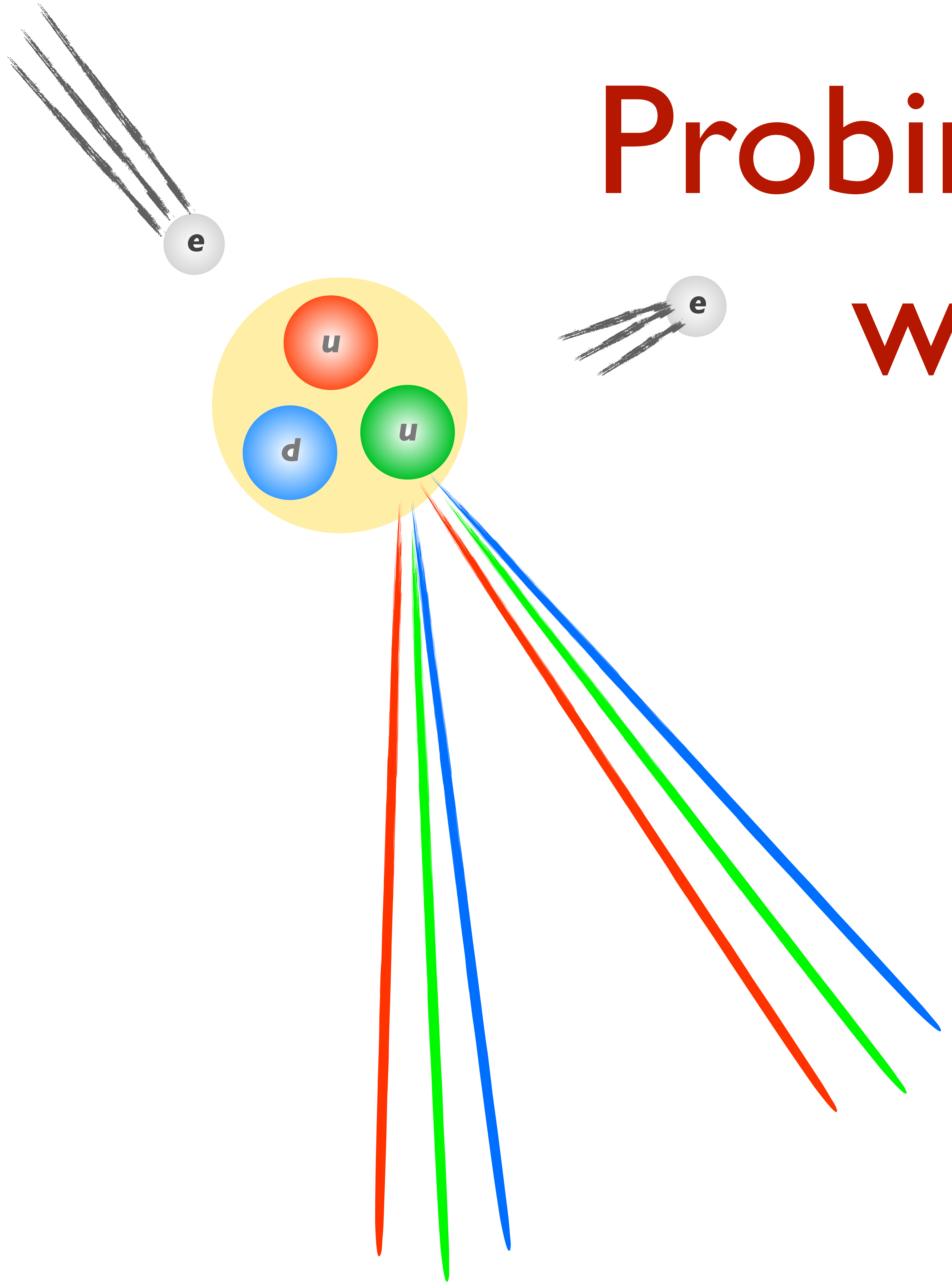


Probing fundamentals of QCD with DIS Event Shapes

Christopher Lee, LANL

in collaboration with June-Haak Ee (LANL), Daekyoung Kang (Fudan), Iain Stewart (MIT)

Heavy-Ion Physics in the EIC Era
INT Workshop
August 9, 2024

INSTITUTE for NUCLEAR THEORY

Los Alamos NATIONAL LABORATORY

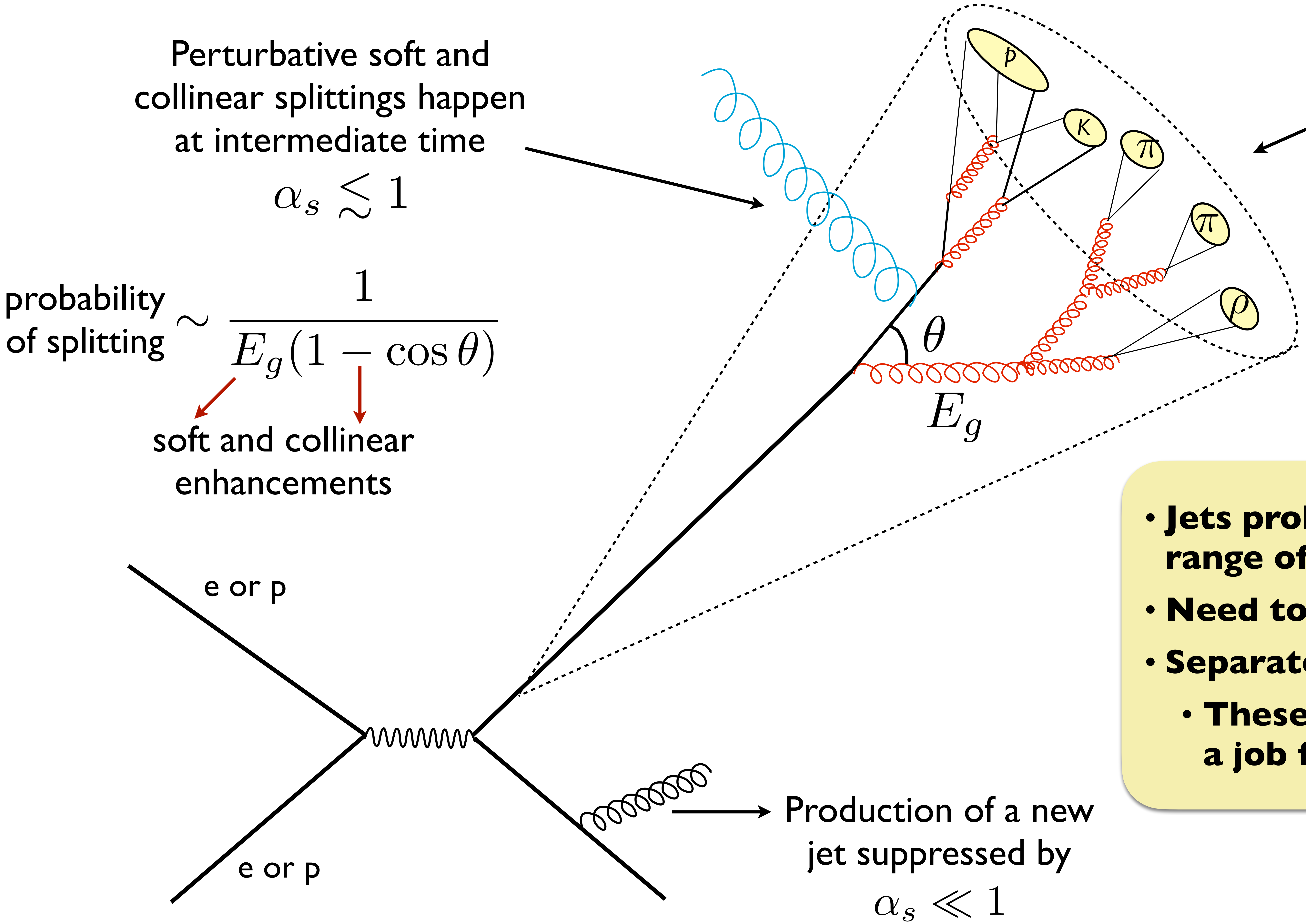
Outline of What is Not in This Talk

- No heavy ions (*p* is heavy enough for me!)
- No actual determination of α_s (but show a path towards it)
- No(t many) results for EIC (will show comparison to HERA data)

Outline of What is in This Talk

- Status of α_s determinations from e^+e^- and DIS jet measurements
- Global measures of *jettiness*: event shapes
- Factorization and resummation in SCET
- Nonperturbative Effects and Universality
- Predictions for DIS event shapes at HERA *and EIC!*
  and sensitivity to α_s

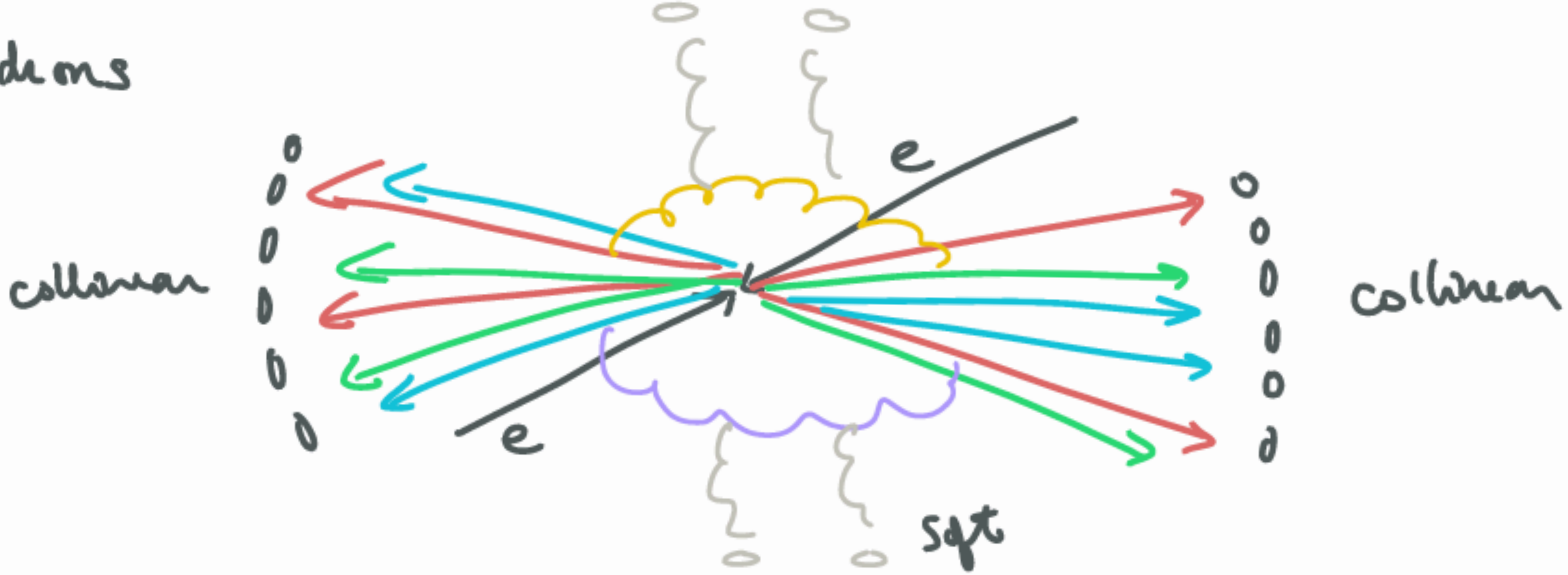
Formation of Jets in QCD



- **Jets probe strong interaction over wide range of scales**
- **Need to resum large perturbative logs**
- **Separate pert. and non-pert. physics**
  - **These are problems of *scale separation:* a job for EFT**

HADRONIC EVENT SHAPES: Global measures of "jetty" structure

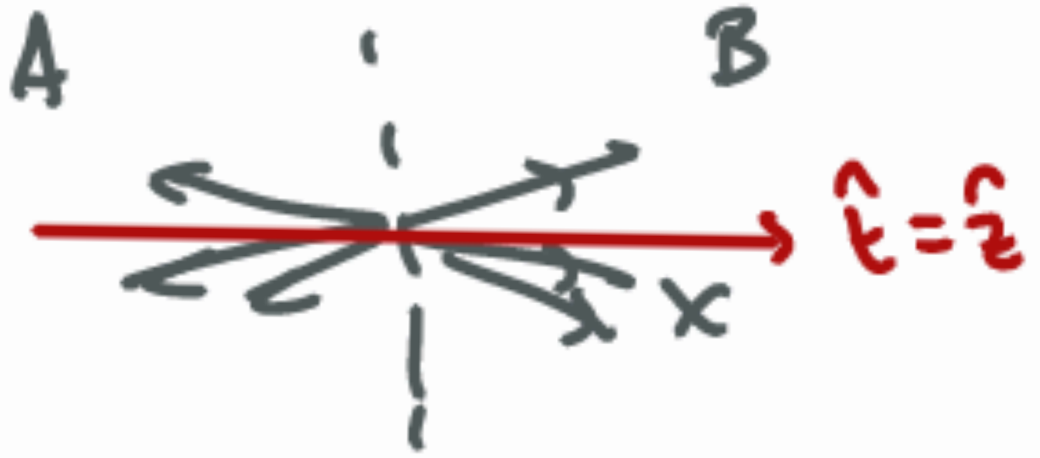
$e^+e^- \to$ hadrons



e.g.

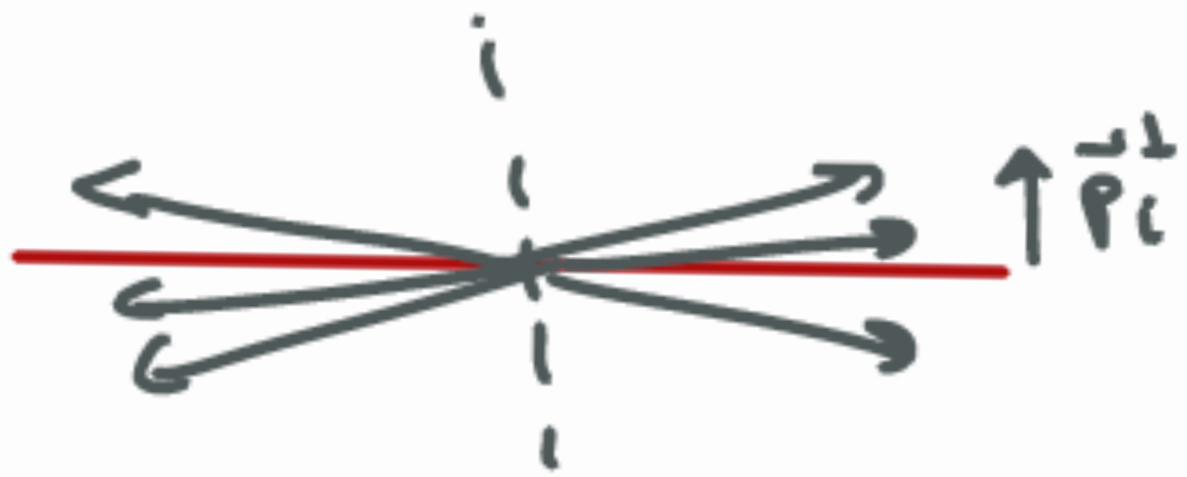
THRUST: $T = \frac{1}{Q} \max_{\hat{t}} \sum_{i \in X} |\vec{p}_i \cdot \hat{t}|$

$= \frac{2}{Q} |\vec{p}_z^{\,A}|$ or $\tau = 1 - T$



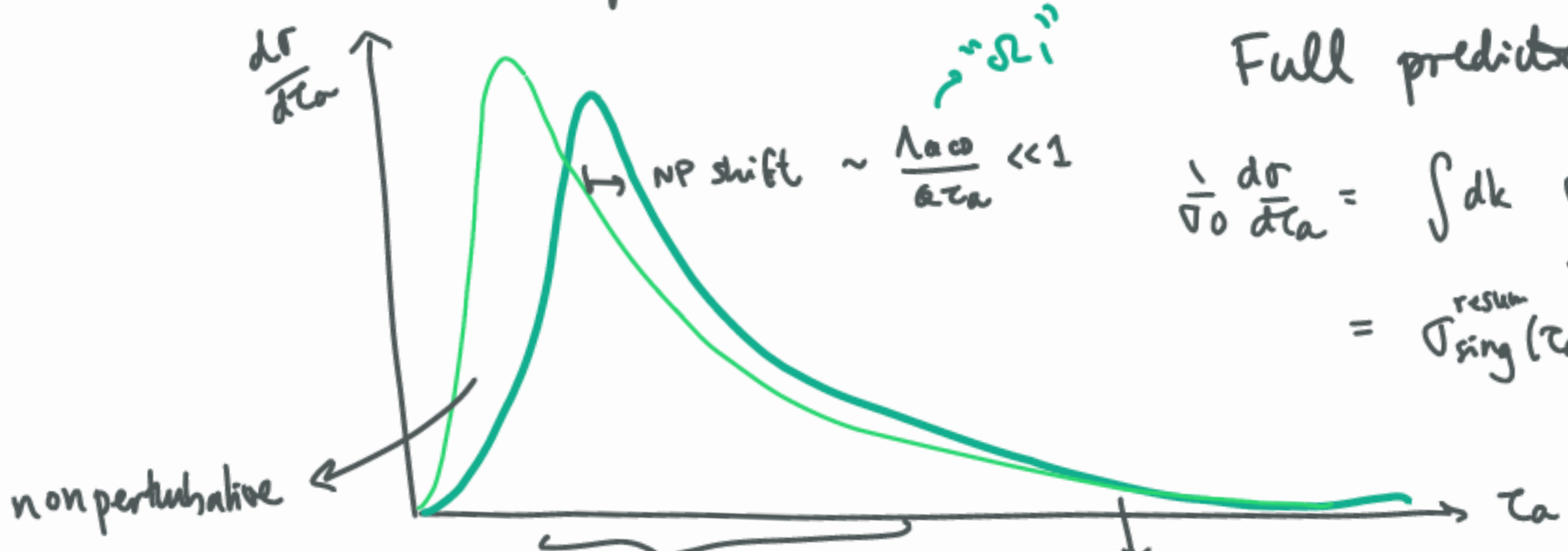
$T = \frac{1}{2}$

BROADENING: $B = \frac{1}{Q} \sum_{i \in X} |\vec{p}_i^{\,\perp}|$



EVENT SHAPES & SENSITIVITY TO α_s

τ_a's and similar event shapes probe QCD effects over wide range of scales, perturbative and nonperturbative:



Full prediction:

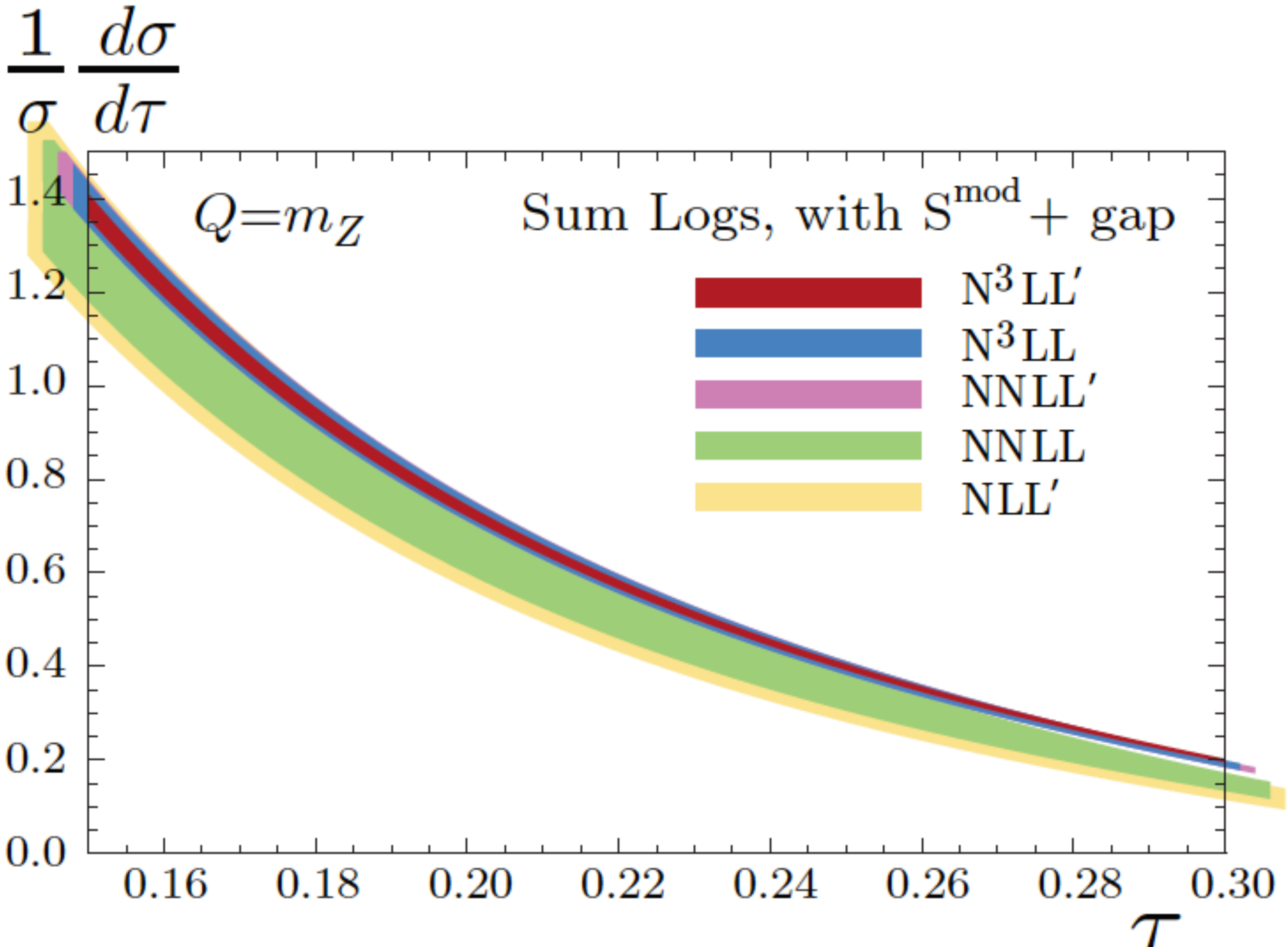
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} = \int dk\, \sigma_{PT}\left(\tau_a - \frac{k}{Q}\right) S_{NP}(k)$$

$$= \sigma^{\text{resum}}_{\text{sing}}(\tau_a; \mu_{H,J,S}) + \sigma^{\text{F.O.}}_{\text{non-sing}}(\tau_a; \mu_{ns})$$

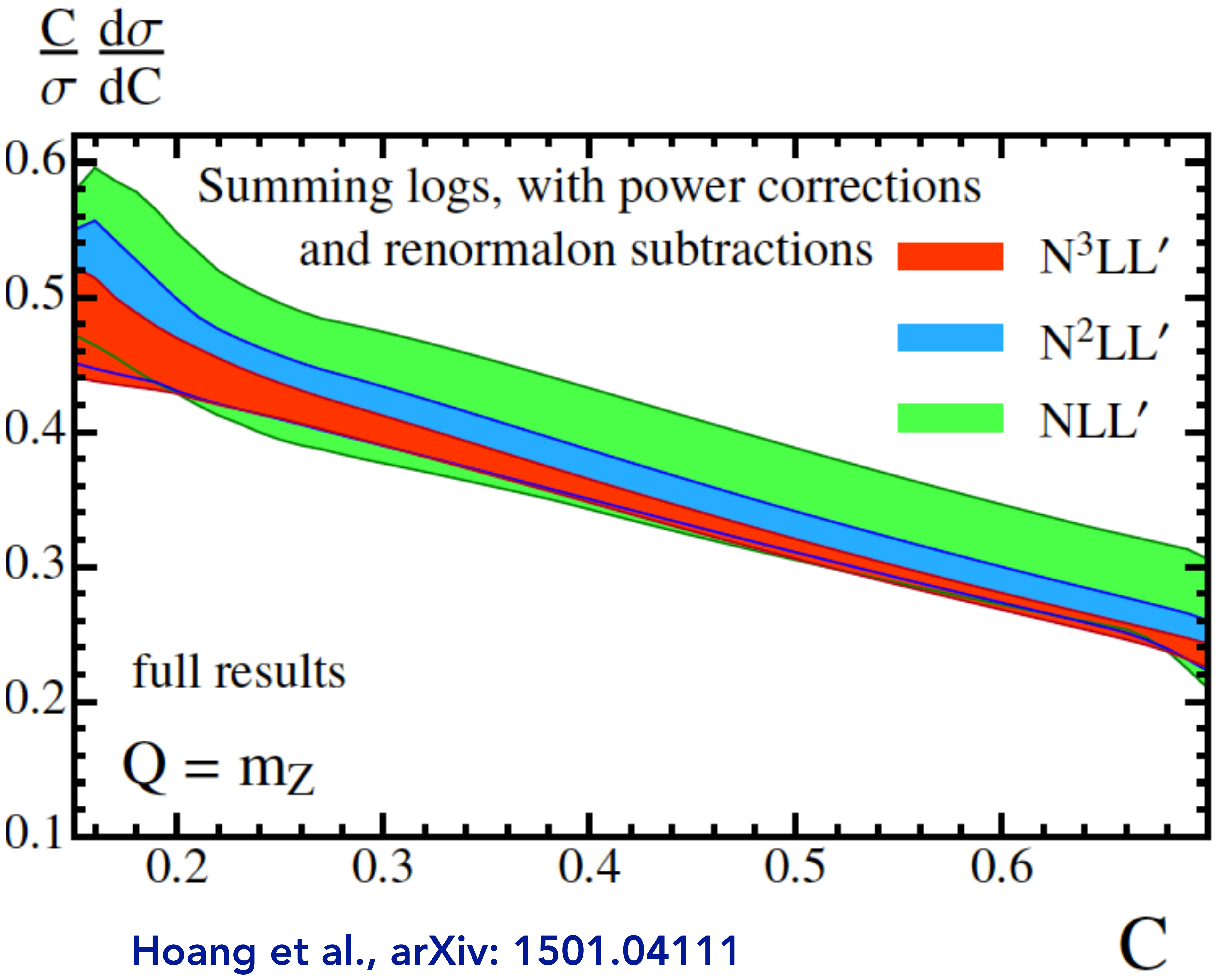
$\sigma_{PT}(\tau - \frac{k}{Q}) \otimes S_{NP}(k)$	expansion in	expansion in $\alpha_s(Q)$
$\frac{\Lambda_{QCD}}{Q\tau} \sim 1$	$\alpha_s(Q\tau), \alpha_s(Q\tau^{\frac{1}{2-a}})$	
↓	↓	↓
SHAPE FUNCTION	RESUMMATION	FIXED ORDER

(e^+e^-) Event shapes to high precision

- **First N³LL′ resummed event shape distributions with state-of-the-art treatment of nonperturbative corrections, e.g.:**



Abbate et al.,arXiv:1006.3080

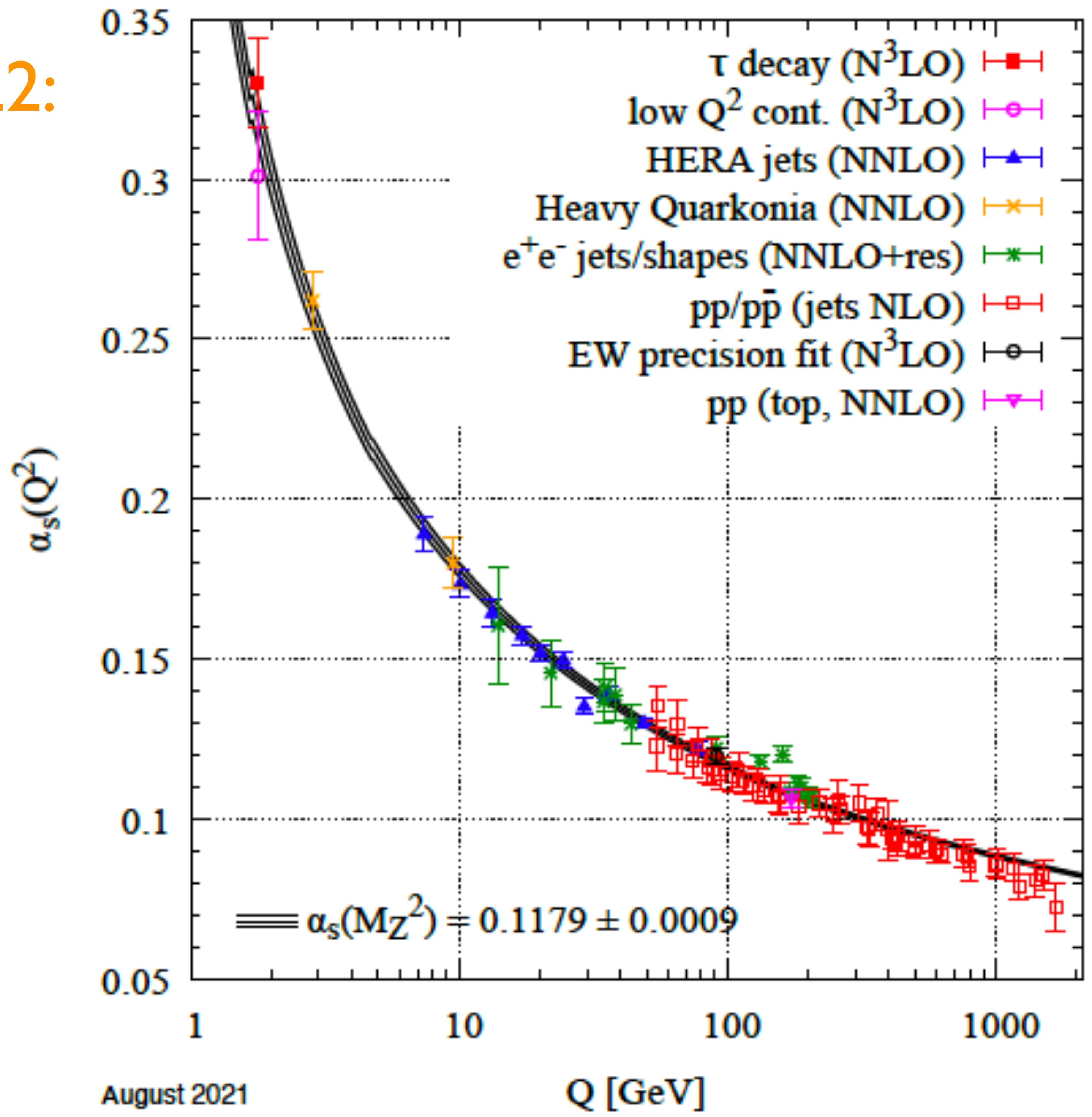


Hoang et al., arXiv: 1501.04111

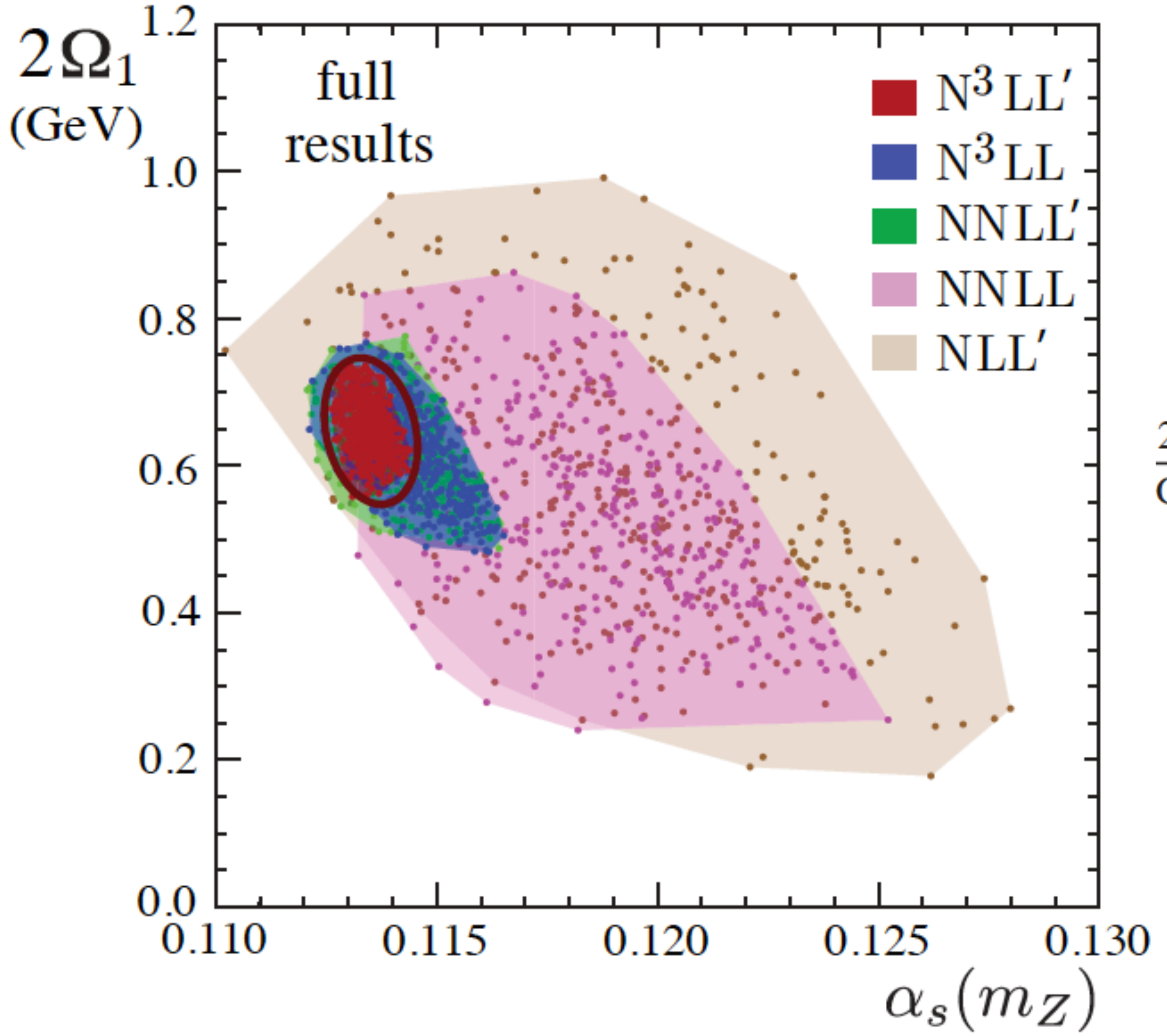
Makes e+e- event shapes one of the most precise ways, in principle, to determine α_s

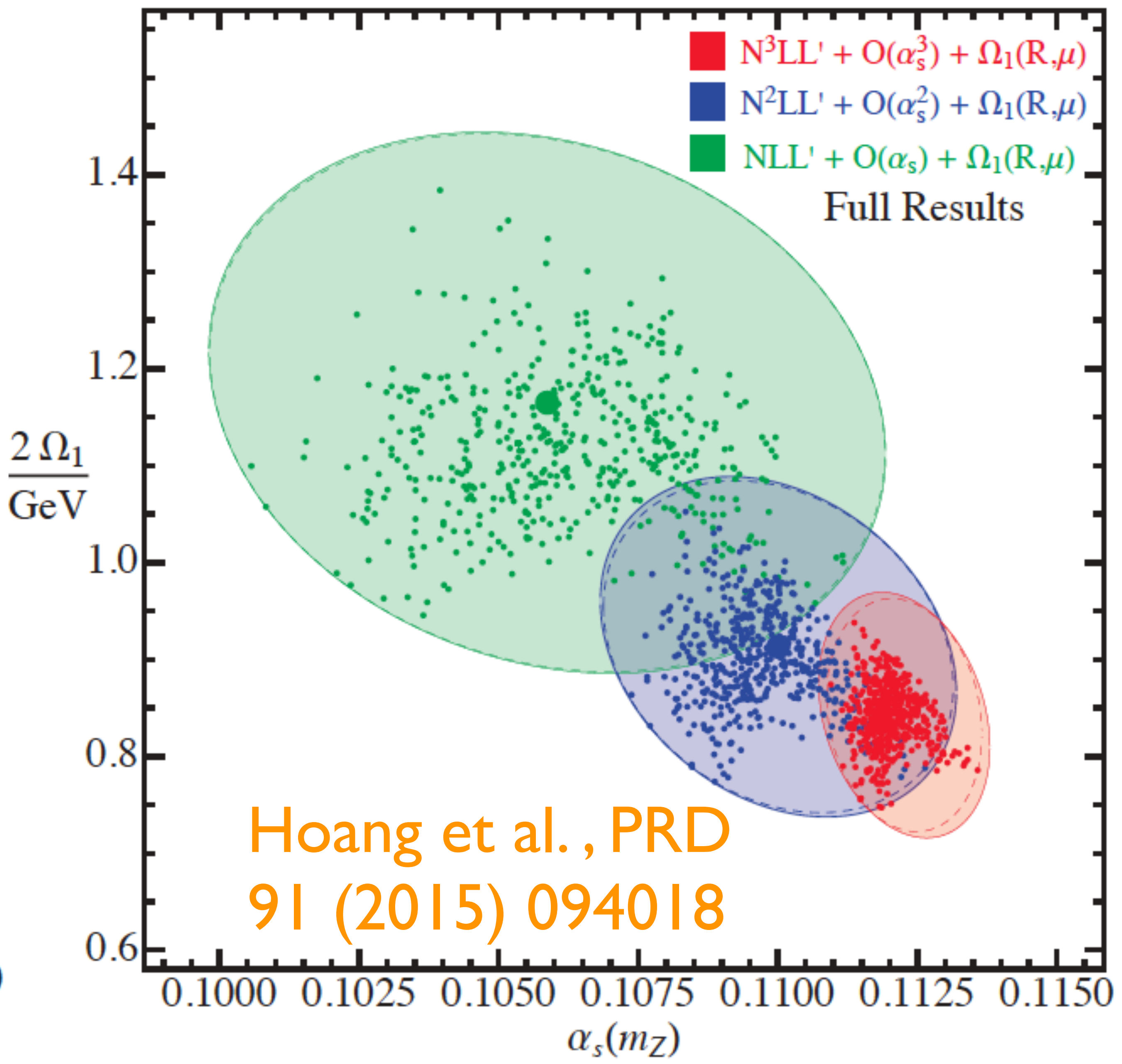
Event shapes and the strong coupling

PDG 2022:



