Transport coefficients in the pre-equilibrium stage

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INT, August 2024

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Outline

- Initial stage of heavy ion collision:
  - Glasma: color fields
  - Bottom-up thermalization: QCD kinetic theory
- Transport: jets and heavy quarks
  - Glasma: field correlators or Wong's equations
    - D. Avramescu, V. Greco, A. Ipp, D. I. Müller and M. Ruggieri, [arXiv:2303.05599 [hep-ph]]
    - D. Avramescu, V. Greco, T.L., H. Mäntysaari, D. I. Müller, in preparation
  - Bottom-up: kinetic theory calculations K. Boguslavski, A. Kurkela, T. L., F. Lindenbauer, J. Peuron
    - arXiv:2303.12520 [hep-ph]
    - arXiv:2303.12595 [hep-ph]
    - arXiv:2312.11252 [hep-ph]
    - arXiv:2312.00447 [hep-ph]

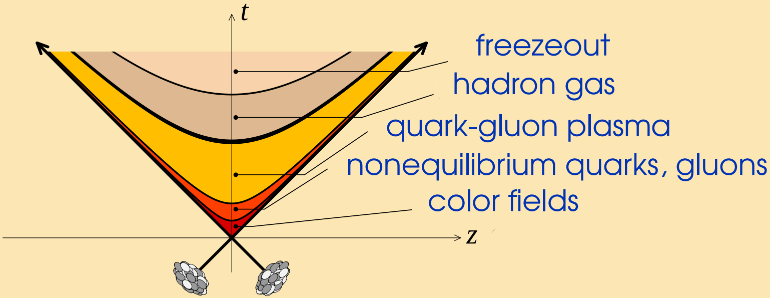
Goal: understand interactions of jets & heavy quarks in pre-equilibrium phase

All results here: boost invariant expansion, transversally infinite system

Heavy ion collision in spacetime

The purpose in heavy ion collisions: to create QCD **matter**,
i.e. system that is large and lives long
compared to the microscopic scale

$$t \gg \frac{1}{T} \qquad L \gg \frac{1}{T} \qquad T > 200\text{MeV}$$



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Concentrate here on the **earliest stage**: glasma and thermalization

Gluon saturation and glasma

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_\mathrm{s} \gg \Lambda_{\mathrm{QCD}}$.

Gluon saturation, Glass and Glasma

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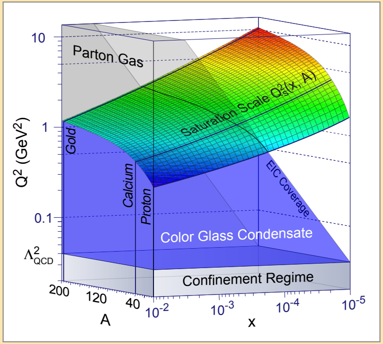
▼

p $\sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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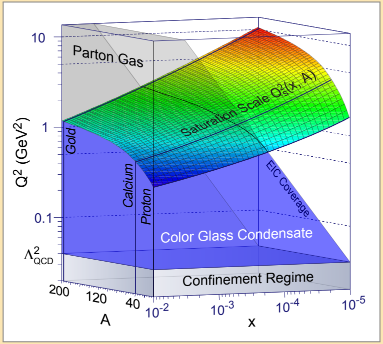

CGC: Effective theory for wavefunction of nucleus

- Large x = color charge ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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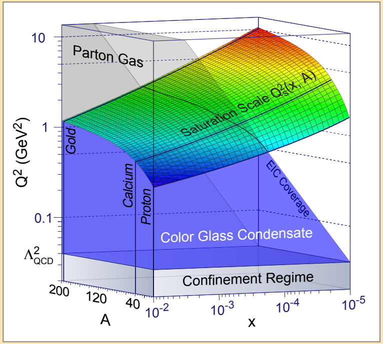

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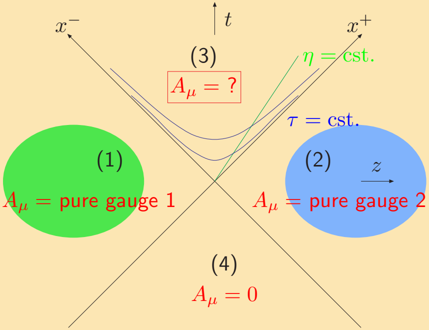
Glasma: field configuration of two colliding sheets of CGC. (Here $y \sim \ln \sqrt{s}$)

How to obtain intitial glasma fields

Now let **two** dense color field systems collide



Need LC gauge fields $A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x}) \partial_i V^\dagger_{(1,2)}(\mathbf{x})$

$V(\mathbf{x})$ = Wilson line for nuclei (1) and (2) (from ρ)

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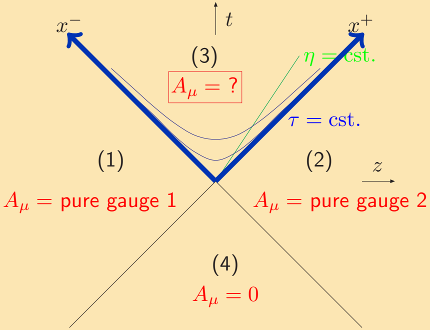

