

TMDs and PDFs of spin-1 hadrons

Shunzo Kumano

Japan Women's University

High Energy Accelerator Research Organization (KEK)

<http://research.kek.jp/people/kumanos/>

**Collaborators: F. E. Close (Oxford), W. Cosyn (FIU), Y.-B. Dong (IHEP),
S. Hino (Saga), T.Y. Kimura (Sokendai), M. Miyama (Saga) ,
M. Sargsian (FIU), Q.-T. Song (Zhengzhou Univ)**

**INT workshop on QCD at the Femtoscale in the Era of Big Data
INT, University of Washington, Seattle, USA, June 10 - July 5, 2024
<https://www.int.washington.edu/programs-and-workshops/24-2a>**

July 13, 2024

My situation

3-4 days / week at the university

2 days / week at the KEK Tsukuba campus

View of Ikebukuro downtown
from my JWU office



Contents

1. Introduction

- Tensor-polarized structure function b_1 , gluon transversity

2. b_1 , gluon transversity, TMDs, PDFs, fragmentation functions, multiparton distribution functions of spin-1 hadrons

- b_1 by “standard” deuteron model [1]
- Tensor-polarized PDFs at hadron accelerator facilities (Drell-Yan) [2]
- Gluon transversity at hadron accelerator facilities (Drell-Yan) [3]
- TMDs and PDFs up to twist 4 [4]
- Twist-2 relation and sum rule for PDFs [5]
- Relations from equation of motion and a Lorentz-invariance relation [6,7]
- Summary of spin-1 PDFs, TMDs, fragmentation functions [8]

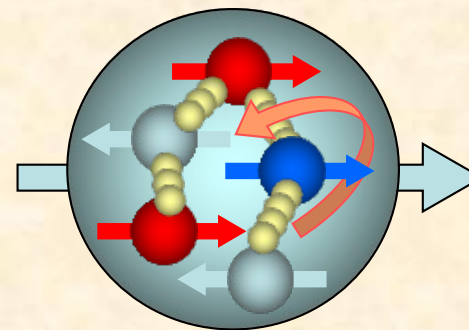
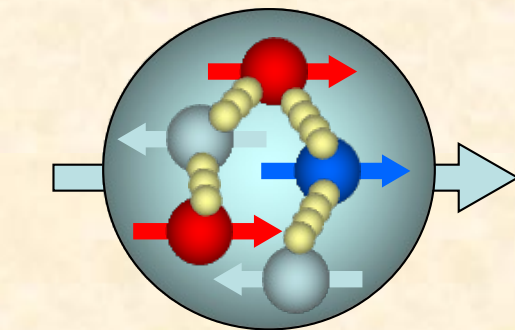
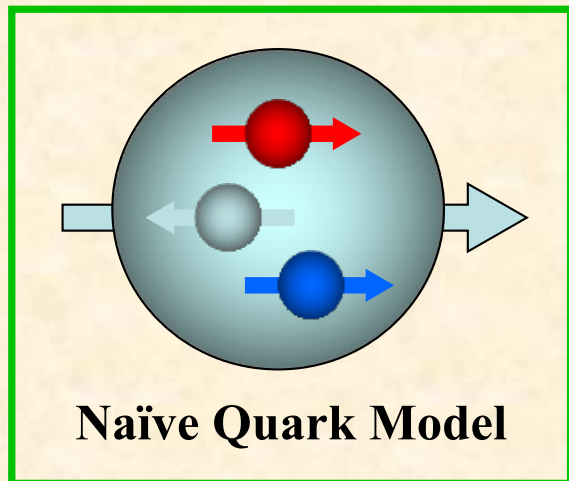
3. Future prospects and summary

- References [1] W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, PRD 95 (2017) 074036.
[2] SK and Qin-Tao Song, PRD 94 (2016) 054022.
[3] PRD 101 (2020) 054011 & 094013.
[4] PRD 103 (2021) 014025.
[5] JHEP 09 (2021) 141.
[6] PLB 826 (2022) 136908.
[7] Qin-Tao Song, PRD 108 (2023) 094041.
[8] SK, arXiv:2406.01180.

Nucleon spin

Almost none of nucleon spin is carried by quarks!

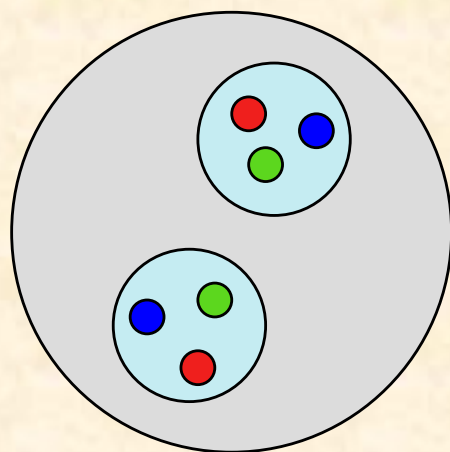
Nucleon spin puzzle!?



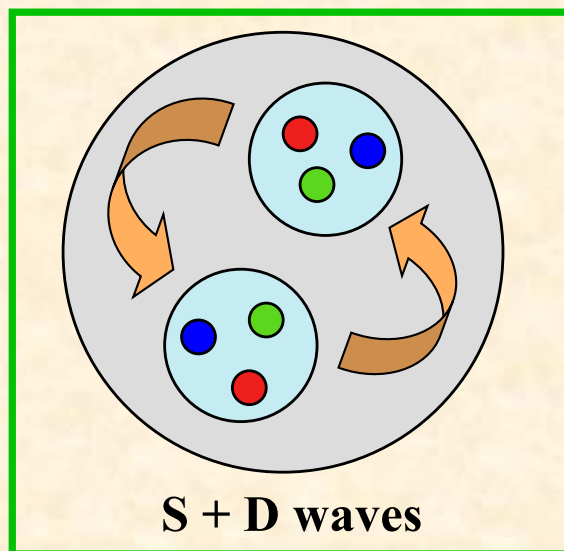
“old” standard model

Tensor structure b_1 (e.g. deuteron)

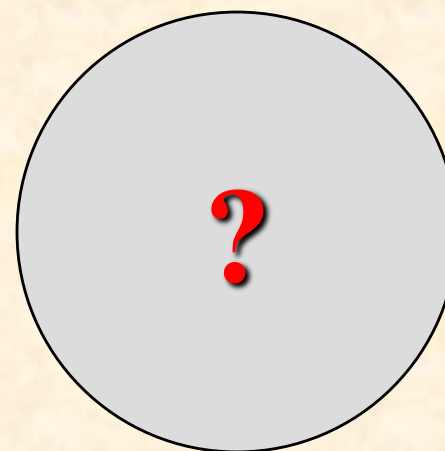
Tensor-structure puzzle!?



$b_1 = 0$



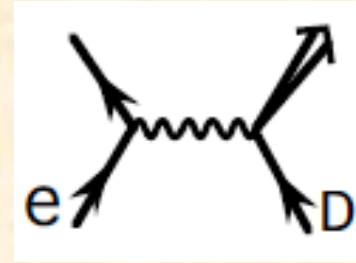
standard model $b_1 \neq 0$



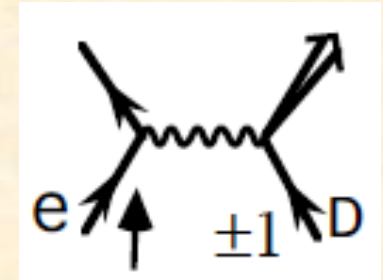
b_1 experiment $\neq b_1$ “standard model”

Structure Functions

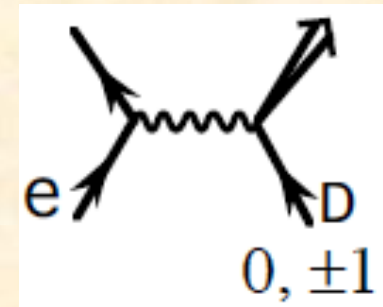
$$F_1 \propto \langle d\sigma \rangle$$



$$g_1 \propto d\sigma(\uparrow, +1) - d\sigma(\uparrow, -1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1) + \sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}[\sigma(+1) + \sigma(-1)]$

Parton Model

$$F_1 = \frac{1}{2} \sum_i e_i^2 (q_i + \bar{q}_i)$$

$$q_i = \frac{1}{3} (q_i^{+1} + q_i^0 + q_i^{-1})$$

$$g_1 = \frac{1}{2} \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$$

$$\Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

$$[q_{\uparrow}^H(x, Q^2)]$$

$$b_1 = \frac{1}{2} \sum_i e_i^2 (\delta_T q_i + \delta_T \bar{q}_i)$$

$$\delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

Gluon transversity $\Delta_T g$

Note on our notations:

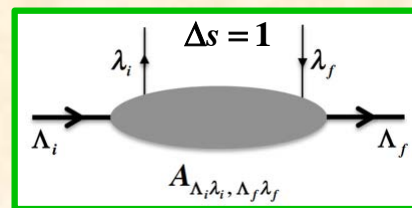
Tensor-polarized gluon distribution: $\delta_T g$

Gluon transversity: $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, +\frac{1}{2} + \frac{1}{2}\right) - A\left(+\frac{1}{2} - \frac{1}{2}, +\frac{1}{2} - \frac{1}{2}\right)$

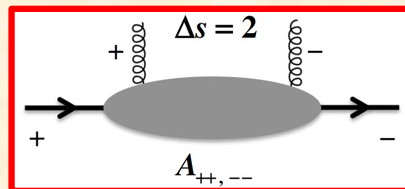
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron:

$\Delta_T g(x) \sim A(+1+1, -1-1)$,

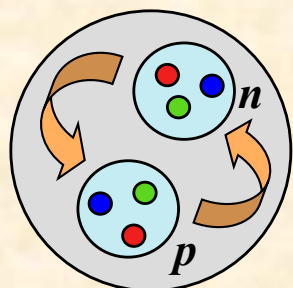
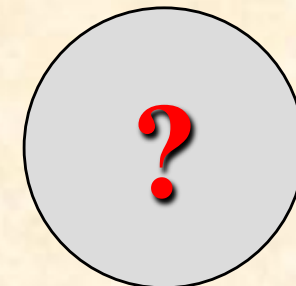
~~$A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$ not possible for nucleon~~



Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1 (\delta_T q, \delta_T g) \neq 0 \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?



S + D waves

Physics beyond “the standard model” in nuclear physics?
(Physics beyond the standard model in particle physics???)

**“Standard” deuteron model
prediction for b_1**

Electron scattering from a spin-1 hadron

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571.

[L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.]

$$W_{\mu\nu} = -F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{v} + g_1 \frac{i}{v} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{v^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma) \quad \text{spin-1/2, spin-1}$$

$$-b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu}) \quad \text{spin-1 only}$$

Note: Obvious factors from $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$ are not explicitly written. $E^\mu =$ polarization vector

$$v = p \cdot q, \quad \kappa = 1 + M^2 Q^2 / v^2, \quad E^2 = -M^2, \quad s^\sigma = -\frac{i}{M^2} \epsilon^{\sigma\alpha\beta\tau} E_\alpha^* E_\beta p_\tau$$

b_1, \dots, b_4 terms are defined so that they vanish by spin average.

$$r_{\mu\nu} = \frac{1}{v^2} \left(q \cdot E^* q \cdot E - \frac{1}{3} v^2 \kappa \right) g_{\mu\nu}, \quad s_{\mu\nu} = \frac{2}{v^2} \left(q \cdot E^* q \cdot E - \frac{1}{3} v^2 \kappa \right) \frac{p_\mu p_\nu}{v}$$

b_1, b_2 terms are defined to satisfy $2xb_1 = b_2$ in the Bjorken scaling limit.

$$t_{\mu\nu} = \frac{1}{2v^2} \left(q \cdot E^* p_\mu E_\nu + q \cdot E^* p_\nu E_\mu + q \cdot E p_\mu E_\nu^* + q \cdot E p_\nu E_\mu^* - \frac{4}{3} v p_\mu p_\nu \right)$$

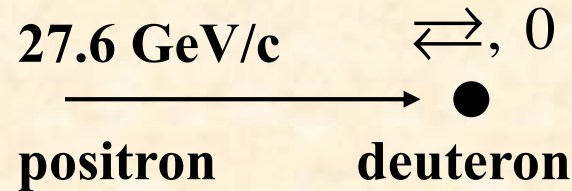
$$u_{\mu\nu} = \frac{1}{v} \left(E_\mu^* E_\nu + E_\nu^* E_\mu + \frac{2}{3} M^2 g_{\mu\nu} - \frac{2}{3} p_\mu p_\nu \right)$$

$2xb_1 = b_2$ in the scaling limit $\sim O(1)$

$$b_3, b_4 = \text{twist-4} \sim \frac{M^2}{Q^2}$$

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.



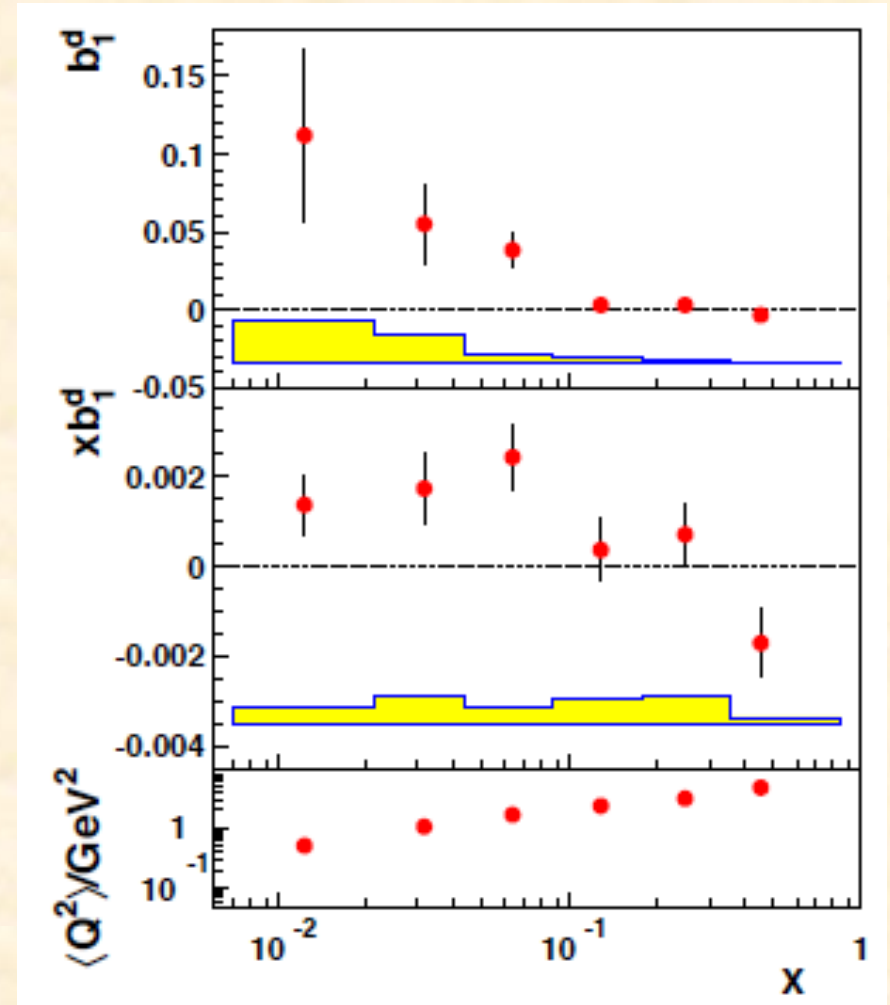
b_1 measurement in the kinematical region

$$0.01 < x < 0.45, \quad 0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$$

b_1 sum in the restricted Q^2 range $Q^2 > 1 \text{ GeV}^2$

$$\int_{0.02}^{0.85} dx b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5 \text{ GeV}^2$



$$\int dx b_1^D(x) = \lim_{t \rightarrow 0} -\frac{5}{24} \frac{t}{M^2} F_Q(t) + \sum_i e_i^2 \int dx \delta_T \bar{q}_i(x) = 0 ?$$

b_1 sum rule: F. E. Close and SK,
PRD 42 (1990) 2377.

$$\int \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} \int dx [u_v - d_v] + \frac{2}{3} \int dx [\bar{u} - \bar{d}] \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

Theory 1: Basic convolution approach

Convolution model: $A_{hH, hH}(x, Q^2) = \int \frac{dy}{y} \sum_s f_s^H(y) \hat{A}_{hs, hs}(x/y, Q^2) \equiv \sum_s f_s^H(y) \otimes \hat{A}_{hs, hs}(y, Q^2)$

$$A_{hH, h'H'} = \varepsilon_{h'}^{*\mu} W_{\mu\nu}^{H'H} \varepsilon_h^\nu, \quad b_1 = A_{+0,+0} - \frac{A_{++,++} + A_{+,-,+}}{2}$$

$$\hat{A}_{+\uparrow,+\uparrow} = F_1 - g_1, \quad \hat{A}_{+\downarrow,+\downarrow} = F_1 + g_1$$

