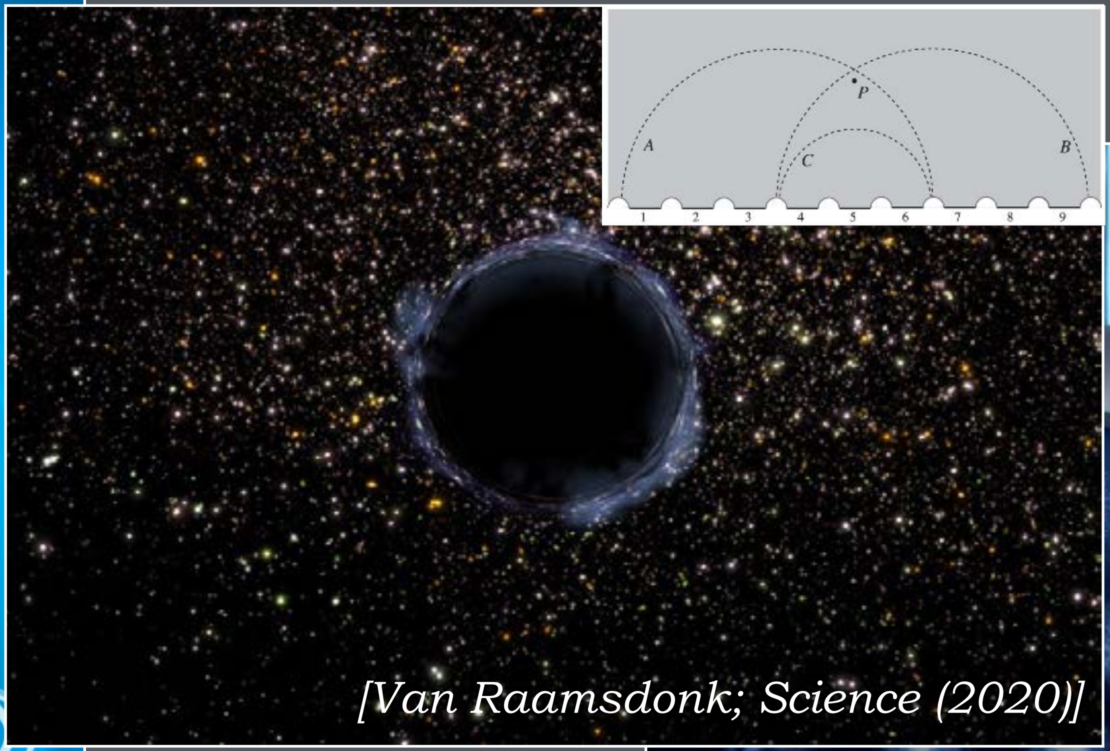
Entanglement, thermalization (and transport) in quark-gluon plasma, in ultra-cold gases, & black holes

Quantum Few- and Many-Body Systems in Universal Regimes, INT, Seattle

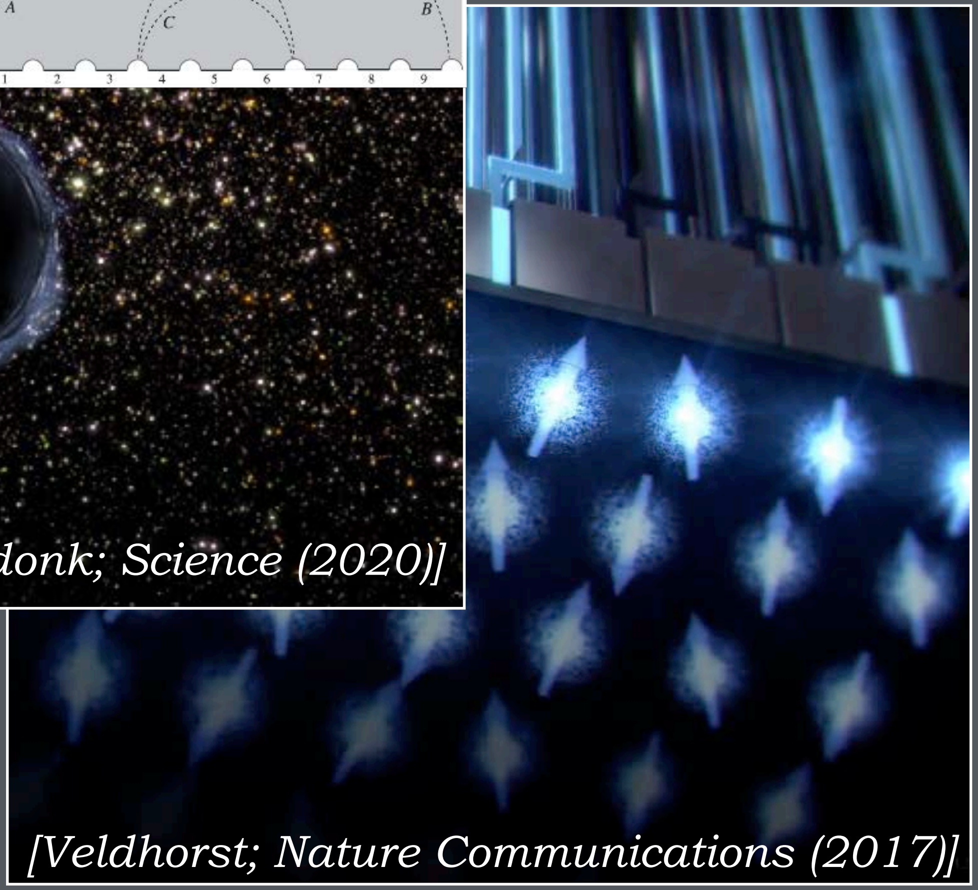
October 23rd, 2024



[STAR Collaboration; Nature (2017)]



[Van Raamsdonk; Science (2020)]



[Veldhorst; Nature Communications (2017)]

Matthias Kaminski
University of Alabama

U.S. DEPARTMENT OF ENERGY

Circular rainbow viewed from plane above Seattle

Circular rainbow viewed from plane above Seattle

Gauge/Gravity Correspondence

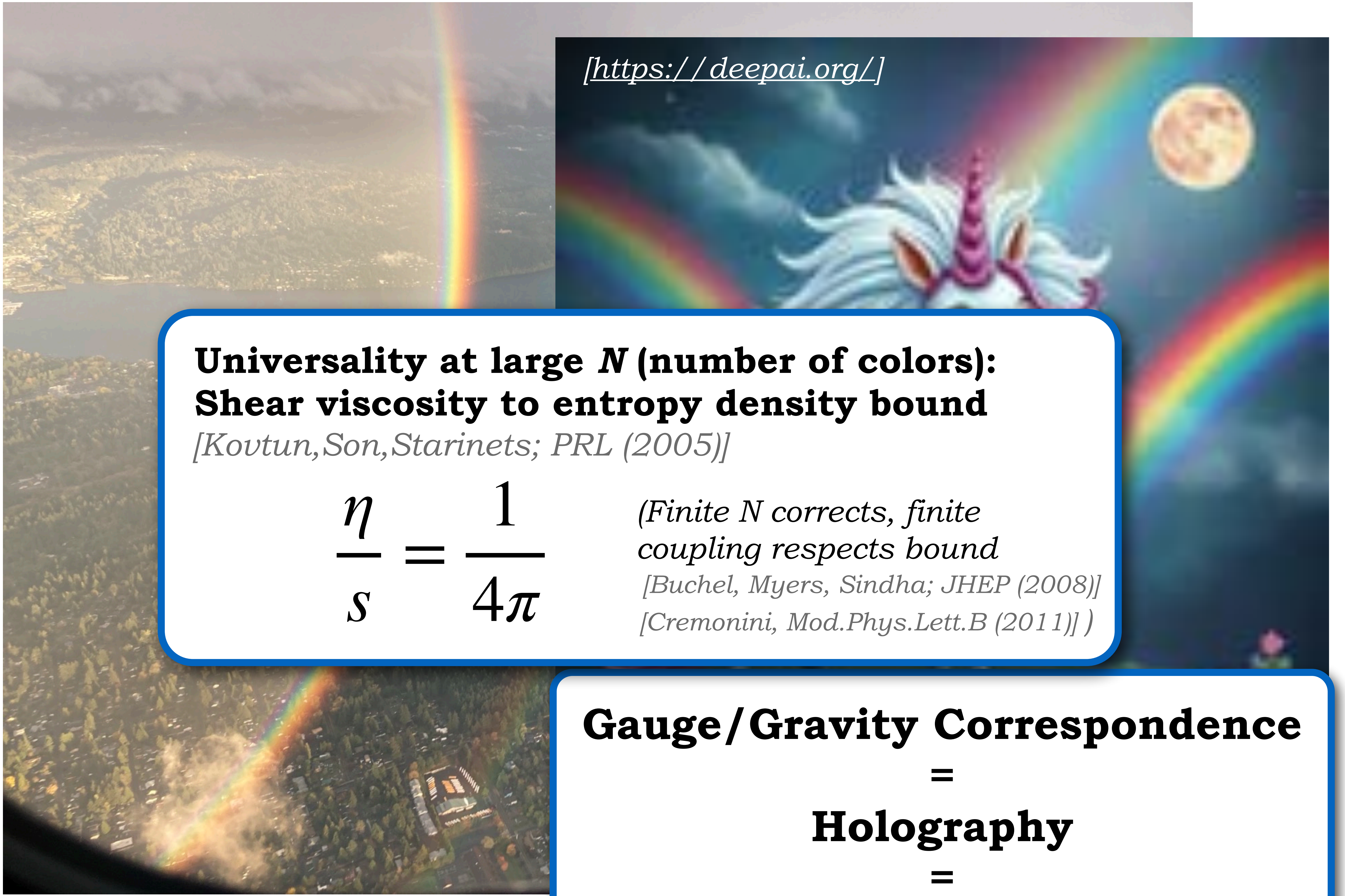
=

Holography

=

AdS/CFT

Circular rainbow viewed from plane above Seattle



Universality at large *N* (number of colors):
Shear viscosity to entropy density bound
[Kovtun,Son,Starinets; PRL (2005)]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

(Finite N corrects, finite coupling respects bound
[Buchel, Myers, Sindha; JHEP (2008)]
[Cremonini, Mod.Phys.Lett.B (2011)])

Gauge/Gravity Correspondence
=
Holography
=
AdS/CFT

Outline

1. **Statistical versus quantum mechanic dynamics**
2. **Holography (far from equilibrium)**
3. **Holographic entanglement entropy (calculation)**
4. **Quantum gravity experiments**

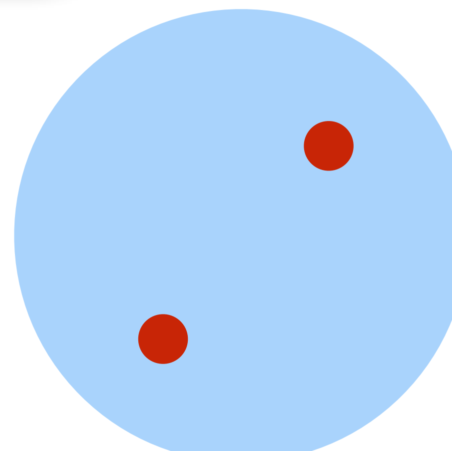
Thermodynamics: entropy increases

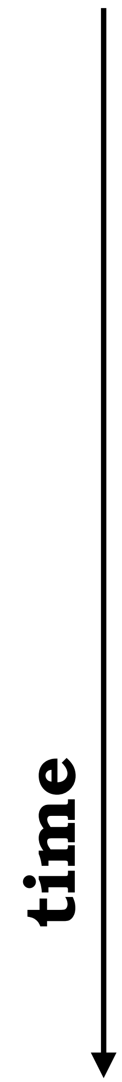
Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Entropy = Non-Information

$S \propto$ # of configurations





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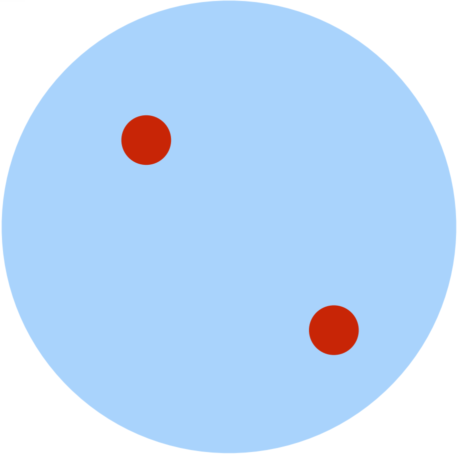
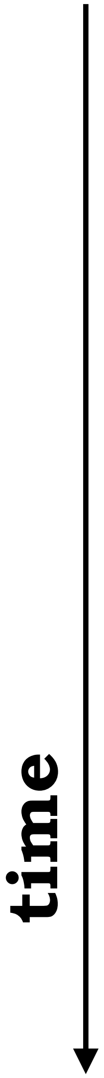


Definition via density matrix

$$\rho = \sum_i p_i |n_i\rangle\langle n_i|$$

$$S = -\,\mathrm{tr}\left(\rho \log \rho\right)$$



Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

[Kaufman et al.; Science (2016)]

"Quantum thermalization through entanglement in an isolated many-body system"

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"Quantum thermalization through entanglement in an isolated many-body system"

Pure state density matrix

$$\rho = 1 \cdot |\text{eigenstate}\rangle\langle\text{eigenstate}|$$

with $\mathrm{tr}\left(\rho^2\right) = 1$.

Vanishing entropy $S = -\,\mathrm{tr}\left(\rho \log \rho\right) = 0$

Unitary time evolution $\rho \to U^{-1}\rho U$ keeps $S=0$.

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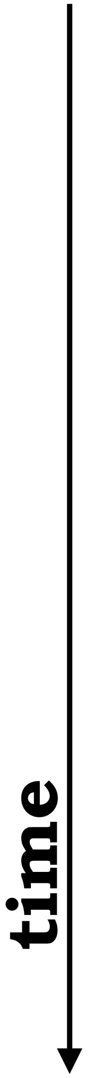
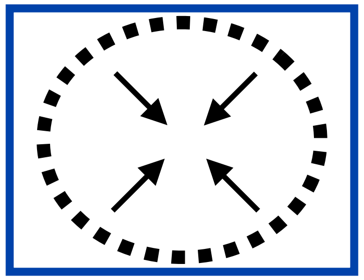
Example 1:
Black hole evaporation

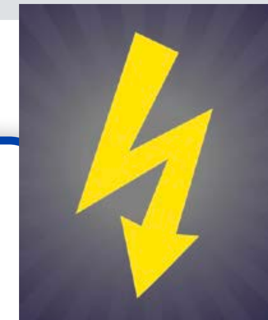
[Almheiri et al.; Rev.Mod.Phys. (2021)]





Pure state. **Zero entropy.**
(inward falling mass shell)

Clash of two fundamental concepts



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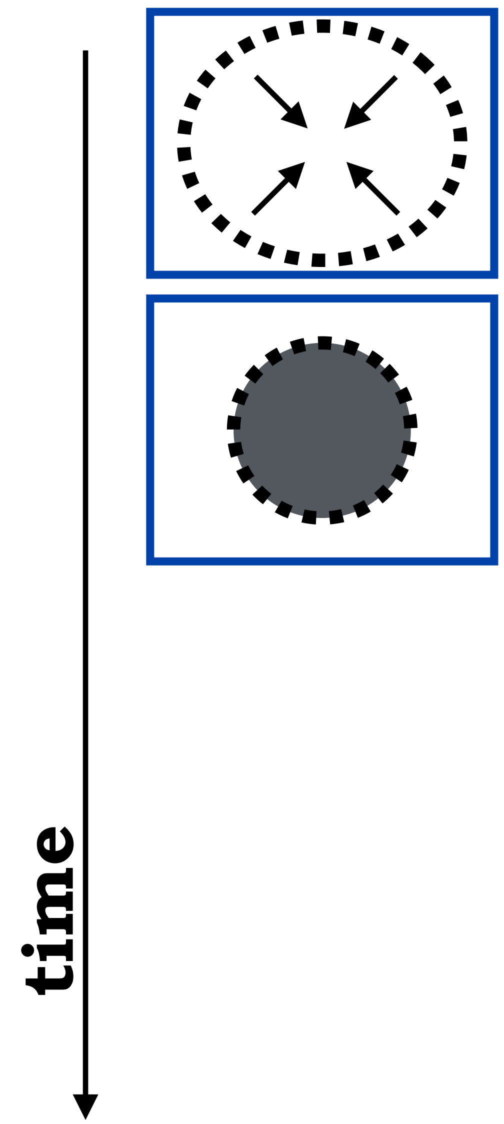

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Black hole is formed.

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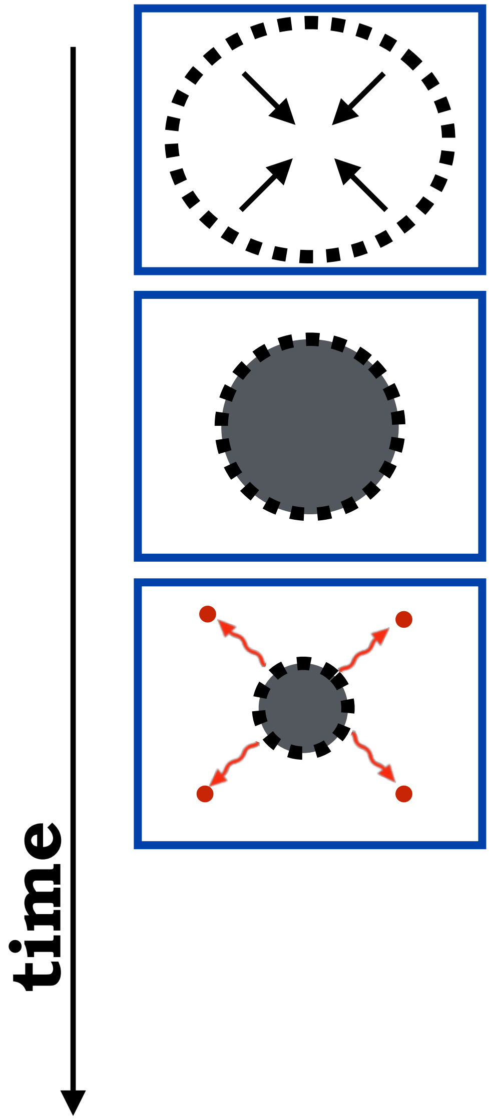

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Hawking radiation,
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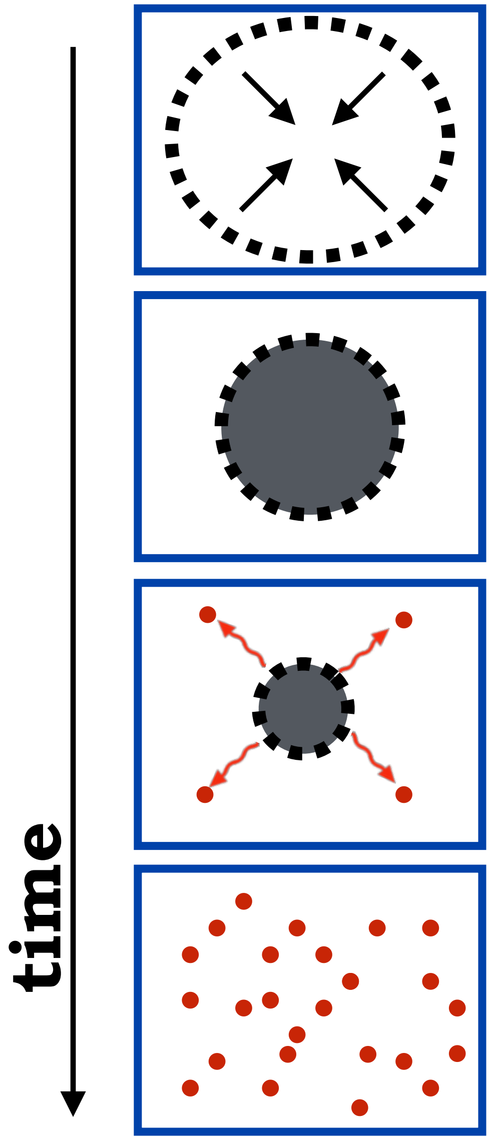

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Hawking radiation,
black hole evaporates.

Thermal Hawking radiation.
Nonzero entropy!?