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$\tau = 0$: match using $[D_\mu, F^{\mu\nu}] = J^\nu$:

$$
\begin{aligned}
A^i\Big|_{\tau=0} &= A^i_{(1)} + A^i_{(2)} \\
A^\eta|_{\tau=0} &= \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}] \\
A^\tau &= 0 \text{ gauge choice}
\end{aligned}
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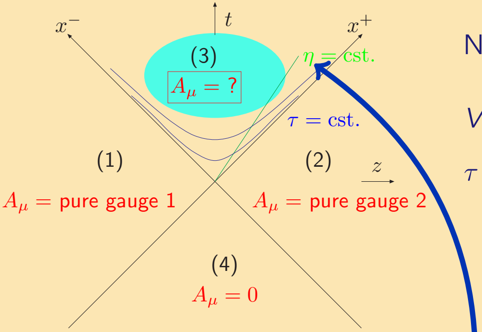

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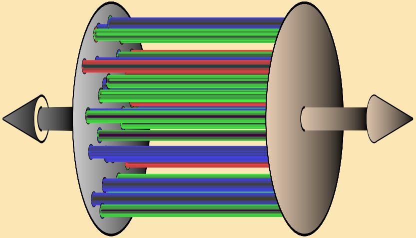
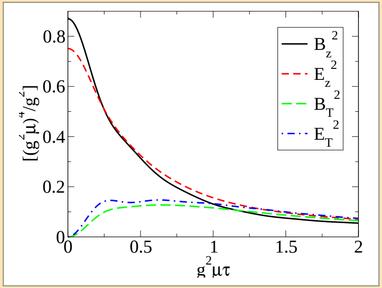
\end{aligned}
$$

$\tau > 0$ Numerical **CYM** or approximations

This is the **glasma** field $\Longrightarrow$ Then average over initial Wilson lines.

Initial glasma fields





- Initial condition is longitudinal E and B field,
  at $\tau \sim 1/Q_s$ evolves to $E_z^2 \sim B_z^2 \sim 2E_x^2 \sim 2B_x^2 \sim 2E_y^2 \sim 2B_y^2$
- Depend on transverse coordinate
  with correlation length $1/Q_s$ $\Longrightarrow$ gluon correlations
- Fix gauge, Fourier-decompose: Gluons with $p_T \sim Q_s$
- Boost-invariant $A_\mu(x)$ $\Longrightarrow$ anisotropic gluons $\langle p_z \rangle \ll \langle p_T \rangle$

Bottom-up thermalization

Bottom-up thermalization

Weak coupling QCD description of Glasma $\Longrightarrow$ QGP Baier, Mueller, Schiff, Son `hep-ph/0009237`

3 stages

1. Overoccupied, classical field stage $(0 \to \star)$: growing anisotropy of hard $\sim Q_s$ modes
2. Bath of soft particles develops $(\star \to \bullet)$
3. Radiative breakup of hard particles $(\bullet \to \blacktriangledown)$

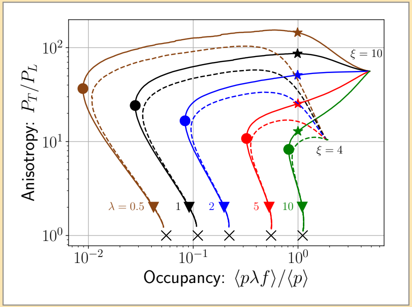
$$\tau_{\text{BMSS}} = \alpha_s^{-13/5} Q_s^{-1}$$

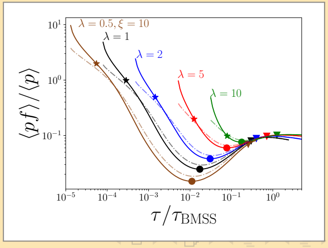
Can be tracked with AMY kinetic theory:

$$-\frac{d}{d\tau} f_{\mathbf{p}} = \mathcal{C}^{2\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}^{1\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}^{\text{exp}}[f_{\mathbf{p}}].$$

Attractor: different initial conditions converge

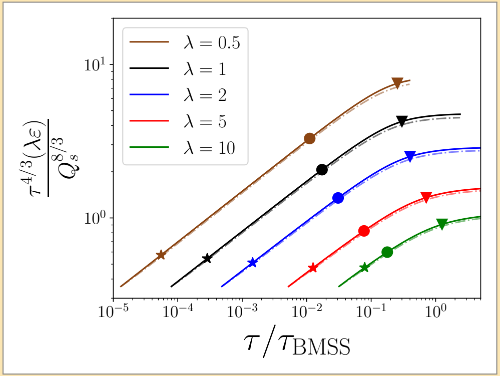
(ξ: initial anisotropy, $\lambda = 4\pi N_c \alpha_s$)