Abbate et al., PRD 83 (2011) 074021

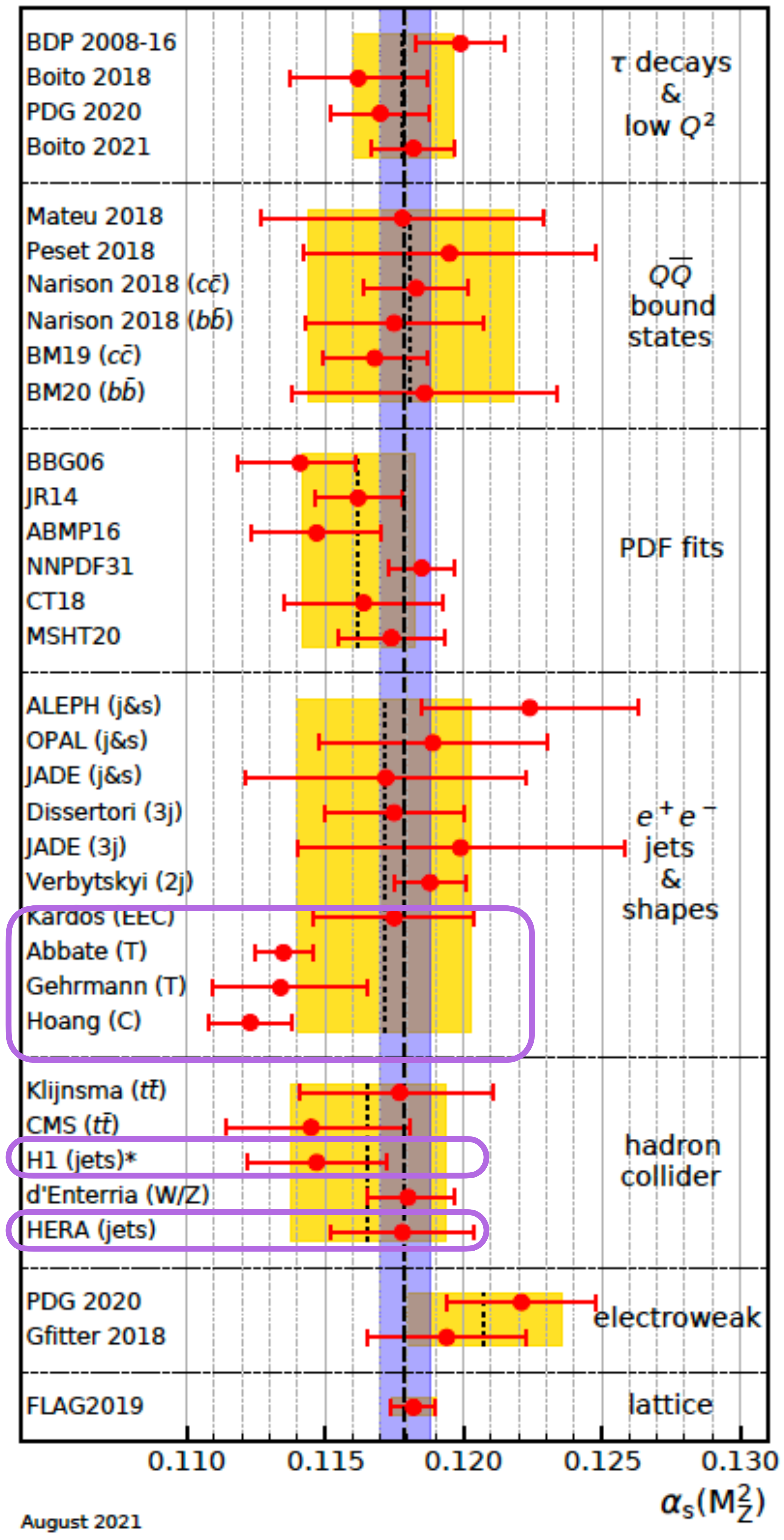




Hoang et al., PRD 91 (2015) 094018

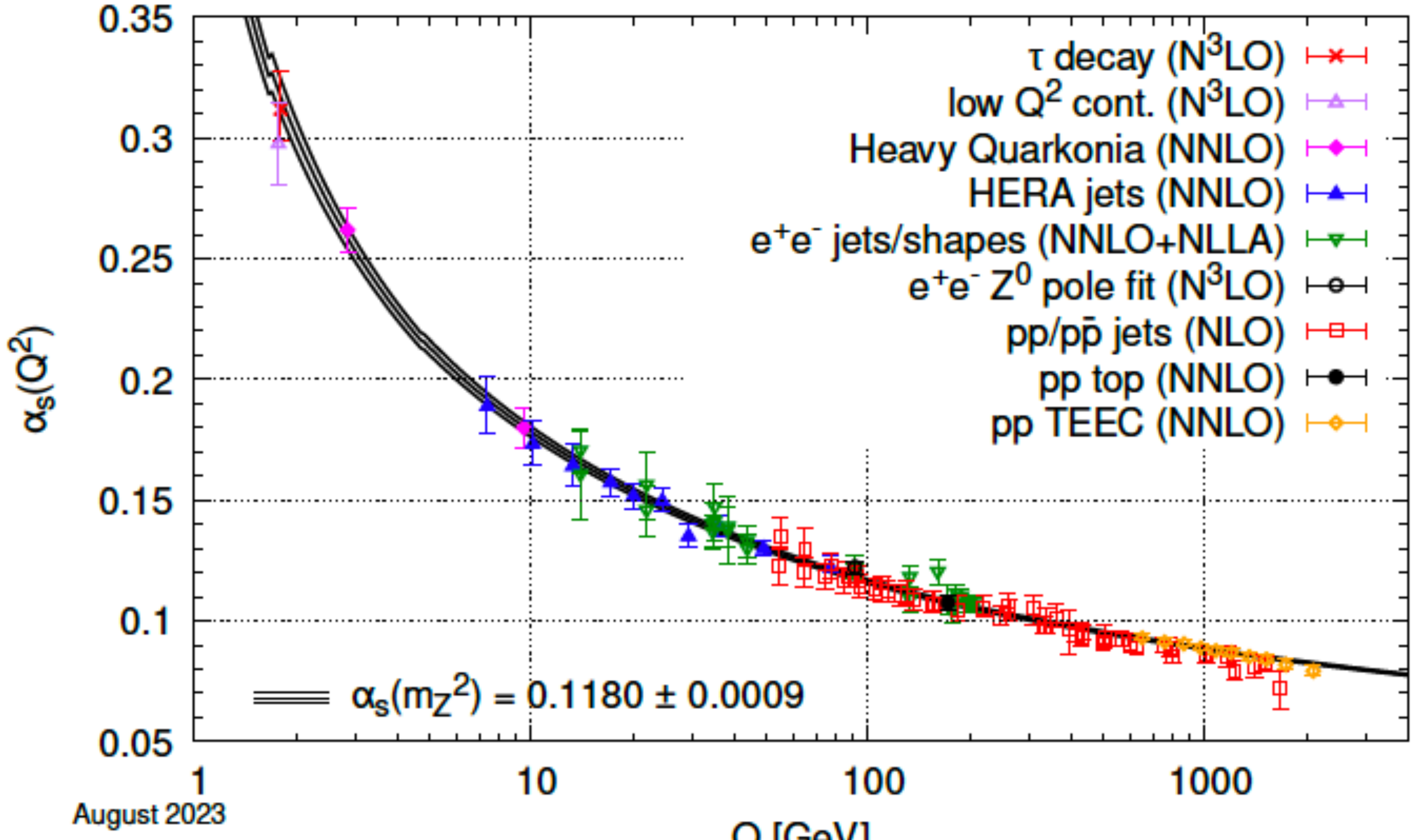
Event shapes

ep:

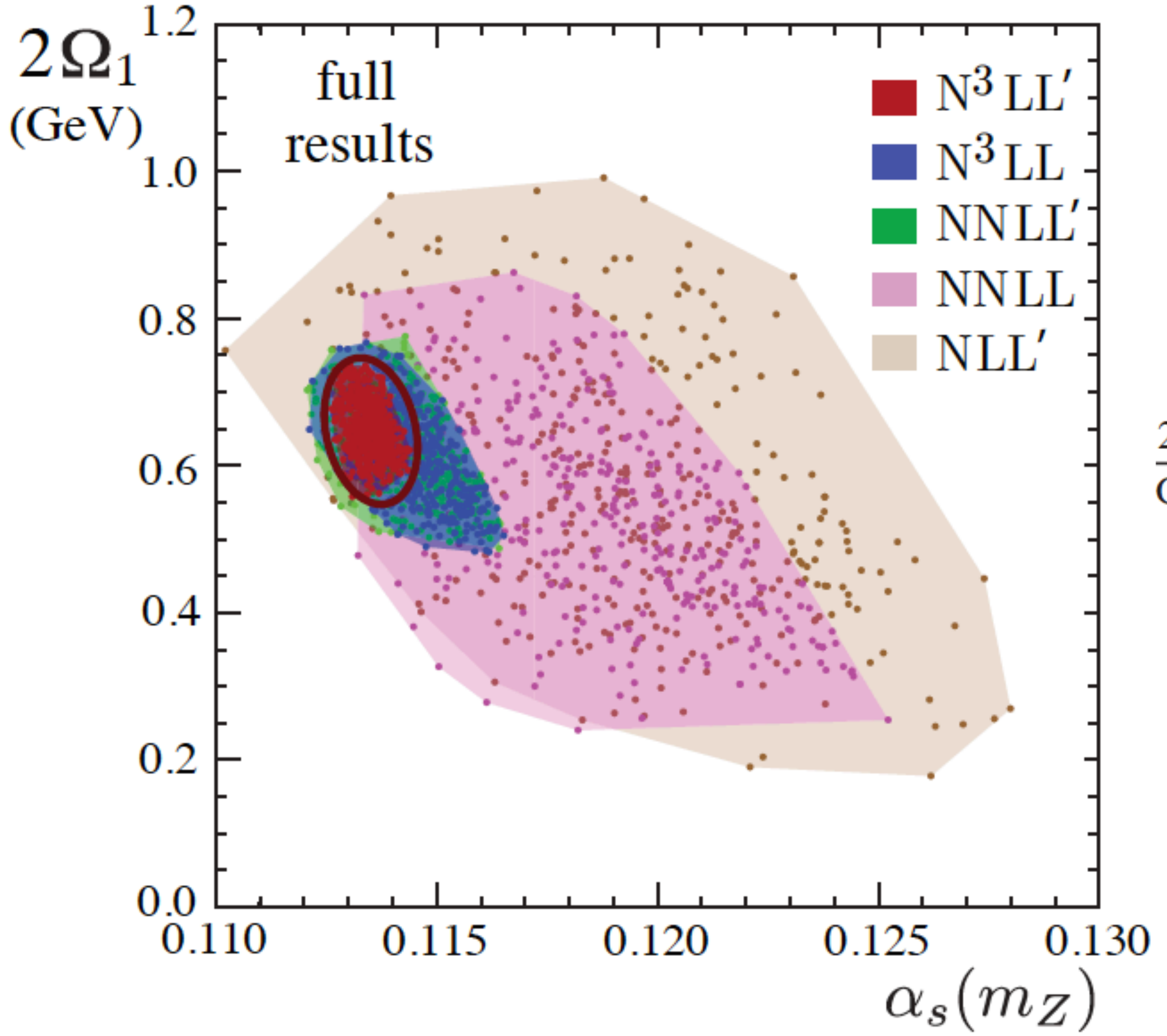


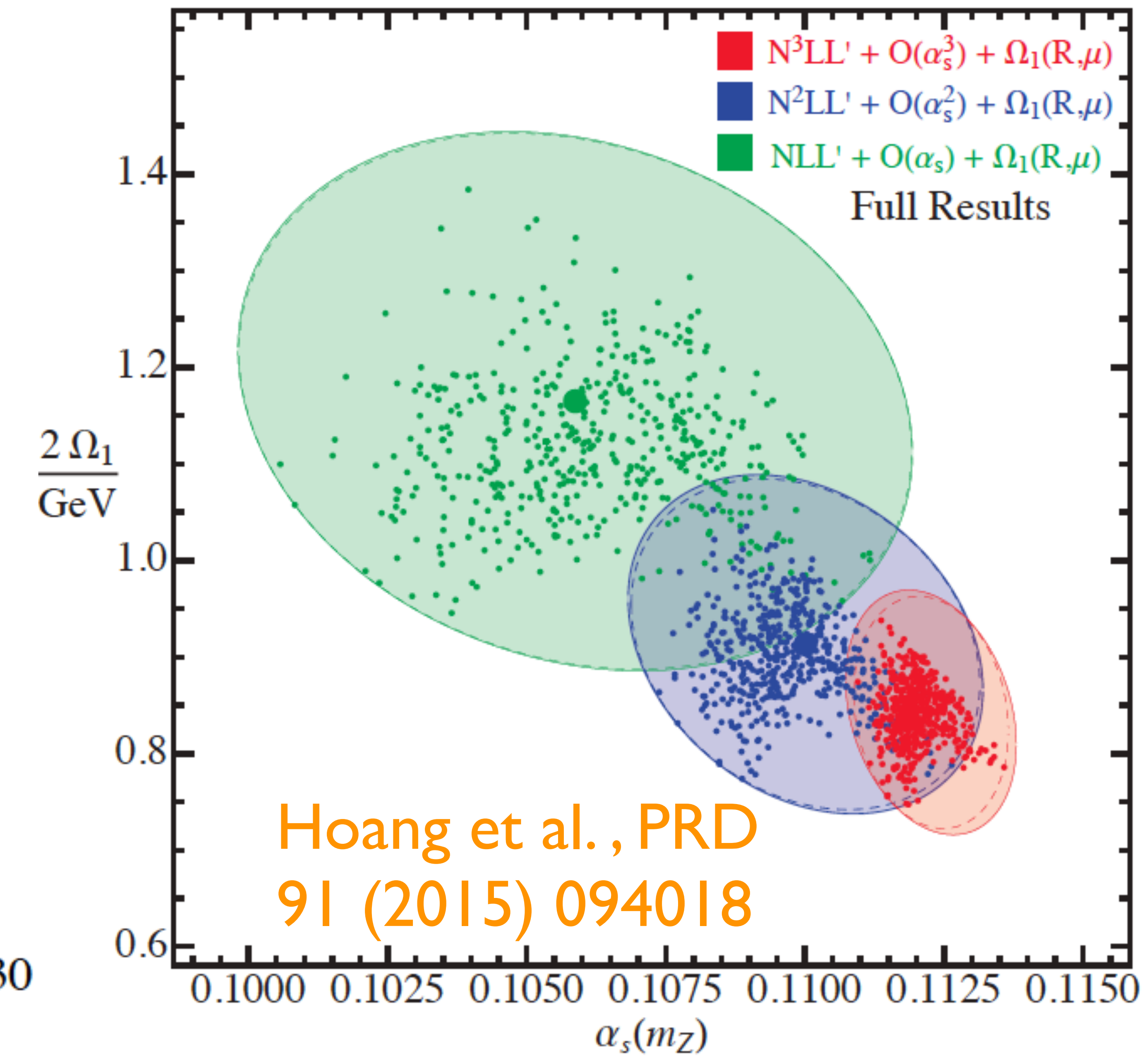
Event shapes and the strong coupling

PDG 2024:

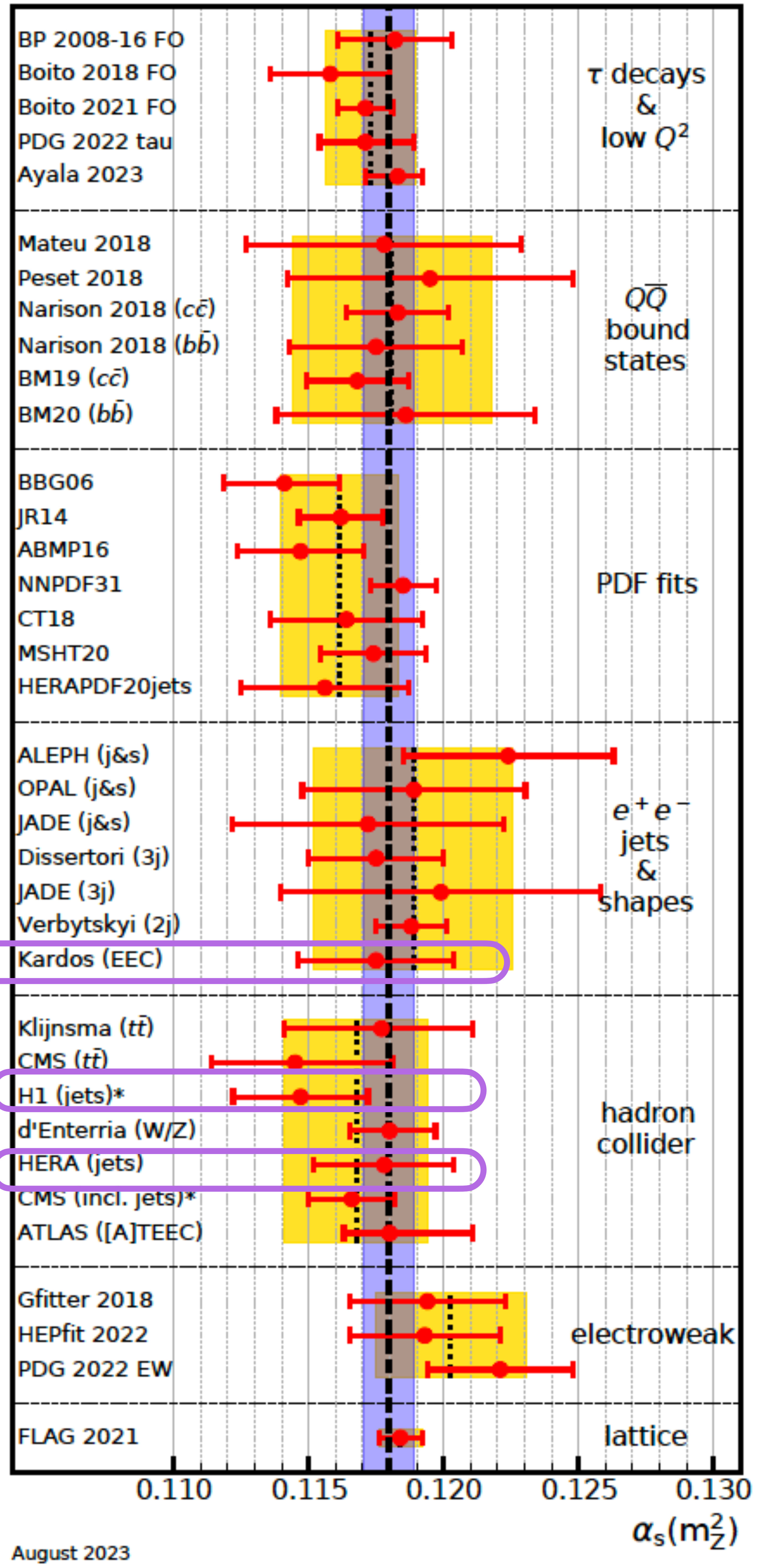


Abbate et al., PRD 83 (2011) 074021





Hoang et al., PRD 91 (2015) 094018



Event shapes??

ep:

Uncertainties underestimated?

CL @ INT 21r-2b

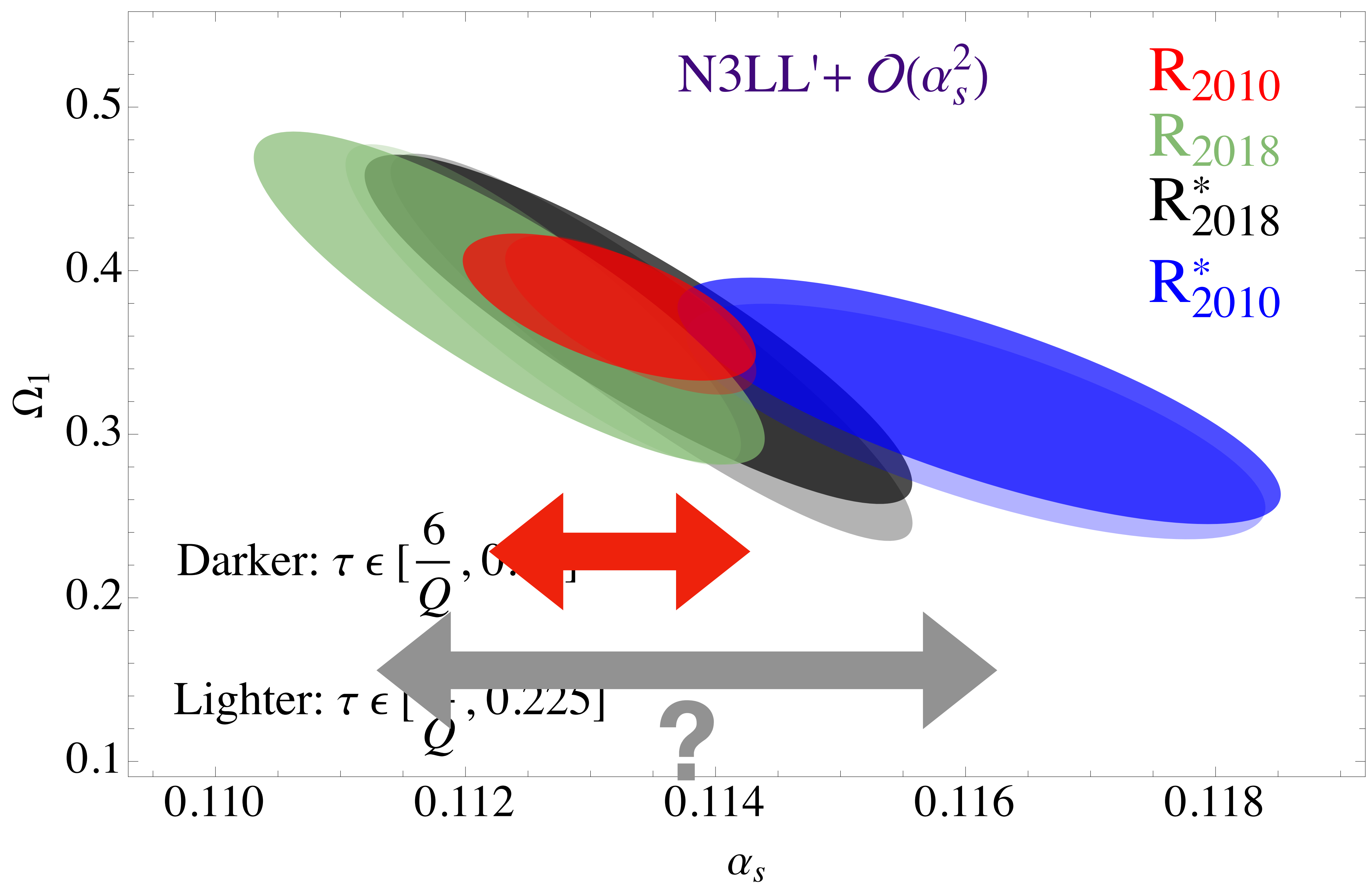
- Some questions have been raised about systematic uncertainties due to understanding of size of nonperturbative power corrections in *3-jet* region

  Caola *et al.* [2108.08897, 2204.02247]
  Nason, Zanderighi [2301.03607]

  See: Benitez-Rathgeb *et al.* [2405.14380]

- and/or uncertainties due to schemes chosen to subtract renormalon ambiguities and/or perturbative scales chosen in the non-singular fixed-order prediction in the 3-jet region that probe the size of unresummed *subleading-power* logs

  Bell, CL, Makris, Talbert, Yan [2311.03990]

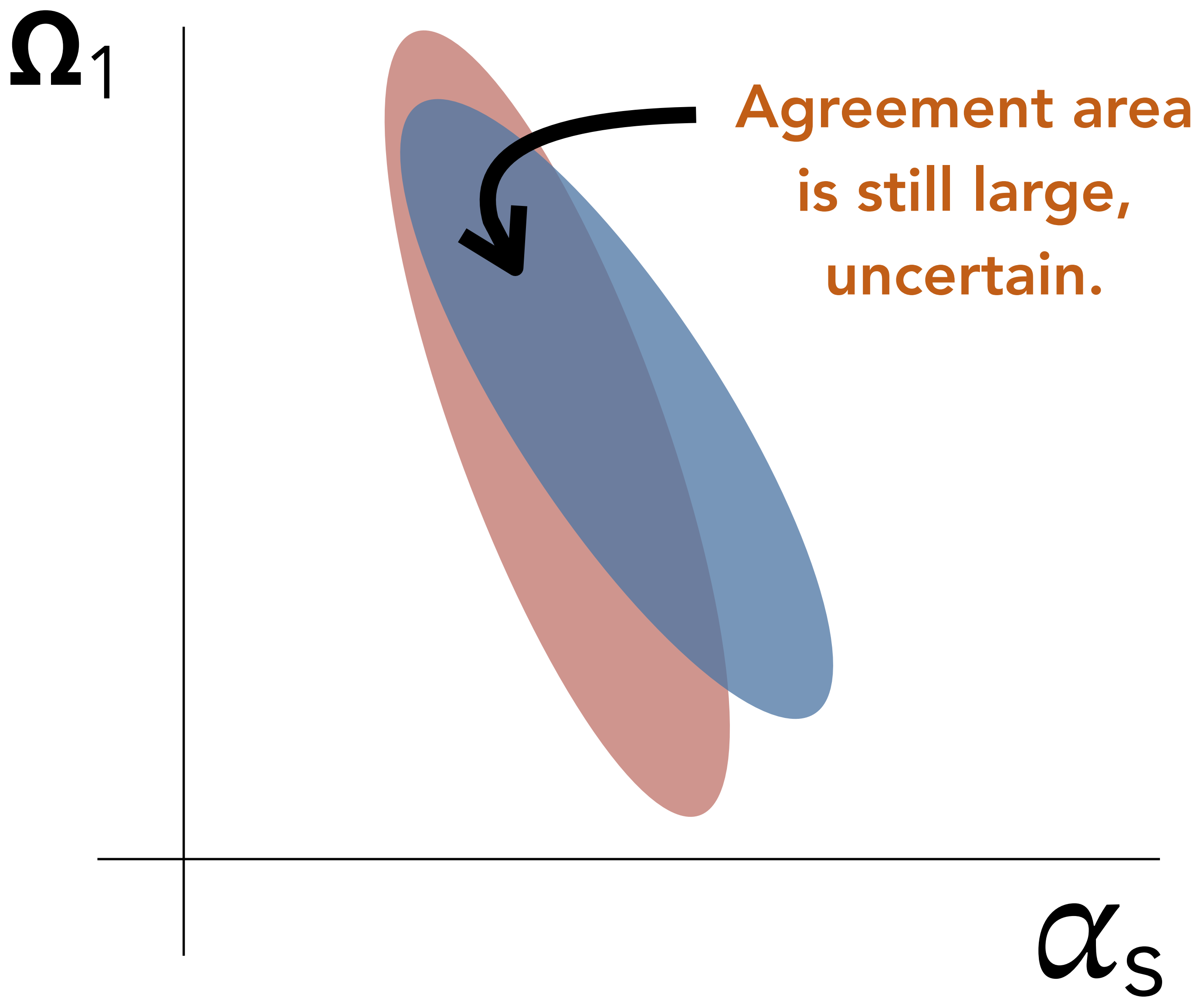


- *Perhaps room for other methods to help resolve!*

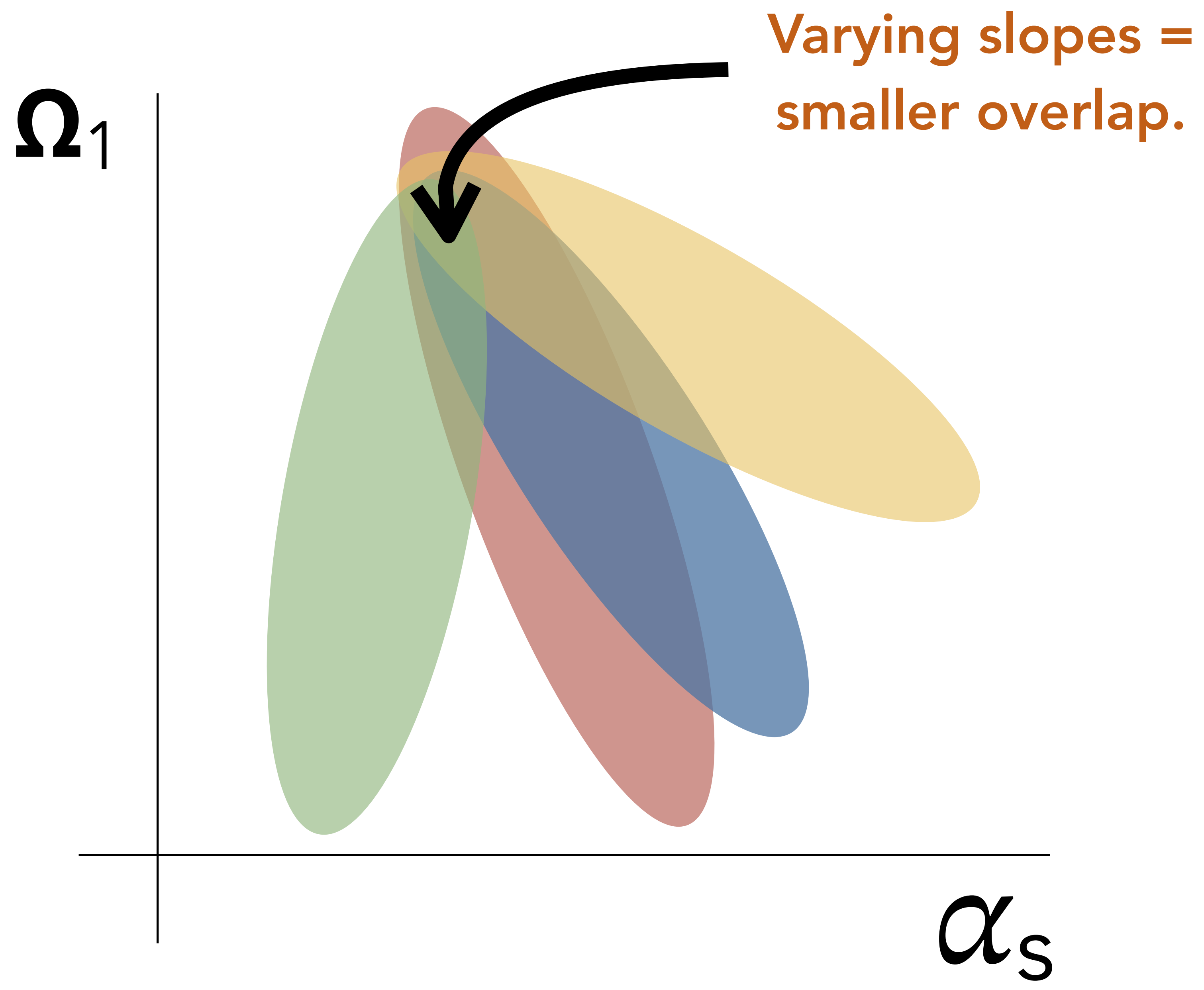
Need to break degeneracies

- In tail region, leading nonperturbative effect is a shift by $c_e\Omega_1/Q$
  c_e is an exact observable dependent coefficient, e.g. angularities c_e= 2/(1-a)

$$\tau_a = \frac{1}{Q}\sum_i |\mathbf{p}_i^\perp| e^{-|\eta_i|(1-a)}$$

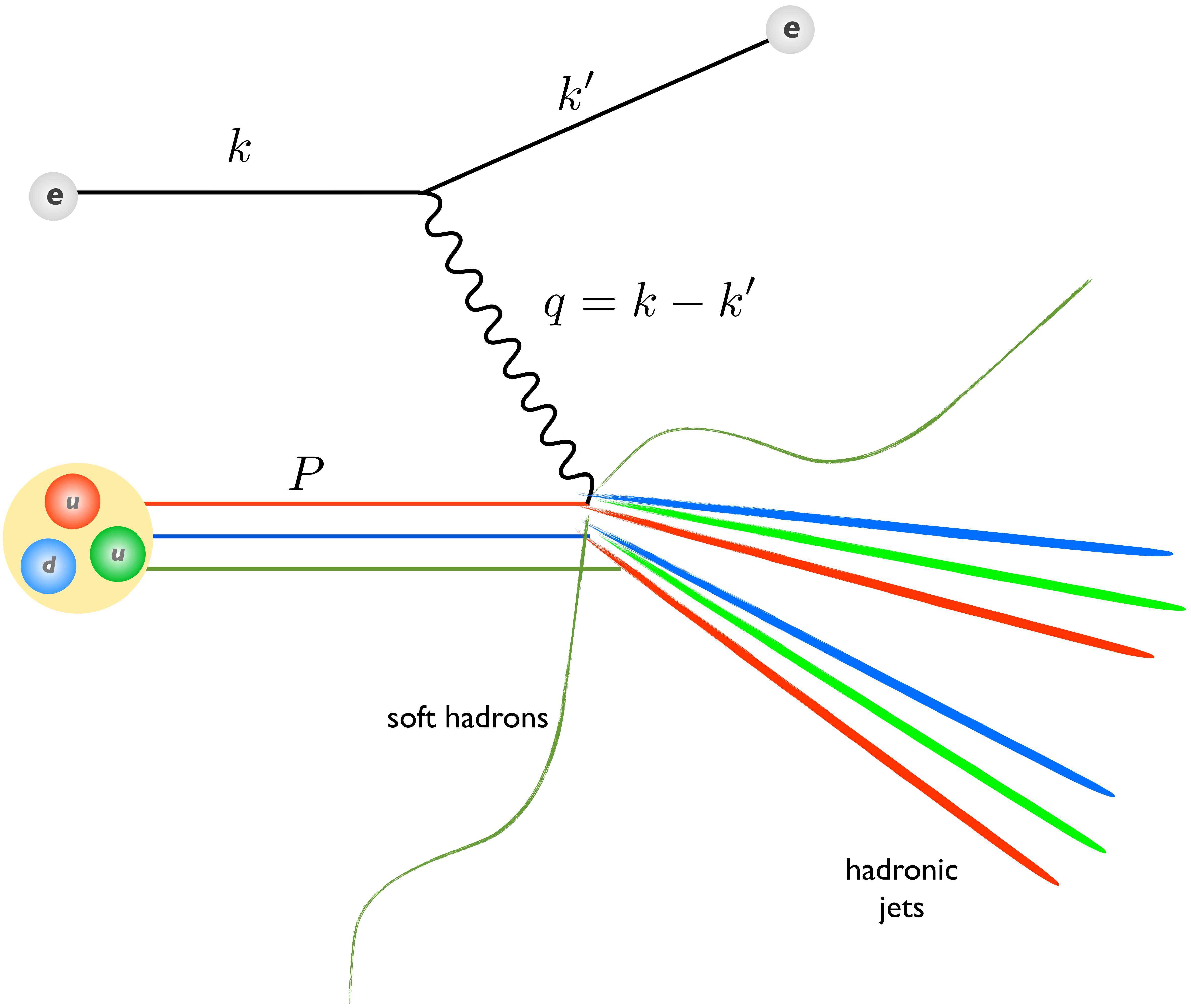


Few observables
or few Q's



Many observables
or many Q's
Readily accessible in DIS

DIS Kinematics



Not as clean as e+e-, but provides single laboratory to vary x, Q to break $\{\alpha_s, \Omega_1\}$ degeneracies

$$s = (k+P)^2$$ squared center-of-mass energy

$$Q^2 = -q^2$$ momentum transfer

$$x = \frac{Q^2}{2P \cdot q}$$ Bjorken scaling variable

$$y = \frac{P \cdot q}{P \cdot k}$$ lepton energy loss in proton rest frame

$$Q^2 = xys$$

$$p_X = q + P$$ total momentum of final hadronic state

$$p_X^2 = \frac{1-x}{x} Q^2$$ invariant mass of final hadronic state

1934-2024

Jets in DIS and the strong coupling

Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	±0.0011	±0.0050	±0.0051	±4.3
HERA average 2007		0.1198	±0.0019	±0.0026	±0.0032	±2.7

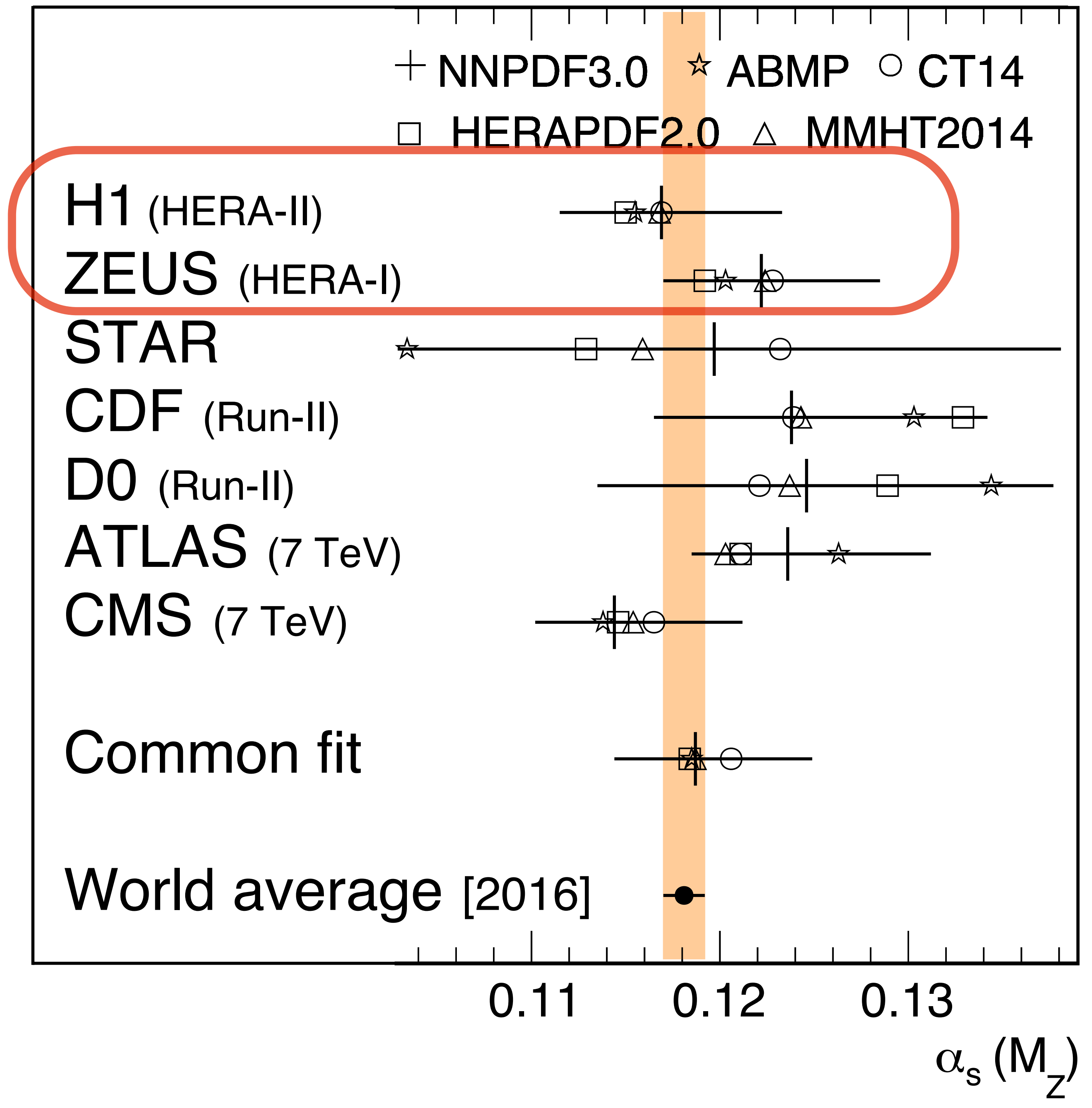
Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

Extractions from exclusive jet cross sections have order 10% uncertainty, dominated by theory

Improve to level of e^+e^-?

