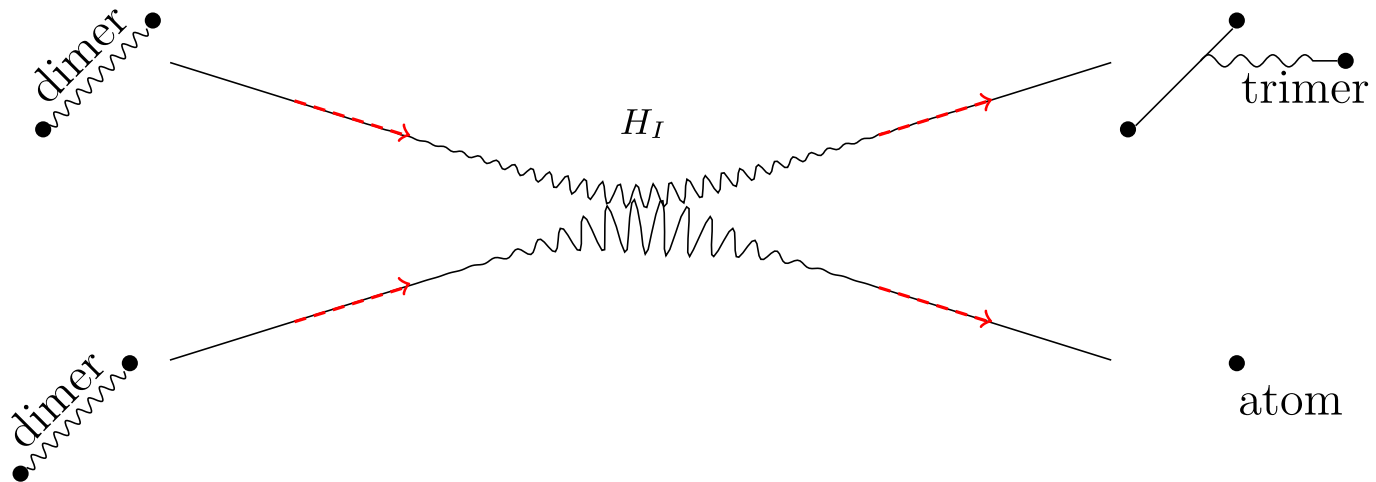


4-body reaction features **universally** correlated with dimer & trimer subsystems

October 9, 2024

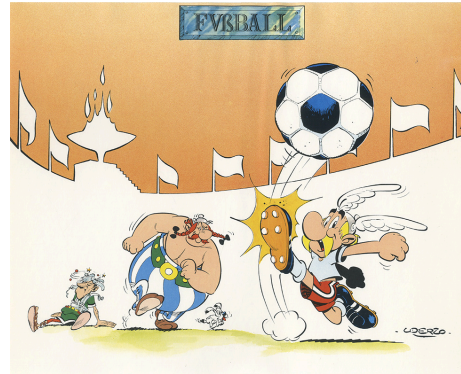


Universal and peculiar features of short-distance-different particles



\emptyset td's (NFL '13) ≈ 2.8

 $\approx 410(15)\text{g}$



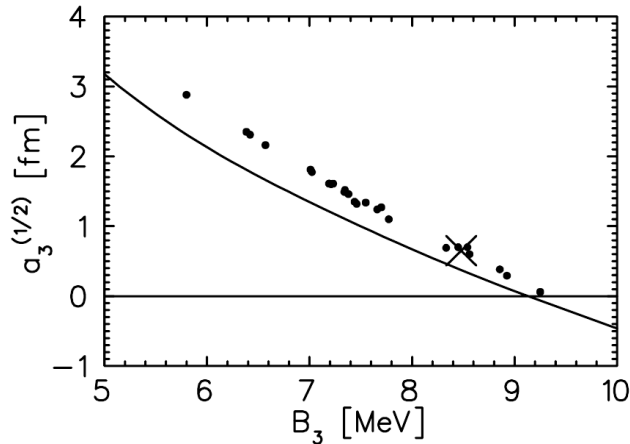
\emptyset goals (UK '13) ≈ 2.8

 $\approx 430(20)\text{g}$



\emptyset tries (AUS '13) ≈ 5.7

 $\approx 430(25)\text{g}$



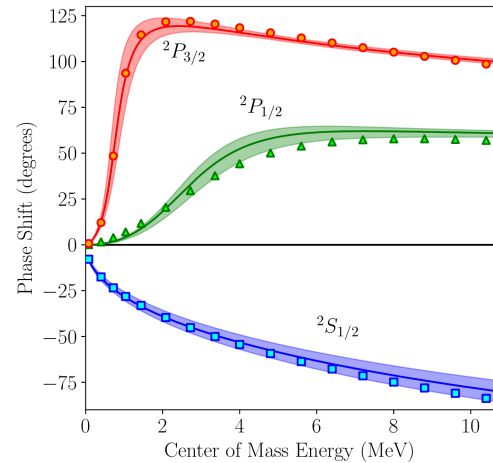
[P.F. Bedaque, H.-W. Hammer, U. van Kolck \(2000\)](#)