Clash of two fundamental concepts

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Example 1:
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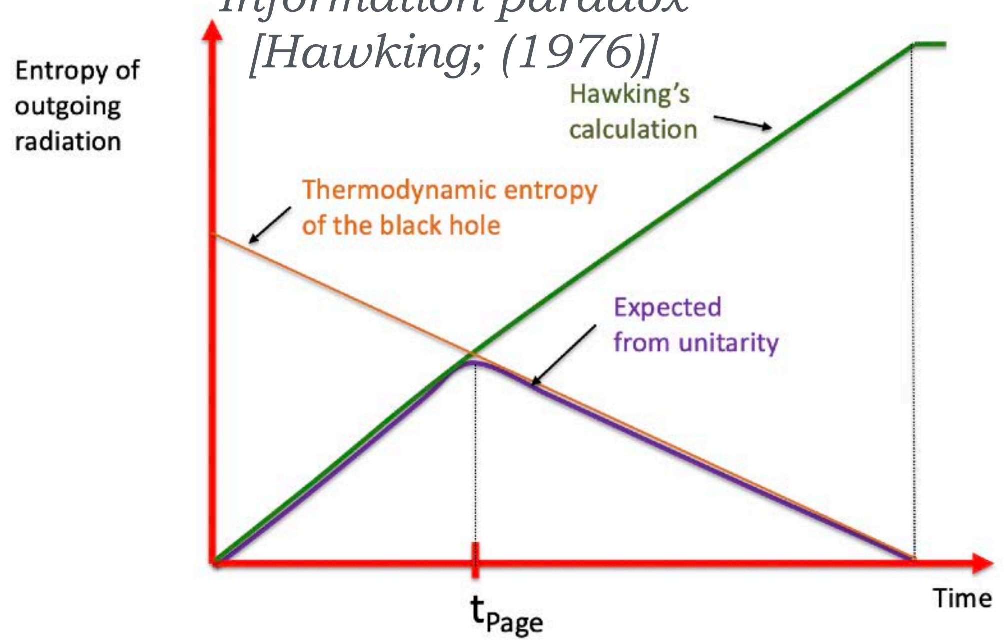
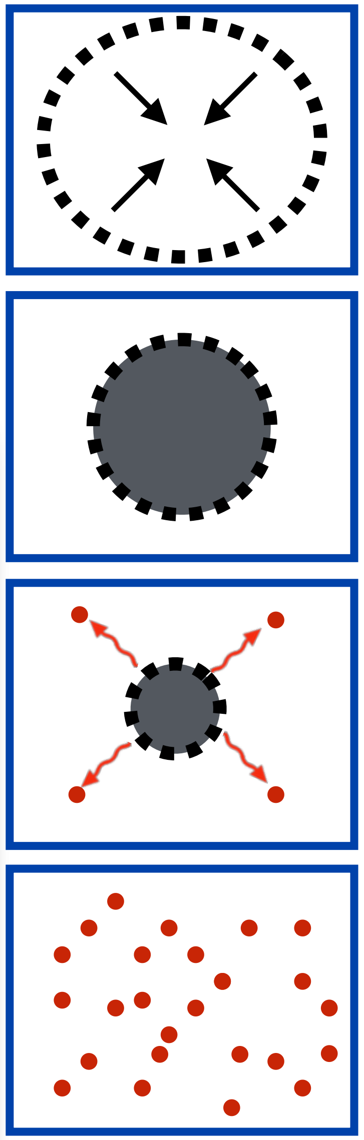
[Almheiri et al.; Rev.Mod.Phys. (2021)]

Page curve (fine-grained S)

Information paradox
[Hawking; (1976)]





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Hawking radiation,
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~~Thermal~~ Hawking radiation.
~~**Nonzero entropy!?**~~

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Example 1:
Black hole evaporation

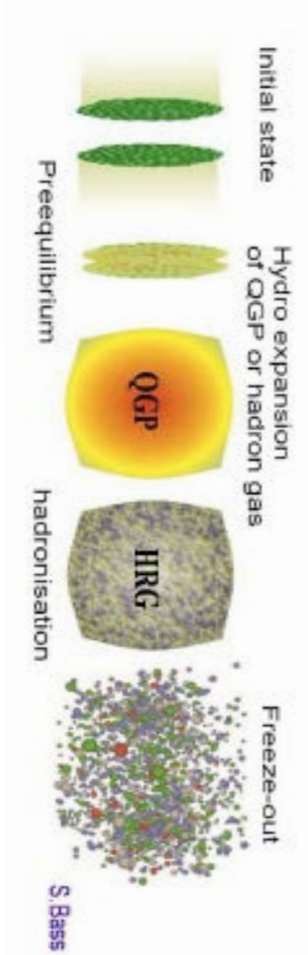
Example 2:
Heavy ion collisions (HIC)

Quantum information approach to HIC
[Kharzeev; Phil.Trans.A.Math.Phys.Eng.Sci. (2021)]

[Zhang et al.; PRD (2021)]

[Florio,Kharzeev; PRD (2021)]

time



Pure state. **Zero entropy.**
(two colliding ions)
[Müller,Schäfer; (2017)]

Particle distributions in detectors look thermal. **Nonzero entropy!?**

Clash of two fundamental concepts

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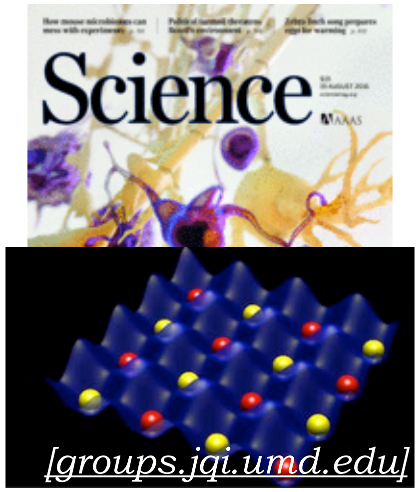
Example 1: Black hole evaporation

Example 2: Heavy ion collisions

Example 3: Ultracold atoms

[Kaufman et al.; Science (2016)]
"Quantum thermalization through entanglement in an isolated many-body system"



- Rb atoms in optical lattice
- six-site Bose-Hubbard system
- quench and microscopy

time

Pure state. **Zero entropy. Remains pure!**

BUT subsystems thermalize! (nonzero entropy)

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

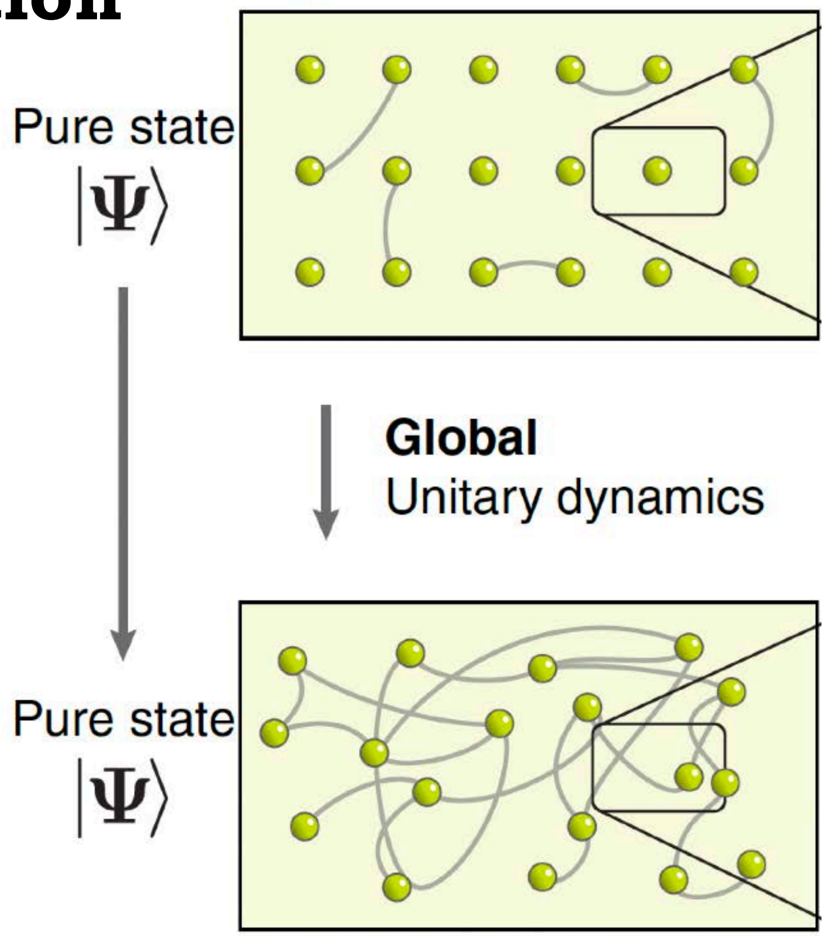
Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

General proposed resolution

- Total system remains pure.
- Subsystems thermalize (nonzero entropy).
- Eigenstate thermalization hypothesis (ETH)? *[Srednicki; (1993)]*

. . . many open questions!



[Kaufman et al.; Science (2016)]

grey links: entanglement

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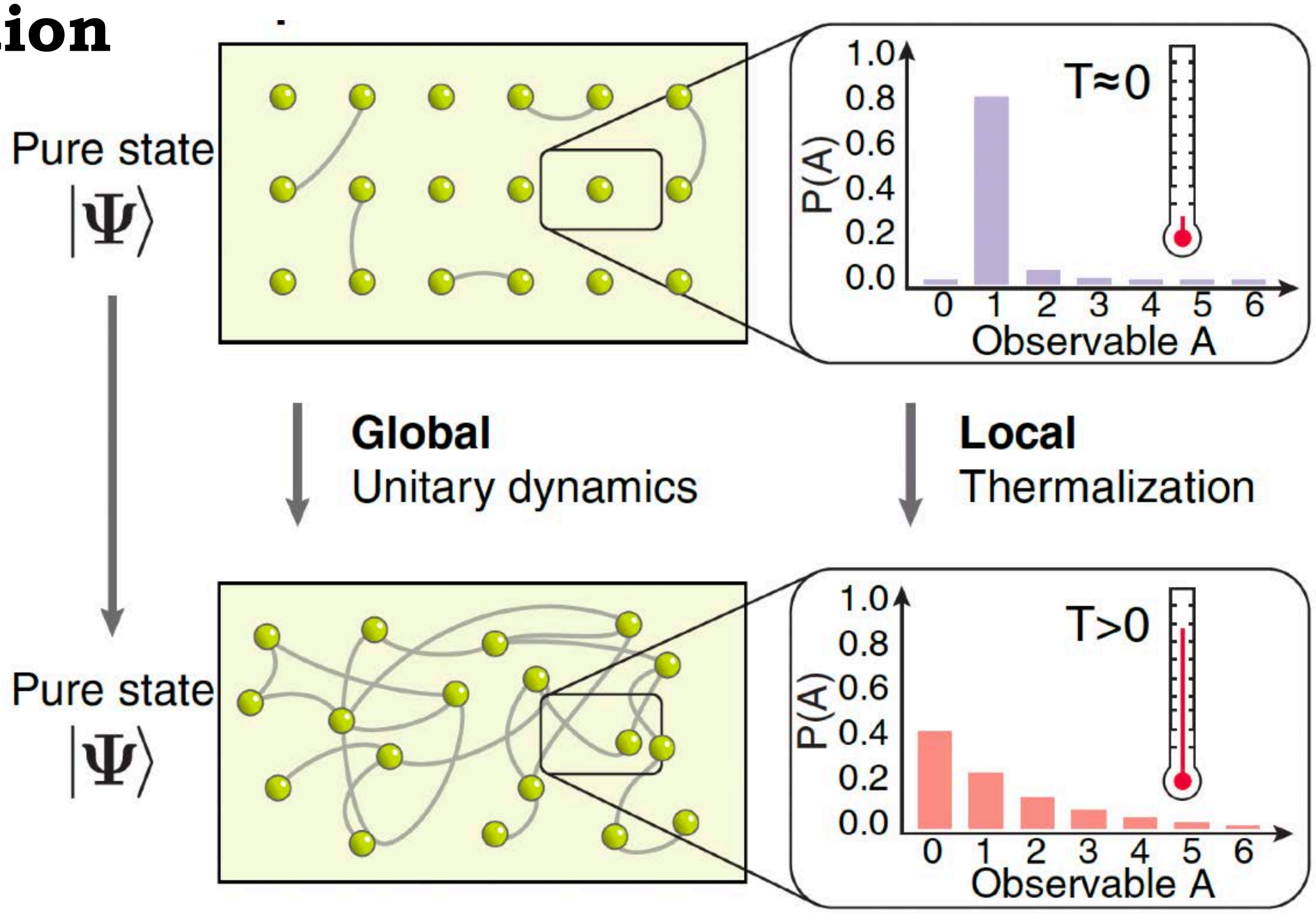

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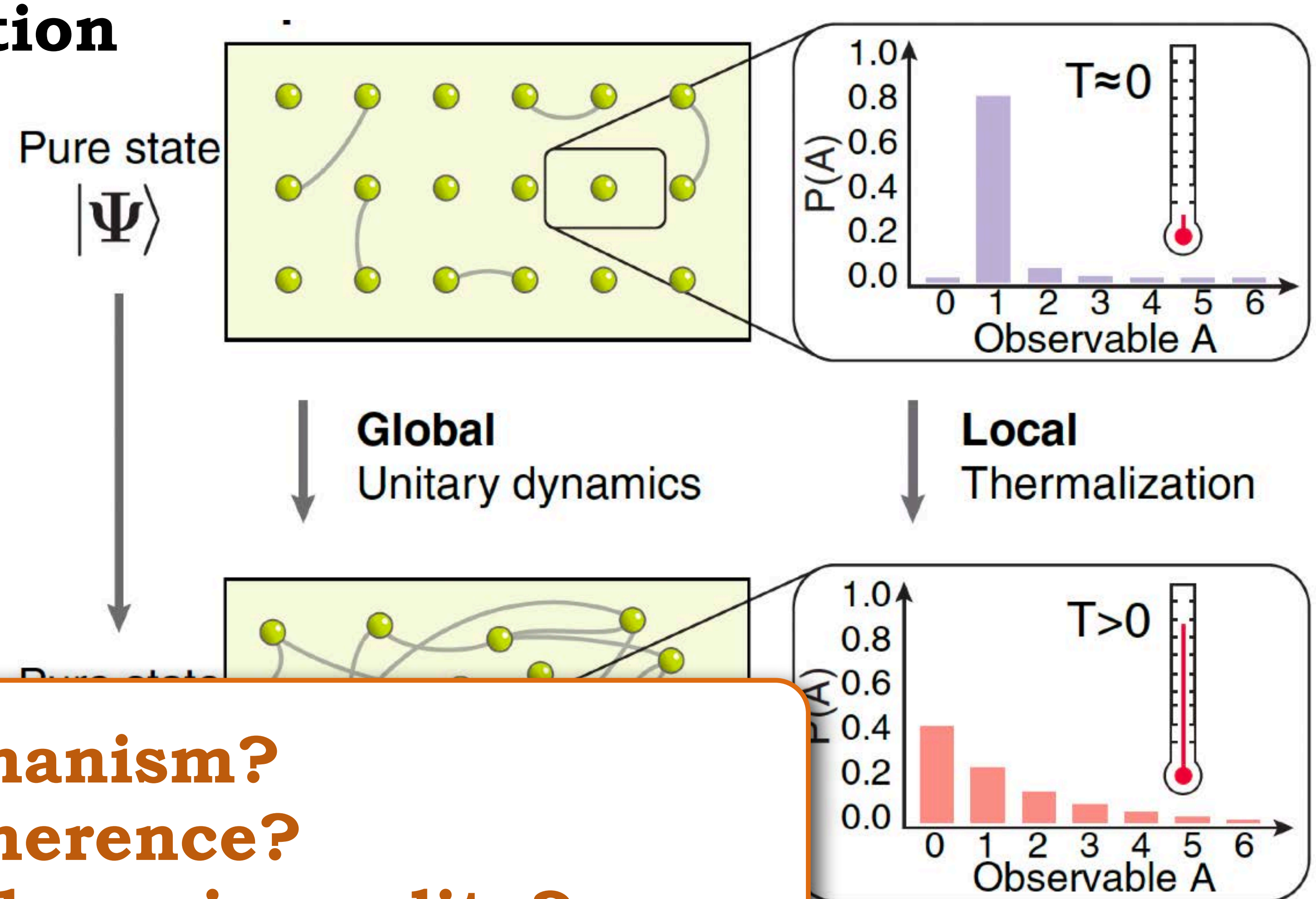

grey links: entanglement

➡**thermalization mechanism?**
➡**entanglement/decoherence?**
➡**few versus many body: universality?**
➡***talks by Qi Zhou and Artem Volosniev***

Physical question

How do black holes and other isolated quantum systems thermalize?

➡ ***talk by Aurel Bulgac (slower than ETH-thermalization)***

Outline

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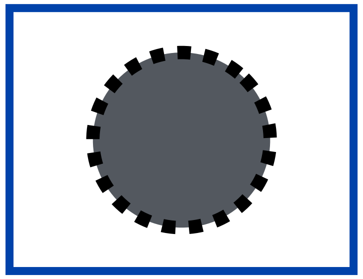
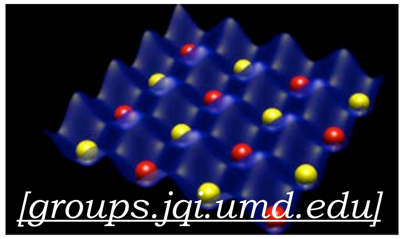
Ion collision/Ultracold atoms ~ Black hole formation?

? ~



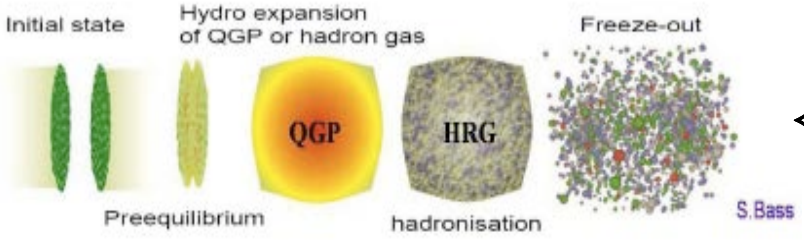


Exact correspondence: Holography (gauge/gravity)

***N=4* Super-Yang-Mills**
in **3+1 dimensions**
with *SU(N)* and λ
(gauge)

$\Leftrightarrow$

Typ II B Superstring Theory
in **(4+1)-dimensional**
Anti de Sitter space
(gravity)



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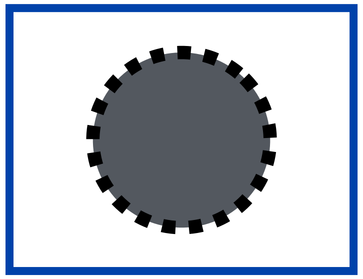

['t Hooft (1993)]
[Susskind; J.Math.Phys. (1995)]
[Maldacena; Adv.Theor.Math.Phys. (1997)]

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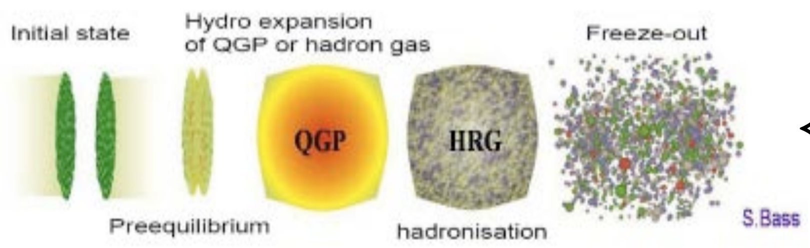

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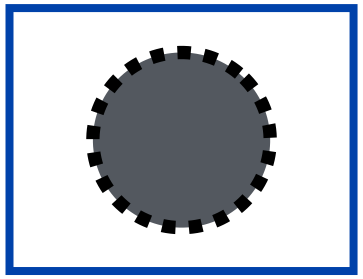

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[Susskind; J.Math.Phys. (1995)]
[Maldacena; Adv.Theor.Math.Phys. (1997)]

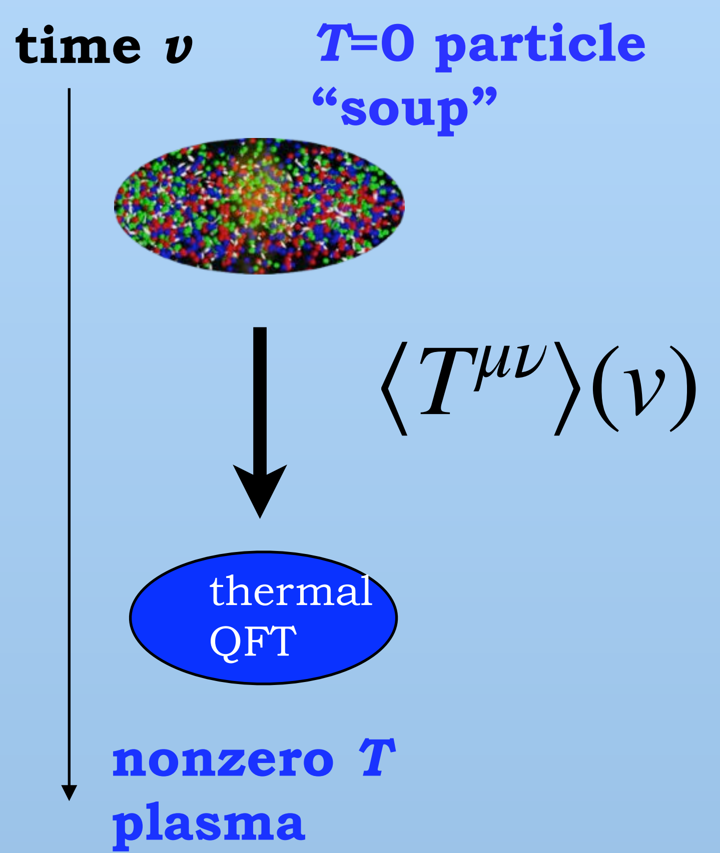
Black hole entropy grows as its **surface area** (not as its volume).

[Bekenstein]
[Hawking]

$$S \propto A_h$$

Far from equilibrium holography

Thermalization in field theory

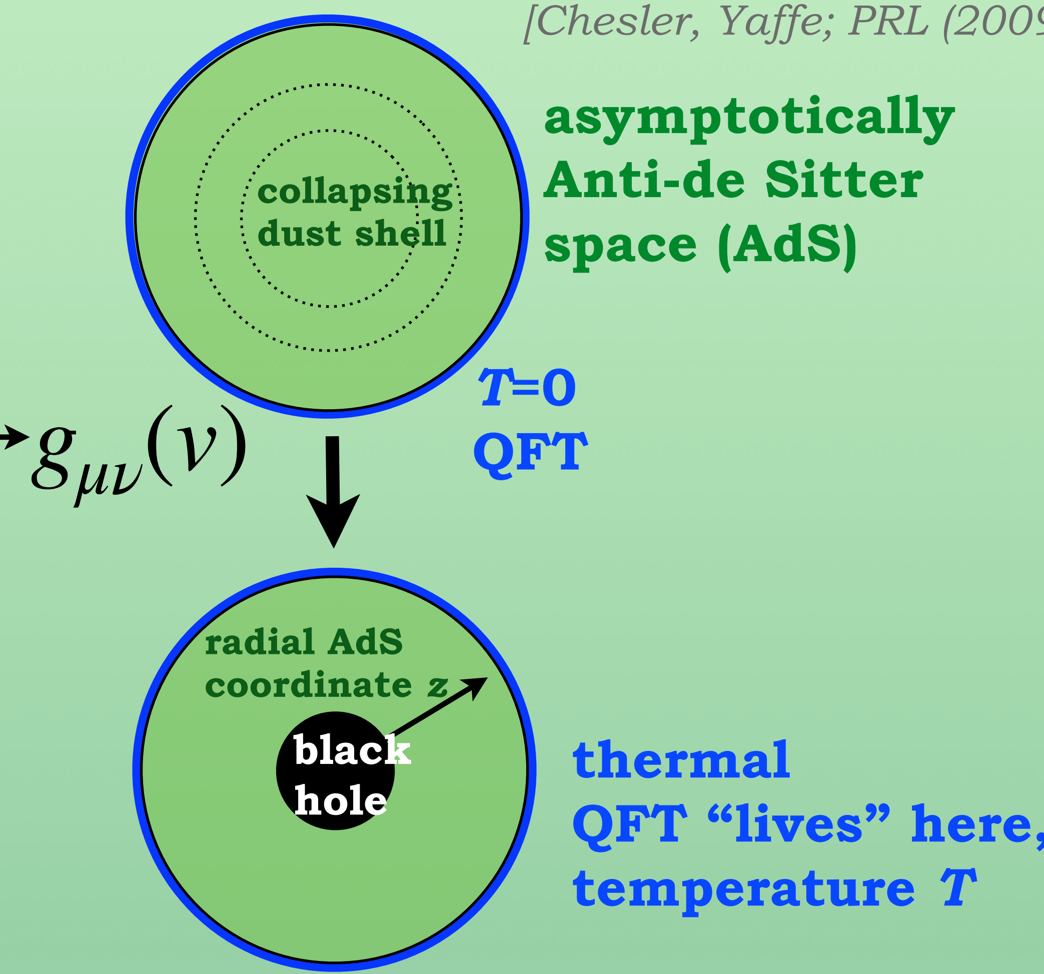


gauge/gravity correspondence

$\langle T^{\mu\nu}\rangle(v) \longleftrightarrow g_{\mu\nu}(v)$

Horizon formation in gravity

[Janik,Peschanski; PRD (2006)]
[Chesler, Yaffe; PRL (2009)]



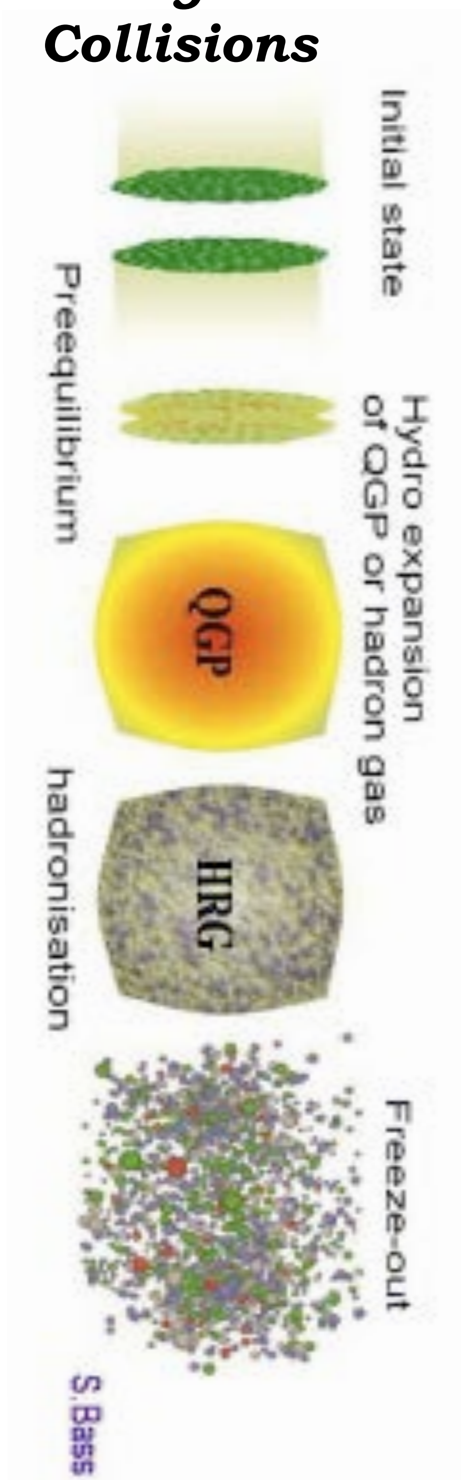
➡**exact/numerical thermalization results from holography**