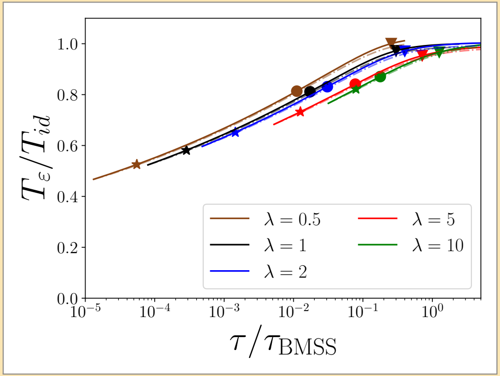




Approach to hydro

- Bjorken hydro $\varepsilon \sim 1/\tau^{4/3}$
- Most of pre-equilibrium: $\varepsilon \sim 1/\tau$

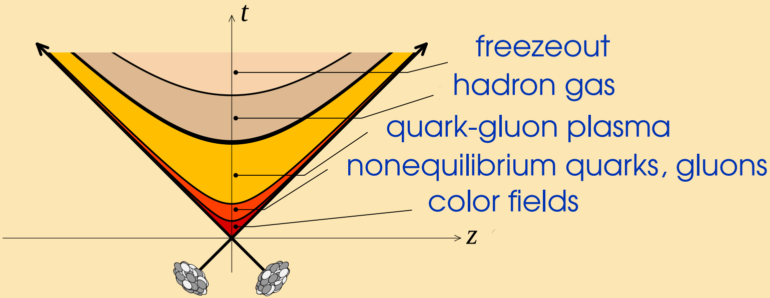




- T_{id}= bkwd extrapolated ideal hydro
- $T_\varepsilon \sim \sqrt[4]{\varepsilon}$

Hard probes of pre-equilibrium phase

Hard probes in pre-equilibrium

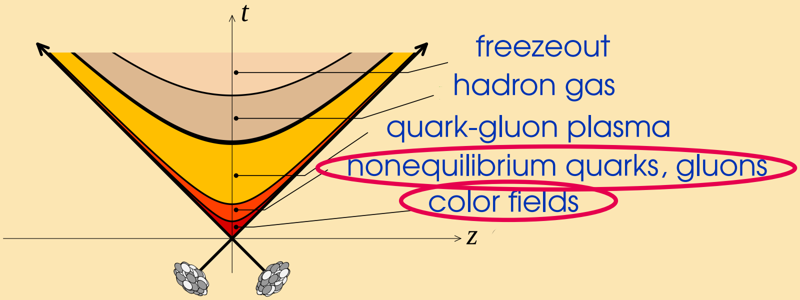


- Timescales for hard $M \sim m_c, p_T$ probes:

$$1/M \ll 1/Q_s \ll t_{\text{therm}}$$

- Hard probes $M \sim m_c, p_T$ created first $\Longrightarrow$ cannot neglect pre-equilibrium
- Even if thermalization is quick, pre-equilibrium is hot, dense $\Longrightarrow$ large effect

Hard probes in pre-equilibrium



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- Even if thermalization is quick, pre-equilibrium is hot, dense $\Longrightarrow$ large effect

Independent scatterings?

- Do hard probes undergo independent multiple scatterings?
- If yes, parametrize medium by

$$\left.\begin{array}{c}\hat{q}\\ \kappa\end{array}\right\} = \frac{d\langle q_\perp^2\rangle}{dt} \quad \left\{\begin{array}{c}\text{jet } (p=\infty)\\ \text{H.Q. } (m=\infty)\end{array}\right.$$

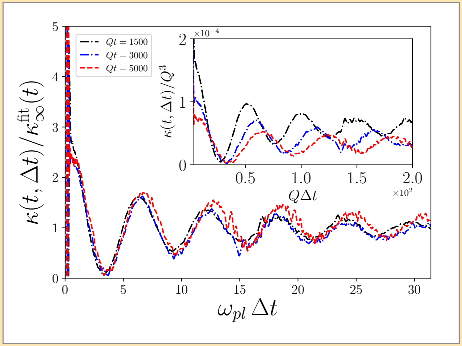
- In kinetic theory yes, by construction

Not obvious in glasma!

E.g. isotropic overoccupied YM

K. Boguslavski, A. Kurkela, T.L. and J. Peuron,

[arXiv:2005.02418 [hep-ph]]



Overoccupation of IR modes ($k \sim m_D$)

$\Longrightarrow$ modifies "p diffusion" picture

(Thermal would be κ =cst)

Hard probes in glasma

Classical particles in CYM: Wong's equations

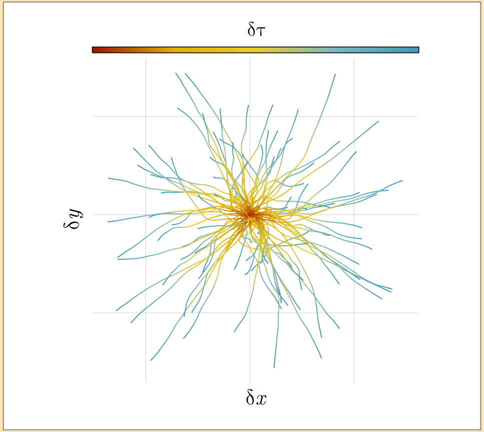
- $p \to \infty$ or $m \to \infty$:
  trajectory does not depend on field
  $\Longrightarrow$ compute Δp from field correlators
  A. Ipp, D. I. Müller and D. Schuh,
  [arXiv:2009.14206 [hep-ph]]
- In general: Wong's equations

