

Studies of GPDs and gravitational form factors at Fermilab and KEK

Neutrino beams

e^+e^- facilities
Hadron beams

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**INT workshop on QCD at the Femtoscale in the Era of Big Data
INT, University of Washington, Seattle, USA, June 10 - July 5, 2024
<https://www.int.washington.edu/programs-and-workshops/24-2a>**

July 17, 2024

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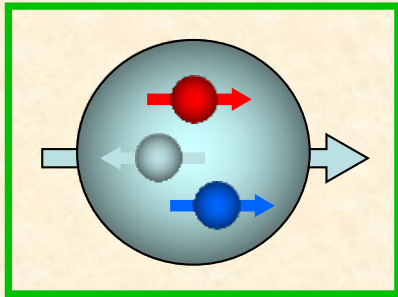
- **Motivations for studying GPDs (Generalized Parton Distributions)**
- **GPDs at neutrino facilities (Fermilab, CERN)**
- **GPDs at hadron accelerator facilities (J-PARC, NICA, GSI, ...)**
- **Timelike GPDs (GDAs) at e^+e^- colliders (KEKB)**
- **Comments on GPDs for exotic hadrons**
(transition GPDs could be used for exotic hadrons)
- **Future prospects on GPD projects**

if I have time

**Motivations for studying
gravitational form factors
and GPDs**

Origin of nucleon spin

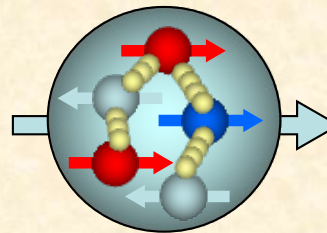
“old” standard model



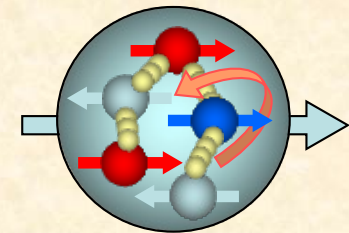
$$p_{\uparrow} = \frac{1}{3\sqrt{2}} \left(uud \left[2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

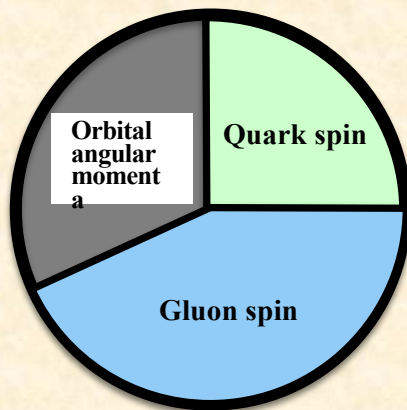
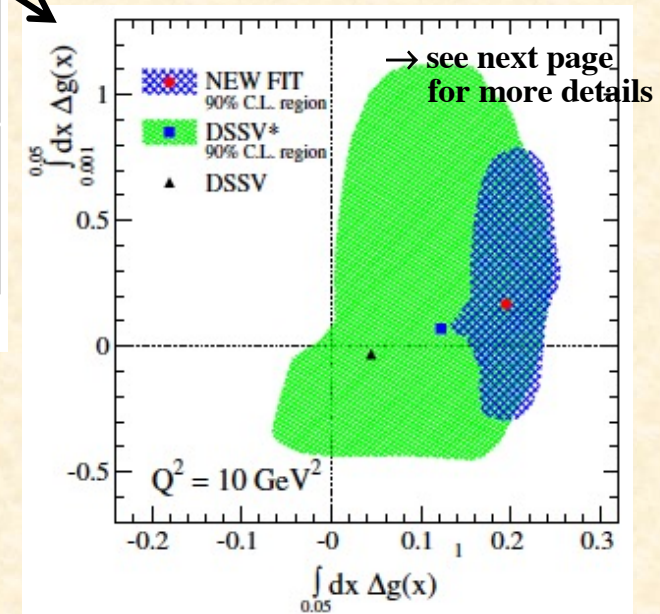
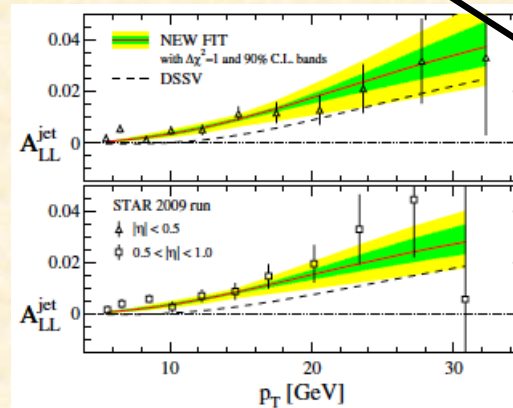
$$\Delta\Sigma = \sum_i \int dx \left[\Delta q_i(x) + \Delta \bar{q}_i(x) \right] \rightarrow 1 \text{ (100\%)}$$



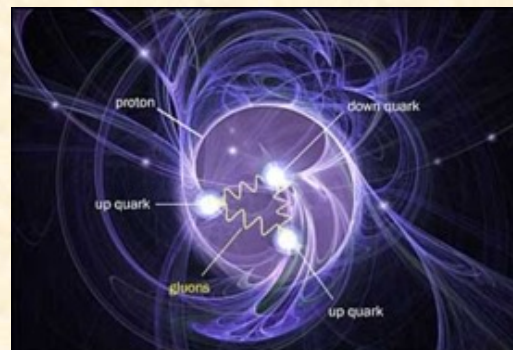
gluon spin



angular momentum



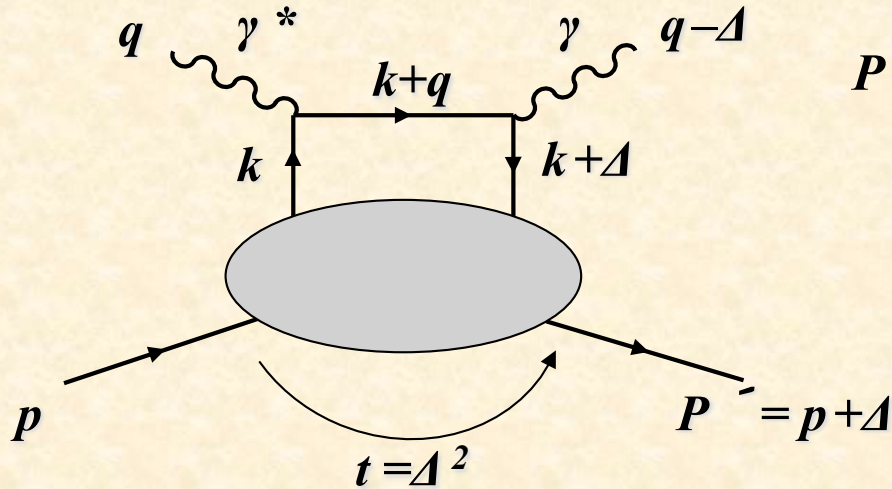
“A possible” spin decomposition



Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_{q,g}$$

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$

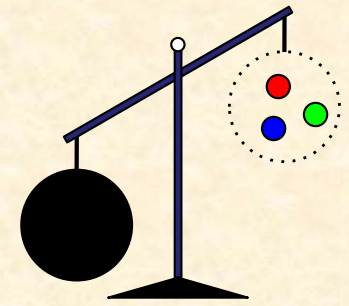
Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

\Rightarrow probe L_q , key quantity to solve the spin puzzle!

Origin of hadron masses

Mass and spin of the nucleon are two of fundamental quantities in physics.



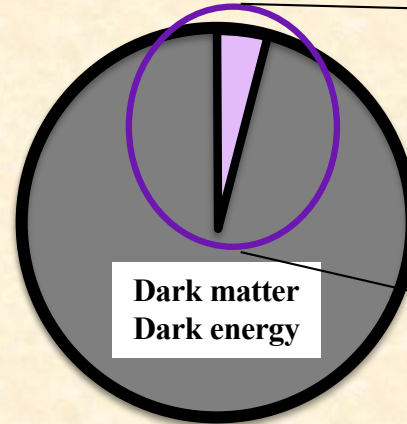
Nucleon mass: $M = \langle p | \int d^3x T^{00}(x) | p \rangle$

Ordinary matter
= Atoms \approx Nucleons

Quark mass

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2} \bar{q}(x) i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\alpha}(x)$$



Dark matter

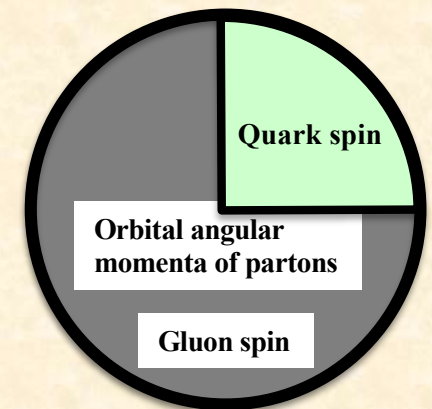


Origin of nucleon mass

Nucleon spin: $\frac{1}{2} = \langle p | J^3 | p \rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2} \epsilon^{3jk} \int d^3x M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x)x^{\mu} - T^{\alpha\mu}(x)x^{\nu}$

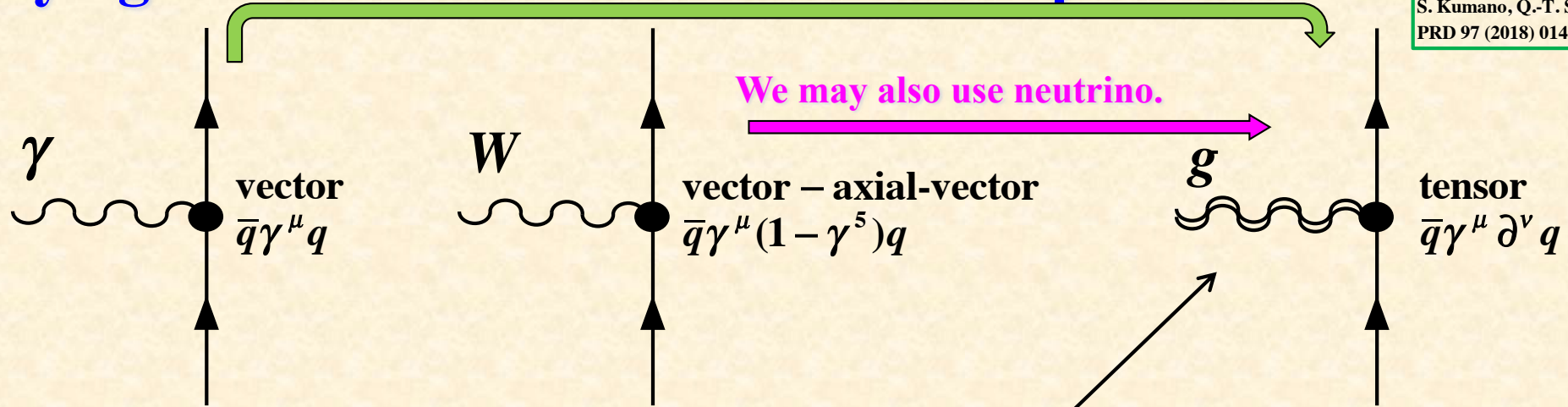


Origin of nucleon spin
("Dark spin")

Why “gravitational” interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev,
PRD 97 (2018) 014020.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2) \gamma^+ q(z/2) | p \rangle \Big|_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

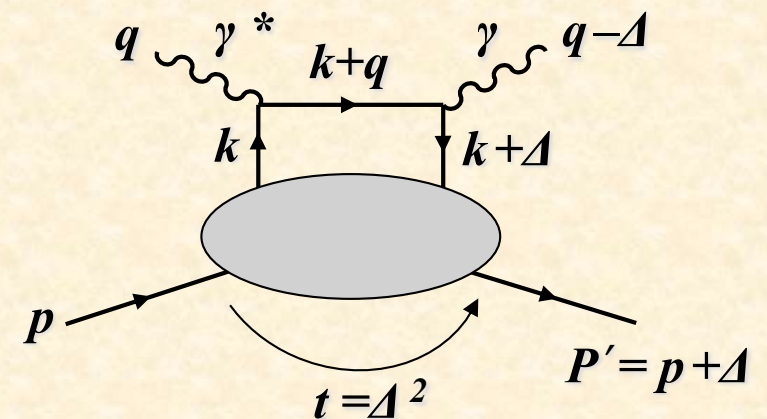
Non-local operator of GPDs/GDAs:

$$\begin{aligned} & (P^+)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z^+=0, \bar{z}_\perp=0} \\ &= \left(i \frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z=0} \\ &= \bar{q}(0) \gamma^+ \left(i \tilde{\partial}^+ \right)^{n-1} q(0) \end{aligned}$$

= energy-momentum tensor of a quark for $n = 2$
(electromagnetic for $n = 1$)

= source of gravity

Virtual Compton or (timelike) two-photon process



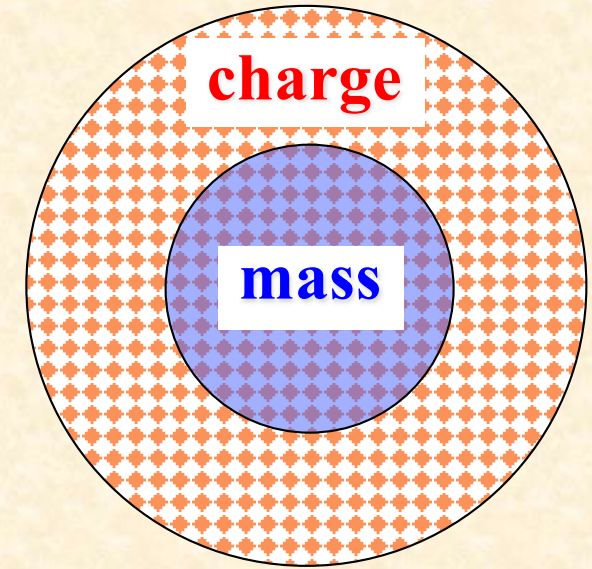
Gravitational form factors and radii for pion

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

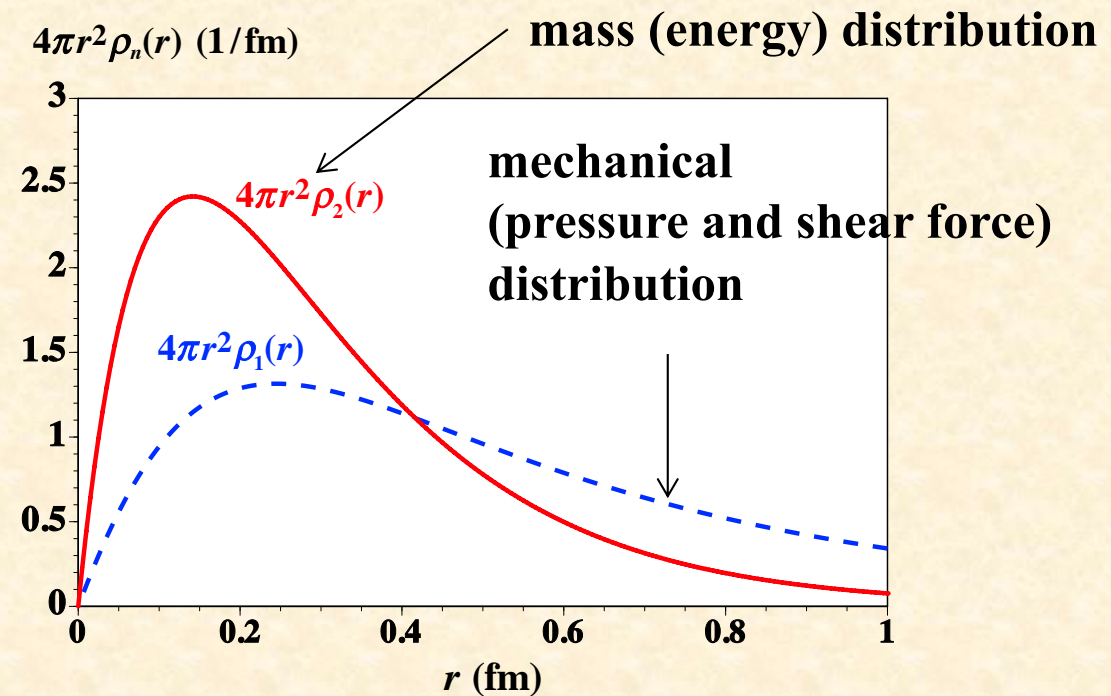
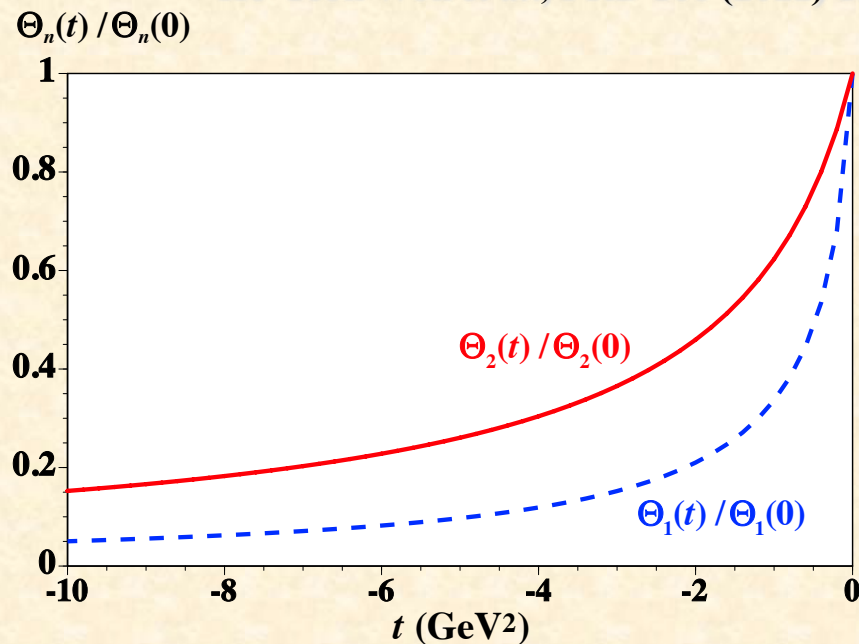
$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

SK, Q.-T. Song, O. Teryaev
PRD 97 (2018) 014020.



Related theoretical studies:

- A. Freese and I. C. Cloet, PRC 100 (2019) 015201;
- P. E. Shanahan and W. Detmold, PRD 99 (2019) 014511;
- C. D. Roberts *et al.*, Prog. Part. Nucl. Phys. 120 (2021) 103883;
- J.-L. Zhang *et al.* PLB 815 (2021) 136158;
- June-Young Kim and Hyun-Chul Kim, PRD 104 (2021) 074019;
- Ho-Yeon Won *et al.*, PRD 106 (2022) 114009.