Momentum distribution: $f^H(y) = \int d^3 p y |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N}\right)$

$$y = \frac{Mp \cdot q}{M_N P \cdot q} \simeq \frac{2p^-}{P^-}, \quad f^H(y) \equiv f_{\uparrow}^H(y) + f_{\downarrow}^H(y)$$

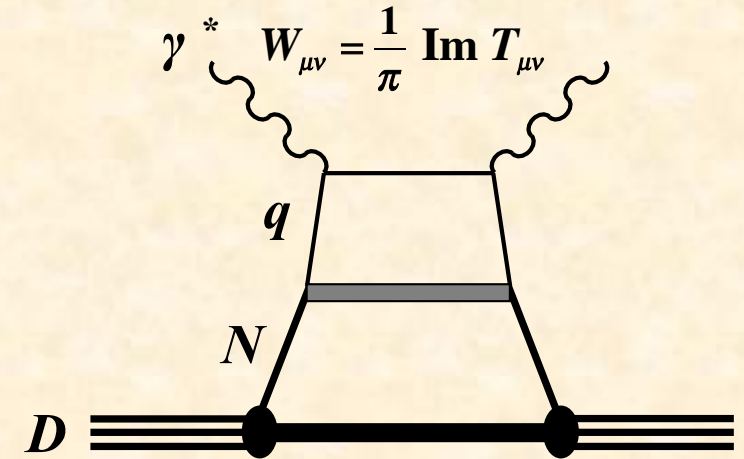
D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$

↓

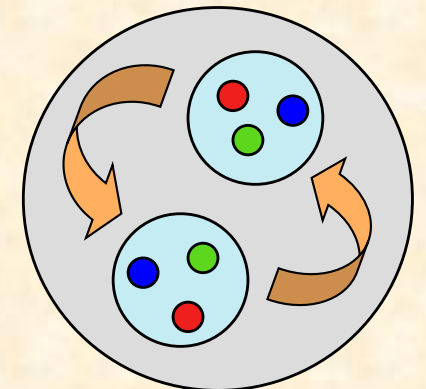
$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2)$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

$$= \int d^3 p y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p) \phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3 \cos^2 \theta - 1) \delta\left(y - \frac{p \cdot q}{M_N \nu}\right)$$

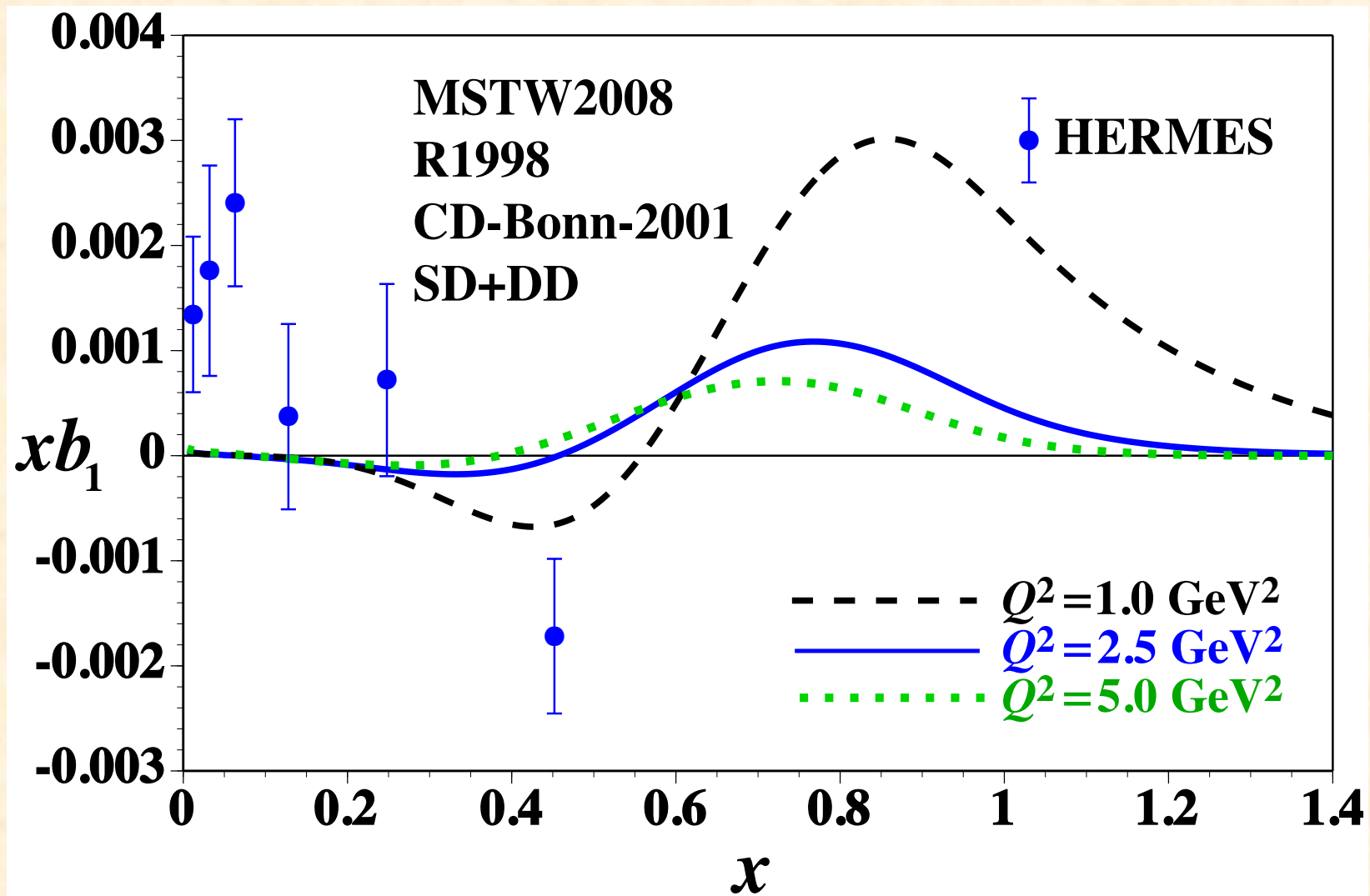


Standard model
of the deuteron



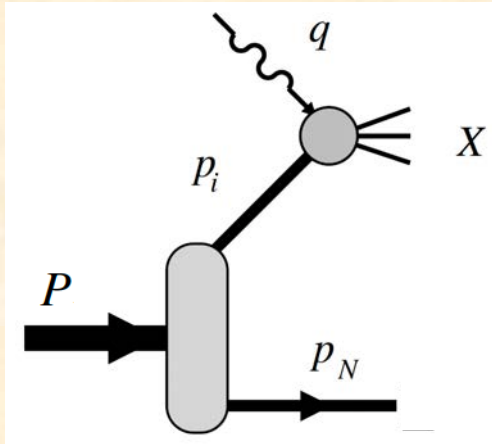
S + D waves

Comparison with HERMES measurements



Theory 2: Virtual nucleon approximation with higher-twist effects

- L. L. Frankfurt and M. I. Strikman, Phys. Rep. 76, 215 (1981);
 B. D. Keister and W. Polyzou, Adv. Nucl. Phys. 20, 225 (1991);
 W. Cosyn and M. Sargsian, Phys. Rev. C 84, 014601 (2011);
 W. Cosyn, W. Melnitchouk, and M. Sargsian, Phys. Rev. C 89, 014612 (2014).
 W. Cosyn and C. Weiss, Phys. Rev. C 102 (2020) 065204.



Virtual nucleon approximation (VNA)

$$W_{\mu\nu}^{\lambda'\lambda}(P, q) = 4(2\pi)^3 \int d\Gamma_N \frac{\alpha_N}{\alpha_i} W_{\mu\nu}^N(p_i, q) \rho_D(\lambda', \lambda)$$

momentum-fractions for interacting (i) and spectator nucleons (N):

$$\alpha_i = \frac{2p_i^-}{P^-}, \quad \alpha_N = \frac{2p_N^-}{P^-} = 2 - \alpha_i, \quad P = p_i + p_N$$

phase space: $d\Gamma_N = \frac{d^3 p_N}{2E_{p_N} (2\pi)^3}$

deuteron density: $\rho_D(\lambda', \lambda) = \sum_{\lambda_N, \lambda'_N} \frac{[\psi_{\lambda'}^D(\vec{k}, \lambda'_N, \lambda_N)]^\dagger \psi_{\lambda}^D(\vec{k}, \lambda'_N, \lambda_N)}{\alpha_N \alpha_i}$

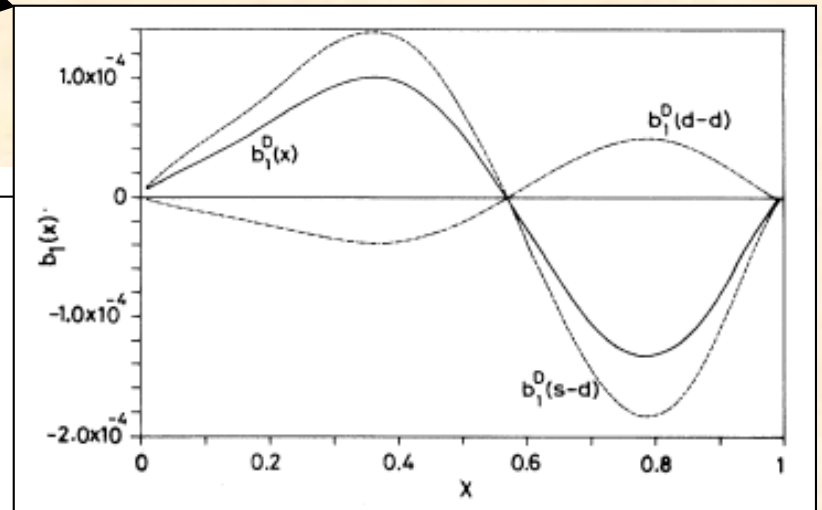
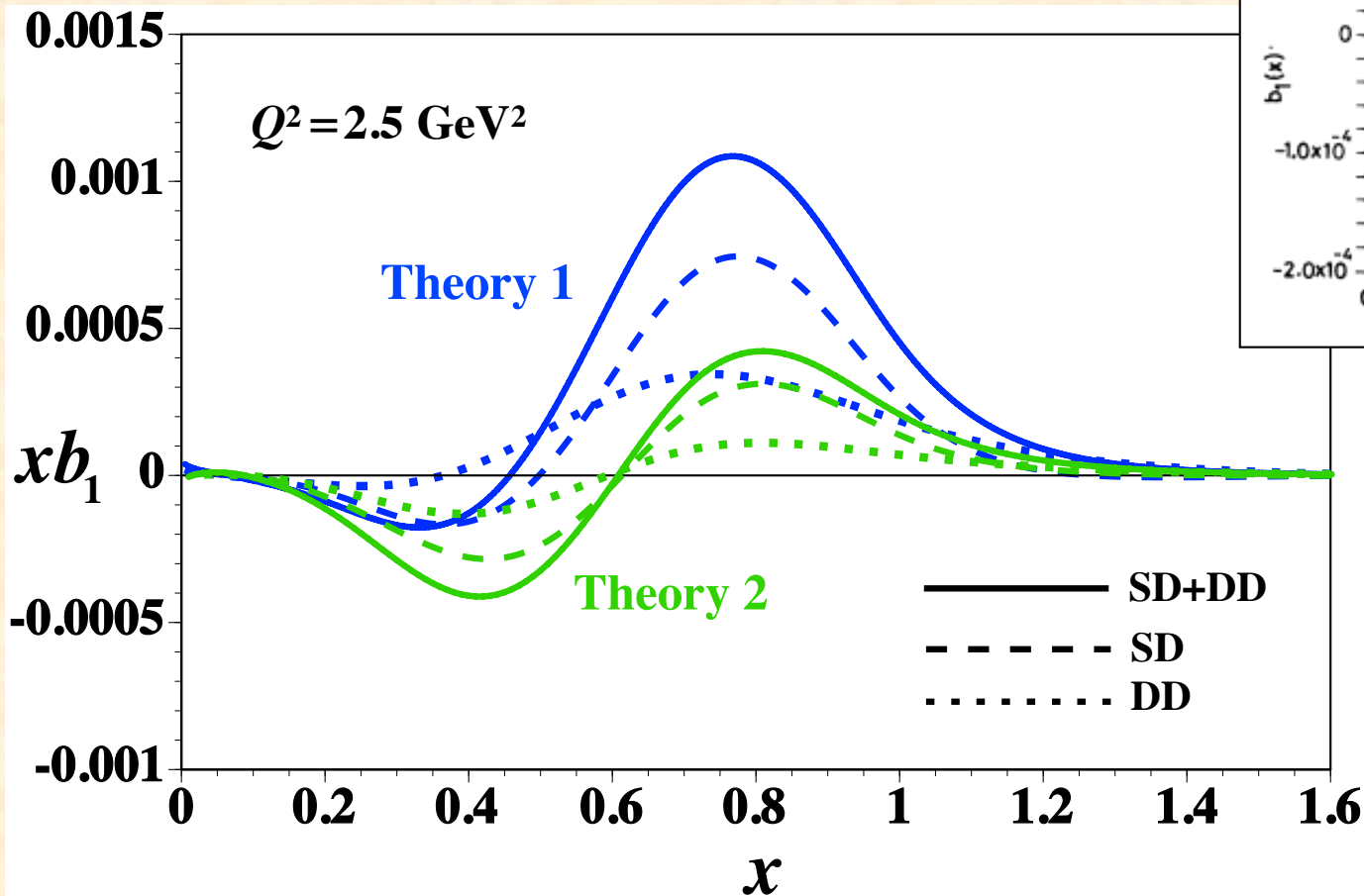
Results on b_1 in the convolution description

Very different from

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571.

H. Khan and P. Hoodbhoy, PRC44 (1991) 1219;

- (1) SD term is opposite,
- (2) $b_1(x)$ exists even at $x > 1$,
- (3) $|b_1(\text{CDKS})| = 10^{-3} \gg |b_1(\text{KH})| = 10^{-4}$.



“Standard-model” prediction for b_1 of deuteron

$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2), \quad y = \frac{Mp \cdot q}{M_N P \cdot q} \approx \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

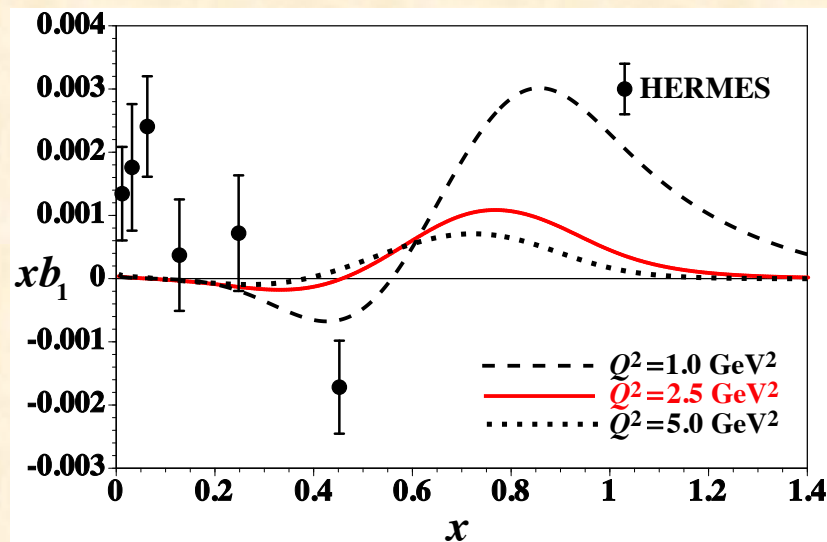
$$= \int d^3 p y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p) \phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3 \cos^2 \theta - 1) \delta \left(y - \frac{p \cdot q}{M_N v} \right)$$

S-D term D-D term

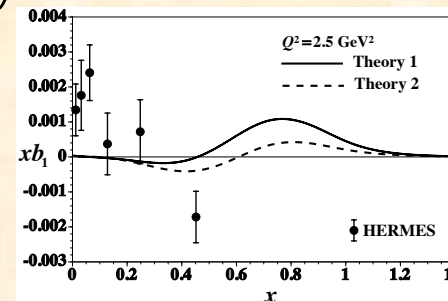
Nucleon momentum distribution:

$$f^H(y) \equiv f_{\uparrow}^H(y) + f_{\downarrow}^H(y) = \int d^3 p y |\phi^H(\vec{p})|^2 \delta \left(y - \frac{E - p_z}{M_N} \right)$$

D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$



W. Cosyn, Yu-Bing Dong, SK, M. Sargsian,
Phys. Rev. D 95 (2017) 074036.



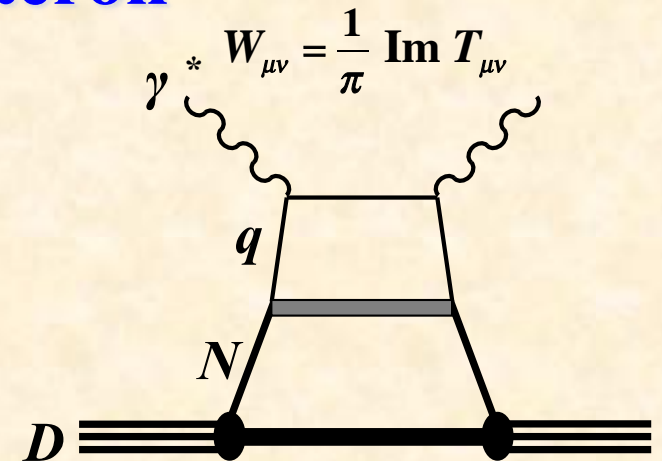
$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$
at $x < 0.5$

Standard convolution model does not work for the deuteron tensor structure!?

