Low-energy photodisintegration of light nuclei within Cluster EFT

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Outline

- Considered light nuclei: ^{9}Be and ^{6}He
- Theoretical ingredients/methods of calculation
- ^{9}Be: Interaction model
- ^{9}Be: Results
- ^{6}He case
- Outlook

Why ^{9}Be?

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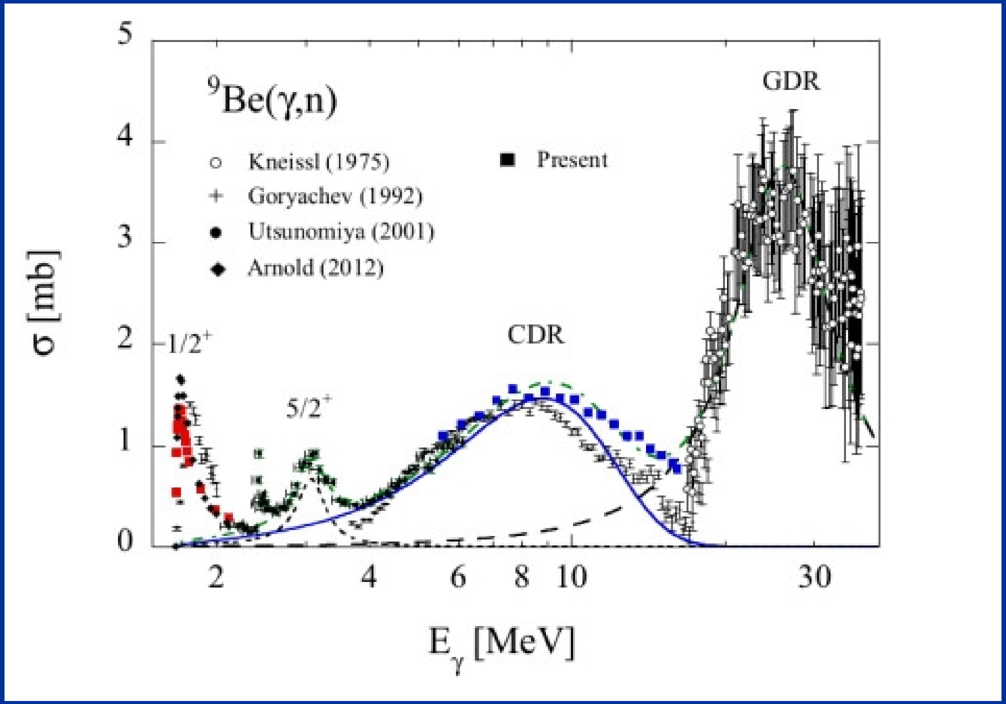

H. Utsunomiya et al.
PRC 92, 064323 (2015)

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- Further interesting additional aspect: Calculation of reaction using potentials derived in cluster effective field theory (cluster EFT)

Cluster EFT

EFT: Based on a separation of scales for low and high energies

Aim: Description of observables in low-energy regime

Inclusion of high-energy effects on low-energy observables via low-energy constants (LECs)

Degrees of freedom for ^{9}Be can be nucleons, but in the low-energy regime also given by two alpha particles plus a neutron. The upper limit of the low-energy regime is the excitation energy of the alpha particle (about 20 MeV).

^{9}Be described as $\alpha\alpha$n system has a shallow binding energy of about 1.5 MeV

Lorentz integral transform (LIT)

The cross section of nuclei excited by external probes (photon, electron, neutrino) is given in terms of response function of the form

$R(E) = \int df\, |\langle f|\, O\, |i\rangle|^2\, \delta(E_f - E_i - E)$ with a given excitation operator O

Consider the LIT: $L(\sigma) = \int dE\, R(E)/[(E - \sigma_R)^2 + \sigma_I^2]$

LIT can be calculated without explicit knowledge of the response function by

$$L(\sigma) = \langle \varphi | \varphi \rangle$$

where φ fulfills $(H - \sigma_R - E_i - \sigma_I)\, |\varphi\rangle = O\, |i\rangle$