Nucleon pressure

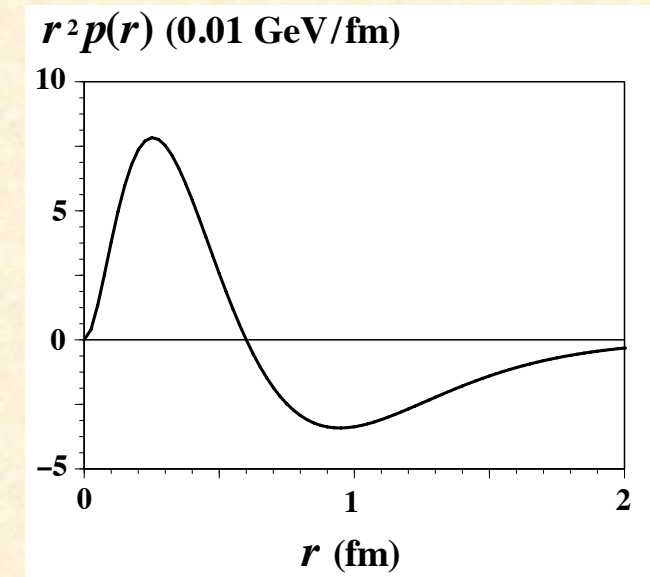
$$\langle N(p') | T_q^{\mu\nu}(0) | N(p) \rangle = \bar{u}(p') \left[A \gamma^{(\mu} \bar{P}^{\nu)} + B \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C} M g^{\mu\nu} \right] u(p)$$

Recent progress

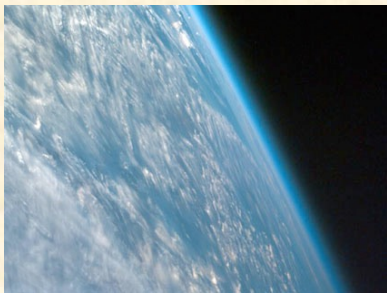
V. D. Burkert, L. Elouadrhiri, and F. X. Girod,
Nature 557 (2018) 396;

M. V. Polyakov and P. Schweitzer,
Int. J. Mod. Phys. A 33 (2018) 1830025;

C. Lorce, H. Moutarde, and A. P. Tranwinski,
Eur. Phys. J. C 79 (2019) 89.



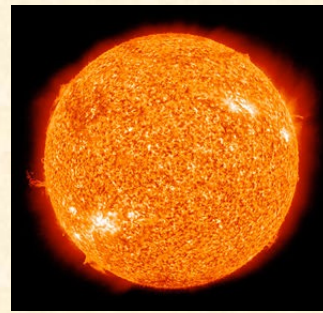
Highest pressure in nature 1 Pa (Pascal) = 1 N/m²



Earth atmosphere
10⁵ Pa = 1000 hPa



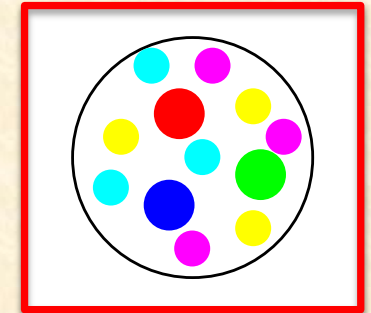
Center of earth
10¹¹ Pa = 100 GPa



Center of Sun
10¹⁶ Pa = 10 PPa



Neutron star
10³⁴ Pa



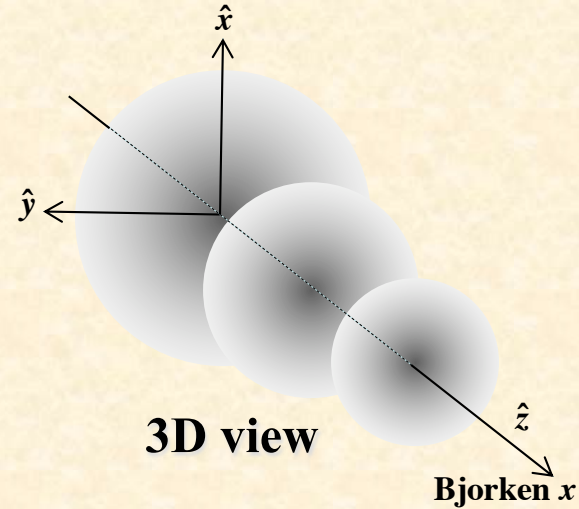
Hadron
10³⁵ Pa

Proton (hadrons) puzzle studies by hadron tomography

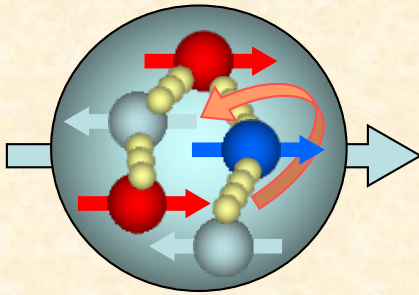
Hadron tomography



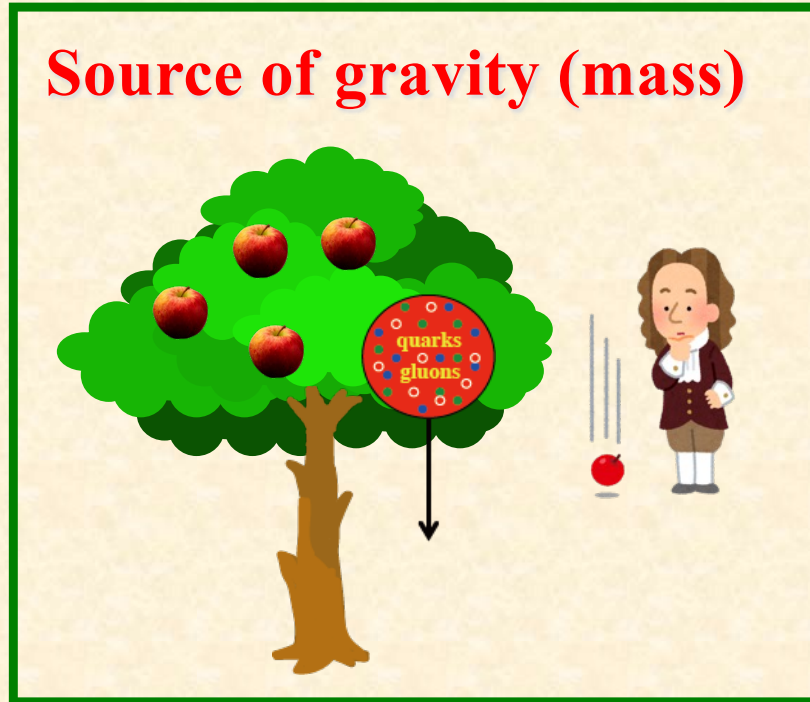
Proton radius puzzle



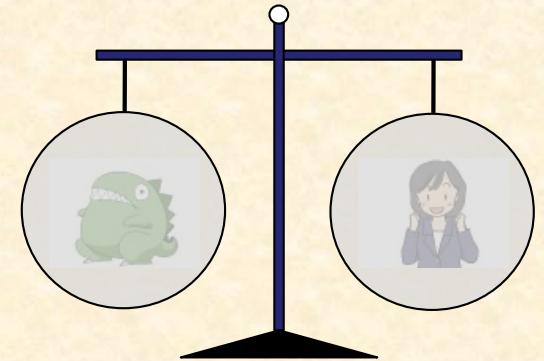
Origin of nucleon spin



Source of gravity (mass)



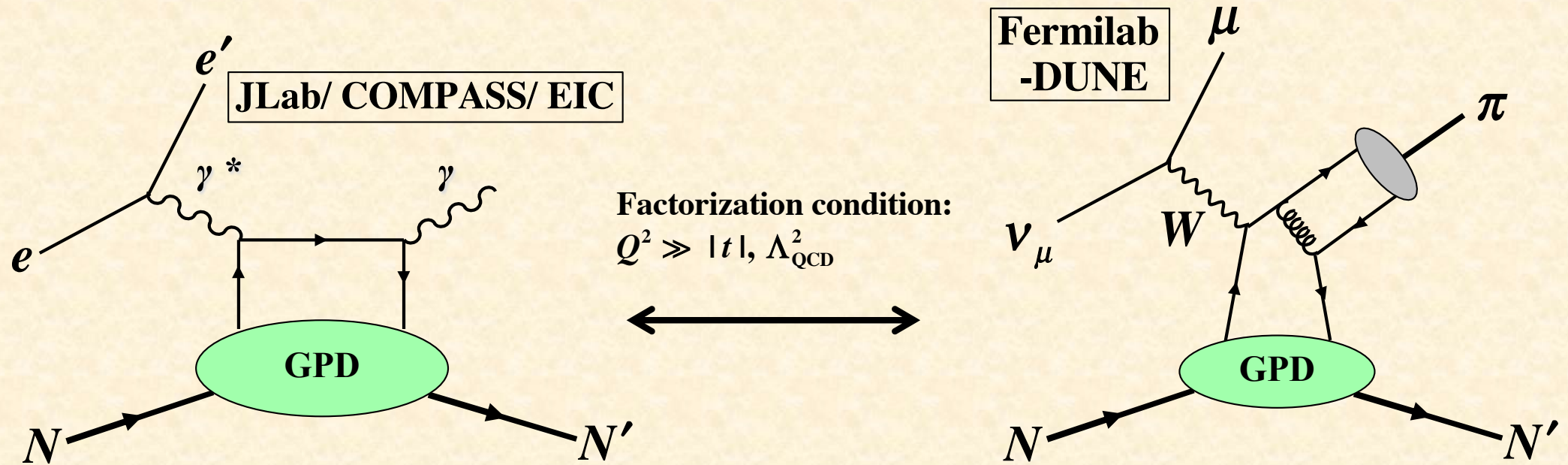
Exotic hadrons



Possible studies on GPDs at neutrino facilities

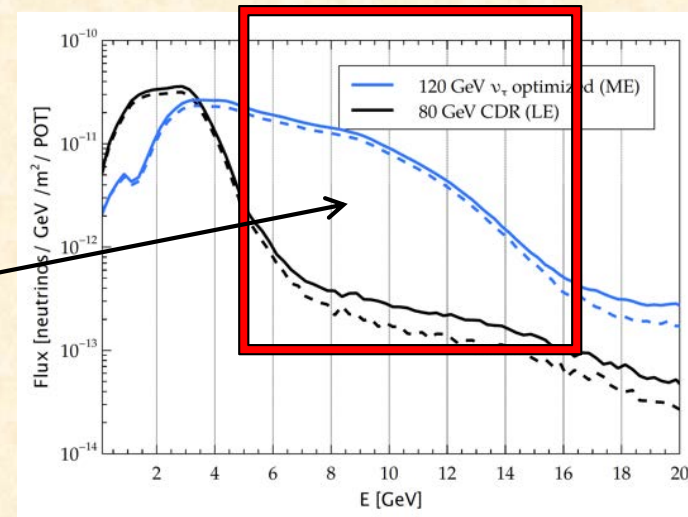
- SK, EPJ Web Conf. 208 (2019) 07003.
- EIC yellow report, R. Abdul Khalek *et al.*, arXiv:2103.05419, Sec. 7.5.2, Neutrino physics by SK and R. Petti.
- SK and R. Petti, PoS (NuFact2021) 092.
- X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie, arXiv:2401.11440.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)



**Deep Underground Neutrino Experiment (DUNE)
at Long-Baseline Neutrino Facility (LBNF)**

**High-energy part of the LBNF ν beam
can be used for the GPD studies.**



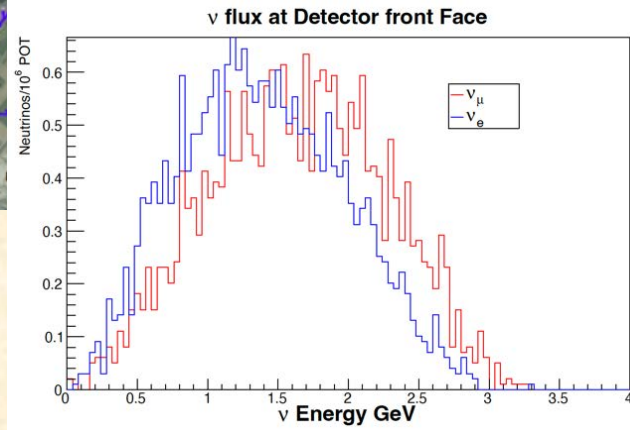
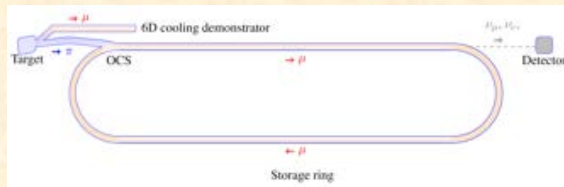
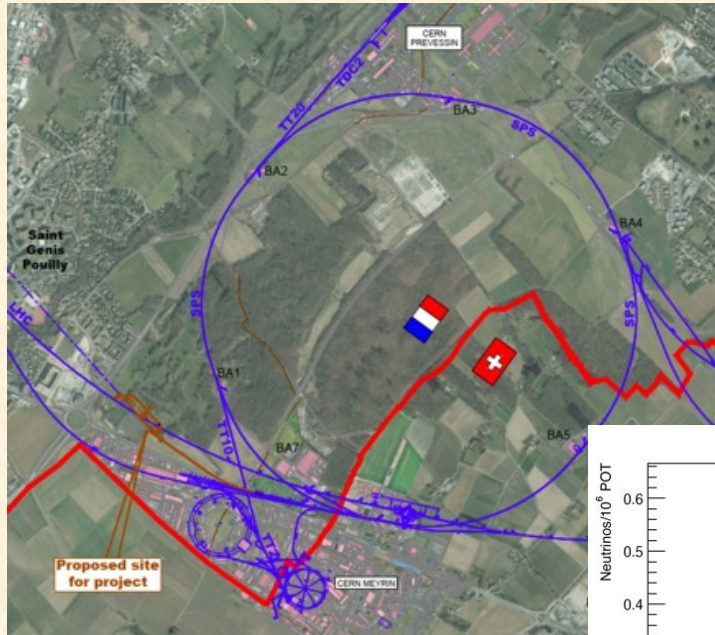
J. Rout et al., PRD 102 (2020) 116018

nuSTORM (Neutrinos from Stored Muons)

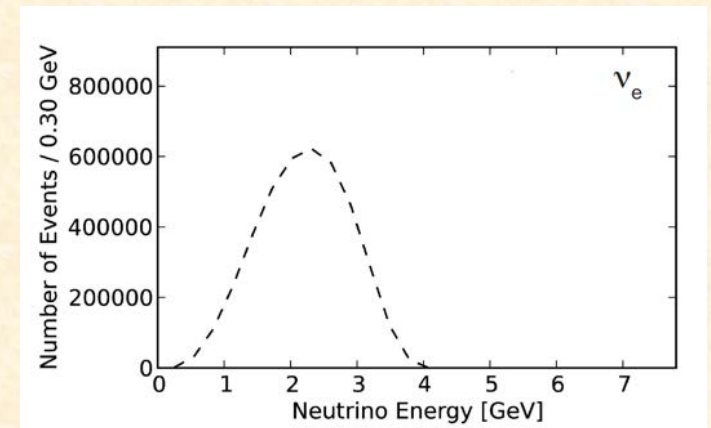
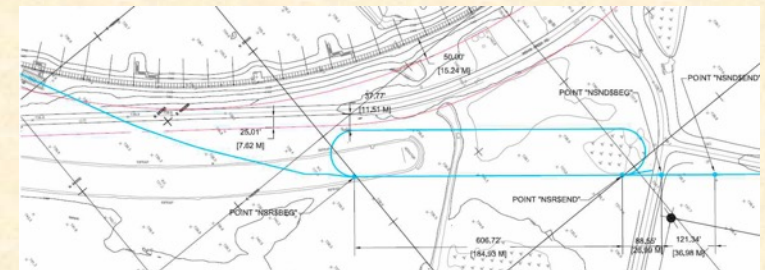
Fermilab

Feasibility Study, C. C. Ahdida *et al.*, (2020);
L. A. Ruso *et al.*, arXiv:2203.07545.

CERN



Letter of Intent, arXiv:1206.0294,
P. Kyberd *et al.* (2012);
Proposal, D. Adey *et al.*, arXiv:1308.6822.
No recent update.



At this stage, the considered beam energy
is not high enough for structure-function studies;
however, high-energy option could be possible.
(personal communications: Xianguo Lu)

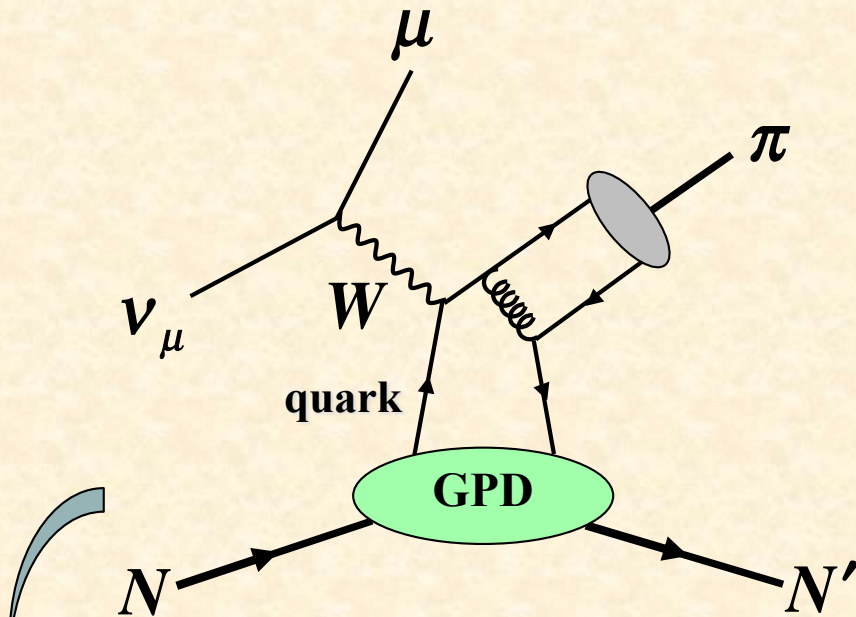
→ SK's talk at the nuSTORM-collaboration meeting
on July, 15, 2024

They could be interested in the higher-energy possibility.