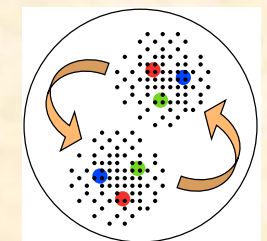
G. A. Miller, PRC 89 (2014) 045203,

Interesting suggestions:

hidden-color, 6-quark, ...
 $|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \dots$



Standard model of the deuteron



Tensor-polarized PDFs
at hadron accelerator facilities
(*e.g.* Fermilab)

Spin asymmetries in the parton model

unpolarized: q_a , longitudinally polarized: Δq_a ,
 transversely polarized: $\Delta_T q_a$, tensor polarized: δq_a

Unpolarized cross section

$$\left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle = \frac{\alpha^2}{4Q^2} (1 + \cos^2 \theta) \frac{1}{3} \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]$$

Spin asymmetries

$$A_{LL} = \frac{\sum_a e_a^2 [\Delta q_a(x_A) \Delta \bar{q}_a(x_B) + \Delta \bar{q}_a(x_A) \Delta q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

$$A_{TT} = \frac{\sin^2 \theta \cos(2\phi) \sum_a e_a^2 [\Delta_T q_a(x_A) \Delta_T \bar{q}_a(x_B) + \Delta_T \bar{q}_a(x_A) \Delta_T q_a(x_B)]}{1 + \cos^2 \theta \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

$$A_{UQ_0} = \frac{\sum_a e_a^2 [q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B)]}{2 \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

$$\begin{aligned} A_{LT} &= A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1} \\ &= A_{LQ_1} = A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0 \end{aligned}$$

M. Hino and SK,
 PRD 59 (1999) 094026;
 60 (1999) 054018.

Advantage of the hadron reaction ($\delta \bar{q}$ measurement)

$$A_{UQ_0} (\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta_T \bar{q}_a(x_B)}{2 \sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}$$

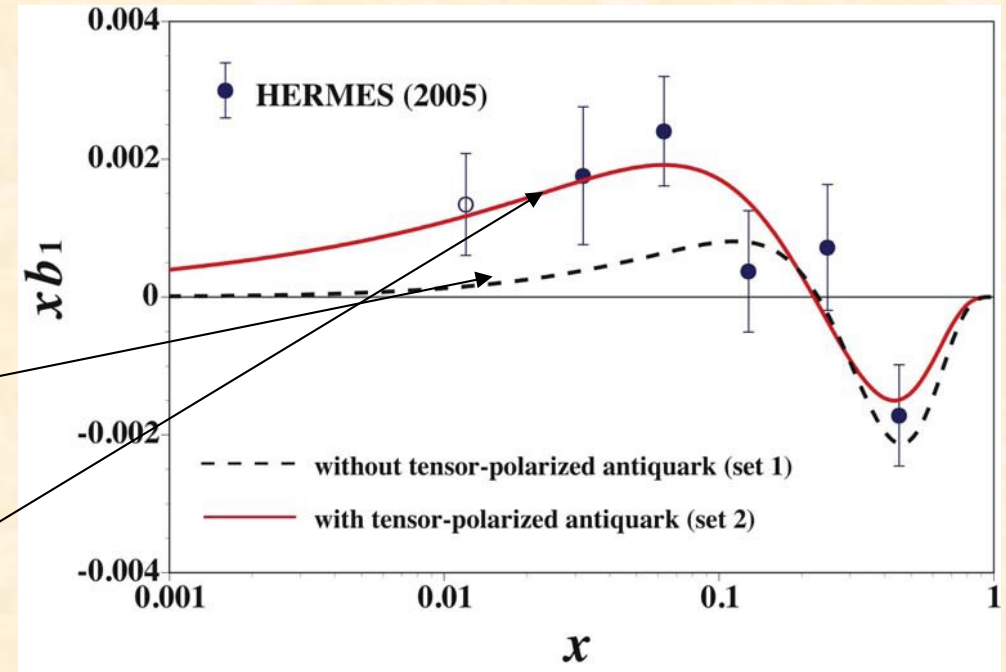
Note: $\delta \neq$ transversity in my notation

Tensor-polarized PDFs

SK, PRD 82 (2010) 017501.

Two-types of fit results:

- set-1 ($\delta_T \bar{q} = 0$): $\chi^2 / \text{d.o.f.} = 2.83$
Without $\delta_T \bar{q}$, the fit is not good enough.
- set-2 ($\delta_T \bar{q} \neq 0$): $\chi^2 / \text{d.o.f.} = 1.57$
With finite $\delta_T \bar{q}$, the fit is reasonably good.



Obtained tensor-polarized distributions

$\delta_T q(x)$, $\delta_T \bar{q}(x)$ from the HERMES data.

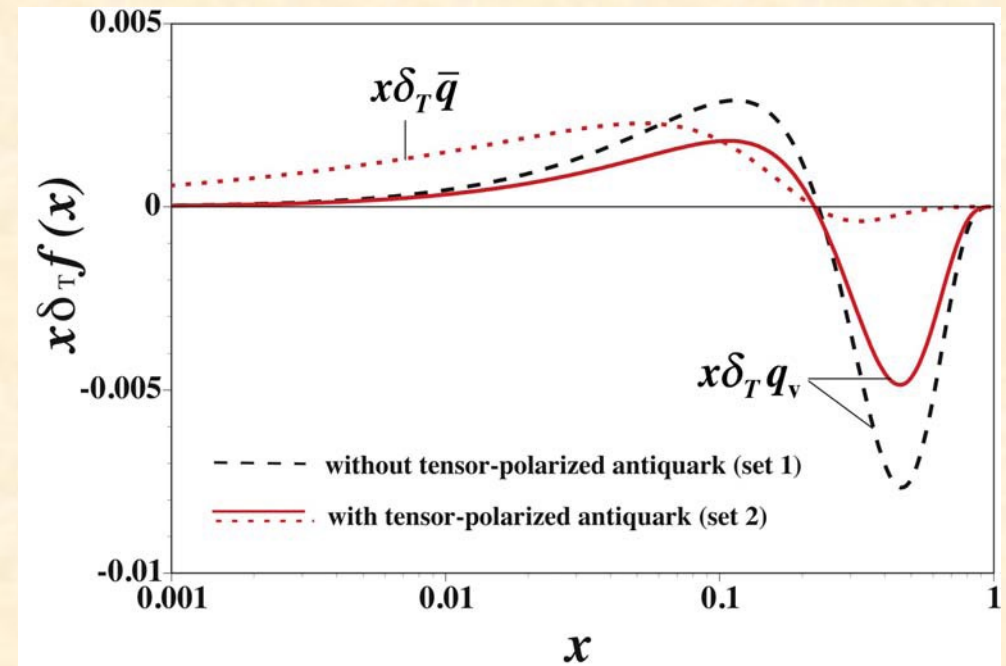
→ They could be used for

- experimental proposals,
- comparison with theoretical models.

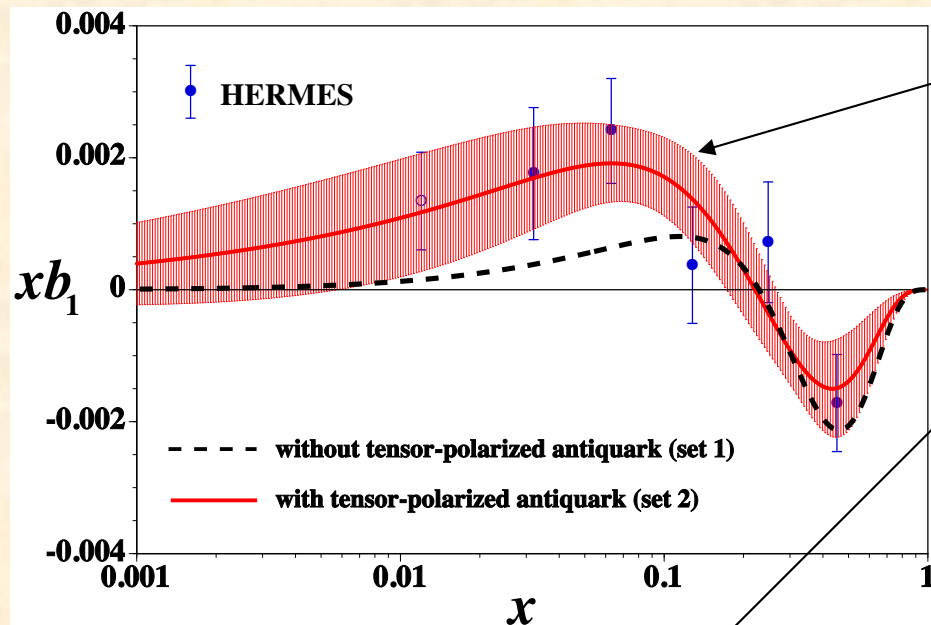
Finite tensor polarization for antiquarks:

$$\int_0^1 dx b_1(x) = 0.058$$

$$= \frac{1}{9} \int_0^1 dx [4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x)]$$

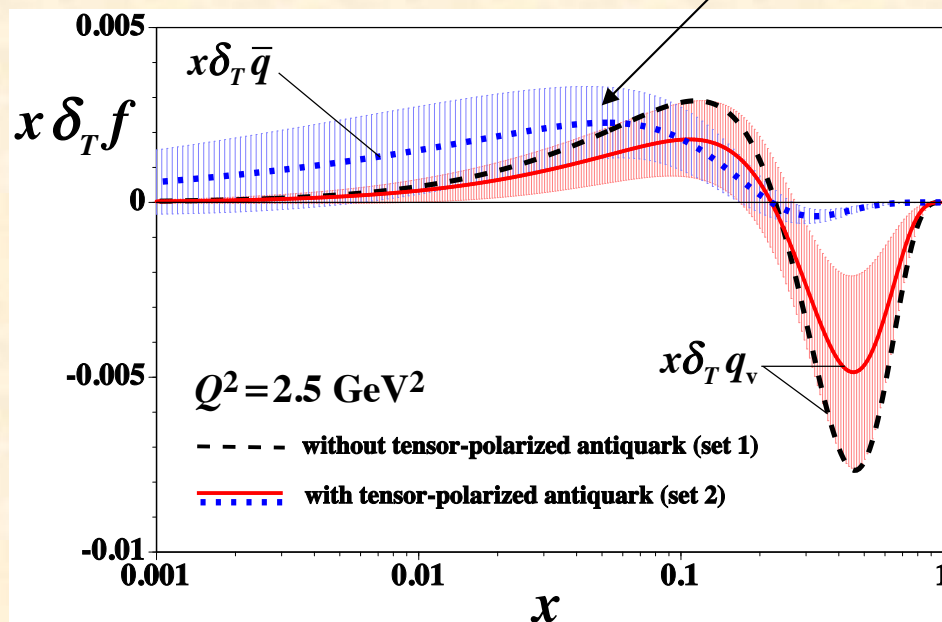


Tensor-polarized PDFs with errors



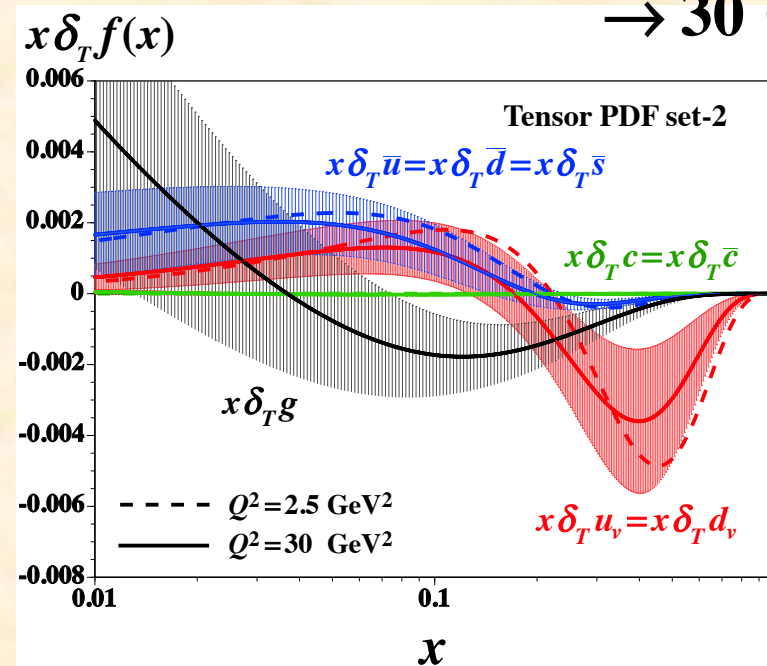
still large errors,
need experimental improvement
→ JLab, EIC, ...

experimental measurement
for antiquark distributions
→ Fermilab, ...



Q^2 evolution

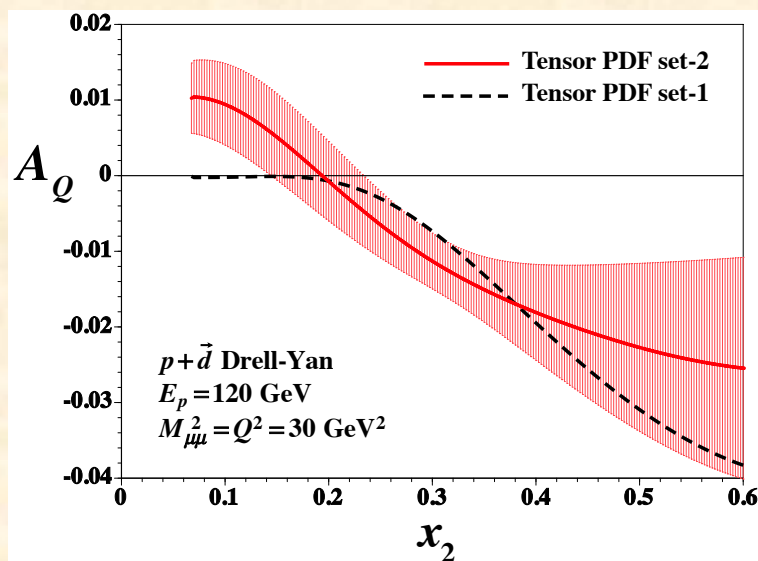
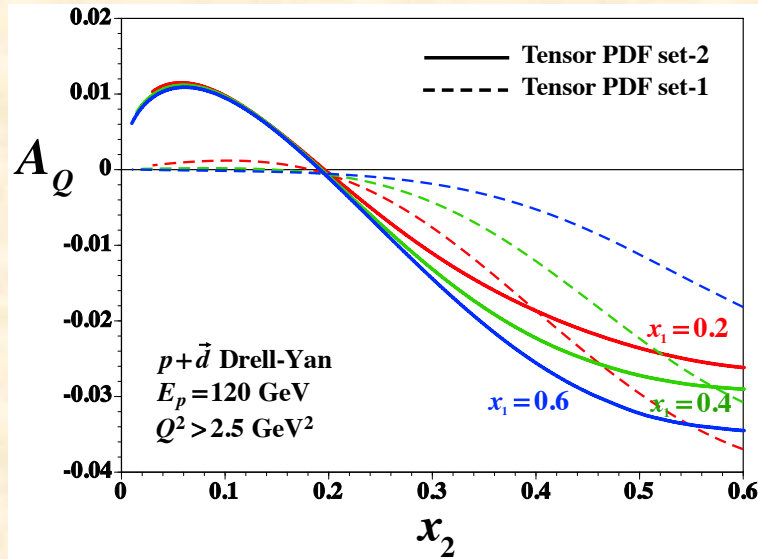
$$Q^2 = 2.5 \text{ GeV}^2 \rightarrow 30 \text{ GeV}^2$$



Tensor-polarized spin asymmetry at Fermilab

$$A_Q = \frac{\sum_a e_a^2 [q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B)]}{\sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}$$

Polarized fixed-target experiments
at the Main Injector



E1039-SpinQuest

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

List of Collaborators:

- D. Geesaman, P. Reimer
Argonne National Laboratory, Argonne, IL 60439
- C. Brown, D. Christian
Fermi National Accelerator Laboratory, Batavia IL 60510
- M. Dieffenthaler, J.-C. Peng
University of Illinois, Urbana, IL 61081
- W.-C. Chang, Y.-C. Chen
Institute of Physics, Academia Sinica, Taiwan
- S. Sawada
KEK, Tsukuba, Ibaraki 305-0801, Japan
- T.-H. Chang
Ling-Tung University, Taiwan
- J. Huang, X. Jiang, M. Leitch, A. Klein, K. Liu, M. Liu, P. McGaughey
Los Alamos National Laboratory, Los Alamos, NM 87545
- E. Beise, K. Nakahara
University of Maryland, College Park, MD 20742
- C. Aidala, W. Lorenzon, R. Raymond
University of Michigan, Ann Arbor, MI 48109-1040
- T. Badman, E. Long, K. Sliker, R. Zielinski
University of New Hampshire, Durham, NH 03824
- R.-S. Guo
National Kaohsiung Normal University, Taiwan
- Y. Goto
RIKEN, Wako, Saitama 351-01, Japan
- L. El Fassi, K. Myers, R. Ransome, A. Tadepalli, B. Tice
Rutgers University, Rutgers NJ 08544
- J.-P. Chen
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606
- K. Nakano, T.-A. Shibata
Tokyo Institute of Technology, Tokyo 152-8551, Japan
- D. Crabb, D. Day, D. Keller, O. Rondon
University of Virginia, Charlottesville, VA 22904

SK and Qin-Tao Song,
PRD 94 (2016) 054022.