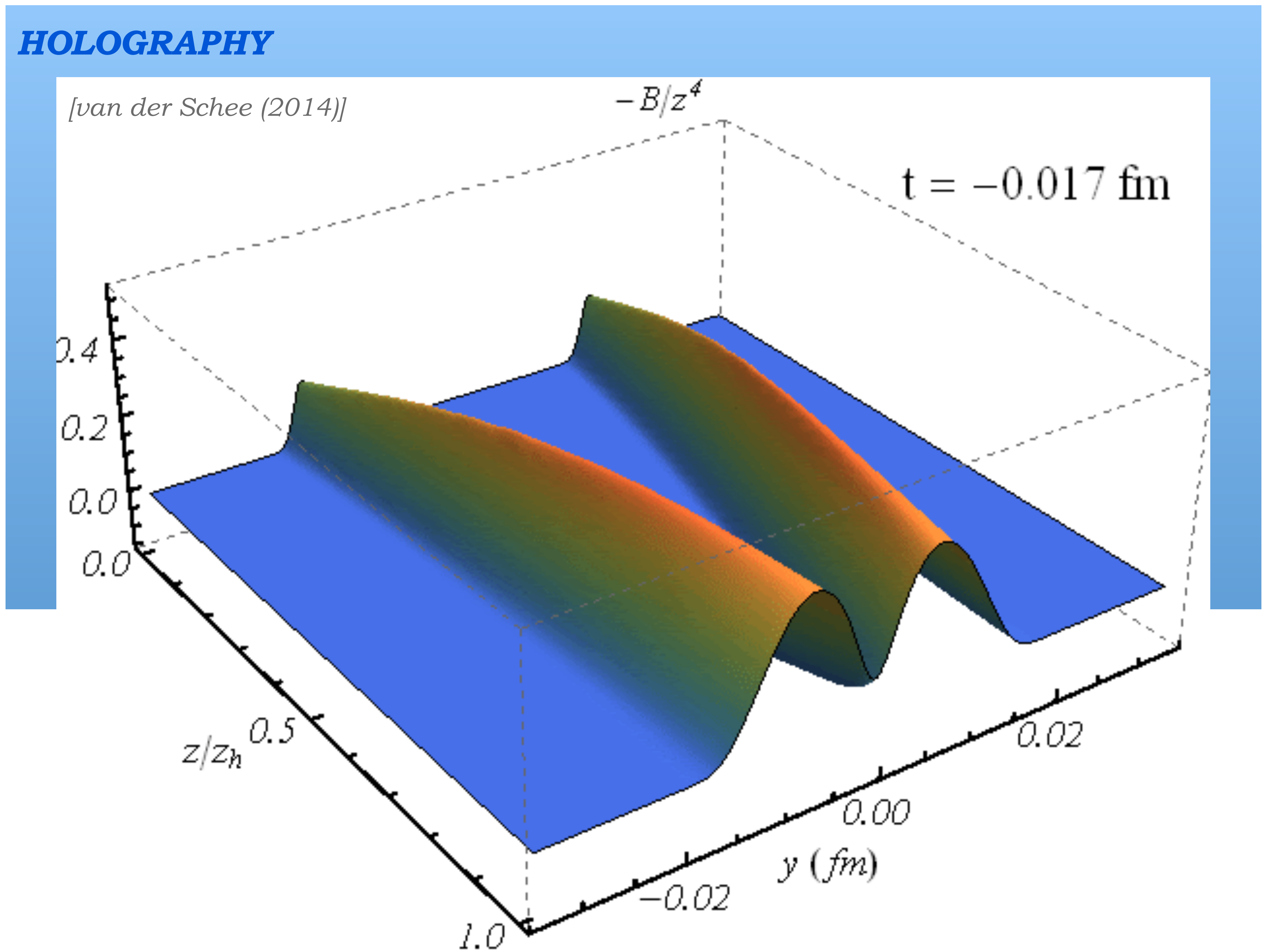
[Cartwright,Kaminski; JHEP (2019)]
[Cartwright,Kaminski; JHEP (2021)]
[Cartwright,Kaminski,Knipfer (2022)]
[Cartwright,Kaminski,Schenke; PRC (2022)]

Holographic heavy ion collision (numerical, large N)

time

Example: heavy ion Collisions





holographic idea: [Janik, Peschanski; PRD (2006)]

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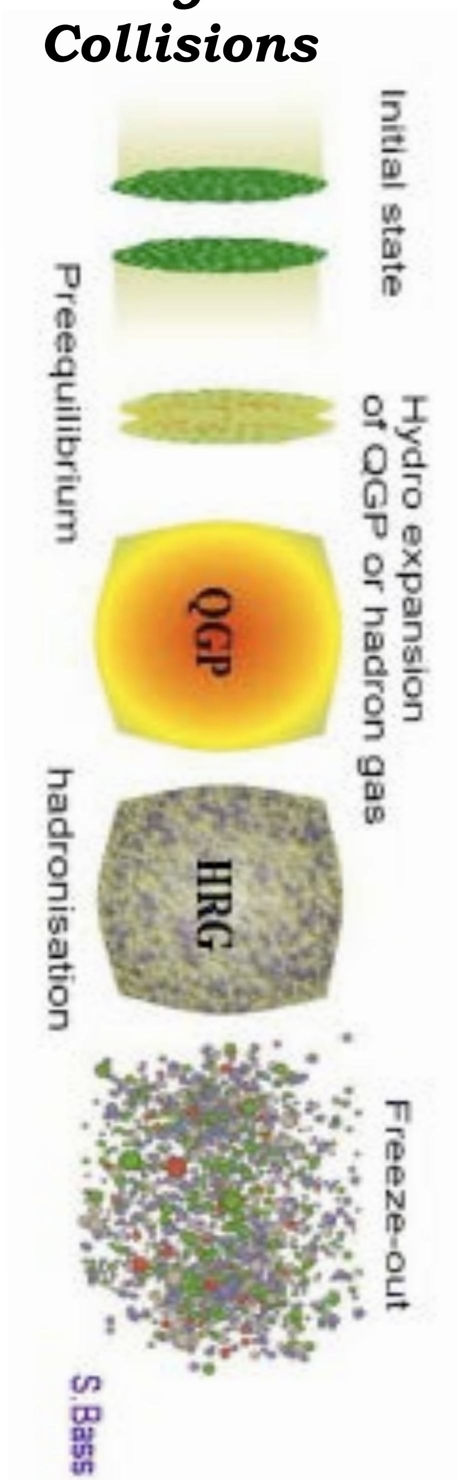
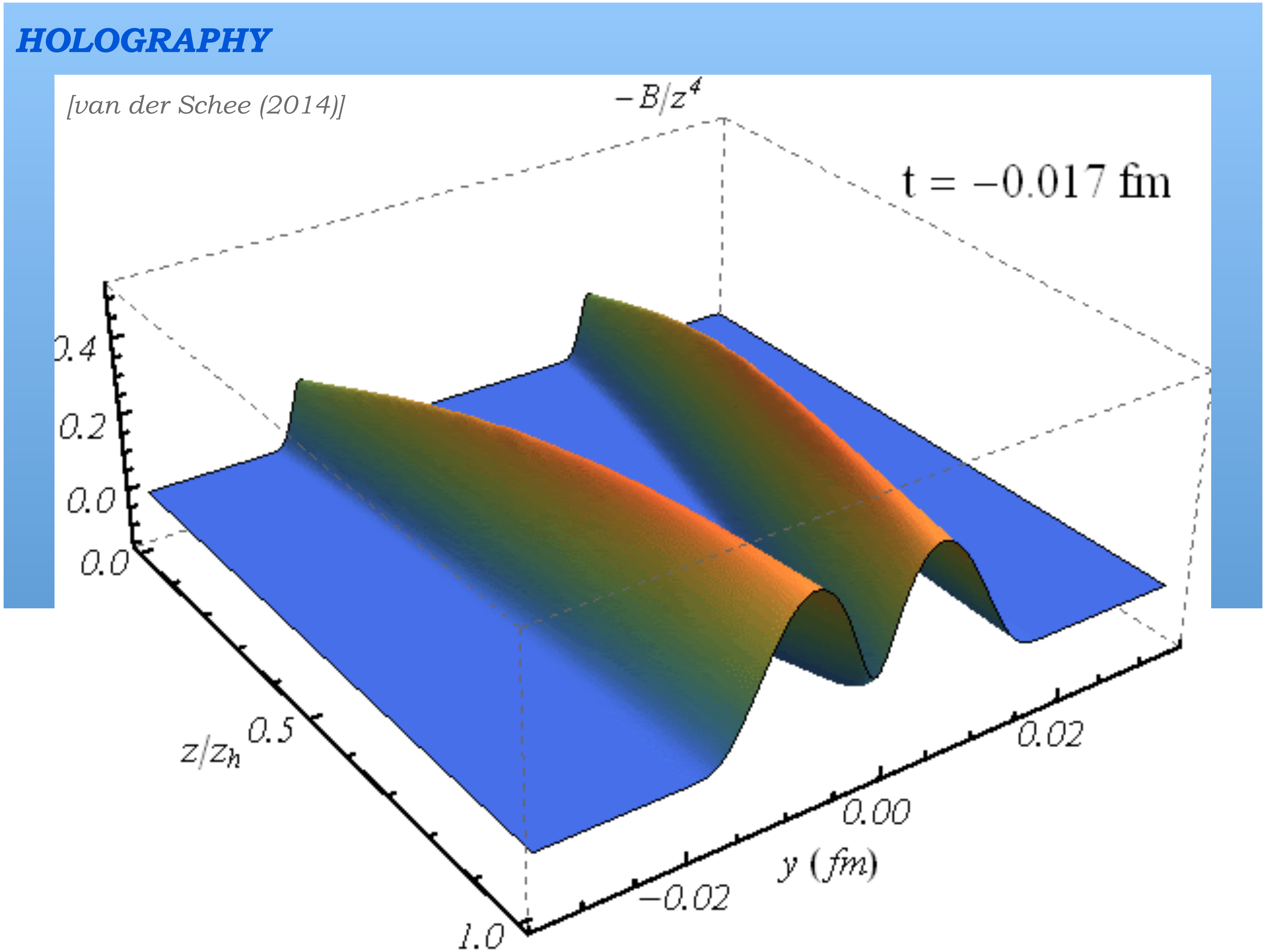
Example: heavy ion Collisions



time



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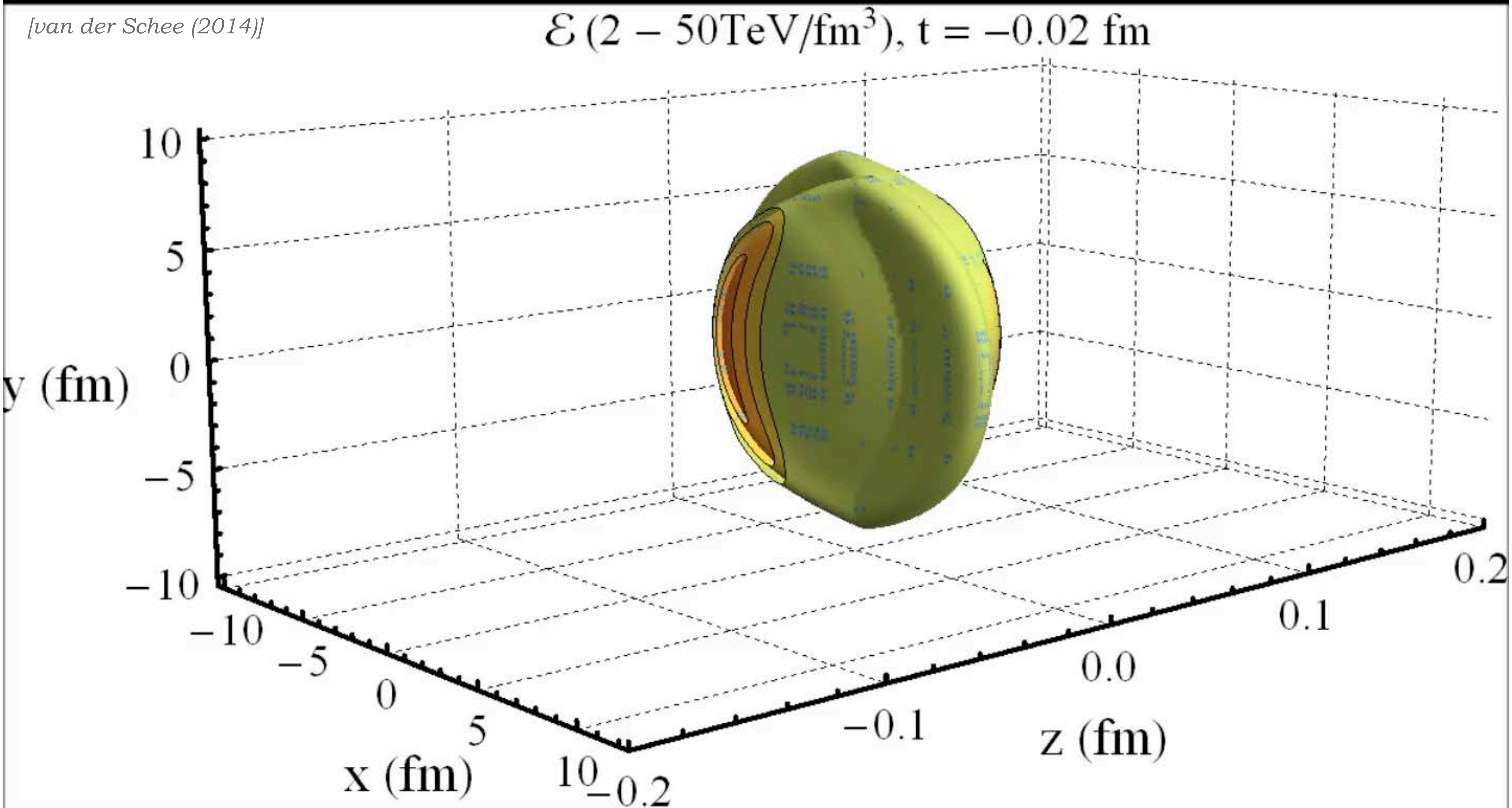
Off-center holographic heavy ion collision



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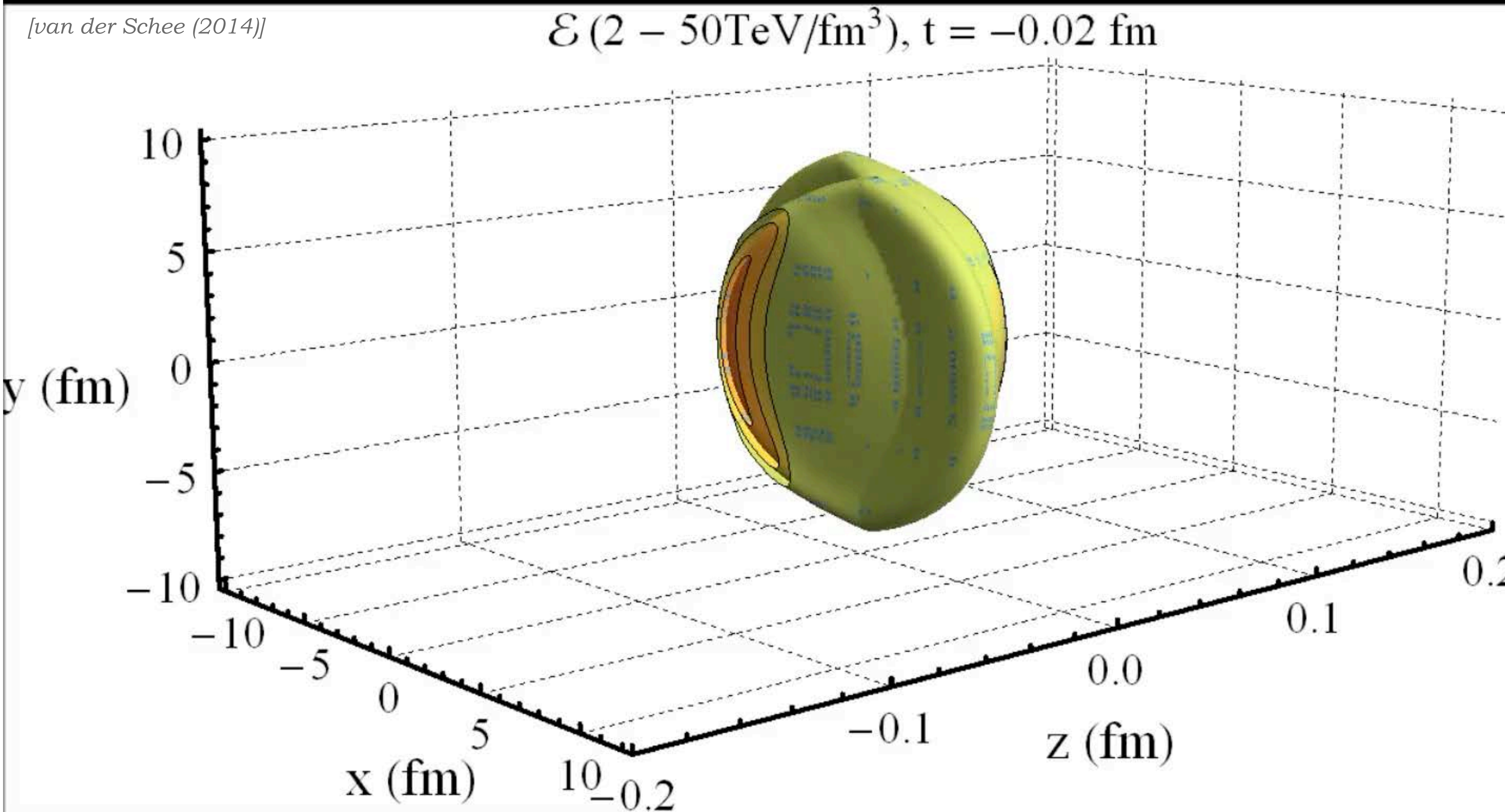

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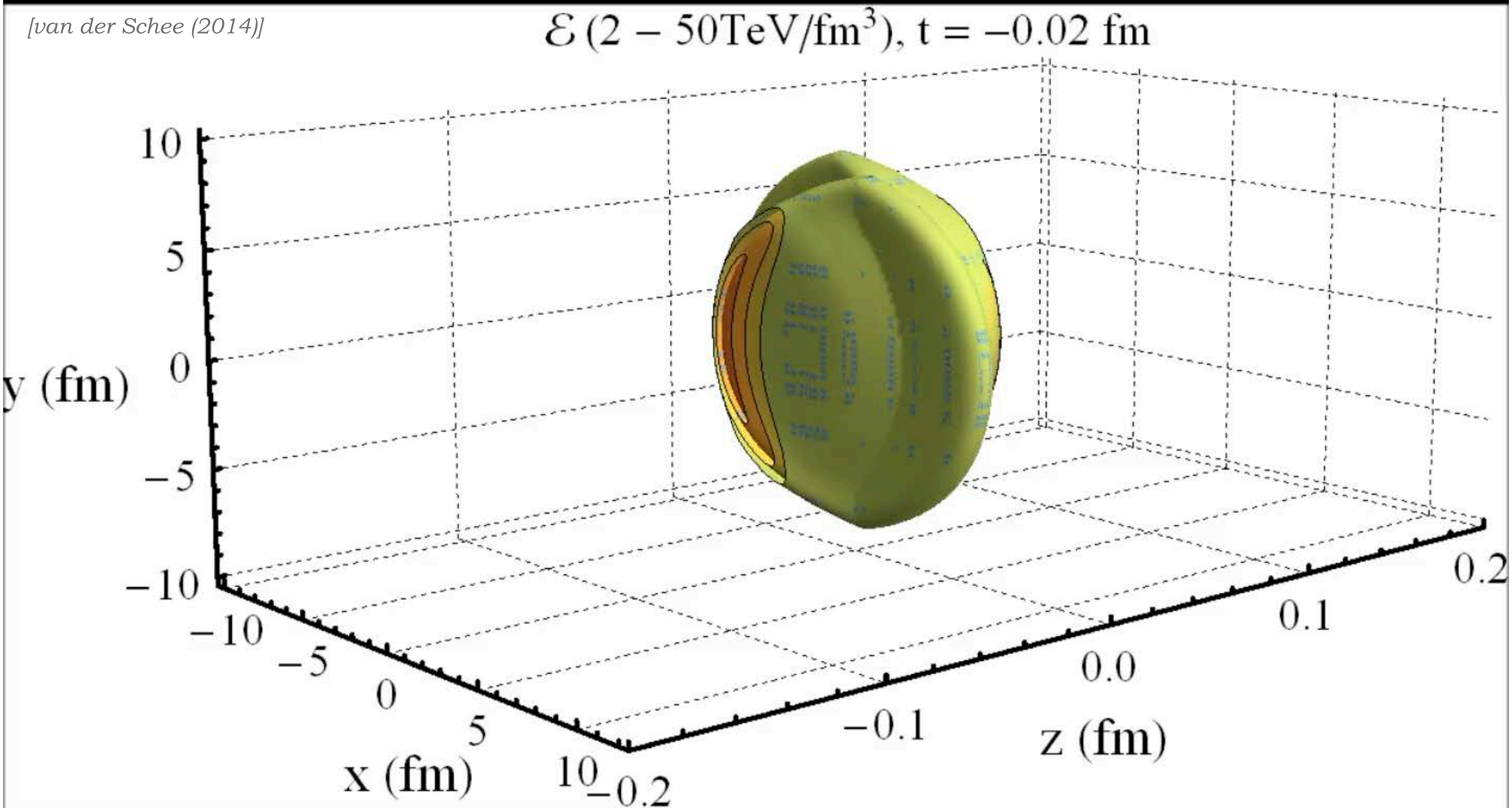
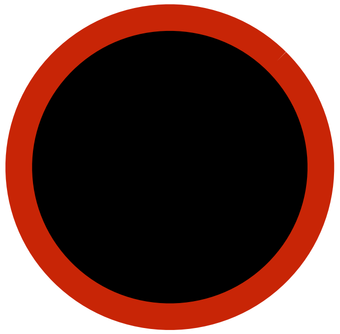


Outline

Entanglement entropy corresponds to minimal surface

RECALL: Black hole entropy grows as its surface area (not as its volume).