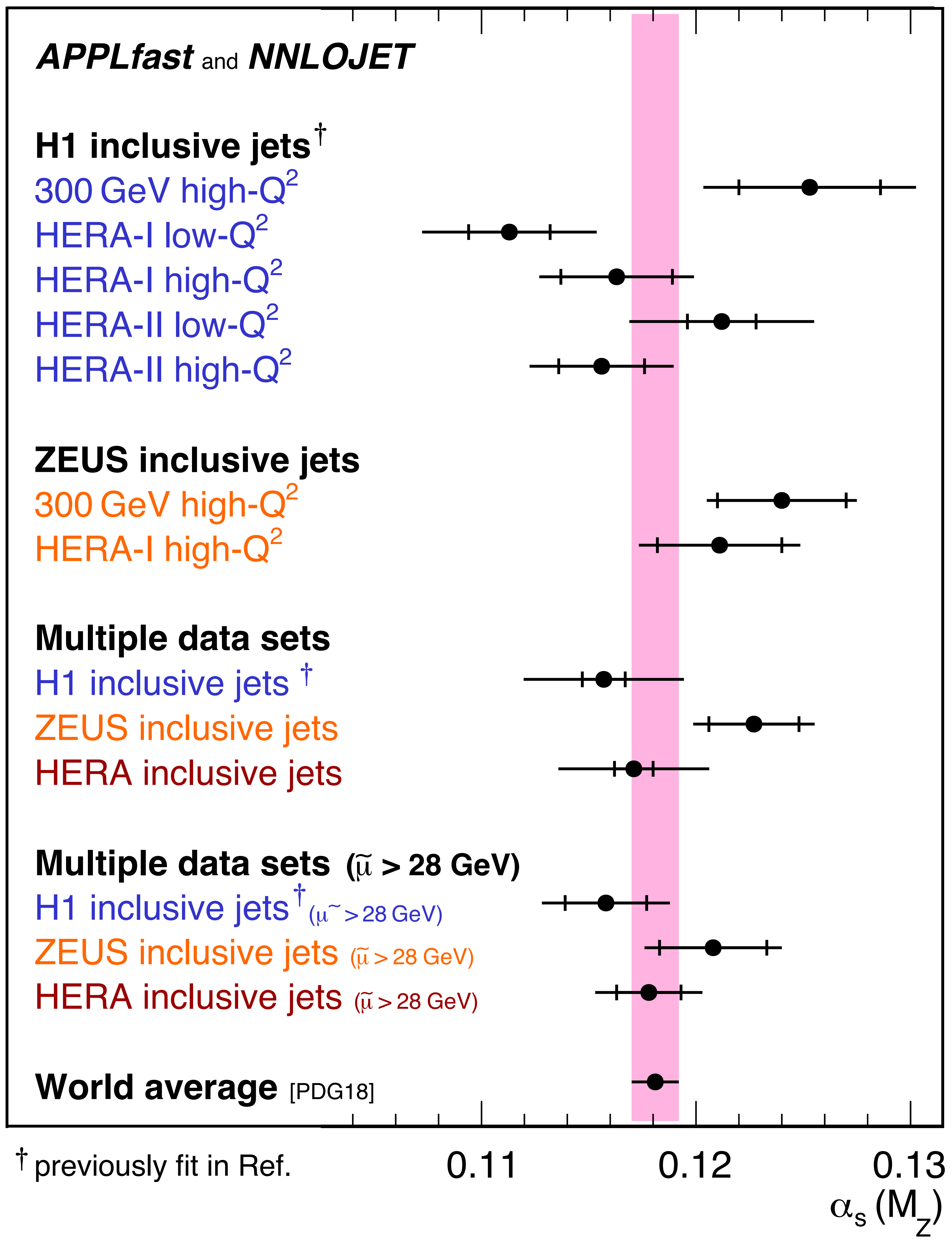
C. Glasman, in the Proceedings of the Workshop on Precision Measurements of α_s [1110.0016]

Jets in DIS and the strong coupling



Britzger *et al.* [1712.00480]

α_s results from HERA inclusive jet data in NNLO



Extractions from inclusive jet cross sections have order 10% uncertainty, exp + theory

Improve to level of e^+e^-?

Britzger *et al.* [1906.05303]

N-jettiness

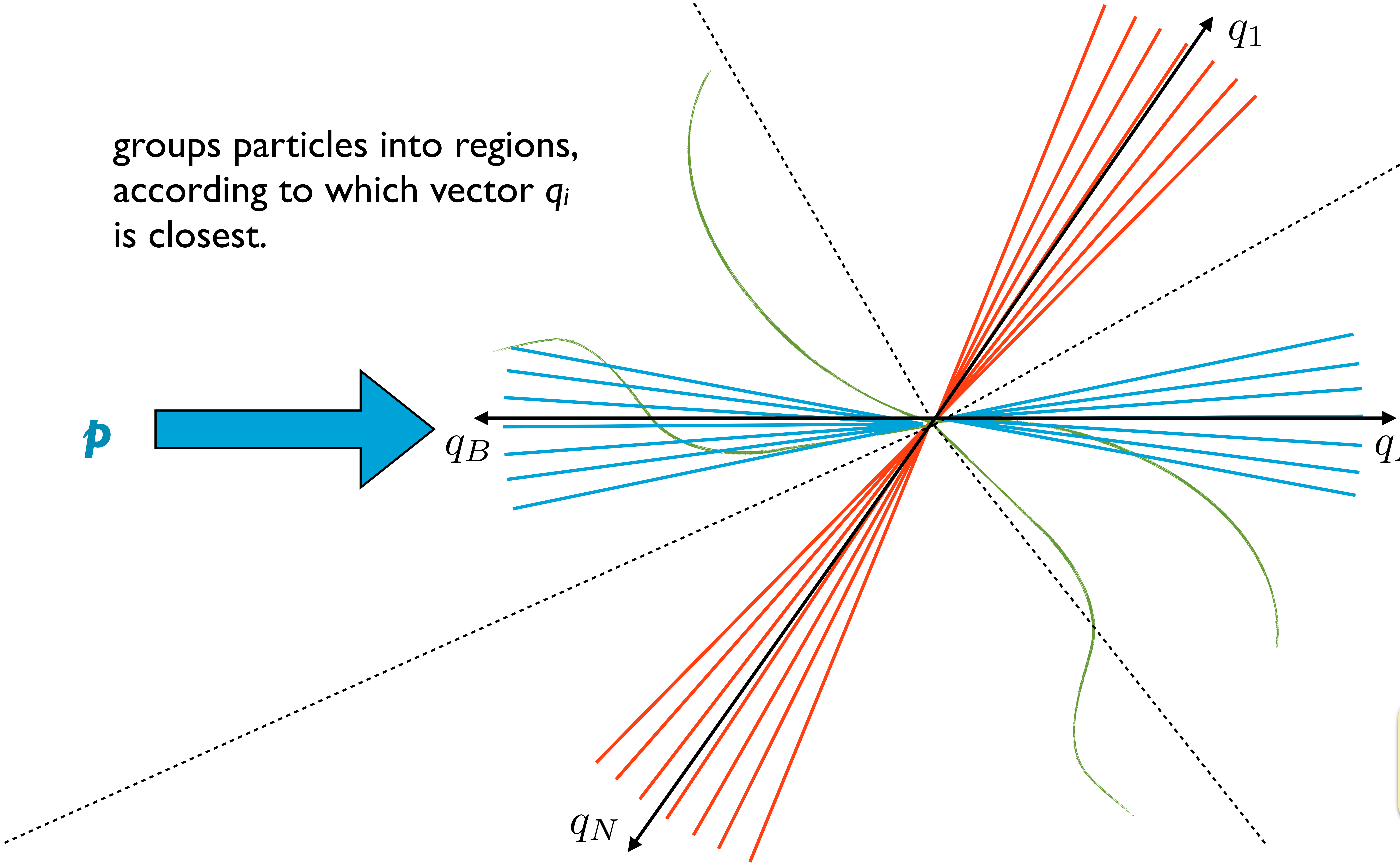
Stewart, Tackmann, Waalewijn (2010)

- A global event shape measuring degree to which final state is N-jet-like.

$$\tau_N = \frac{2}{Q^2}\sum_k \min\{q_A \cdot p_k, q_B \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k\}$$

 # beams **# jets**

groups particles into regions, according to which vector q_i is closest.

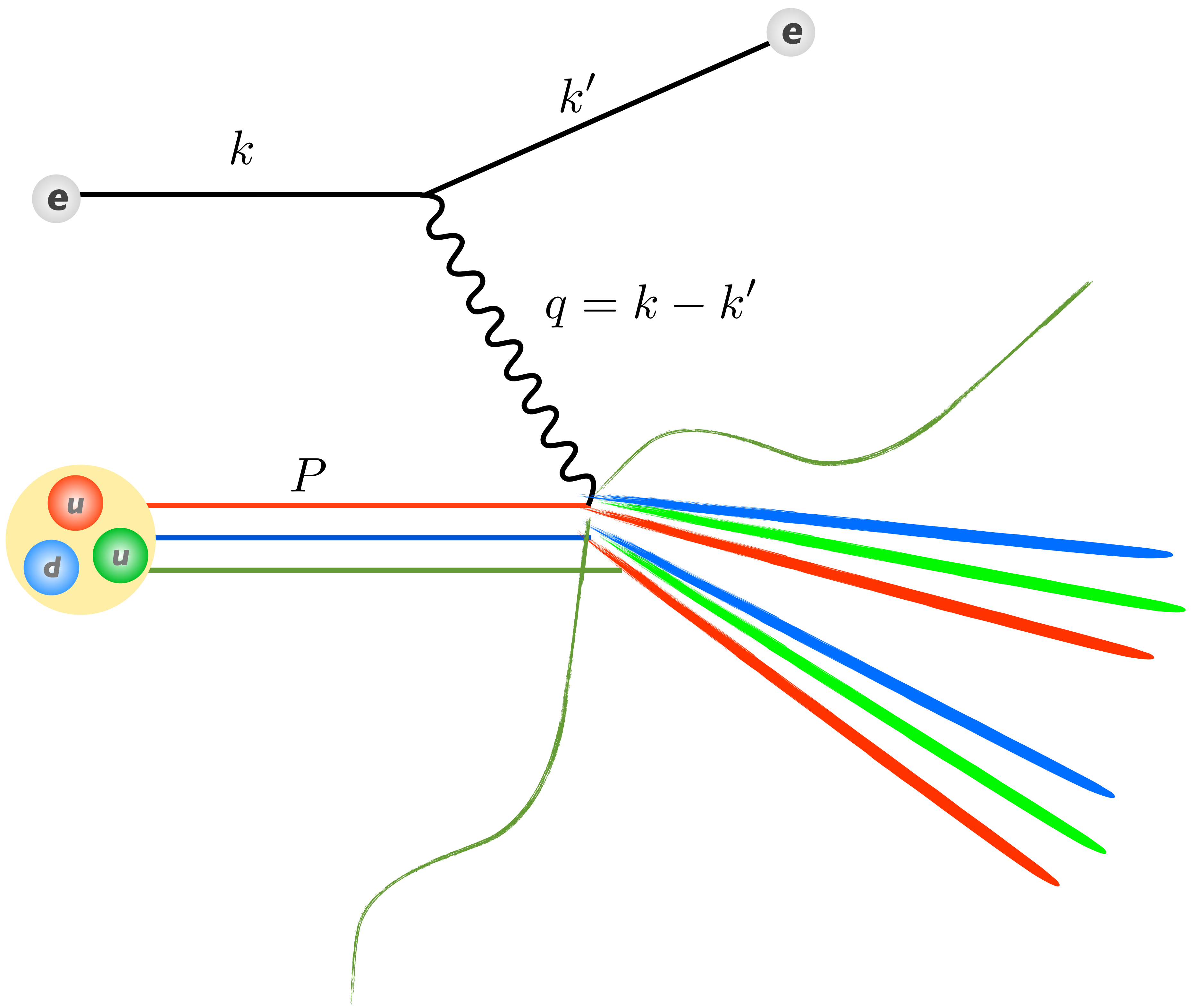


Factorization and Resummation-friendly

1-Jettiness in DIS

D. Kang, CL, I. Stewart [1303.6952]

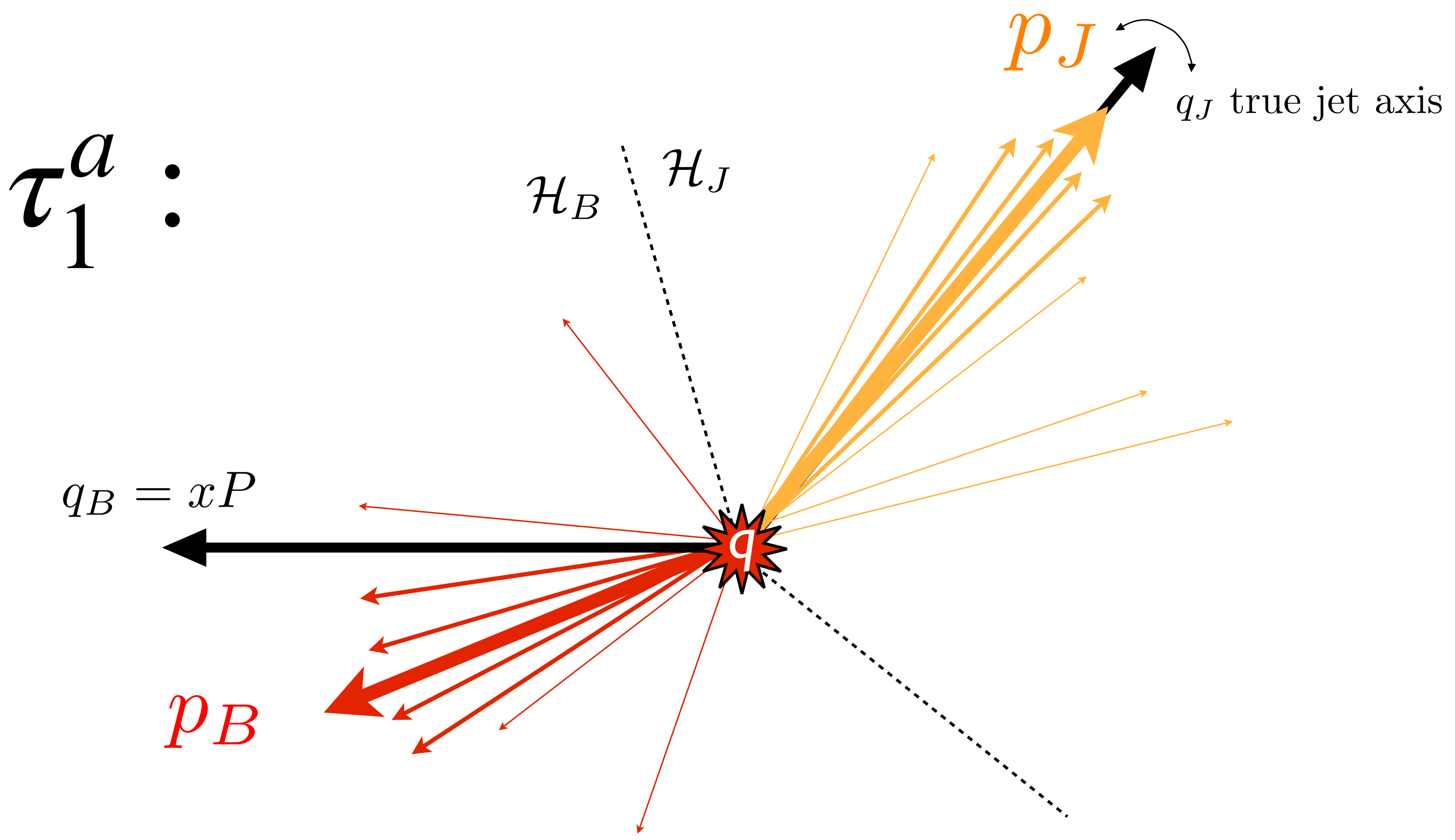
also Z. Kang, Liu, Mantry, Qiu [1204.5469, 1303.3063,1312.0301]
Considered eA collisions too



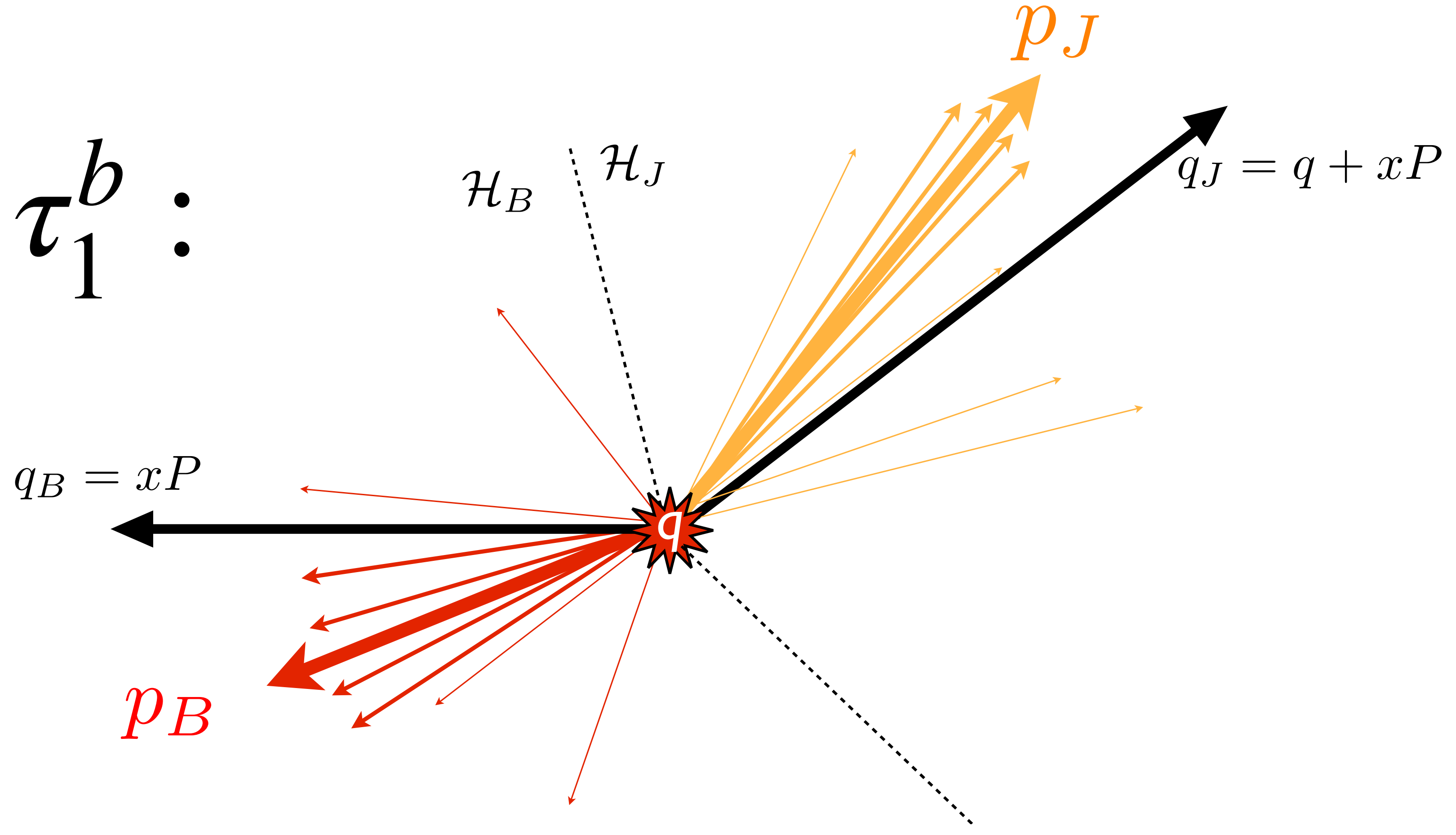
- "1-jettiness" in DIS measures final states with beam radiation + one additional jet

$$\tau_1 = \frac{2}{Q^2} \sum_{i\in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

τ_1^a :



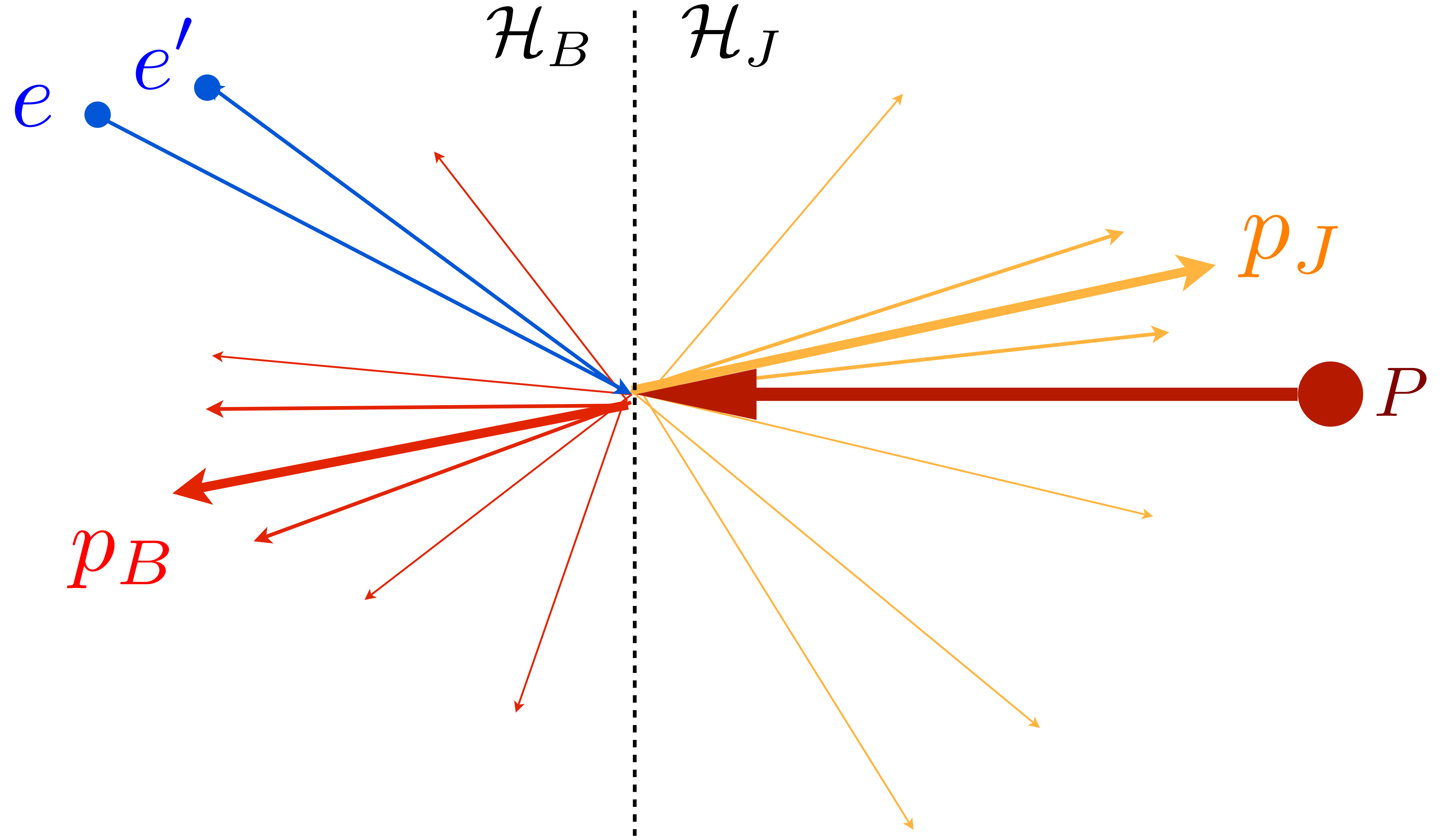
τ_1^b :



- Different choices of axes are possible: different sensitivity to ISR transverse momentum

DIS thrust

τ_1^b :



In the Breit frame:

$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

$$\overset{\text{Breit}}{=} 1 - \frac{2}{Q}\sum_{i\in\mathcal{H}_J^b} p_z^i$$

$$q_B = xP$$

$$q_J = q + xP$$

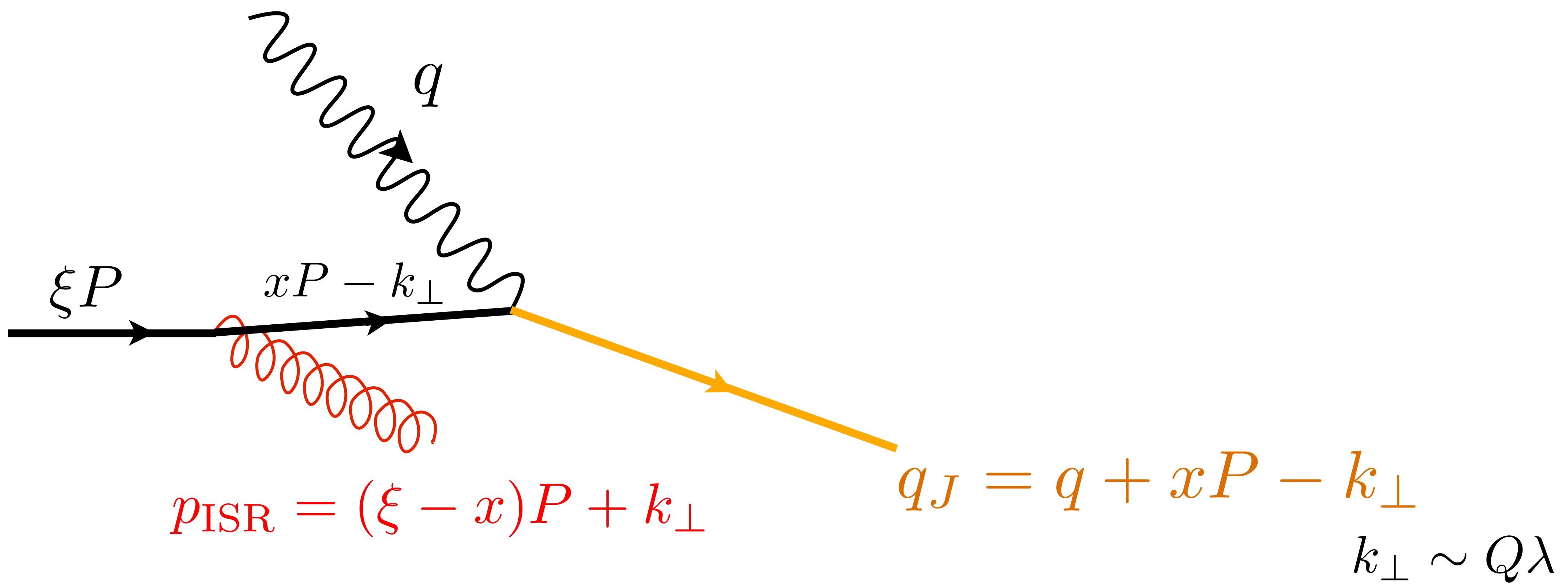
same as DIS thrust of Antonelli, Dasgupta, Salam (1999)

ultimately depends only on momentum in jet or "current" hemisphere

(thanks to momentum conservation)

(not true of τ_1^a)

sensitive to ISR transverse momentum:



D. Kang, CL, Stewart
[1407.6706]

Fixed-order computation

Cross section:

$$\frac{d\sigma}{dx\, dQ^2\, d\tau} = L_{\mu\nu}(x, Q^2) W^{\mu\nu}(x, Q^2, \tau)$$

Hadronic tensor:

$$W^{\mu\nu}(x, Q^2, \tau) = \int d^4x\, e^{iq\cdot x} \langle P | J^{\mu\dagger}(x) \delta(\tau - \hat{\tau}) J^\nu(0) | P \rangle$$

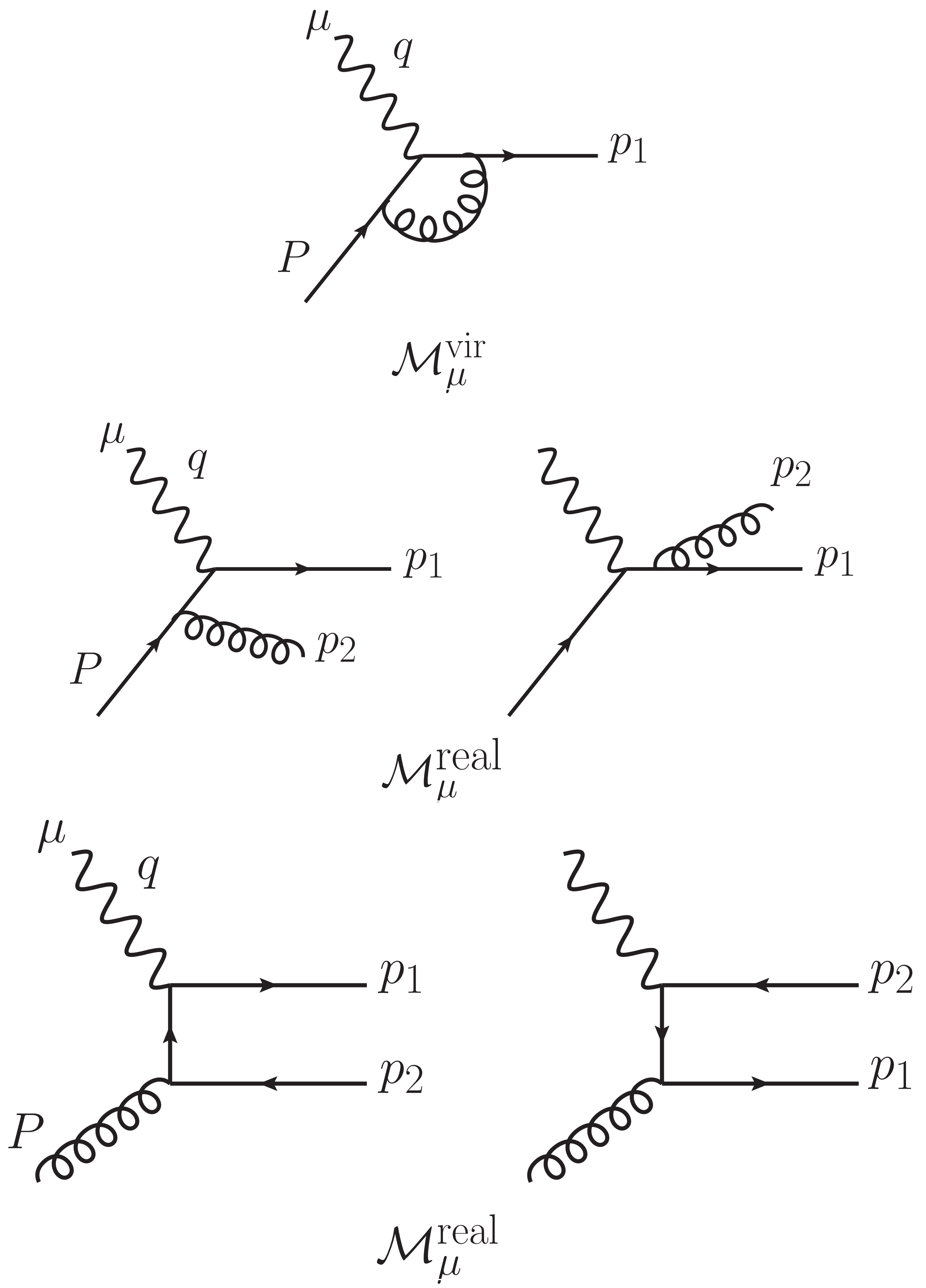
Measure thrust of final state:

$$W^j_{\mu\nu}(x, Q^2, \tau) = \frac{1}{s_j} \sum_n \int d\Phi_n \mathcal{M}^*_\mu(j(P) \to p_1 \dots p_n) \mathcal{M}_\nu(j(P) \to p_1 \dots p_n) \times (2\pi)^D \delta^D(P + q - \sum_i p_i) \delta\Big(\tau - \tau(\{p_1 \dots p_n\})\Big) \equiv \sum_n W^{j[n]}_{\mu\nu}$$

2-particle phase space:

$$W^{j[2]}_{\mu\nu} = \frac{1}{s_j} \frac{1}{8\pi} \left(\frac{4\pi}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left(\frac{x}{1-x}\right)^\epsilon \int_0^1 \frac{dv}{v^\epsilon (1-v)^\epsilon} \mathcal{M}^*_\mu \mathcal{M}_\nu \delta\Big(\tau - \tau(x, v)\Big)$$

Diagrams to $\mathcal{O}(\alpha_s)$:



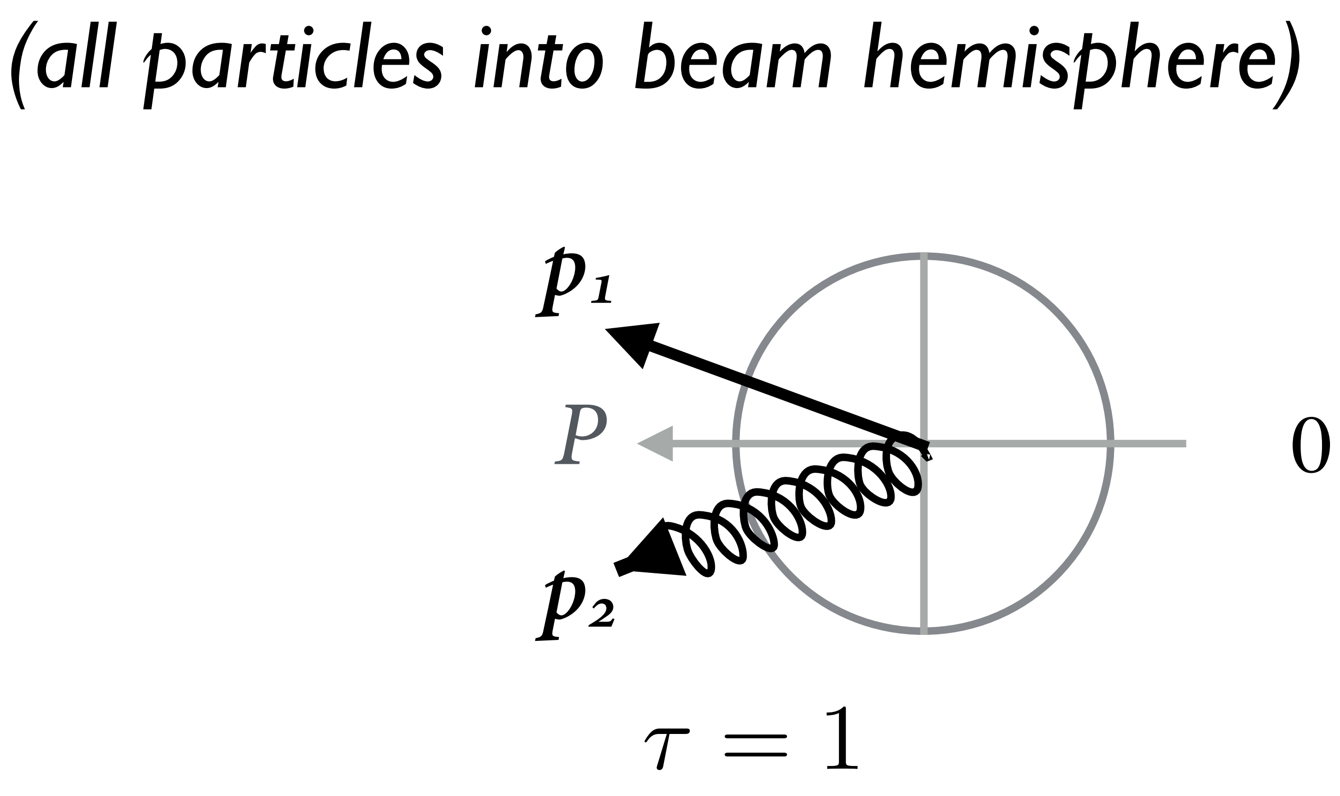
$$p_1^\mu = Qv\frac{n_z^\mu}{2} + Q(1-v)\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} + p_\perp^\mu$$