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \frac{p^\mu}{p^\tau},$$

$$\frac{\mathrm{D}p^\mu}{\mathrm{d}\tau} = \frac{g}{T_R}\,\mathrm{tr}\{QF^{\mu\nu}\}\frac{p_\nu}{p^\tau}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}\tau} = -\mathrm{i}g[A_\mu, Q]\frac{p^\mu}{p^\tau}$$

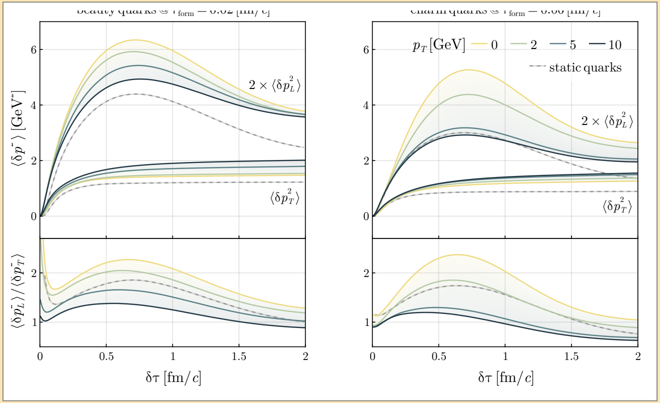
$\Longrightarrow$ solve numerically



D. Avramescu, V. Greco, A. Ipp, D. I. Müller and M. Ruggieri, [arXiv:2303.05599 [hep-ph]]

Momentum broadening in glasma

D. Avramescu, V. Greco, A. Ipp, D. I. Müller and M. Ruggieri, [arXiv:2303.05599 [hep-ph]]



(Note typical glasma: anisotropy between L,T w.r.t. beam)

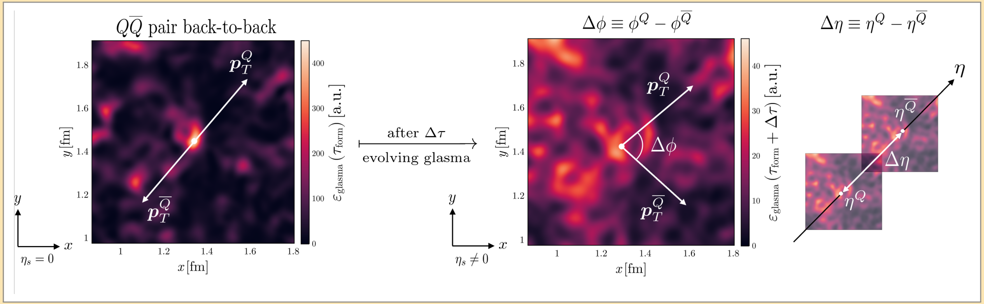
- **Coherent**, not independent scatterings (which would be $\delta p^2 \sim \tau$)
- Not meaningful to extract κ, $\hat{q}$ and do H.Q. diffusion/jet quenching

$\Longrightarrow$ Directly simulate physical observables in glasma

Angular correlations for heavy quark pairs

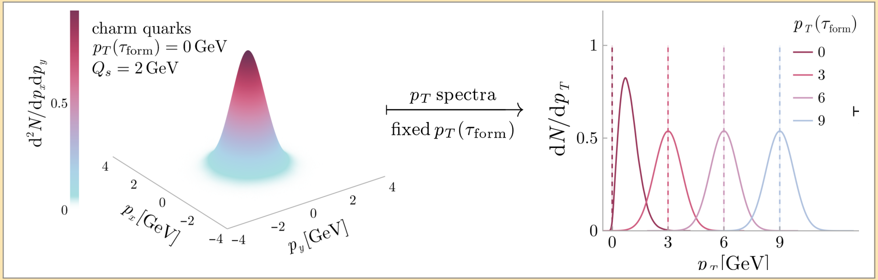
D. Avramescu, V. Greco, T.L., H. Mäntysaari, D. I. Müller, in preparation

- Motivation: prospect of measuring $D\bar{D}$ azimuthal correlations.
- Presumably flow and non-flow contribute
- Here: medium modification to non-flow back-to-back correlations
- Initialize $Q\bar{Q}$ pair with $\Delta\phi = \pi$, $\Delta\eta = 0$, follow with Wong's equations