Phillips line:

low-energy deuteron-neutron amplitude = $f(B_3)$

$$a_{\text{dimer-dimer}} / a_{\text{atom-atom}} \approx 0.6$$

[D.S. Petrov, C. Salomon, and G.V. Shlyapnikov \(2003\)](#)



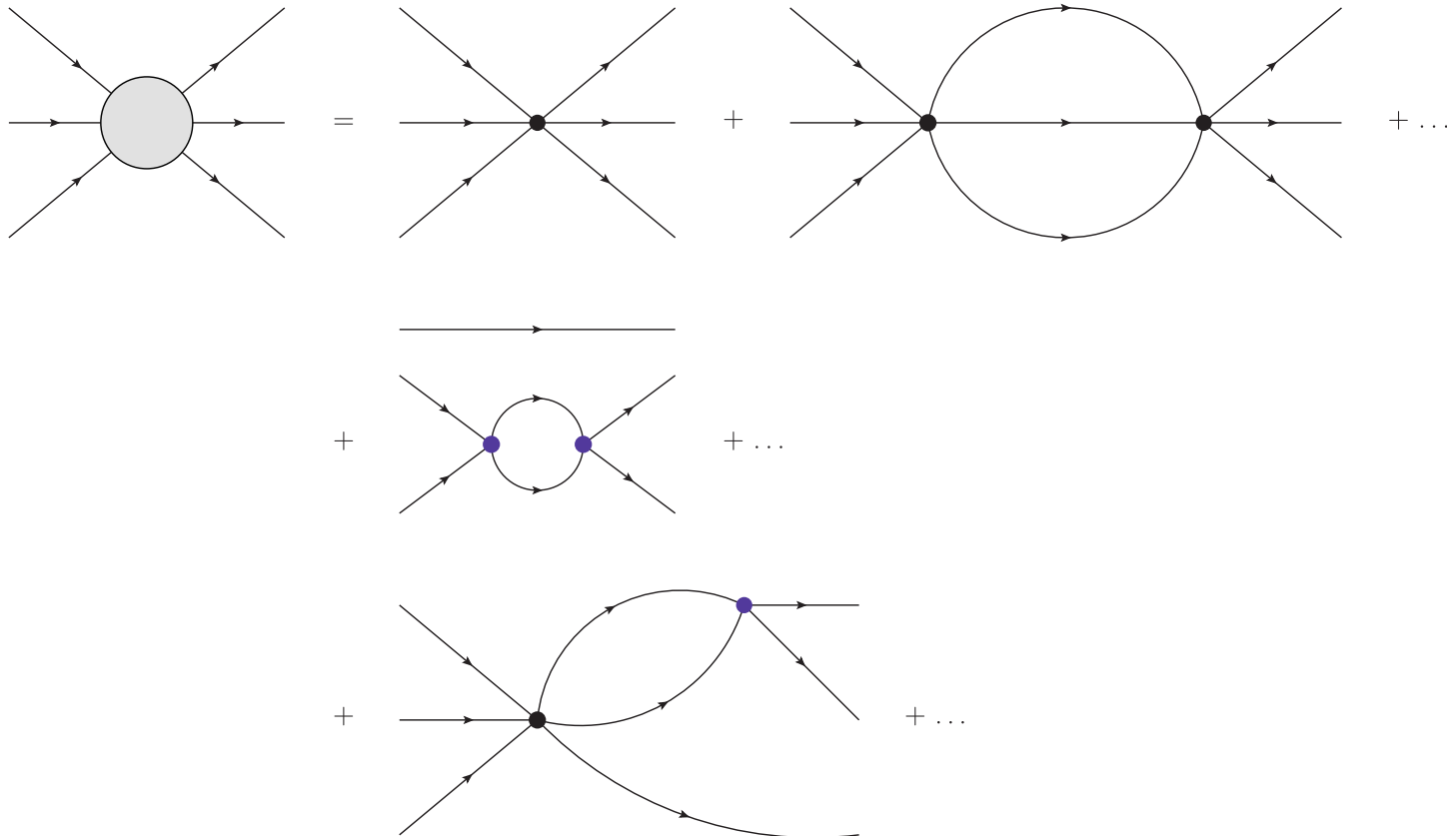
[K. Kravvaris et al. \(2020\)](#)

neutron- α :