- Norm of φ exists thus φ can be calculated with bound-state methods
- we use expansions of ground state $|i\rangle$ and LIT state $|\varphi\rangle$ in hyperspherical harmonics (HH)

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Solution with HH basis leads to discretized continuum states $|n\rangle$ with

- $L(\sigma) = \Sigma_n\, |\langle n|\, O\, |i\rangle|^2 / [(E_n - \sigma_R)^2 + \sigma_I^2]$

If spectrum E_n is sufficiently dense: $R(E) \longrightarrow \sigma_I / \pi\;\; L(\sigma)$

Inversion of the LIT

- LIT is calculated for a fixed σ_I in many σ_R points

Express the searched response function formally on a basis set with M basis basis functions $f_m(E)$ and open coefficients c_m with correct threshold behaviour for the $f_m(E)$ (e.g., $f_m = f_{thr}(E) \exp(-\alpha E/m)$). If specific structures, like narrow resonances, are present allow for basis functions $f_m(E)$ with such a structure, e.g. Lorentzians with variable position and width

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- Make a LIT of the basis functions and determine coefficents c_m by a fit to the calculated LIT

- Increase M up to the point that a sufficient convergence is obtained (structures with too small widths or uncontrolled oscillations should not be present)

Hyperspherical harmonics (HH)

A-body problem: A-1 intrinsic positions/momenta + cm position/momentum

In case of HH the 3A-3 intrinsic variables are given by a hyperradius ρ or a hypermomentum Q and 3A-4 angles described by Ω

The HH basis consists of a hyperradial/hypermomentum part and a hyperspherical part. The expansion is given by

$$\sum_{[K]n} Y_{[K]}(\Omega)\, R_{[K]n}(\rho/Q)$$

K is the grand-angular quantum number and [K] stands for a set of quantum numbers associated with K

We use nonsymmetrized HH (NSHH) in momentum space

Collaboration for ^{9}Be calculation:

Ylenia Capitani, Elena Filandri, Chen Ji,
Giuseppina Orlandini

^{9}Be described as $\alpha\alpha n$: $\alpha\alpha$ and αn interactions?

Consider effective range expansion for phase shift $\delta_l(k)$

$$k^{2l+1}\delta_l(k) \cot(\delta_l) = -1/a_l + r_l k^2/2 + O_l k^4 + \ldots$$

with scattering length a_l and effective range r_l

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LO Cluster EFT for resonant partial waves: choose LECs such that experimental results for a_l and r_l are described

s-wave resonance $^1S_0$ for $\alpha\alpha$ (^{8}Be)

$a_0 = -1920$ fm, $r_0 = 1.1$ fm

p-wave resonance $^2P_{3/2}$ for αn (^{5}He)

$a_1 = -62.951$ fm^3, $r_1 = -0.882$ fm^{-1}

and s-wave $^2S_{1/2}$ for αn

$a_0 = 2.464$ fm, $r_0 = 1.385$ fm

Cluster EFT

Potentials in momentum space

$$V(\mathbf{p},\mathbf{p}') = \sum_\ell V_\ell(p,p')\,(2\ell+1)\,P_\ell \cos(\Theta_{\mathbf{pp}'})$$

$$V_\ell(p,p') = g(p)\,g(p')\,p^\ell\,p'^\ell\,[\lambda_0 + \lambda_1 (p^2 + p'^2)]$$

where **p** and **p'** are the relative momenta of the 2-body system

and g(p) is a cutoff: $g(p) = \exp(-p^4/\Lambda^4)$

Make similar expansion for t-matrix

$$\mathbf{t}_\ell(p,p') = g(p)g(p')\,p^\ell\,p'^\ell\,[\tau_{00}(E)+\tau_{10}(E)\,p^2+\tau_{01}(E)\,p'^2+\tau_{11}(E)\,p^2p'^2]$$