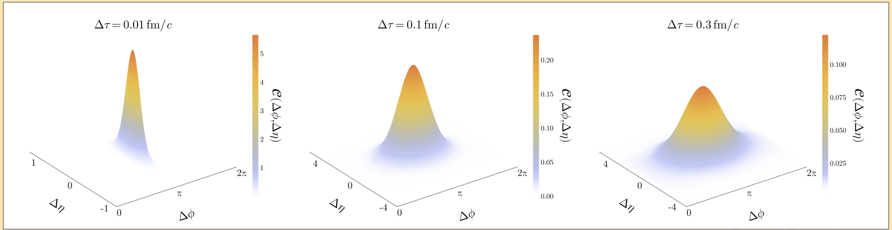


Momentum broadening

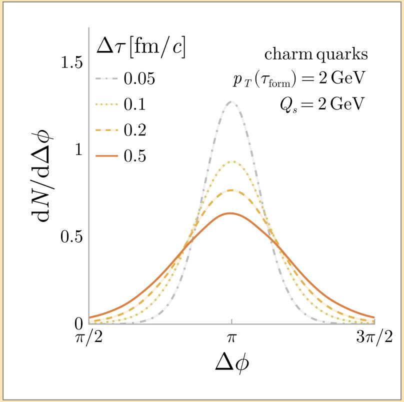
Momentum of quark broadens as a $\sim$ Gaussian:

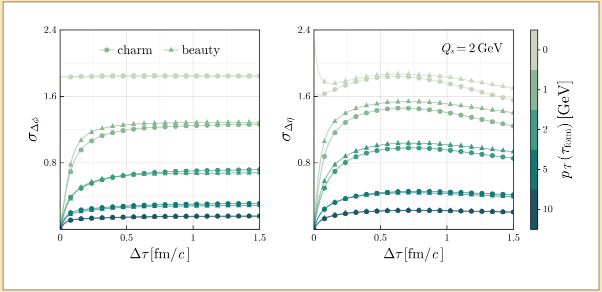


Which results in $\Delta\phi$, $\Delta\eta$ decorrelation with time:



Azimuthal decorrelation



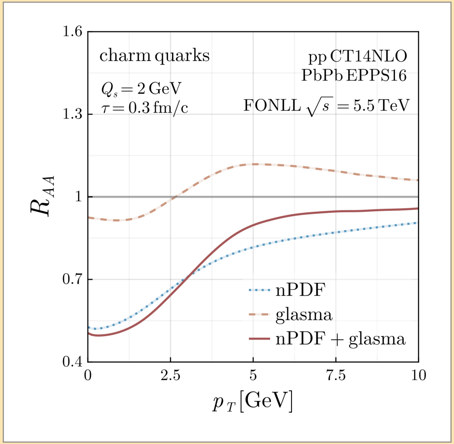


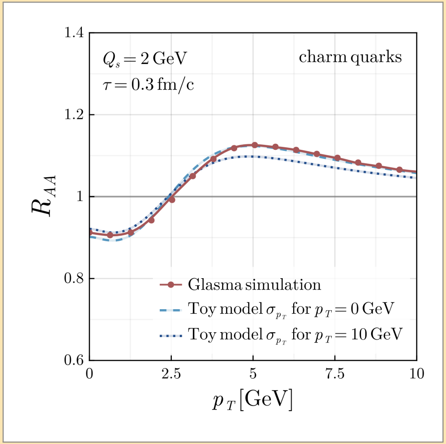
- Significant $\Delta\phi$ broadening
- Widths $\sigma_{\Delta\phi}, \sigma_{\Delta\eta}$ naturally decrease with p_T

  (δp_T^2 roughly p_T-independent)

Nuclear modification ratio

Also calculate R_{AA} with FONLL spectrum + glasma (with or without nPDFs)





- Significant effect on R_{AA}, but not as large as nPDF
- Gaussian toy model (width extracted from δp_T^2) is a good description

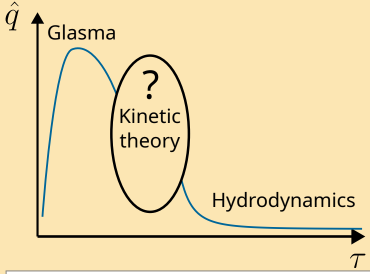
Hard probes during bottom-up thermalization

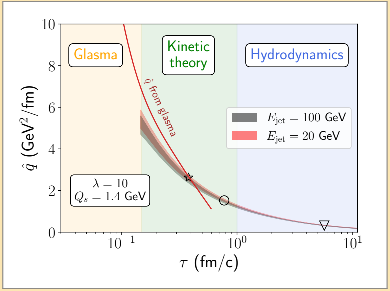
Kinetic theory: transport coefficients

Kinetic theory:
independent scatterings by construction

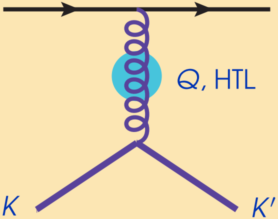
$$\left.\begin{array}{c}\hat{q}\\ \kappa\end{array}\right\} = \frac{d\left\langle q_\perp^2\right\rangle}{dt} \quad \left\{\begin{array}{l}\text{jet } (p=\infty)\\ \text{H.Q. } (m=\infty)\end{array}\right.$$

- Standard for a long time:
  $\hat{q}$, κ in thermal system
  $\Longrightarrow$ Input for jet quenching, H.Q. diffusion
- Aim: complete the picture
  from the glasma to hydrodynamics