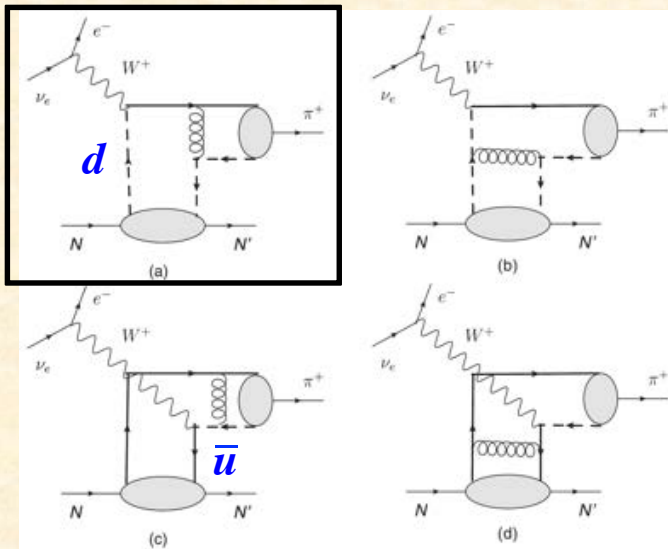
Recent work on pion production in neutrino reaction for GPD studies

B. Pire, L. Szymanowski, and J. Wagner,
Phys. Rev. D 95, 114029 (2017).

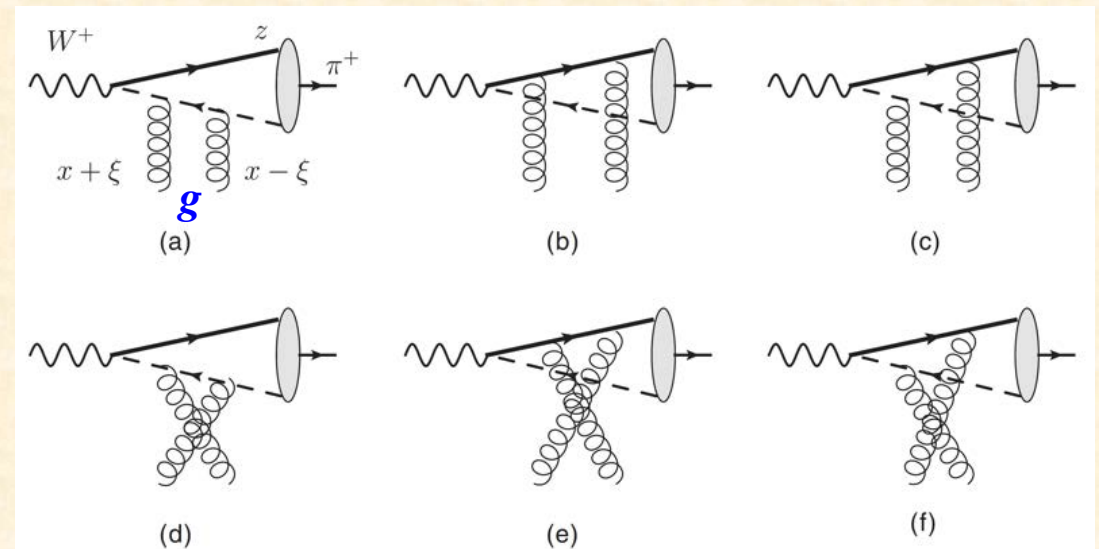
There are several processes to contribute to the pion-production cross section, including the gluon GPD terms.



Quark GPDs



Gluon GPDs



Cross section formalism

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \rightarrow \ell^- N' \pi)}{dy dQ^2 dt d\phi} = \Gamma \varepsilon \sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32 (2\pi)^4 (s - m_N^2)^2 y (1-\varepsilon) \sqrt{1+4x^2 m_N^2 / Q^2}}$$

$$\sigma_L = \varepsilon_L^{\mu\nu} W_{\mu\nu} \varepsilon_L^\nu = \frac{1}{Q^2} \left[(1-\xi^2) \left\{ |C_q \mathcal{H}_q + C_g \mathcal{H}_g|^2 + |C_q \tilde{\mathcal{H}}_q|^2 \right\} + \frac{\xi^4}{1-\xi^2} \left\{ |C_q \mathcal{E}_q + C_g \mathcal{E}_g|^2 + |C_q \tilde{\mathcal{E}}_q|^2 \right\} \right. \\ \left. - 2\xi^2 \operatorname{Re} \left\{ (C_q \mathcal{H}_q + C_g \mathcal{H}_g)(C_q \mathcal{E}_q + C_g \mathcal{E}_g)^* \right\} - 2\xi^2 \operatorname{Re} \left\{ C_q \tilde{\mathcal{H}}_q (C_q \tilde{\mathcal{E}}_q)^* \right\} \right]$$

Quark contributions

$$T_q = -i \frac{C_q}{2Q} N(p') \left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \tilde{\mathcal{H}}_q \hat{n} \gamma_5 - \tilde{\mathcal{E}}_q \frac{\gamma_5 n \cdot \Delta}{2m_N} \right] N(p)$$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz \phi_\pi(z)}{1-z} \int dx \frac{F_q(x, \xi, t)}{x - \xi + i\varepsilon}$$

= (pion distribution amplitude) · (quark GPD)

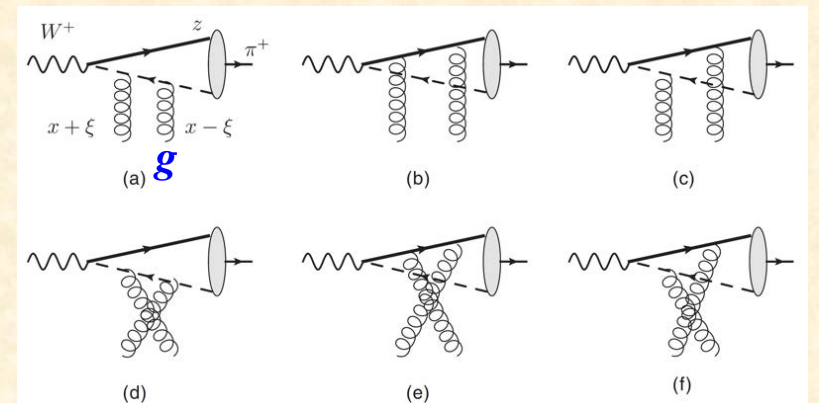
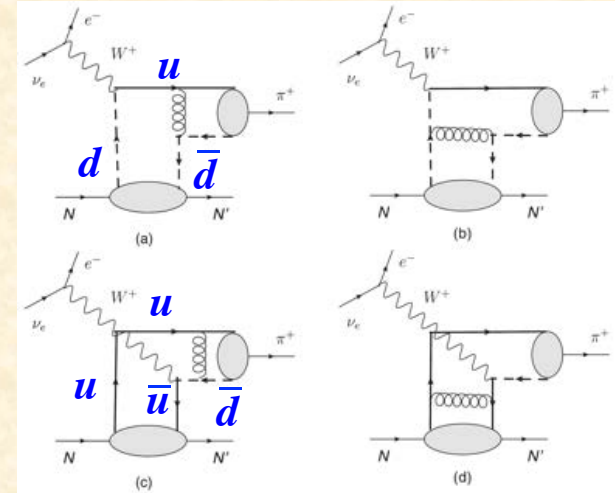
$$F_q(x, \xi, t) \equiv F_d(x, \xi, t) - F_u(-x, \xi, t)$$

$$F = H, E, \tilde{H}, \tilde{E}$$

Gluon contributions

$$T_g = -i \frac{C_g}{2Q} N(p') \left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} \right] N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x, \xi, t)}{x - \xi + i\varepsilon}$$



GK (Goloskokov-Kroll) - 2013 parametrization

P. Kroll, H. Moutarde, F. Sabatie,
Eur. Pjys. J. C 73 (2013) 2278.

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0}$$

$$= \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0}$$

$$= \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

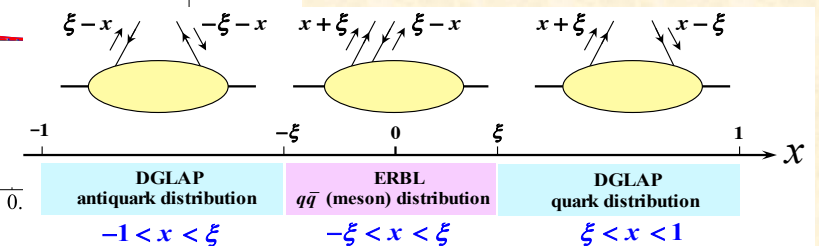
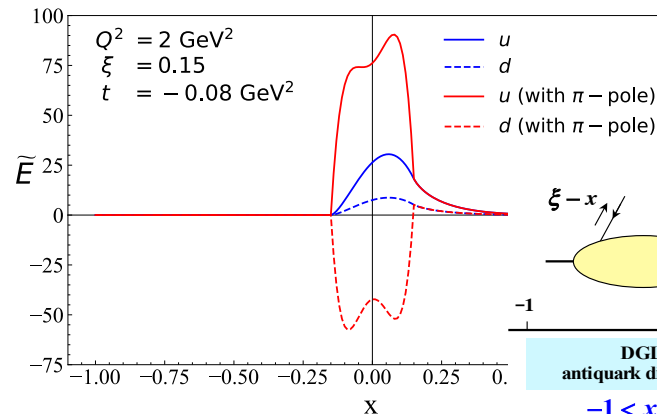
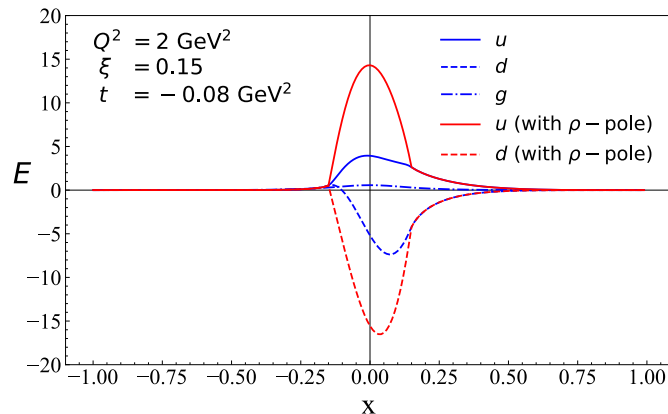
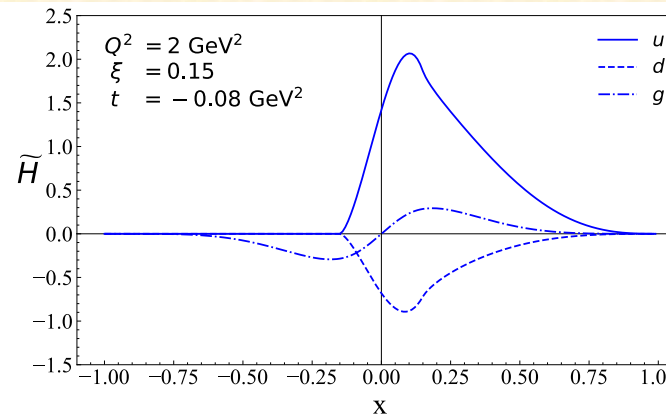
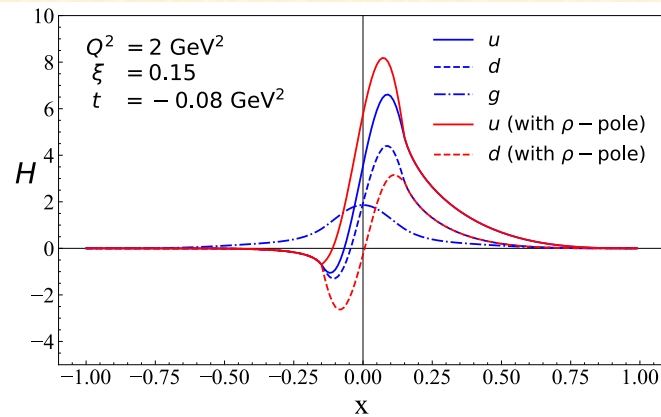
$$F_i(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) f_i(\beta, \alpha, t) + D_i(x', t) \Theta(\xi^2 - x^2)$$

$$f_i(\beta, \alpha, t) = F_i(\beta, \xi = 0, t = 0) e^{t p_{h_i}(\beta)} \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{[(1-|\beta|)^2 - \alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\Theta(\xi^2 - x^2) = \begin{cases} 1 & \xi^2 > x^2 \\ 0 & \xi^2 < x^2 \end{cases}, \quad p_{h_i}(\beta) = -\alpha'_{h_i} \ln \beta + b_{h_i}$$

$$F_i(\beta, \xi = 0, t = 0) = \beta^{-\delta_i} (1-\beta)^{2n_i+1} \sum_{j=0}^3 c_{f_j} \beta^{j/2},$$

parameters determined by global analysis



Cross sections

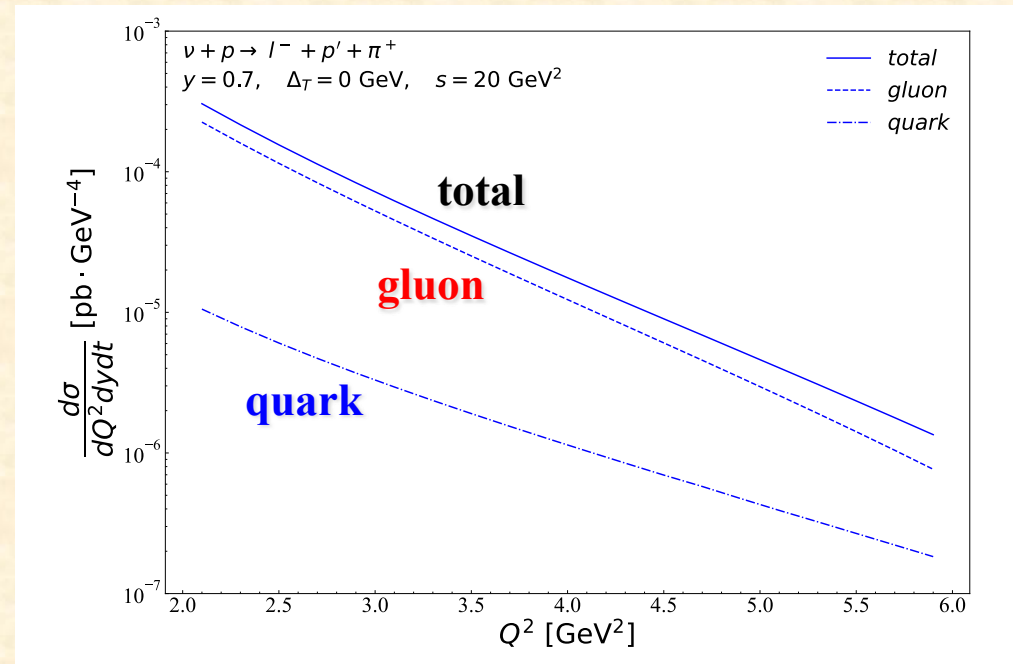
π^+ production: $\nu p \rightarrow \ell^- \pi^+ p$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz \phi_\pi(z)}{1-z} \int dx \frac{F_q(x, \xi, t)}{x - \xi + i\epsilon} \quad \text{gluon} \gg \text{quark}$$

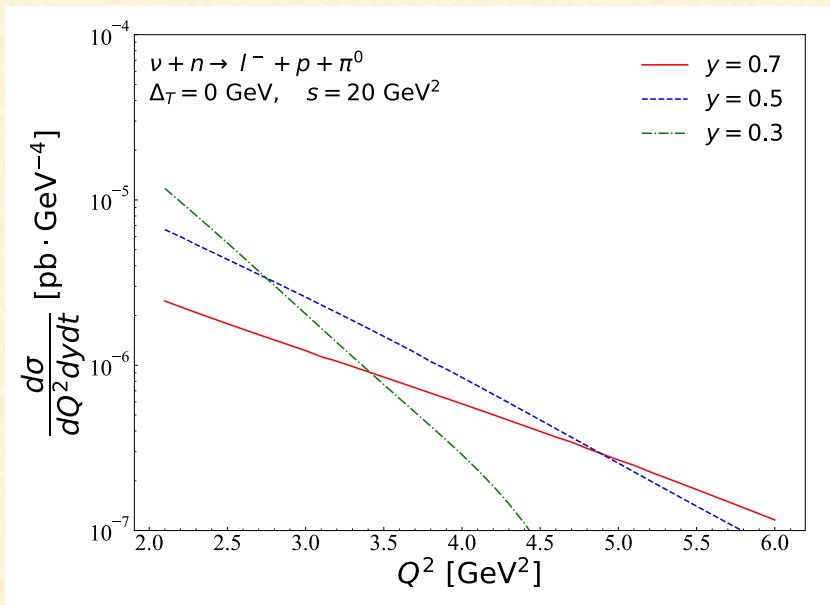
$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x, \xi, t)}{x - \xi + i\epsilon}$$

$$\frac{\mathcal{F}_q}{\mathcal{F}_g} \sim \frac{\xi}{8} = \frac{0.1 \sim 0.3}{8} = 0.01 \sim 0.04$$

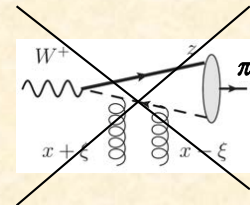
= a few % $\ll 1$



π^0 production: $\nu n \rightarrow \ell^- \pi^0 p$



no gluon



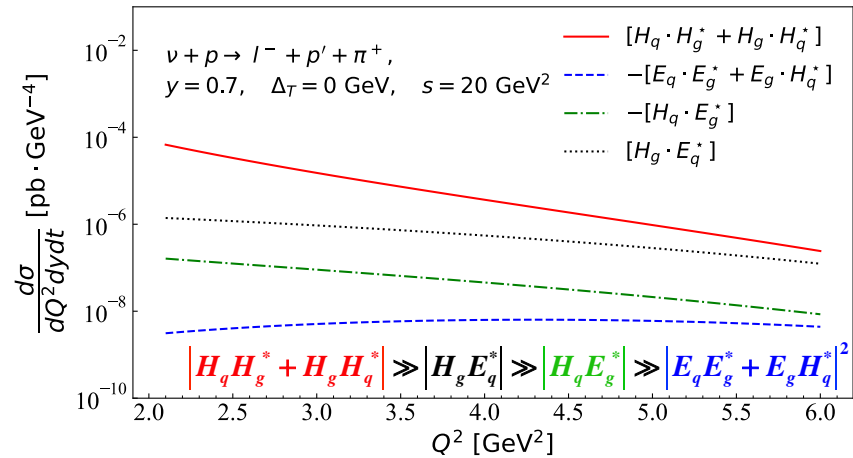
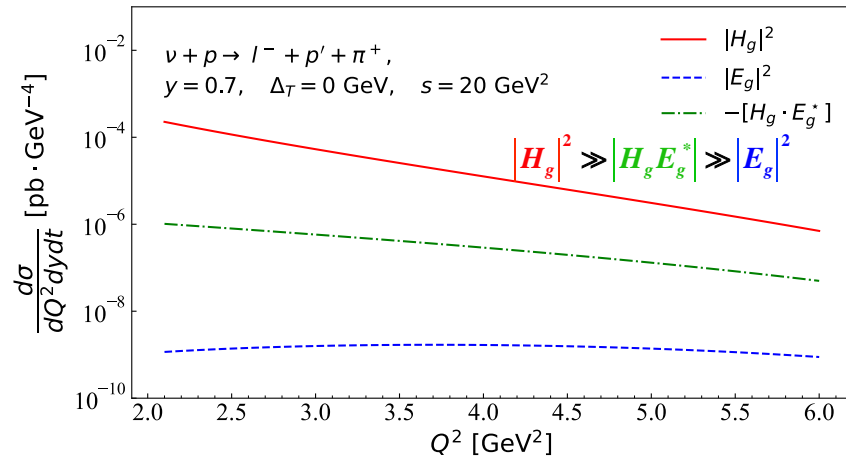
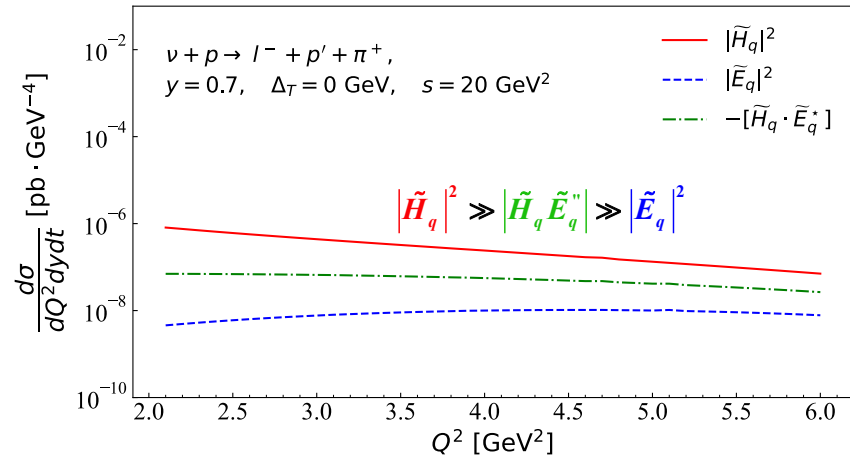
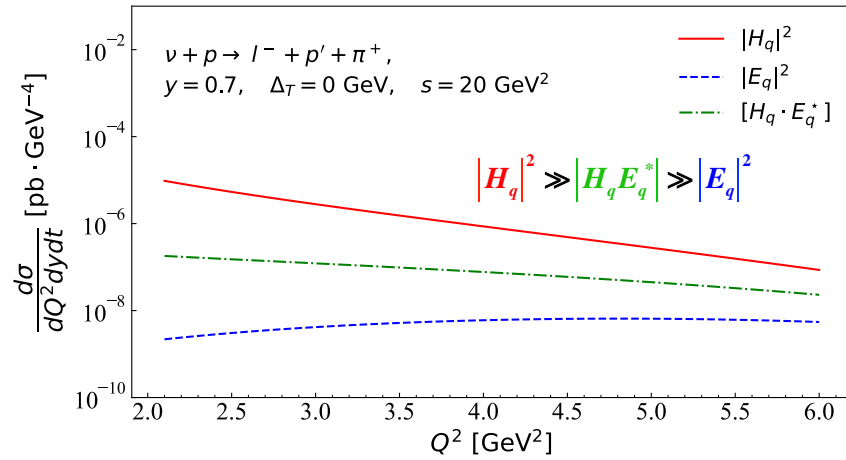
no gluon for π^0

Neutrino GPD studies are complementary to the charged-lepton projects.