resonance poles = $f(?)$

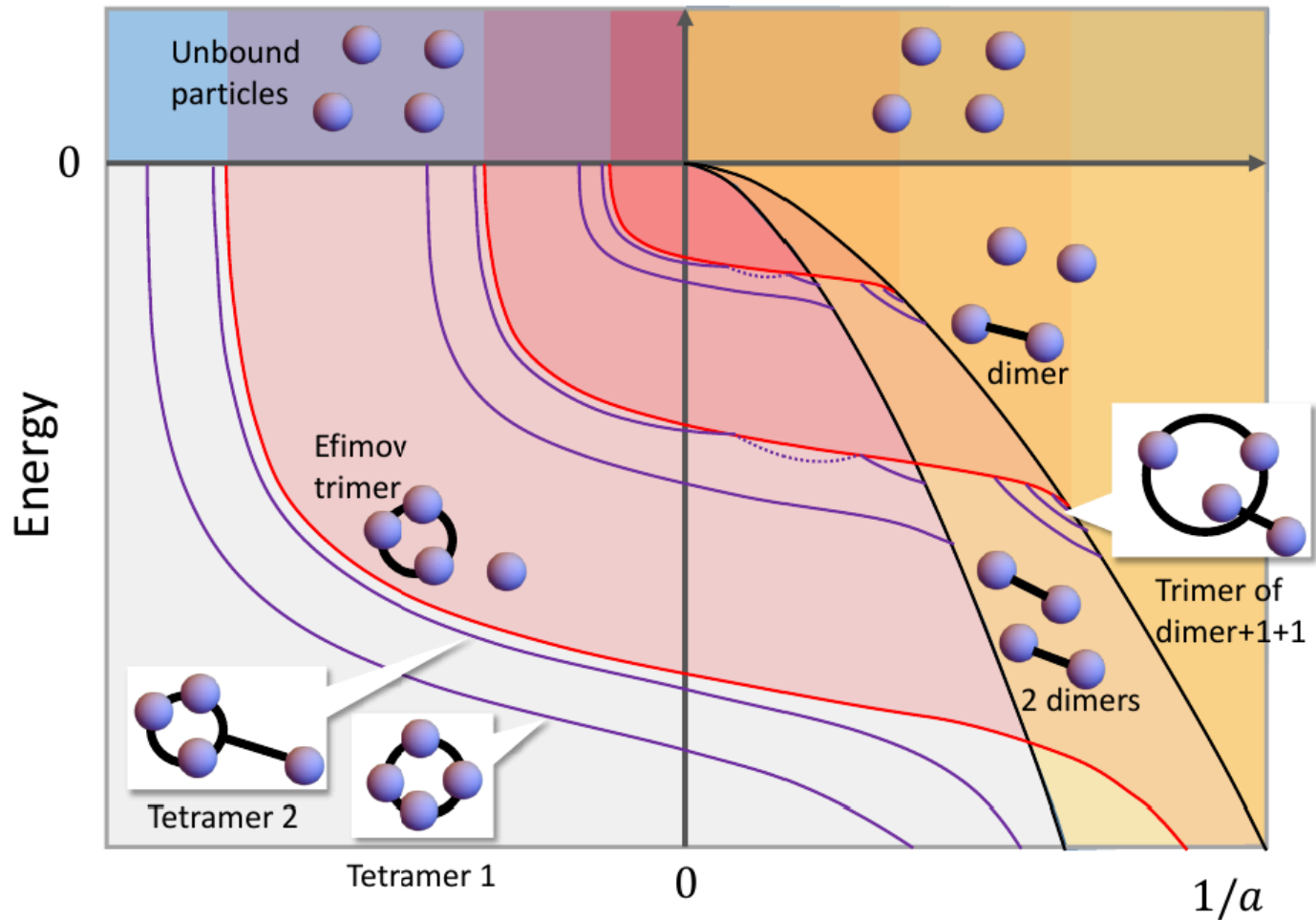
The universal minimum \hat{V} : **structure** and **input**

$$\begin{aligned}\Psi(E) &= C + C^2 \cdot L_1(E) + C^3 \cdot L_2(E) + \dots \\ &= C(\lambda) + C(\lambda)^2 \cdot L_{1,\lambda}(E) + C^3 \cdot L_{2,\lambda}(E) + \dots \\ &\stackrel{!}{=} B_{\text{exp}} = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle}\end{aligned}$$