$$S \propto A_h$$



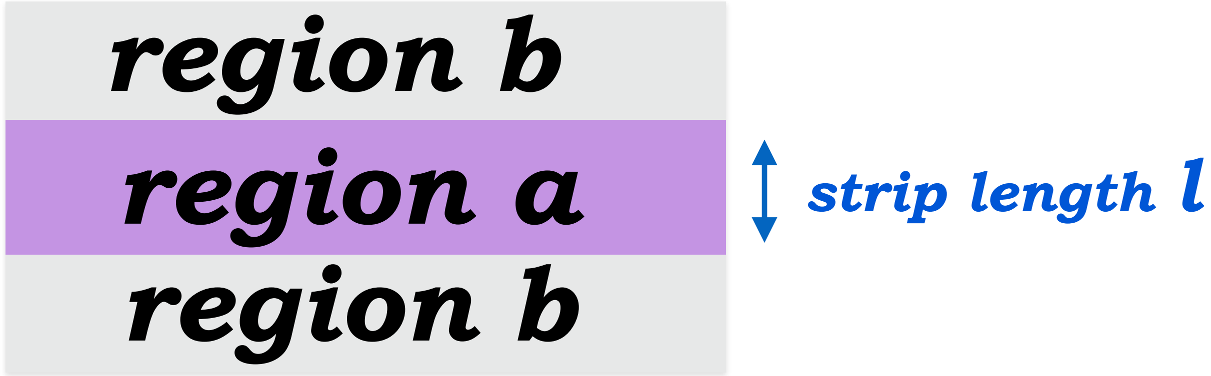
Entanglement entropy in quantum field theory corresponds to a particular minimal surface in gravity theory

[Ryu,Takayanagi; JHEP (2006)]

Calculation of entanglement entropy in 2D CFT from minimal surface in AdS3

Definition: entanglement entropy

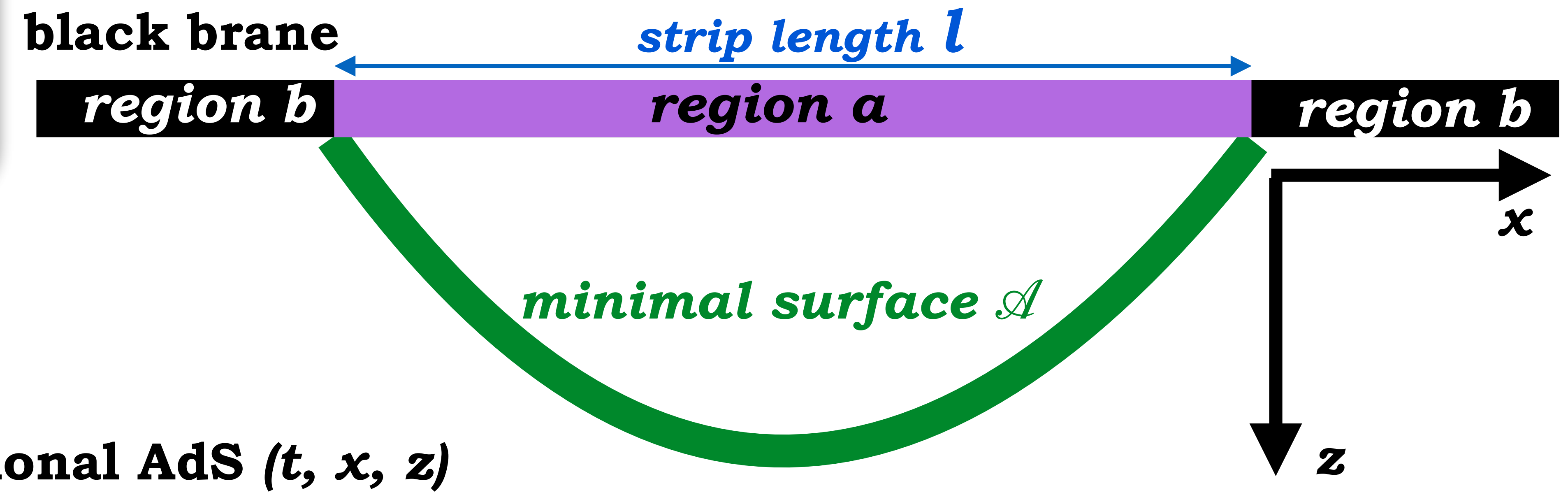
$$S_a = -\mathrm{Tr}\rho_a \log \rho_a , \qquad \rho_a = \mathrm{Tr}_b \left|\psi\right\rangle \left\langle\psi\right|$$



Holographically dual definition

[Ryu,Takayanagi; JHEP (2006)]

$$S_a = \frac{1}{4G}\mathscr{A}$$



Example: 3-dimensional AdS *(t, x, z)*
at a given time *t*,
minimal surface is shortest path
which is also called a geodesic

Calculation of entanglement entropy in 2D CFT from minimal surface in AdS3

Example: 3-dimensional AdS in Eddington-Finkelstein coordinates
(t, x, z) —> (v, x, z)

Calculate minimal surface = shortest path = geodesic

Metric: $$ds^2 = \frac{1}{z^2}\left(-dv^2 - 2dzdv + d\vec{x}^2\right)$$

Clever parametrization of surface:

$$X^\alpha(\sigma) \;=\; (Z(\sigma), V(\sigma), X(\sigma))$$

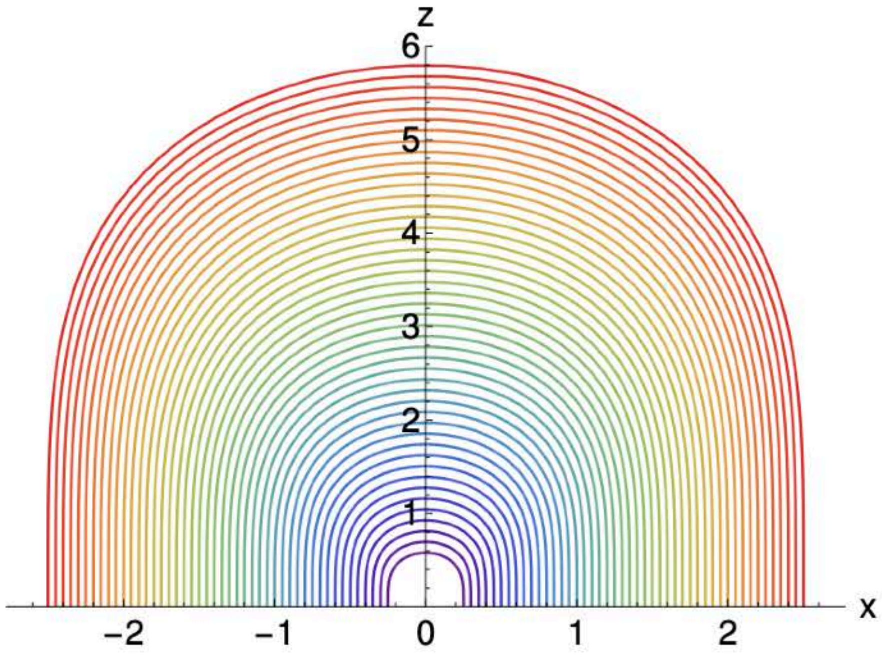
Geodesic equation:

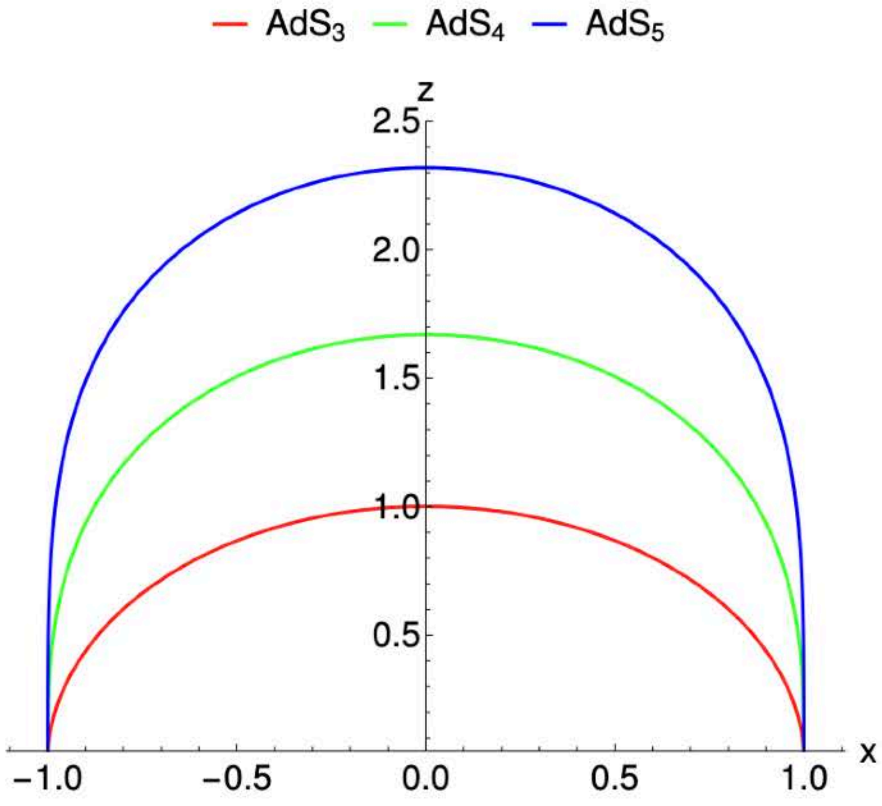
$$\ddot{X}^\alpha(\sigma) + \Gamma^\alpha_{\beta\gamma}(X^\delta(\sigma))\dot{X}^\beta(\sigma)\dot{X}^\gamma(\sigma) = J(\sigma)\dot{X}^\alpha(\sigma)$$

Solution: minimal surfaces

$$\begin{aligned} Z(\sigma) &= \frac{l}{2}\left(1-\sigma^2\right), \\ V(\sigma) &= v_0 - Z(\sigma), \\ X(\sigma) &= \frac{l}{2}\sigma\sqrt{2-\sigma^2} \end{aligned}$$

in AdS5, different *l*



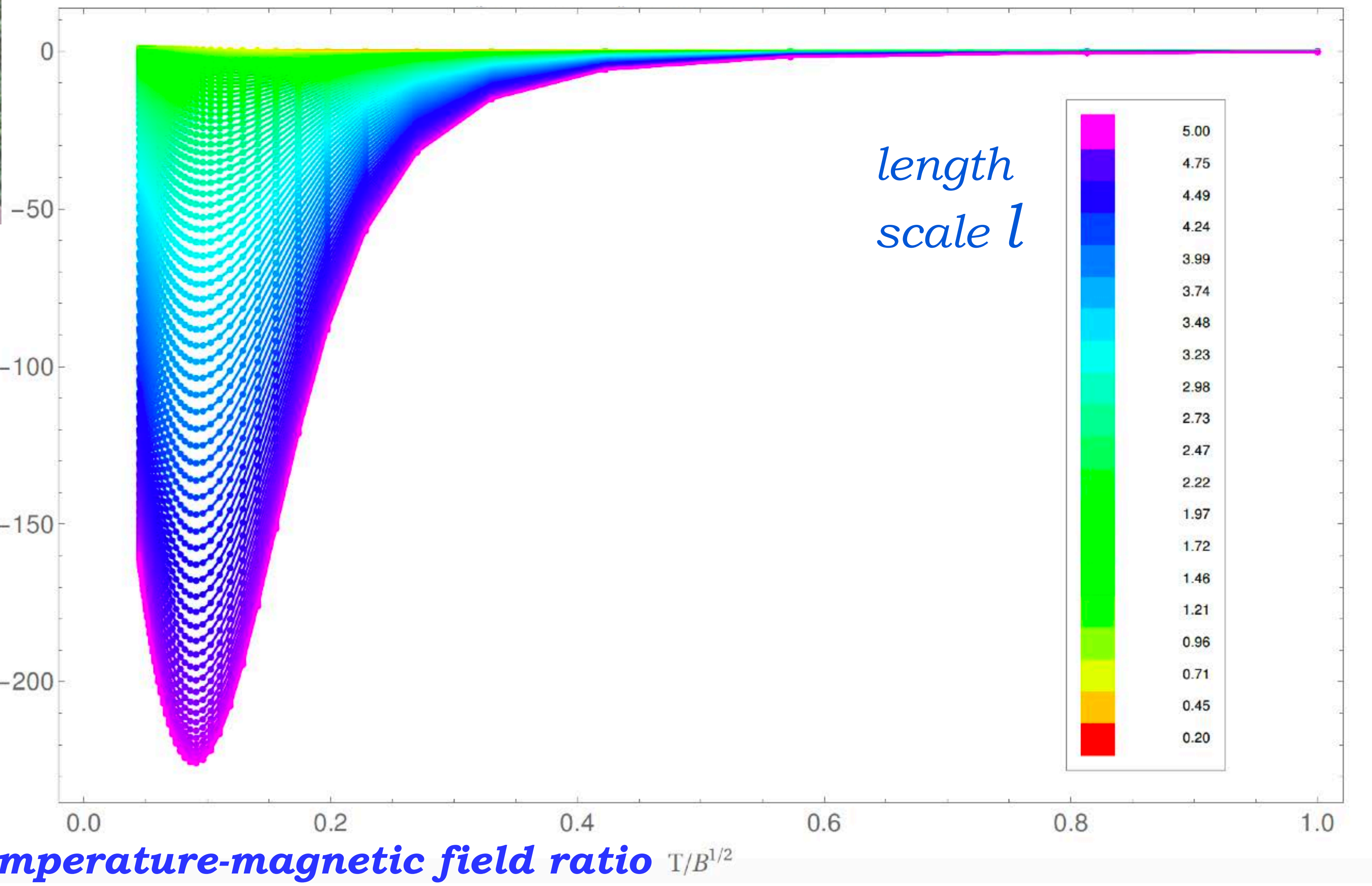


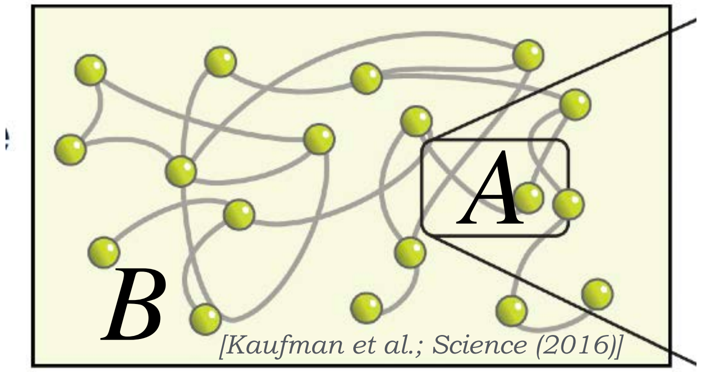
Effect of chiral anomaly and magnetic field on entanglement

Calculation: strongly coupled *N*=4 Super-Yang-Mills theory in strong *B*; compute minimal surfaces in AdS5

Entropic c-function (with anomaly minus without anomaly)

*[**Cartwright**, Kaminski; JHEP (2021)]*





Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\, \rho$$

Entanglement entropy:

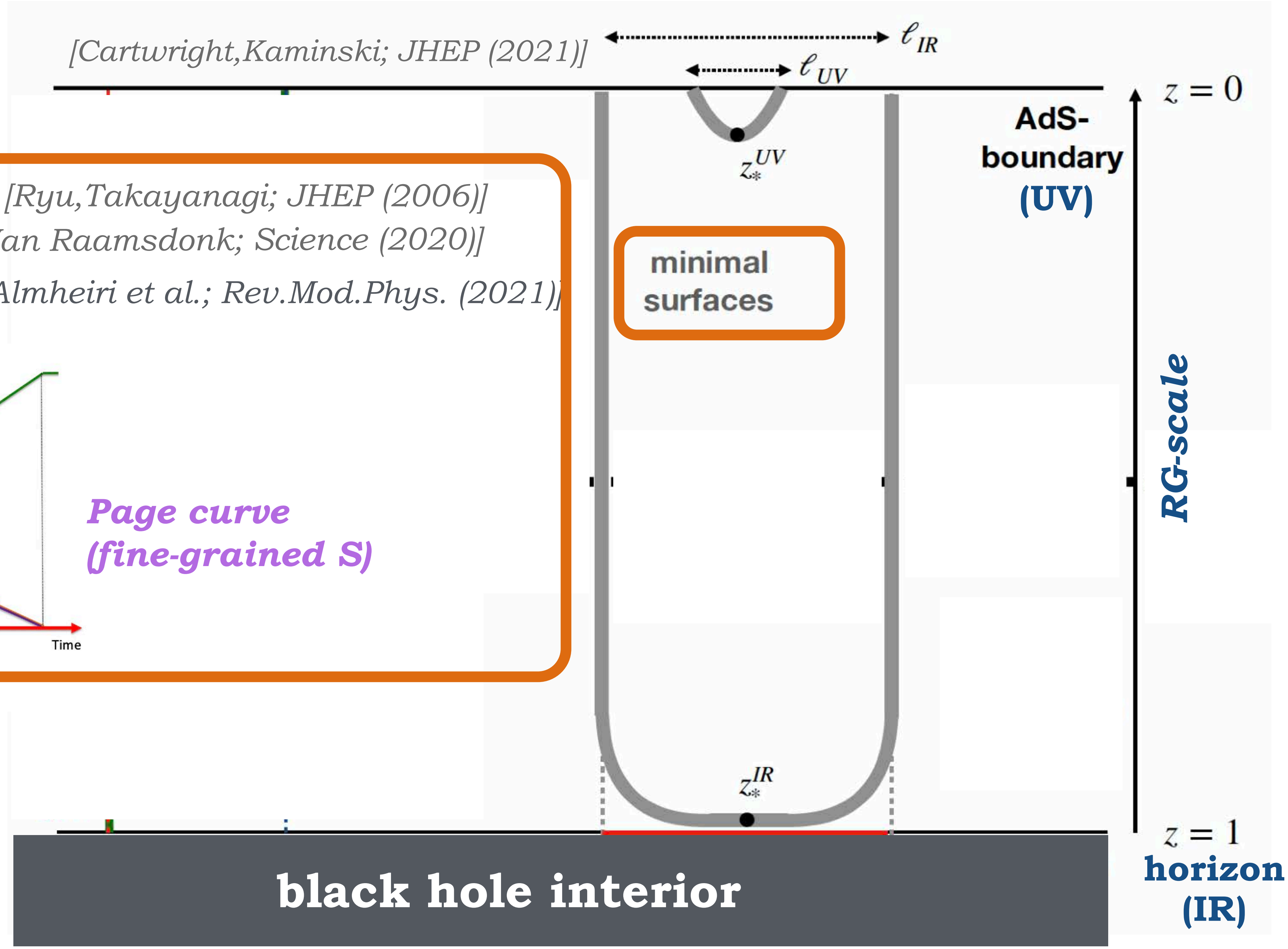
$$S_A = -\,\mathrm{tr}\left(\rho_A \log \rho_A\right)$$

Entropic c-function:

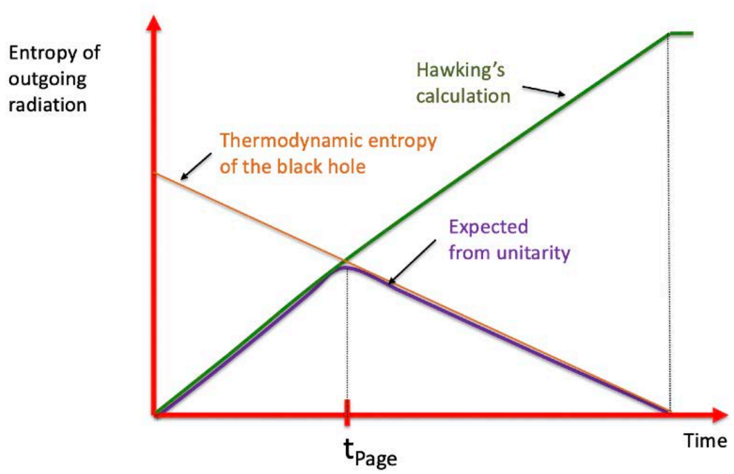
$$c \propto \frac{\partial S_A}{\partial l}$$

➡anomaly effect peaked at $T/B^{1/2} = 0.1$

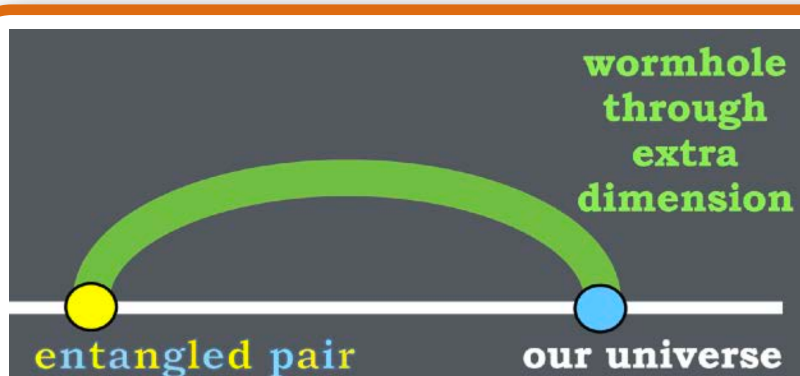
Geometric picture: three faces of *minimal surfaces*



1) ***Entanglement entropy:*** *[Ryu,Takayanagi; JHEP (2006)]*
2) ***Emergent spacetime:*** *[Van Raamsdonk; Science (2020)]*
3) ***Fine-grained entropy:*** *[Almheiri et al.; Rev.Mod.Phys. (2021)]*



Page curve (fine-grained S)



➡ Entanglement = Spacetime

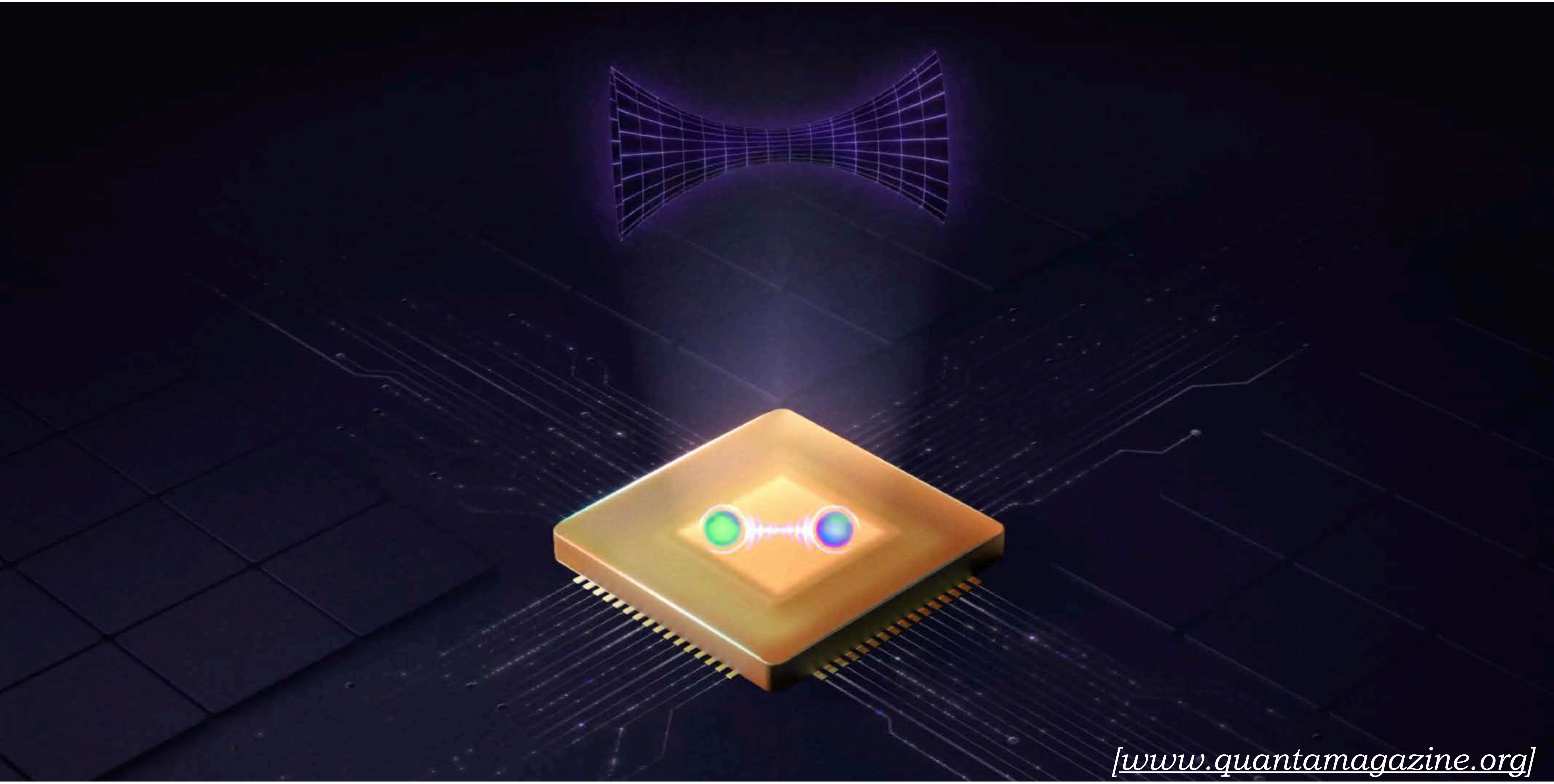
[Van Raamsdonk; Gen.Rel.Grav. (2010)]

Outline

1. Statistical versus quantum mechanic dynamics
2. Holography (far from equilibrium)
3. Holographic entanglement entropy (calculation)
4. **Quantum gravity experiments**
   - **indirect: traversable wormhole on quantum computer**
   - **direct: gauge/gravity correspondence on electric circuit board**

Indirect Experiment: Simulation on Quantum Computer

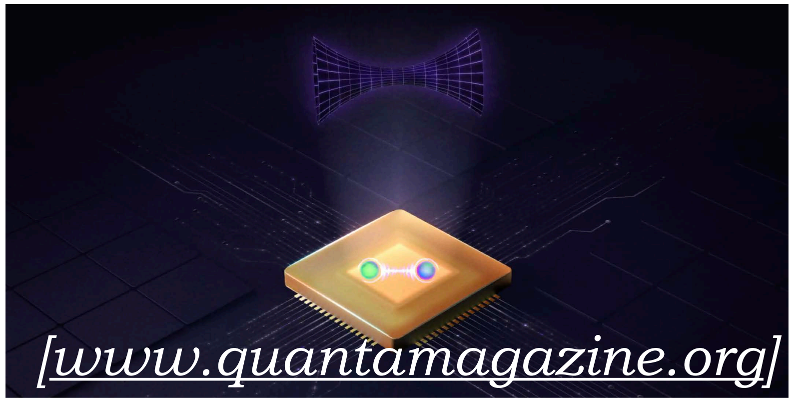
[Jafferis et al.; Nature (2022)]



[www.quantamagazine.org]

Examine spacetime with "quantum computing glasses".