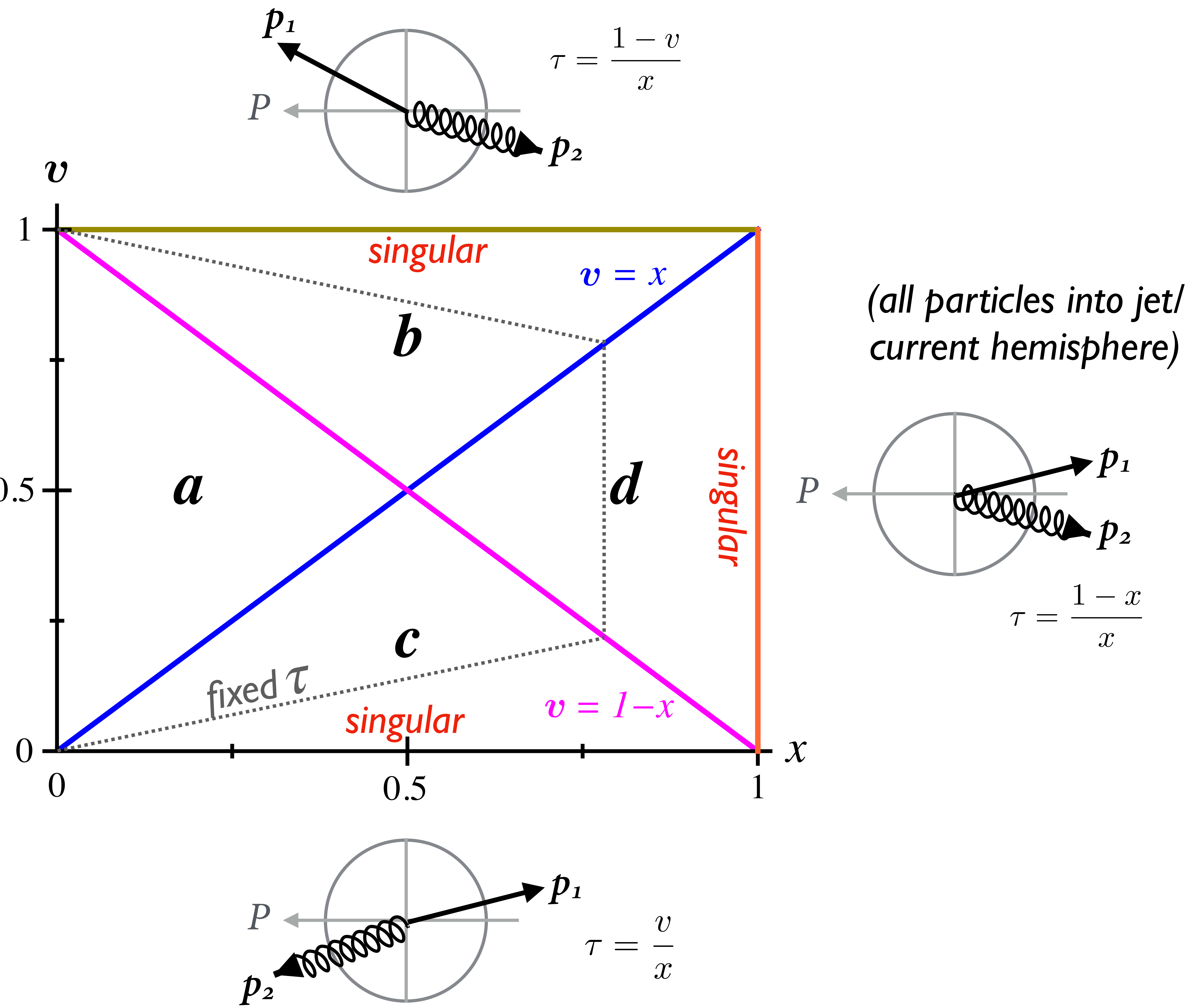
$$p_2^\mu = Q(1-v)\frac{n_z^\mu}{2} + Qv\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} - p_\perp^\mu$$

Fixed-order computation

D. Kang, CL, Stewart
[1407.6706]

2-particle phase space:





$$p_1^\mu = Qv\frac{n_z^\mu}{2} + Q(1-v)\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} + p_\perp^\mu$$

$$p_2^\mu = Q(1-v)\frac{n_z^\mu}{2} + Qv\frac{1-x}{x}\frac{\bar{n}_z^\mu}{2} - p_\perp^\mu$$

Fixed-order results

D. Kang, CL, Stewart [1407.6706]

Structure functions:

$$W^{\mu\nu}(x,Q^2,\tau) = 4\pi\left[T_1^{\mu\nu}\mathcal{F}_1(x,Q^2,\tau) + T_2^{\mu\nu}\frac{\mathcal{F}_2(x,Q^2,\tau)}{P\cdot q}\right] \qquad T_1^{\mu\nu} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\,, \quad T_2^{\mu\nu} = \left(P^\mu - q^\mu\frac{P\cdot q}{q^2}\right)\left(P^\nu - q^\nu\frac{P\cdot q}{q^2}\right)$$

$$\mathcal{F}_L \equiv \mathcal{F}_2 - 2x\mathcal{F}_1$$

Group into singular and non-singular parts:

$$F_1(x,Q^2,\tau) = \sum_{i\in\{q,\bar q,g\}} (A_i + B_i)$$

$$F_L(x,Q^2,\tau) = \sum_{i\in\{q,\bar q,g\}} 4x\,A_i\,.$$

(integrated:)

$$F(x,Q^2,\tau) = \int_0^\tau d\tau' \mathscr{F}(x,Q^2,\tau)$$

$$F_i = F_i^{\rm sing} + F_i^{\rm ns}$$

Singular terms:

$$A_{q,g}^{\rm sing} = 0\,,$$

$$B_q^{\rm sing} = \sum_f Q_f^2\left\{f_q(x)\left[\frac{1}{2} - \frac{\alpha_s C_F}{4\pi}\left(\frac{9}{2} + \frac{\pi^2}{3} + 3\ln\tau + 2\ln^2\tau\right)\right]\right.$$
$$\left. + \frac{\alpha_s C_F}{4\pi}\int_x^1 \frac{dz}{z} f_q(x/z)\left[\mathcal{L}_1(1-z)\,(1+z^2) + (1-z) + P_{qq}(z)\ln\frac{Q^2\tau}{\mu^2}\right]\right\}$$

$$B_g^{\rm sing} = \sum_f Q_f^2 \frac{\alpha_s T_F}{2\pi}\int_x^1 \frac{dz}{z} f_g(x/z)\left[1 - P_{qg}(z) + P_{qg}(z)\ln\frac{Q^2\tau(1-z)}{\mu^2}\right].$$

Need $\ln\tau$ **resummation**

Fixed-order results

Non-singular terms to $\mathcal{O}(\alpha_s)$:

D. Kang, CL, Stewart [1407.6706]

For $\mathcal{O}(\alpha_s^2)$ we use NLOJet++

Z. Nagy [hep-ph/0307268]

$$A_q^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s C_F}{4\pi}\Big\{\Theta_0 \int_x^{\frac{1}{1+\tau}} dz\, f_q(\tfrac{x}{z})(2z\tau - 1) + \int_x^1 dz\, f_q(\tfrac{x}{z})\Big\},$$

$$A_g^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s T_F}{\pi}\Big\{\Theta_0 \int_x^{\frac{1}{1+\tau}} dz f_g(\tfrac{x}{z})(2z\tau - 1)(1-z) + \int_x^1 dz\, f_g(\tfrac{x}{z})(1-z)\Big\},$$

$$B_q^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s C_F}{4\pi}\Big\{\Theta_0 \int_x^{\frac{1}{1+\tau}} \frac{dz}{z} f_q(\tfrac{x}{z})\Big[\frac{1-4z}{2(1-z)}(2z\tau - 1) + P_{qq}(z)\ln\frac{z\tau}{1-z\tau}\Big]$$
$$+ f_q(x)(3\ln\tau + 2\ln^2\tau) + \int_x^1 \frac{dz}{z} f_q(\tfrac{x}{z})\Big[\mathcal{L}_0(1-z)\frac{1-4z}{2} - P_{qq}(z)\ln z\tau\Big]\Big\},$$

$$B_g^{\rm ns} = \sum_f Q_f^2 \frac{\alpha_s T_F}{2\pi}\Big\{\Theta_0 \int_x^{\frac{1}{1+\tau}} \frac{dz}{z} f_g(\tfrac{x}{z})\Big[-(2z\tau - 1) + P_{qg}(z)\ln\frac{z\tau}{1-z\tau}\Big]$$
$$- \int_x^1 \frac{dz}{z} f_g(\tfrac{x}{z})[1 + P_{qg}(z)\ln z\tau]\Big\},$$

$$\Theta_0 \equiv \Theta_0(\tau, x) \equiv \theta(\tau)\theta(1-\tau)\theta\Big(\frac{1-x}{x} - \tau\Big)$$

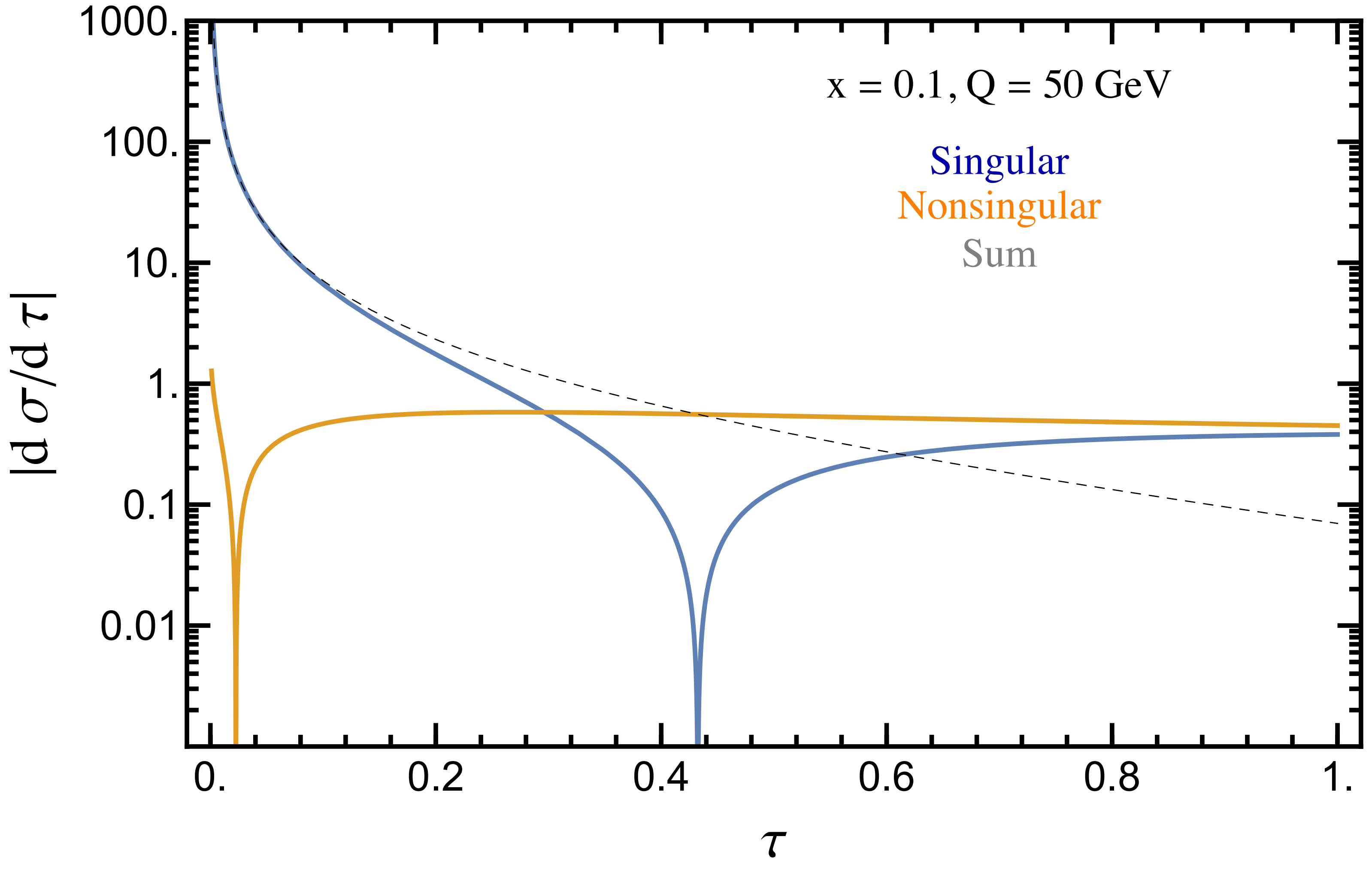
$$P_{qq}(z) \equiv \Big[\theta(1-z)\frac{1+z^2}{1-z}\Big]_+ = (1+z^2)\mathcal{L}_0(1-z) + \frac{3}{2}\delta(1-z)$$

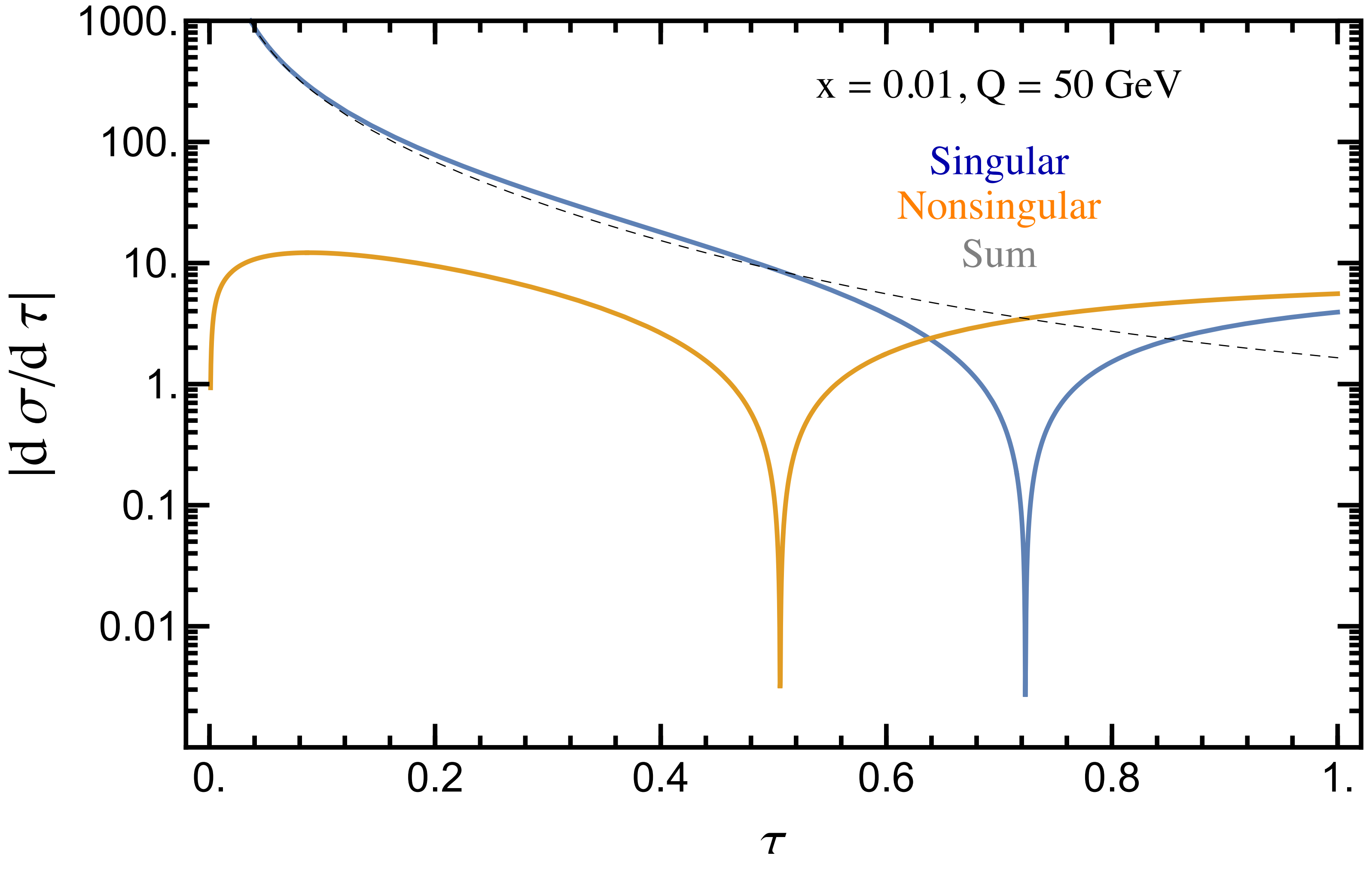
$$P_{qg}(z) \equiv \theta(1-z)[(1-z)^2 + z^2].$$

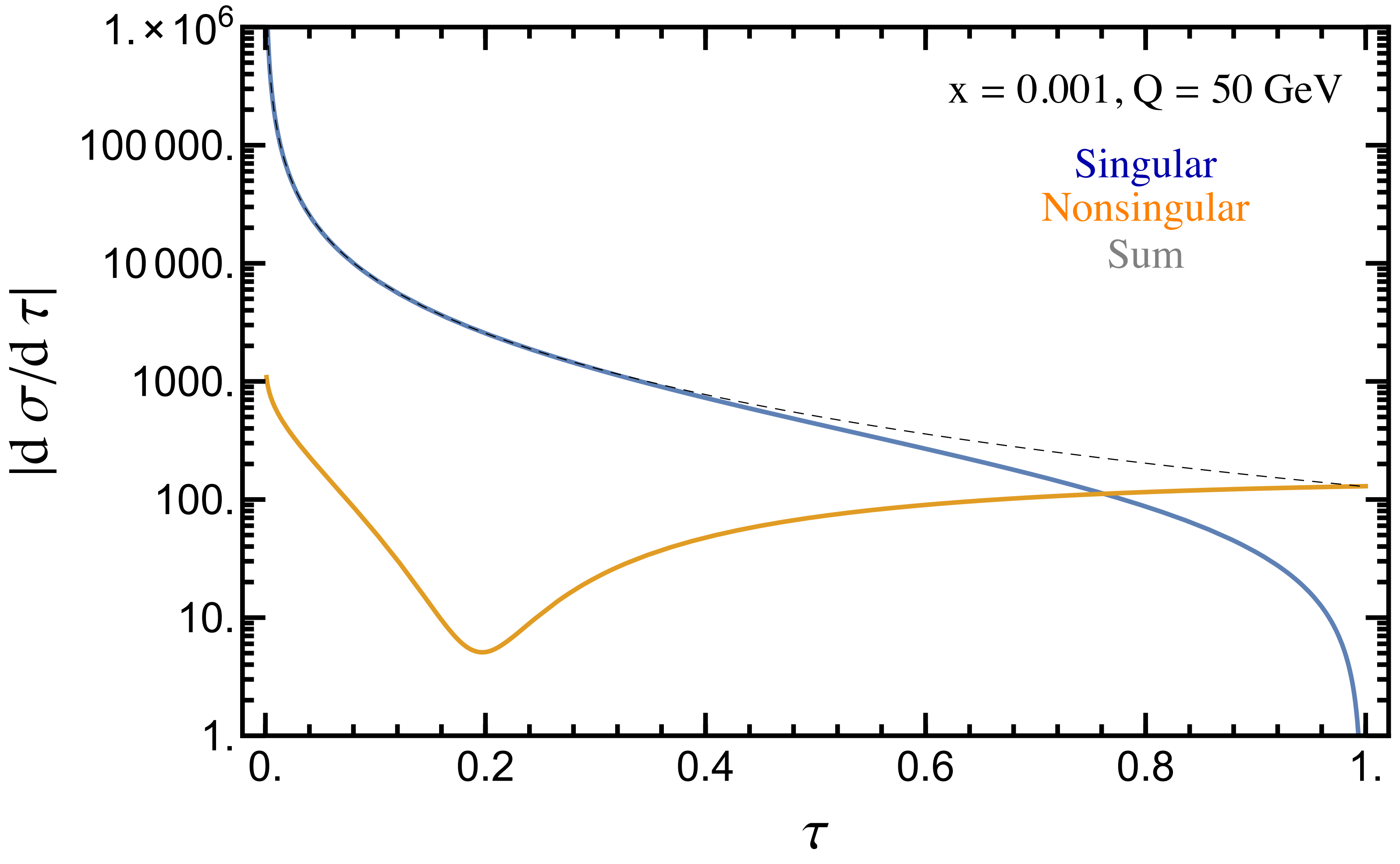
Singular vs. non-singular

D. Kang, CL, Stewart (2014)

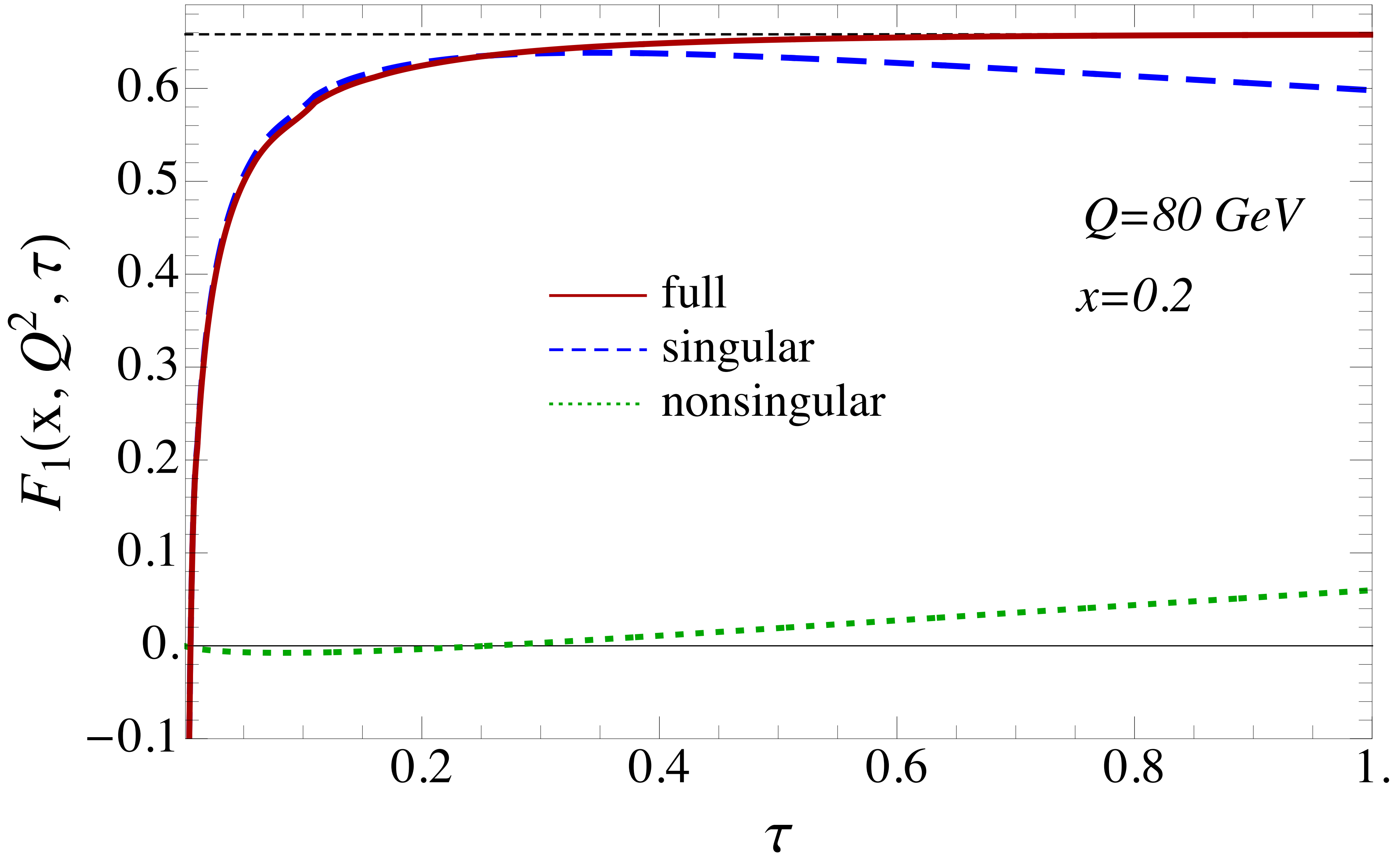
Contributions to differential thrust spectrum:







Add up to total integrated cross section:



Singular vs. non-singular

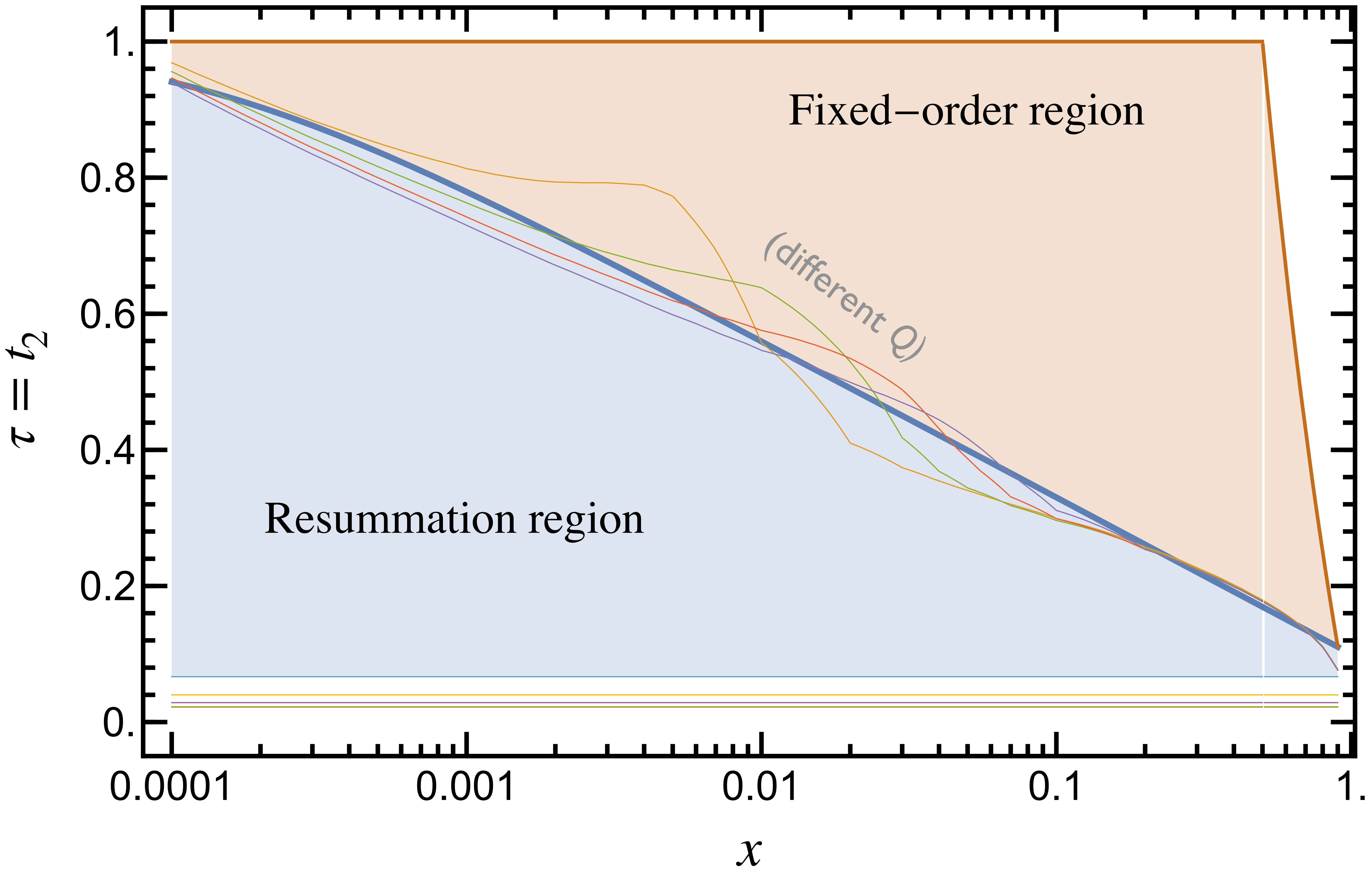
Region where resummation is important is thus a function of *x*:

Crossing point between singular and non-singular contributions is, empirically, about:

$$t_2 = \frac{1 - \log(x + x_c)}{10}$$

$$x_c = 0.0001234$$

[Based on $\mathcal{O}(\alpha_s)$ results; Remains similar at $\mathcal{O}(\alpha_s^2)$]



Large Logs

- If we calculate event shape τ cross section in QCD perturbation theory, we will find:

$$\int_0^\tau d\tau \frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau} \sim \Big[1 + \frac{\alpha_s}{4\pi}\Big(F_{12}\ln^2\tau - F_{11}\ln\tau + F_{10}\Big) + \Big(\frac{\alpha_s}{4\pi}\Big)^2\Big(F_{24}\ln^4\tau + F_{23}\ln^3\tau + F_{22}\ln^2\tau + F_{21}\ln\tau + F_{20}\Big) + \dots\Big]$$

- In the narrow-jet limit $\tau \rightarrow 0$ the logs grow large and spoil the perturbative expansion. Reorganize the expansion:

$$\begin{aligned}\ln\sigma(\tau) \sim \;& \alpha_s(\ln^2\tau + \ln\tau) \\ &+ \alpha_s^2(\ln^3\tau + \ln^2\tau + \ln\tau) \\ &+ \alpha_s^3(\ln^4\tau + \ln^3\tau + \ln^2\tau + \ln\tau) \\ &+ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots\end{aligned}$$

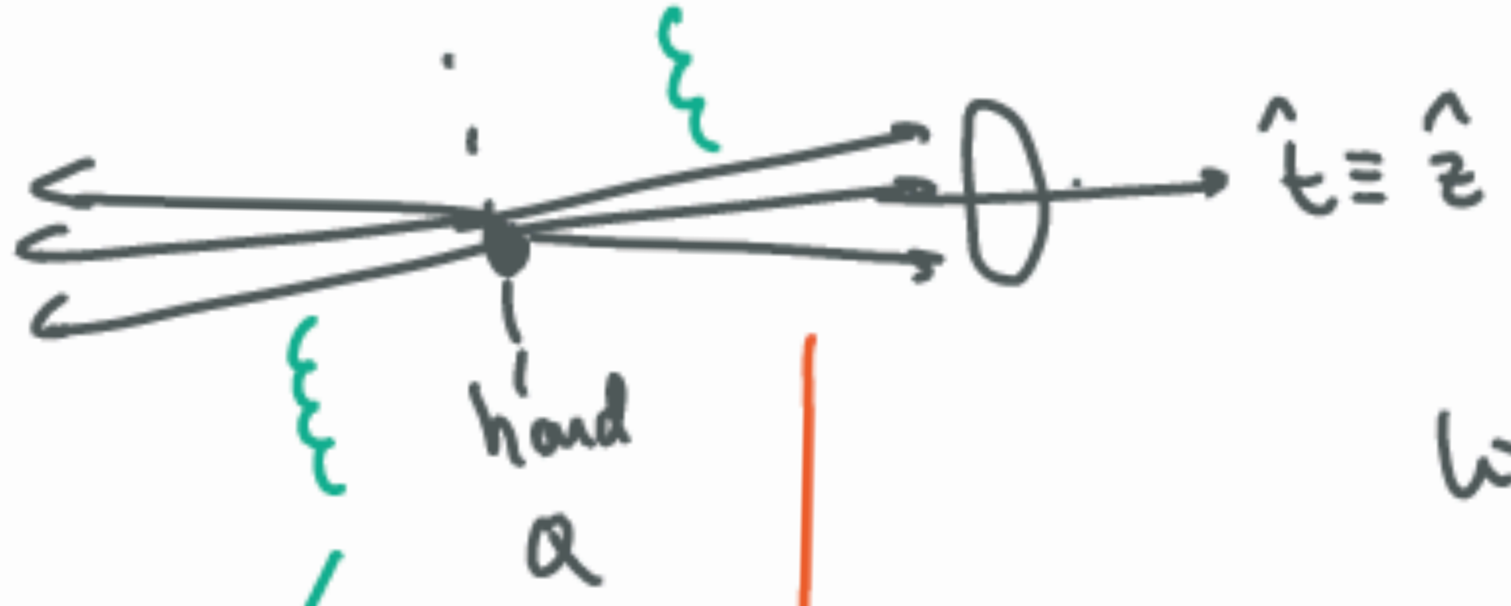
Leading Log (LL)	Next-to-Leading Log (NLL)	NNLL	N3LL
$\sim \alpha^{-1}$	~ 1	$\sim \alpha$	$\sim \alpha^2$

New power counting when $\ln\tau \sim \frac{1}{\alpha}$: $\sim \alpha^{-1}$ $\quad \sim 1$ $\quad \sim \alpha$ $\quad \sim \alpha^2$

- **These logs are of large ratios of disparate physical scales**
- **Need to identify and factor these scales**
- **Use RG evolution to resum the logs**

Momentum scales

Thrust: $M^2 = M_A^2 + M_B^2 = Q^2 \tau \quad (\text{if} \ll Q^2)$



$n = (1, +\hat{z})$
$\bar{n} = (1, -\hat{z})$

light-cone coordinates:

$$p^\mu = (\bar{n}\cdot p,\, n\cdot p,\, \vec{p}_\perp)$$

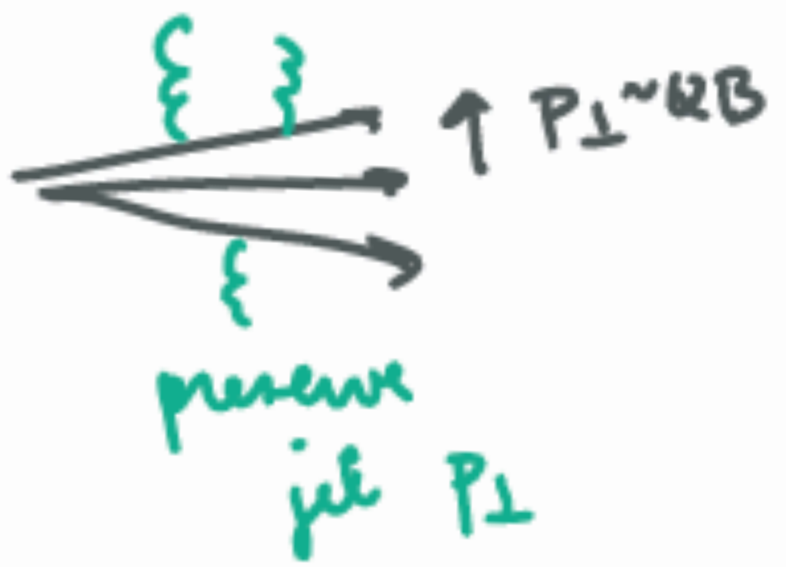
preserve jet mass

Collinear $p_c \sim \left(Q, \frac{M^2}{Q}, M\right) \sim Q(1, \tau, \sqrt{\tau})$

↕ same

soft $k_s \sim \left(\frac{M^2}{Q}, \frac{M^2}{Q}, \frac{M^2}{Q}\right) \sim Q(\tau, \tau, \tau)$

Broadening:



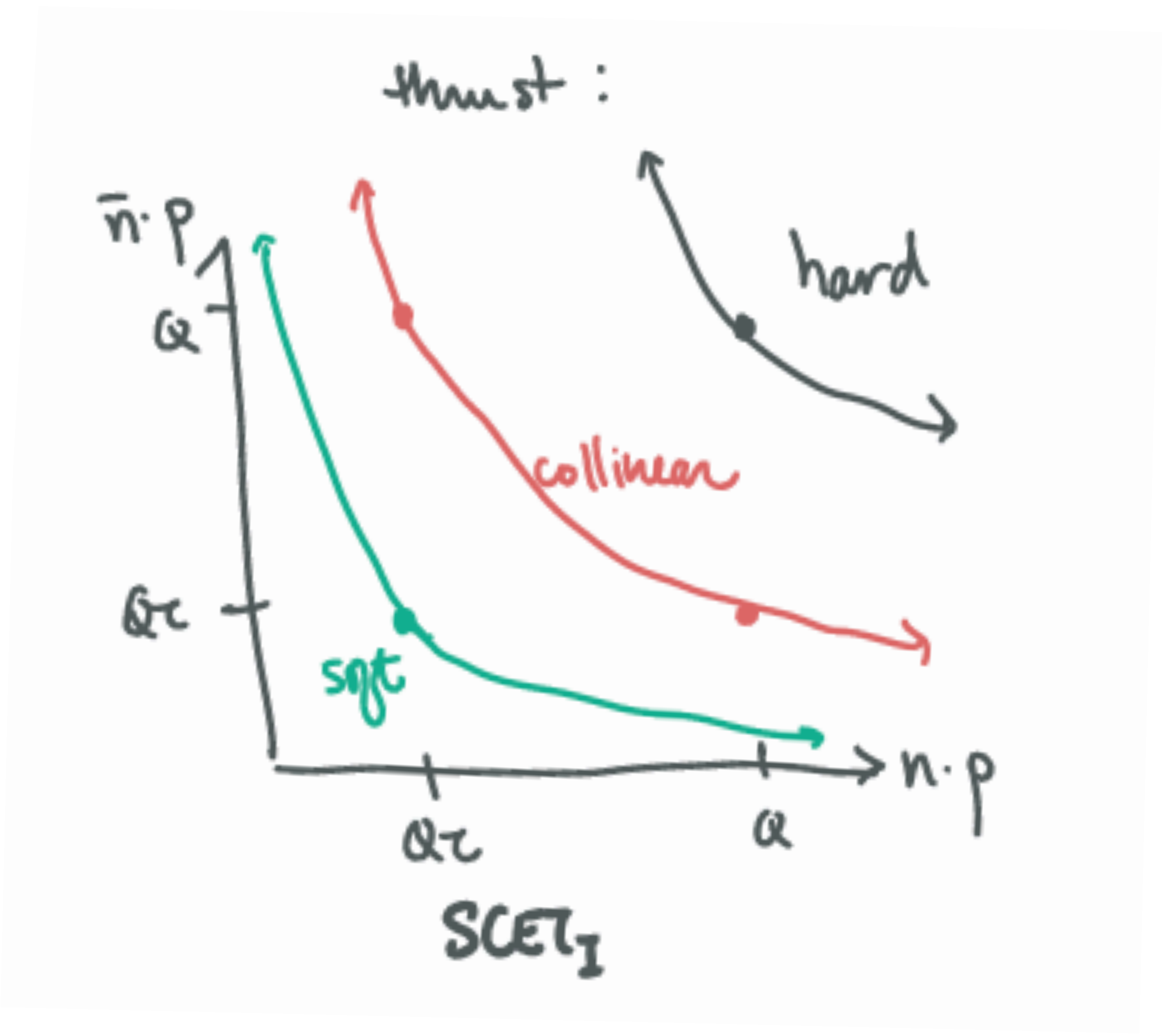
coll $p_c \sim (Q, QB^2, QB)$

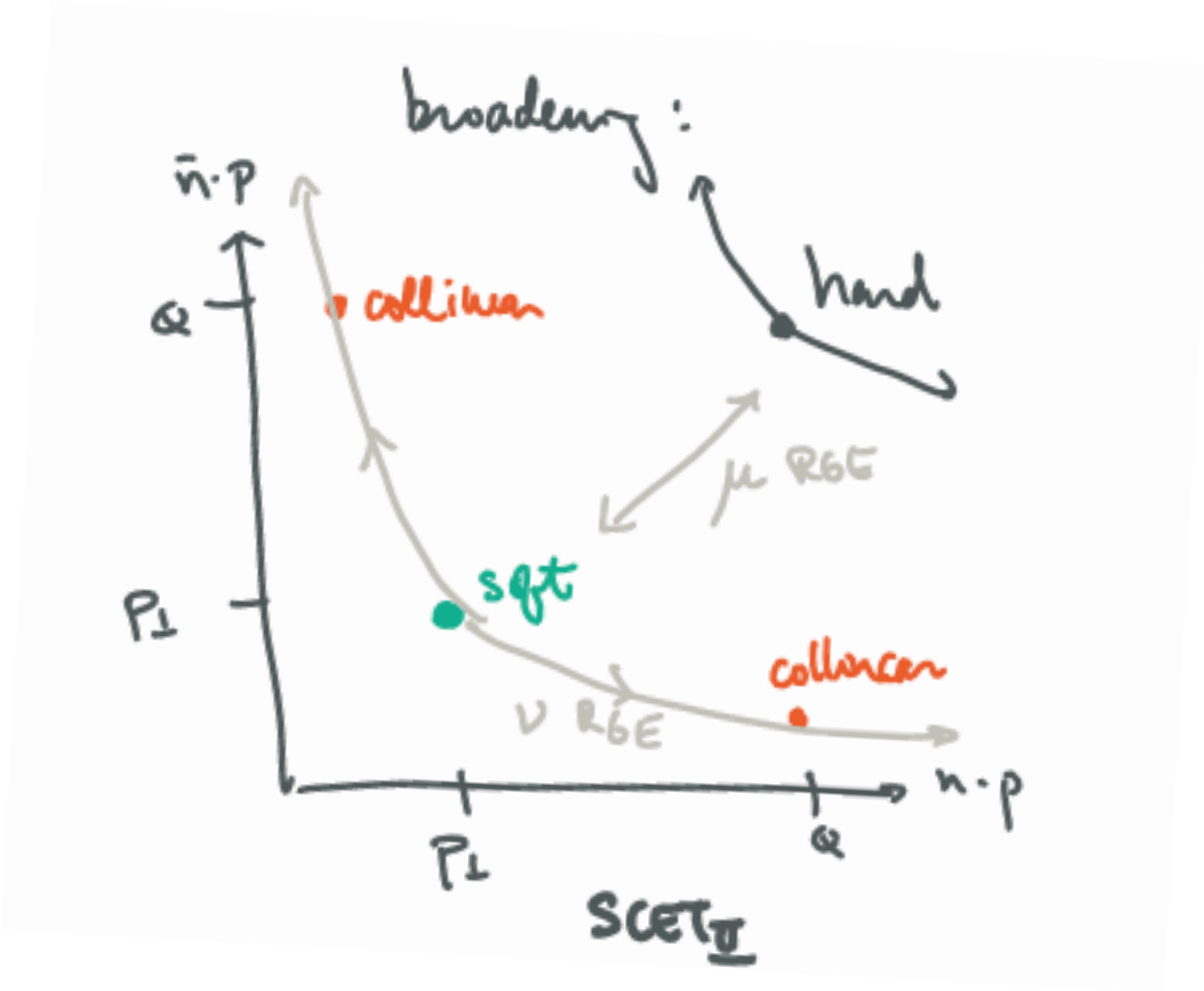
↕ same

soft $k_s \sim Q(B, B, B)$

SCET modes

Bauer, Fleming, Luke, Pirjol, Stewart (2000-02)





Chiu, Jain, Neill, Rothstein (2011-12)

Factorization Theorem for DIS thrust

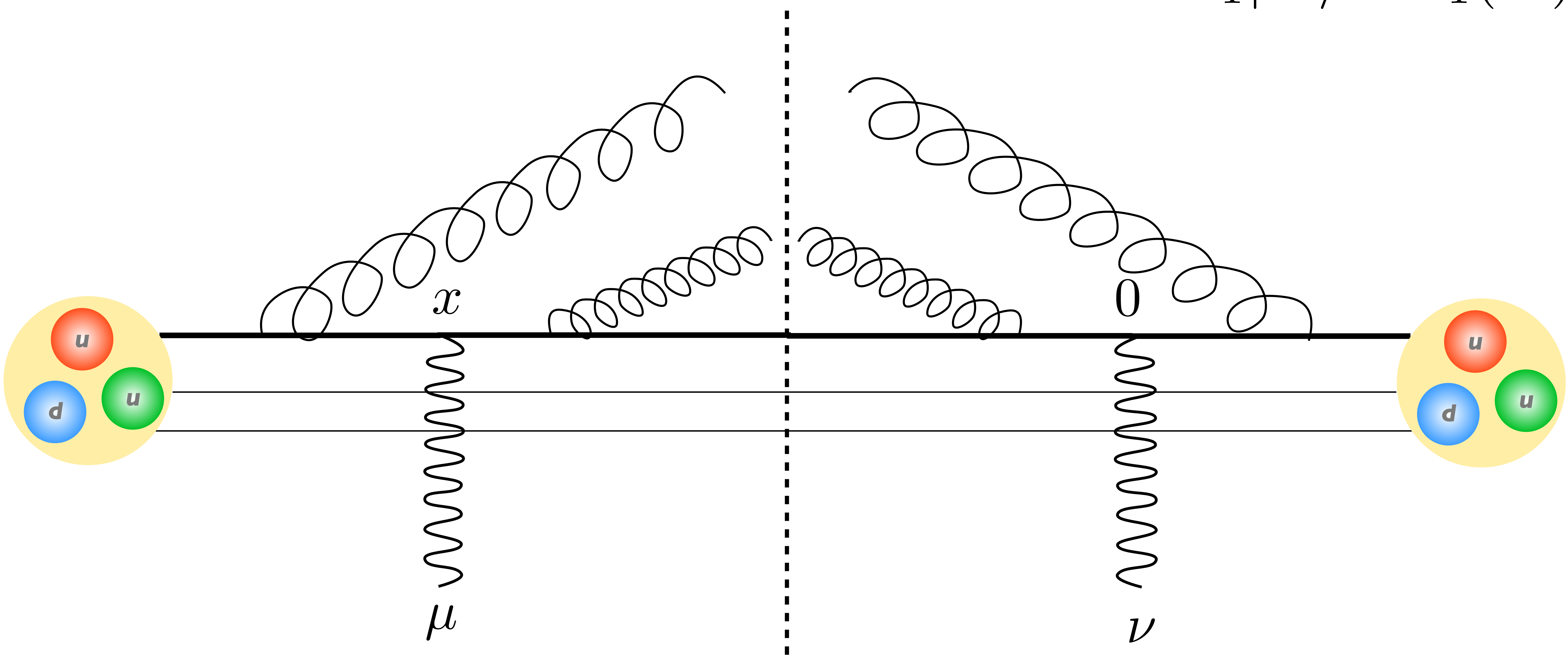
Start in QCD:

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2)W^{\mu\nu}(x,Q^2,\tau_1)$$

leptonic tensor hadronic tensor

$$W^{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x}\langle P|\bar{q}\gamma^\mu q(x)\delta(\tau_1-\hat{\tau}_1)\bar{q}\gamma^\nu q(0)|P\rangle$$

$$\hat{\tau}_1|X\rangle = \tau_1(X)|X\rangle$$



Measure τ_1 of particles crossing the cut

Factorization Theorem for DIS thrust

Match onto 2-jet operators in SCET:

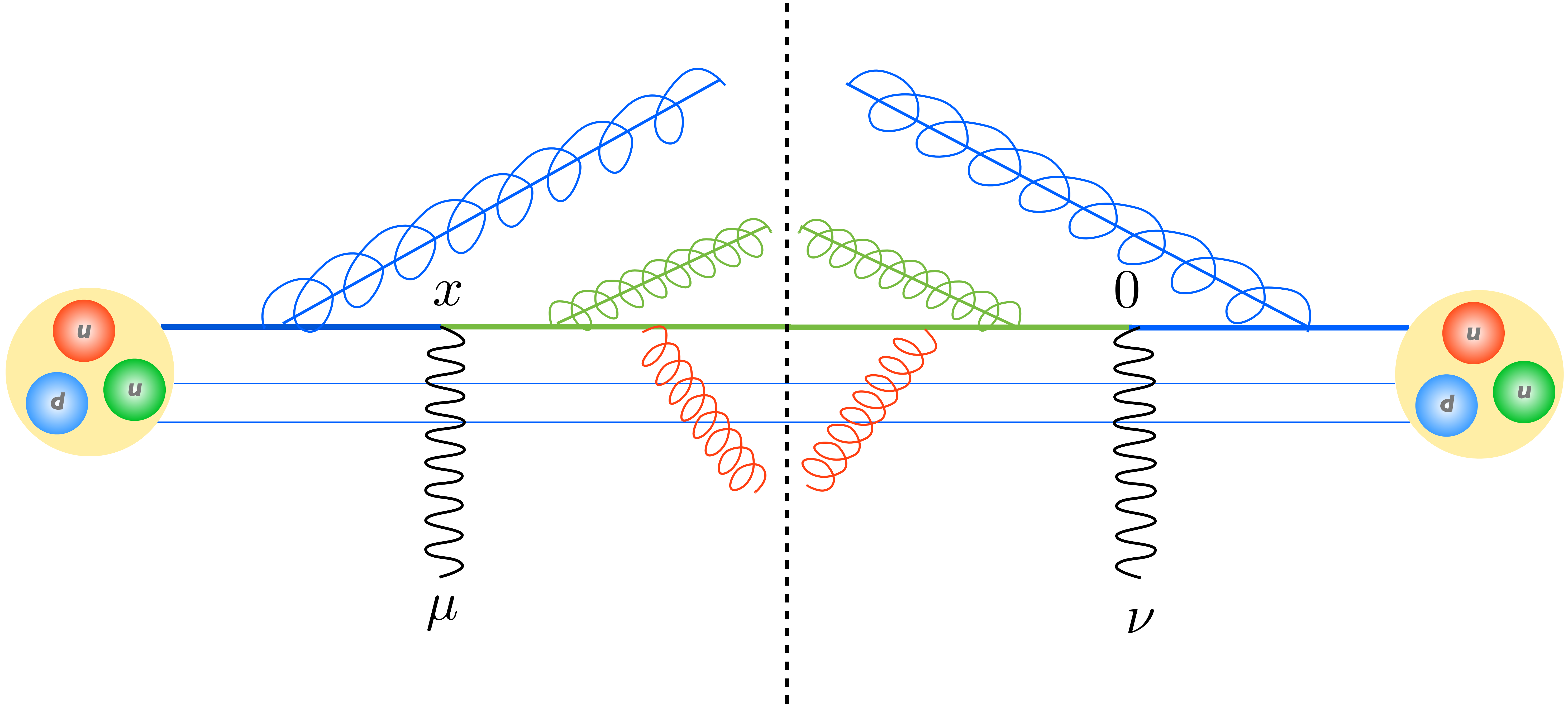
$$W_{\mu\nu}(x, Q^2, \tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2 - \tilde{p}_1)\cdot x} C^*_\mu(\tilde{p}_1, \tilde{p}_2) C_\mu(\tilde{p}_1, \tilde{p}_2)$$
$$\times \langle P_{n_B} | \bar{\chi}_{n_2,\tilde{p}_2}(x) \overline{T}[Y^\dagger_{n_2}(x) Y_{n_1}(x)] \chi_{n_1,\tilde{p}_1}(x)$$
$$\times \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times \bar{\chi}_{n_1,\tilde{p}_1}(0) T[Y^\dagger_{n_1}(0) Y_{n_2}(0)] \chi_{n_2,\tilde{p}_2}(0) | P_{n_B} \rangle$$

collinear jet operators in SCET

$$\chi_n = [W_n \xi_n]$$

collinear Wilson line — collinear quark field

$Y_{n_{1,2}}$ soft gluon Wilson lines



Factorization Theorem for DIS thrust

Match onto 2-jet operators in SCET:

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C_\mu^*(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$
$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y_{n_2}^\dagger(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$
$$\times \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times \bar{\chi}_{n_1,\tilde{p}_1}(0)T[Y_{n_1}^\dagger(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

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collinear Wilson line — collinear quark field

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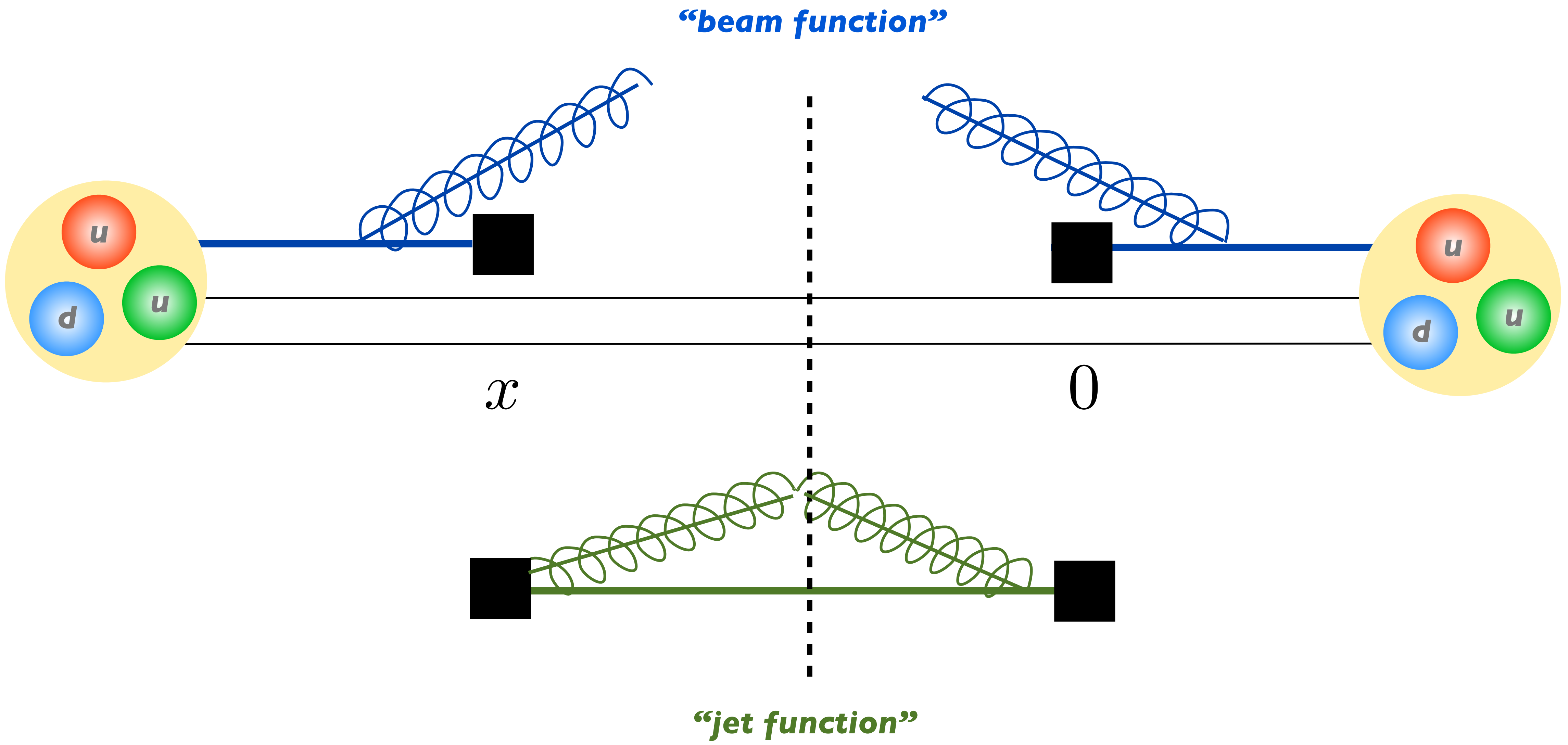
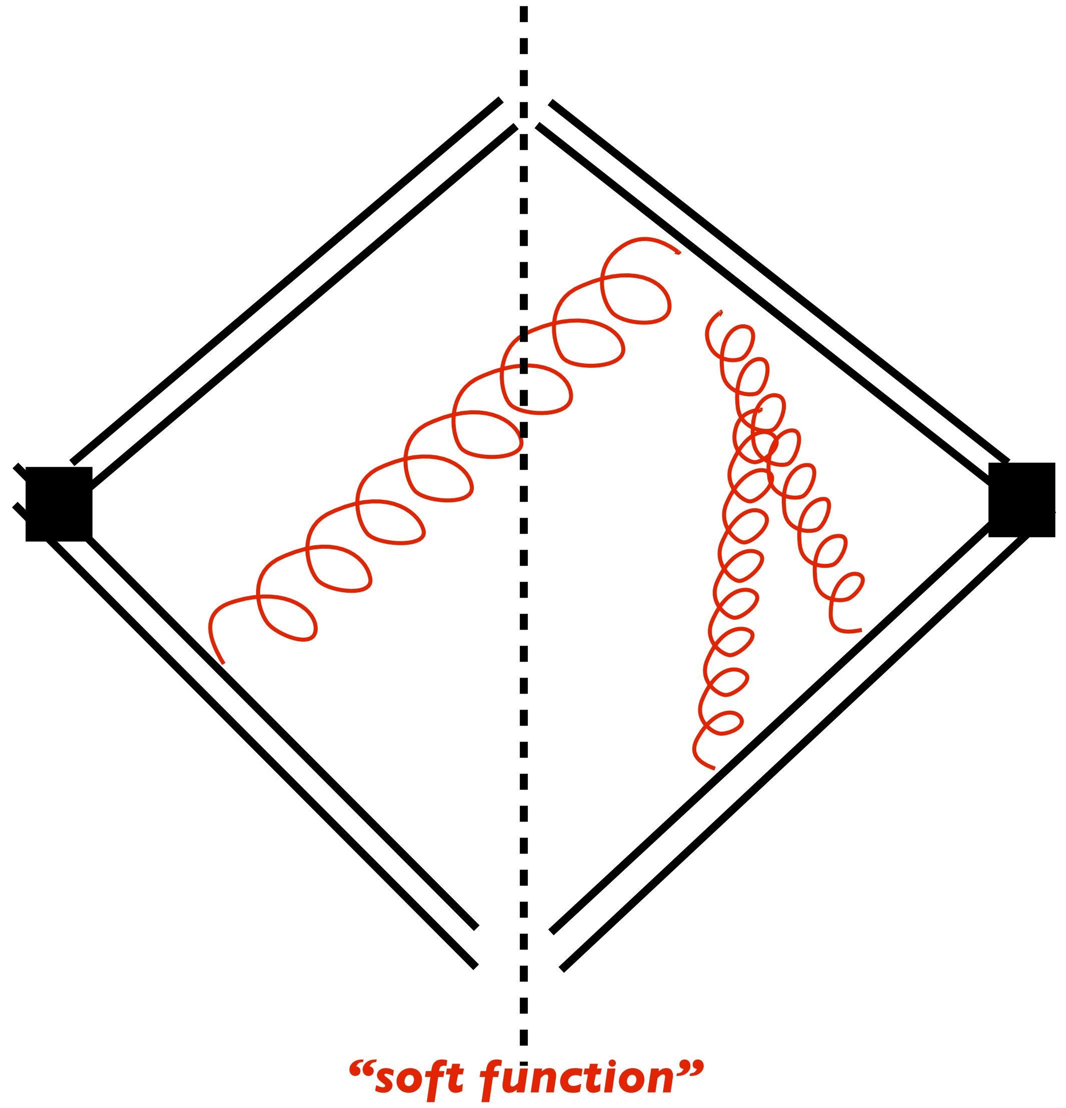


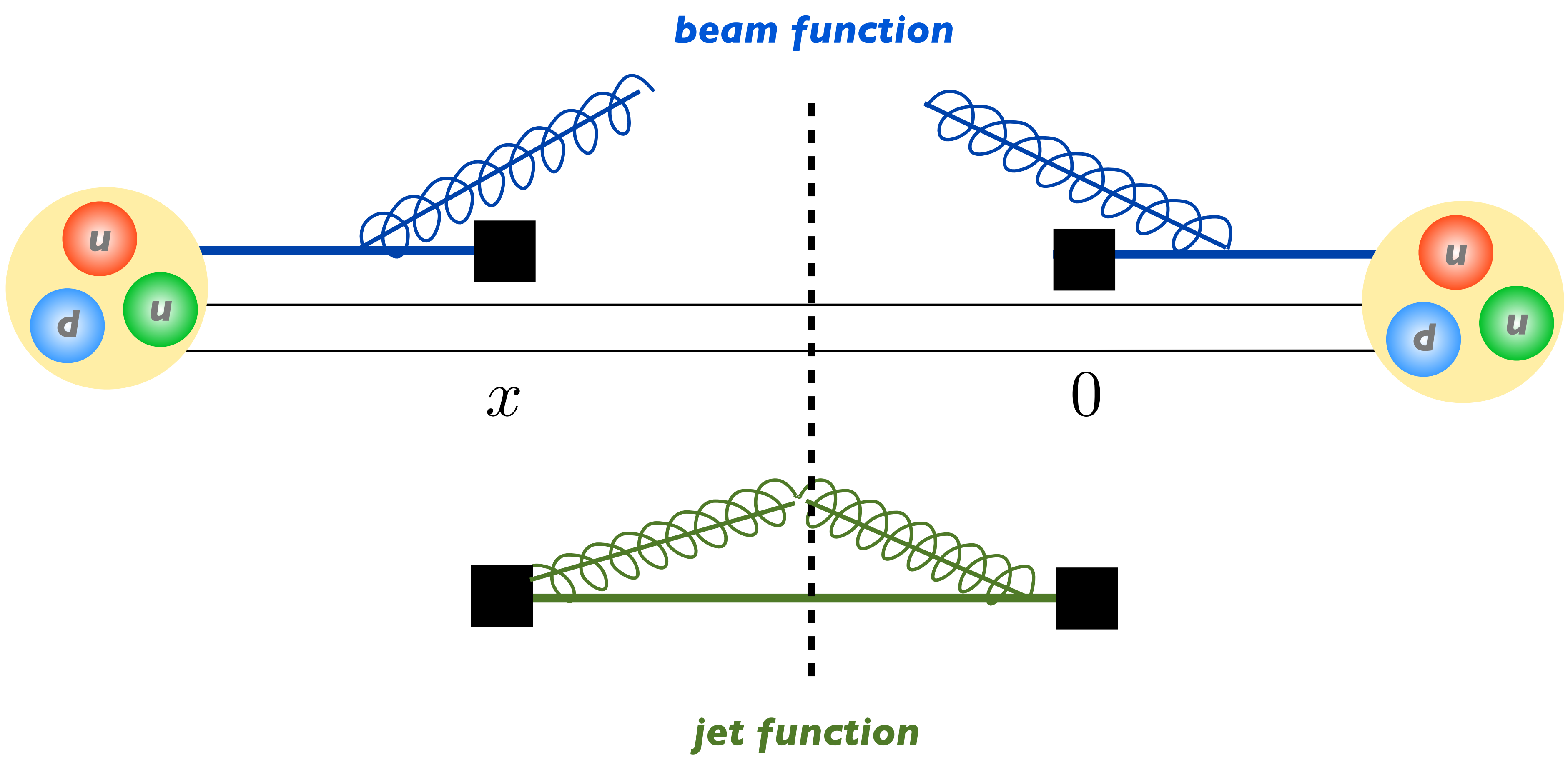


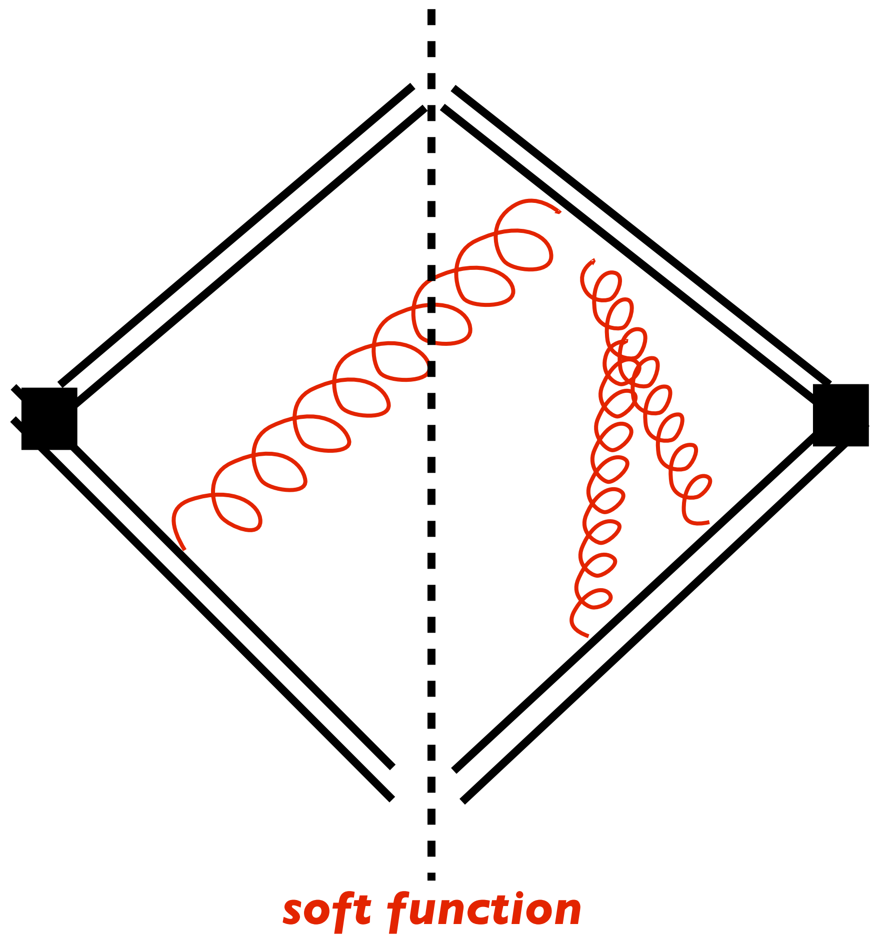
Factorization Theorem for DIS thrust

Factor collinear and soft matrix elements:

$$
\begin{aligned}
W_{\mu\nu}(x,Q^2,\tau_1) = &\int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S\; C^*(Q^2,\mu)C(Q^2,\mu)\;\delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big) \\
&\times \langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B} Y_{n'_J}](0)|0\rangle \\
&\times \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle \\
&\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle
\end{aligned}
$$

(+ permutations)





$$
\begin{aligned}
\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = &\,H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \\
&\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)S(k_S,\mu)
\end{aligned}
$$

Factorization Theorem for DIS thrust

Factor collinear and soft matrix elements:

hard function

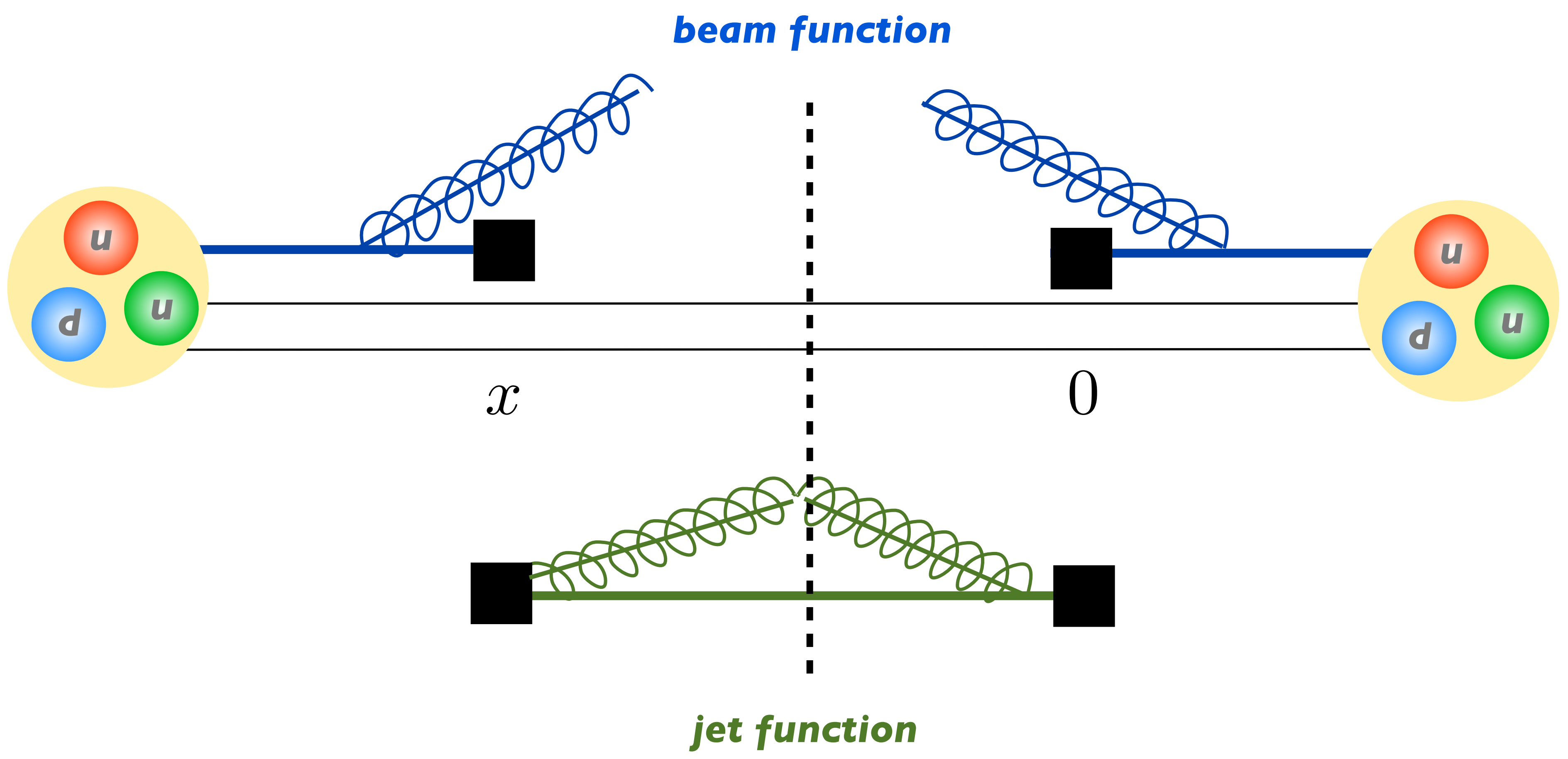
$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S \, C^*(Q^2,\mu)C(Q^2,\mu)\,\delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big)$$

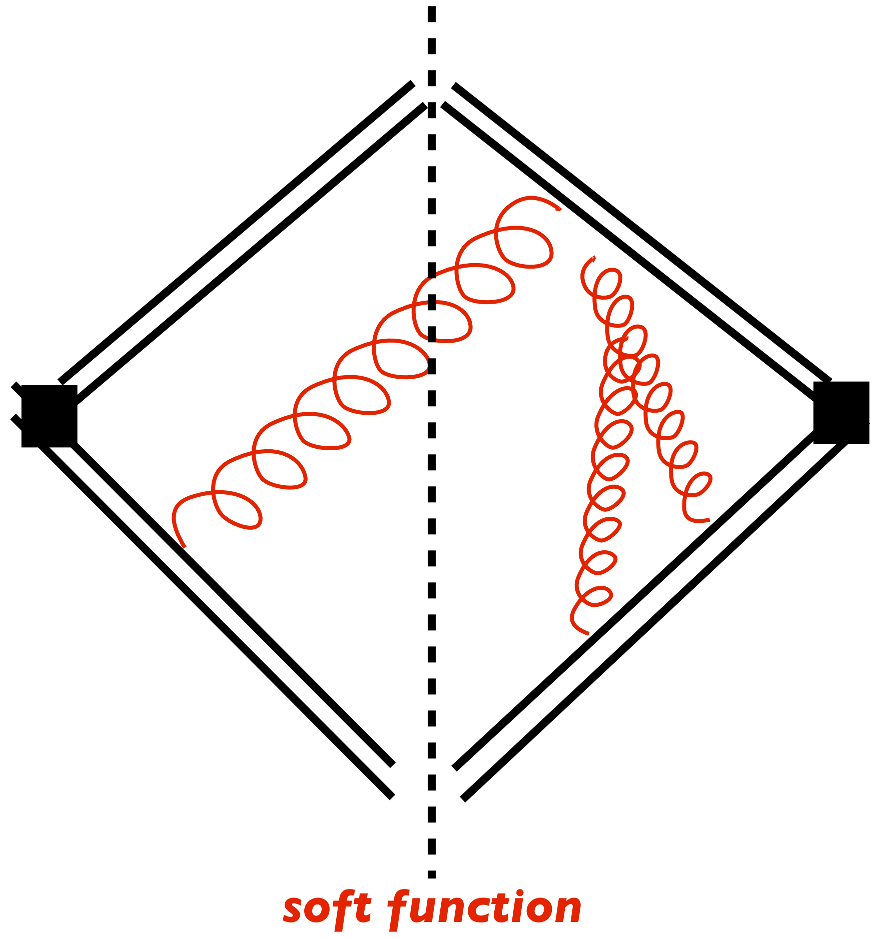
soft function $\times \langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B}Y_{n'_J}](0)|0\rangle$

beam function $\times \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$

jet function $\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$

(+ permutations)

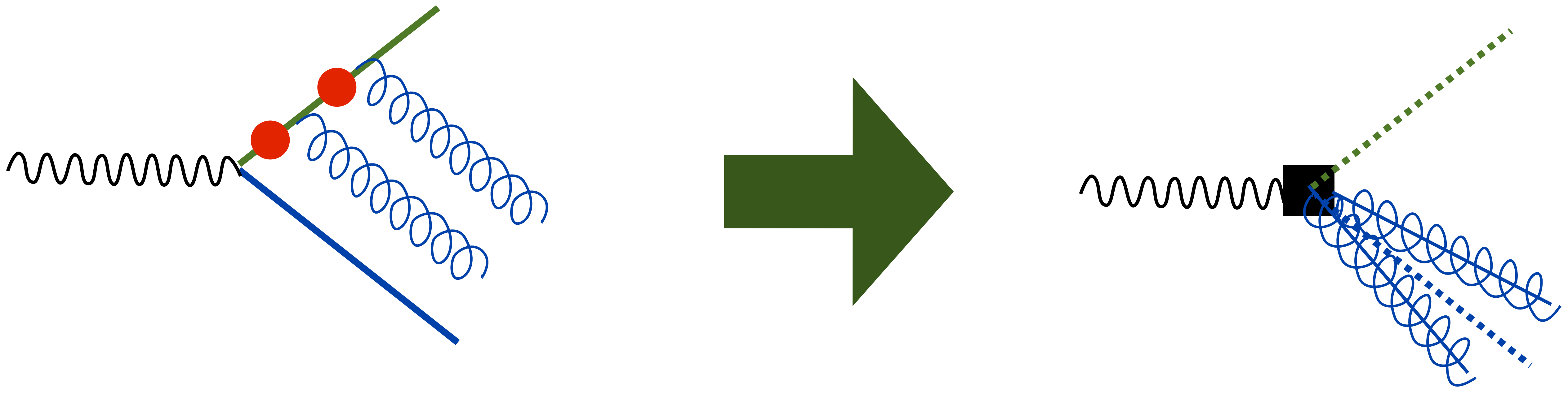




$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S\,\delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)S(k_S,\mu)$$