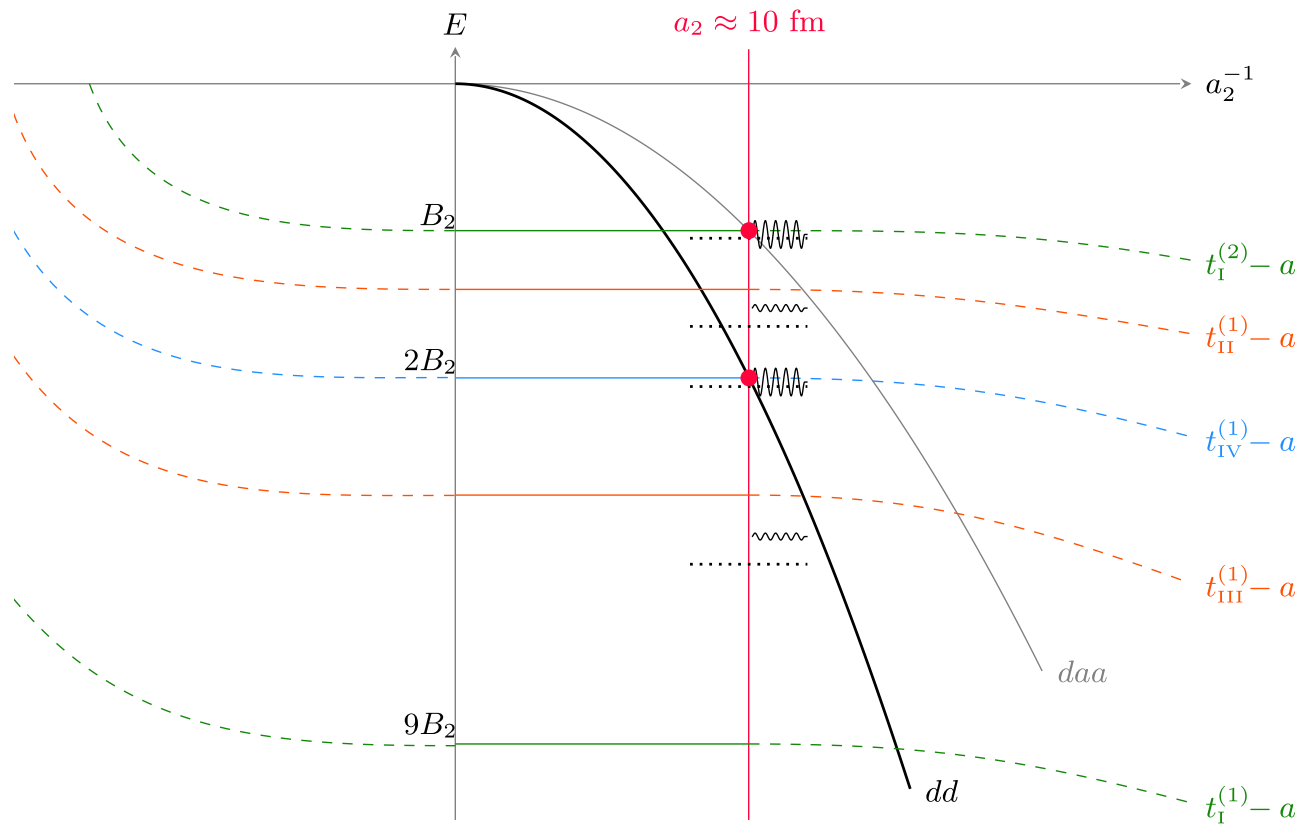
What do we know about **universal** phenomena in the 4-boson system?

$a_2 \rightarrow \infty$ i.e. **no** 2-body scale



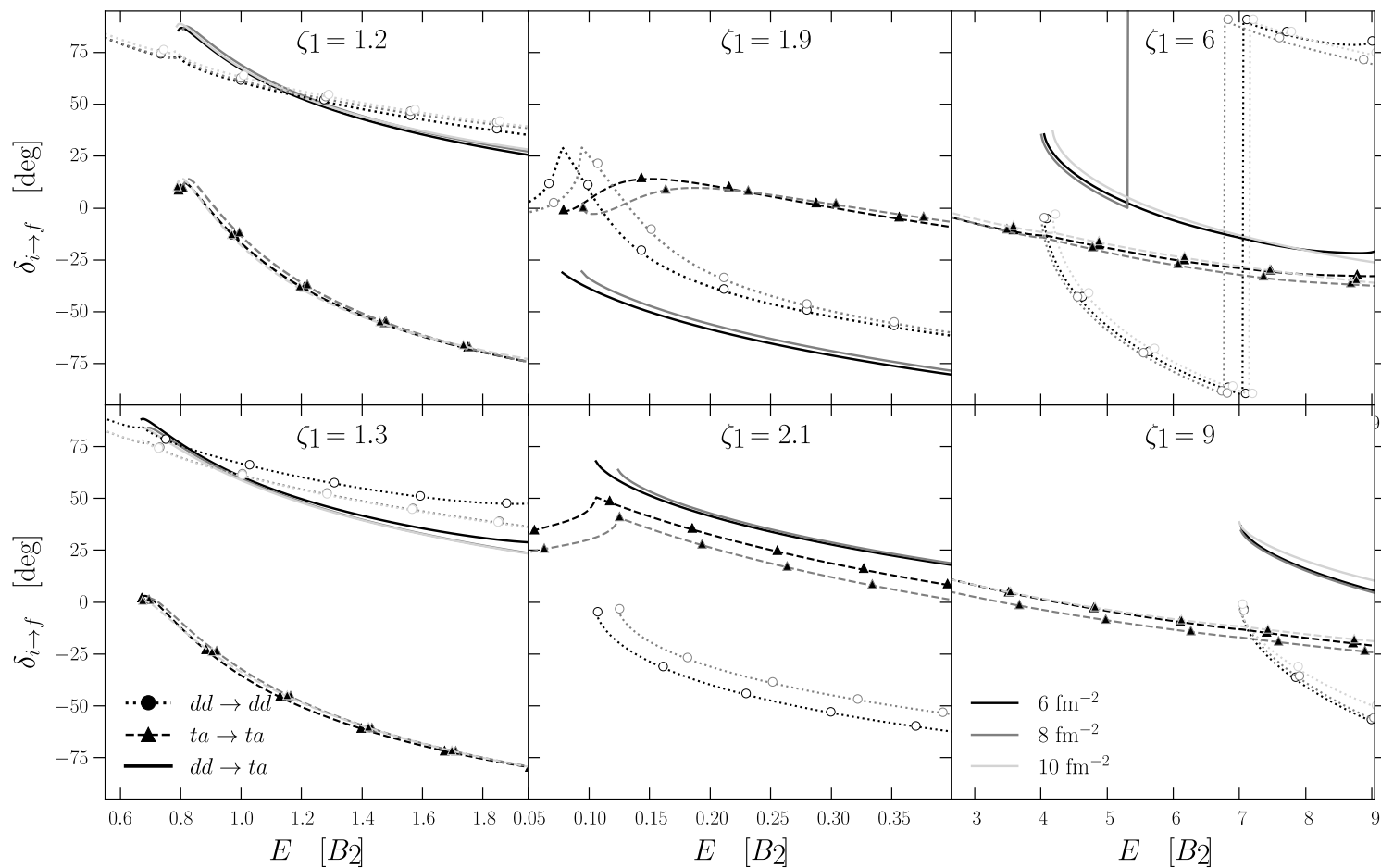
How does the **minimal** theory describe 4-component-fermion reactions?