- Gluon GPDs could be probed in charged-pion production.
- Quark GPDs could be probed in π^0 production.
- Flavor dependence of quark GPDs could be investigated.

Contribution of each term to the π^+ -production cross section

$$\frac{d\sigma(\nu_\ell N \rightarrow \ell^- N' \pi)}{dy dQ^2 dt d\phi} \propto \frac{1}{Q^2} \left[(1 - \xi^2) \left\{ |C_q \mathcal{H}_q + C_g \mathcal{H}_g|^2 + |C_q \tilde{\mathcal{H}}_q|^2 \right\} + \frac{\xi^4}{1 - \xi^2} \left\{ |C_q \mathcal{E}_q + C_g \mathcal{E}_g|^2 + |C_q \tilde{\mathcal{E}}_q|^2 \right\} \right. \\ \left. - 2\xi^2 \operatorname{Re} \left\{ (C_q \mathcal{H}_q + C_g \mathcal{H}_g)(C_q \mathcal{E}_q + C_g \mathcal{E}_g)^* \right\} - 2\xi^2 \operatorname{Re} \left\{ C_q \tilde{\mathcal{H}}_q (C_q \tilde{\mathcal{E}}_q)^* \right\} \right]$$

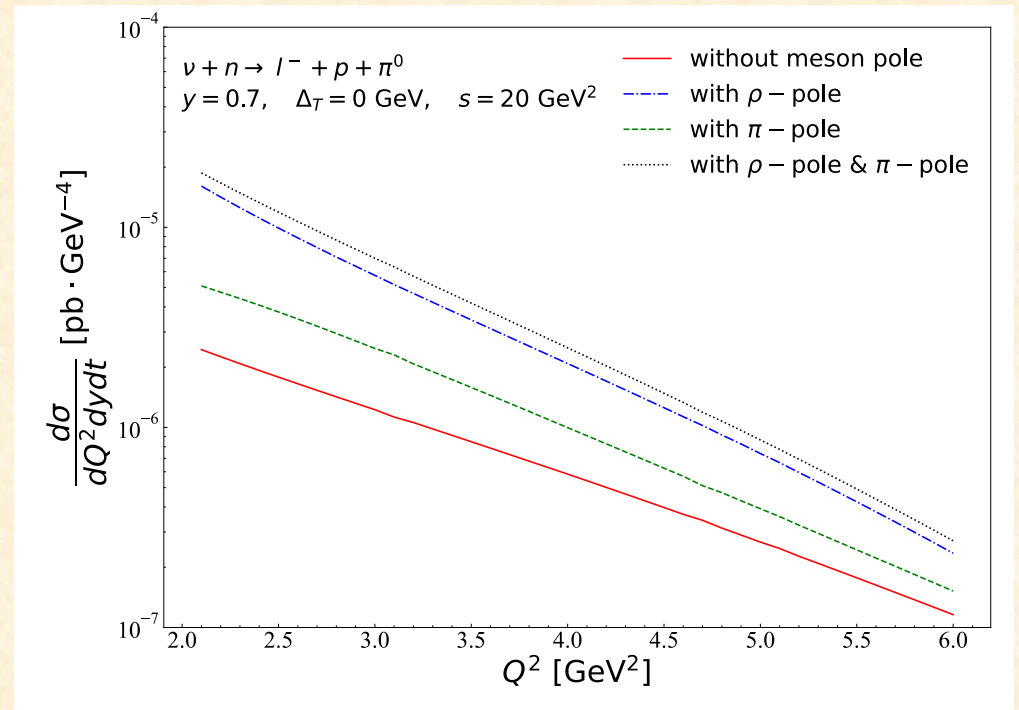
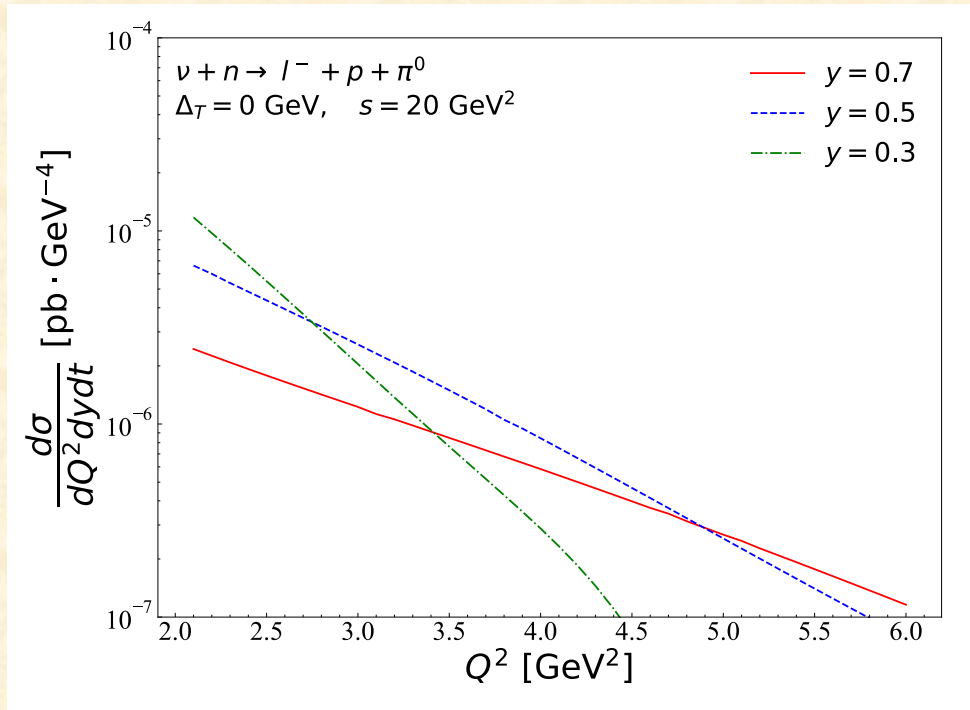


$$H_g > H_q > \tilde{H}_q > E_q, \tilde{E}_q, E_g$$

- π^+ production is sensitive to gluon \mathcal{H}_g .

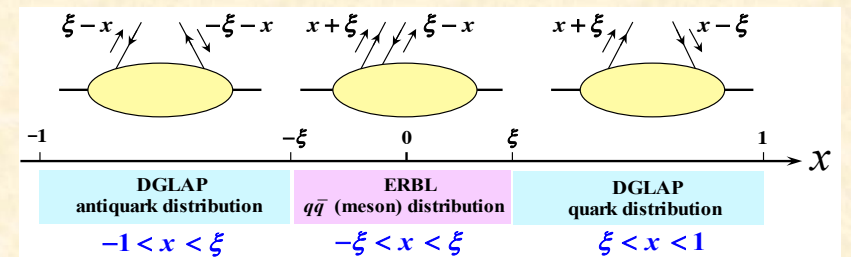
Contribution of each term to the π^0 -production cross section

$$\frac{d\sigma(\nu_\ell N \rightarrow \ell^- N' \pi)}{dy dQ^2 dt d\phi} \propto \frac{1}{Q^2} \left[(1 - \xi^2) \left\{ |C_q \mathcal{H}_q + \cancel{C_g \mathcal{H}_g}|^2 + |C_q \tilde{\mathcal{H}}_q|^2 \right\} + \frac{\xi^4}{1 - \xi^2} \left\{ |C_q \mathcal{E}_q + \cancel{C_g \mathcal{E}_g}|^2 + |C_q \tilde{\mathcal{E}}_q|^2 \right\} \right. \\ \left. - 2\xi^2 \operatorname{Re} \left\{ (C_q \mathcal{H}_q + \cancel{C_g \mathcal{H}_g})(C_q \mathcal{E}_q + \cancel{C_g \mathcal{E}_g})^* \right\} - 2\xi^2 \operatorname{Re} \left\{ C_q \tilde{\mathcal{H}}_q (C_q \mathcal{E}_q)^* \right\} \right]$$



$$\cancel{\mathcal{H}_g} > \mathcal{H}_q > \tilde{\mathcal{H}}_q > \mathcal{E}_q, \tilde{\mathcal{E}}_q, \cancel{\mathcal{E}_g}$$

- π^0 production is sensitive to gluon \mathcal{H}_q .
- GPDs in the ERBL (Efremov-Radyushkin-Brodsky-Lepage) region are probed.



Gravitational form factors, Prospects on neutrino GPD project

Nucleon mass: $M = \langle N(p) | \int d^3x T^{00}(x) | N(p) \rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2} \bar{q}(x) i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F_{\alpha}^{\nu}(x) = T_q^{\mu\nu}(x) + T_g^{\mu\nu}(x)$$

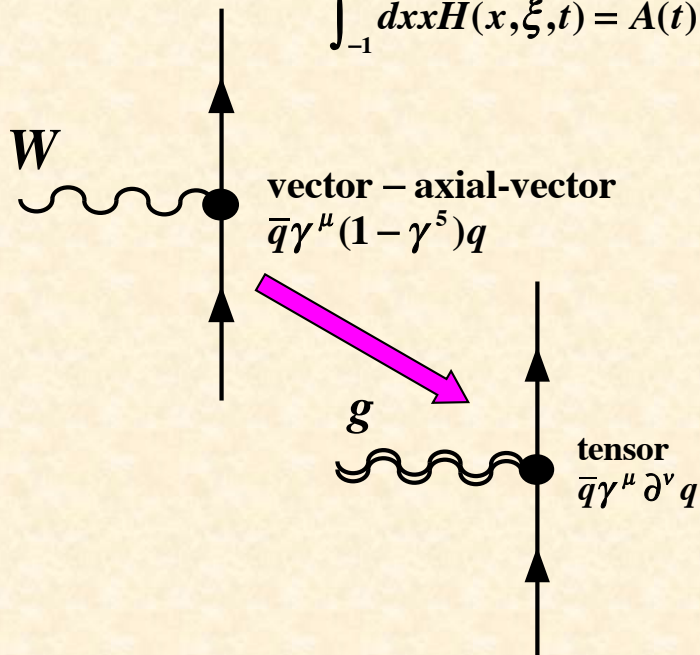
Gravitational form factors: A, B, C, D

$$\langle N(p') | \int d^3x T^{\mu\nu}(x) | N(p) \rangle = u(p') \left[A \gamma^{\{\mu} \bar{P}^{\nu\}} + B \frac{\bar{P}^{\{\nu} i \sigma^{\nu\}\alpha} \Delta_{\alpha}}{2M} + C M g^{\nu\nu} + D \frac{\Delta^{\mu} \Delta^{\nu} - g^{\nu\nu} \Delta^2}{M} \right] u(p)$$

$$T^{00}: \quad \langle N(p') | \int d^3x T^{00}(x) | N(p) \rangle = 2ME \left[A(t) - \frac{t}{4M^2} \{A(t) - 2B(t) + D(t)\} \right]$$

GPDs and gravitational form factors:

$$\int_{-1}^1 dx x H(x, \xi, t) = A(t) + \xi^2 D(t), \quad \int_{-1}^1 dx x E(x, \xi, t) = B(t) - \xi^2 D(t)$$



- **Neutrino-scattering experiments (LBNF) are valuable and complementary to JLab, COPMASS, KEK-B, and the other facility projects in the sense that the cross sections are sensitive to quark flavor.**

- **This project is already in progress.**

The new detector, which was the basis of various GPD measurements, was selected by the DUNE collaboration to be part of the near detector complex. (2021, R. Petti).
 nuSTORM at CERN?

Hadron accelerator facilities on GPDs

(including future possibilities)

High-energy hadron physics experiments

CERN
(LHC, **COMPASS/AMBER**,
LHeC, FCC, CLIC, nuSTORM)

Fermilab
(SeaQuest, **SpinQuest, DUNE**)

GSI (FAIR)

KM3NeT

JINR (NICA)

Baikal GVD

IHEP (BEPC, CEPC)

IMP (HIAF, EicC)

BNL
(RHIC, **EIC**)

JLab

ILC

KEK
(**KEKB, J-PARC**)

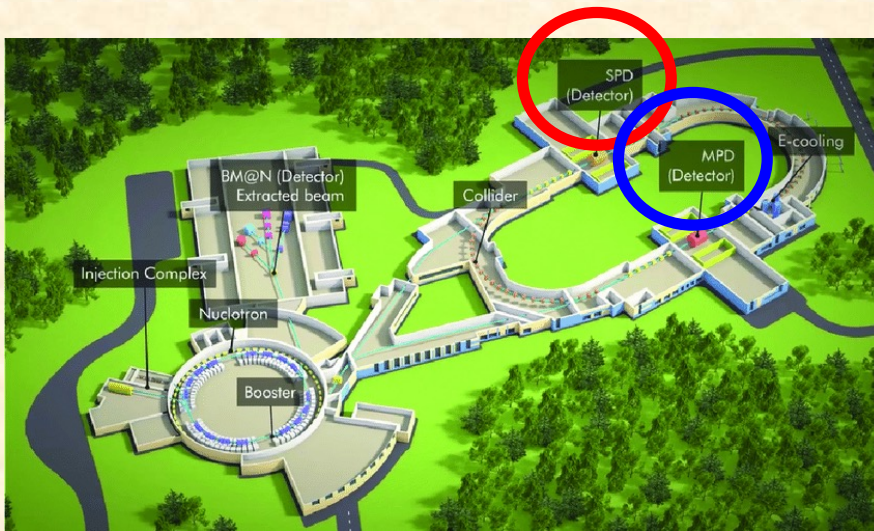
IceCube



Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

Nuclotron-based Ion Collider facility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)

MPD (MultiPurpose Detector for heavy ion physics)

$$\vec{p} + \vec{p}: \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$$

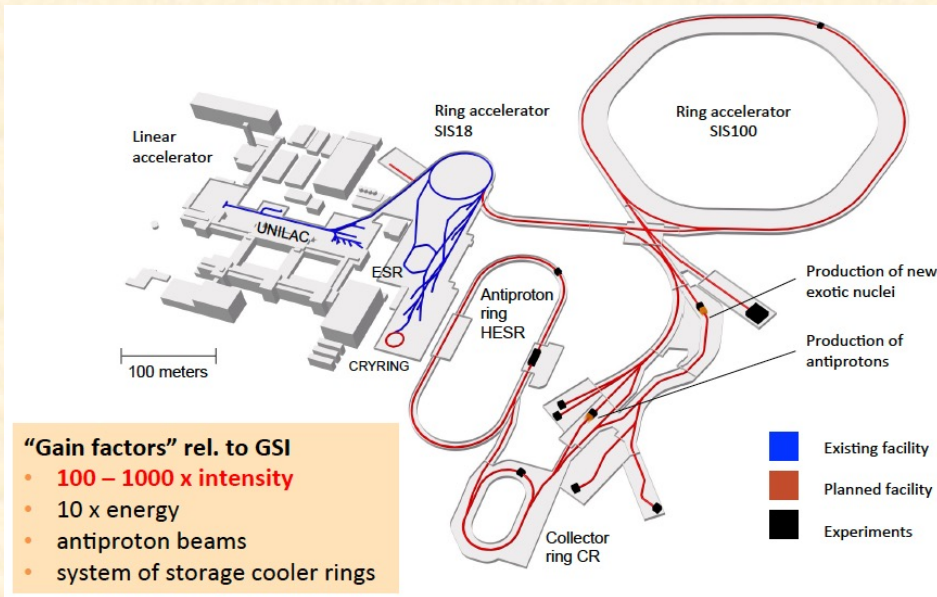
$$\vec{d} + \vec{d}: \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$$

$\vec{p} + \vec{d}$ is also possible.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.* (NICA project), arXiv:2011.15005, Progress in Nuclear and Particle Physics in press.

Unique opportunity in high-energy spin physics,
especially on the deuteron spin physics.

GSI-FAIR



- "Gain factors" rel. to GSI**
- 100 – 1000 x intensity
 - 10 x energy
 - antiproton beams
 - system of storage cooler rings

APPA

- Atomic Physics and Fundamental Symmetries,
- Plasma Physics,
- Materials Research,
- Radiation Biology,
- Cancer Therapy with Ion Beams / Space Research

CBM

- Dense and Hot Nuclear Matter

NUSTAR

- Nuclear Structure and Reaction Studies with nuclei far off stability,
- Physics of Explosive Nucleosynthesis (r-process)

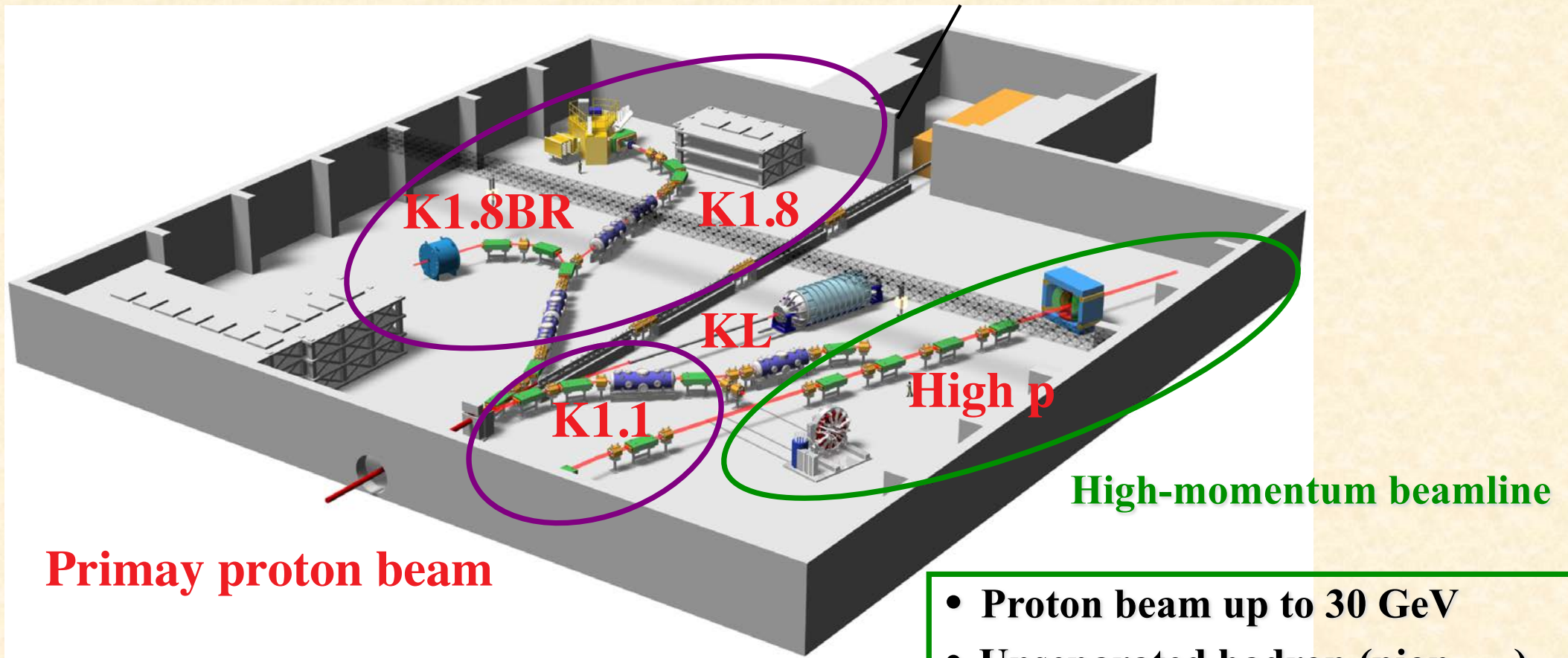
PANDA

- Hadron Structure & Dynamics with cooled antiproton beams



Hadron facility

(Low energy) Kaon and pion experiments are done at these beamlines.