Indirect Experiment: Simulation on Quantum Computer


[www.quantamagazine.org]

[Jafferis et al.; Nature (2022)]

IBM/Quantinuum Competitors:
[Shapoval et al.; Quantum (2023)]

Principle

Quantumgravity problem

send a signal through a wormhole in Jackiw-Teitelboim (JT) Gravity

→ **gauge/gravity correspondence** →

Quantum mechanics problem

quantum teleportation protocol in Sachdev-Ye-Kitaev (SYK) model

↓ **machine learning: learn SYK** ↓

Simplified SYK problem

machine represents problem???

(Hamiltonian of seven Majorana fermions with five fully-commuting terms)

← **Google's Sycamore solves machine-simplified problem** ←

Signal!

SYK Model of Majorana Fermions

from [Jafferis et al.; Nature (2022)]

Here, we study the dynamics of traversable wormholes through a many-body simulation of an SYK system of N fermions[2,3]. The traversable wormhole protocol is equivalent to a quantum teleportation protocol in the large-N semiclassical limit (Fig. 1c). Explicitly, given left and right Hamiltonians H_L and H_R with N Majorana fermions ψ on each side, the SYK model with q couplings is given by

$$H_{L,R} = \sum_{1 \le j_1 < \cdots < j_q \le N} J_{j_1 \ldots j_q} \psi^{j_1}_{L,R} \cdots \psi^{j_q}_{L,R}, \qquad (1)$$

where the couplings are chosen from a Gaussian distribution with mean zero and variance $J^2(q-1)!/N^{q-1}$. We choose $q = 4$ and demonstrate gravitational physics at sufficiently small N, sparsifying $J_{j_1 \ldots j_q}$ to enable experimental implementation.

Applying the learning process, we produce a large population of sparse Hamiltonians showing the appropriate interaction sign dependence (Fig. 2a). We select the learned Hamiltonian

$$\begin{aligned} H_{L,R} = & -0.36\psi^1\psi^2\psi^4\psi^5 + 0.19\psi^1\psi^3\psi^4\psi^7 - 0.71\psi^1\psi^3\psi^5\psi^6 \\ & +0.22\psi^2\psi^3\psi^4\psi^6 + 0.49\psi^2\psi^3\psi^5\psi^7, \end{aligned} \qquad (3)$$

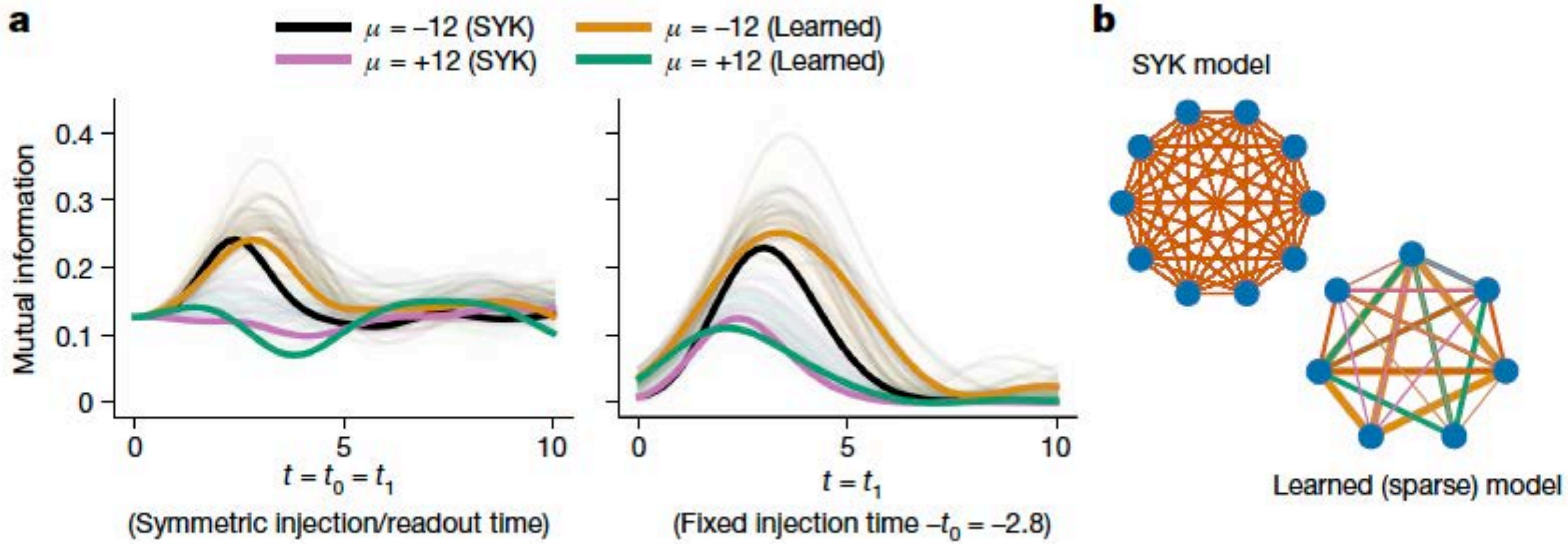
which requires seven of the original $N = 10$ SYK model fermions, where ψ^j denotes the Majorana fermions of either the left or the right systems.

Machine Learning (ML): Simplify the Problem

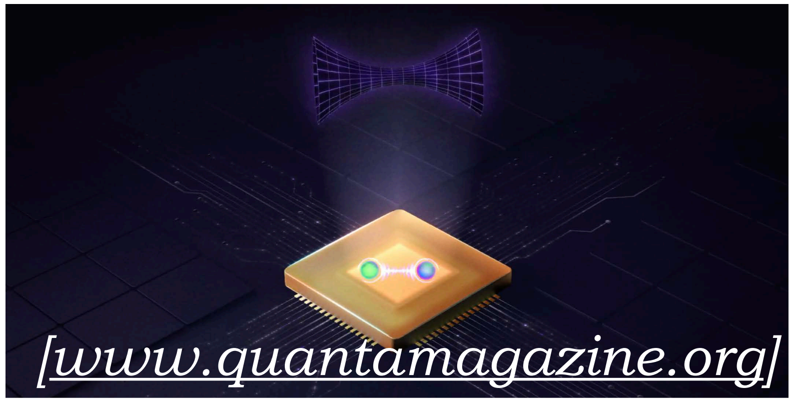
[Jafferis et al.; Nature (2022)]

Use of ML

- use machine learning techniques to construct a sparsified SYK model, experimentally realized with 164 two-qubit gates on a nine-qubit circuit



Potential shortcomings of Jafferis et al.'s experiment



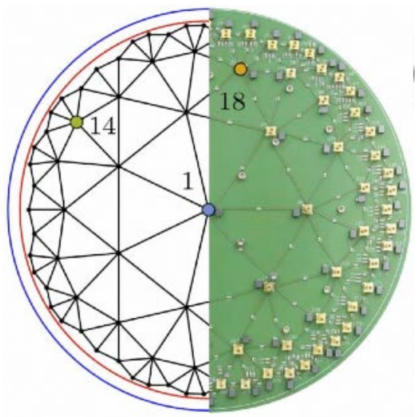
[Jafferis et al.; Nature (2022)]

Comment criticizing this experiment

[Kobrin/Schuster/Yao; preprint (2023)]

- **Problem 1:** learned Hamiltonian does not exhibit thermalization
- **Problem 2:** resembles SYK only for operators used in ML training
- **Problem3:** perfect size winding is generic feature of small-size models

see also response to criticism: [Jafferis et al.; (2023)]



Principle

Quantumgravity problem

send a signal through a wormhole in Jackiw-Teitelboim (JT) Gravity

gauge/ gravity corres- pondence →

Quantum mechanics problem

quantum teleportation protocol in Sachdev-Ye-Kitaev (SYK) model

↓ **machine learning: learn SYK**

Simplified SYK problem

machine represents problem???

← **Google's Sycamore solves machine-simplified problem**

↓ **Quantumgravity quantum information on quantum circuit**

Signal!

Direct Experiment: Black Hole on Electrical Circuit

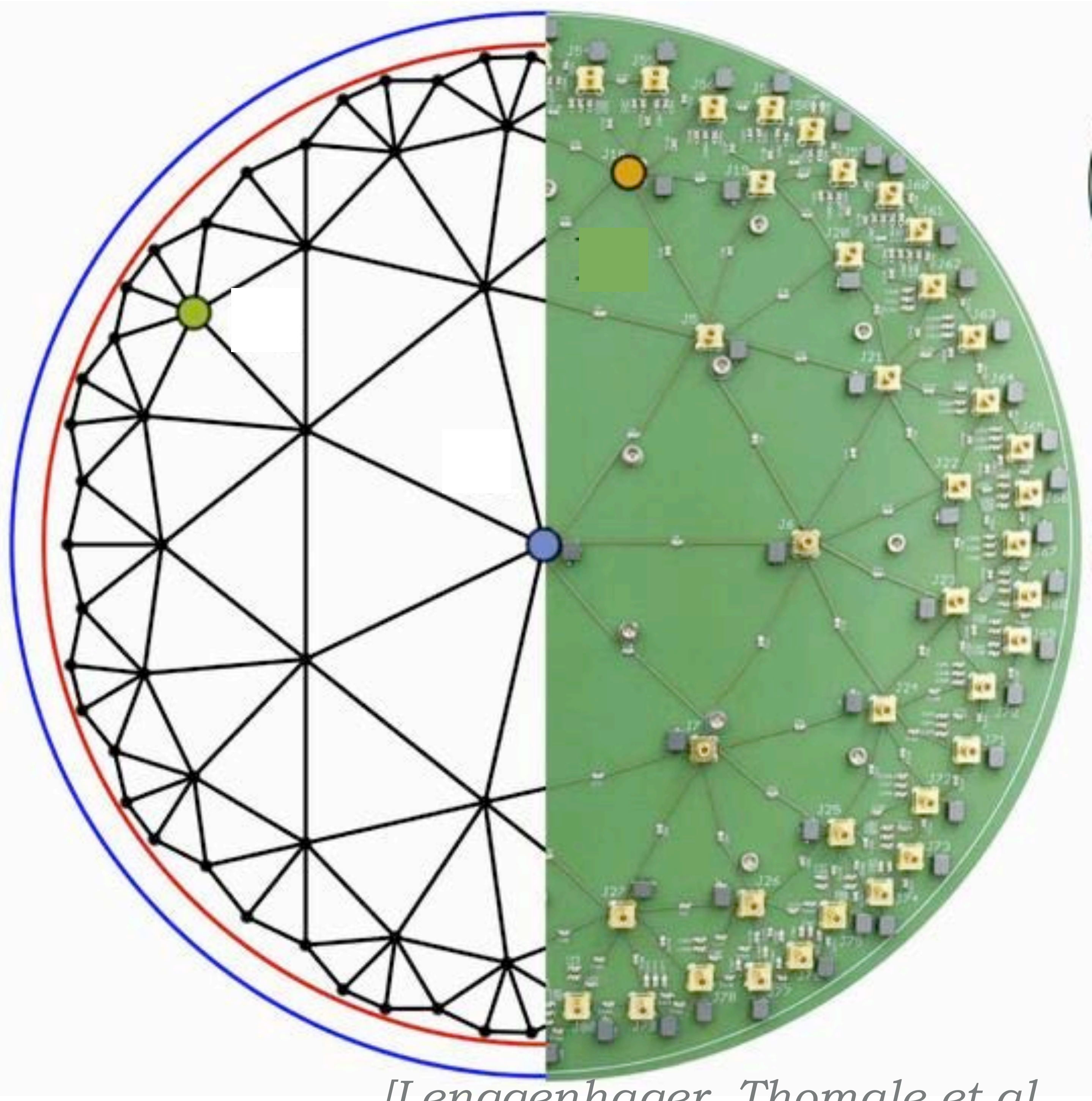
[Dey, Chen, Kaminski, et al.; PRL (2024)]

Model black hole on electrical hyperbolic circuit board

Voltage on circuit satisfies Klein-Gordon equation for massive scalar in AdS

[Basteiro et al.; PRL (2023)]

Testing holography in the lab

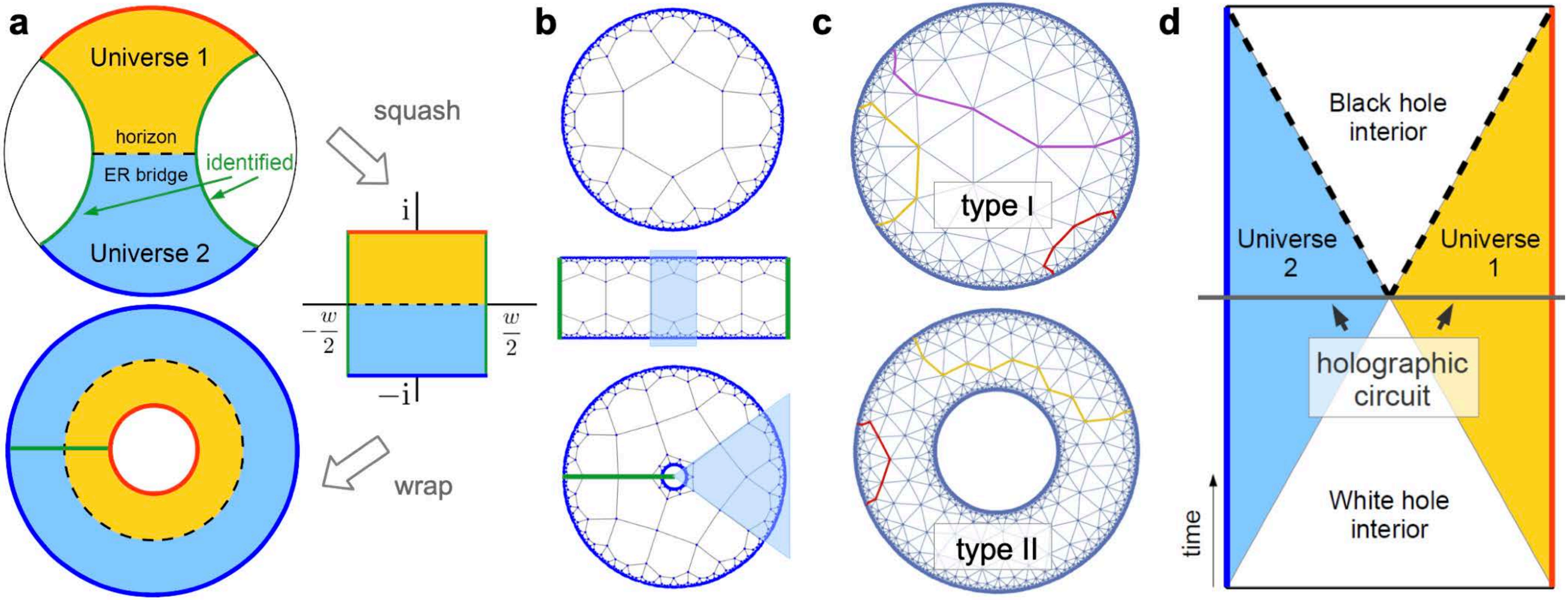


[Lenggenhager, Thomale et al., Nat. Commun. (2022)]

Direct Experiment: Black Hole on Electrical Circuit

[Dey, Chen, Kaminski, et al.; PRL (2024)]

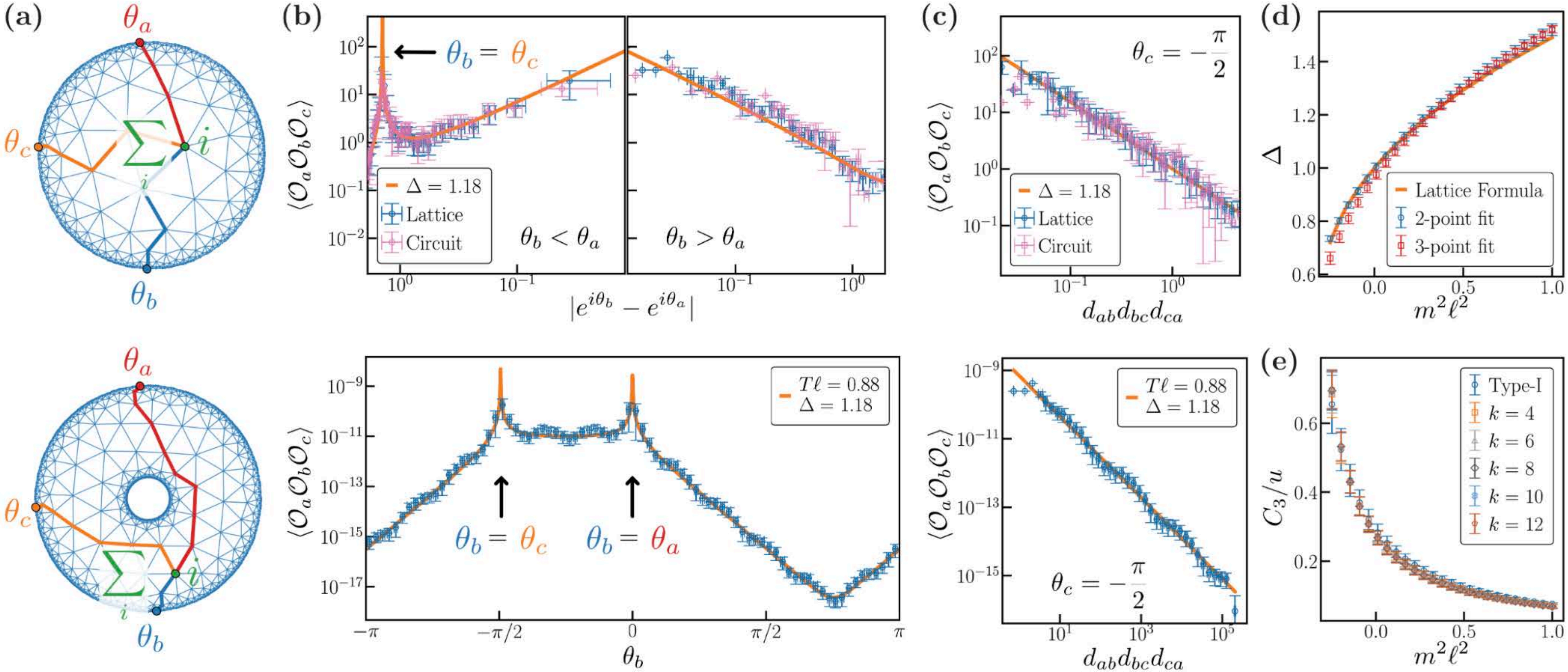
Realize wormhole on classical electric circuit:



Direct Experiment: Black Hole on Electrical Circuit

[Dey, Chen, Kaminski, et al.; PRL (2024)]

Three point functions measured on circuit (gravity side):



CFT expectation (gauge side):

$$\langle \mathcal{O}_a \mathcal{O}_b \mathcal{O}_c \rangle \simeq \frac{C_3}{(d_{ab} d_{ac} d_{bc})^{\Delta}} \qquad d_{ab} = \begin{cases} |e^{i\theta_a} - e^{i\theta_b}| & \text{(type-I)} \\ \frac{\sinh(\pi T \ell |\theta_a - \theta_b|)}{\pi T \ell} & \text{(type-II)} \end{cases}$$

Discussion

SUMMARY

- isolated quantum systems thermalize in similar ways
- entanglement = spacetime

OUTLOOK

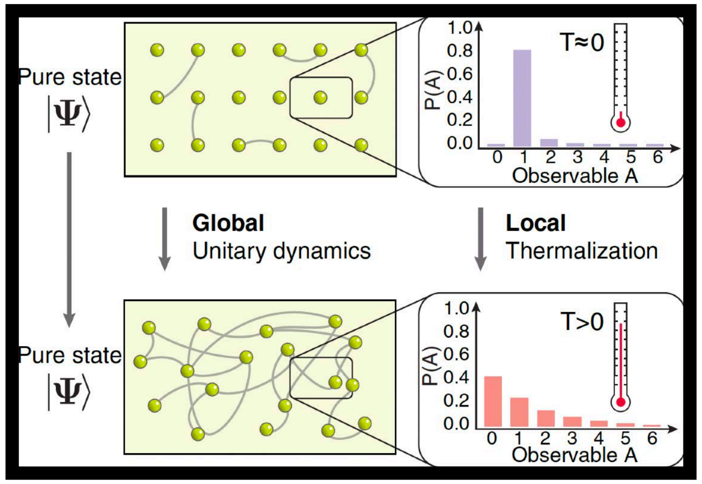
- entanglement entropy in holography far-from-equilibrium
- finite *N* corrections: get closer to few-body dynamics
- demonstrate “entanglement=spacetime” in experiments

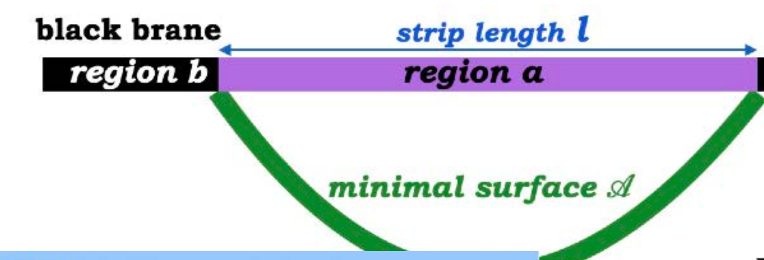
  (spacetime emerging from entangled quantum bits?)