Gluon transversity
at hadron accelerator facilities
(*e.g.* Fermilab)

Gluon transversity $\Delta_T g$

Note on our notations:

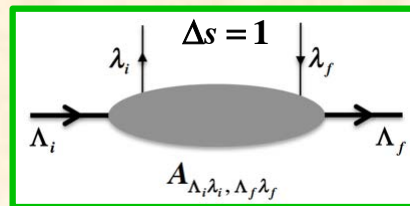
Tensor-polarized gluon distribution: $\delta_T g$

Gluon transversity: $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, +\frac{1}{2} + \frac{1}{2}\right) - A\left(+\frac{1}{2} - \frac{1}{2}, +\frac{1}{2} - \frac{1}{2}\right)$

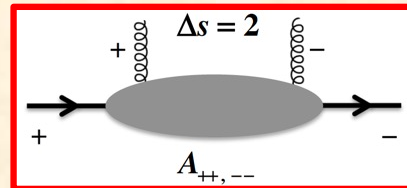
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron:

$\Delta_T g(x) \sim A(+1+1, -1-1)$,

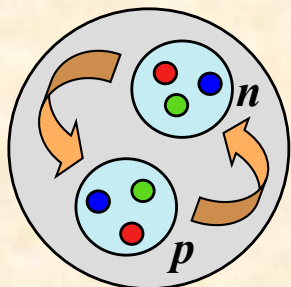
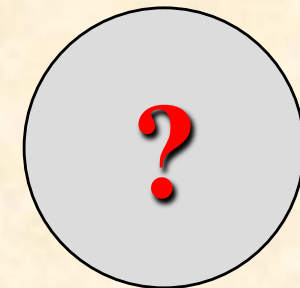
~~$A\left(+\frac{1}{2} + 1, -\frac{1}{2} - 1\right)$ not possible for nucleon~~



Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1 (\delta_T q, \delta_T g) \neq 0 \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?



S + D waves

Physics beyond “the standard model” in nuclear physics?
(Physics beyond the standard model in particle physics???)

Letter of Intent at Jefferson Lab (middle 2020's)

Jefferson Lab,
Electron accelerator ~12 GeV



LoI, arXiv:1803.11206

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

Electron scattering with polarized-deuteron target

$$\left. \frac{d\sigma}{dx dy d\phi} \right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4} \left[xy^2 F_1(x, Q^2) + (1-y)F_2(x, Q^2) - \frac{1}{2}x(1-y)\Delta(x, Q^2) \cos(2\phi) \right]$$

$$\Delta(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3} \Delta_T g(y, Q^2)$$

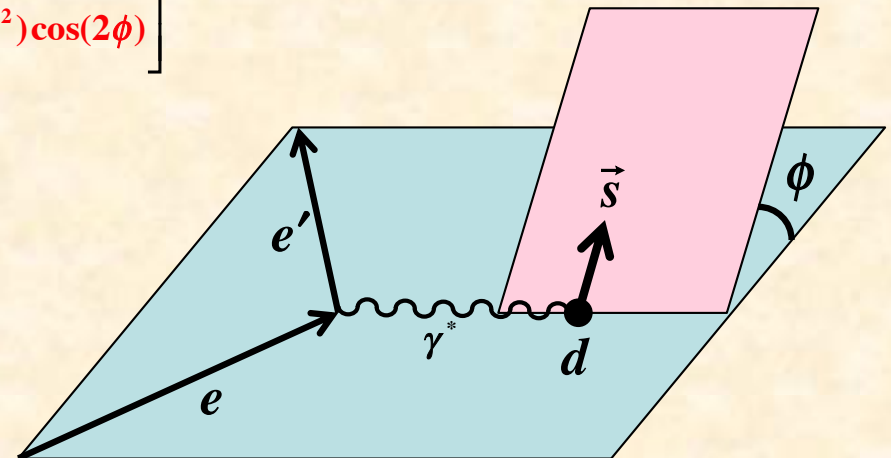
By looking at the deuteron-polarization angle ϕ ,
the quark transversty $\Delta_T g$ can be measured.

Lattice QCD estimates:

W. Detmold and P. E. Shanahan,

PRD 94 (2016) 014507; 95 (2017) 079902.

For development of polarized deuteron target,
see D. Keller, D. Crabb, D. Day
Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.



Our motivation by considering the JLab experiment

We proposed to use hadron accelerator facilities for studying the gluon transversity.

Advantages:

- Independent experiment from JLab
- Different kinematical regions: larger Q^2 , smaller x
- Hadron facilities are often useful for probing gluon distributions (namely a leading effect).
- Hadron cross sections are generally larger (not for Drell-Yan).
- The gluon transversity could be measured in a different form from the integral $\int_x^1 \frac{dy}{y^3} \Delta_T q(y, Q^2)$ in the JLab experiment.

→ In our PRD 101 (2020) 054011 & 094013 , we proposed proton-deuteron Drell-Yan process by considering the Fermilab-E1039.

However, our formalism is validated for Drell-Yan experiments at any other facilities.



Fermilab-MI



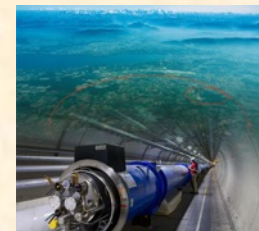
NICA



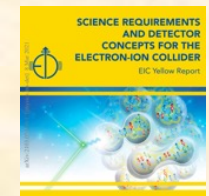
GSI-FAIR



J-PARC

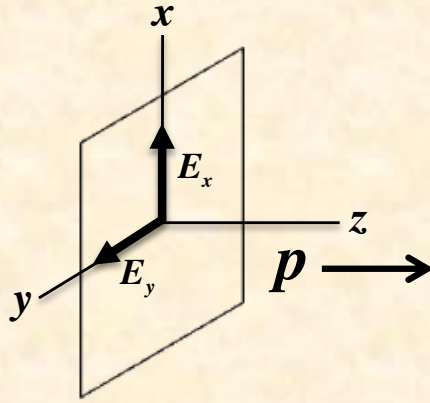


LHC (fixed target)
COMPASS/AMBER



EIC
/EicC

Gluon transversity distribution in deuteron



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ e^{i x p^+ \xi^-} \left\langle p E_x \left| A^x(0) A^x(\xi) - A^y(0) A^y(\xi) \right| p E_x \right\rangle_{\xi^+ = \bar{\xi}_T = 0}$$

$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1, 0, 0)$, $\vec{E}_y = \vec{\varepsilon}_y = (0, 1, 0)$

Spin and tensor of the deuteron

$$S^\mu = \frac{1}{M} \varepsilon^{\mu\nu\alpha\beta} p_\nu \text{Im}(E_\alpha^* E_\beta), \quad T^{\mu\nu} = -\frac{1}{3} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) - \text{Re}(E^{\mu*} E^\nu)$$

$$E^\mu = (0, \vec{E}), \quad \vec{E}_\pm = \frac{1}{\sqrt{2}} (\mp 1, -i, 0), \quad \vec{E}_0 = (0, 0, 1)$$

- $\vec{E}_+, \vec{E}_0, \vec{E}_-$: Spin states with z -components of spin $s_z = +1, 0, -1$
- $\vec{E}_x = (1, 0, 0), \vec{E}_y = (0, 1, 0)$: **Linear polarizations**
 → to measure gluon transversity

- (1) Prepare $s_x = 0$ [$\vec{E}_x = (1, 0, 0)$] by taking the quantization axis x and $s_y = 0$ [$\vec{E}_y = (0, 1, 0)$] by taking the quantization axis y .
- (2) Combination of transverse polarizations.

Transverse polarization

Linear polarization

$$S = (S_T^x, S_T^y, S_L),$$

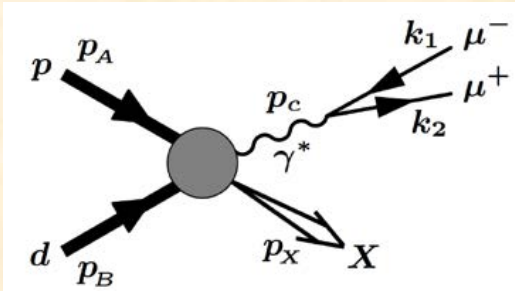
$$T = \frac{1}{2} \begin{pmatrix} -\frac{2}{3} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3} S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3} S_{LL} \end{pmatrix} \quad S_{TT}^{xy} = S_{LT}^x = S_{LT}^y = 0$$

| Polarizations | \vec{E} | S_T^x | S_T^y | S_L | S_{LL} | S_{TT}^{xx} |
|-------------------|---------------------------------|---------|---------|-------|----------------|----------------|
| Longitudinal $+z$ | $\frac{1}{\sqrt{2}}(-1, -i, 0)$ | 0 | 0 | +1 | $+\frac{1}{2}$ | 0 |
| Longitudinal $-z$ | $\frac{1}{\sqrt{2}}(+1, -i, 0)$ | 0 | 0 | -1 | $+\frac{1}{2}$ | 0 |
| Transverse $+x$ | $\frac{1}{\sqrt{2}}(0, -1, -i)$ | +1 | 0 | 0 | $-\frac{1}{4}$ | $+\frac{1}{2}$ |
| Transverse $-x$ | $\frac{1}{\sqrt{2}}(0, +1, -i)$ | -1 | 0 | 0 | $-\frac{1}{4}$ | $+\frac{1}{2}$ |
| Transverse $+y$ | $\frac{1}{\sqrt{2}}(-i, 0, -1)$ | 0 | +1 | 0 | $-\frac{1}{4}$ | $-\frac{1}{2}$ |
| Transverse $-y$ | $\frac{1}{\sqrt{2}}(-i, 0, +1)$ | 0 | -1 | 0 | $-\frac{1}{4}$ | $-\frac{1}{2}$ |
| Linear x | $(1, 0, 0)$ | 0 | 0 | 0 | $+\frac{1}{2}$ | -1 |
| Linear y | $(0, 1, 0)$ | 0 | 0 | 0 | $+\frac{1}{2}$ | +1 |

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Proton-deuteron Drell-Yan cross section

Drell-Yan cross section



$$d\sigma_{pd \rightarrow \mu^+ \mu^- X} = \int_0^1 dx_a \int_0^1 dx_b f_a(x_a) f_b(x_b) d\hat{\sigma}_{ab \rightarrow \mu^+ \mu^- d}, \quad M_{ab \rightarrow \mu^+ \mu^- d} = eM_{\gamma^* \rightarrow \mu^+ \mu^-} \frac{-1}{Q^2} eM_{ab \rightarrow \gamma^* d}$$

In terms of lepton tensor $L^{\mu\nu}$ and hadron tensor $W_{\mu\nu}$

$$\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}}{d\tau dq_T^2 d\phi dy} = \frac{\alpha^2}{12\pi^2 Q^4} \left[\int d\Phi_2(q; k_1, k_2) 2L^{\mu\nu} \right] W_{\mu\nu}$$

$$\text{dilepton phase space: } d\Phi_2(q; k_1, k_2) = \delta^4(q - k_1 - k_2) \frac{d^3 k_1}{2E_1 (2\pi)^3} \frac{d^3 k_2}{2E_2 (2\pi)^3}$$

$$L^{\mu\nu} = 2(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - k_1 \cdot k_2 g^{\mu\nu})$$

$$W_{\mu\nu} = \sum_{\text{spin, color}} \bar{\sum}_q e_q^2 \int_{\min(x_a)}^1 dx_a \frac{\pi}{p_g^-(x_a - x_1)} \text{Tr} \left[\Gamma_{\nu\beta} \left\{ \Phi_{q/A}(x_a) + \Phi_{\bar{q}/A}(x_a) \right\} \hat{\Gamma}_{\mu\alpha} \Phi_{g/B}^{\alpha\beta}(x_b) \right], \quad \hat{\Gamma}_{\nu\beta} = \gamma^0 \Gamma_{\nu\beta} \gamma^0$$

Collinear correlation functions

Refs. A. Bacchetta and P. J. Mulders, Phys. Rev. D 62 (2000) 114004,

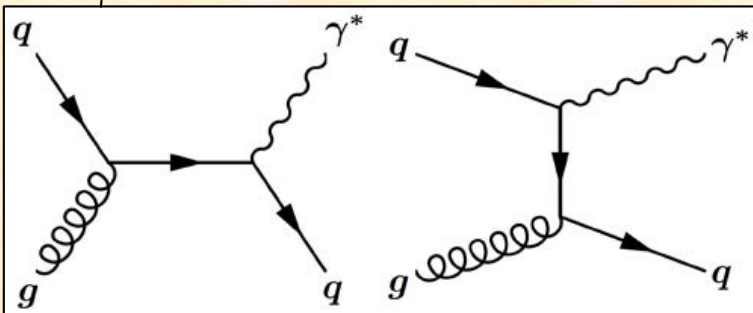
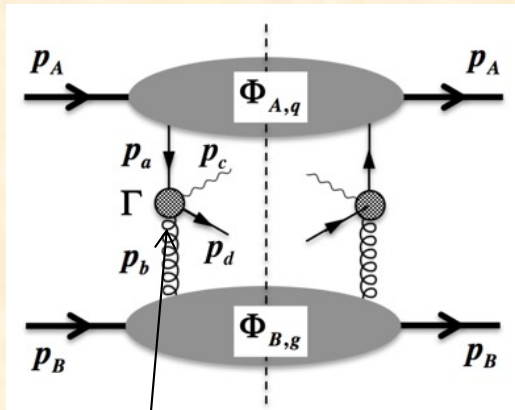
D. Boer et al., JHEP 10 (2016) 013,

T. van Daal, arXiv:1812.07336 (Ph.D. Thesis).

$$\Phi_{q/A}(x_a) = \frac{1}{2} \left[\bar{n} f_{1,q/A}(x_a) + \gamma_5 \bar{n} S_{A,L} g_{1,q/A}(x_a) + \bar{n} \gamma_5 \not{s}_{A\perp} h_{1,q/A}(x_a) \right]$$

$$\Phi_{q/B}(x_b) = \frac{1}{2} \left[n f_{1,q/B}(x_b) + \gamma^5 n S_{B,L} g_{1,q/B}(x_b) + i\sigma_{\mu\nu} \gamma^5 n^\mu S_{B,T}^\nu h_{1,q/B}(x_b) + n S_{LL} f_{1LL,q/B}(x_b) + \sigma_{\mu\nu} n^\nu S_{B,LT}^\mu h_{1LT,q/B}(x_b) \right]$$

$$\Phi_{g/B}^{ij}(x_b) = \frac{1}{2} \left[-g_T^{ij} f_{1,g/B}(x_b) + i\epsilon_T^{ij} S_{B,L} g_{1LL,g/B}(x_b) - g_T^{ij} S_{B,LL} f_{1LL,g/B}(x_b) + S_{B,TT}^{ij} h_{1TT,g/B}(x_b) \right]$$



Gluon transversity: $\Delta_T g = h_{1TT,g}$
(Sorry to use two different notations in a talk.)