Primary proton beam

High-momentum beamline

- Proton beam up to 30 GeV
- Unseparated hadron (pion, ...) beam up to 15~20 GeV

Possible studies on GPDs at hadron accelerator facilities

**SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;**

**T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.**

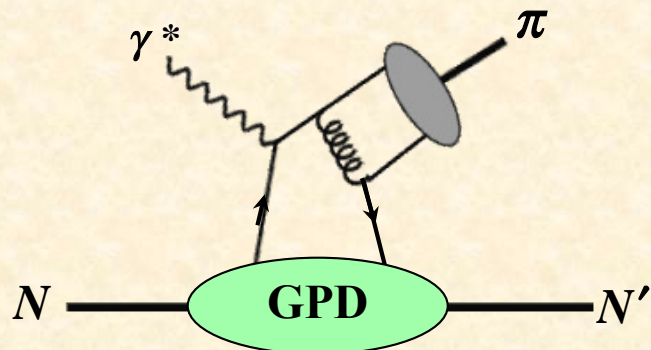
J-PARC LoI 2019-07, J.-K. Ahn *et al.* (2019).

J-PARC proposal under preparation (2024),

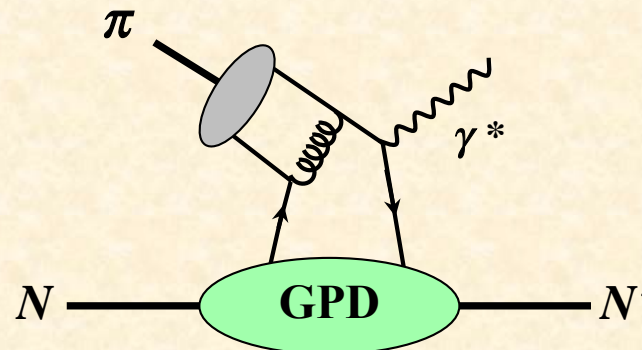
**Please get in touch with W.-C. Chang, N. Tomida
if you are interested in this project.**

GPD projects at JLab /EIC and J-PARC

JLab / EIC



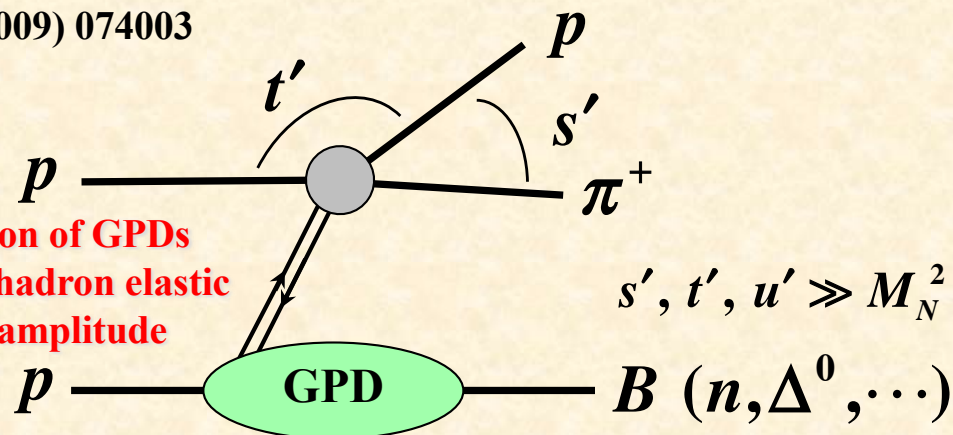
J-PARC



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003

Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude

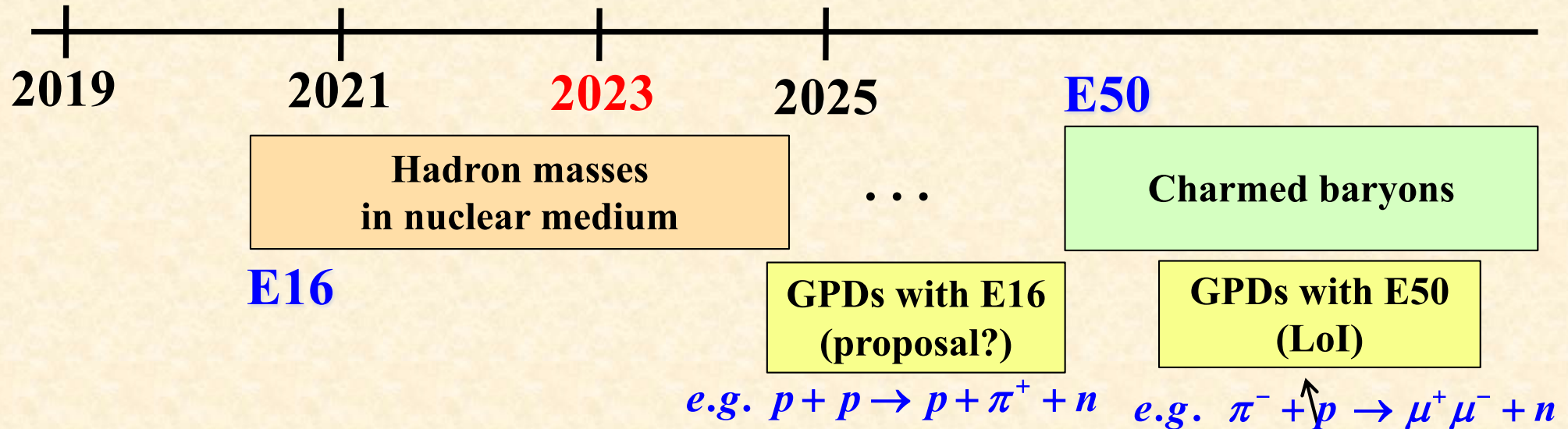


$$s', t', u' \gg M_N^2$$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$$\begin{aligned} \pi + N &\rightarrow \gamma + \gamma + N' \\ h + M_B &\rightarrow h' + \gamma + M_D \\ h + M_B &\rightarrow h' + M_C + M_D \end{aligned}$$

Physics of J-PARC high-momentum beamline



Proposal

Electron pair spectrometer at the J-PARC 50-GeV PS to explore the chiral symmetry in QCD

April 28, 2006
June 07, 2006 rev.1

S. Yokkaichi¹, H. En'yo, M. Naruki, R. Muto, T. Tabaru
RIKEN

K. Ozawa, H. Hamagaki
Center for Nuclear Study, Graduate School of Science, University of Tokyo

K. Shigaki
Graduate School of Science, Hiroshima University

S. Sawada, M. Sekimoto
High Energy Accelerator Research Organization (KEK)

F. Sakuma, K. Aoki
Department of Physics, Kyoto University

KEK/J-PARC-PAC 2012-19

Charmed Baryon Spectroscopy via the (π, D^{*+}) reaction

Y. Morino, T. Nakano,* H. Noumi¹,* K. Shirotori, Y. Sugaya, and T. Yamaga
*Research Center for Nuclear Physics (RCNP), Osaka University,
10-1, Mihogaoka, Ibaraki, Osaka, 567-0047, Japan*

K. Ozawa²
*Institute of Particle and Nuclear Studies(IPNS),
High Energy Accelerator Research Organization (KEK),
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

T. Ishikawa
*Research Center for Electron Photon Science,
Tohoku University, 1-2-1, Mikamine,
Taihaku-ku, Sendai, Miyagi 982-0826, Japan*

Y. Miyachi
*Physics Department, Yamagata University, 1-4-12,
Kojirakawa-machi, Yamagata 990-8560, Japan*

K. Tanida
*Department of Physics and Astronomy,
Seoul National University, Seoul 151-747, Korea*

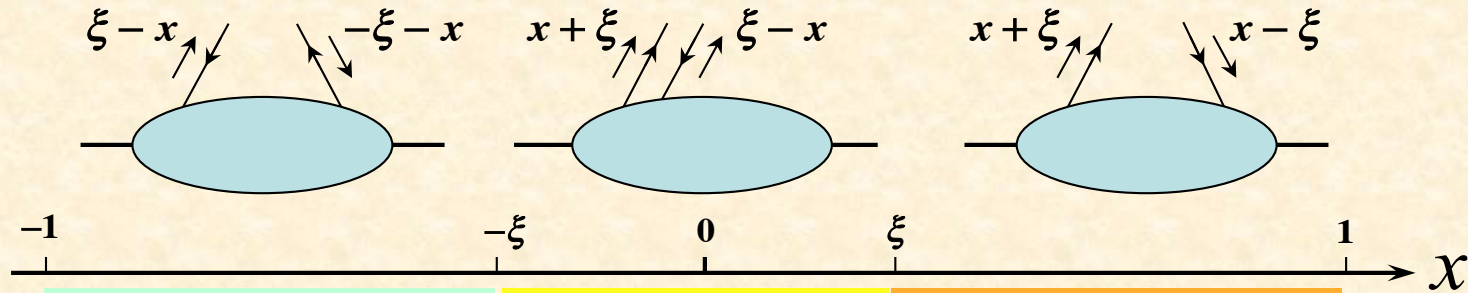
There is a possibility
for high-energy hadron physics,
including nucleon structure, ...

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process
at J- PARC

JungKeun Ahn,¹ Sakiko Ashikag,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki Noumi,^{13,8,1} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,1} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

GPDs in different x regions and GPDs at hadron facilities



$$-1 < x < \xi \quad (x + \xi < 0, x - \xi < 0)$$

$$\xi < x < 1 \quad (x + \xi > 0, x - \xi > 0)$$

$$-\xi < x < \xi \quad (x + \xi > 0, x - \xi < 0)$$

Quark distribution

Emission of quark with momentum fraction $x+\xi$
 Absorption of quark with momentum fraction $x-\xi$

$q\bar{q}$ (meson)-like distribution amplitude

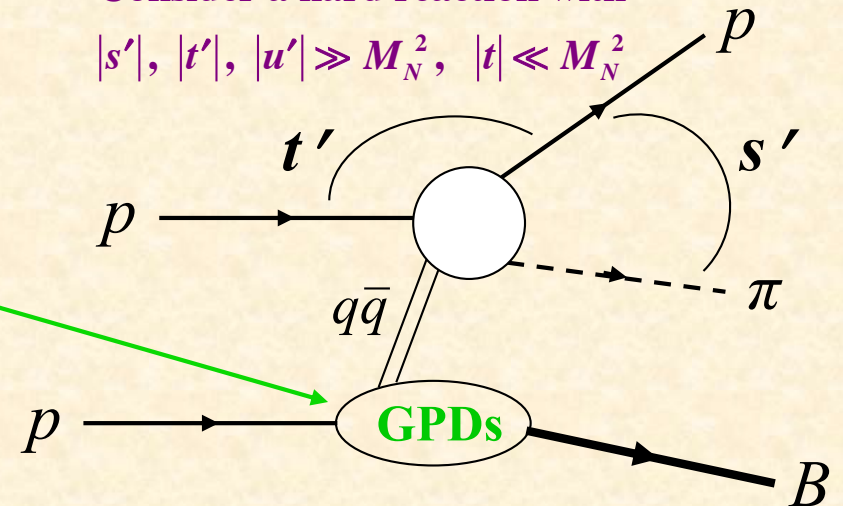
Emission of quark with momentum fraction $x+\xi$
 Emission of antiquark with momentum fraction $\xi-x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi-x$
 Absorption of antiquark with momentum fraction $-\xi-x$

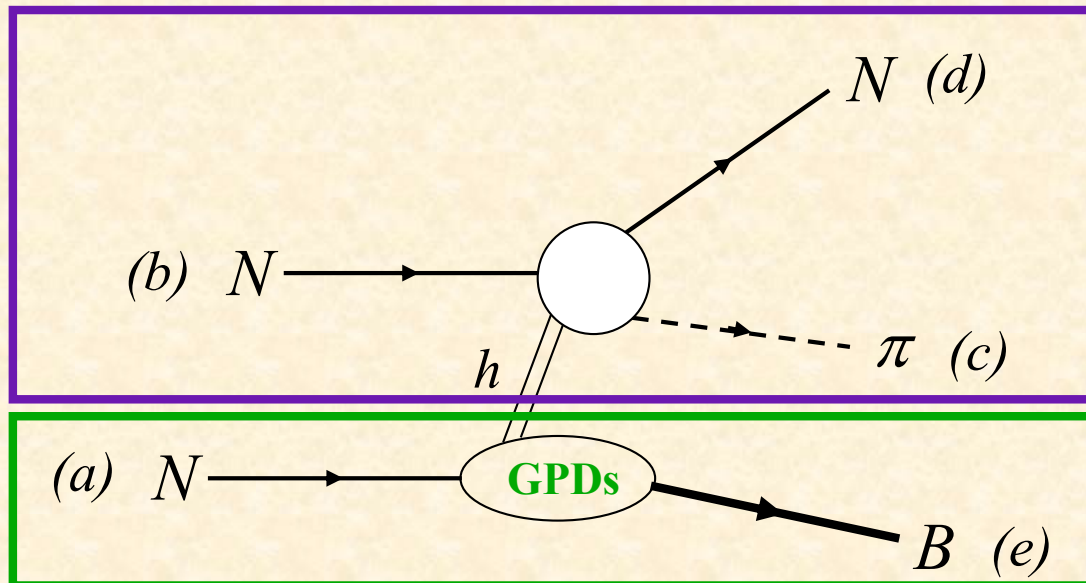
Consider a hard reaction with

$$|s'|, |t'|, |u'| \gg M_N^2, \quad |t| \ll M_N^2$$



Efremov-Radyushkin
 -Brodsky-Lepage (ERBL) region

Cross section estimates for $N + N \rightarrow N + \pi + B$



$\frac{d\sigma(s', t')}{dt'}$ so as to explain
BNL-AGS experimental data on
 $\pi + p \rightarrow \pi + p, \pi + p \rightarrow \rho + p$

This part is expressed by GPDs.

Purposes of our studies:

- (1) The ultimate purpose is to extract the GPDs in the ERBL region by measurements at hadron facilities in addition to lepton ones.
- (2) Since our work is the first one to point out the GPD studies at hadron reactions, we estimate the order of magnitude of cross sections simply by using meson-pole expressions of the GPDs.
→ For experimental feasibility studies.

Cross section estimate (ξ dependence)

$$\frac{d\sigma_{NN \rightarrow N\pi B}}{d\xi dt dt'} \propto \frac{d\sigma_{MN \rightarrow \pi N}}{dt'} \left[8(1 - \xi^2) \{H(x, \xi, t)\}^2 + 16\xi^2 H(x, \xi, t)E(x, \xi, t) - \frac{t}{m_N^2} (1 + \xi)^2 \{E(x, \xi, t)\}^2 \right. \\ \left. + 8(1 - \xi^2) \{\tilde{H}(x, \xi, t)\}^2 + 18\xi^2 \tilde{H}(x, \xi, t)\tilde{E}(x, \xi, t) - \frac{2t\xi^2}{m_N^2} \{\tilde{E}(x, \xi, t)\}^2 \right]$$

Skewness parameter: $\xi = \frac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

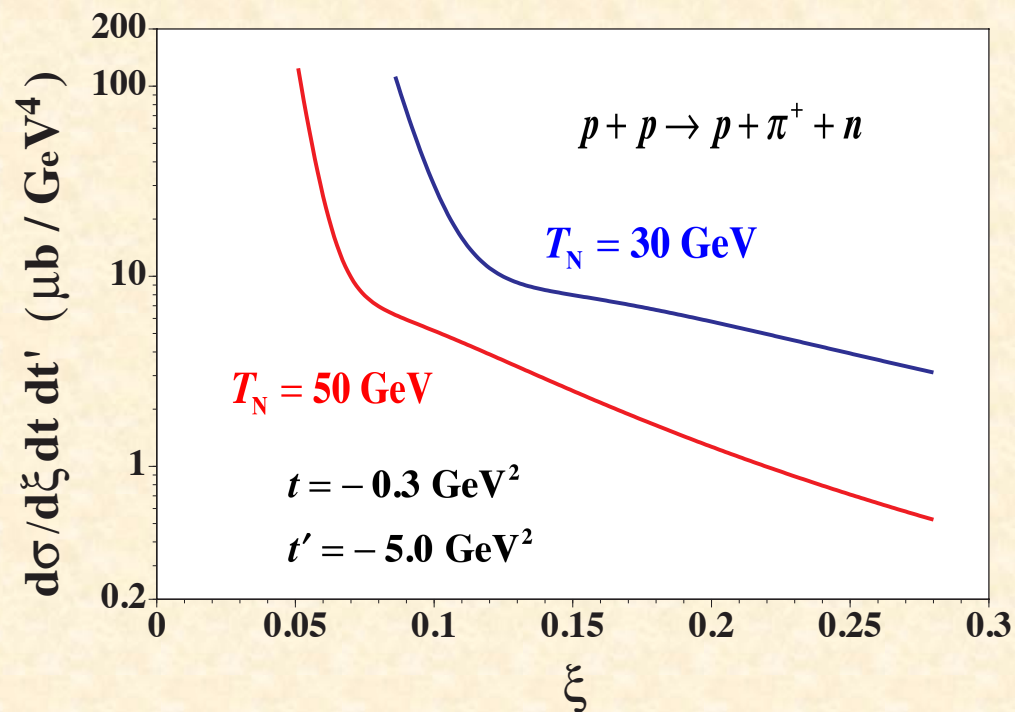
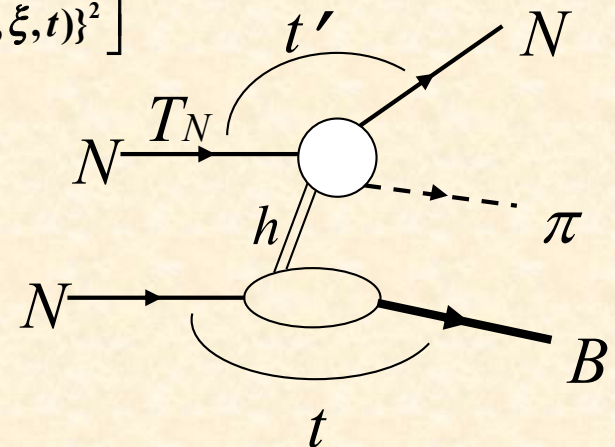
$\frac{d\sigma}{d\xi dt dt'}$ $\left(\frac{\mu\text{b}}{\text{GeV}^2} \right)$ as a function of ξ

at fixed $T_N = 30$ (50) GeV,

$t = -0.3 \text{ GeV}^2$, $t' = -5 \text{ GeV}^2$.