Calculating transport coefficients



Momentum broadening from interactions with medium particles:

$$\begin{matrix}\hat{q} \\ \kappa\end{matrix} \sim \int_{\mathbf{k}\mathbf{k}'\mathbf{p}'} \frac{q_T^2}{E_{\mathbf{p}}} (2\pi)^4 \delta^4(P + K - P' - K') \, |\mathcal{M}|^2 \, f(\mathbf{k}) \, (1 + f(\mathbf{k}')) \, ,$$

- κ: heavy quark $P = (M, \mathbf{0})$, $M \to \infty$
- $\hat{q}$: energetic jet $P^2 = 0, p \to \infty$ (need cutoff $\hat{q} \sim \ln \Lambda_\perp$)

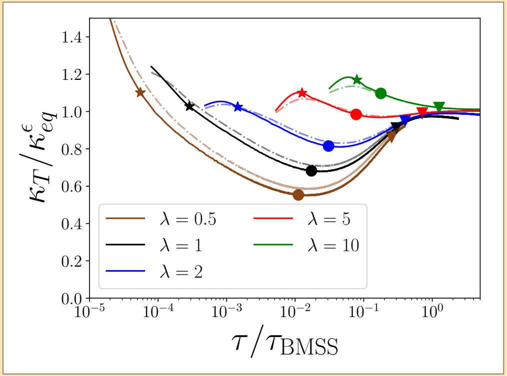
These limits: **medium properties**, not probe

Result: κ

K. Boguslavski, A. Kurkela, T. L., F. Lindenbauer, J. Peuron, arXiv:2303.12520 [hep-ph]

Compare to thermal system with same ε
(Landau matching,
thermal with same m_D or T_* is much further)

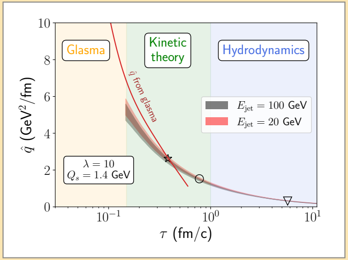
- Enhancement first (overoccupied)
- Then suppression (underoccupied)
- Larger $\lambda = 4\pi N_c \alpha_s$:
  behavior smoothed out

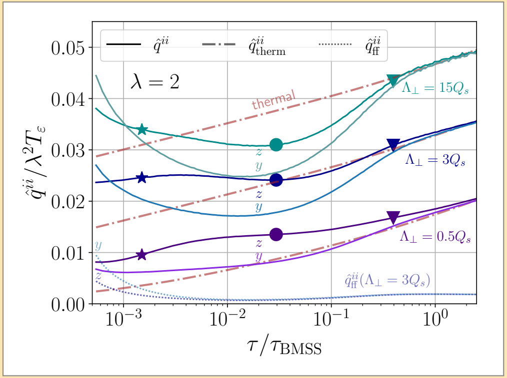


Result: $\hat{q}$

K. Boguslavski, A. Kurkela, T. L., F. Lindenbauer, J. Peuron, arXiv:2303.12595 [hep-ph]

- Large cutoff $\Lambda_\perp$:
  Enhancement first, then suppression
- Smaller $\Lambda_\perp$:
  smoother, overall enhancement

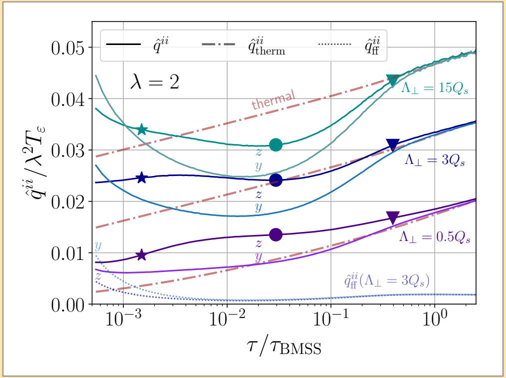


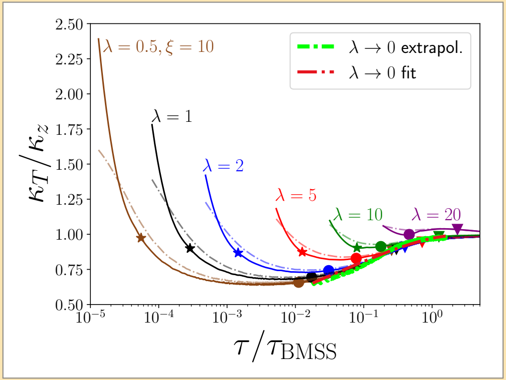


- $\varepsilon \sim 1/\tau$ large
- At end of BMSS ▼: $\hat{q} \approx$ JETSCAPE estimate (can match by tuning $\Lambda_\perp$)

Anisotropy

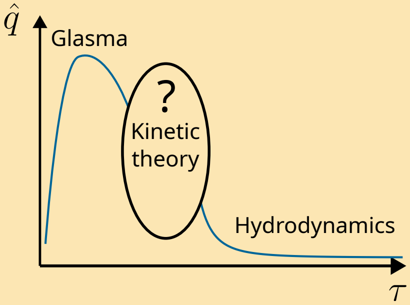
- Inital overoccupied: $\kappa_T > \kappa_L$, $\hat{q}_T > \hat{q}_L$ $\Longrightarrow$ Bose enhancement, Glasma
- Then underoccupied $\kappa_T < \kappa_L$, $\hat{q}_T < \hat{q}_L$ $\Longrightarrow$ Anisotropy of f