On-shell: $p=p'=k$, $E=k^2/2\mu$

On-shell t-matrix: $\mathbf{t}_\ell^{on}(E) = g^2(k)\, k^{2\ell} \left[\tau_{00}(E)+k^2\left(\tau_{10}(E)+\tau_{01}(E)\right)+ k^4\tau_{11}(E)\right]$

Put t-matrix in Lippmann-Schwinger equation:

$$t_\ell(p,p') = v_\ell(p,p') + \int d^3q/(2\pi)^3\, v_\ell(p,q)\left[p^2/2\mu - q^2/2\mu + i\varepsilon\right]^{-1} t_\ell(q,p')$$

and insert the expressions for $t_\ell(p,p')$ and $v_\ell(p,p')$
then solve LS equation analytically

Note: Because of Coulomb potential $\alpha\alpha$ case is more complicated.

Here the t-matrix can be splitted into $t = t_C + t_{SC}$

where t_C is the t-matrix connected to the pure Coulomb interaction, while t_{SC} is the one associated to the Coulomb-distorted short-range interaction

Compare the analytic solution for the on-shell T-matrix to the effective range expansion (here given without Coulomb)

$$k^{2\ell}/t_\ell^{on}(E) = -\mu/2\pi \;\; (-1/a_\ell + r^2_{\ell,e} k^2 - i k^{2\ell+1} + \ldots), \qquad E = k^2/2\mu,$$

$\Rightarrow$

Quadratic eqs. with two solutions for LECs λ_0 and λ_1 for any value of Λ

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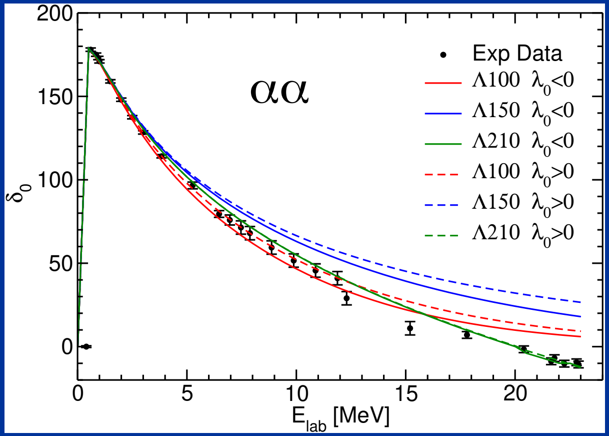

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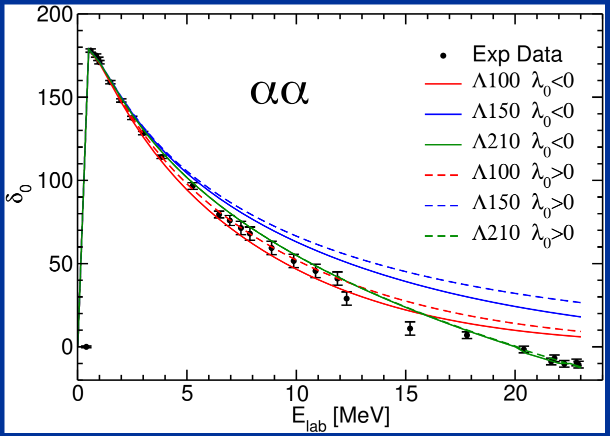


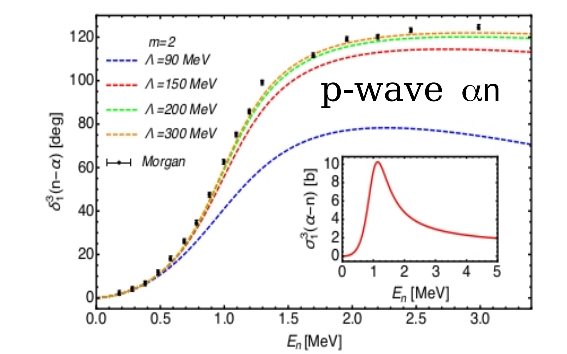


FIGURE 4.2: Phase shifts $\delta_1^3(E_n)$ ($l=1$, $J=3/2$) with experimental data from Morgan and Walter [29] and in the inset the cross section $\sigma_1^3(E_n)$ obtained with $\Lambda = 300$ MeV.