Hard and Jet Functions

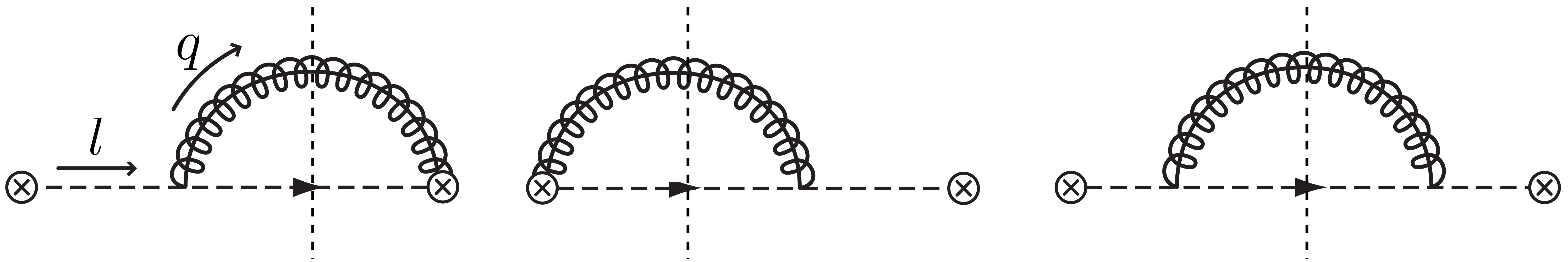
Hard function:



$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu) C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right) + \ldots$$

known to 3 loops

Jet function:



$$J(t,\mu) = \delta(t) + \frac{\alpha_s(\mu) C_F}{4\pi}\left\{(7-\pi^2)\delta(t) - \frac{3}{\mu^2}\left[\frac{\mu^2\theta(t)}{t}\right]_+ + \frac{4}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\right\} + \ldots$$

known to 3 loops

Beam Function and PDFs

transverse momentum dependent beam function:

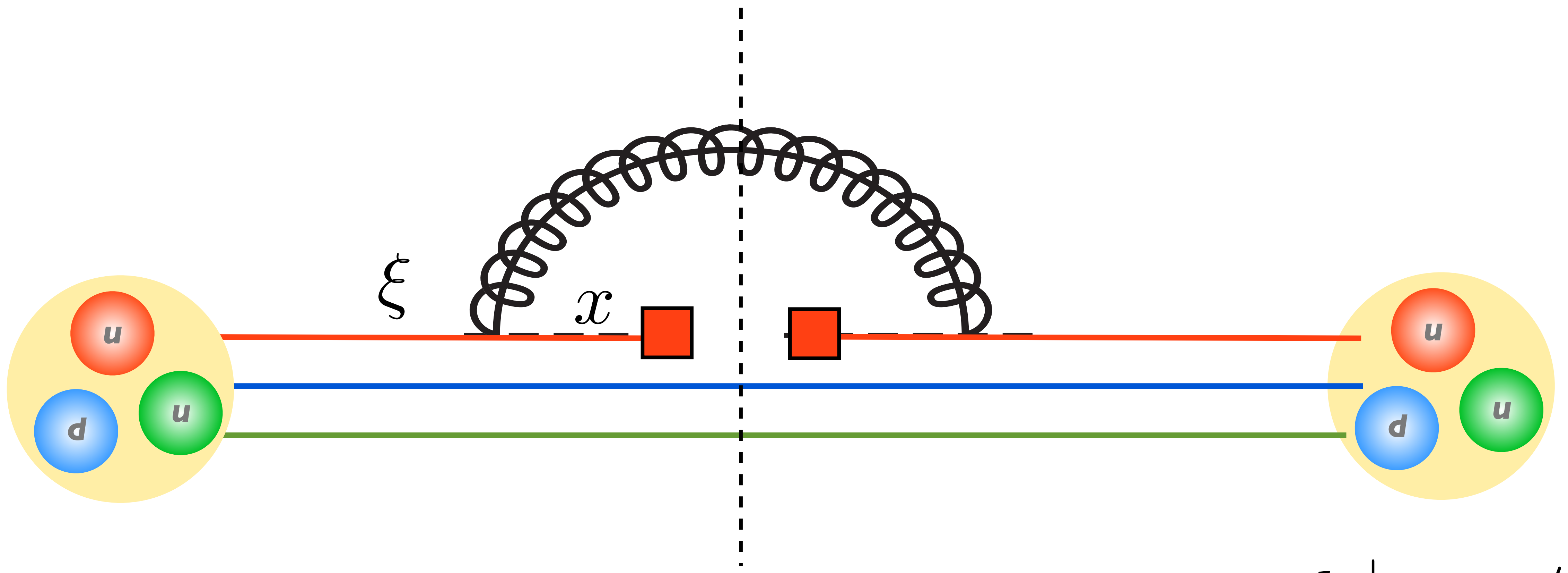
$$B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{ik^+ y^-/2} \langle P_n(P^-)| \bar{\chi}_n\Big(y^- \frac{n}{2}\Big) \delta(xP^- - \bar{n}\cdot\mathcal{P}) \delta(k_\perp^2 - \mathcal{P}_\perp^2) \chi_n(0) |P_n(P^-)\rangle$$

match onto PDF

$$f(x,\mu) = \theta(\omega) \langle P_n(P^-)| \bar{\chi}_n(0) \delta(xP^- - \bar{n}\cdot\mathcal{P}) \chi_n(0) |P_n(P^-)\rangle$$

$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(\xi, \mu)$$

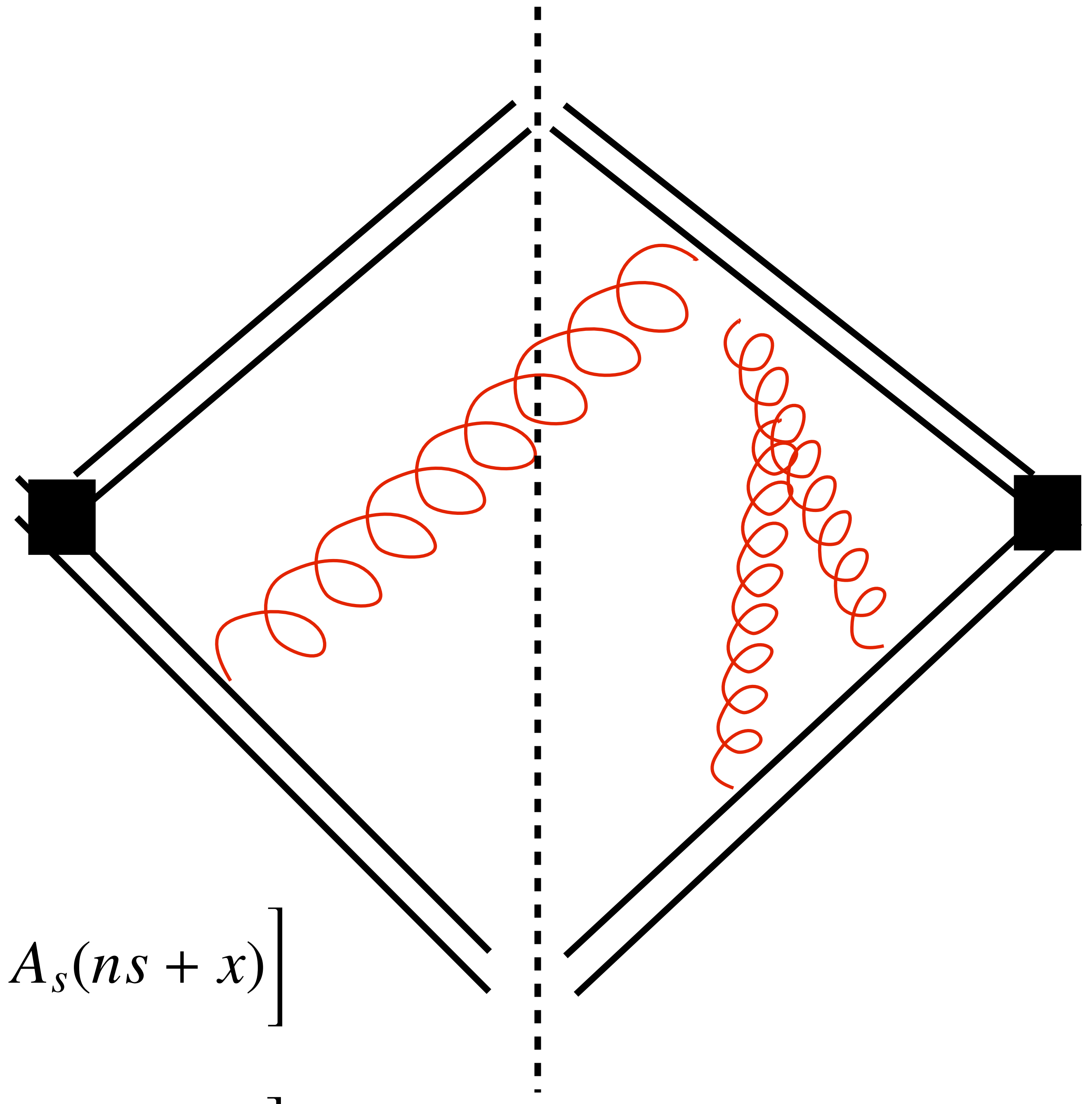
known to 2 loops; anomalous dimension known to 3 loops



Measure small light-cone momentum $k^+ = t/P^-$
and transverse momentum $\mathbf{k}_\perp$
of initial state radiation

Soft function

- Soft functions for e+e- dijets, DIS 1-jettiness, and *pp* beam thrust:



$$S_2(\ell_1,\ell_2,\mu) = \frac{1}{N_C}\,\mathrm{Tr}\sum_{i\in X_s}\left|\langle X_s|\,T[Y_n^{\pm\dagger}(0)Y_{\bar n}^{\pm}(0)]\,|0\rangle\right|^2$$
$$\times\,\delta\Big(\ell_1-\sum_{i\in X_s}\theta(\bar n\cdot k_i-n\cdot k_i)\,n\cdot k_i\Big)\,\delta\Big(\ell_2-\sum_{i\in X_s}\theta(n\cdot k_i-\bar n\cdot k_i)\,\bar n\cdot k_i\Big),$$

e⁺e⁻:	$++$
DIS:	$--$
***pp*:**	$+-$

$$Y_n^{+\dagger}(x) = P\exp\Big[ig\int_0^\infty ds\, n\cdot A_s(ns+x)\Big]$$
$$Y_n^{-}(x) = P\exp\Big[ig\int_{-\infty}^0 ds\, n\cdot A_s(ns+x)\Big],$$

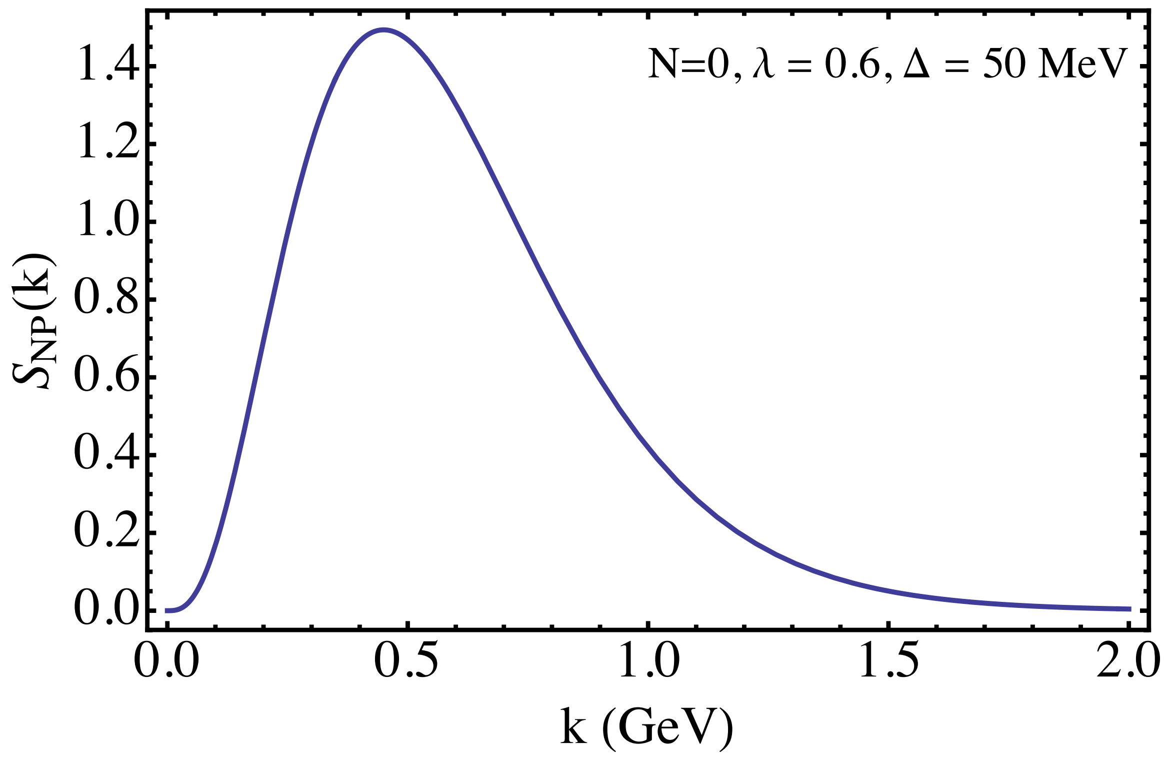
- Perturbatively, it is known that $S_2^{ee} = S_2^{ep} = S_2^{pp}$ to at least $\mathcal{O}(\alpha_s^2)$ D. Kang, Labun, CL (2015); Boughezal, Liu, Petriello (2015)

$$S(k_s,\mu) = \int d\ell_1 d\ell_2\,\delta(k-\ell_1-\ell_2)\,S_2(\ell_1,\ell_2,\mu)$$

$$\int S(k_s,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left\{-\frac{8C_F}{1-a}\ln^2\frac{k}{\mu} + c_S^1(a)\right\}$$

Nonperturbative corrections

- In general, soft function expressed as convolution of perturbative part and nonperturbative shape function:



$$S(k,\mu) = \int dk'\, S_{\mathrm{PT}}(k-k',\mu)\, f_{\mathrm{mod}}(k' - 2\overline{\Delta}_a)$$

- For large enough $\tau\ (k_S)$, leading effect is a shift:

$$\langle e\rangle = \langle e\rangle_{\mathrm{PT}} + c_e \frac{\Omega_1}{Q}$$

c_e observable dependent, calculable coefficient

Ω_1 universal nonperturbative parameter

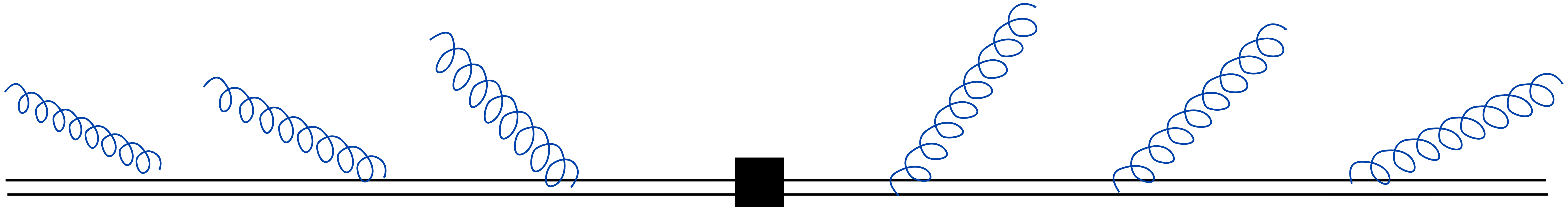
Amongst DIS event shapes:
Kang, CL, Stewart [1303.6952]
relation to *pp* jet mass::
Stewart, Tackmann, Waalewijn [1405.6722]

- Rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

$$\Omega_1 = \frac{1}{N_C}\mathrm{Tr}\langle 0|\overline{Y}_{\bar n}^\dagger Y_n^\dagger \mathcal{E}_T(\eta) Y_n \overline{Y}_{\bar n}|0\rangle$$

Lee, Sterman [hep-ph/0611061]

↓ "energy flow" operator

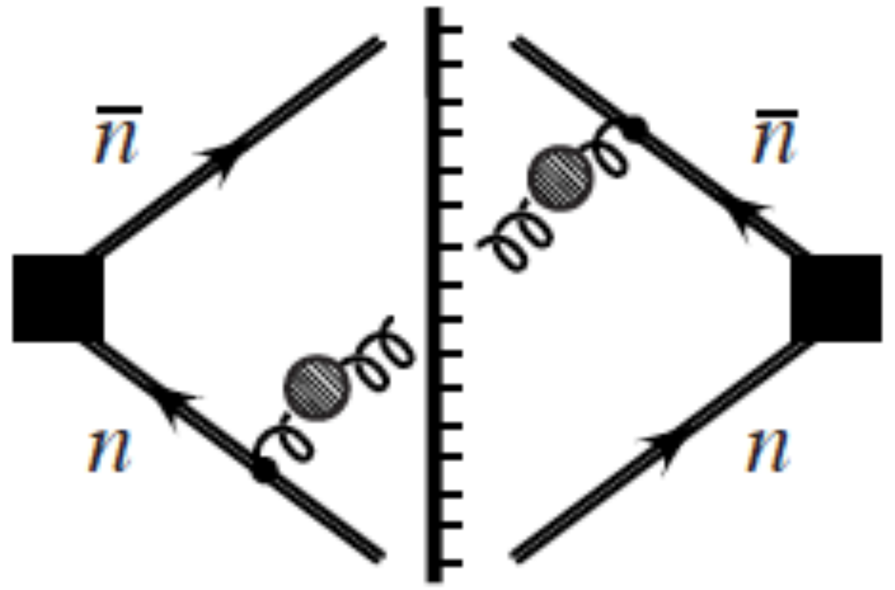


soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

Non-perturbative effects and gapped soft function

- However, both the perturbative soft function and gap parameter suffer renormalon ambiguities.

$$S(k,\mu) = \int dk' \, S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k'-2\overline{\Delta}_a)$$



- $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity in gap $\overline{\Delta}_a$

- Subtract a series with the same/canceling ambiguity from both PT and NP pieces:

$$\overline{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \xrightarrow{\text{Laplace space}} \widetilde{S}(\nu,\mu) = \left[e^{-2\nu\Delta_a(\mu)}\,\widetilde{f}_{\rm mod}(\nu)\right]\left[e^{-2\nu\delta_a(\mu)}\,\widetilde{S}_{\rm PT}(\nu,\mu)\right]$$

renormalon free *renormalon free*

R_{gap} scheme

[0803.4214]
[0806.3852]

- Choosing the ***R_{gap}* scheme** to cancel the leading renormalon,

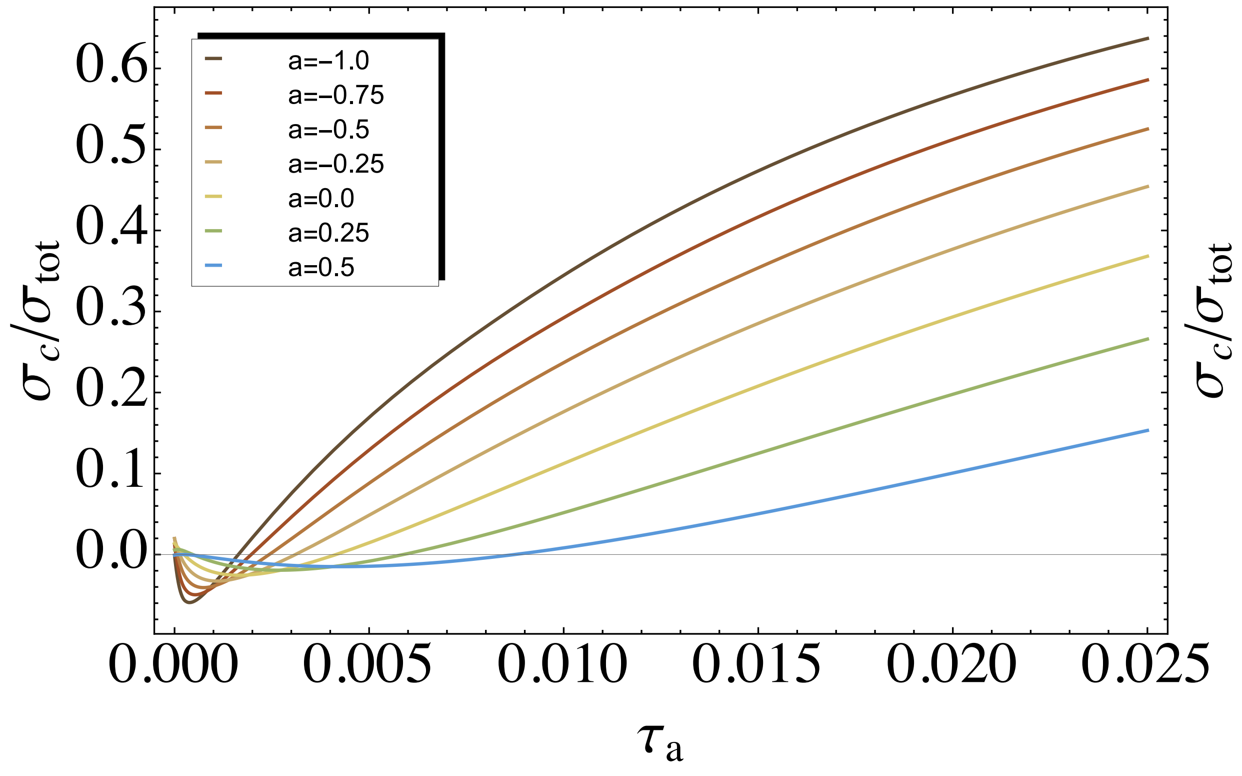
$$Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widehat{S}_{\rm PT}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})}=0 \quad\longrightarrow\quad \delta_a(\mu,R)=\frac{1}{2}Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widetilde{S}_{\rm PT}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})},$$

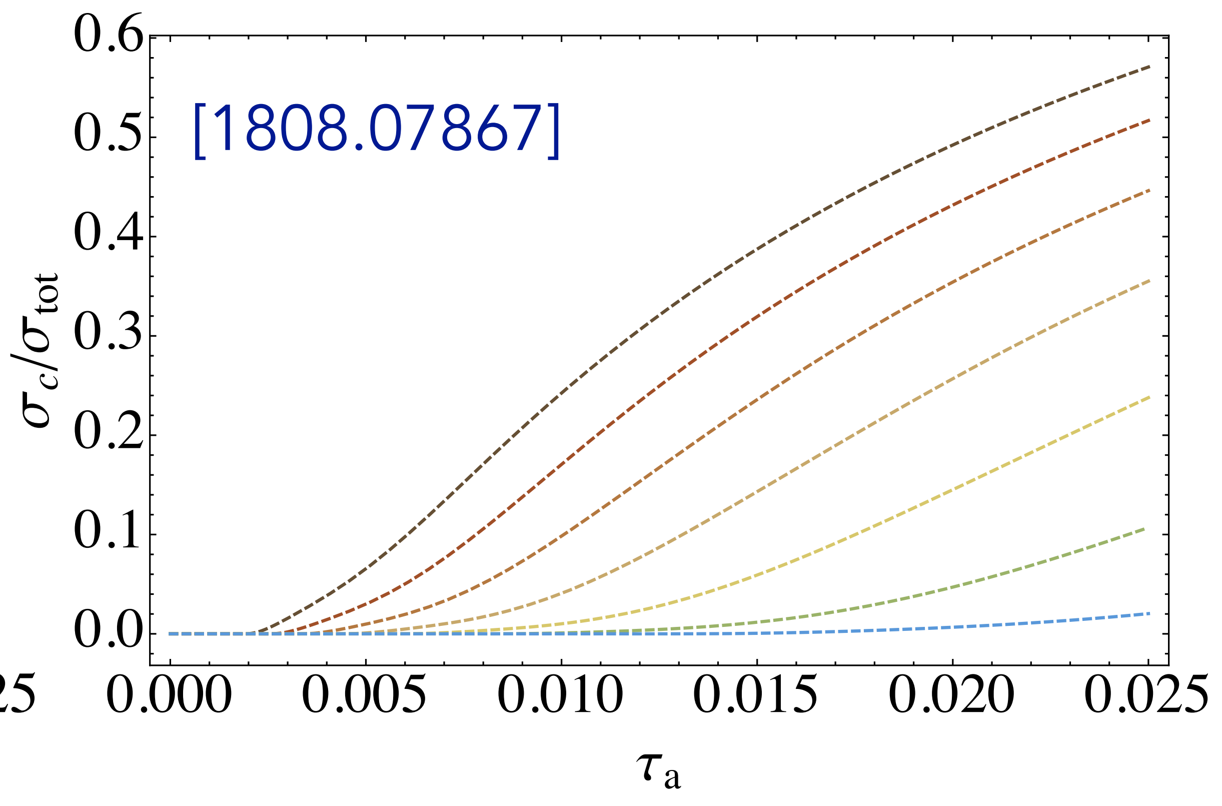
$$\widehat{S}_{\rm PT}(\nu,\mu)=e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)$$

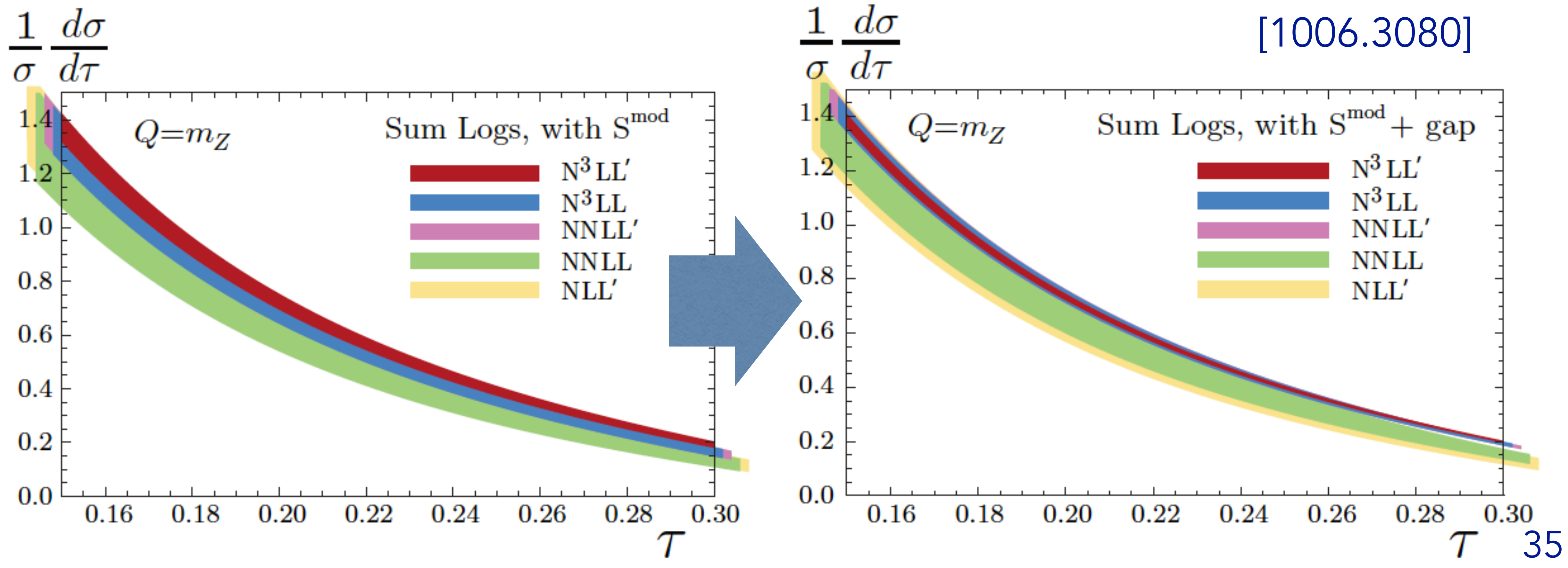
Gapped and renormalon free soft function
$$S(k,\mu)=\int dk'\,S_{\rm PT}(k-k',\mu)\left[e^{-2\delta_a(\mu,R)\frac{d}{dk'}}\,f_{\rm mod}(k'-2\Delta_a(\mu,R))\right]$$

Final cross section is expanded order-by-order in bracketed term
$$\frac{1}{\sigma_0}\sigma(\tau_a)=\int dk\,\sigma_{\rm PT}\Big(\tau_a-\frac{k}{Q}\Big)\left[e^{-2\delta_a(\mu_S,R)\frac{d}{dk}}f_{\rm mod}(k-2\Delta_a(\mu_S,R))\right]$$

- Improves small τ_a behavior and perturbative convergence:







Evolution and resummation

- Easier to discuss in terms of Laplace transforms (or Fourier transforms to position space)

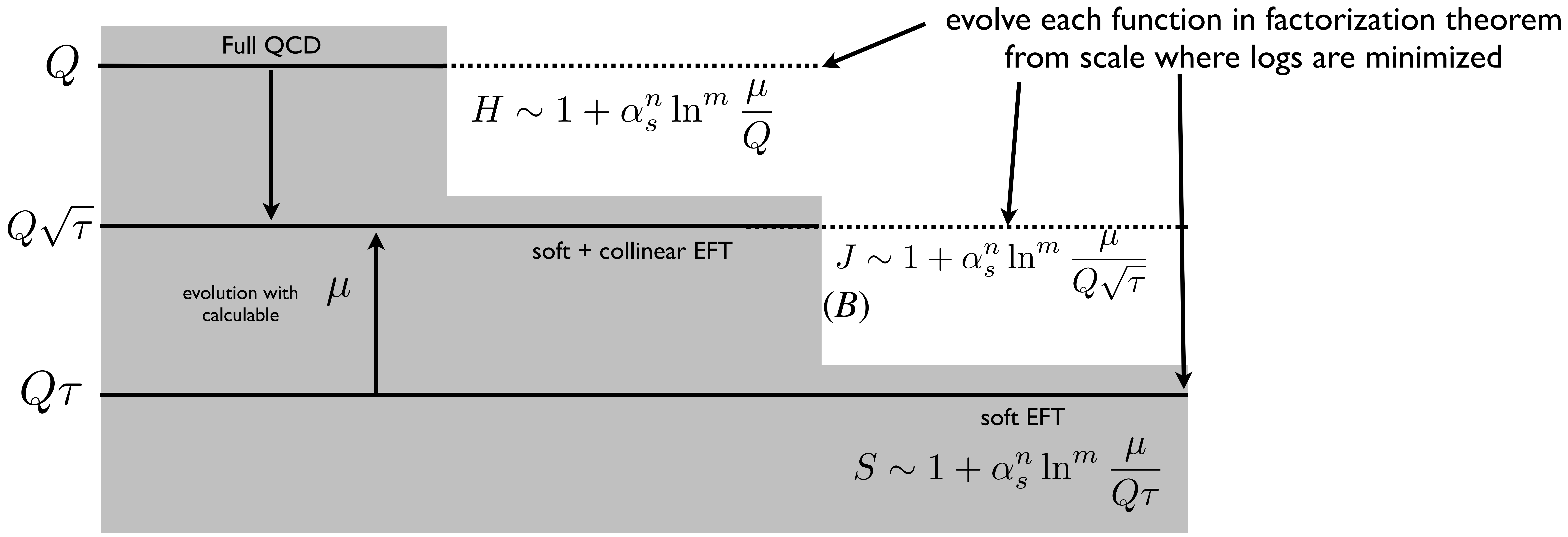
$$\tilde{\sigma}(\nu) = \int_0^\infty d\tau\, e^{-\nu\tau}\sigma(\tau)$$

- Turns factorization theorem into a simple product:

$$\tilde{\sigma}(\nu) = H(Q^2,\mu)\tilde{J}(Q^2/\nu,\mu)\tilde{B}(Q^2/\nu,x,\mu)\tilde{S}(Q/\nu,\mu)$$

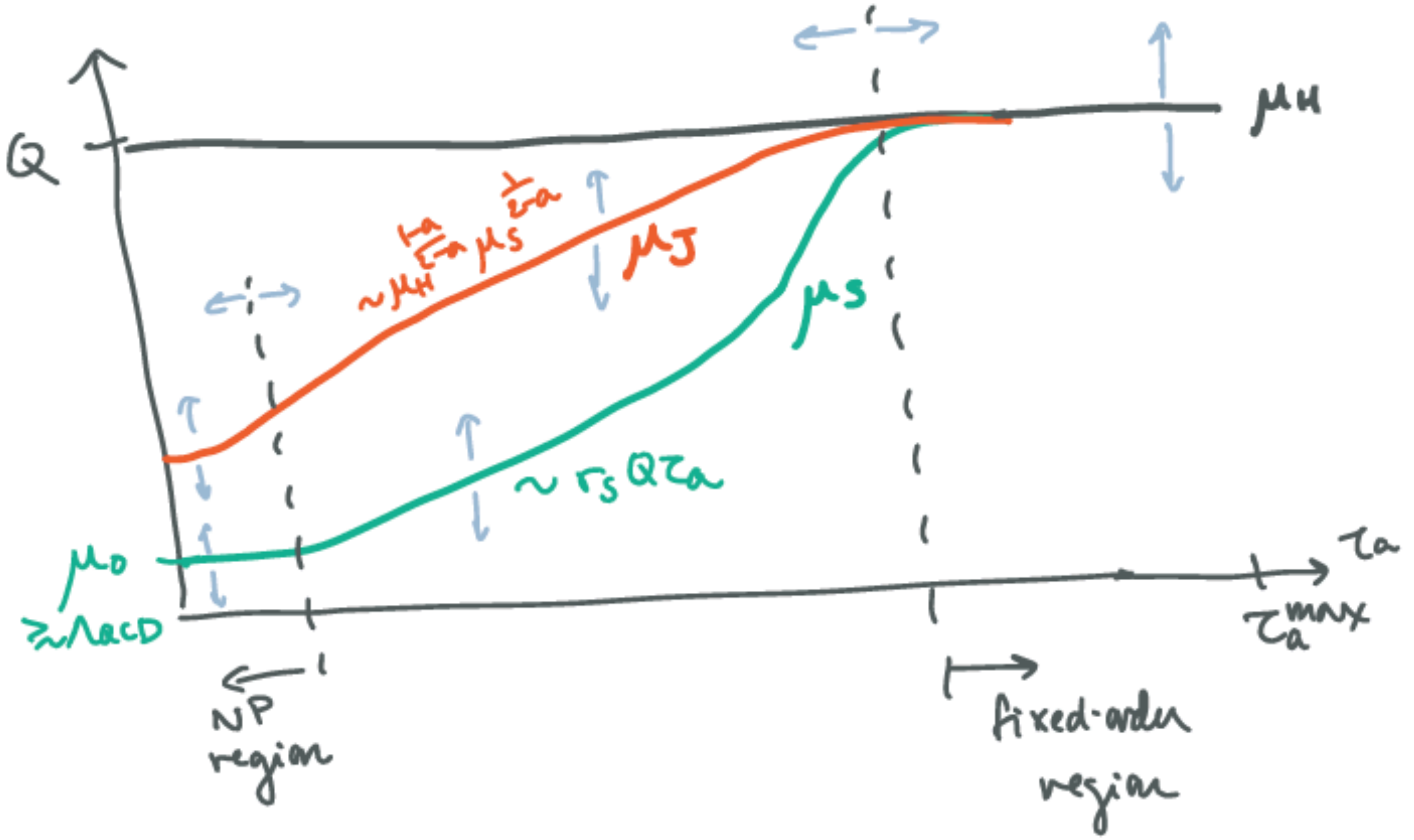
- RGE obeyed by Laplace-space jet and soft functions:

$$\mu\frac{d}{d\mu}\tilde{F}(Q^j/\nu,\mu) = \gamma_F(Q^j/\nu,\mu)\tilde{F}(Q^j/\nu,\mu)$$



Scale profiles

Freedom to choose μ_H, μ_J, μ_S (and μ_{ns}, R) allows not only log resummation but robust estimate of perturbative theory uncertainties in each region:



- allows scales to merge in fixed order region before $\tau_a = \tau_a^{max}$ (< 1)
- stable NP region to convolve shape function
- variation of all parameters to fully probe theory uncertainty