Proton-deuteron Drell-Yan cross section

SK and Qin-Tao Song,
PRD 101 (2020) 054011 & 094013.

Drell-Yan cross section

$$\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}(E_x - E_y)}{d\tau dq_T^2 d\phi dy} = \frac{\alpha^2 \alpha_s C_F q_T^2}{6\pi s^3} \cos(2\phi) \int_{\min(x_a)}^1 dx_a \frac{1}{(x_a x_b)^2 (x_a - x_1)(\tau - x_a x_2)^2} \sum_q e_q^2 x_a [q_A(x_a) + \bar{q}_A(x_a)] x_b \Delta_T g_B(x_b)$$

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad \min(x_a) = \frac{x_1 - \tau}{1 - x_2}, \quad x_b = \frac{x_a x_2 - \tau}{x_a - \tau}$$

= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

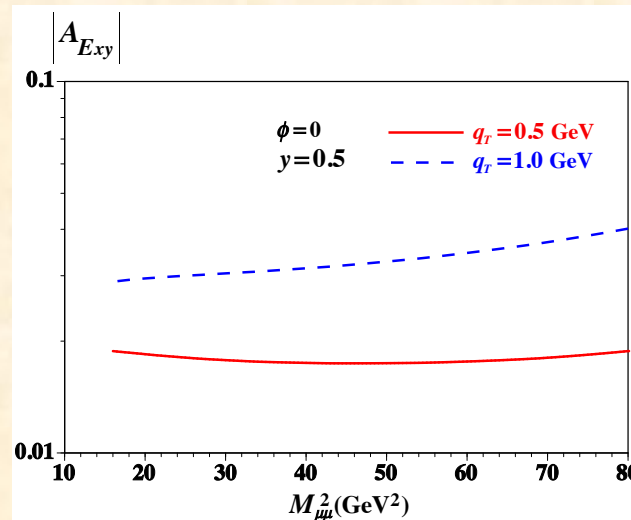
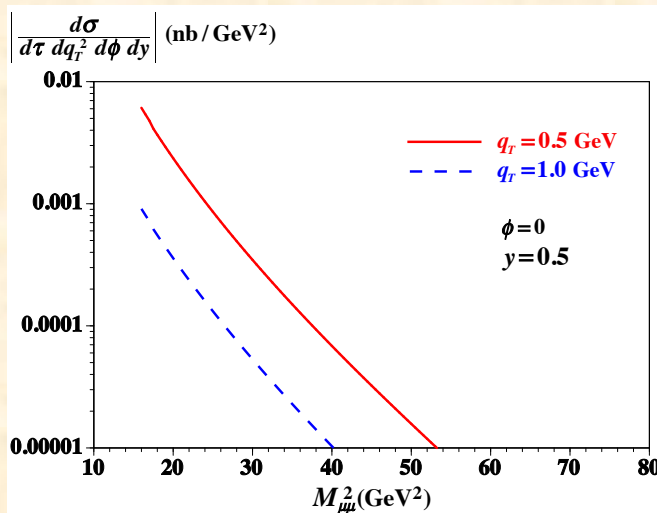
- Consider the Fermilab-E1039 experiment with the proton beam of $p = 120$ GeV

- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g = \Delta g_p + \Delta g_n$ (or $\frac{\Delta g_p + \Delta g_n}{2}$, $\frac{\Delta g_p + \Delta g_n}{4}$)

- CTEQ14 for $q(x) + \bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squared ($M_{\mu\mu}^2 = Q^2$) dependence

Spin asymmetry: $A_{E_{xy}} = \frac{\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}(E_x) - \frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}(E_y)}{d\tau dq_T^2 d\phi dy}}{\frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}(E_x) + \frac{d\sigma_{pd \rightarrow \mu^+ \mu^- X}(E_y)}{d\tau dq_T^2 d\phi dy}}$



**New proposal
at Fermilab-PAC (2023, D. Keller)**

TMDs and PDFs for spin-1 hadrons up to twist 4

**Note: Higher-twist effects are sizable at a few GeV^2 Q^2
in tensor-polarized structure functions,
W. Cosyn, Yu-Bing Dong, SK, M. Sargsian,
PRD 95 (2017) 074036.**

TMD correlation functions for spin-1 hadrons

Correlation functions

Spin vector: $S^\mu = S_L \frac{P^+}{M} \bar{n}^\mu - S_L \frac{M}{2P^+} n^\mu + S_T^\mu$

Tensor: $T^{\mu\nu} = \frac{1}{2} \left[\frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \bar{n}^\mu \bar{n}^\nu + \frac{P^+}{M} \bar{n}^{\langle\mu} S_{LT}^{\nu\rangle} - \frac{2}{3} S_{LL} (\bar{n}^{\langle\mu} n^{\nu\rangle} - g_T^{\mu\nu}) + S_{TT}^{\mu\nu} - \frac{M}{2P^+} n^{\langle\mu} S_{LT}^{\nu\rangle} + \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu \right]$

Tensor part (twist-2): [Bacchetta, Mulders, PRD 62 \(2000\) 114004](#)

$$\Phi(k, P, T) = \left(\frac{A_{13}}{M} I + \frac{A_{14}}{M^2} \not{P} + \frac{A_{15}}{M^2} \not{\mathcal{K}} + \frac{A_{16}}{M^3} \sigma_{\rho\sigma} P^\rho k^\sigma \right) k_\mu k_\nu T^{\mu\nu} + \left[A_{17} \gamma_\nu + \left(\frac{A_{18}}{M} P^\rho + \frac{A_{19}}{M} k^\rho \right) \sigma_{\nu\rho} + \frac{A_{20}}{M^2} \varepsilon_{\nu\rho\sigma} P^\rho k^\sigma \gamma^\tau \gamma_5 \right] k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ dependent terms are added for up to twist 4.

[For the spin-1/2 nucleon: [Goeke, Metzand, Schlegel, PLB 618 \(2005\) 90](#); [Metz, Schweitzer, Teckentrup, PLB 680 \(2009\) 141](#).]

[Kumano-Song-2021](#), for the details see [PRD 103 \(2021\) 014025](#)

$$\Phi(k, P, T | n) = \left(\frac{A_{13}}{M} I + \frac{A_{14}}{M^2} \not{P} + \frac{A_{15}}{M^2} \not{\mathcal{K}} + \frac{A_{16}}{M^3} \sigma_{\rho\sigma} P^\rho k^\sigma \right) k_\mu k_\nu T^{\mu\nu} + \left[A_{17} \gamma_\nu + \left(\frac{A_{18}}{M} P^\rho + \frac{A_{19}}{M} k^\rho \right) \sigma_{\nu\rho} + \frac{A_{20}}{M^2} \varepsilon_{\nu\rho\sigma} P^\rho k^\sigma \gamma^\tau \gamma_5 \right] k_\mu T^{\mu\nu}$$

[Bacchetta
-Mulders \(2000\)](#)

**New terms
in our paper
(2021)**

$$\begin{aligned} & + \left(\frac{B_{21}M}{P \cdot n} k_\mu + \frac{B_{22}M^3}{(P \cdot n)^2} n_\mu \right) n_\nu T^{\mu\nu} + i \gamma_5 \varepsilon_{\mu\rho\sigma} P^\rho \left(\frac{B_{23}}{(P \cdot n)M} k^\tau n^\sigma k_\nu + \frac{B_{24}M}{(P \cdot n)^2} k^\tau n^\sigma n_\nu \right) T^{\mu\nu} \\ & + \left[\frac{B_{25}}{P \cdot n} \not{n} k_\mu k_\nu + \left(\frac{B_{26}M^2}{(P \cdot n)^2} \not{n} + \frac{B_{28}}{P \cdot n} \not{P} + \frac{B_{30}}{P \cdot n} \not{\mathcal{K}} \right) k_\mu n_\nu + \left(\frac{B_{27}M^4}{(P \cdot n)^3} \not{n} + \frac{B_{29}M^2}{(P \cdot n)^2} \not{P} + \frac{B_{31}M^2}{(P \cdot n)^2} \not{\mathcal{K}} \right) n_\mu n_\nu + \frac{B_{32}M^2}{P \cdot n} \gamma_\mu n_\nu \right] T^{\mu\nu} \\ & - \left[\varepsilon_{\mu\rho\sigma} \gamma^\tau P^\rho \left(\frac{B_{34}}{P \cdot n} n^\sigma k_\nu + \frac{B_{33}}{P \cdot n} k^\sigma n_\nu + \frac{B_{35}M^2}{(P \cdot n)^2} n^\sigma n_\nu \right) + \varepsilon_{\lambda\rho\sigma} k^\lambda \gamma^\tau P^\rho n^\sigma \left(\frac{B_{36}}{P \cdot n M^2} k_\mu k_\nu + \frac{B_{37}}{(P \cdot n)^2} k_\mu n_\nu + \frac{B_{38}M^2}{(P \cdot n)^3} n_\mu n_\nu \right) \right] \gamma_5 T^{\mu\nu} \\ & + \varepsilon_{\mu\rho\sigma} k^\tau P^\rho n^\sigma \left(\frac{B_{39}}{(P \cdot n)^2} k_\nu + \frac{B_{40}M^2}{(P \cdot n)^3} n_\nu \right) \not{n} \gamma_5 T^{\mu\nu} \\ & + \sigma_{\rho\sigma} \left[P^\rho k^\sigma \left(\frac{B_{41}}{(P \cdot n)M} k_\mu n_\nu + \frac{B_{42}M}{(P \cdot n)^2} n_\mu n_\nu \right) + P^\rho n^\sigma \left(\frac{B_{43}}{(P \cdot n)M} k_\mu k_\nu + \frac{B_{44}M}{(P \cdot n)^2} k_\mu n_\nu + \frac{B_{45}M^3}{(P \cdot n)^3} n_\mu n_\nu \right) \right] T^{\mu\nu} \\ & + \sigma_{\rho\sigma} \left[k^\rho n^\sigma \left(\frac{B_{46}}{(P \cdot n)M} k_\mu k_\nu + \frac{B_{47}M}{(P \cdot n)^2} k_\mu n_\nu + \frac{B_{48}M^3}{(P \cdot n)^3} n_\mu n_\nu \right) \right] T^{\mu\nu} + \sigma_{\mu\sigma} \left[n^\sigma \left(\frac{B_{49}M}{P \cdot n} k_\nu + \frac{B_{50}M^3}{(P \cdot n)^2} n_\nu \right) + \left(\frac{B_{51}M}{P \cdot n} P^\sigma + \frac{B_{52}M}{P \cdot n} k^\sigma \right) n_\nu \right] T^{\mu\nu} \end{aligned}$$

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$

Twist-3 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2} \text{Tr} [\Phi^{[\Gamma]}(x, k_T, T) \Gamma] = \frac{1}{2} \text{Tr} \left[\int dk^- \Phi(k, P, T | n) \Gamma \right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2} F^\perp(x, k_T^2)$$

$$\Phi^{[\gamma^i]}(x, k_T, T) = \frac{M}{P^+} \left[f_{LL}^\perp(x, k_T^2) \frac{S_{LL} k_T^i}{M} + f'_{LT}(x, k_T^2) S_{LT}^i - f_{LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} - f'_{TT}(x, k_T^2) \frac{S_{TT}^i k_{Tj}}{M} + f_{TT}^\perp(x, k_T^2) \frac{k_T^i k_T \cdot S_{TT} \cdot k_T}{M^3} \right]$$

$$\Phi^{[1]}(x, k_T, T) = \frac{M}{P^+} \left[e_{LL}(x, k_T^2) S_{LL} - e_{LT}^\perp(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + e_{TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \right]$$

$$\Phi^{[i\gamma_5]}(x, k_T, T) = \frac{M}{P^+} \left[e_{LT}(x, k_T^2) \frac{S_{LT\mu} \epsilon_T^{\mu\nu} k_{T\nu}}{M} - e_{TT}(x, k_T^2) \frac{S_{TT\mu\rho} k_T^\rho \epsilon_T^{\mu\nu} k_{T\nu}}{M^2} \right]$$

$$\Phi^{[\gamma^i \gamma_5]}(x, k_T, T) = \frac{M}{P^+} \left[-g_{LL}^\perp(x, k_T^2) \frac{S_{LL} \epsilon_T^i k_{Tj}}{M} - g'_{LT}(x, k_T^2) \epsilon_T^i S_{LTj} + g_{LT}^\perp(x, k_T^2) \frac{\epsilon_T^i k_{Tj} S_{LT} \cdot k_T}{M^2} + g'_{TT}(x, k_T^2) \frac{\epsilon_T^i S_{TTj} k_T^l}{M} - g_{TT}^\perp(x, k_T^2) \frac{\epsilon_T^i k_{Tj} k_T \cdot S_{TT} \cdot k_T}{M^3} \right]$$

$$\Phi^{[\sigma^{+-}]}(x, k_T, T) = \frac{M}{P^+} \left[h_{LL}(x, k_T^2) S_{LL} - h_{LT}(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + h_{TT}(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \right]$$

$$\Phi^{[\sigma^{ij}]}(x, k_T, T) = \frac{M}{P^+} \left[h_{LT}^\perp(x, k_T^2) \frac{S_{LT}^i k_T^j - S_{LT}^j k_T^i}{M} - h_{TT}^\perp(x, k_T^2) \frac{S_{TT}^i k_{Tj} k_T^k - S_{TT}^j k_{Tk} k_T^i}{M^2} \right]$$

*2, *3 Because of the time-reversal invariance, the collinear PDFs $g_{LT}(x)$ and $h_{LL}(x)$ do not exist. However, the corresponding new collinear fragmentation functions $G_{LT}(z)$ and $H_{LL}(z)$ should exist. (see our PRD paper for the details)

| Quark \ Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^+ \gamma_5$ | | σ^{ij}, σ^{-+} | |
|----------------|--|--|---------------------|------------------------|--|--|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f^\perp [e] | | | g^\perp | | [h] |
| L | | f_L^\perp [e _L] | g_L^\perp | | [h _L] | |
| T | | f_T, f_T^\perp [e _T , e _T ^\perp] | g_T, g_T^\perp | | [h _T], [h _T ^\perp] | |
| LL | f_{LL}^\perp [e _{LL}] | | | g_{LL}^\perp | | [h _{LL}] |
| LT | f_{LT}, f_{LT}^\perp [e _{LT} , e _{LT} ^\perp] | | | g_{LT}, g_{LT}^\perp | | [h _{LT}], [h _{LT} ^\perp] |
| TT | f_{TT}, f_{TT}^\perp [e _{TT} , e _{TT} ^\perp] | | | g_{TT}, g_{TT}^\perp | | [h _{TT}], [h _{TT} ^\perp] |

New TMDs

[· · ·] = chiral odd

| Quark \ Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^+ \gamma_5$ | | σ^{ij}, σ^{-+} | |
|----------------|--------------------------|-------|---------------------|-------|----------------------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | [e] | | | | | |
| L | | | | | [h _L] | |
| T | | | g_T | | | |
| LL | [e _{LL}] | | | | | *3 |
| LT | f_{LT} | | | *2 | | |
| TT | | | | | | |

New collinear PDFs

Twist-4 TMDs for spin-1 hadrons

may skip

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2} \text{Tr} \left[\Phi^{[\Gamma]}(x, k_T, T) \Gamma \right] = \frac{1}{2} \text{Tr} \left[\int dk^- \Phi(k, P, T | n) \Gamma \right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2} F^\perp(x, k_T^2)$$

$$\Phi^{[\gamma^-]}(x, k_T, T) = \frac{M^2}{P^{+2}} \left[f_{3LL}(x, k_T^2) S_{LL} - f_{3LT}(x, k_T^2) \frac{S_{LT} \cdot k_T}{M} + f_{3TT}(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \right]$$

$$\Phi^{[\gamma^- \gamma_5]}(x, k_T, T) = \frac{M^2}{P^{+2}} \left[g_{3LL}(x, k_T^2) \frac{S_{LT\mu} \epsilon_T^{\mu\nu} k_{T\nu}}{M} + g_{3TT}(x, k_T^2) \frac{S_{TT\mu\rho} k_T^\rho \epsilon_T^{\mu\nu} k_{T\nu}}{M^2} \right]$$

$$\Phi^{[\sigma^{i-}]}(x, k_T, T) = \frac{M^2}{P^{+2}} \left[h_{3LL}^\perp(x, k_T^2) \frac{S_{LL} k_T^i}{M} + h'_{3LT}(x, k_T^2) S_{LT}^i - h_{3LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} - h'_{3TT}(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} + h_{3TT}^\perp(x, k_T^2) \frac{k_T^i k_T \cdot S_{TT} \cdot k_T}{M^3} \right]$$