Conclusions

- Pre-equilibrium stage short, but hot
  $\Longrightarrow$ Significant effect on hard observables
- Glasma:
  - Classical Yang Mills
    + classical colored particles
  - ***p*** broadening coherent,
    not independent multiple scattering
- Bottom-up thermalization
  - QCD kinetic theory:
    trace system from glasma to hydro
  - $\hat{q}$, κ within $\sim$ 30% of thermal system
    @ same energy density
- Both stages: anisotropy w.r.t. beam:
  measurable?



EKT $\hat{q}$ parametrizations available in 2303.12595, κ upon request

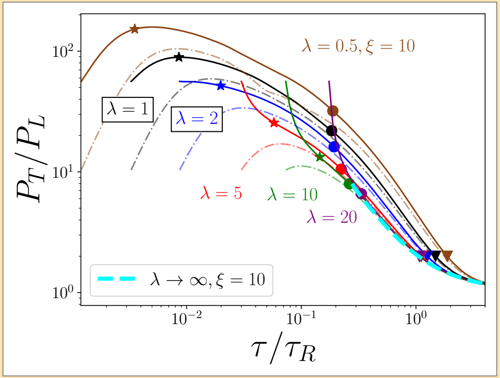
Attractors

Two "limiting attractors"

$$\tau_R(\lambda, \tau) = \frac{4\pi \frac{\eta}{s}}{T_\varepsilon}$$

(T_ε from energy density)

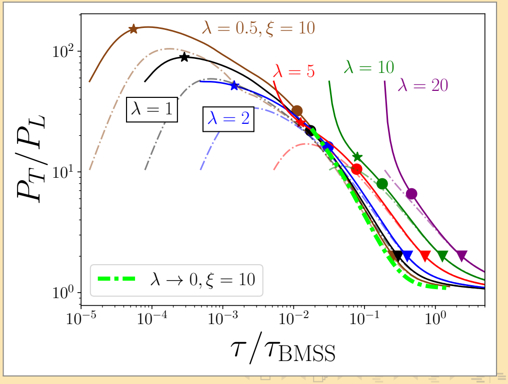
- Isotropization rate near equilibrium
- "Hydro attractor" in literature



$$\tau_{\rm BMSS} = \alpha_s^{-13/5}/Q_s$$

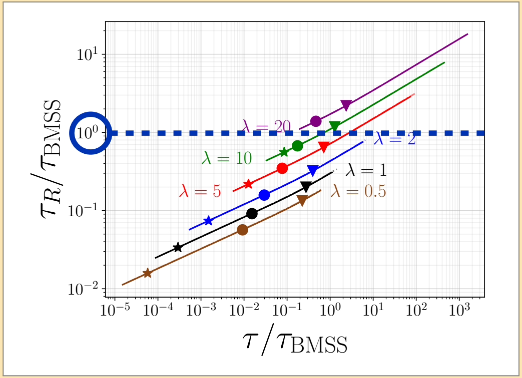
- Weak coupling QCD thermalization
- Timescale for rough isotropy

(Then hydro attractor takes over)



How different are the timescales?

- Weak coupling: timescales different
- Viscous hydro (relevant scale τ_R) follows from EKT (relevant scale τ_{BMSS}) Contradiction? No!
- $\lambda \ll 1 \implies \tau_R \ll \tau_{\text{BMSS}}$ First spend **long** time in BMSS regime then short time on hydro attractor



(τ_R depends on τ, because $\varepsilon(\tau)$ changes)

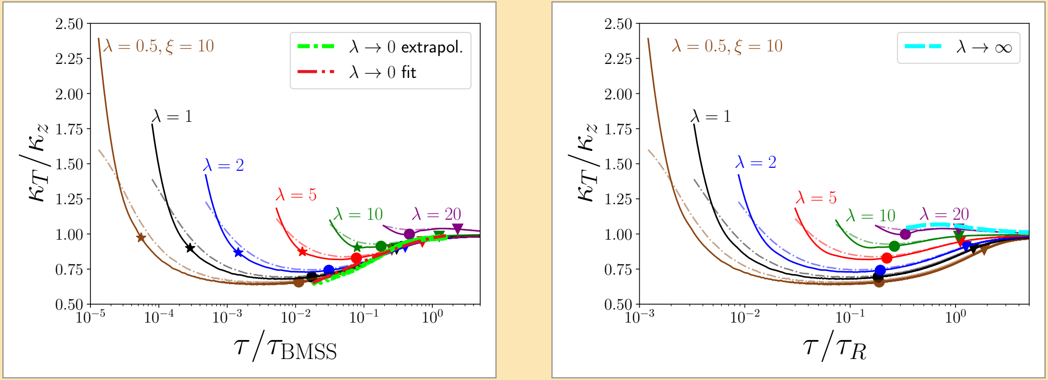
BMSS regime can matter more than hydro attractor for hard probes.