αn s-wave potential

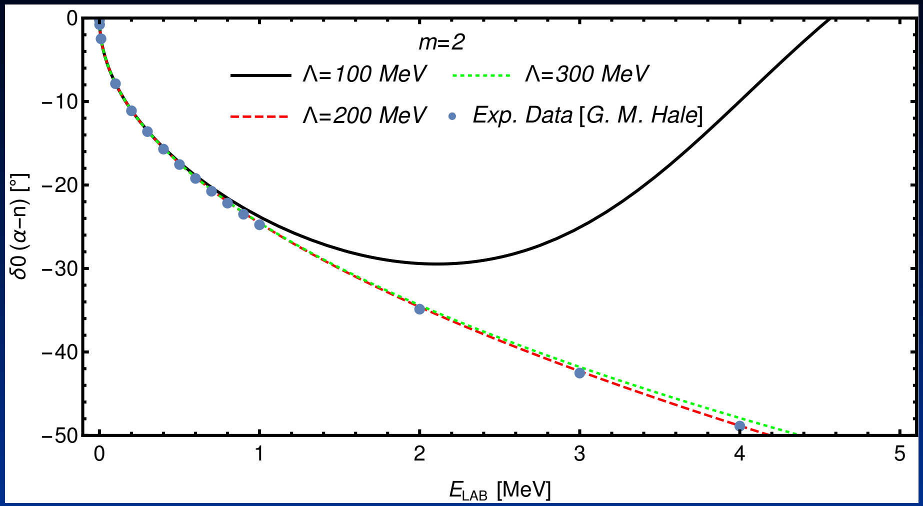
Problem: cluster ansatz leads to a deep bound state for αn system and adds spurious eigenvalues to the $\alpha\alpha$n three-body Hamiltonian which need to be removed

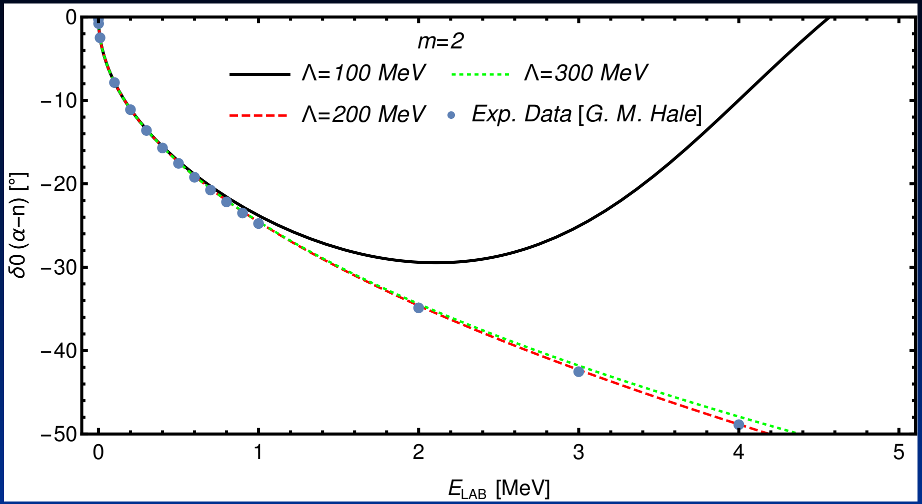
We remove this unphysical states with the so-called projection technique: one replaces the potential $V(p,p')$ by

$$V(p,p') + \Gamma \phi(p)\phi(p')$$

where $\phi(p)$ is the wavefunction of the αn deep bound state.
Note: the additional potential term has no effect on the αn phase shifts
The parameter Γ should go to infinity (in practice: Γ has to be chosen sufficiently large)