*4 Because of the time-reversal invariance, $h_{3LT}(x)$ does not exist; however, the corresponding new collinear fragmentation function $H_{3LT}(z)$ should exist because the time-reversal invariance does not have to be imposed.

| Quark \ Hadron | γ^- | | $\gamma^- \gamma_5$ | | σ^{i-} | |
|----------------|------------|----------------|---------------------|-----------|----------------------------|------------------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_3 | | | | | $[h_3^\perp]$ |
| L | | | g_{3L} | | $[h_{3L}^\perp]$ | |
| T | | f_{3T}^\perp | g_{3T} | | $[h_{3T}], [h_{3T}^\perp]$ | |
| LL | f_{3LL} | | | | | $[h_{3LL}^\perp]$ |
| LT | f_{3LT} | | | g_{3LT} | | $[h_{3LT}], [h_{3LT}^\perp]$ |
| TT | f_{3TT} | | | g_{3TT} | | $[h_{3TT}], [h_{3TT}^\perp]$ |

New TMDs

$[\dots] = \text{chiral odd}$

| Quark \ Hadron | γ^- | | $\gamma^- \gamma_5$ | | σ^{i-} | |
|----------------|------------|-------|---------------------|-------|---------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_3 | | | | | |
| L | | | g_{3L} | | | |
| T | | | | | $[h_{3T}]$ | |
| LL | f_{3LL} | | | | | |
| LT | | | | | | *4 |
| TT | | | | | | |

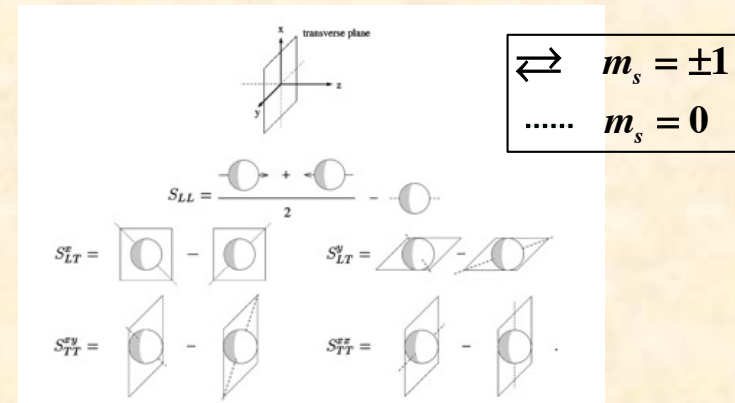
New collinear PDFs

TMDs and their sum rules for spin-1 hadrons

see our PRD paper
for the details

Twist-2 TMDs Bacchetta-Mulders, PRD 62 (2000) 114004.

| Quark \ Hadron | U (γ^+) | | L ($\gamma^+\gamma_5$) | | T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) | |
|----------------|------------------|----------------|--------------------------|-----------|--|------------------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_1 | | | | | $[h_1^\perp]$ |
| L | | | g_{1L} | | | $[h_{1L}^\perp]$ |
| T | | f_{1T}^\perp | g_{1T} | | $[h_1], [h_{1T}^\perp]$ | |
| LL | f_{1LL} | | | | | $[h_{1LL}^\perp]$ |
| LT | f_{1LT} | | | g_{1LT} | | $[h_{1LT}], [h_{1LT}^\perp]$ |
| TT | f_{1TT} | | | g_{1TT} | | $[h_{1TT}], [h_{1TT}^\perp]$ |



Time-reversal invariance in collinear correlation functions (PDFs)

$$\int d^2k_T \Phi_{T\text{-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T h_{1LT}(x, k_T^2) = 0,$$

$$\int d^2k_T g_{LT}(x, k_T^2) = 0,$$

$$\int d^2k_T h_{LL}(x, k_T^2) = 0,$$

$$\int d^2k_T h_{3LT}(x, k_T^2) = 0$$

Twist-3 TMDs SK and Qin-Tao Song, PRD 103 (2021) 014025.

| Quark \ Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^+\gamma_5$ | | σ^{ij}, σ^{-+} | |
|----------------|--|--|--------------------|------------------------|----------------------------|----------------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_1^\perp $[e]$ | | | g^\perp | | $[h]$ |
| L | | f_L^\perp $[e_L]$ | g_L^\perp | | | $[h_L]$ |
| T | | f_T, f_T^\perp $[e_T, e_T^\perp]$ | g_T, g_T^\perp | | $[h_T], [h_T^\perp]$ | |
| LL | f_{LL}^\perp $[e_{LL}]$ | | | g_{LL}^\perp | | $[h_{LL}]$ |
| LT | f_{LT}, f_{LT}^\perp $[e_{LT}, e_{LT}^\perp]$ | | | g_{LT}, g_{LT}^\perp | | $[h_{LT}], [h_{LT}^\perp]$ |
| TT | f_{TT}, f_{TT}^\perp $[e_{TT}, e_{TT}^\perp]$ | | | g_{TT}, g_{TT}^\perp | | $[h_{TT}], [h_{TT}^\perp]$ |

Twist-4 TMDs

| Quark \ Hadron | γ^- | | $\gamma^-\gamma_5$ | | σ^{i-} | |
|----------------|------------|----------------|--------------------|-----------|----------------------------|------------------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_3 | | | | | $[h_3]$ |
| L | | | g_{3L} | | | $[h_{3L}^\perp]$ |
| T | | f_{3T}^\perp | g_{3T} | | $[h_{3T}], [h_{3T}^\perp]$ | |
| LL | f_{3LL} | | | | | $[h_{3LL}^\perp]$ |
| LT | f_{3LT} | | | g_{3LT} | | $[h_{3LT}], [h_{3LT}^\perp]$ |
| TT | f_{3TT} | | | g_{3TT} | | $[h_{3TT}], [h_{3TT}^\perp]$ |

New fragmentation functions (FFs) for spin-1 hadrons

see arXiv:2201.05397

Corresponding fragmentation functions exist for the spin-1 hadrons simply by changing function names and kinematical variables.

Collinear FFs:
X. Ji, PRD 49, 114 (1994).

TMD distribution functions: $f, g, h, e; x, k_T, S, T, M, n, \gamma^+, \sigma^{i+}$
 \Downarrow

TMD fragmentation functions: $D, G, H, E; z, k_T, S_h, T_h, M_h, \bar{n}, \gamma^-, \sigma^{i-}$

Collinear FFs, twist 2

| Quark Hadron | U (γ^+) | | L ($\gamma^+\gamma_5$) | | T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) | |
|-----------------|------------------|-------|--------------------------|-------|--|-------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D_1 | | | | | |
| L | | | G_{1L} | | | |
| T | | | | | $[H_1]$ | |
| LL | D_{1LL} | | | | | |
| LT | | | | | | $[H_{1LT}]$ |
| TT | | | | | | |

Collinear FFs, twist 3

| Quark Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^i\gamma_5$ | | σ^{ij}, σ^{-+} | |
|-----------------|--------------------------|-------|--------------------|----------|----------------------------|------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | $[E]$ | | | | | |
| L | | | | | $[H_L]$ | |
| T | | | G_T | | | |
| LL | $[E_{LL}]$ | | | | | $[H_{LL}]$ |
| LT | D_{LT} | | | G_{LT} | | |
| TT | | | | | | |

Collinear FFs, twist 4

| Quark Hadron | γ^- | | $\gamma^-\gamma_5$ | | σ^{i-} | |
|-----------------|------------|-------|--------------------|-------|---------------|-------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D_3 | | | | | |
| L | | | G_{3L} | | | |
| T | | | | | $[H_{3T}]$ | |
| LL | D_{3LL} | | | | | |
| LT | | | | | | $[H_{3LT}]$ |
| TT | | | | | | |

TMD FFs, twist 2 [] = chiral odd

| Quark Hadron | U (γ^+) | | L ($\gamma^+\gamma_5$) | | T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) | |
|-----------------|------------------|------------|--------------------------|-----------|--|---------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D_1 | | | | | $[H_1^+]$ |
| L | | | G_{1L} | | $[H_{1L}^+]$ | |
| T | | D_{1T}^+ | G_{1T} | | $[H_1], [H_{1T}^+]$ | |
| LL | D_{1LL} | | | | | $[H_{1LL}^+]$ |
| LT | D_{1LT} | | | G_{1LT} | $[H_{1LT}], [H_{1LT}^+]$ | |
| TT | D_{1TT} | | | G_{1TT} | $[H_{1TT}], [H_{1TT}^+]$ | |

TMD FFs, twist 3

| Quark Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^i\gamma_5$ | | σ^{ij}, σ^{-+} | |
|-----------------|--|--------------------------------|--------------------|--------------------|----------------------------|------------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D^+ $[E]$ | | | G^+ | | $[H]$ |
| L | | D_L^+ $[E_L]$ | G_L^+ | | $[H_L]$ | |
| T | | D_T, D_T^+ $[E_T, E_T^+]$ | G_T, G_T^+ | | $[H_T], [H_T^+]$ | |
| LL | D_{LL}^+ $[E_{LL}]$ | | | G_{LL}^+ | | $[H_{LL}]$ |
| LT | D_{LT}, D_{LT}^+ $[E_{LT}, E_{LT}^+]$ | | | G_{LT}, G_{LT}^+ | | $[H_{LT}], [H_{LT}^+]$ |
| TT | D_{TT}, D_{TT}^+ $[E_{TT}, E_{TT}^+]$ | | | G_{TT}, G_{TT}^+ | | $[H_{TT}], [H_{TT}^+]$ |

TMD FFs, twist 4

| Quark Hadron | γ^- | | $\gamma^-\gamma_5$ | | σ^{i-} | |
|-----------------|------------|------------|--------------------|-----------|--------------------------|---------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D_3 | | | | | $[H_3^+]$ |
| L | | | G_{3L} | | $[H_{3L}^+]$ | |
| T | | D_{3T}^+ | G_{3T} | | $[H_{3T}], [H_{3T}^+]$ | |
| LL | D_{3LL} | | | | | $[H_{3LL}^+]$ |
| LT | D_{3LT} | | | G_{3LT} | $[H_{3LT}], [H_{3LT}^+]$ | |
| TT | D_{3TT} | | | G_{3TT} | $[H_{3TT}], [H_{3TT}^+]$ | |

New TMD FFs

PDFs for spin-1 hadrons

Twist-2 PDFs

| Quark \ Hadron | U (γ^+) | | L ($\gamma^+\gamma_5$) | | T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) | |
|----------------|------------------|-------|--------------------------|-------|--|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_1 | | | | | |
| L | | | $g_{1L}(g_1)$ | | | |
| T | | | | | $[h_1]$ | |
| LL | $f_{1LL}(b_1)$ | | | | | |
| LT | | | | | | *1 |
| TT | | | | | | |

*1: $h_{1LT}(x)$, *2: $g_{LT}(x)$, *3: $h_{LL}(x)$, *4: $h_{3LT}(x)$

Because of the time-reversal invariance, the collinear PDF vanishes.

However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function should exist as a collinear fragmentation function.

[] = chiral odd

Twist-3 PDFs

| Quark \ Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^+\gamma_5$ | | σ^{ij}, σ^{-+} | |
|----------------|--------------------------|-------|--------------------|-------|----------------------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | $[e]$ | | | | | |
| L | | | | | $[h_L]$ | |
| T | | | g_T | | | |
| LL | $[e_{LL}]$ | | | | | *3 |
| LT | f_{LT} | | | *2 | | |
| TT | | | | | | |

Twist-4 PDFs

| Quark \ Hadron | γ^- | | $\gamma^-\gamma_5$ | | σ^{i-} | |
|----------------|------------|-------|--------------------|-------|---------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_3 | | | | | |
| L | | | g_{3L} | | | |
| T | | | | | $[h_{3T}]$ | |
| LL | f_{3LL} | | | | | |
| LT | | | | | | *4 |
| TT | | | | | | |

Summary on Spin-1 TMDs and PDFs

TMDs of spin-1 hadrons

- TMDs: interdisciplinary field of physics
- We proposed **new 30 TMDs and 3 PDFs in twist 3 and 4.**
- **New sum rules for TMDs.**
- **New TMD fragmentation functions.**

Twist-3 TMD: $f_{LL}^\perp, e_{LL}, f_{LT}, f_{LT}^\perp, e_{1T}, e_{1T}^\perp, f_{TT}, f_{TT}^\perp, e_{TT}, e_{TT}^\perp,$
 $g_{LL}^\perp, g_{LT}, g_{LT}^\perp, g_{TT}, g_{TT}^\perp, h_{1L}, h_{LT}, h_{LT}^\perp, h_{TT}, h_{TT}^\perp$

Twist-4 TMD: $f_{3LL}, f_{3LT}, f_{3TT}, g_{3LT}, f_{3TT}, h_{3LL}^\perp, h_{3LT}, h_{3LT}^\perp, h_{3TT}, h_{3TT}^\perp$

Twist-3 PDF: e_{LL}, f_{LT}

Twist-4 PDF: f_{3LL}

Sum rules: $\int d^2k_T g_{LT}(x, k_T^2) = \int d^2k_T h_{LL}(x, k_T^2) = \int d^2k_T h_{3LL}(x, k_T^2) = 0$

TMD distribution functions: $f, g, h, e; x, k_T, S, T, M, n, \gamma^+, \sigma^{i+}$
 \Downarrow

TMD fragmentation functions: $D, G, H, E; z, k_T, S_h, T_h, M_h, \bar{n}, \gamma^-, \sigma^{i-}$

Analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule

Twist-3 PDFs

| Quark \ Hadron | U (γ^+) | | L ($\gamma^+\gamma_5$) | | T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) | |
|----------------|------------------|-------|--------------------------|-------|--|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_1 | | | | | |
| L | | | $g_{1L}(g_1)$ | | | |
| T | | | | | $[h_1]$ | |
| LL | $f_{1LL}(b_1)$ | | | | | |
| LT | | | | | | *1 |
| TT | | | | | | |

Twist-2 PDFs

| Quark \ Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^+\gamma_5$ | | σ^{ij}, σ^{-+} | |
|----------------|--------------------------|-------|--------------------|-------|----------------------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | $[e]$ | | | | | |
| L | | | | | $[h_L]$ | |
| T | | | g_T | | | |
| LL | $[e_{LL}]$ | | | | | *3 |
| LT | | | | | *2 | |
| TT | | | | | | |

[] = chiral odd

We derived analogous relations to Wandzura-Wilczek relation and Burkhardt-Cottingham sum rule for f_{LT} and f_{1LL} .

SK and Qin-Tao Song, JHEP 09 (2021) 141.

For spin-1/2 nucleons,

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \quad (\text{Wandzura-Wilczek relation}), \quad \int_0^1 dx g_2(x) = 0 \quad (\text{Burkhardt-Cottingham sum rule})$$

For tensor-polarized spin-1 hadrons, we obtained

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y), \quad \int_0^1 dx f_{2LT}^+(x) = 0, \quad f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$$

$$\int_0^1 dx f_{LT}^+(x) = 0 \quad \text{if} \quad \int_0^1 dx f_{1LL}^+(x) = \frac{2}{3} \int_0^1 dx b_1^+(x) = 0$$

Existence of multiparton distribution functions: $F_{G,LT}(x_1, x_2)$, $G_{G,LT}(x_1, x_2)$, $H_{G,LL}^1(x_1, x_2)$, $H_{G,TT}(x_1, x_2)$

Relations from equation of motion and Lorentz-invariance relation for spin-1 hadrons

SK and Qin-Tao Song,
PLB 826 (2022) 136908.