At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π - and ρ -like intermediate states.

For the details, please look at SK, M. Strikman, K. Sudoh, PRD 80 (2009) 074003.



Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re} \{ \tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t) \} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_\pi^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_5 q(z/2) | p(p) \rangle_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_5 q_u(z/2) | p(p) \rangle_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t)]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t)]$$

**T. Sawada, W.-C. Chang, SK, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.**

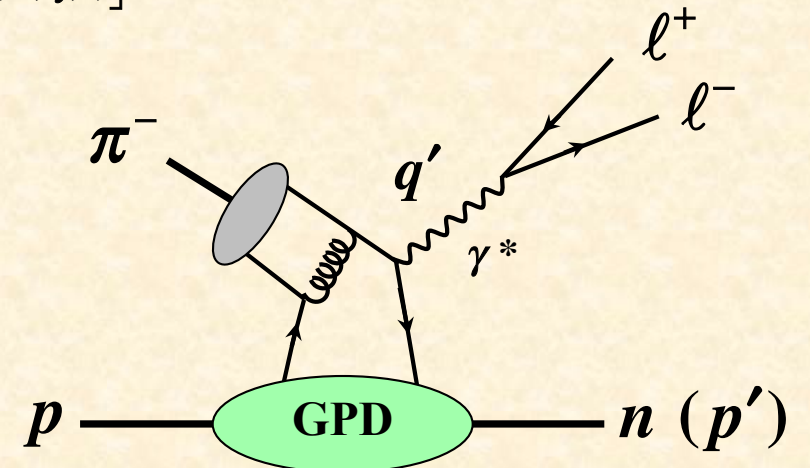
LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process

at J-PARC

JungKeun Ahn,¹ Sakiko Ashikaga,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niyama,¹² Hiroyuki Noumi,^{13,8,1} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,4} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

LoI for a J-PARC experiment

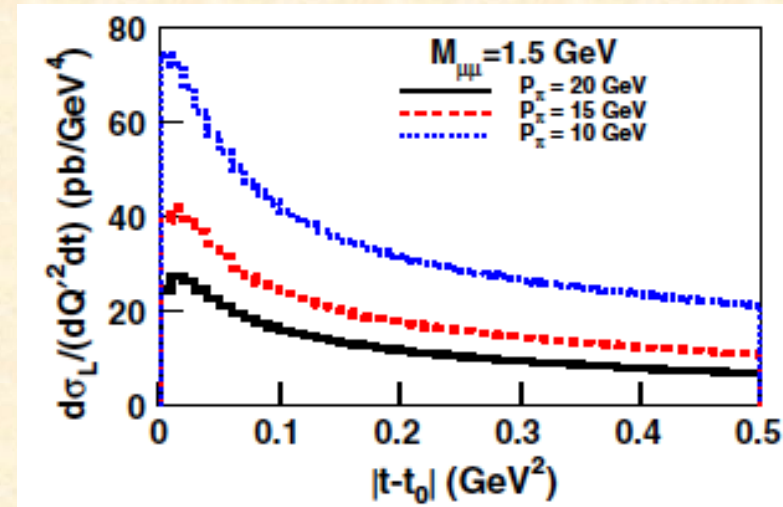
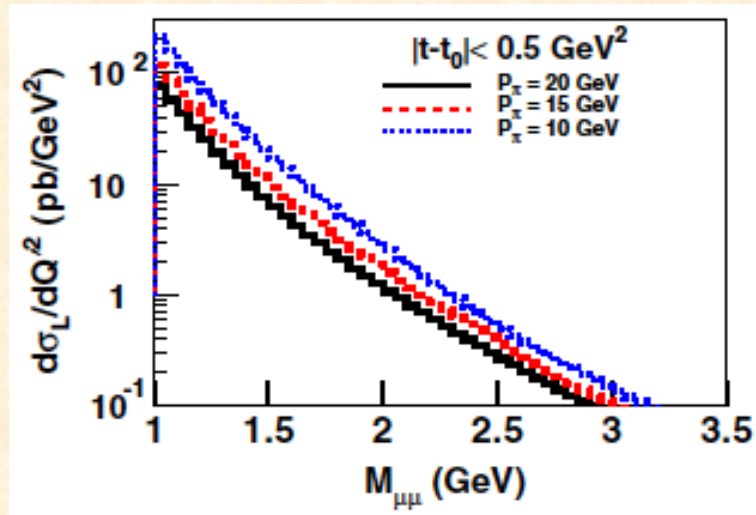


$$\pi^- (\bar{u}d) + p(uud) \rightarrow n(udd) + \gamma^* (\rightarrow \ell^+ \ell^-)$$

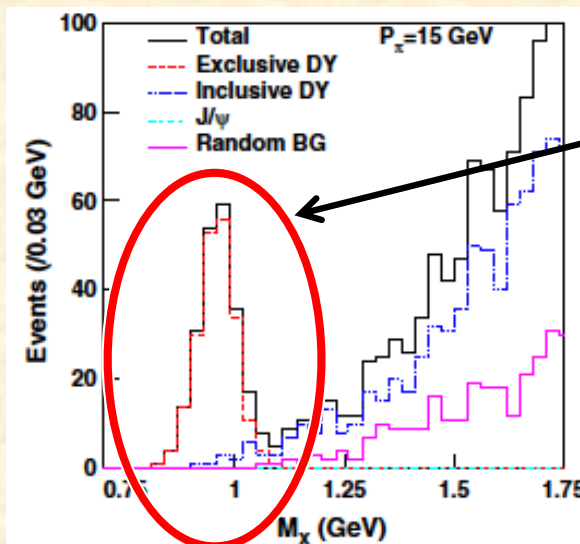
Expected Drell-Yan events at J-PARC

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$



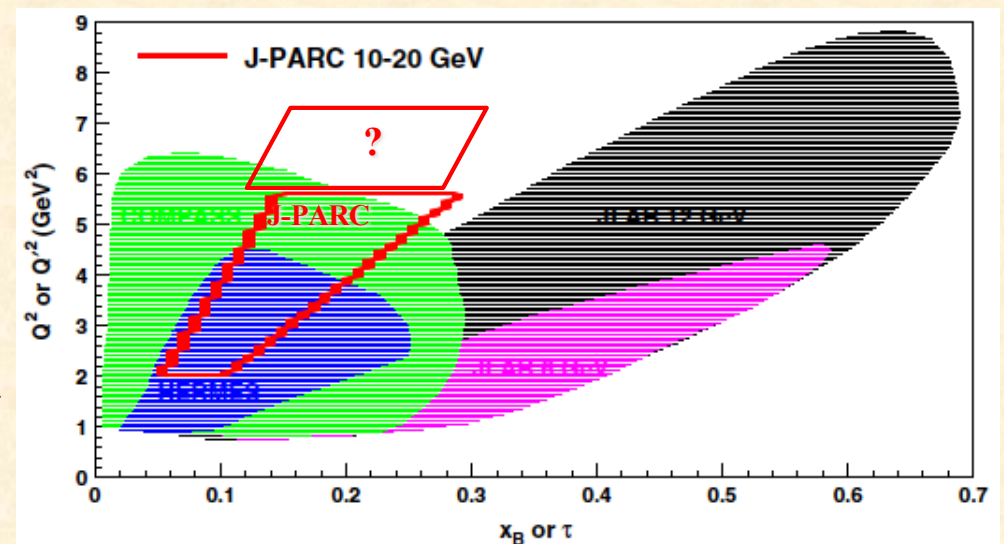
Missing mass



**Exclusive
Drell-Yan**

$$M_X^2 = (q + p - q')^2$$

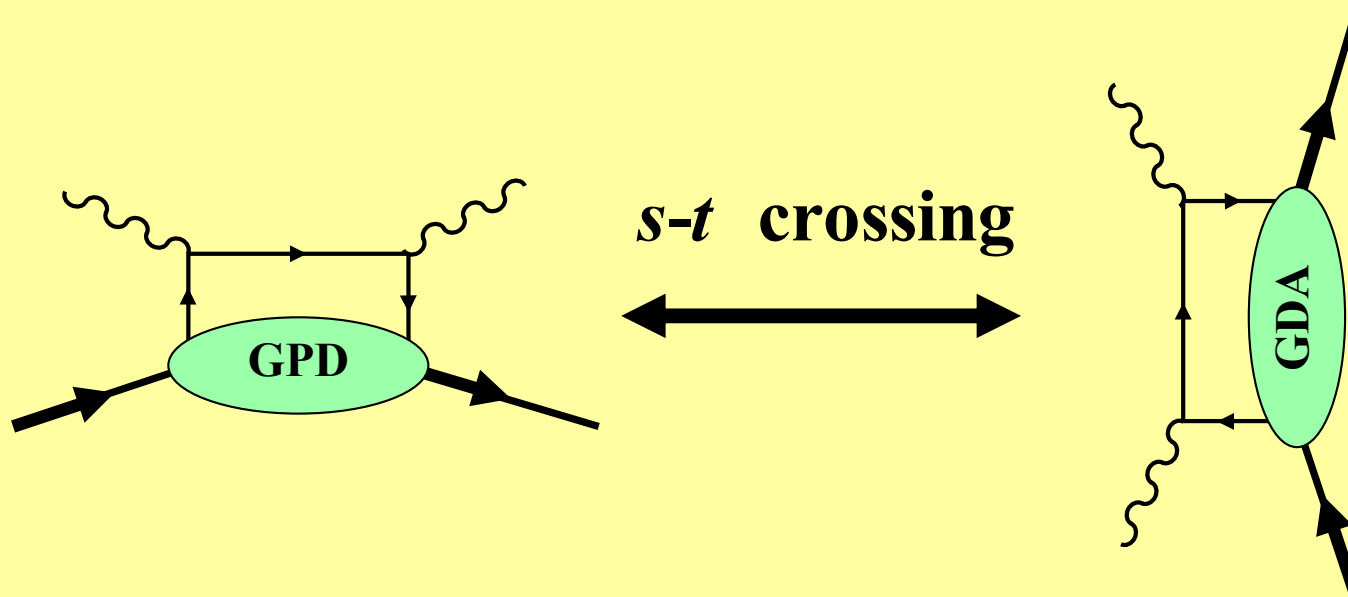
$$q = p_\pi, \quad p = p_p, \quad q = p_{\mu^+\mu^-}$$



Generalized Distribution Amplitudes (GDAs) and extraction of gravitational form factors from KEKB data

Spacelike GPDs

GDA = Timelike GPDs



SK, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

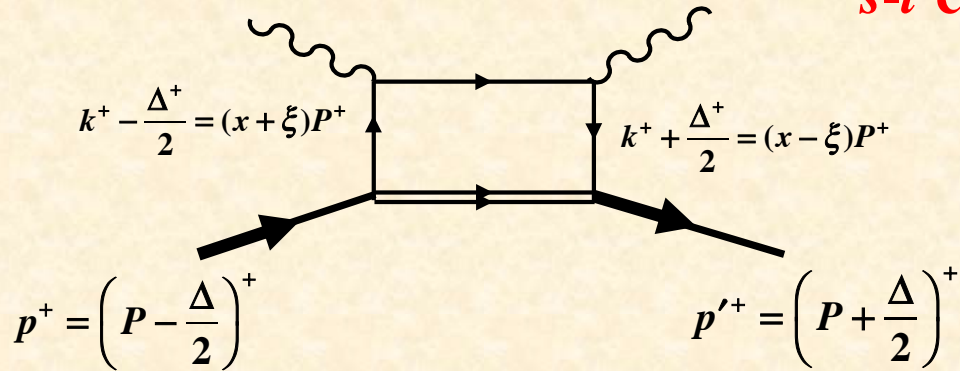
GPD $H_q^h(x, \xi, t)$ and GDA (= timelike GPD) $\Phi_q^{hh}(z, \zeta, W^2)$

GPD: $H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

$H_q^h(x, \xi, t)$



JLab / COMPASS

$P = \frac{p+p'}{2}, \quad \Delta = p' - p$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

\longleftrightarrow
s-t crossing

$\Phi_q^{hh}(z, \zeta, W^2)$

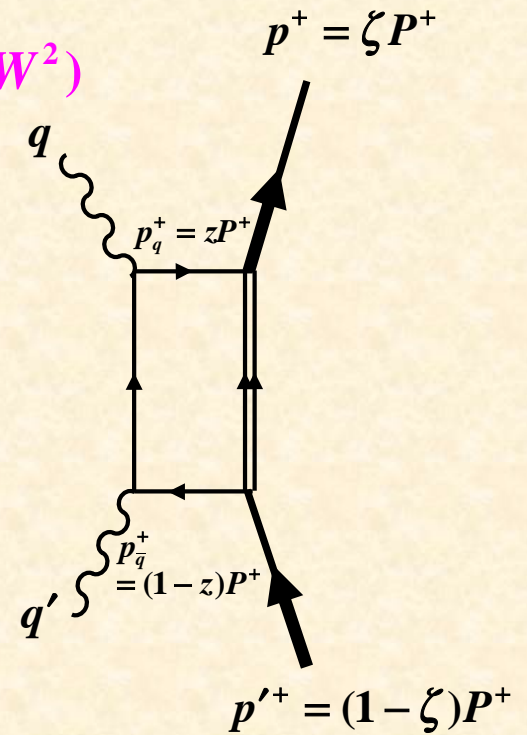
$z \Leftrightarrow \frac{1-x/\xi}{2}$
 $\zeta \Leftrightarrow \frac{1-1/\xi}{2}$
 $W^2 \Leftrightarrow t$

KEKB

Bjorken variable for γ^* : $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{p'^+} = \frac{1 + \beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$



Cross section for $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i \int d^4\xi e^{-i\xi \cdot q} \langle \pi(p) \pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

GDA (timelike GPD): $\Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) \pi(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle_{y^+=0, \vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1) \zeta (1-\zeta) F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{[1 + (s - 4m_\pi^2) / \Lambda^2]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

- Resonances: There exist resonance contributions to the cross section.

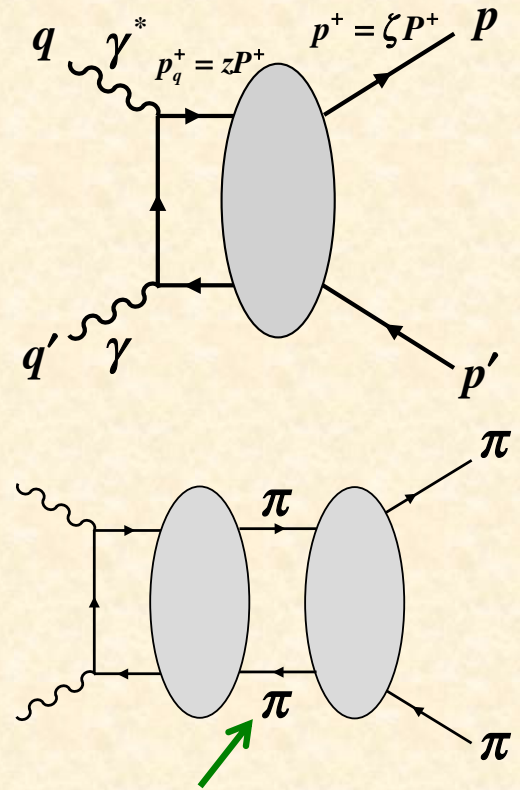
$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18 N_f z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$P_2(x) = \frac{1}{2} (3x^2 - 1)$$

$$\tilde{B}_{10}(W) = \text{resonance} [f_0(500), f_0(980)] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance} [f_2(1270)] + \text{continuum}$$

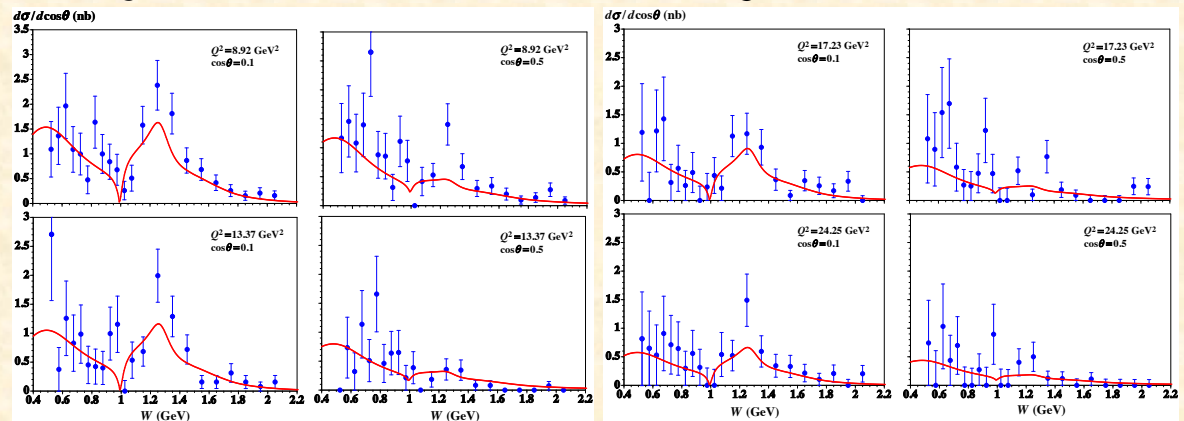
Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.



Including intermediate resonance contributions

$Q^2 = 8.92, 13.37 \text{ GeV}^2$

$Q^2 = 17.23, 24.25 \text{ GeV}^2$



Gravitational form factors and radii for pion

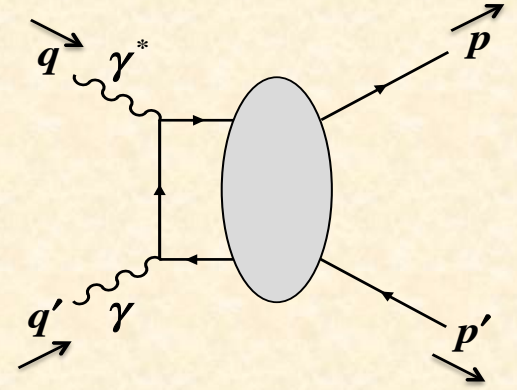
$$\int_0^1 dz (2z-1) \Phi_q^{\pi^0\pi^0}(z, \zeta, s) = \frac{2}{(P^+)^2} \langle \pi^0(p) \pi^0(p') | T_q^{++}(0) | 0 \rangle$$

$$\langle \pi^0(p) \pi^0(p') | T_q^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[(s g^{\mu\nu} - P^\mu P^\nu) \Theta_{1,q}(s) + \Delta^\mu \Delta^\nu \Theta_{2,q}(s) \right]$$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}, \Theta_{2,q}$: gravitational form factors for pion



See also Hyeon-Dong Son,
Hyun-Chul Kim, PRD90 (2014) 111901.