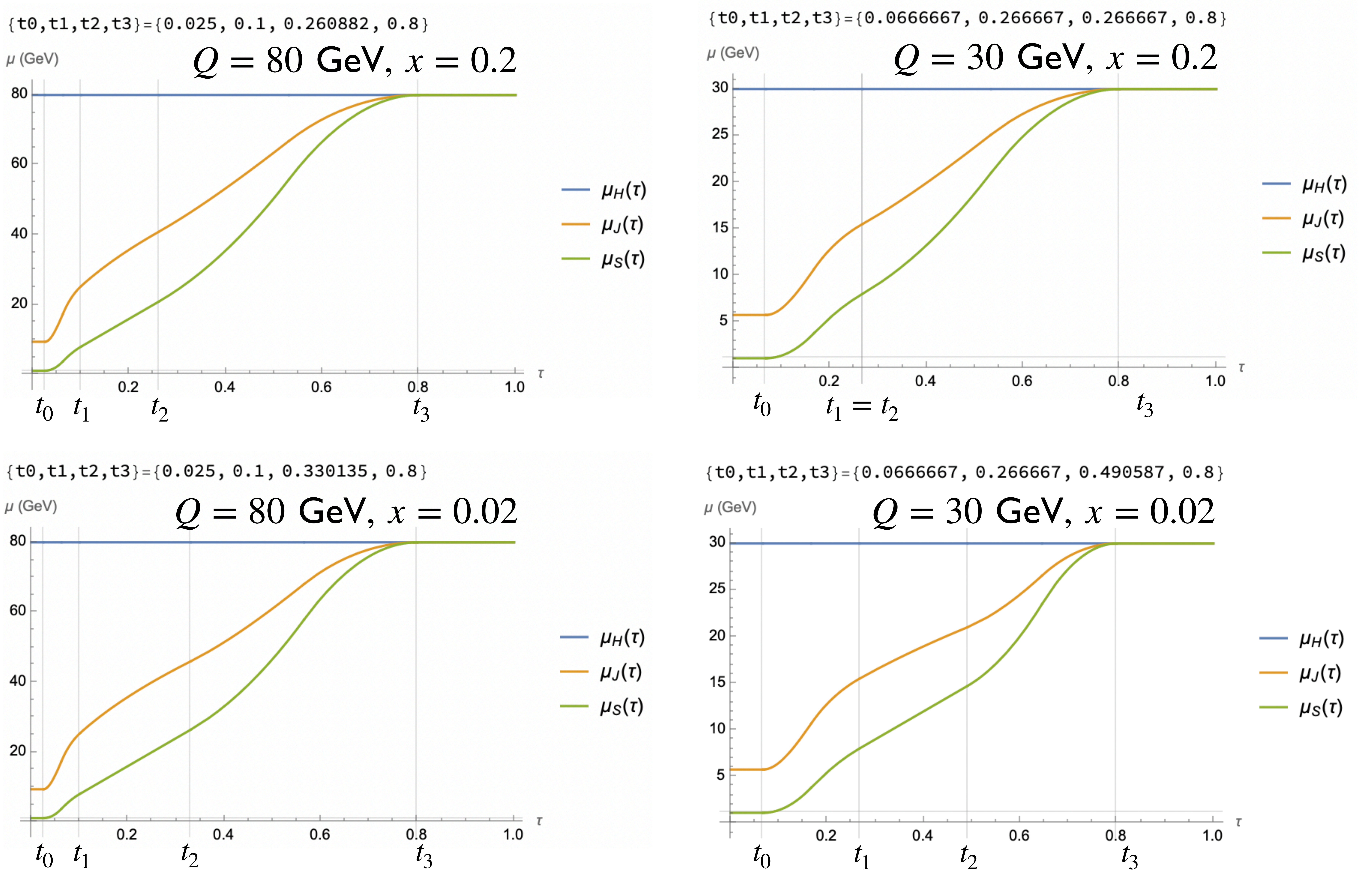
Scale profiles

For DIS, these regions depend on Q, *x*:

$\tau < t_0$: nonperturbative

$t_1 < \tau < t_2$:
canonical resummation

$\tau > t_3$: fixed-order



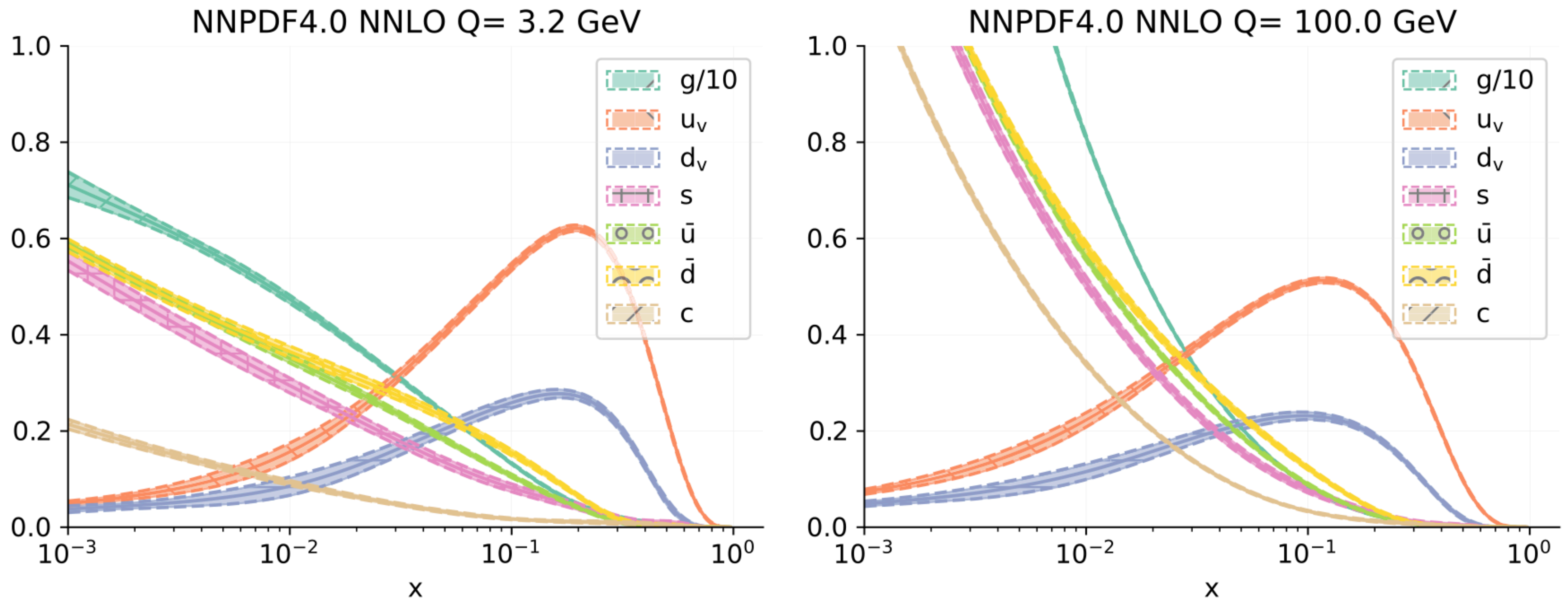
SCET FT for τ_1^b: PDF

τ_1^b **quark beam function**: $\hat{B}_q(t_B, x, \mu) = \int d^2\mathbf{p}_\perp \mathscr{B}_q(t_B - \mathbf{p}_\perp^2, x, \mathbf{p}_\perp^2, \mu)$

PDF for parton j

$\mathscr{B}_q(t, x, \mathbf{k}_\perp^2, \mu)$ is the $k_\perp$-dep. beam function, $\mathscr{B}_i(t, x, \mathbf{k}_\perp^2, \mu) = \mathscr{I}_{ij}(t, x/\xi, \mathbf{k}_\perp^2, \mu) \otimes_\xi f_j(\xi, \mu)$

- Gluon PDF could get very large as $x \to 0$.



- we use NNPDF4.0 NNLO PDF set implemented in LHAPDF.
- PDFs are determined w.r.t. α_s value.
- Should change PDFs for different α_s simultaneously.

332700	NNPDF40_nnlo_as_01160	(tarball)	(info file)	101	1
332900	NNPDF40_nnlo_as_01170	(tarball)	(info file)	101	1
333100	NNPDF40_nnlo_as_01175	(tarball)	(info file)	101	1
333300	NNPDF40_nnlo_as_01185	(tarball)	(info file)	101	1
333500	NNPDF40_nnlo_as_01190	(tarball)	(info file)	101	1
333700	NNPDF40_nnlo_as_01200	(tarball)	(info file)	101	1

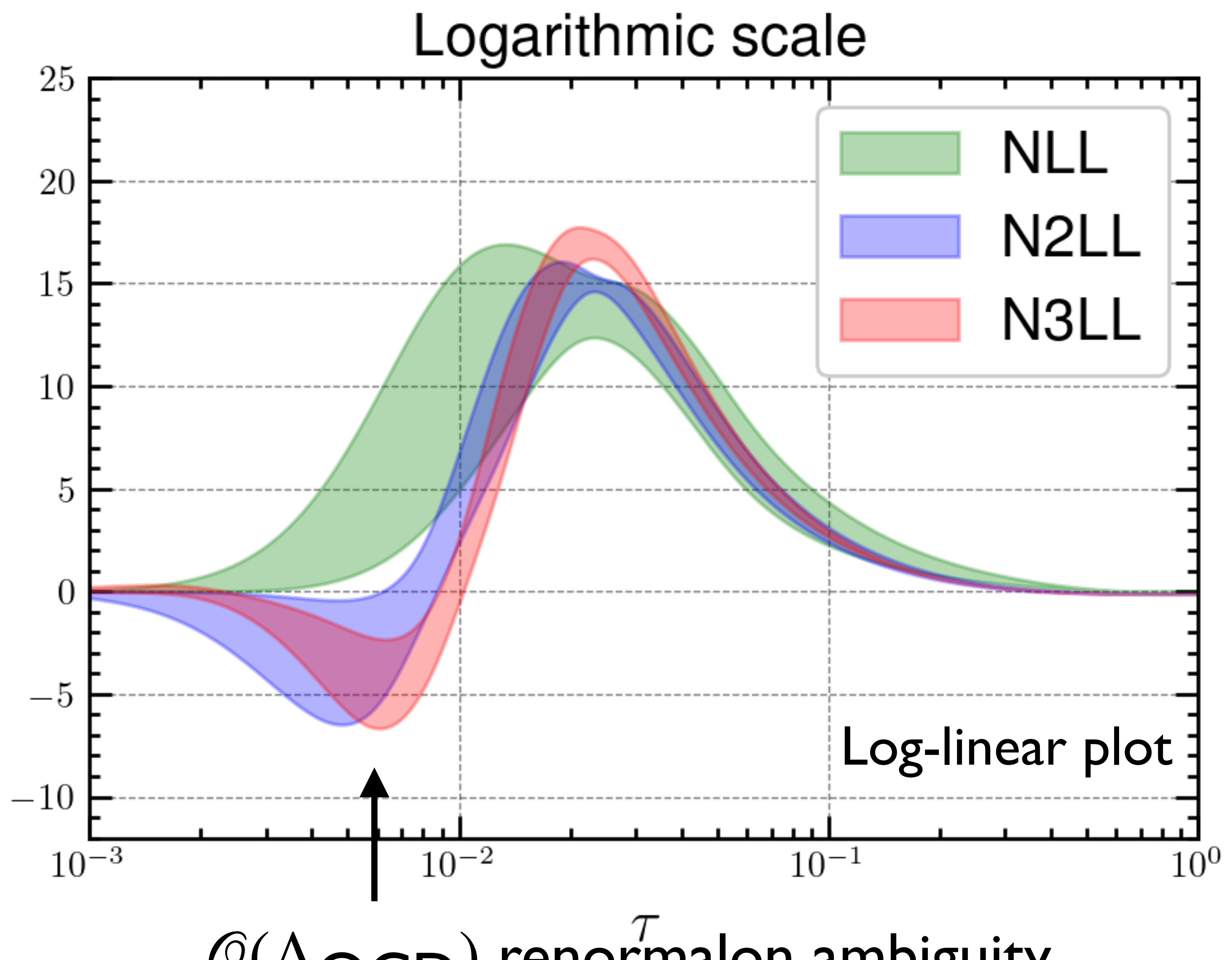
DIS thrust cross sections

Ee, Kang, CL, Stewart [2024]

At N3LL + $\mathcal{O}(\alpha_s^2)$:

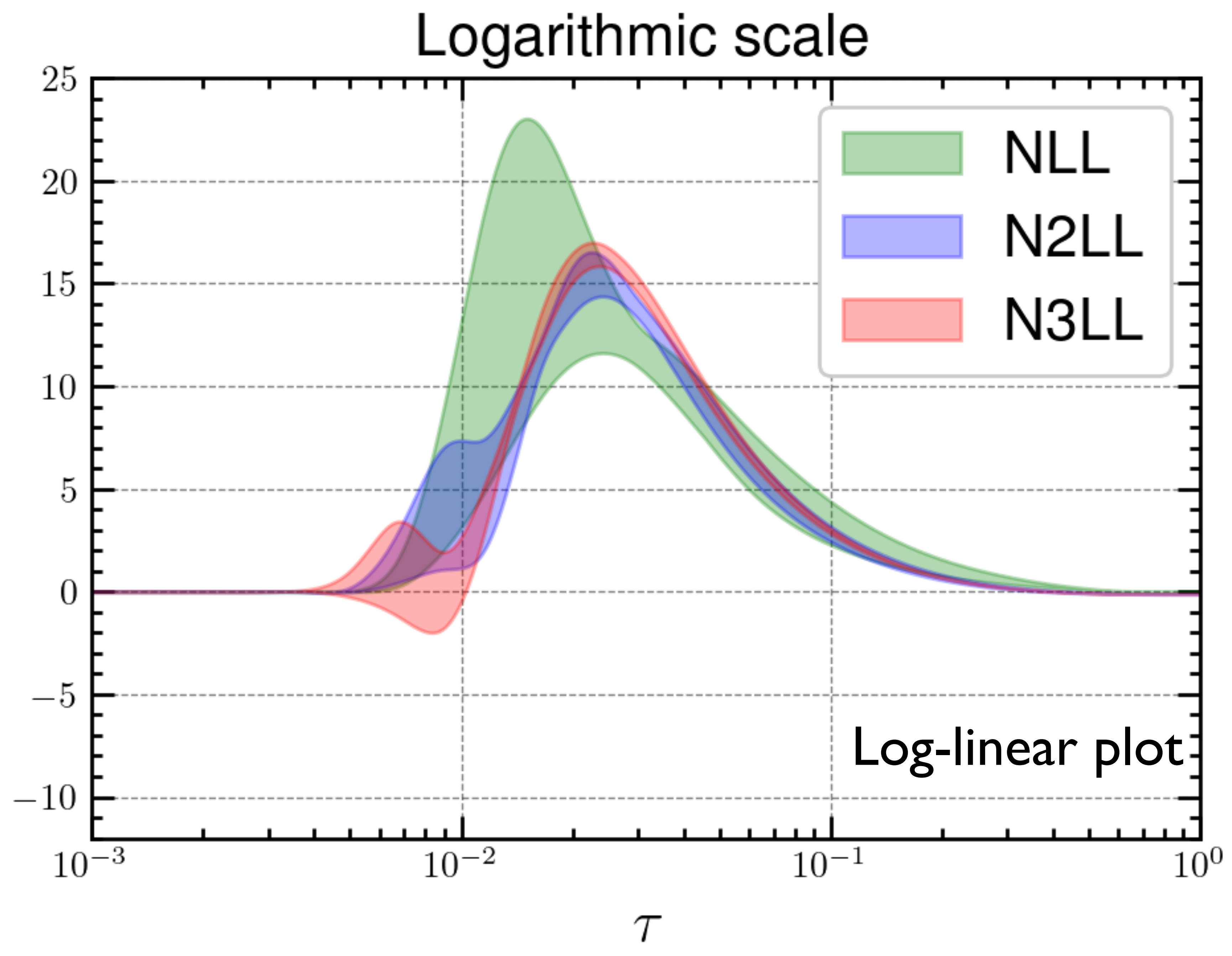
At $x = 0.2$ and $Q = 80$ GeV $\sqrt{s} = 300$ GeV

Before renormalon subtraction
(shape function only)



$\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon ambiguity

After renormalon subtraction
(standard Rgap)



DIS thrust cross sections

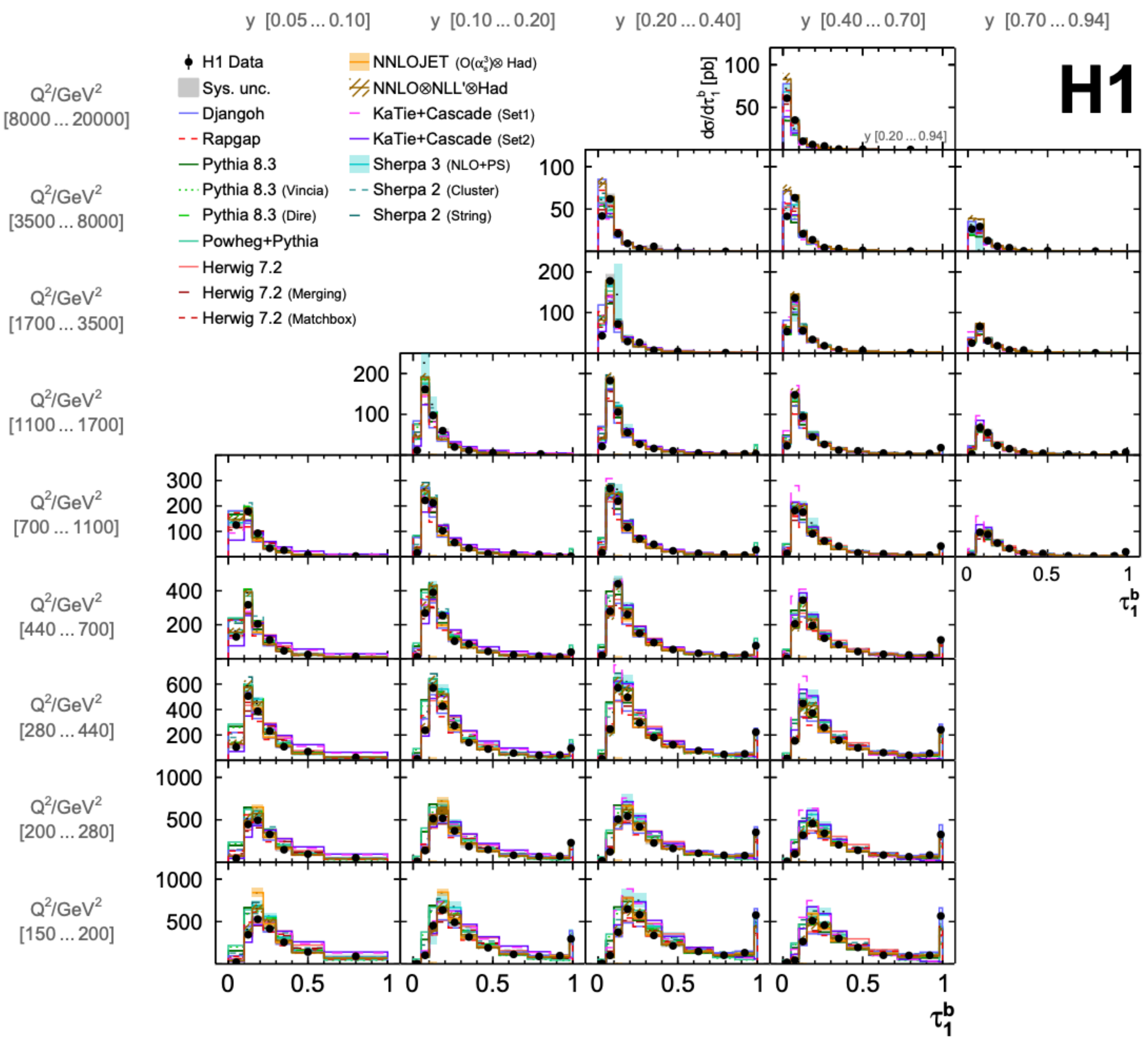
Ee, Kang, CL, Stewart [2024]

At N³LL + $\mathcal{O}(\alpha_s^2)$: $Q = 50$ GeV, $x = 0.2, \sqrt{s} = 140$ GeV

Probable fit window for $\{\alpha_s, \Omega_1\}$

Empty current hemisphere events at $\tau = 1$

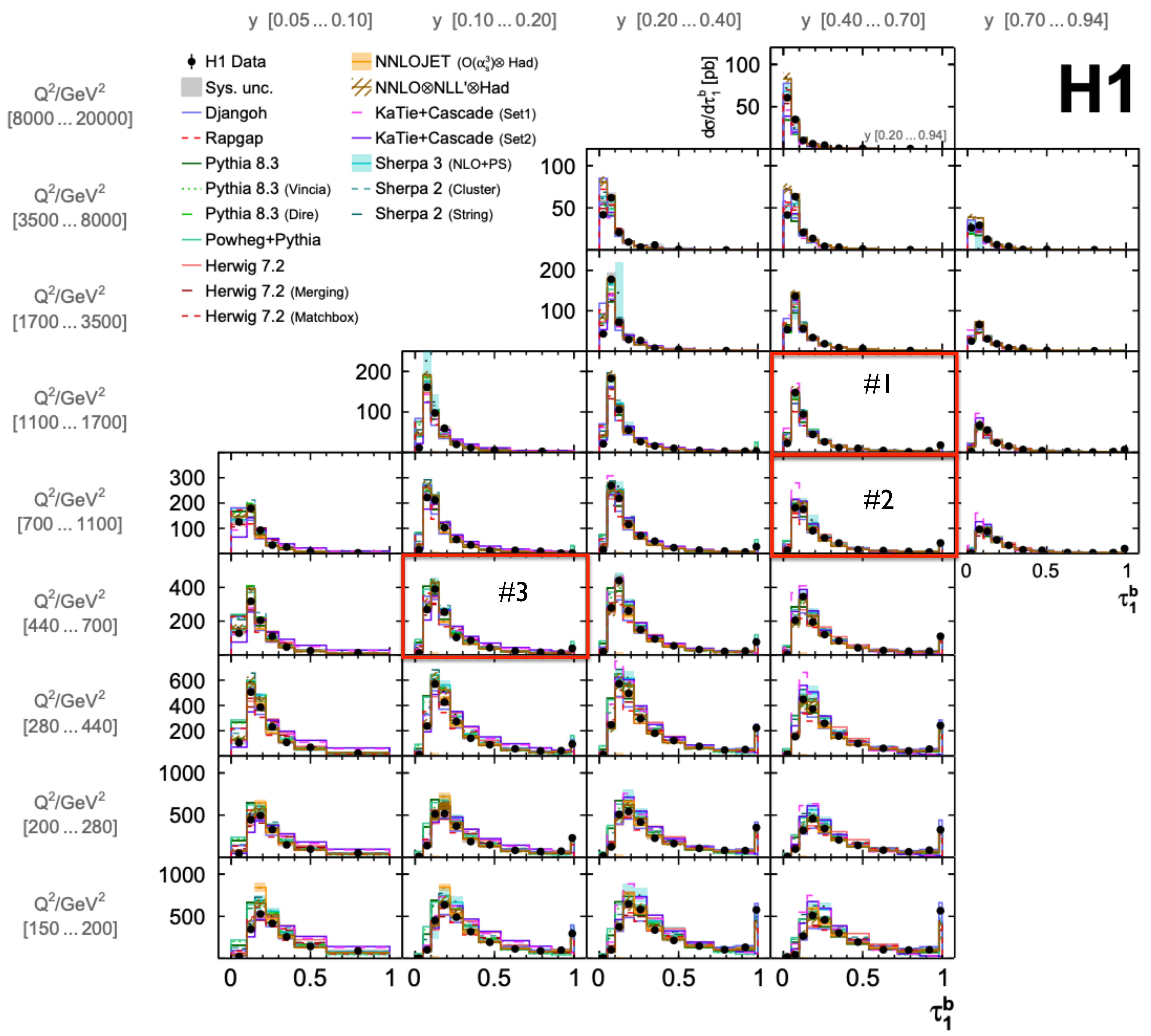
HERA data



H1 Collaboration
[2403.10109]

HERA data

H1 Collaboration
[2403.10109]



Ee, Kang, CL, Stewart [2024]

Theory vs HERA:

H1 Collaboration [2403.10109]

- Bin #1
  $1100 < Q^2/\text{GeV} < 1700$,
  $0.4 < y < 0.7$:

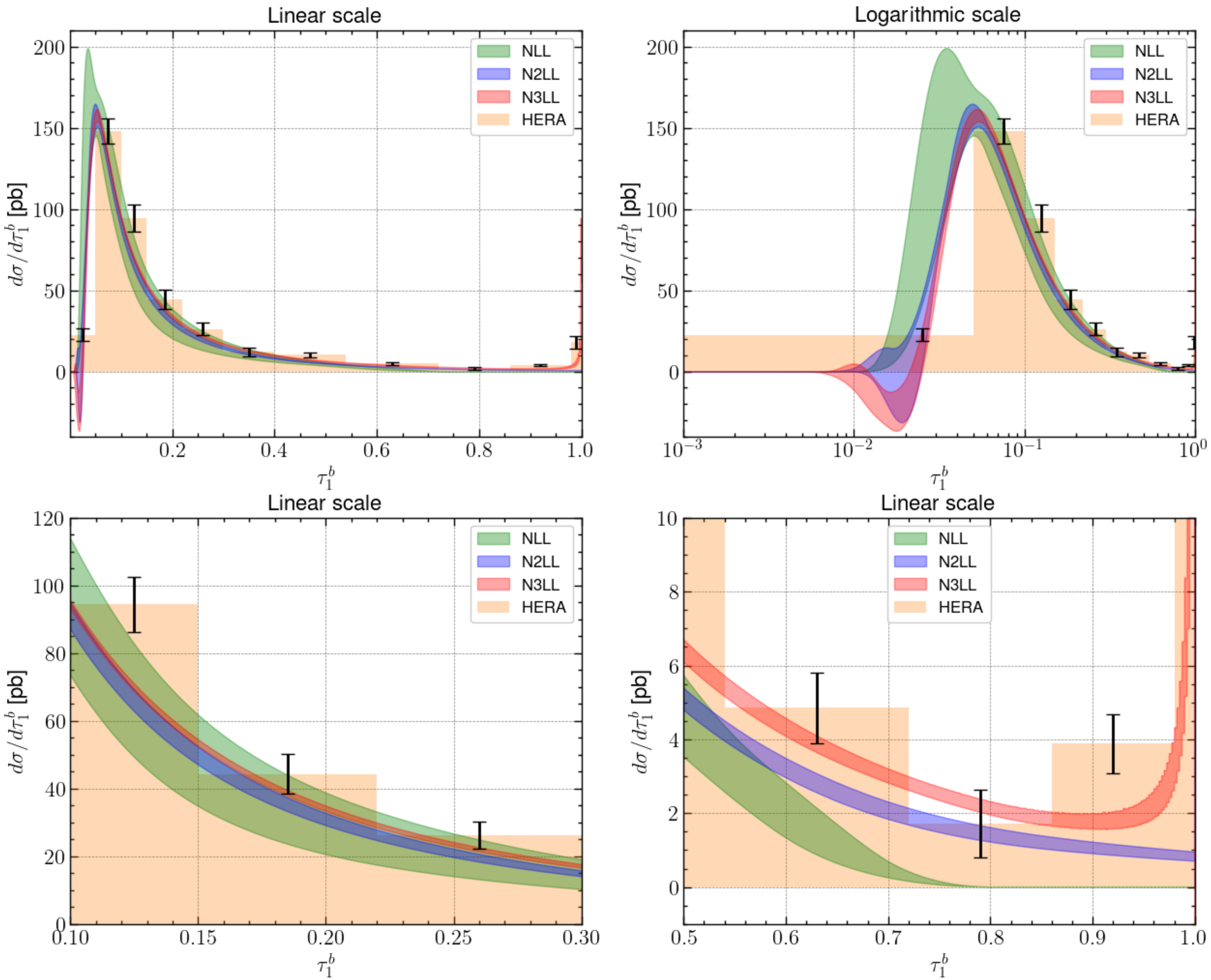
$x_{\text{mean}} \approx 0.025$

$Q_{\text{mean}} \approx 37$ GeV

$t_1 \approx \dfrac{8 \text{ GeV}}{Q_{\text{mean}}} \approx 0.21$

$t_2 = t_2(x_{\text{mean}}) \approx 0.47$

$\alpha_s(M_Z) = 0.118,\ \Omega_1 = 350$ MeV



Ee, Kang, CL, Stewart [2024]

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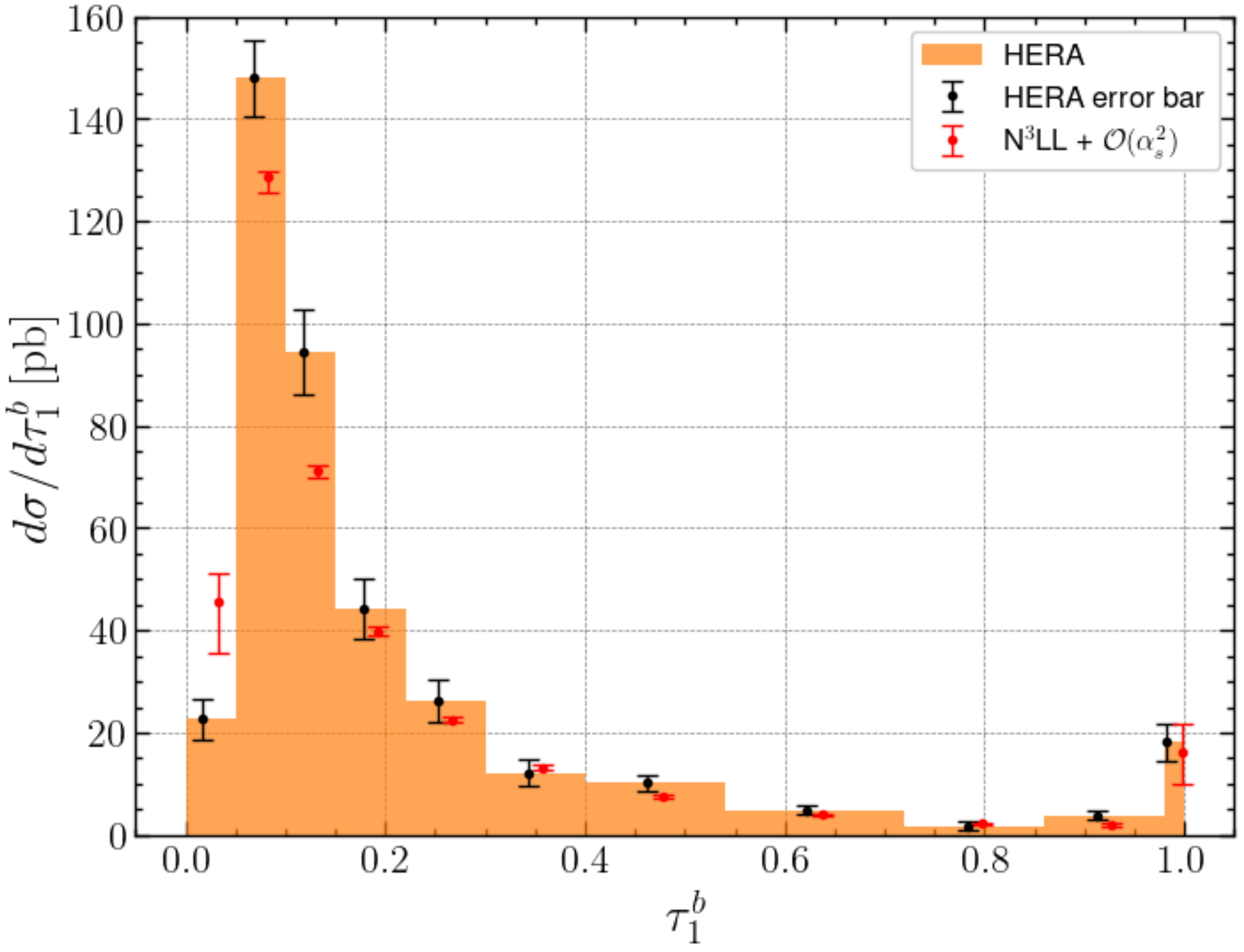
Binned equivalently:



HERA total cross section
23.04(90) pb

Theory total cross section: $20.82^{+1.17}_{-0.85}$ pb

Ee, Kang, CL, Stewart [2024]

Theory vs HERA:

H1 Collaboration [2403.10109]

- Bin #2
  $440 < Q^2/\text{GeV} < 700$,
  $0.1 < y < 0.2$:

$x_{\text{mean}} \approx 0.016$

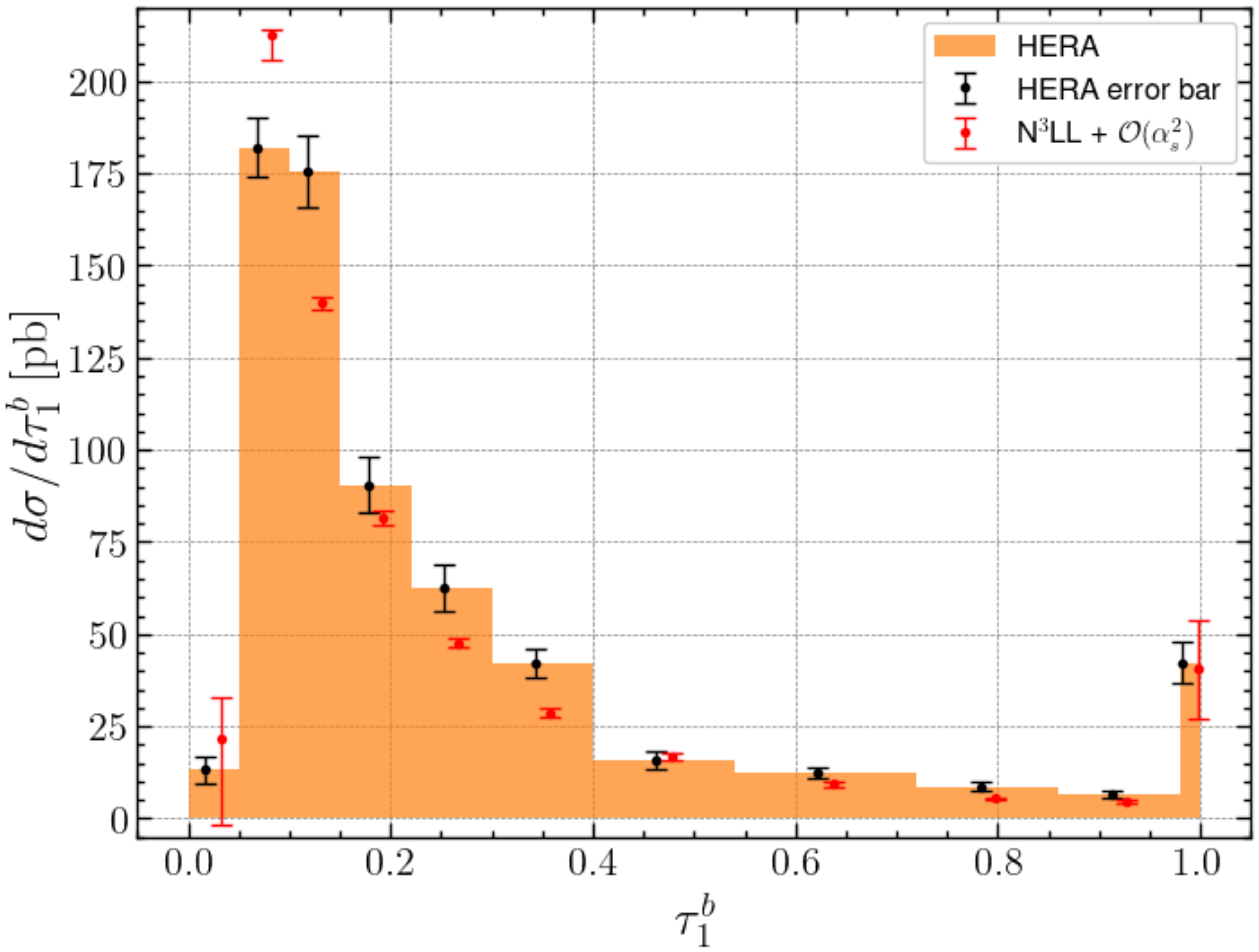
$Q_{\text{mean}} \approx 30$ GeV

$t_1 \approx \dfrac{8 \text{ GeV}}{Q_{\text{mean}}} \approx 0.27$

$t_2 = t_2(x_{\text{mean}}) \approx 0.51$

$\alpha_s(M_Z) = 0.118,\ \Omega_1 = 350$ MeV

Binned equivalently:



HERA total cross section
83.6(2.1) pb

Theory total cross section: $72.10^{+6.23}_{-4.61}$ pb

Ee, Kang, CL, Stewart [2024]

Theory vs HERA:

H1 Collaboration [2403.10109]

- Bin #3
  $700 < Q^2/\text{GeV} < 1100$,
  $0.4 < y < 0.7$:

$x_{\text{mean}} \approx 0.037$

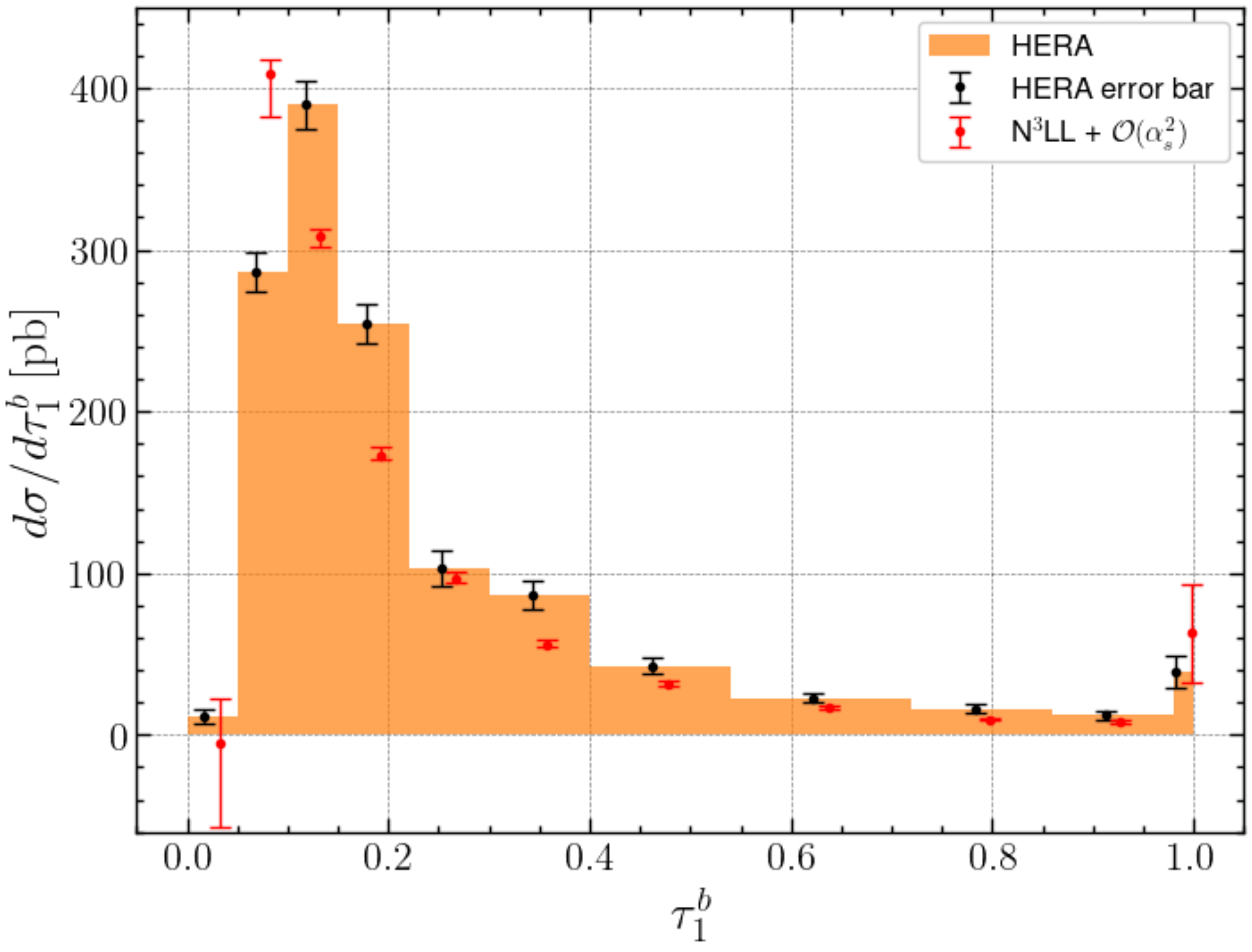
$Q_{\text{mean}} \approx 24$ GeV

$$t_1 \approx \frac{8\ \text{GeV}}{Q_{\text{mean}}} \approx 0.34$$

$$t_2 = t_2(x_{\text{mean}}) \approx 0.43$$

$\alpha_s(M_Z) = 0.118,\ \Omega_1 = 350$ MeV

Binned equivalently:




```
HERA total cross section
41.4(1.2) pb
```

Theory total cross section: $37.38^{+2.63}_{-1.75}$ pb

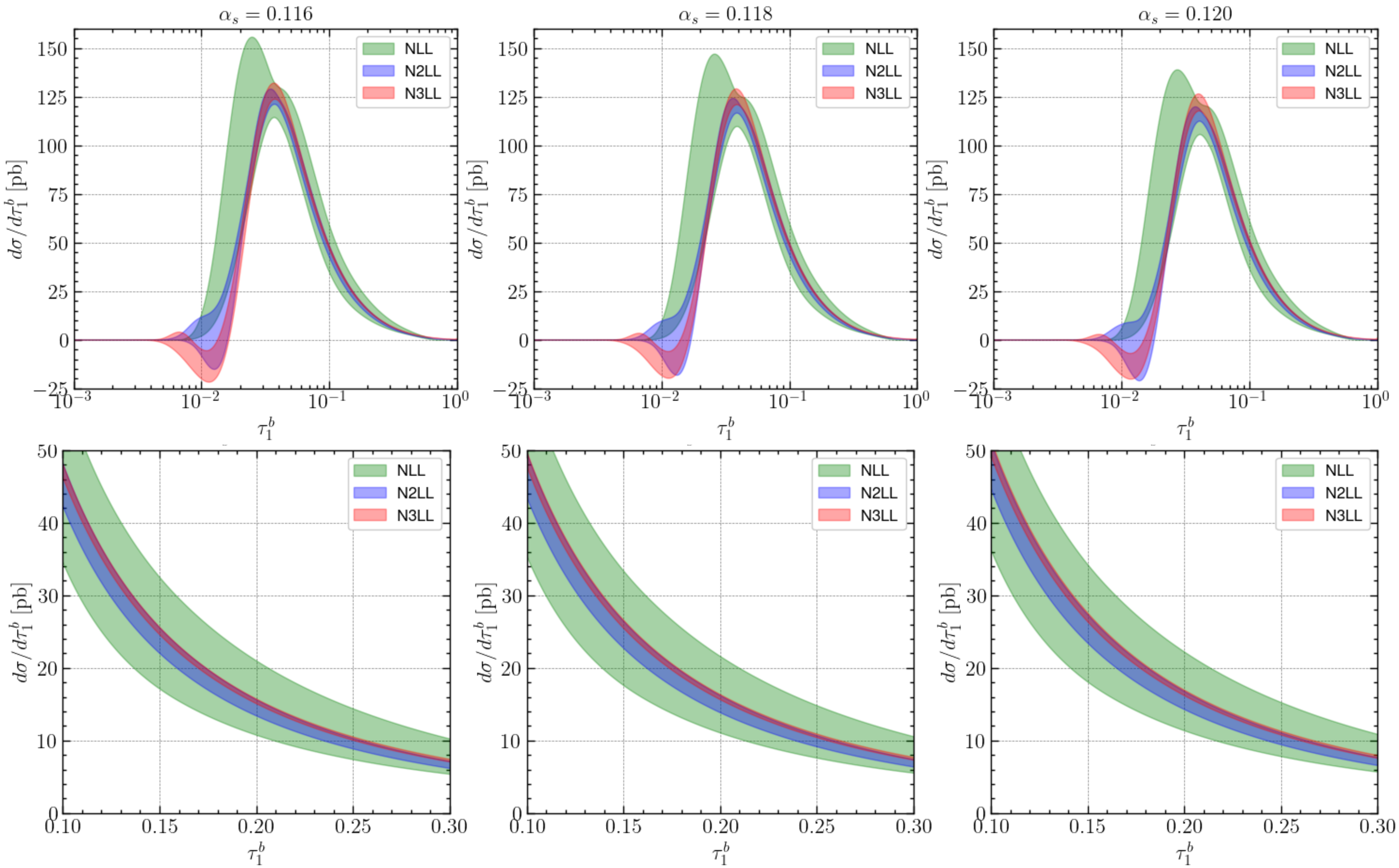
Ee, Kang, CL, Stewart [2024]

Sensitivity to α_s

- $\sqrt{s} = 300$ GeV, $Q = 50$ GeV, $x = 0.05$

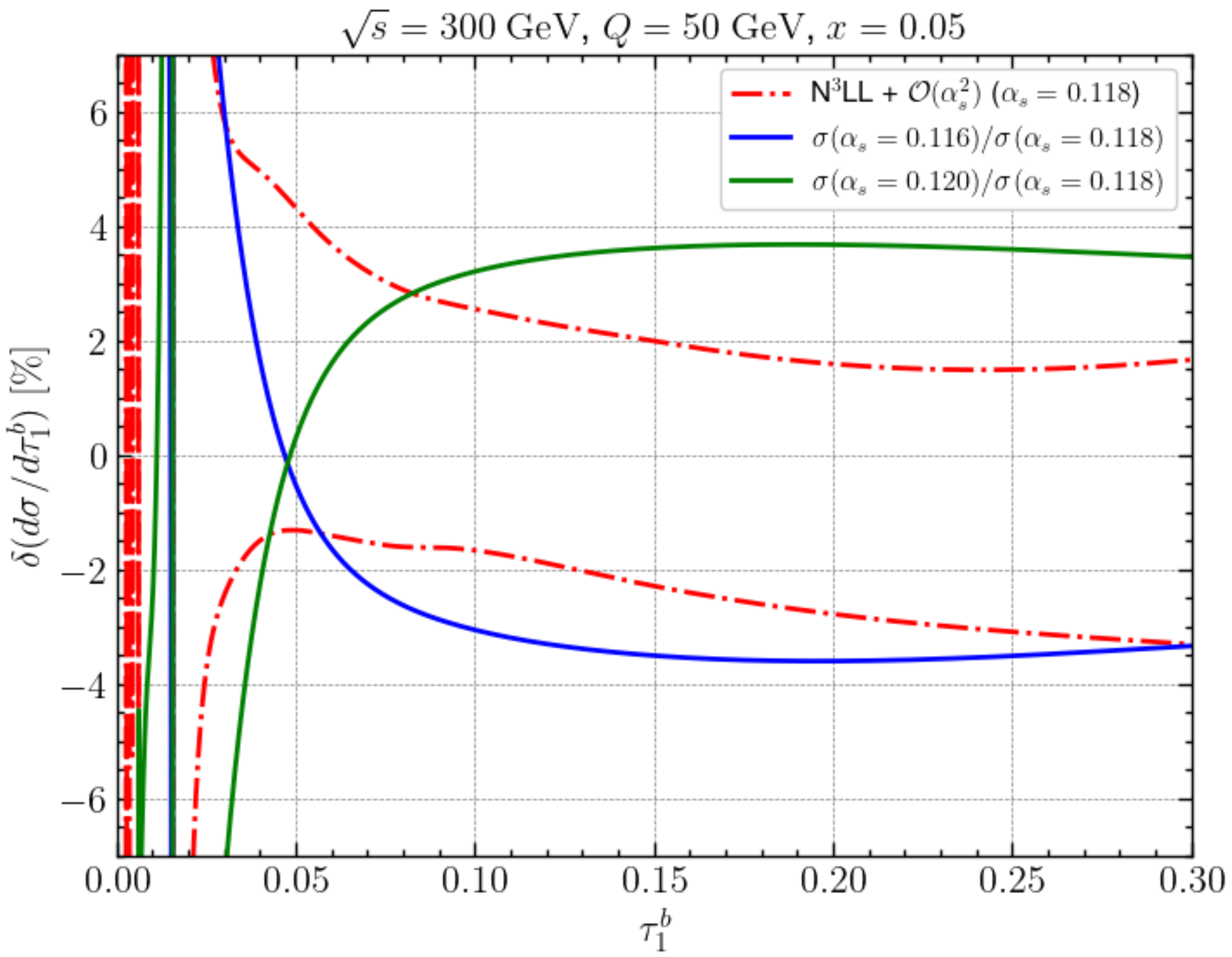
- Peak:

- Tail:



Ee, Kang, CL, Stewart [2024]

Sensitivity to α_s

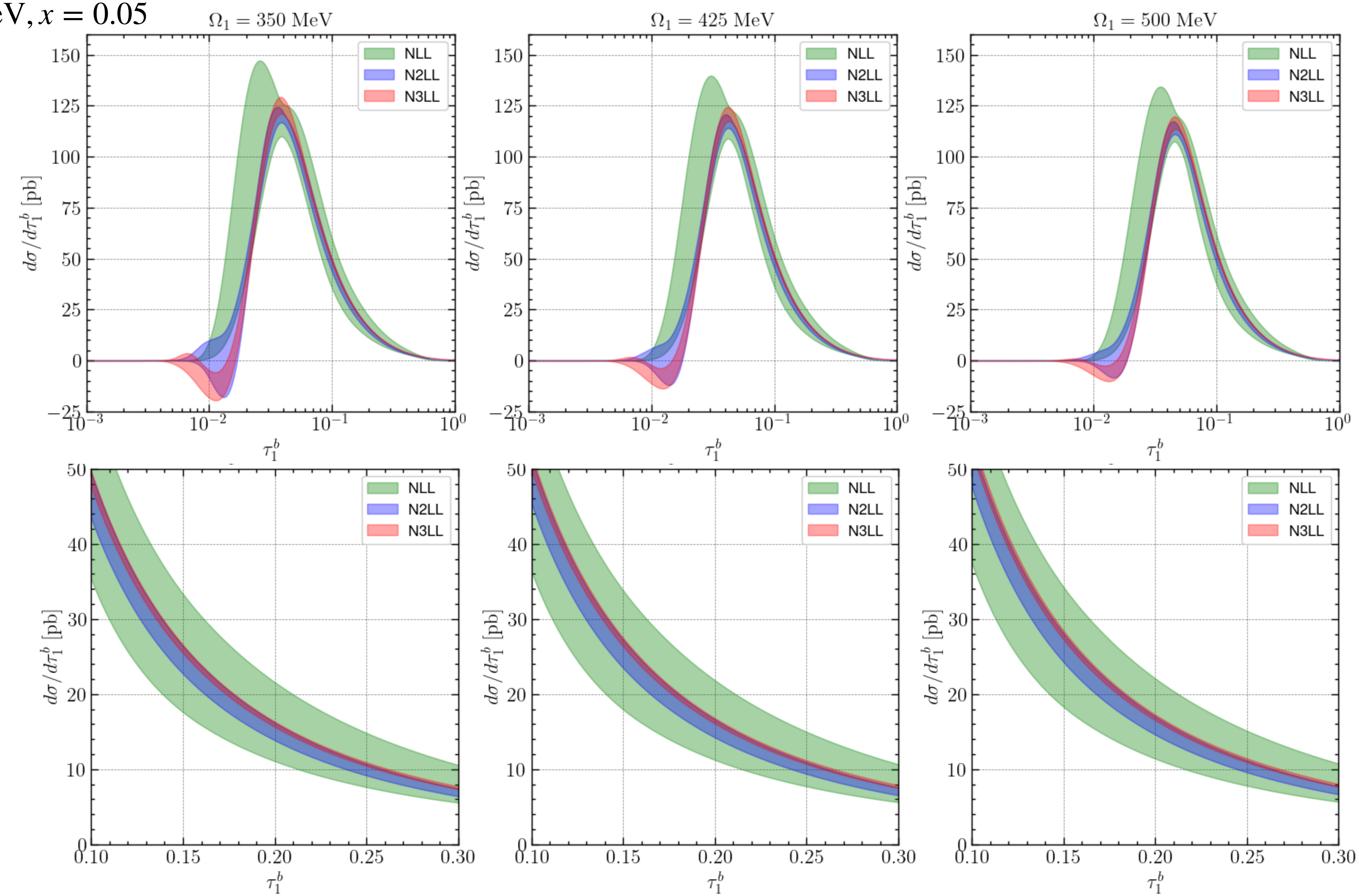


- At single x, Q we have potential percent-level sensitivity to α_s

Sensitivity to Ω_1

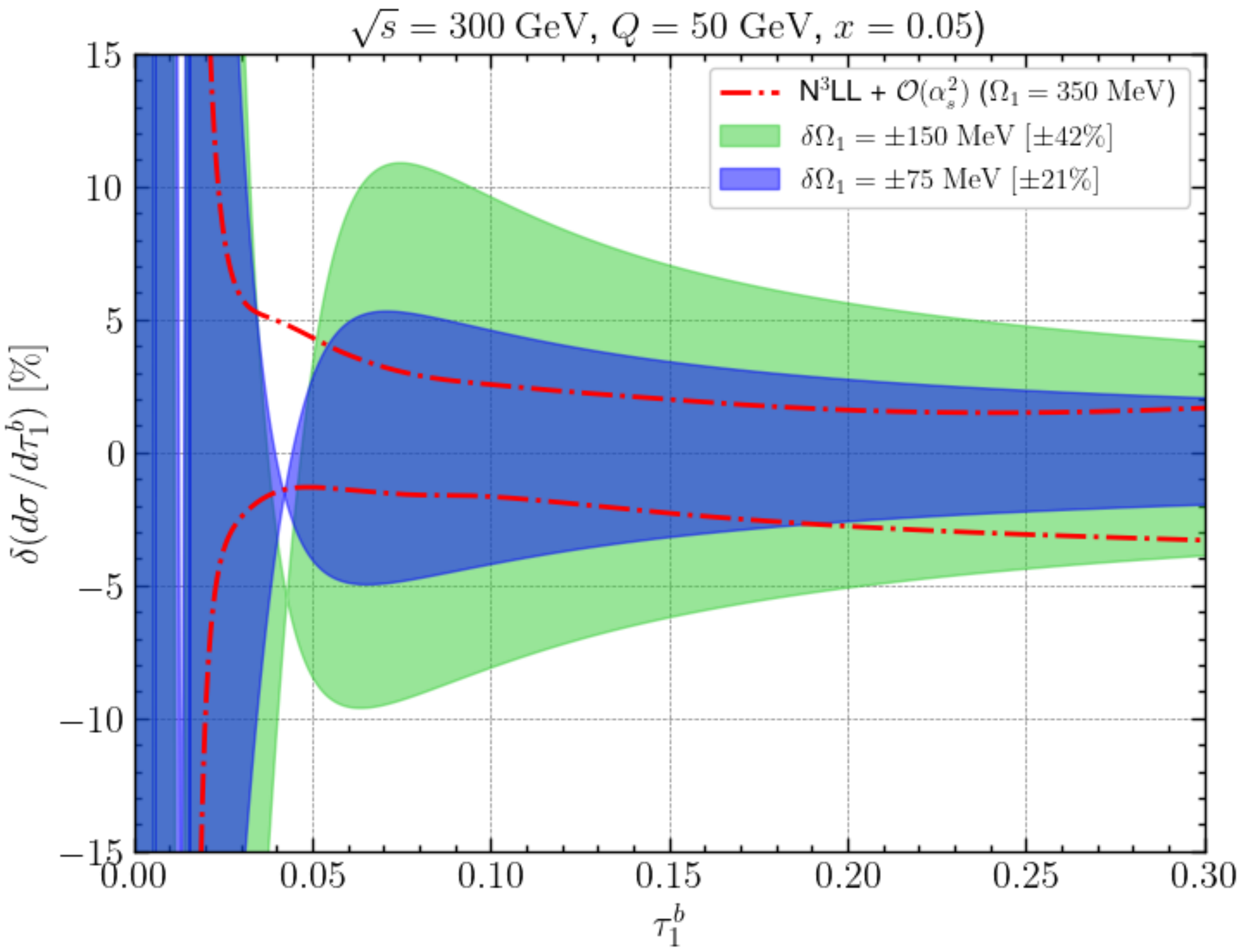
Ee, Kang, CL, Stewart [2024]

- $\sqrt{s} = 300$ GeV, $Q = 50$ GeV, $x = 0.05$
  - Peak:
  - Tail:



Ee, Kang, CL, Stewart [2024]

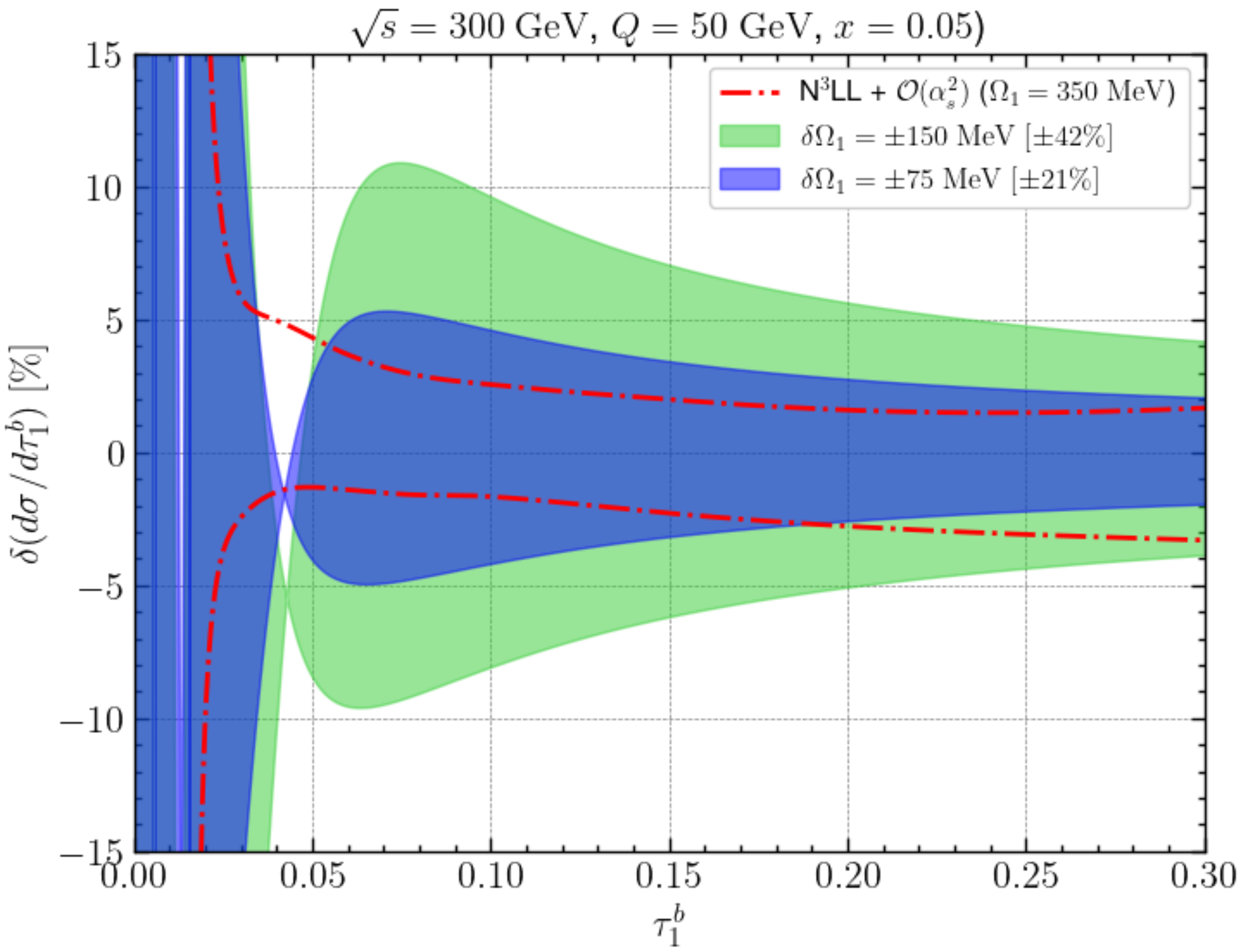
Sensitivity to Ω_1



- At single x, Q we have potential 10%-level sensitivity to Ω_1

Ee, Kang, CL, Stewart [2024]

Sensitivity to Ω_1



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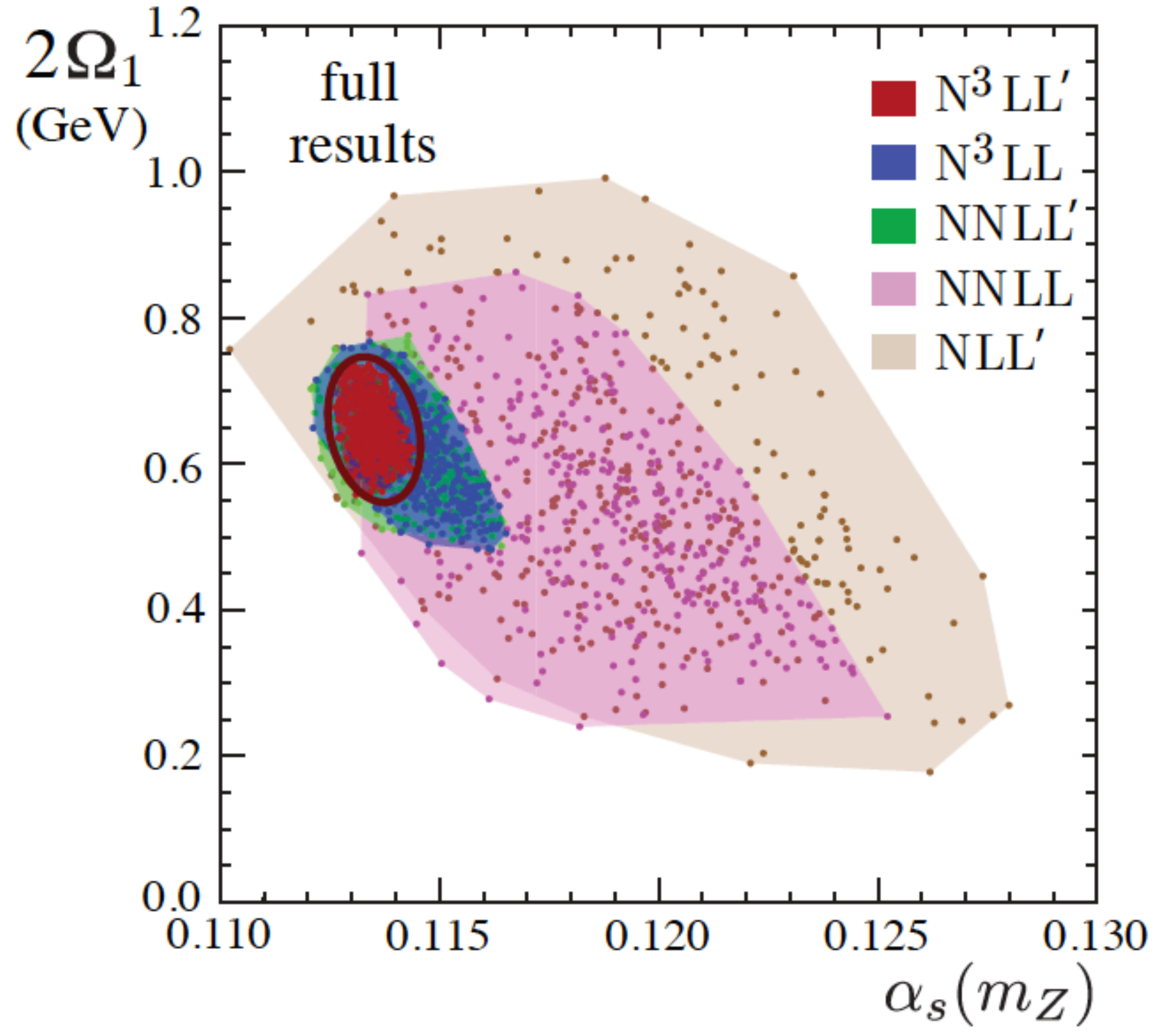
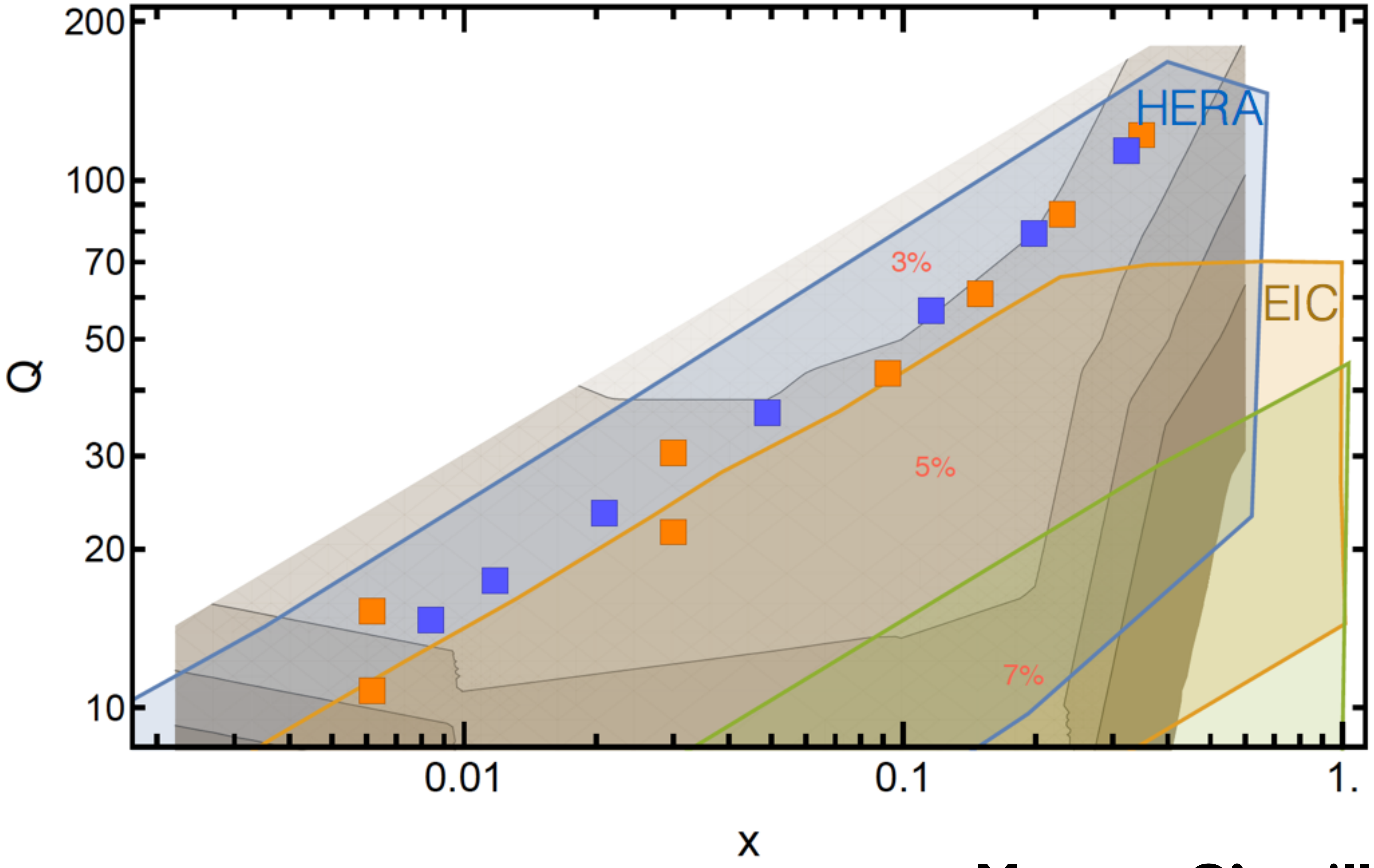


Experimental reach

Current theoretical uncertainty vs. HERA or EIC coverage:

EIC Yellow Report [2103.05419]



Many x, Q's will help break degeneracies between $\{\alpha_s, \Omega_1\}$, improve precision on both

Outlook

- $N^3LL + \mathcal{O}(\alpha_s^2)$ resummed + fixed-order predictions for DIS thrust available, our results to appear soon
- Event shapes in DIS promising candidates for precision determination of strong coupling, PDFs, and hadronization corrections, complementary/ orthogonal to e^+e^- and other determinations
- Results from HERA encourage this promise as we enter the EIC era

Performance improvement: Summary

- We optimized our codes in mostly two ways, one as the bicubic interpolator (both for fixed and resummed) and the other as the convolution integrations (for resummed).

<u>Fixed-order singular (5 scale variations)</u>

Original codes: ~ 40 mins

↓

New codes: ~ 10 mins

<u>Resummed singular (17 scale variations)</u>

Original codes: 13 hours

↓

New codes: ~ 7 hours

But turned out the convolution integrations are subject to the numerical instabilities due to the singularities at the boundaries.

↓

New codes: ~ 2.5 hours

And this is numerically stable!

Optimized bicubic interpolator for the interpolated beam function

Use the integration method implementing tanh-sinh method (mpmath)

Contrast with other work

Cao, Kang, Liu, Mantry
[2401..01941]

- Same theoretical accuracy, but different definition of DIS 1-jettiness

1) $$\tau_1 = \sum_k \min\left\{\frac{2q_B \cdot p_k}{Q_B}, \frac{2q_J \cdot p_k}{Q_J}\right\}$$

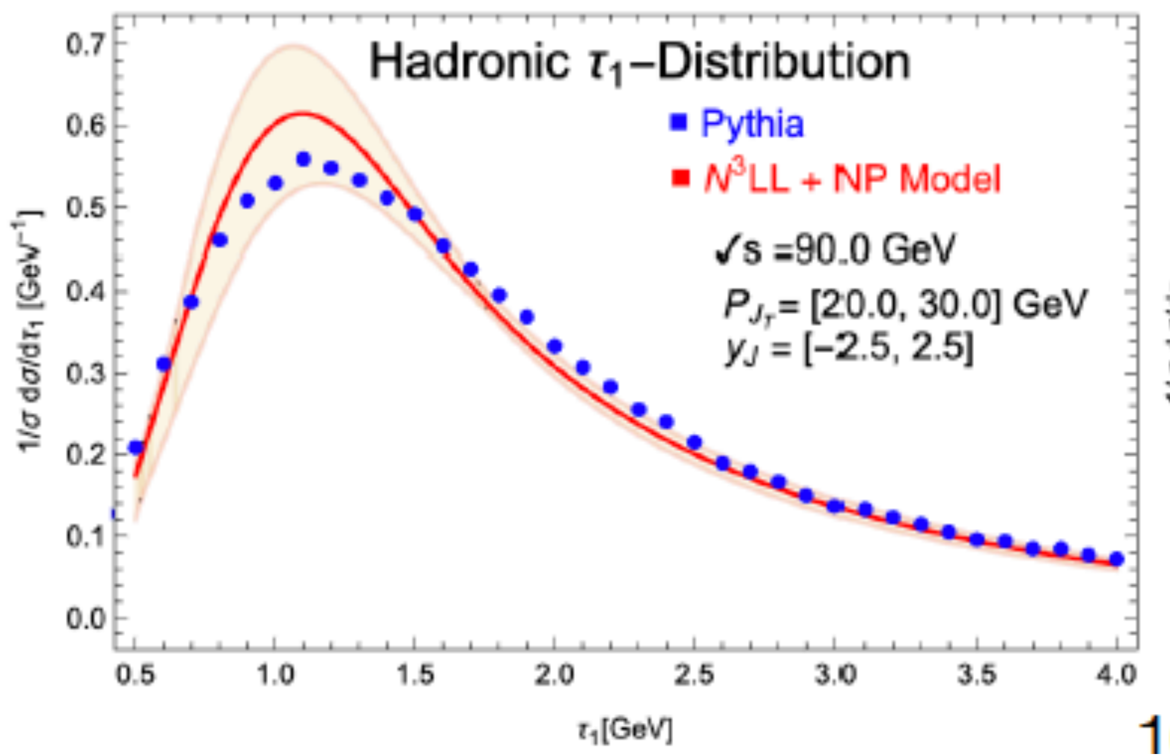
$$q_B = xP, \qquad Q_B = x\sqrt{s}.$$

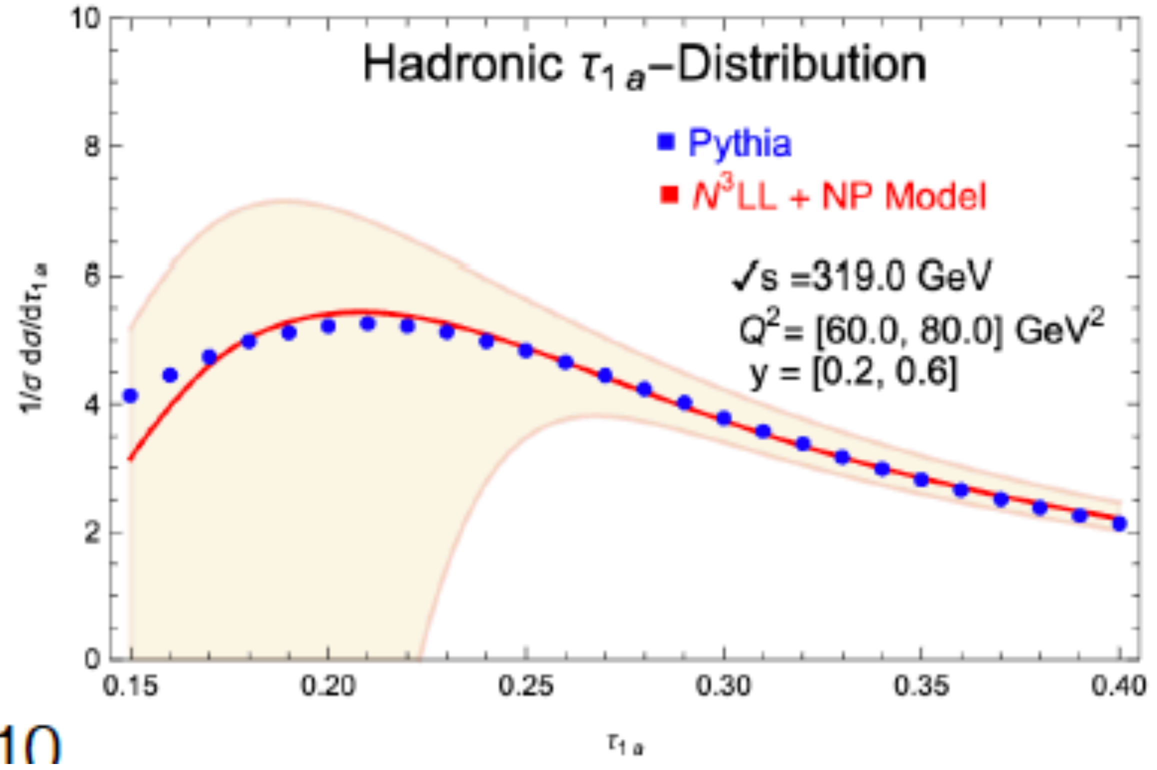
$$Q_J = 2K_{J_T}\cosh y_K, \qquad q_J = (K_{J_T}\cosh y_K, \vec{K}_{J_T}, K_{J_T}\sinh y_K).$$

2) $$\tau_{1a} = \sum_k \min\left\{\frac{2q_B \cdot p_k}{Q^2}, \frac{2q_J \cdot p_k}{Q^2}\right\}.$$

- Requires use of jet algorithm

- The jet axis is chosen to be along the actual jet momentum found from jet algorithm
  $\rightarrow \mathbf{p}_\perp^2$ is not convoluted between jet and beam functions, so written in terms of the ordinary beam function





- Does not make use of universal Ω_1 in shape function
- Simpler profile functions
- No renormalon subtractions

Singular vs. non-singular

Region where resummation is important is thus a function of *x*:

Crossing point between singular and non-singular contributions is, empirically, about:

$$t_2 = \frac{1 - \log(x + x_c)}{10}$$

$$x_c = 0.0001234$$