$$a_2 < \infty$$



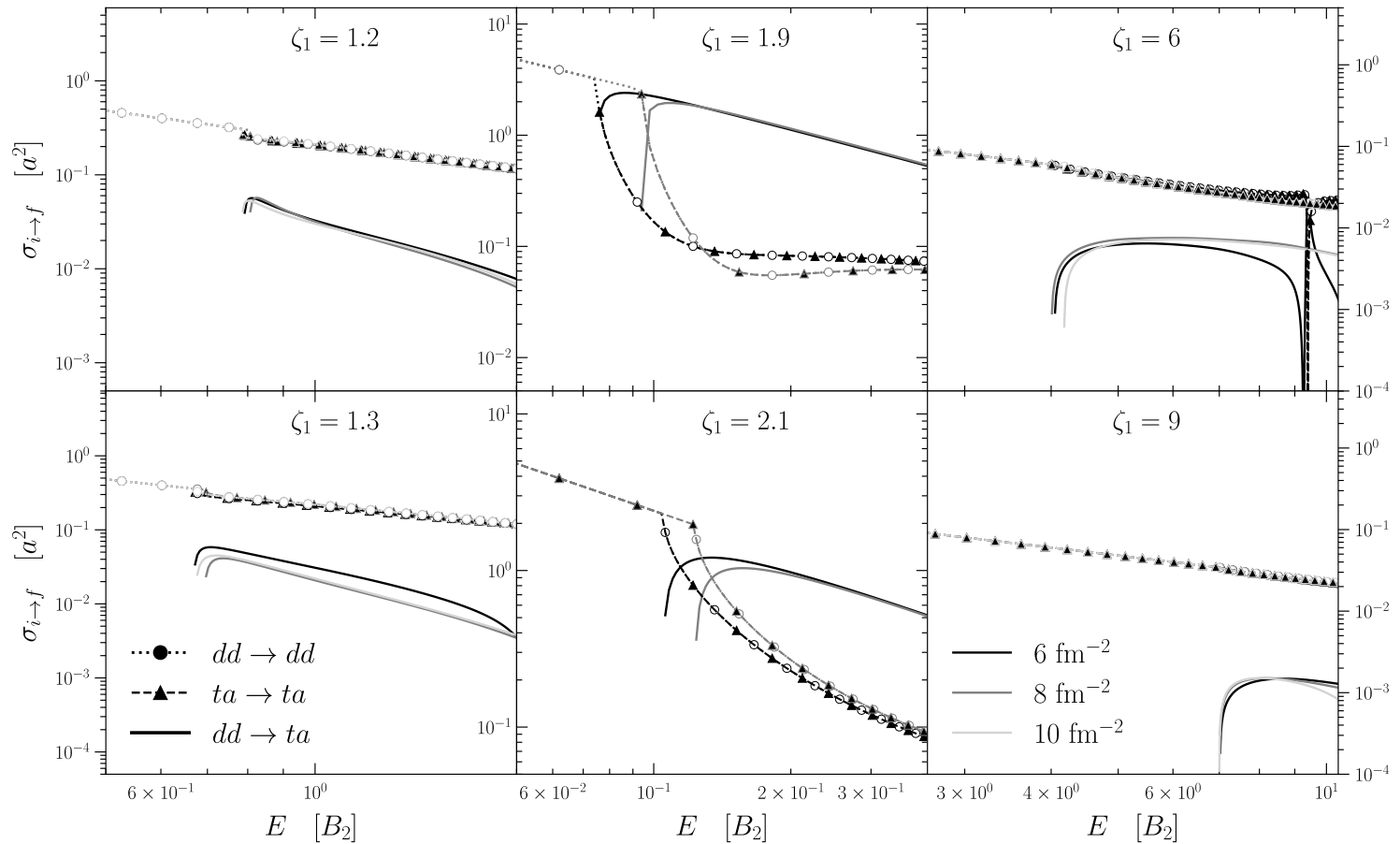
Phase-shift parameters of the 2-channel S-matrix