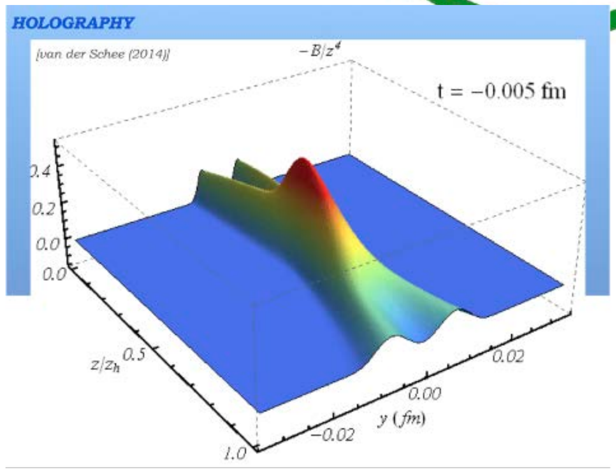
  ➡improve indirect traversable wormhole on quantum computer

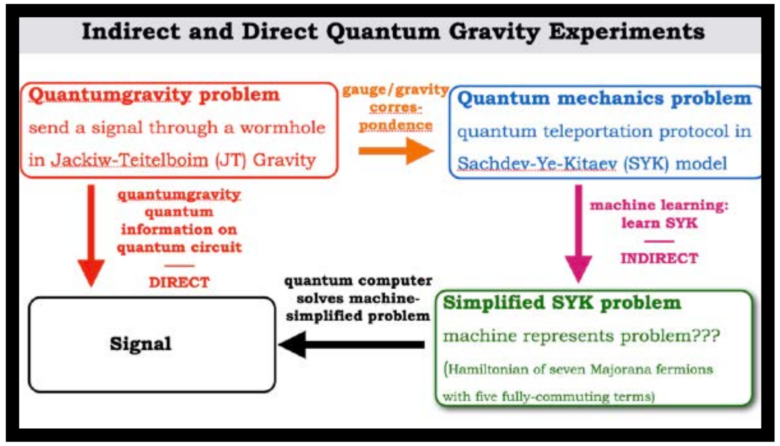
  ➡directly simulate quantum gravity on quantum computer
- use machine learning methods (experiments, ML spacetime)

  ➡*talk by Jane Kim*

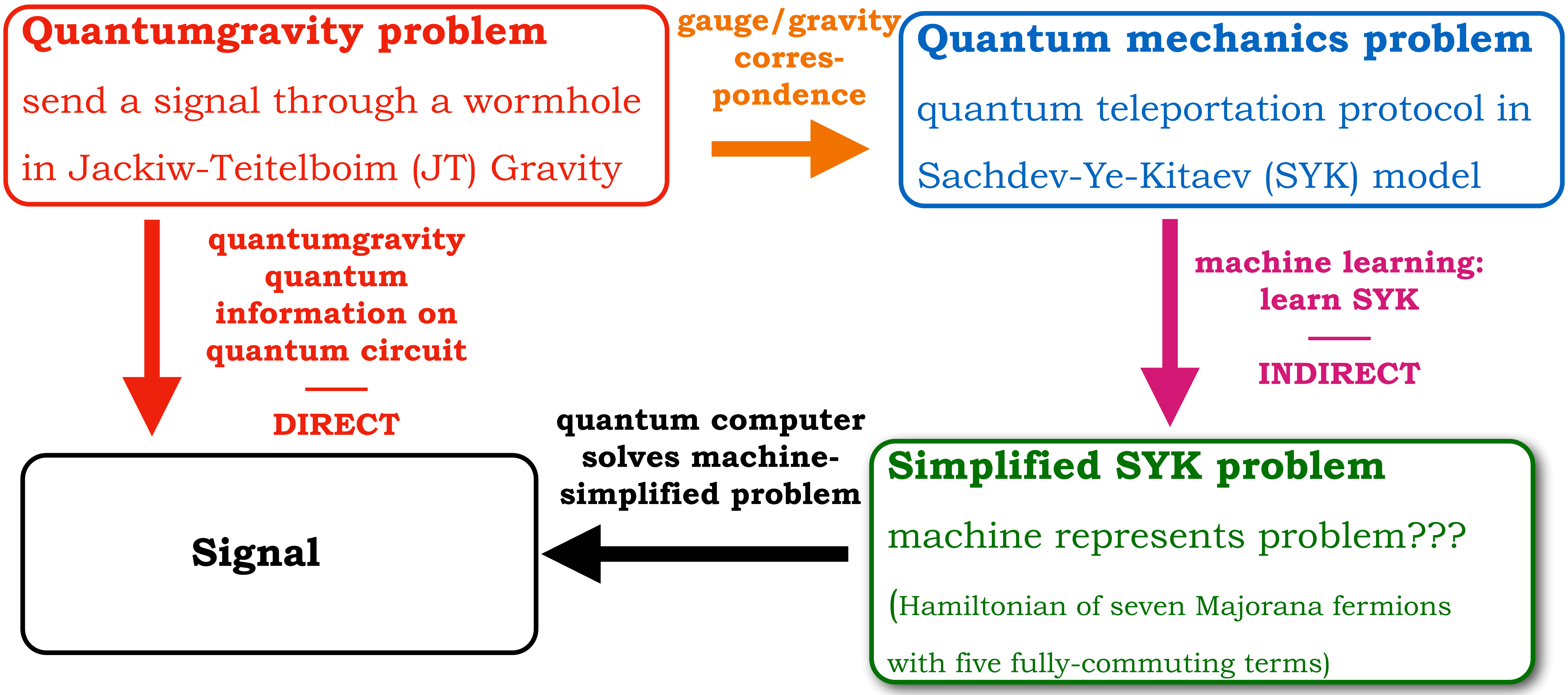




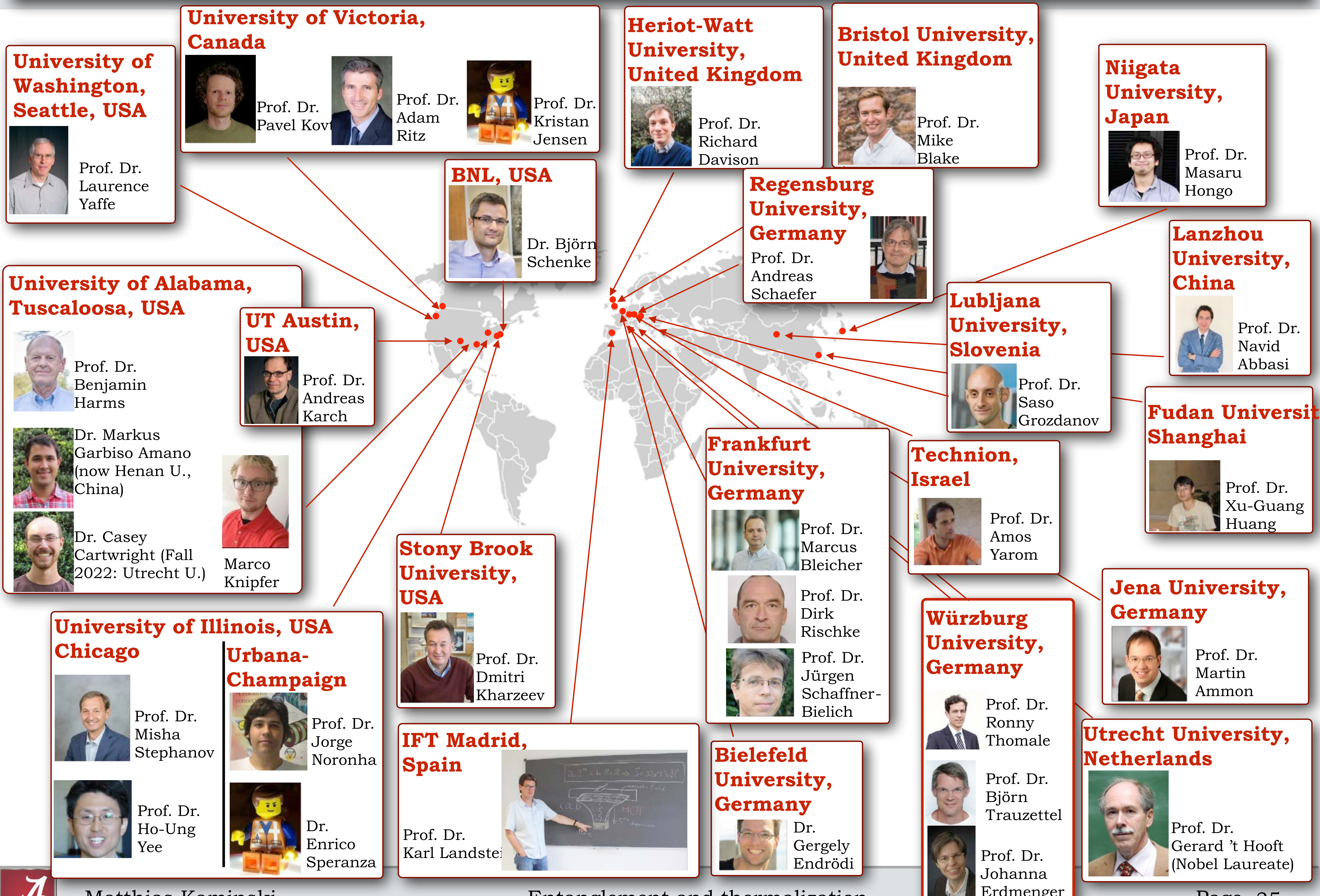




Indirect and Direct Quantum Gravity Experiments



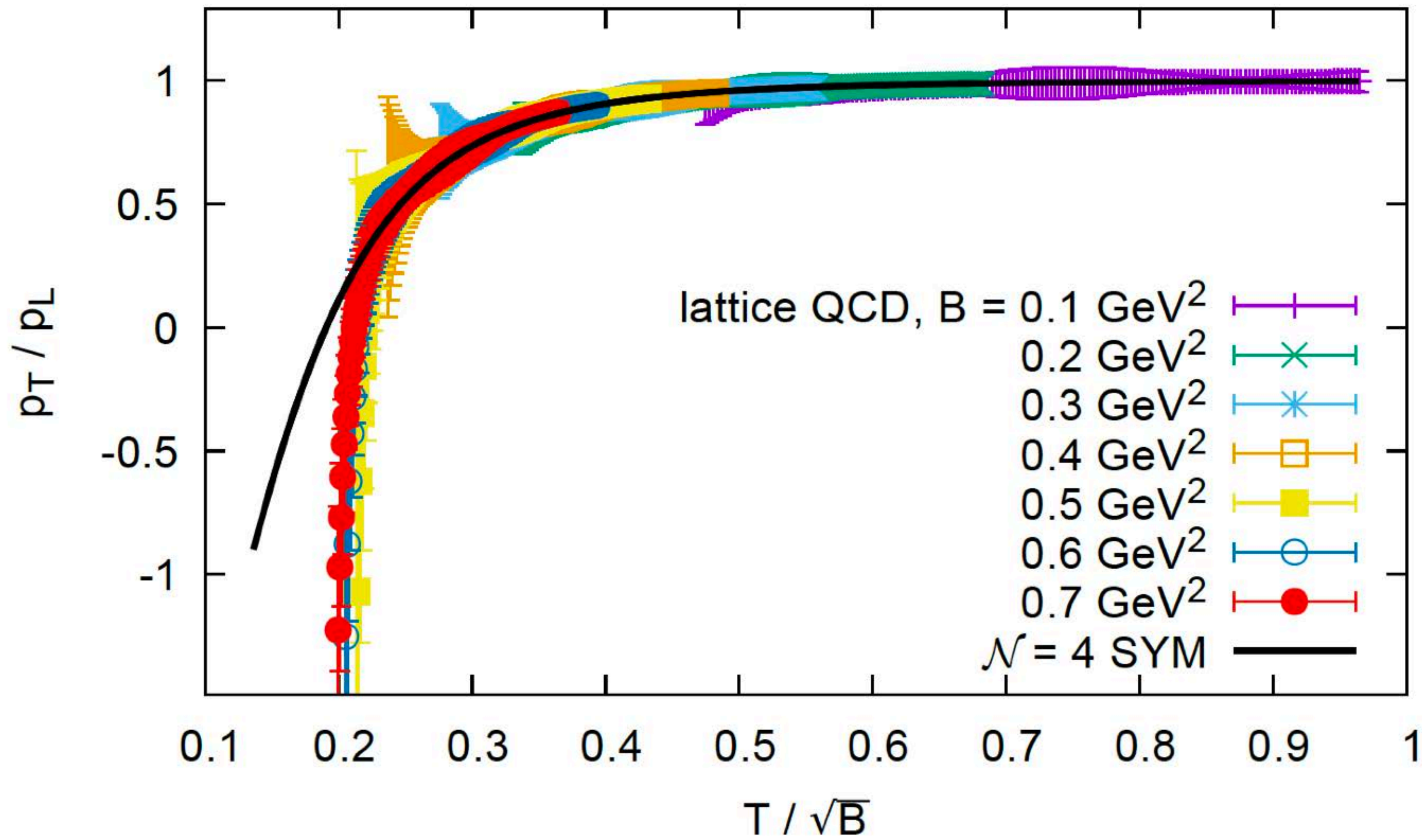
Thanks to my collaborators (since 2012)



APPENDIX

APPENDIX: Universal magneto response in LQCD and N=4 SYM with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $$p_{\rm T} = -\frac{L_{\rm T}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm T}}$$

longitudinal pressure: $$p_{\rm L} = -\frac{L_{\rm L}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm L}}$$

$F_{\rm QCD}$ ... *free energy*

$L_{\rm T}$ ... *transverse system size*

$L_{\rm L}$ ... *longitudinal system size*

V ... *system volume*

Direct Experiment: Black Hole on Electrical Circuit

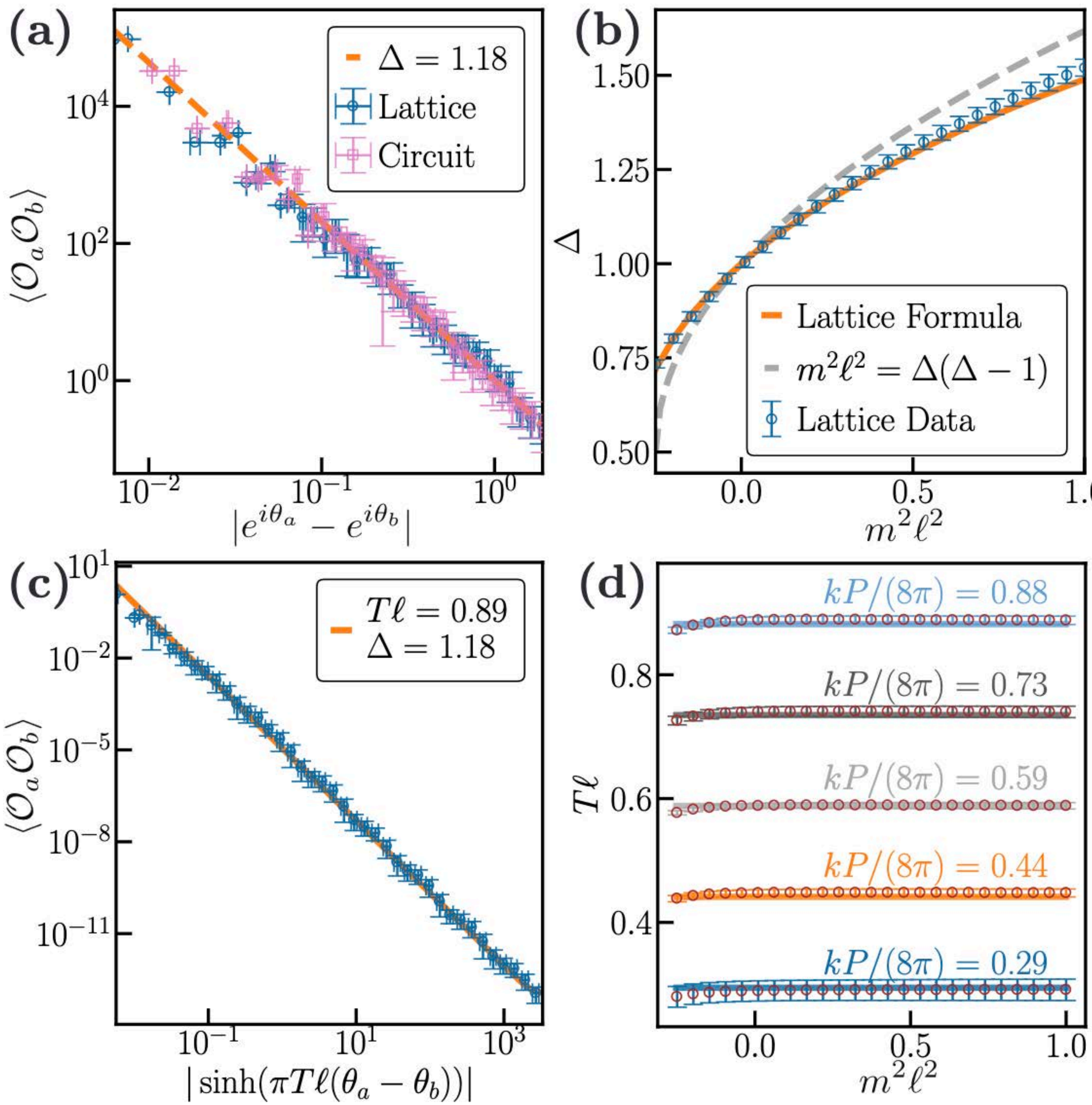
[Dey, Chen, Kaminski, et al.; PRL (2024)]

CFT expectation (gauge side):

$$\langle \mathcal{O}_a \mathcal{O}_b \rangle \simeq \frac{1}{(d_{ab})^{2\Delta}}$$

$$d_{ab} = \begin{cases} |e^{i\theta_a} - e^{i\theta_b}| & \text{(type-I)} \\ \frac{\sinh(\pi T\ell|\theta_a - \theta_b|)}{\pi T \ell} & \text{(type-II)} \end{cases}$$

Two point functions measured on circuit (gravity side):



c-Function

Zamolodchikov's c-theorem in 2D

$$(c)_{\rm UV} \geq (c)_{\rm IR}$$

energy-momentum tensor:

$$\langle T^a{}_a \rangle = -\frac{c}{12} R$$

trace anomaly

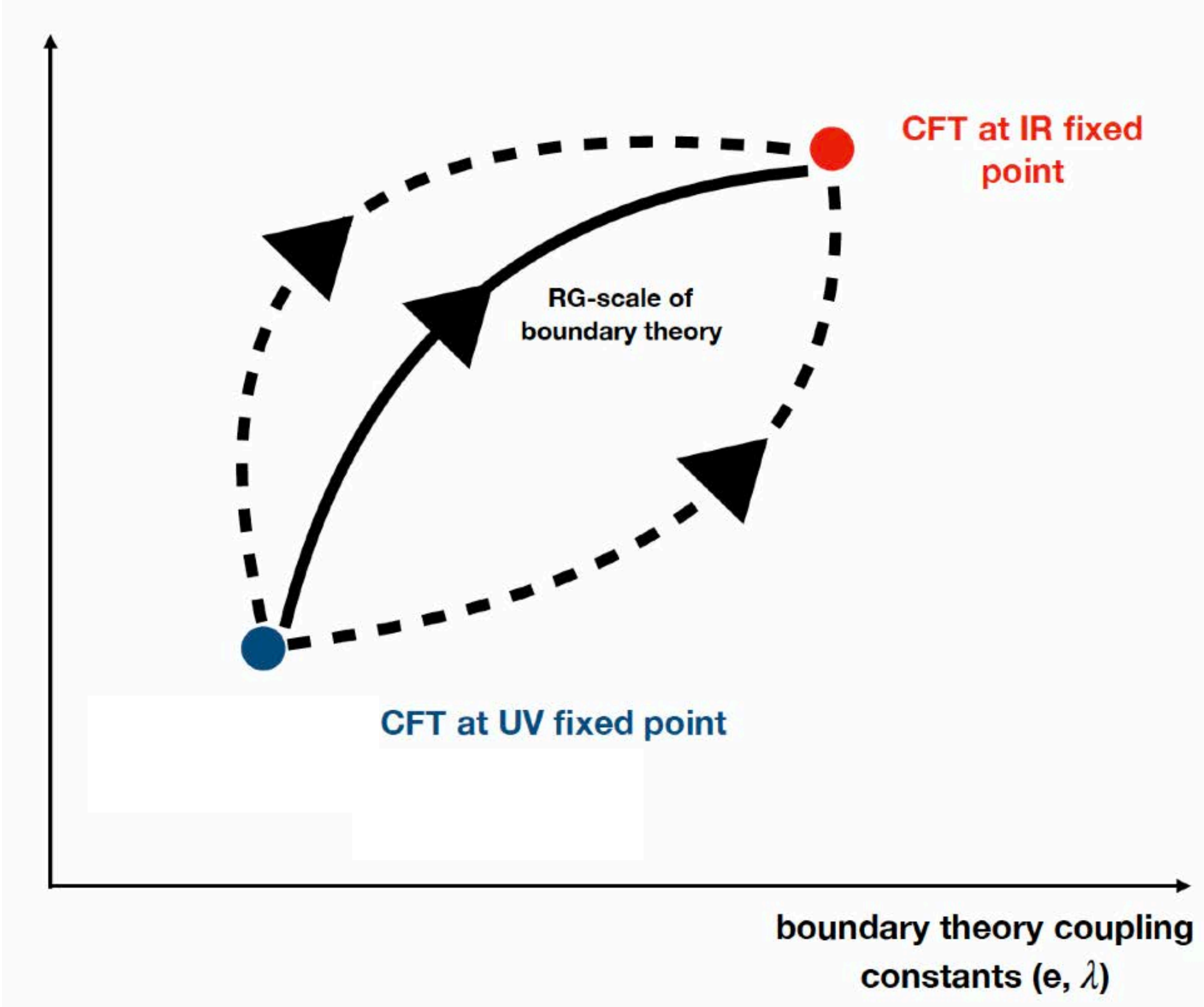
[Zamolodchikov; JETP Lett.(1986)]

c-theorem in 4D (the a-theorem)

$$(a_4)_{\rm UV} - (a_4)_{\rm IR}$$

$$\langle T^b{}_b \rangle = \frac{c_{TT}}{16\pi^2} \mathcal{C}^2 - \frac{a_4}{16\pi^2} \mathcal{E} - \frac{1}{4} F^2$$

[Komargodski,Schwimmer; (2011)]
[Cardy; Phys.Lett.B(1988)]
[Osborn; Phys.Lett.B(1988)]



➡ **IR/UV fixed points: c-function equals central charge of IR/UV CFT**

➡ **c-function measures degrees of freedom**

➡ **take a CFT: *c-function constant***

Entropic c-function

2D

$$c_2 = 3\ell \frac{\delta S_a}{\delta \ell}$$

entanglement entropy

[Casini,Huerta; Phys.Lett.B (2004)]