We plot on log scale. E.g. if $\hat{q} \sim \epsilon(\tau) \sim 1/\tau$
$0.1\text{fm} < \tau < 1\text{fm}$ and $1\text{fm} < \tau < 10\text{fm}$ contribute equally to $\int d\tau \hat{q}(\tau)$

Attractors for $\hat{q}$ and κ

κ anisotropy, 2 attractors

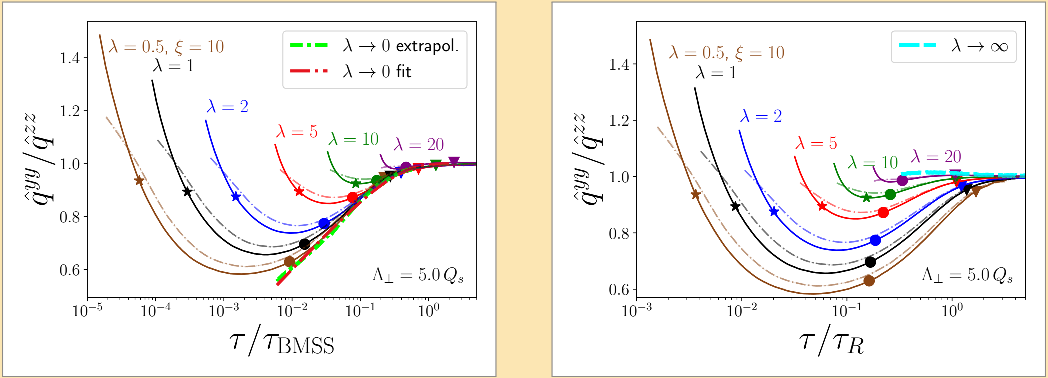
Anisotropy of κ, scaling with the two attractor timescales



Weak coupling BMSS is a better description, over larger range in τ

$\hat{q}$ anisotropy, 2 attractors

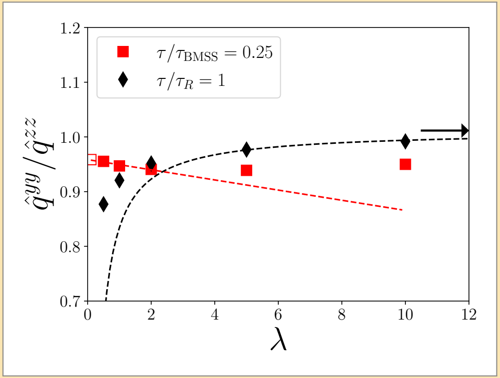
Anisotropy of $\hat{q}$, scaling with the two attractor timescales

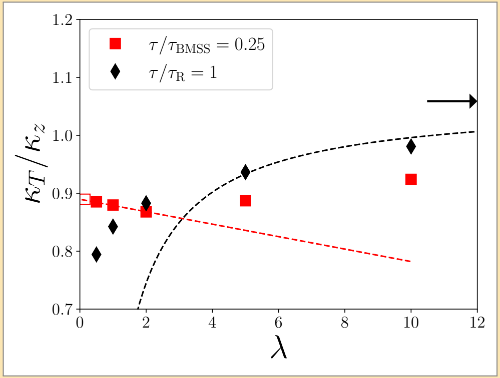


Weak coupling BMSS is a better description, over larger range in τ

Extrapolating to weak and strong coupling

How do we construct the attractor curves?





- Take fixed value of τ/τ_{BMSS} or τ/τ_R
- Linear fit in λ or $1/\lambda$, separately for each τ.
- For BMSS also provide a parametrization of the τ-dependence ("$\lambda \to 0$ fit" in plot)

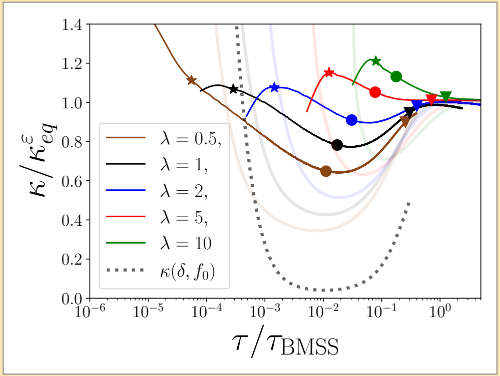
Relevant microscopic scales

- Occupation number f
- Coupling α_s
- Anisotropy $\delta \sim \sqrt{\frac{\langle p_z^2 \rangle}{\langle p_T^2 \rangle}}$
- Hard scale $p_T^2 \sim Q_s^2$

From these estimate

- Energy density $\varepsilon \sim \delta Q_s^4 f$
- Debye scale $m_D^2 \sim \alpha_s \delta \, Q_s^2 f$
- Soft mode eff. temperature $T_* \sim Q_s(f+1)$
- $\kappa \sim m_D^2 T_*$

Understanding the systematics



(Light: T_*, m_D from EKT, dashed: f, δ from EKT)