$$S_{if} = \eta_{if} e^{2i\delta_{if}}$$

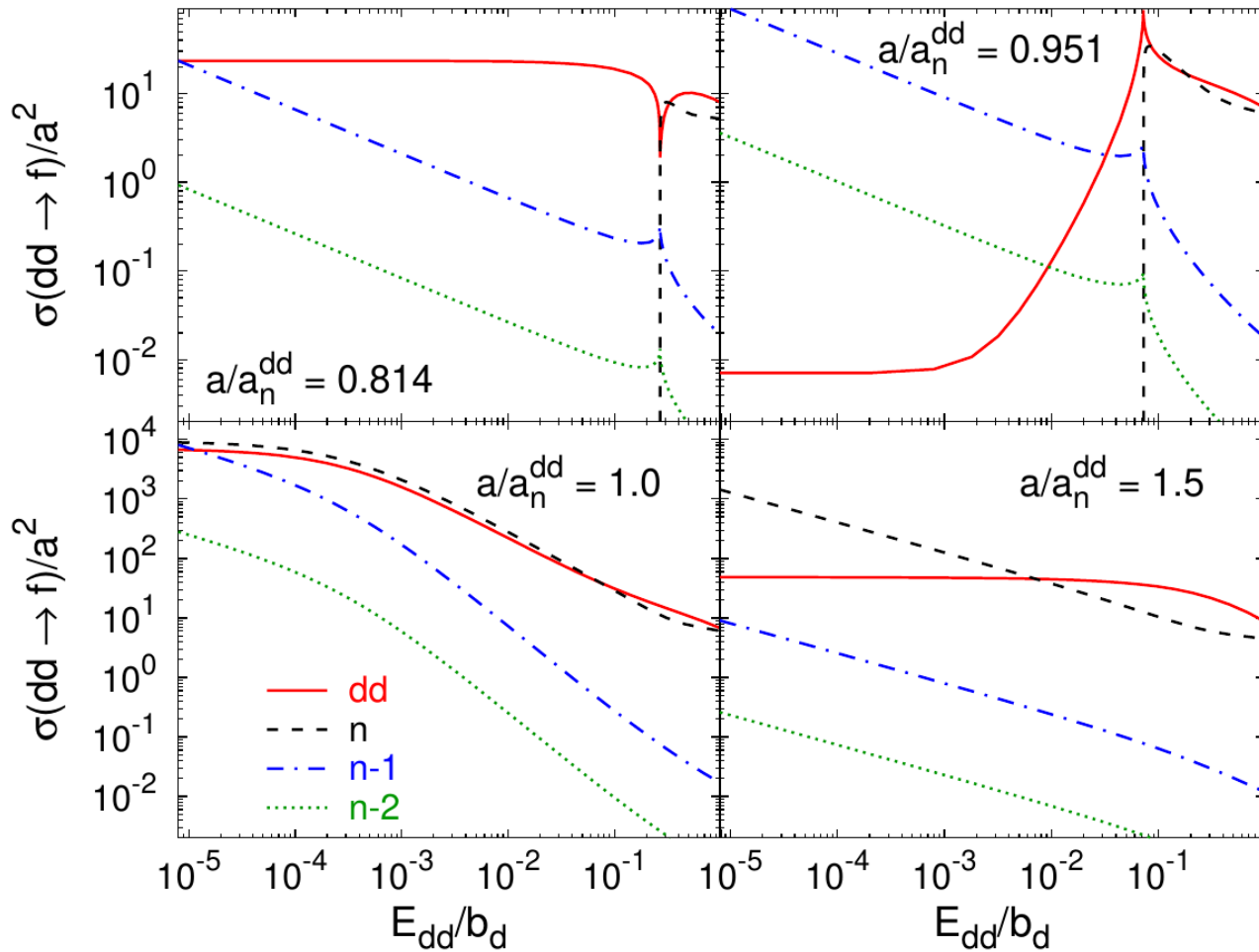


Diagonal- and mixing-strength parameters of the 2-channel S-matrix

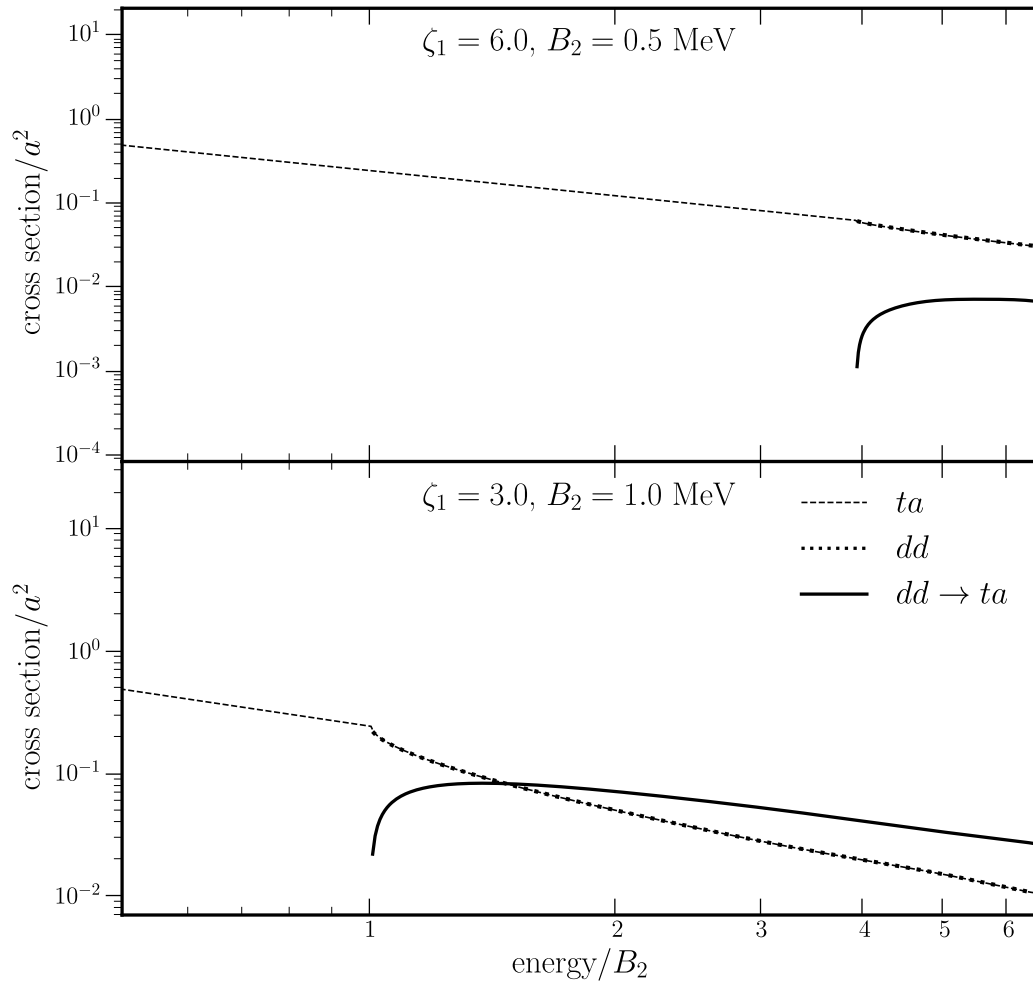
$$S_{if} = \eta_{if} e^{2i\delta_{if}}$$



S. Mondal, R. Goswami, U. Raha, JK (2024)



Reaction vs. elastic strength with different atom-atom scattering lengths a_2



Algorithm for the analytic parametrization of inter-cluster potential with 2-, 3-, &c. couplings

- Expand in *realistic* DoF's

$$\Psi = \hat{\mathcal{A}} \left\{ \sum_i \phi(A_i) \phi(B_i) F_i(\mathbf{R}_i) + \sum_j \phi(A_j) \phi(B_j) \phi(C_j) F_j(\mathbf{R}_{1j}, \mathbf{R}_{2j}) + \dots + \sum_m c_m \chi_m \right\} Z(\mathbf{R}_{c.m.})$$