- $x f_{LT}(x) - \int_{-1}^{+1} dy [F_{D,LT}(x,y) + G_{D,LT}(x,y)] = 0$, $x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathcal{P} \int_{-1}^{+1} dy \frac{F_{G,LT}(x,y) + G_{G,LT}(x,y)}{x-y} = 0$
- $x e_{LL}(x) - 2 \int_{-1}^{+1} dy H_{D,LL}^\perp(x,y) - \frac{m}{M} f_{1LL}(x) = 0$, $x e_{LL}(x) - 2 \mathcal{P} \int_{-1}^{+1} dy \frac{H_{G,LL}^\perp(x,y)}{x-y} - \frac{m}{M} f_{1LL}(x) = 0$

and the Lorentz-invariance relation

- $\frac{df_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2 \mathcal{P} \int_{-1}^{+1} dy \frac{F_{G,LT}(x,y)}{(x-y)^2} = 0$

Lorentz invariance
= frame independence of twist-3 observables

transverse-momentum moment of TMD: $f^{(1)}(x) = \int d^2 k_T \frac{\vec{k}_T^2}{2M^2} f(x, k_T^2)$

Twist-2 PDFs

| Quark Hadron | U (γ^+) | | L ($\gamma^+ \gamma_5$) | | T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$) | |
|-----------------|------------------|-------|---------------------------|-------|---|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_1 | | | | | |
| L | | | $g_{1L}(g_1)$ | | | |
| T | | | | | $[h_1]$ | |
| LL | $f_{1LL}(b_1)$ | | | | | |
| LT | | | | | | |
| TT | | | | | | |

Twist-3 PDFs

| Quark Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^+ \gamma_5$ | | σ^{ij}, σ^{i+} | |
|-----------------|--------------------------|-------|---------------------|-------|----------------------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | $[e]$ | | | | | |
| L | | | | | $[h_L]$ | |
| T | | | g_T | | | |
| LL | $[e_{LL}]$ | | | | | |
| LT | f_{LT} | | | | | $*1$ |
| TT | | | | | | |

Twist-3 TMDs

| Quark Hadron | U (γ^+) | | L ($\gamma^+ \gamma_5$) | | T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$) | |
|-----------------|------------------|----------------|---------------------------|-----------|---|------------------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | f_1 | | | | | $[h_1^\perp]$ |
| L | | | g_{1L} | | | $[h_{1L}^\perp]$ |
| T | | f_{1T}^\perp | g_{1T} | | $[h_1], [h_{1T}^\perp]$ | |
| LL | f_{1LL} | | | | | $[h_{1LL}^\perp]$ |
| LT | f_{1LT} | | | g_{1LT} | | $[h_{1LT}], [h_{1LT}^\perp]$ |
| TT | f_{1TT} | | | g_{1TT} | | $[h_{1TT}], [h_{1TT}^\perp]$ |

[] = chiral odd

Relations on fragmentation functions

Qin-Tao Song,
PRD 108 (2023) 094041.

- $E_{LL}(z) + iH_{LL}(z) - \frac{m_q}{M} z D_{1LL}(z) = 2z \left[-iH_{1LL}^{(1)}(z) + \mathcal{P} \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{H_{G,LL}^\perp(z, z_1)}{1/z - 1/z_1} \right]$
- $D_{LT}(z) + iG_{LT}(z) + i\frac{m_q}{M} z H_{1LT}(z) = -z \left[iG_{1LT}^{(1)}(z) - \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{G_{G,LL}(z, z_1)}{1/z - 1/z_1} \right] - z \left[D_{1LT}^{(1)}(z) + \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{D_{G,LT}(z, z_1)}{1/z - 1/z_1} \right]$
- $iH_{1TT}^{(1)}(z) + \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{H_{G,TT}(z, z_1)}{1/z - 1/z_1} = 0$
- $\frac{3}{2} D_{1LL}(z) - D_{LT}(z) - z \left(1 - z \frac{d}{dz} \right) D_{1LT}^{(1)}(z) = -2 \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{\text{Re}[D_{G,LT}(z, z_1)]}{(1/z - 1/z_1)^2}$
- $H_{LL}(z) + 2H_{1LT}(z) + z \left(1 - z \frac{d}{dz} \right) H_{1LL}^{(1)}(z) = -2 \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{\text{Im}[H_{G,LL}^\perp(z, z_1)]}{(1/z - 1/z_1)^2}$
- $G_{LT}(z) + z \left(1 - z \frac{d}{dz} \right) G_{1LT}^{(1)}(z) = -2 \int_z^\infty \frac{dz_1}{(z_1)^2} \frac{\text{Im}[H_{G,LT}(z, z_1)]}{(1/z - 1/z_1)^2}$

Twist-2 TMD FFs

Twist-2 FFs [] = chiral odd

| Quark Hadron | U (γ^+) | | L ($\gamma^+ \gamma_5$) | | T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$) | |
|-----------------|------------------|-----------|---------------------------|-------|---|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D_1 | | | | | |
| L | | | G_{1L} | | | |
| T | | | | | $[H_1]$ | |
| LL | | D_{1LL} | | | | |
| LT | | | | | $[H_{1LT}]$ | |
| TT | | | | | | |

Twist-3 FFs

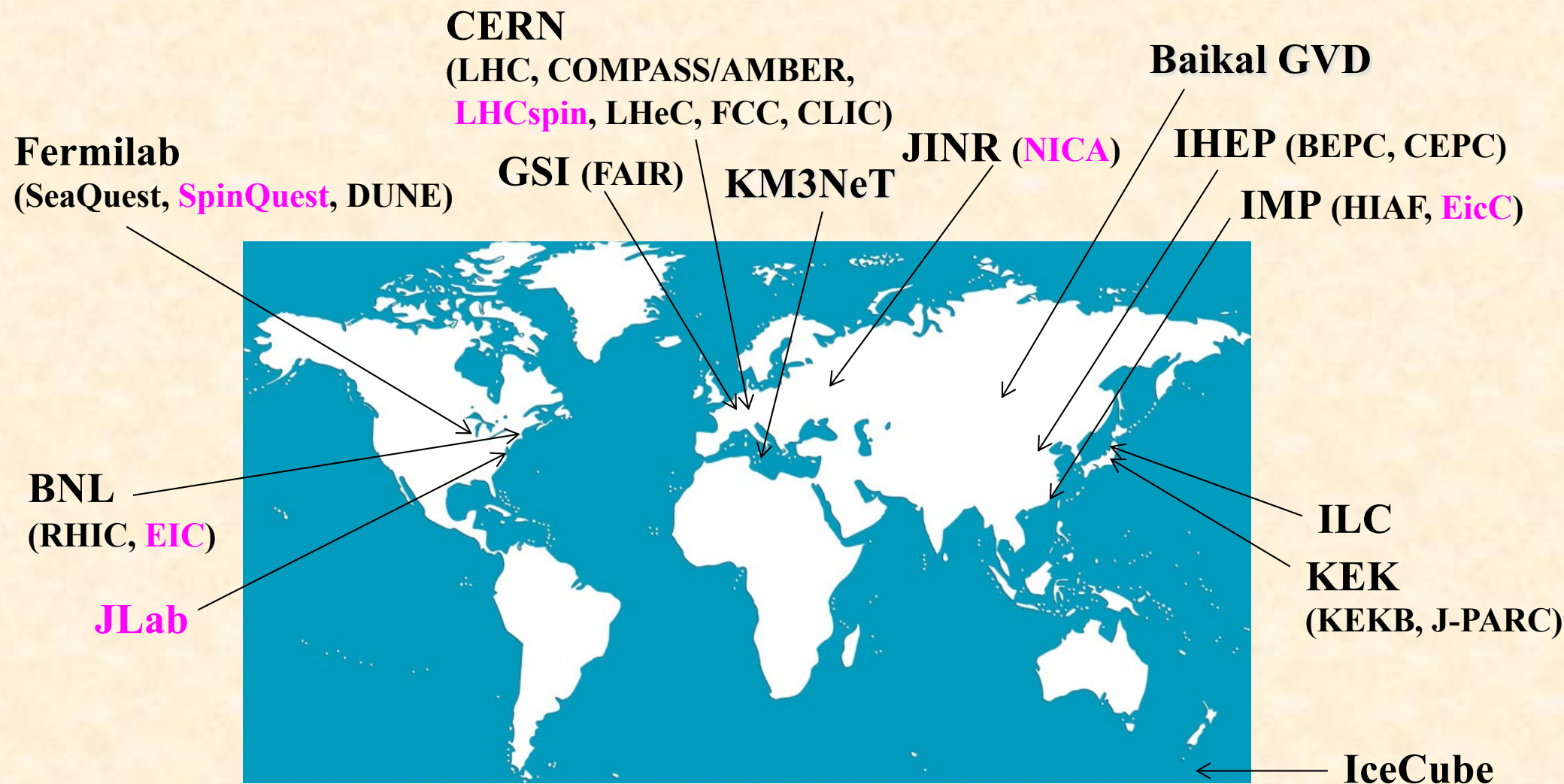
| Quark Hadron | $\gamma^i, 1, i\gamma_5$ | | $\gamma^i \gamma_5$ | | σ^{ij}, σ^{i+} | |
|-----------------|--------------------------|-------|---------------------|-------|----------------------------|-------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | $[E]$ | | | | | |
| L | | | | | $[H_L]$ | |
| T | | | G_T | | | |
| LL | $[E_{1L}]$ | | | | $[H_{LL}]$ | |
| LT | D_{1LT} | | G_{1LT} | | | |
| TT | | | | | | |

Twist-2 TMD FFs

| Quark Hadron | U (γ^+) | | L ($\gamma^+ \gamma_5$) | | T ($i\sigma^{i+} \gamma_5 / \sigma^{i+}$) | |
|-----------------|------------------|----------------|---------------------------|-------|---|-------------------|
| | T-even | T-odd | T-even | T-odd | T-even | T-odd |
| U | D_1 | | | | | $[H_1^\perp]$ |
| L | | | G_{1L} | | $[H_{1L}^\perp]$ | |
| T | | D_{1T}^\perp | G_{1T} | | $[H_1], [H_{1T}^\perp]$ | |
| LL | D_{1LL} | | | | | $[H_{1LL}^\perp]$ |
| LT | D_{1LT} | | G_{1LT} | | $[H_{1LT}^\perp]$ | $[H_{1LT}^\perp]$ |
| TT | D_{1TT} | | G_{1TT} | | $[H_{1TT}^\perp]$ | $[H_{1TT}^\perp]$ |

Future prospects and summary

High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities.

JLab PAC-38 (Aug. 22-26, 2011) proposal, PR12-11-110

The Deuteron Tensor Structure Function b_1

2011

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),
K. Allada, A. Camsonne, A. Deur, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Rondon (co-spokesperson)
DOI: PR12-13-011

The Deuteron Tensor Structure Function b_1

2023

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

K. Allada, A. Camsonne, J.-P. Chen,[†]
A. Deur, D. Gaskell, M. Jones, C. Keith, J. Pierce,
P. Solvignon,[†] S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

O. Rondon Aramayo,[†] D. Crabb, D. B. Day,
C. Hanretty, D. Keller,[†] R. Lindgren, S. Liuti, B. Norum,
Zhihong Ye, X. Zheng
University of Virginia, Charlottesville, VA 22903

N. Kalantarians[†]
Hampton University, Hampton VA 23668

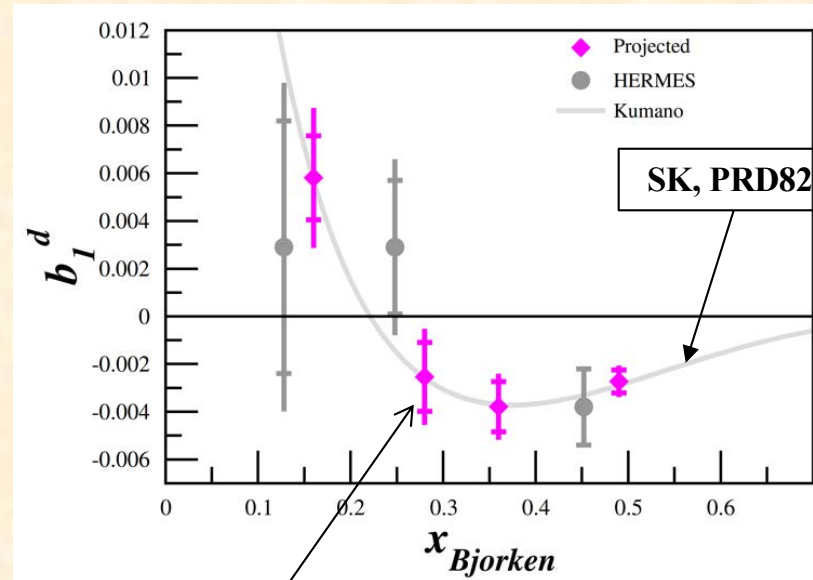
T. Badman, J. Calarco, J. Dawson,
S. Phillips, E. Long,[†] K. Slifer^{†,‡}, R. Zielinski
University of New Hampshire, Durham, NH 03861

G. Ron
Hebrew University of Jerusalem, Jerusalem

W. Bertozzi, S. Gilad, J. Huang
A. Kelleher, V. Sulkosky
Massachusetts Institute of Technology, Cambridge, MA 02139

Ru
Seor

Approved!



Expected errors by JLab

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan
Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon
University of Virginia, Charlottesville, VA 22904

J. Pierce
Oak Ridge National Laboratory, Oak Ridge, TN 37831



Experimental possibility at Fermilab in 2020's

**Polarized fixed-target experiments
at the Main Injector,
Proton beam = 120 GeV** © Fermilab



Fermilab-E1039 (SpinQuest)

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

List of Collaborators:

D. Geesaman, P. Reimer
Argonne National Laboratory, Argonne, IL 60439
C. Brown, D. Christian
Fermi National Accelerator Laboratory, Batavia IL 60510
M. Diefenthaler, J.-C. Peng
University of Illinois, Urbana, IL 61081
W.-C. Chang, Y.-C. Chen
Institute of Physics, Academia Sinica, Taiwan
S. Sawada
KEK, Tsukuba, Ibaraki 305-0801, Japan
T.-H. Chang
Ling-Tung University, Taiwan
J. Huang, X. Jiang, M. Leitch, A. Klein, K. Liu, M. Liu, P. McGaughey
Los Alamos National Laboratory, Los Alamos, NM 87545
E. Beise, K. Nakahara
University of Maryland, College Park, MD 20742
C. Aidala, W. Lorenzon, R. Raymond
University of Michigan, Ann Arbor, MI 48109-1040
T. Badman, E. Long, K. Slifer, R. Zielinski
University of New Hampshire, Durham, NH 03824
R.-S. Guo
National Kaohsiung Normal University, Taiwan
Y. Goto
RIKEN, Wako, Saitama 351-01, Japan
L. El Fassi, K. Myers, R. Ransome, A. Tadepalli, B. Tice
Rutgers University, Rutgers NJ 08544
J.-P. Chen
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606
K. Nakano, T.-A. Shibata
Tokyo Institute of Technology, Tokyo 152-8551, Japan
D. Crabb, D. Day, D. Keller, O. Rondon
University of Virginia, Charlottesville, VA 22904

**Fermilab experimentalists are interested
in the gluon transversity by replacing
the E1039 proton target for the deuteron one.
(Spokesperson of E1039: D. Keller)
However, there was no theoretical formalism
until our work.**

**SK and Q.-T. Song,
PRD 101 (2020) 054011 & 094013**

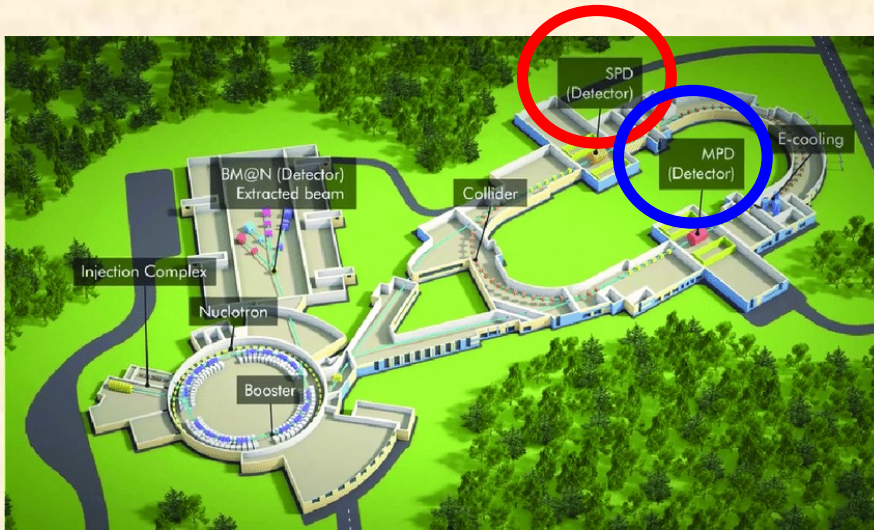
The Transverse Structure of the Deuteron with Drell-Yan

D. Keller¹

¹*University of Virginia, Charlottesville, VA 22904*

Proposal for a Fermilab-PAC in 2023.

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)

MPD (MultiPurpose Detector for heavy ion physics)

$$\vec{p} + \vec{p} : \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$$

$$\vec{d} + \vec{d} : \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$$

$\vec{p} + \vec{d}$ is also possible.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbutov *et al.* (NICA project), Nucl. Part. Phys. 119 (2021) 103858.

Unique opportunity in high-energy spin physics,
especially on the deuteron spin physics.

→ Theoretical formalisms need to be developed.