ℓ: *length scale (inverse energy scale)*

4D

$$a_4 = \beta_4 \frac{\ell^3}{H^2} \frac{\partial S_a}{\partial \ell}$$

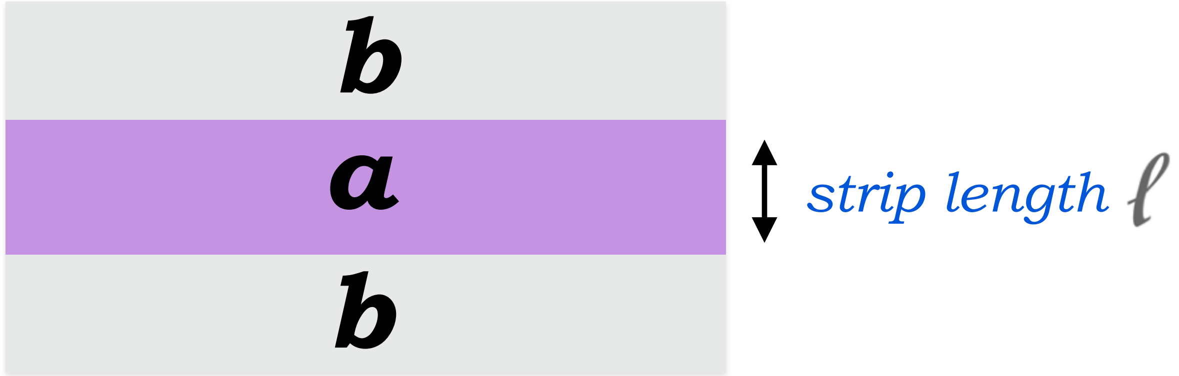
[Nishioka,Takayanagi; JHEP (2007)]
[Myers,Sinha; JHEP (2011)]

H: *IR-regulator*
β_4: *known constant*

➡ c-function defined by entanglement entropy

Definition

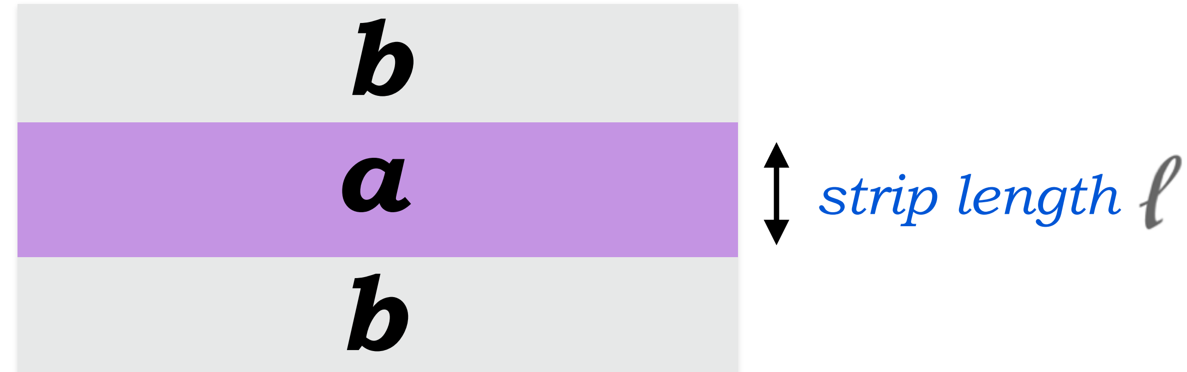
$$S_a = -\mathrm{Tr}\rho_a \log \rho_a , \quad \rho_a = \mathrm{Tr}_b \left|\psi\right\rangle \left\langle\psi\right|$$



Holographic entanglement entropy

Definition

$$S_a = -\mathrm{Tr}\rho_a \log \rho_a, \qquad \rho_a = \mathrm{Tr}_b \left|\psi\right\rangle \left\langle\psi\right|$$

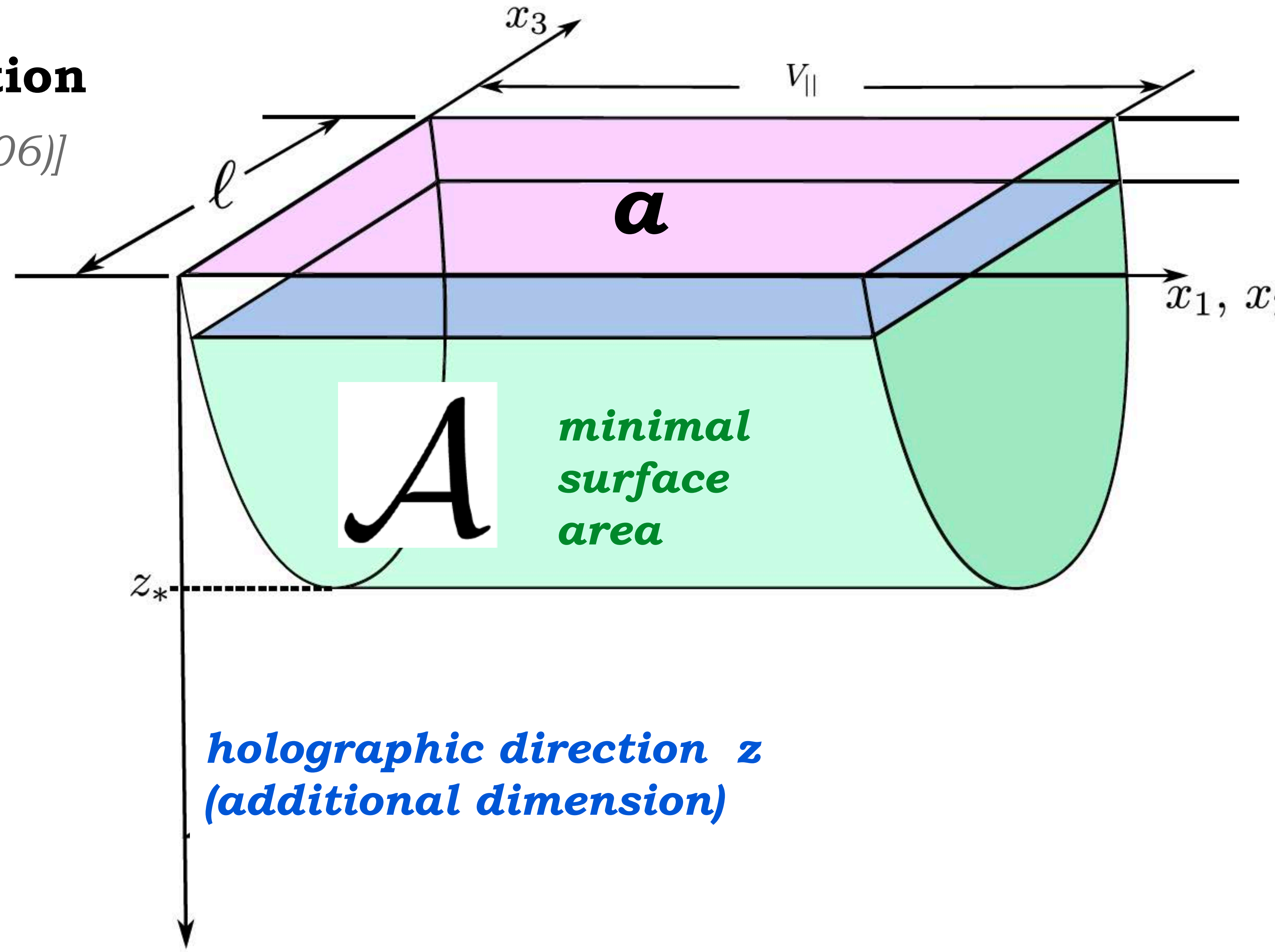


Holographically dual definition

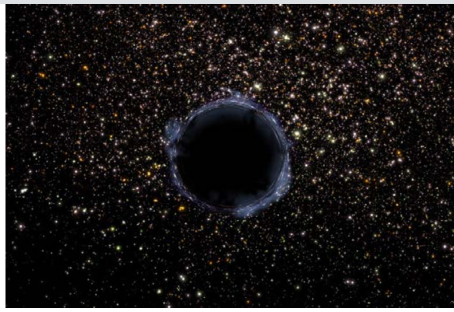
[Ryu,Takayanagi; JHEP (2006)]

$$S_a = \frac{1}{4G_5}\mathcal{A}$$

G_5 is the 5-dimensional gravitational constant of Anti de Sitter spacetime



Gravity dual to *N=4* SYM theory with magnetic field



Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Einstein-Maxwell action encodes N=4 Super-Yang-Mills theory with axial ***U(1) gauge symmetry***

5-dimensional Chern-Simons term ***encodes chiral anomaly***

Einstein-Maxwell equations

$$R_{\mu\nu} + 4g_{\mu\nu} = \frac{1}{2}\left(F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{6}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

$$\nabla_\mu F^{\mu\nu} = -\frac{\gamma}{8\sqrt{-g}}\epsilon^{\nu\alpha\beta\lambda\sigma}F_{\alpha\beta}F_{\lambda\sigma}\,.$$

Solution: charged magnetic black brane metric *[D'Hoker, Kraus; JHEP (2010)]*

- magnetic extension of a (charged) Reissner-Nordstrom black brane

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(\frac{\mathrm{d}z^2}{U(z)} - U(z)\mathrm{d}t^2 + v(z)^2\left(\mathrm{d}x_1^2 + \mathrm{d}x_2^2\right) + w(z)^2\left(\mathrm{d}x_3^2 + c(z)\mathrm{d}t\right)^2\right)$$

with numerically known solutions for U, v, w, c

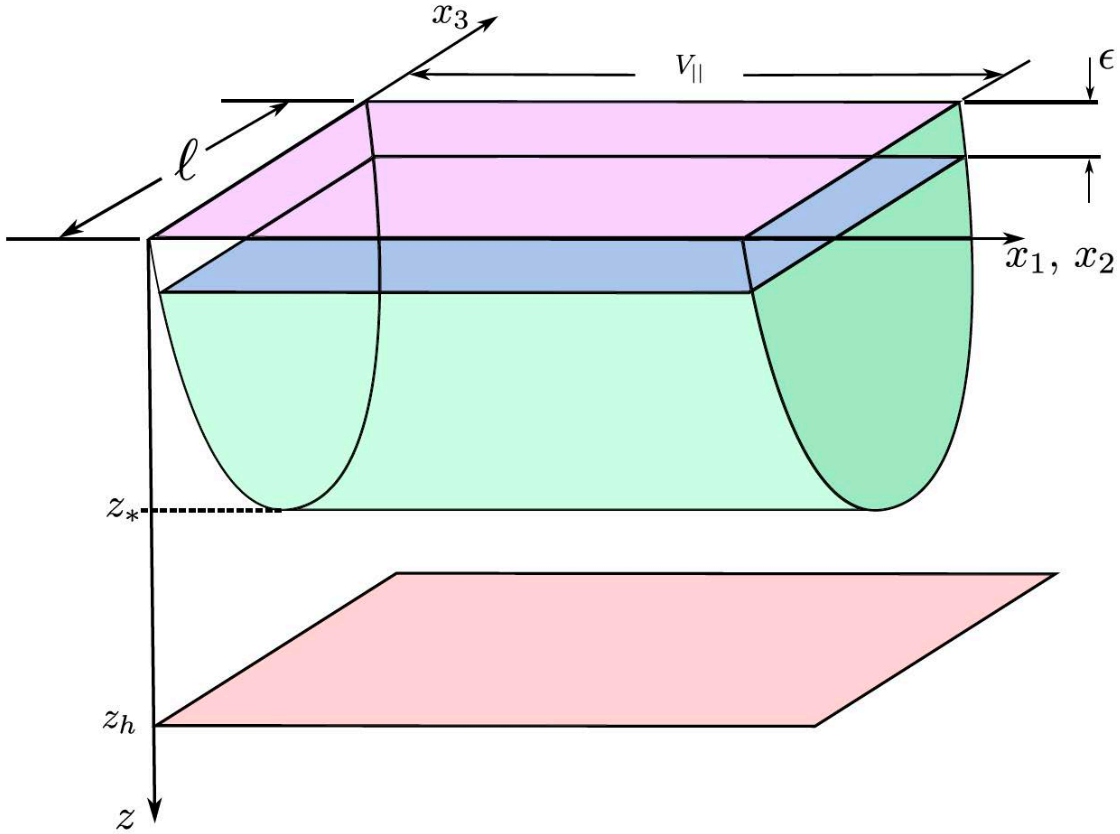
Gravitational *calculation*

➡ **calculate a *geodesic* in conformally deformed AdS metric**

	Transverse	Longitudinal
Embedding Coordinates	$\chi^\mu = (z(\sigma), t(\sigma), x_1(\sigma), x_2, x_3)$	$\chi^\mu = (z(\sigma), t(\sigma), x_1, x_2, x_3(\sigma))$
Surface Coordinates	$\sigma^i = (\sigma, x_2, x_3)$	$\sigma^i = (\sigma, x_1, x_2)$

Recall:

$$S_a = \frac{1}{4G_5}\mathcal{A}$$



Entanglement entropy

$$S_a = \frac{1}{4G_5} V_{||} \int d\sigma \sqrt{\frac{v(z(\sigma))^4 \left(w(z(\sigma))^2 x_3'(\sigma)^2 + \frac{z'(\sigma)^2}{U(z(\sigma))} \right)}{z(\sigma)^6}}$$

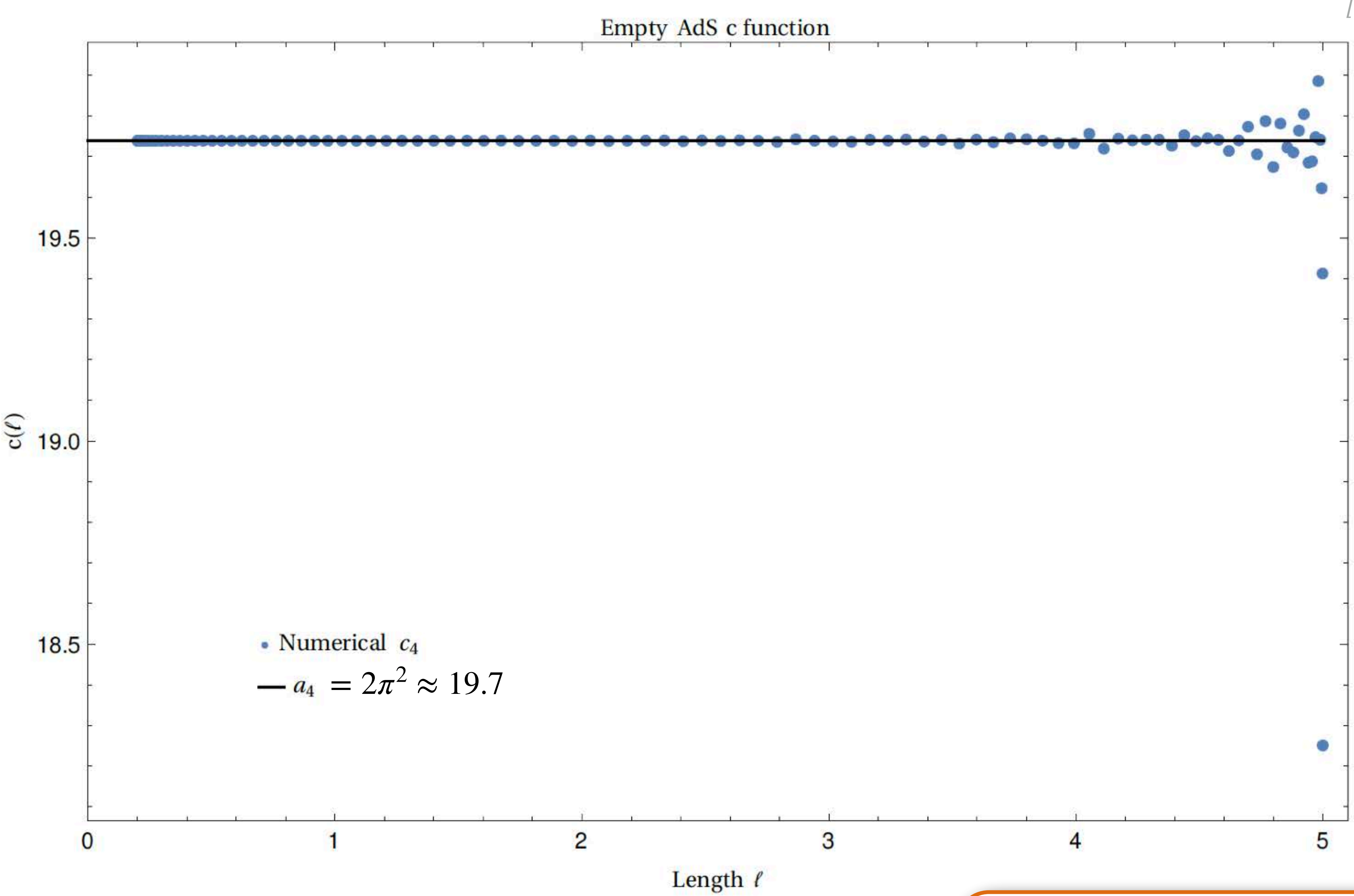
minimal surface area $\mathcal{A}$

$$V_{||} = \int_{-a}^{a} \int_{-b}^{b} dx_1 dx_3$$

Reminder: metric is $ds^2 = \frac{1}{z^2}\left(\frac{dz^2}{U(z)} - U(z)dt^2 + v(z)^2 \left(dx_1^2 + dx_2^2\right) + w(z)^2 \left(dx_3^2 + c(z)dt\right)^2 \right)$

Entropic c-function *in N=4 SYM vacuum state*

[Cartwright, Kaminski; arXiv: 2107.12409]



zero temperature,

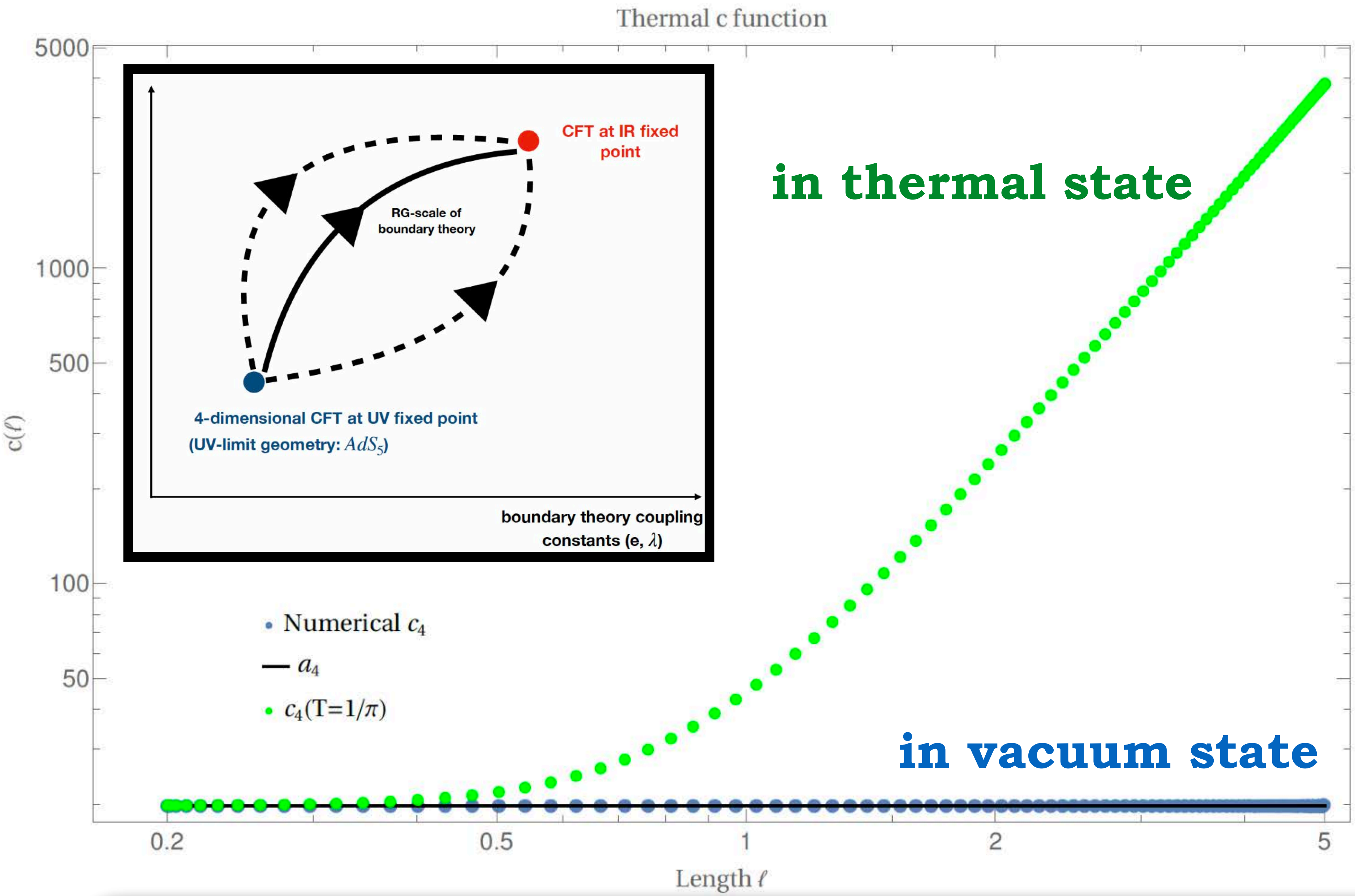
no magnetic field,

vanishing charge

➡ c-function at all scales equal to central charge of *N=4* SYM, which is a CFT

Entropic c-function *increases* in *thermal* state

[Cartwright,Kaminski; JHEP (2021)]



$$a_4 = \beta_4 \frac{\ell^3}{H^2} \frac{\partial S_a}{\partial \ell}$$

[Casini,Huerta; Phys.Lett.B (2004)]

[Nishioka,Takayanagi; JHEP (2007)]

[Myers,Sinha; JHEP (2011)]

H: IR-regulator

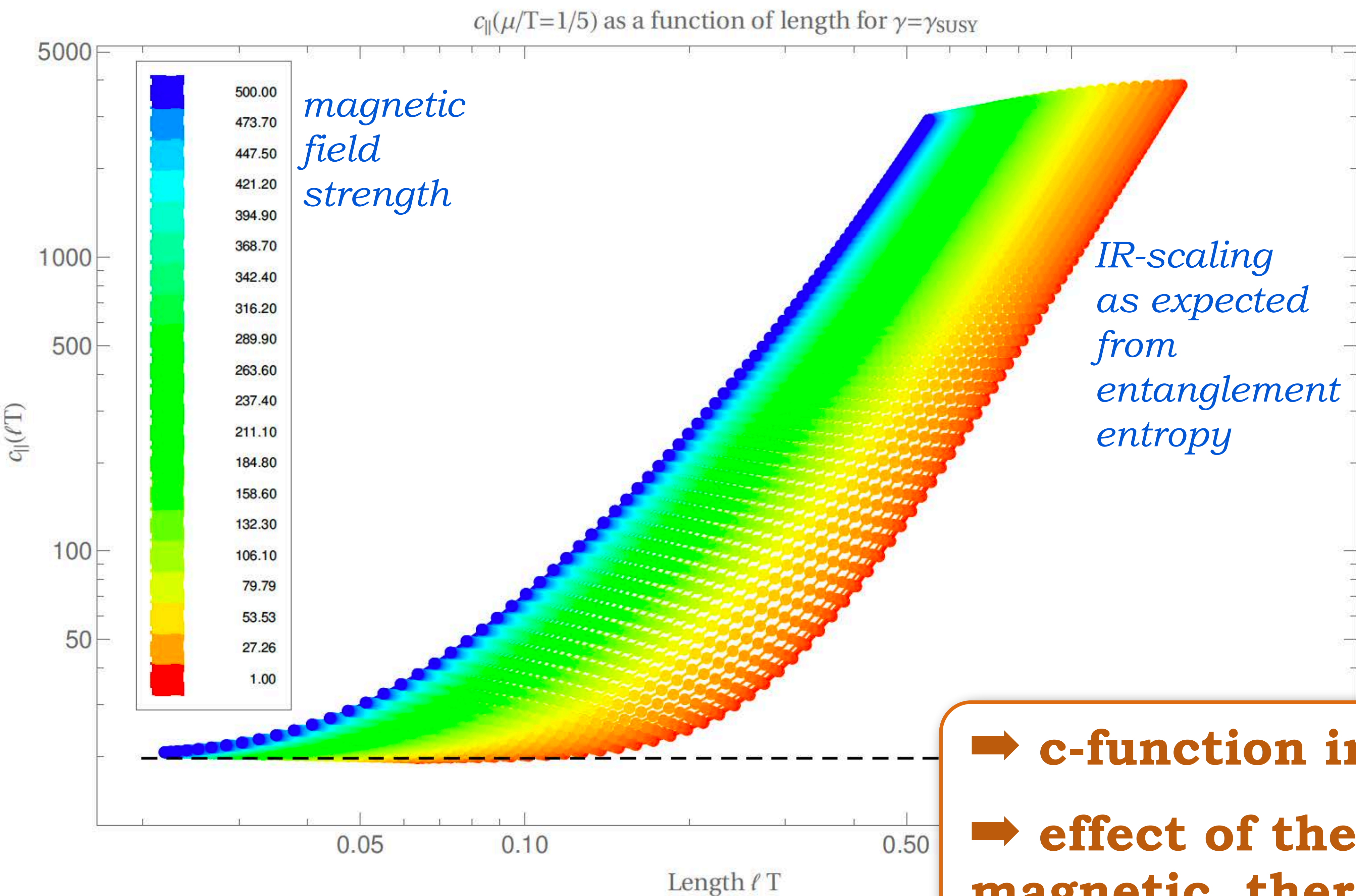
β_4: known constant

- ➡ **c-function increases in thermal state (violates c-theorem?)**
- ➡ **constant in vacuum of CFT (c-theorem valid)**
- ➡ **IR limit: thermal entropy**