- Obtain these states explicitly within "your" theory

$$\hat{H} \phi_A^{(n)} = e_A^{(n)} \phi_A^{(n)}$$

- "Freeze" the asymptotic states

$$\delta \Psi \stackrel{!}{=} \delta F_i$$

- Integrate-out/average-over internal DoF's

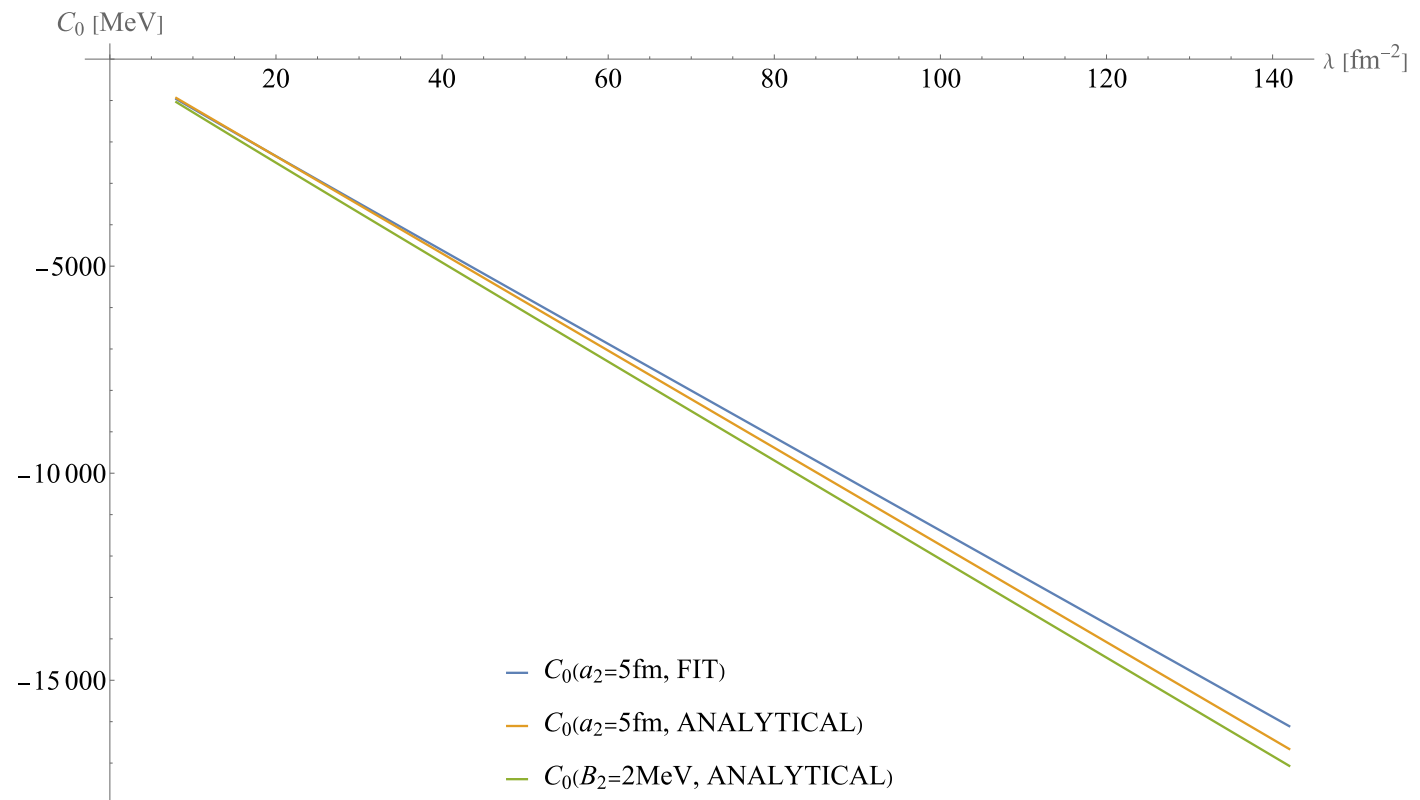
$$\left(\hat{T}_{\mathbf{R}} - E_{\text{rel}} + \mathbb{N}^{-1} \langle \phi_A \phi_B | \hat{V} | \phi_A \phi_B \rangle \right) \chi(\mathbf{R}) - \mathbb{N}^{-1} \int d\mathbf{R}' \left[\langle \phi_A \phi_B | \left(\hat{T}_{\mathbf{R}} - E_{\text{rel}} + \hat{V} \right) \hat{\mathcal{A}} \{ | \phi_A \phi_B \rangle \delta(\mathbf{R} - \mathbf{R}') \} \right] \chi(\mathbf{R}') = 0$$

- Obtain inter-cluster dynamics with a potential matrix parametrized with "microscopic" observables

$$\sum_{n=1}^{N_{\text{loc}}} \hat{\eta}_n e^{-w_n \mathbf{R}^2} \chi(\mathbf{R}) - \sum_{n=1}^{N_{\text{loc}}} \int \left\{ \hat{\zeta}_n e^{-a_n \mathbf{R}^2 - b_n \mathbf{R} \cdot \mathbf{R}' - c_n \mathbf{R}'^2} \right\} \chi(\mathbf{R}') d\mathbf{R}' = 0$$

with $\hat{\eta}_n, \hat{\zeta}_n, w_n, a_n, b_n, c_n$ dependent upon $C_{nn}(\lambda), C_{nm}(\lambda), \alpha(\lambda), E_{\text{rel}}, A, B$

2-body contact-interaction strength's regulator dependence



dimer-atom scattering with an effective inter-cluster potential regulator dependence