Progress in Particle and Nuclear Physics 119 (2021) 103858

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbutov^a, A. Bacchetta^{b,c}, M. Butenschoen^d, F.G. Celiberto^{b,c,e,f}, U. D'Alesio^{g,h}, M. Deka^a, I. Denisenko^a, M.G. Echevarria^a, A. Efremov^a, N.Ya. Ivanov^{g,i}, A. Guskov^{g,j,k}, A. Karpishkov^{l,m}, Ya. Klopov^{n,m}, B.A. Kniehl^d, A. Kotzinian^{l,o}, S. Kumano^p, J.P. Lansberg^q, Keh-Fei Liu^r, F. Murgia^h, M. Nefedov^s, B. Parsamyan^{u,v,w}, C. Pisano^{g,h}, M. Radici^c, A. Rymbekova^a, V. Saleev^{l,d}, A. Shipilova^{l,d}, Qin-Tao Song^g, O. Teryaev^a

Spin-1 deuteron experiments from the middle of 2020's

JLab



The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38
(Update to LCG-11-003)

J.-P. Chen (co-spokesperson), P. Solvigson (co-spokesperson),
K. Allada, A. Camerotto, A. Dang, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Roudnev (co-spokesperson)
Doual B. Day, Hovhannes Bagdasaryan, Charles Hanretty
Richard Lindgren, Blaise Nooren, Zhibong Ye
University of Virginia, Charlottesville, VA 22903

K. Sissler (co-spokesperson), A. Atkias, T. Badman,
J. Calarco, J. Maxwell, S. Phillips, R. Zietzma
University of New Hampshire, Durham, NH 03824

J. Dunne, D. Dutta
Mississippi State University, Mississippi State, MS 39762

G. Ron

Hebrew University of Jerusalem, Jerusalem

W. Bertozzi, S. Glash,
A. Kelleher, V. Sulkosky
Massachusetts Institute of Technology, Cambridge, MA 02139

K. Adhikari

Old Dominion University, Norfolk, VA 23529

R. Gilman

Rutgers, The State University of New Jersey, Piscataway, NJ 08854

Seonho Choi, Hoyoung Kang, Hyekyo Kang, Yoonim Oh
Seoul National University, Seoul 151-747 Korea

**Proposal (approved),
Experiment: middle of 2020's**

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan
Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon
University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

Fermilab



The Transverse Structure of the Deuteron with Drell-Yan

D. Keller¹

¹University of Virginia, Charlottesville, VA 22904

**Proposal,
Fermilab-PAC: 2022
Experiment: 2020's**

NICA



Progress in Particle and Nuclear Physics 119 (2021) 103858

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

A. Arbuzov^a, A. Bacchetta^{b,c}, M. Butenschön^d, E.G. Celiberto^{b,c,d},
U. D'Alesio^{b,c}, M. Deka^a, I. Denisenko^a, M.G. Echevarria^a, A. Efremov^a,
N.Ya. Ivanov^{a,d}, A. Guskov^{a,b,e}, A. Karpishkov^{a,d}, Ya. Klopot^{a,b}, B.A. Kniehl^d,
A. Kotzinian^g, S. Kumano^h, J.P. Lansberg^h, Keh-Fei Liuⁱ, F. Murgia^h,
M. Nefedovⁱ, B. Parsamyan^{a,b,c}, C. Pisano^{b,d}, M. Radici^g, F. Rathmann^h,
V. Saleev^h, A. Shipilova^a, Qin-Tao Songⁱ, O. Teryaev^a

**Prog. Nucl. Part. Phys.
119 (2021) 103858,
Experiment: middle of 2020's**

LHCspin



CERN-ESPP-Note-2018-111

The LHCSpin Project

C. A. Aidala¹, A. Bacchetta^{2,3}, M. Boglione^{4,5}, G. Bozzi^{2,3}, V. Carassiti^{6,7}, M. Chiosso^{1,5}, R. Cimino⁸,
G. Ciullo^{6,7}, M. Conalbrigo^{6,7}, U. D'Alesio^{10,9}, P. Di Nezza⁸, R. Engels¹¹, K. Grigoryev¹¹, D. Keller¹²,
P. Lenzi^{6,7}, S. Lini¹³, A. Metz¹³, P.J. Mulders^{14,15}, F. Murgia¹⁶, A. Nass¹, D. Panzeri^{14,16},
L. L. Pappalardo^{6,7}, B. Pasquini^{2,3}, C. Pisano¹⁰, M. Radici³, F. Rathmann¹¹, D. Reggiani¹⁷, M. Schlegel¹⁸,
S. Scopetta^{19,20}, E. Steffens²¹, A. Vasiliev²²

**arXiv:1901.08002,
Experiment: ~2028**

2030's EIC/EicC

SCIENCE REQUIREMENTS
AND DETECTOR
CONCEPTS FOR THE
ELECTRON-ION COLLIDER
EIC Yellow Report

R. Abdul Khalek et al.
Nucl. Phys. A 1026 (2022) 122447.

**D. P. Anderle et al.,
Front. Phys. 16 (2021) 64701.**

Frontiers in Physics

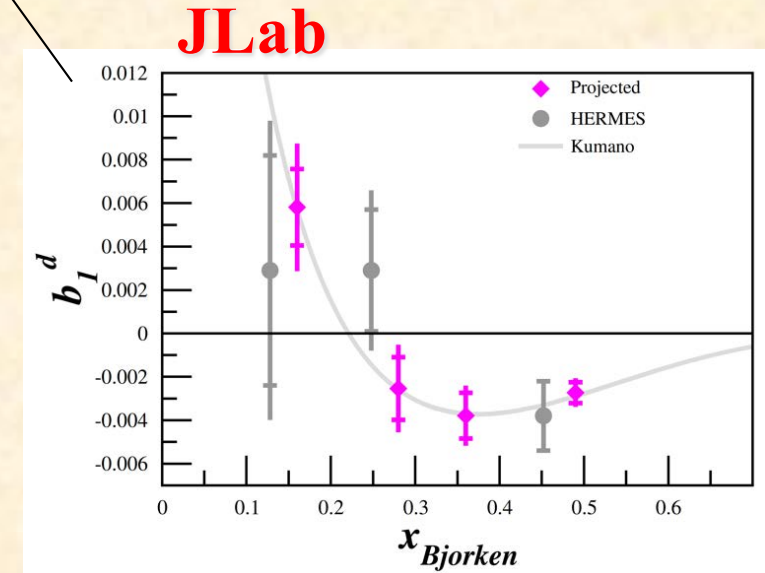
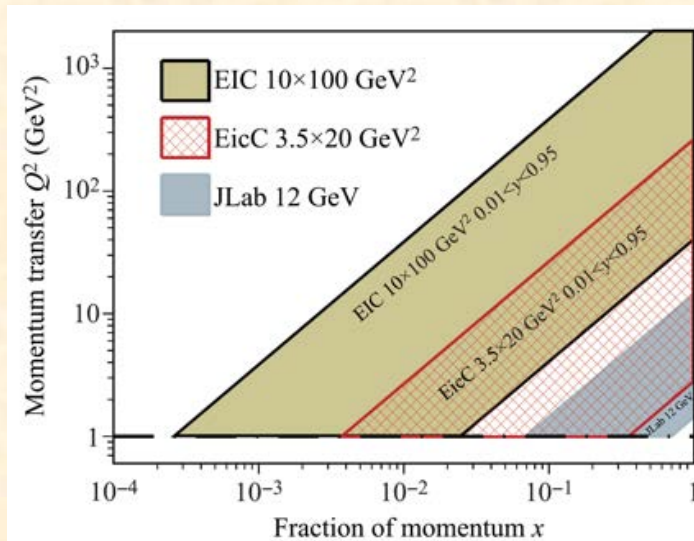
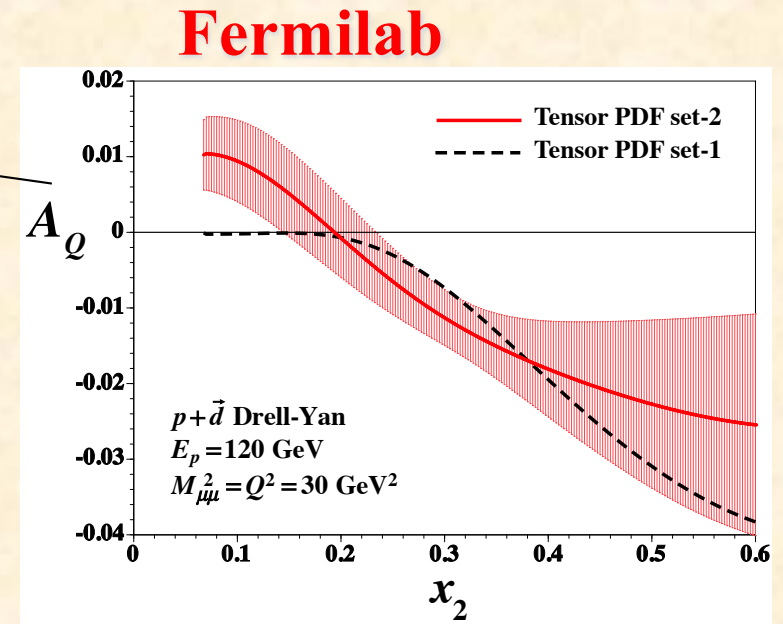
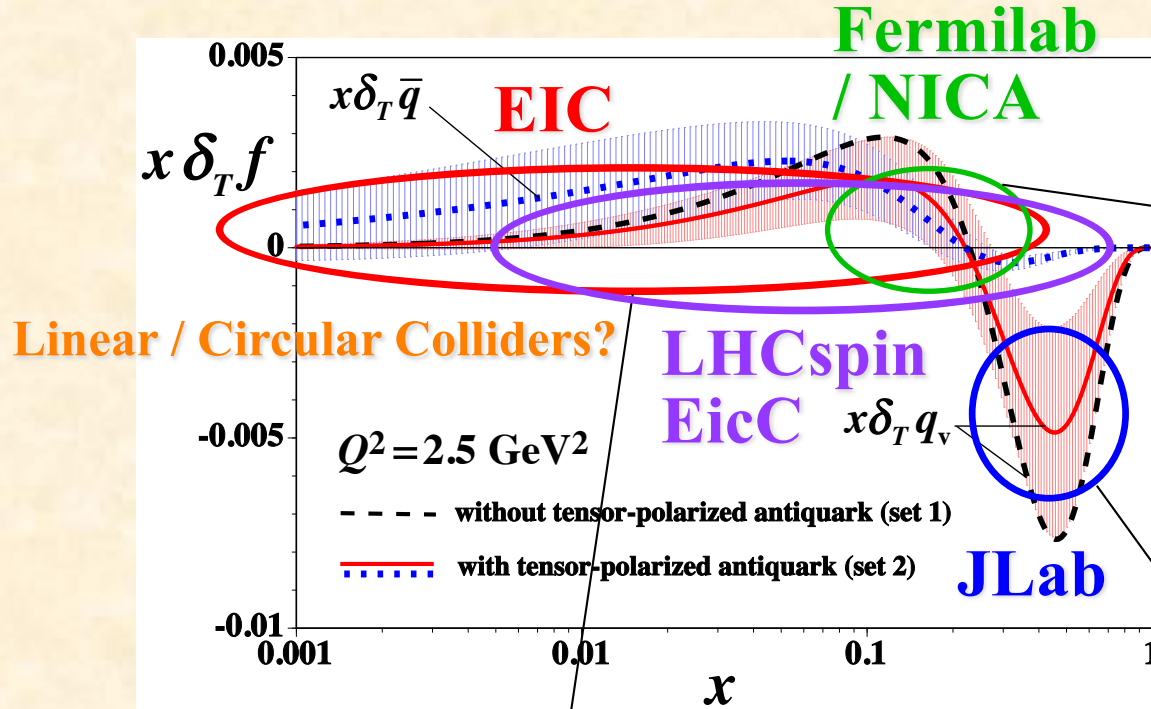
<https://doi.org/10.1007/s11467-021-1062-0>

REVIEW ARTICLE

Electron-ion collider in China

Danielo P. Anderle¹, Valerio Bertone², Xu Cao^{3,4}, Lei Chang⁵, Ningbo Chang⁶, Gu Chen⁷,
Xurong Chen^{8,9}, Zhaojun Chen¹⁰, Zhufang Cui¹¹, Lingyun Dai¹², Weitian Deng¹³, Minghui Ding¹⁴,
Xu Feng¹⁵, Chang Gong¹⁶, Longcheng Gu¹⁷, Feng-Kun Guo^{18,19}, Chenglong Han²⁰, Jun He²¹,
Tie-Jun Hou²², Hongxin Huang²³, Yin Huang²⁴, Kreshimir Kumeriki²⁵, L. P. Kaptanov^{26,27},
Demin Li²⁸, Hongbo Li²⁹, Minsong Li³⁰, Xuejiao Li³¹, Yulin Liang³², Zuoqiang Liang³³, Chen Liu³⁴,
Chuan Liu³⁵, Guoming Liu³⁶, Jie Liu³⁷, Litung Liu³⁸, Xiang Liu³⁹, Tianbo Liu⁴⁰, Xiaofeng Luo⁴¹,
Zhen Lyu⁴², Boqiang Ma⁴³, Fu Ma⁴⁴, Jianping Ma^{45,46}, Yuguang Ma^{47,48,49}, Lijun Mao⁵⁰,
Cédric Mezzadri⁵¹, Hervé Moutarde⁵², Jiahui Ping⁵³, Sixue Qin⁵⁴, Hang Ren⁵⁵, Craig D. Roberts⁵⁶,
Juan Rojas^{57,58}, Guodong Shen⁵⁹, Chao Shi⁶⁰, Qintao Song⁶¹, Hao Sun⁶², Pavel Sznajder⁶³,
Enke Wang⁶⁴, Fan Wang⁶⁵, Qian Wang⁶⁶, Rong Wang⁶⁷, Ruiru Wang⁶⁸, Taofeng Wang⁶⁹, Wei Wang⁷⁰,
Xiaoyu Wang⁷¹, Xiaoyun Wang⁷², Jiajun Wu⁷³, Xinggong Wu⁷⁴, Lei Xia⁷⁵, Bowen Xiao^{76,77},
Guoqing Xiao⁷⁸, Jun-Jun Xiao⁷⁹, Yaping Xiao⁸⁰, Hongxi Xing⁸¹, Hushan Xu⁸², Nu Xu^{83,84},
Shusheng Xu⁸⁵, Mengshi Yan⁸⁶, Wenbiao Yan⁸⁷, Wenchong Yan⁸⁸, Xuhui Yan⁸⁹, Jiancheng Yang⁹⁰,
Yi-Bo Yang⁹¹, Zhi Yang⁹², Deliang Yao⁹³, Zhihong Ye⁹⁴, Peilin Yin⁹⁵, C.-P. Yuan⁹⁶, Wentong Zhan⁹⁷,
Jianhui Zhang⁹⁸, Jinlong Zhang⁹⁹, Pengning Zhang¹⁰⁰, Yifei Zhang¹⁰¹, Chao-Hsi Chang¹⁰²,
Zhenyu Zhang¹⁰³, Hongwei Zhao¹⁰⁴, Kuang-Ta Chao¹⁰⁵, Qiang Zhao¹⁰⁶, Yuxiang Zhao¹⁰⁷,
Zhongqiao Zhao¹⁰⁸, Liang Zheng¹⁰⁹, Jian Zhou¹¹⁰, Xiang Zhou¹¹¹, Xiaorong Zhou¹¹²,
Bingsong Zou¹¹³, Liping Zou¹¹⁴

x regions of b_1 in 2020's and 2030's



Summary

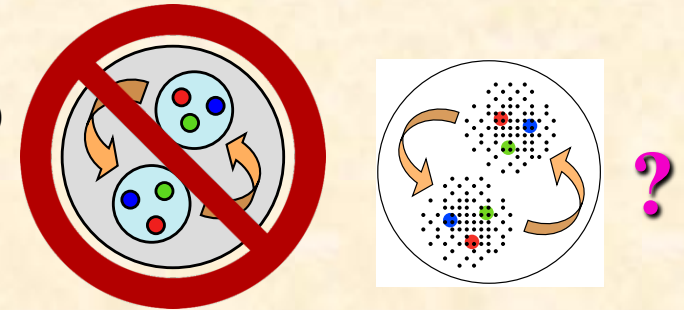
Spin-1 structure functions of the deuteron (additional spin structure to nucleon spin)

- Tensor structure in quark-gluon degrees of freedom
- Tensor-polarized structure function b_1 and PDFs, gluon transversity

Experiments at JLab, Fermilab, NICA, LHCspin/AMBER, EIC/EicC, ...

- New signature beyond “standard” hadron physics?

(beyond the standard model in particle physics???)



standard model

- TMDs up to twist 4

- Higher-twist effects could be sizable at a few GeV^2 Q^2

→ Our relations (WW-like, BC-like, from eq. of motion, Lorentz invariance) could become valuable for future experimental analyses.

There are various experimental projects on the polarized spin-1 deuteron in 2020's and 2030', and “exotic” hadron structure could be found by focusing on the spin-1 nature.

- There is no nuclear effect in ρ and ϕ mesons, so that the gluon transversity, for example, could be sensitive to new physics?!

The End

The End