[Zamolodchikov; JETP Lett.(1986)]
[Komargodski,Schwimmer; (2011)]
[Osborn; Phys.Lett.B(1988)]
[Cardy; Phys.Lett.B(1988)]

Entropic c-Function *increases* in *thermal* state

[Cartwright, Kaminski; arXiv: 2107.12409]



now with temperature, magnetic field, charge, chiral anomaly

➡ **c-function increases**

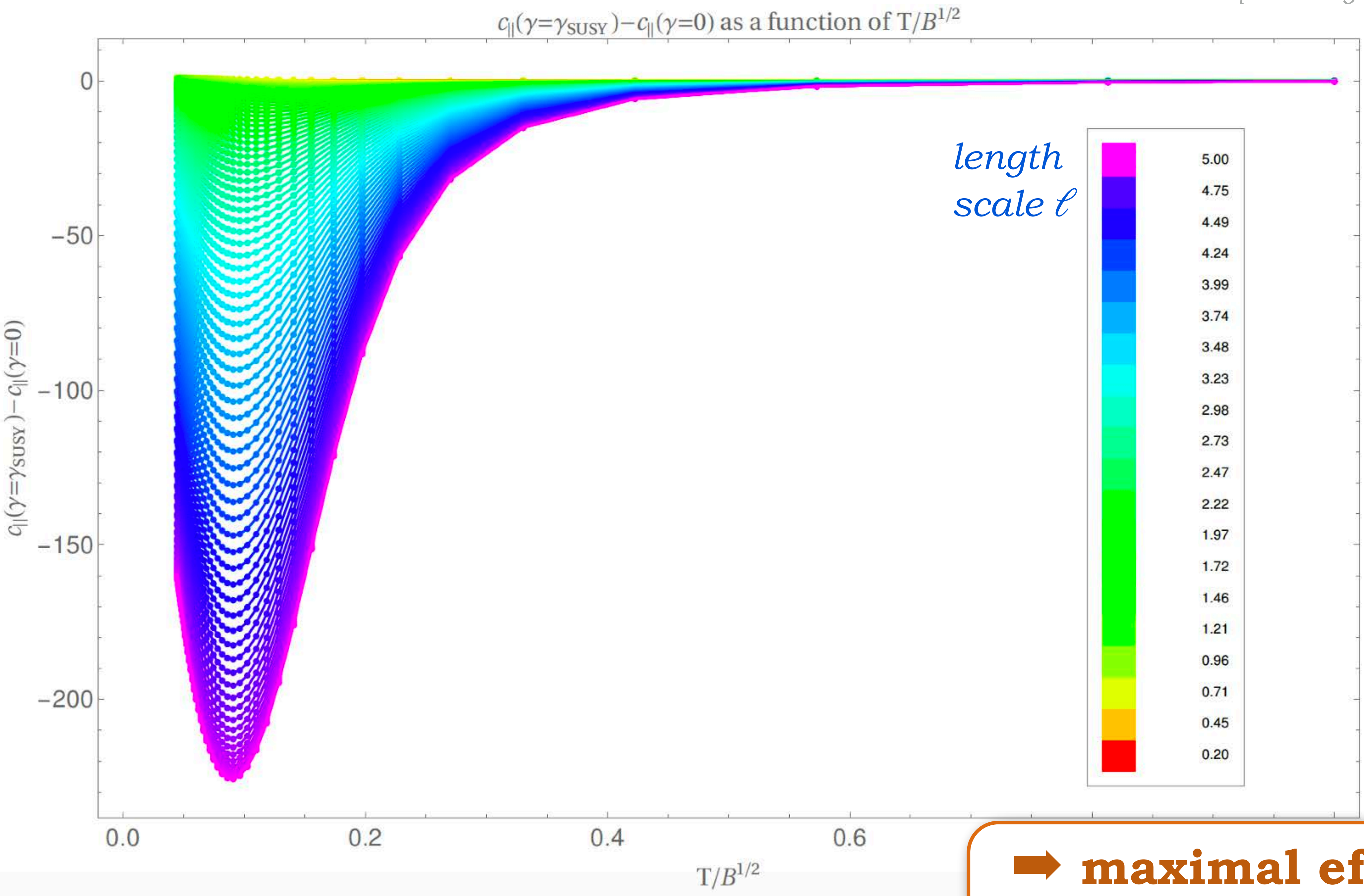
➡ **effect of the charged, magnetic, thermal state**

➡ **IR limit: thermal entropy**

➡ **proposal: measure of occupation number**

Effect of the chiral anomaly

[Cartwright, Kaminski; arXiv: 2107.12409]

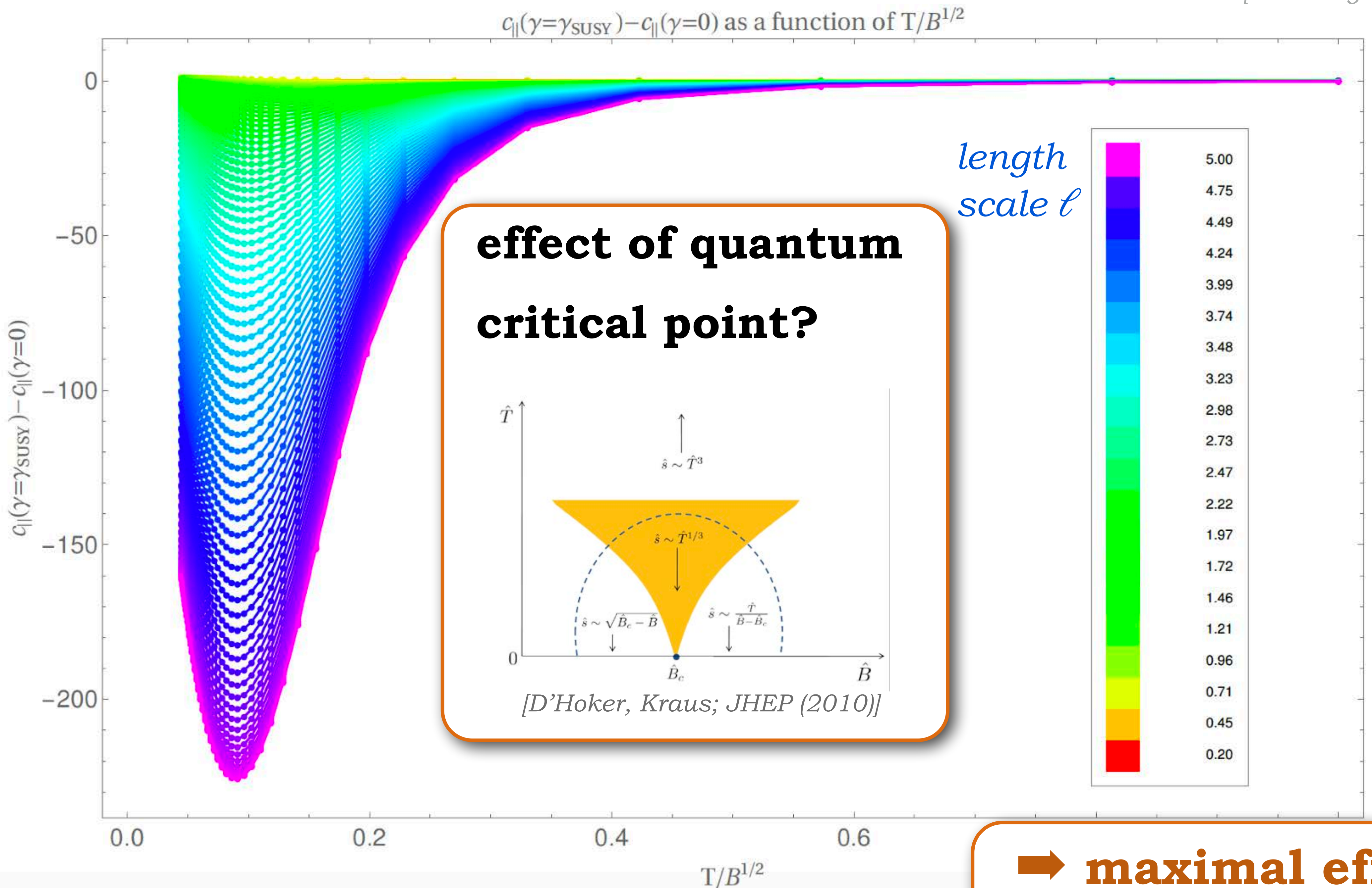


now with temperature, magnetic field, charge, chiral anomaly

➡ maximal effect at 0.1 (thermal entropy has no maximum)

Effect of the chiral anomaly

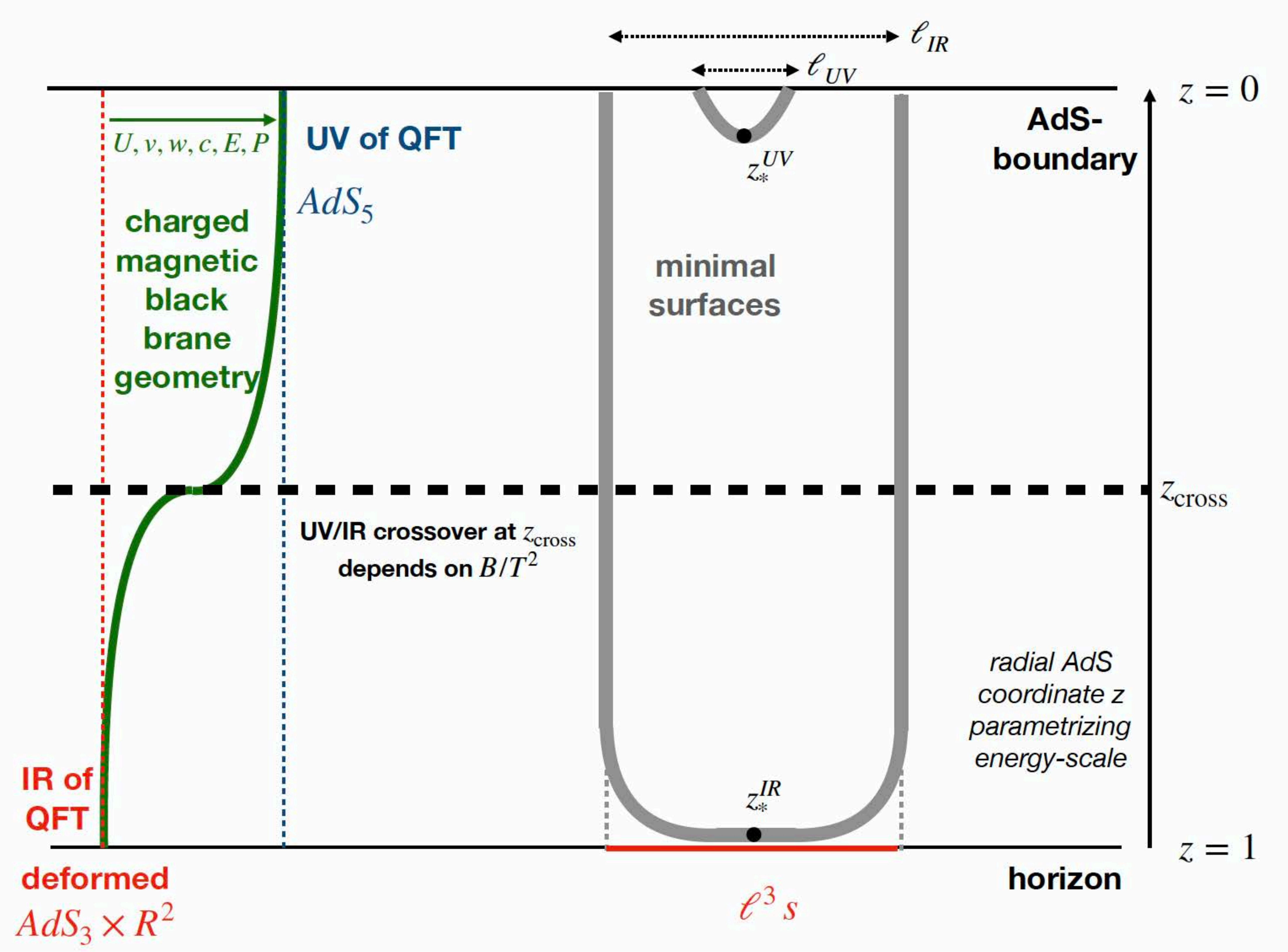
[Cartwright, Kaminski; arXiv: 2107.12409]



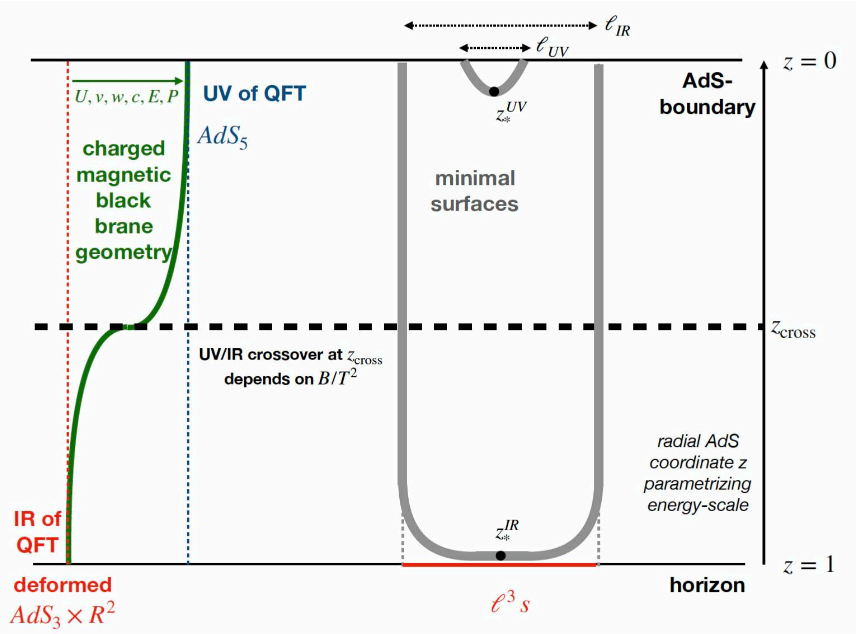
now with temperature, magnetic field, charge, chiral anomaly

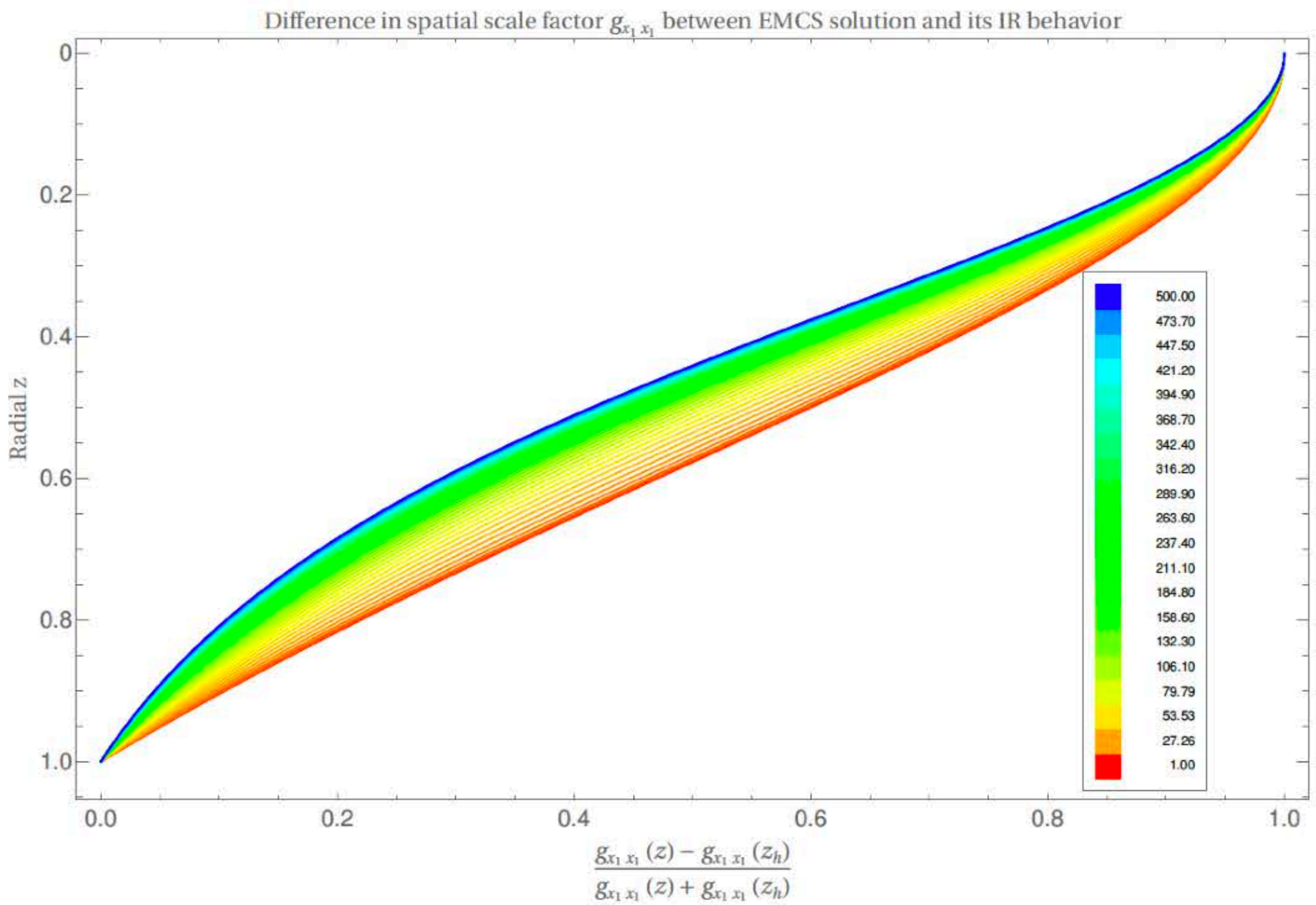
➡ maximal effect at 0.1 (thermal entropy has no maximum)

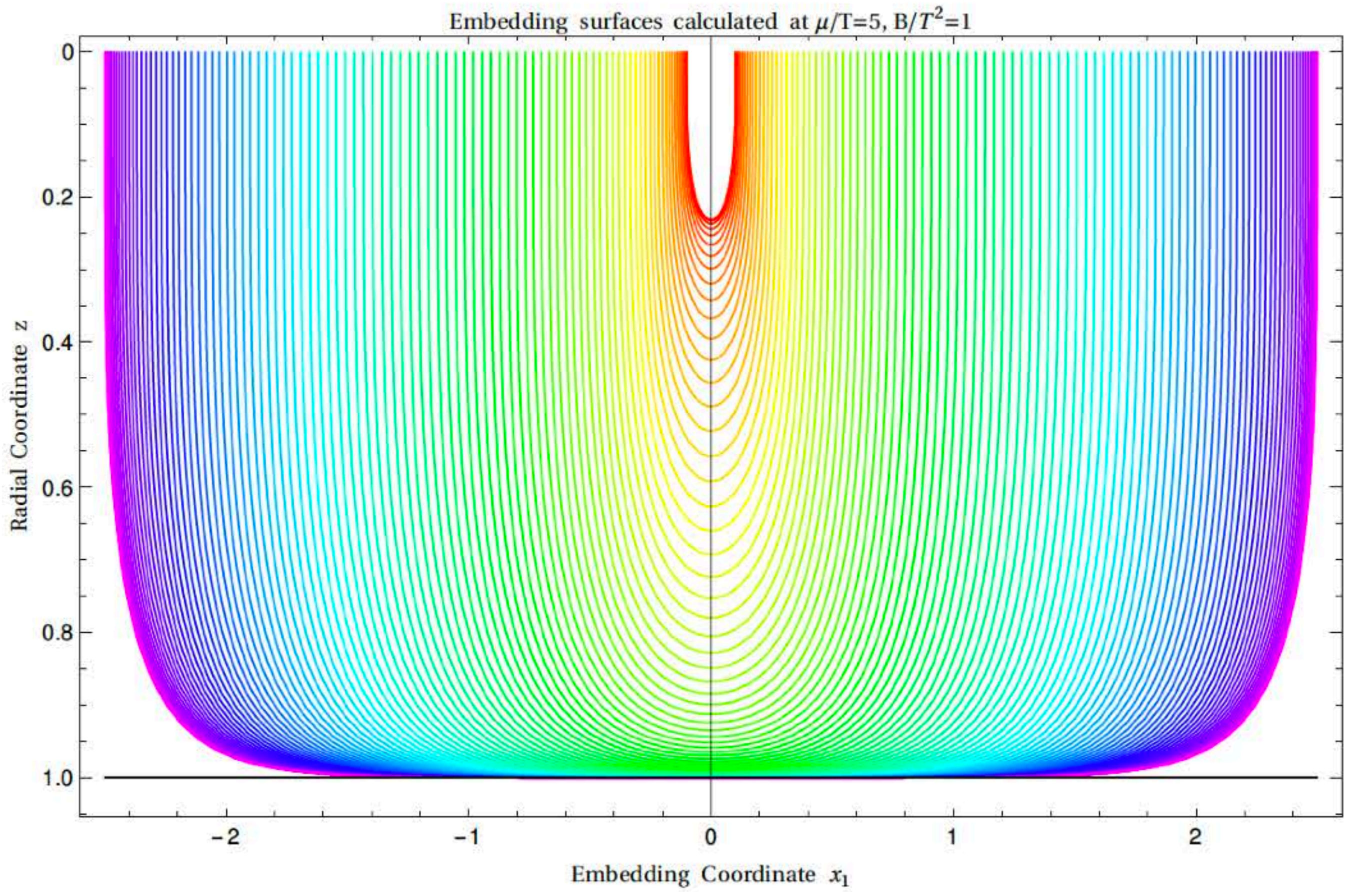
Schematic picture: probing energy scales



Numerical data confirming schematic picture







Thermal entropy