Analysis of $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ cross section

- ⇒ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z, \zeta, s)$
- ⇒ Timelike gravitational form factors $\Theta_{1,q}(s), \Theta_{2,q}(s)$
- ⇒ Spacelike gravitational form factors $\Theta_{1,q}(t), \Theta_{2,q}(t)$
- ⇒ Gravitational radii of pion

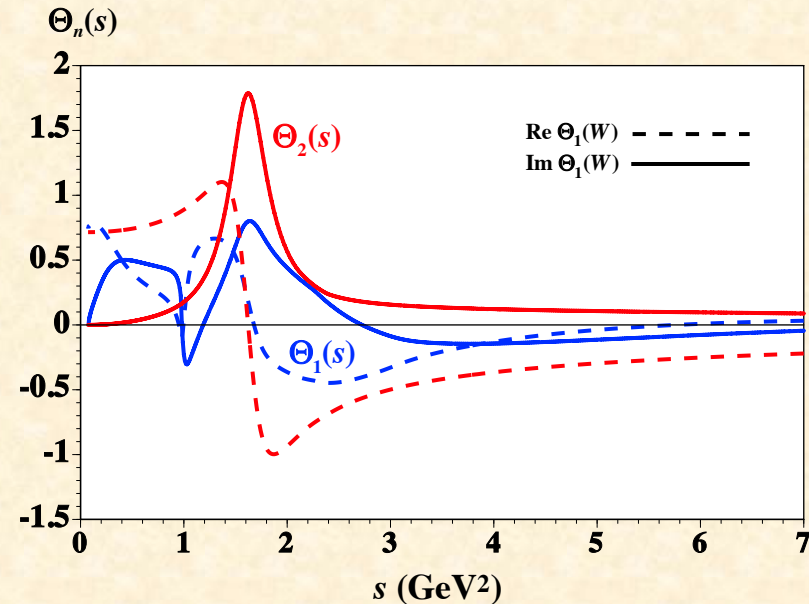
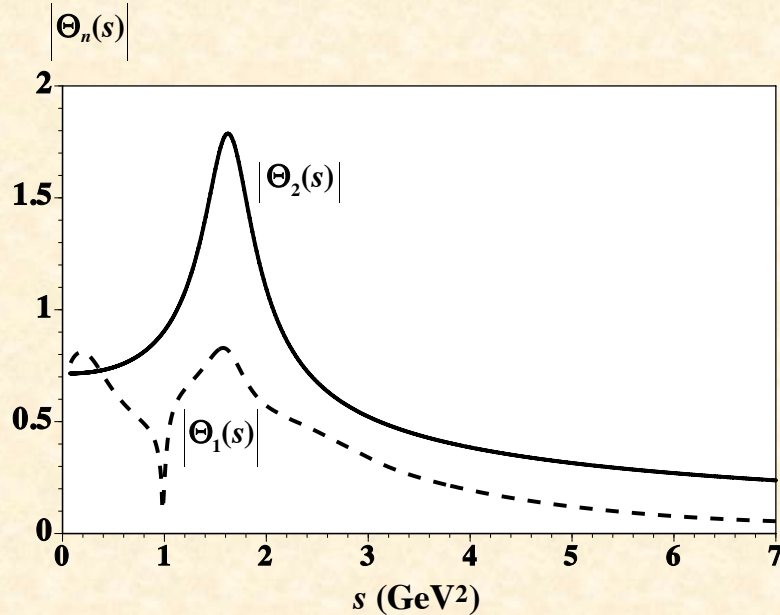
Gravitational form factors:

Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009;

Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;

K. Tanaka, JHEP 01 (2019) 120.



Spacelike gravitational form factors and radii for pion

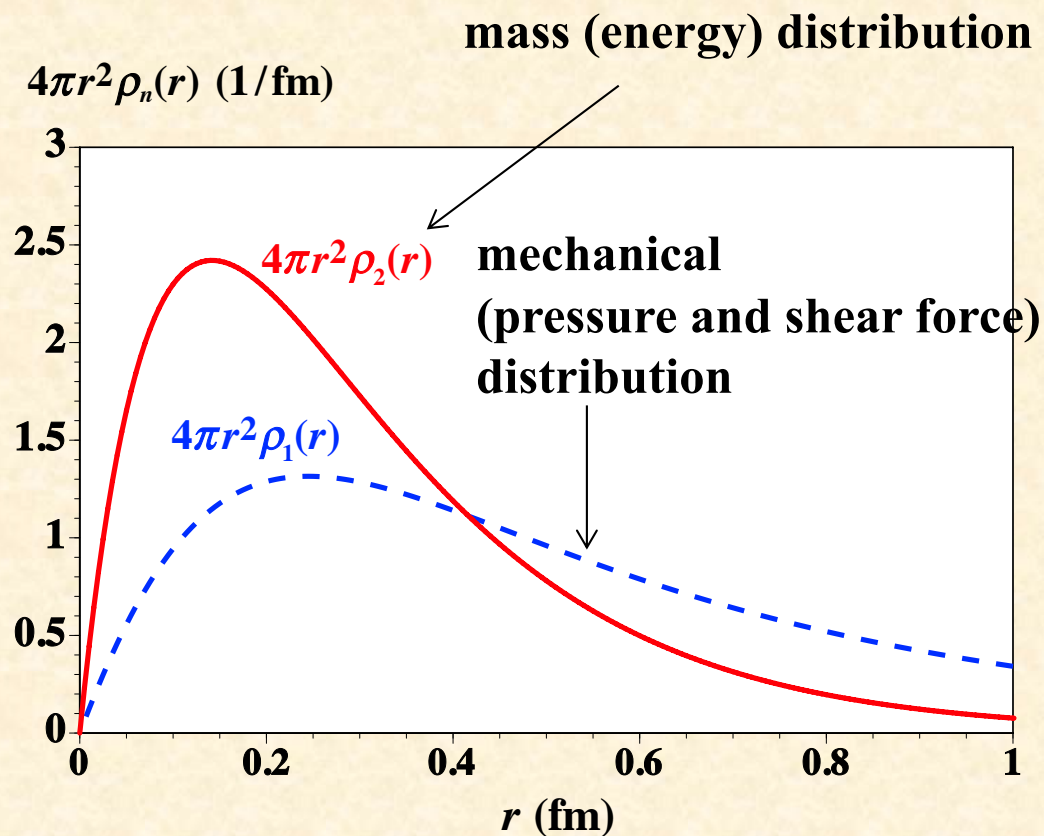
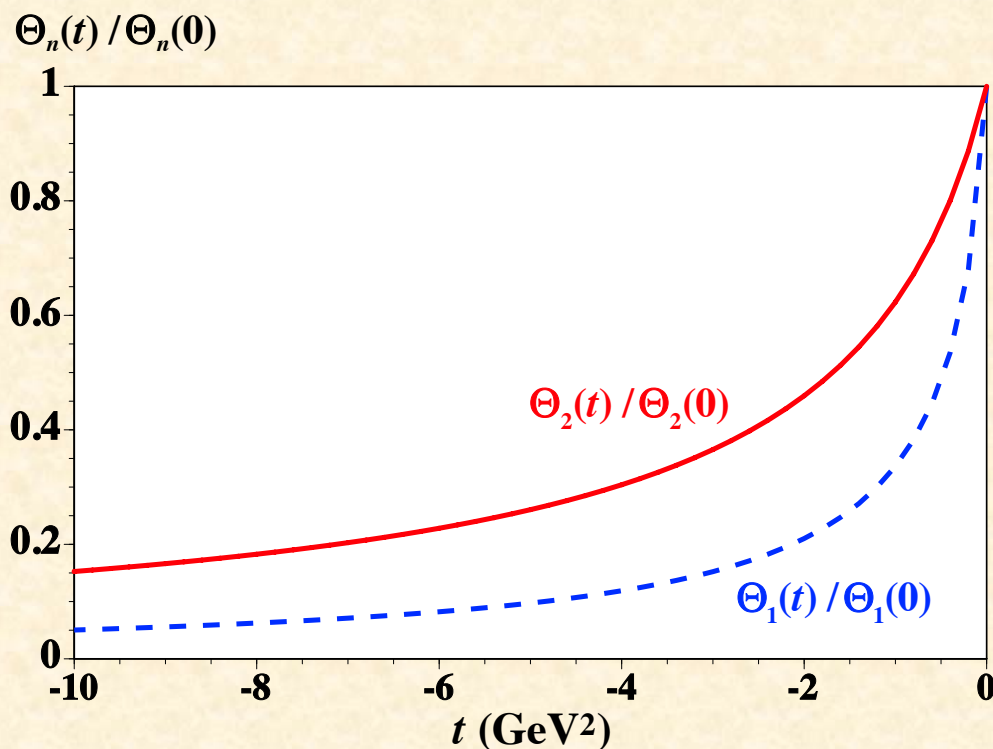
$$F(s) = \Theta_1(s), \Theta_1(s), \quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im} F(s)}{\pi(s-t-i\epsilon)}, \quad \rho(r) = \frac{1}{(2\pi)^3} \int d^3 q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2} \frac{1}{r} \int_{4m_\pi^2}^{\infty} ds e^{-\sqrt{s}r} \text{Im} F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$



GPDs for exotic hadrons

**(If transition GPDs could be studied,
this exotic-hadron project becomes realistic.)**

**H. Kawamura and SK,
Phys. Rev. D 89 (2014) 054007.**

Constituent counting rule for exotic hadrons:

H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;

W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006.

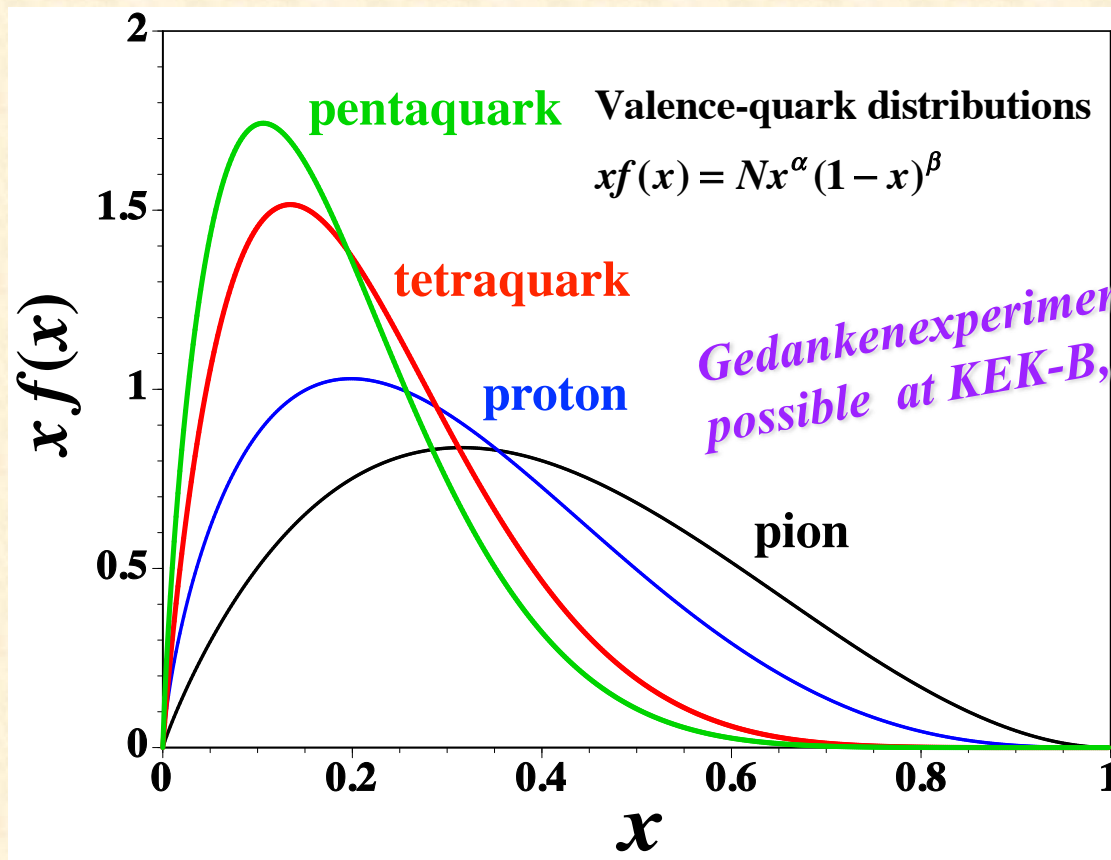
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

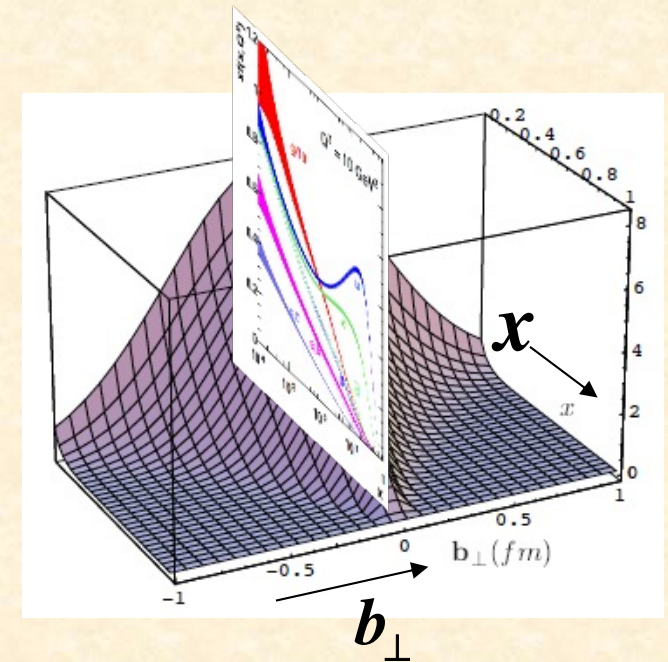
M. Guidal, M.V. Polyakov,
A.V. Radyushkin, M. Vanderhaeghen,
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent counting rule at $x \rightarrow 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx x f(x)$

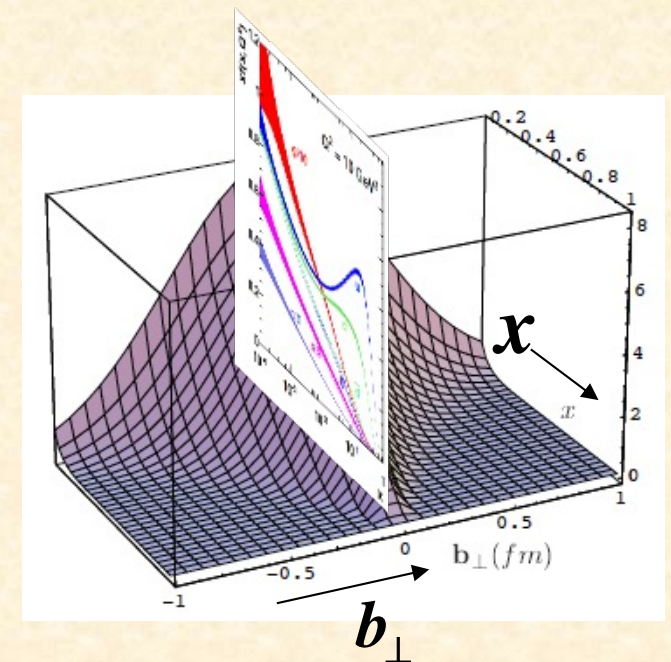
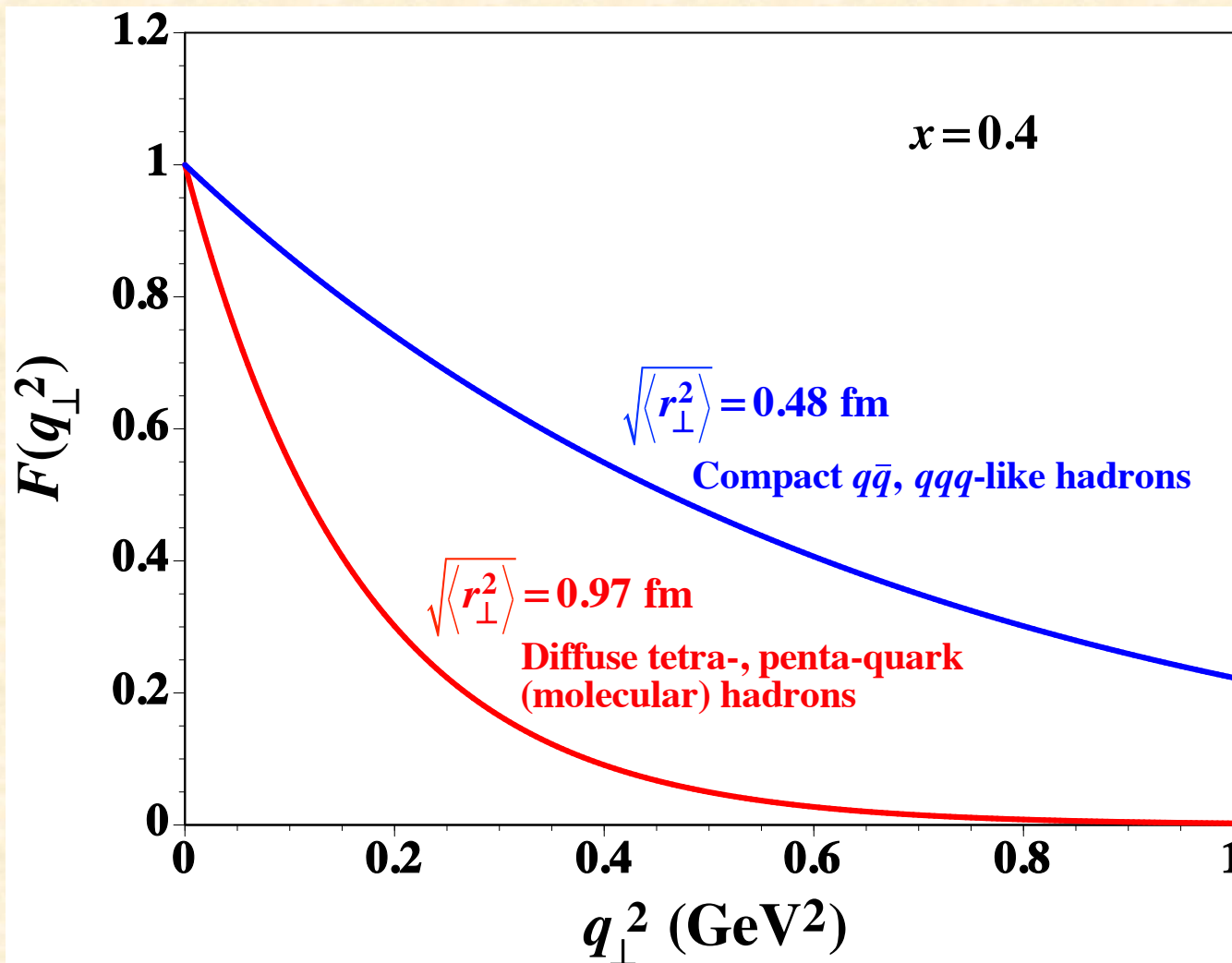


Gedankenexperiment, but possible at KEK-B, ILC, ...



Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$



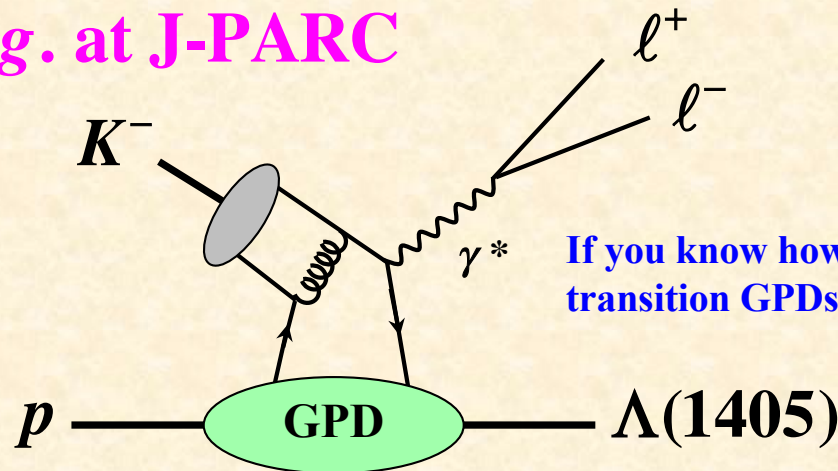
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed quantity = GDAs at KEKB, Linear Collider

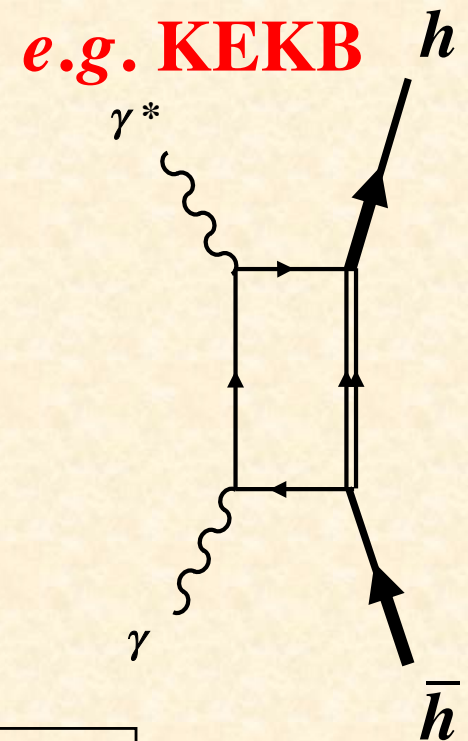
e.g. at J-PARC



If you know how to handle this kind of transition GPDs $N \rightarrow \Lambda$, please inform me.

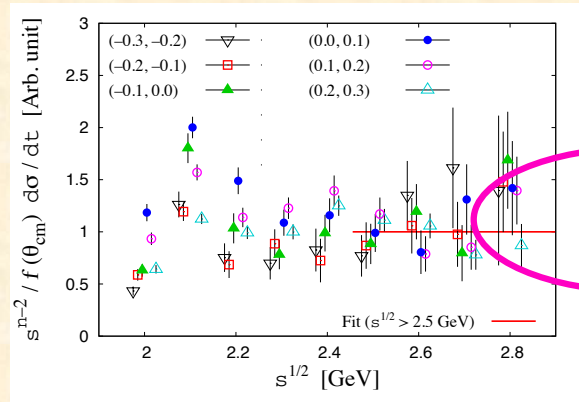
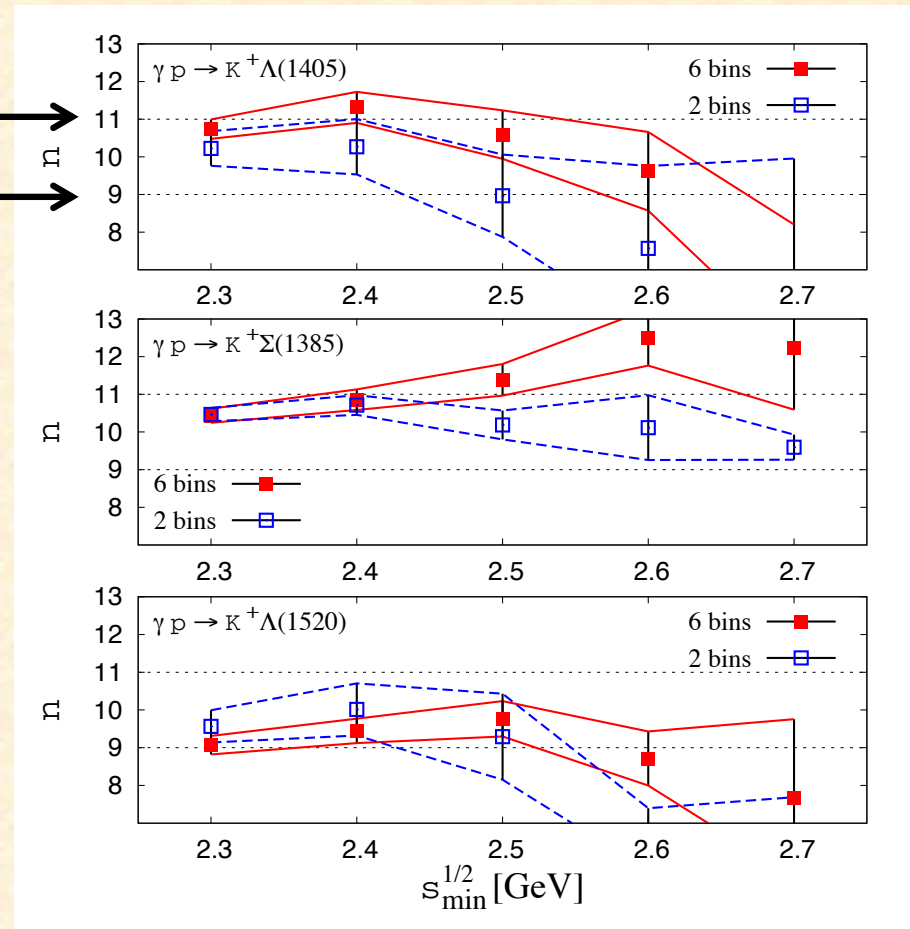
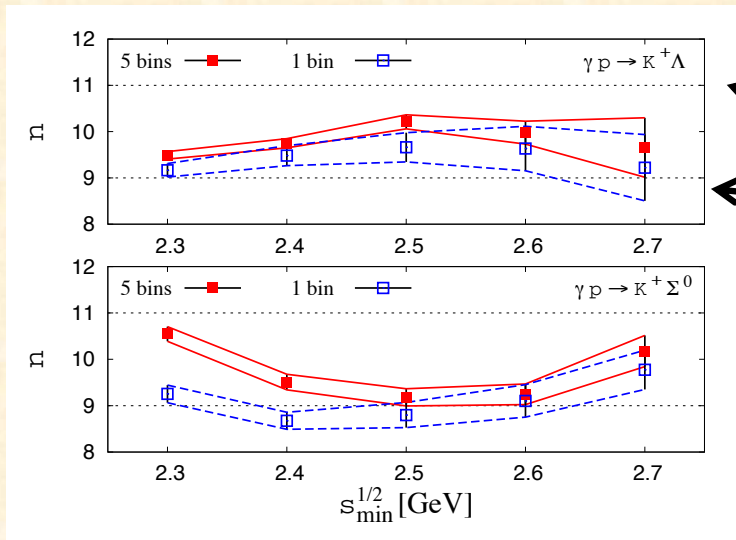
$$K^- (\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate



See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

JLab hyperon productions including $\Lambda(1405)$



Range of
12 GeV JLab!

- Λ . $\Lambda(1520)$ and Σ seem to be consistent with ordinary baryons with $n = 3$.
- $\Lambda(1405)$ looks penta-quark at low energies but $n \sim 3$ at high energies???
- $\Sigma(1385)$: $n = 5$???

→ In order to clarify the nature of $\Lambda(1405)$ [$qqq, \bar{K}N, qqqq\bar{q}$], the JLab 12-GeV experiment plays an important role!

W.-C. Chang, SK, T. Sekihara,
PRD 93 (2016) 034006.

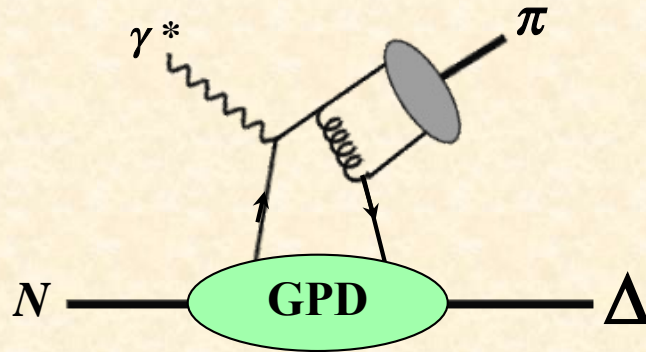
Transition GPDs for exotic hadrons

S. Diehl *et al.* (SK, 15th author),

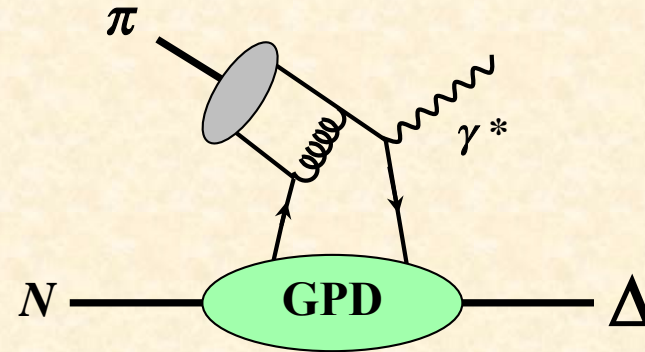
arXiv:2405.15386, submitted for Eur. Phys. J. A

Transition GPDs from N to Δ

JLab / EIC



J-PARC

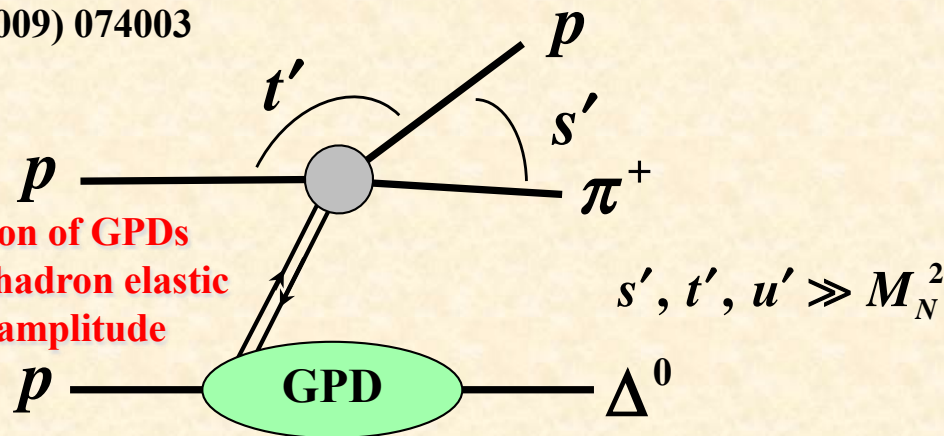


In future

$$K^- + p \rightarrow \Lambda_{1405} + \gamma^* ?$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003

Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude



J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

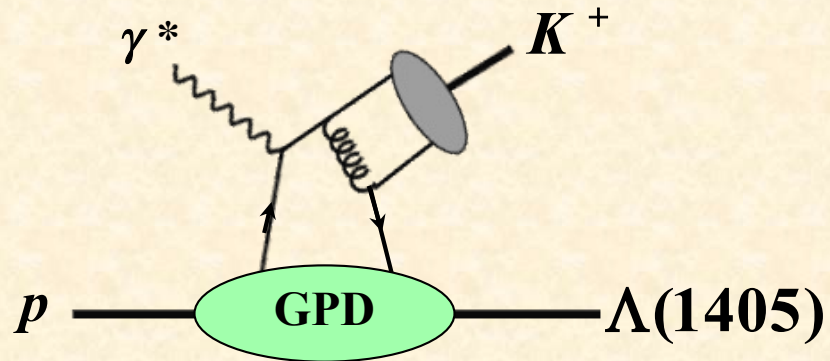
$$\pi + N \rightarrow \gamma + \gamma + N'$$

$$h + M_B \rightarrow h' + \gamma + M_D$$

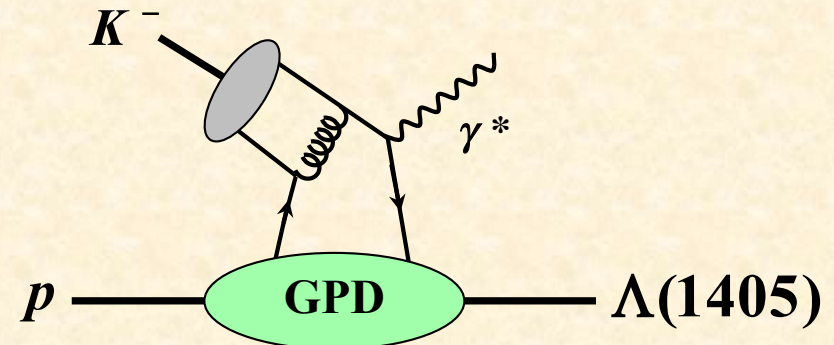
$$h + M_B \rightarrow h' + M_C + M_D$$

Transition GPDs for exotic hadrons

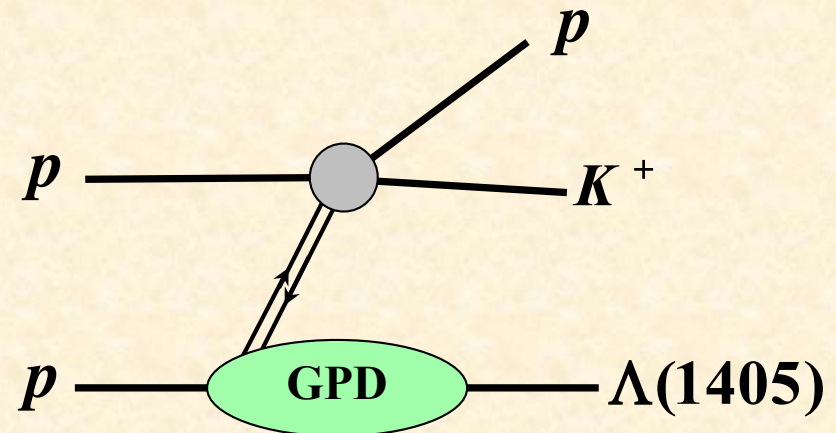
JLab / EIC



J-PARC



However, there is no theoretical study on the $N \rightarrow \Lambda(1405)$ transition GPDs at this stage.



Future prospects on GPD projects

High-energy hadron physics experiments

CERN
(LHC, **COMPASS/AMBER**,
LHeC, FCC, CLIC, nuSTORM)

Fermilab
(SeaQuest, **SpinQuest, DUNE**)

GSI (FAIR)

KM3NeT

JINR (NICA)

Baikal GVD

IHEP (BEPC, CEPC)

IMP (HIAF, EicC)

BNL
(RHIC, **EIC**)

JLab

ILC

KEK
(**KEKB, J-PARC**)

IceCube



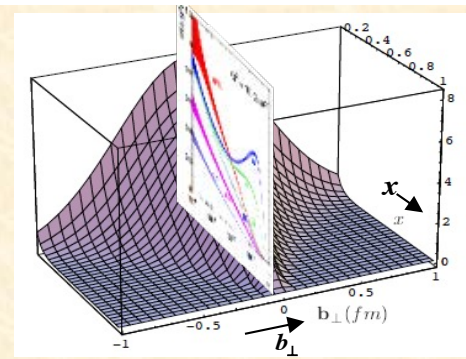
Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

By hadron tomography

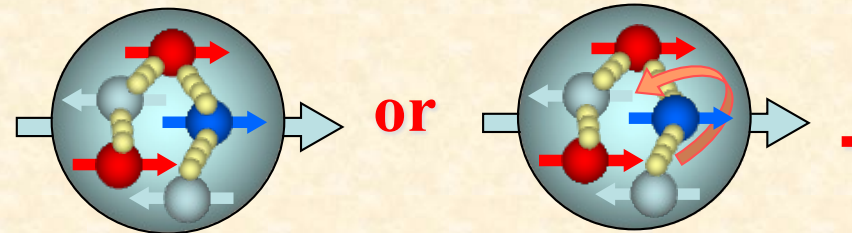


3D view
of hadrons

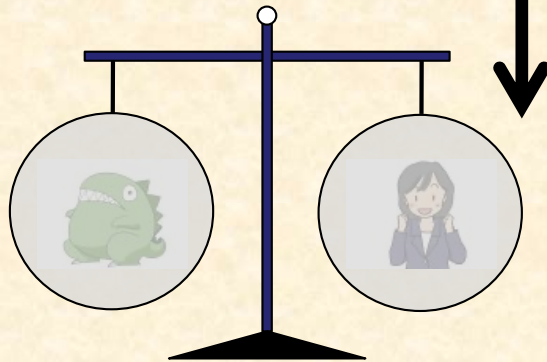


Origin of nucleon spin

By the tomography, we determine



Exotic hadrons



By tomography,
we determine

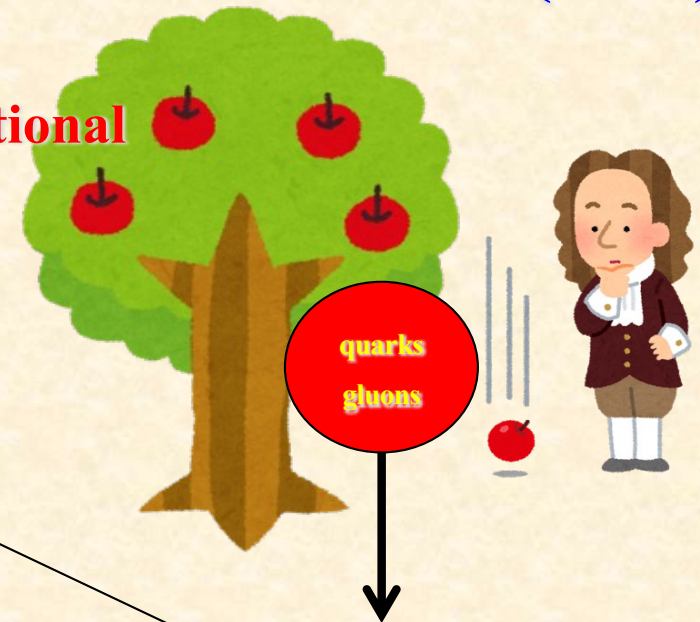


or



Origin of gravitational source (mass)

By tomography,
we determine gravitational
sources in terms of
quarks and gluons.



Summary on GPDs

Hadron-tomography and gravitational form factors

- **Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom**
- **Puzzle to find the origin of nucleon spin**
- **Exotic hadron candidates could be studied in the same tomography method.**
- **There are world-wide lepton and hadron accelerator facilities which has been used and could be used in future for our studies. In addition to the JLab/EIC type electron scattering projects, the GPD studies are possible by neutrino and hadron beam facilities and e^+e^- colliders.**

Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.



The End

The End