αn s-wave phase shift





αn s-wave phase shift

Γ	e_0	e_1	e_2	e_3	e_4	e_5
0	−12.25	0.3149	1.266	2.871	5.168	8.212
1	−11.25	0.3149	1.266	2.871	5.168	8.212
5	−7.245	0.3149	1.266	2.871	5.168	8.212
10	−2.245	0.3149	1.266	2.871	5.168	8.212
15		0.3149	1.266	2.871	5.168	8.212
20		0.3149	1.266	2.871	5.168	8.212
50		0.3149	1.266	2.871	5.168	8.212
250		0.3149	1.266	2.871	5.168	8.212
2000		0.3149	1.266	2.871	5.168	8.212

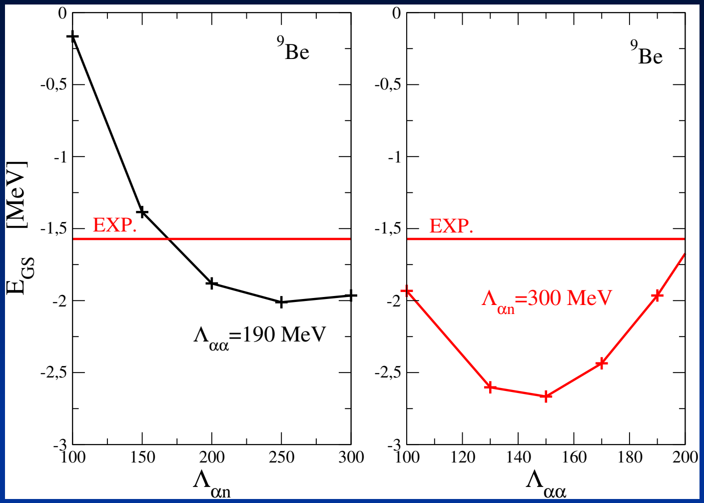
Units MeV

Diagonalization of Hamiltonian
Low-energy spectrum

Conclusion Γ = 15 MeV is sufficient to exclude unphysical αn bound state

BUT: In ααn states Γ has to be chosen considerably higher

Cutoff dependence of ^{9}Be ground-state energy



Wave function is calculated via expansion in hyperspherical harmonics (HH) in momentum space

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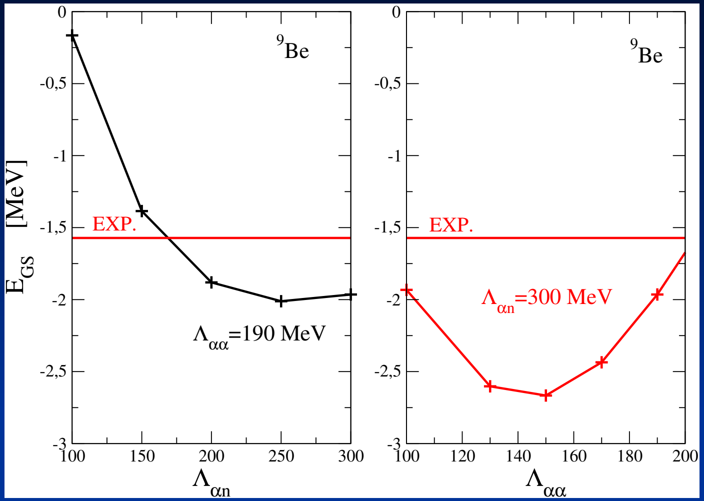

Include 3-body force: $V_3 = \lambda_3 \exp\left[-Q^2/\Lambda_3^2\right] \exp\left[-(Q'^2)/\Lambda_3^2\right]$

Q and Q' are hypermomenta

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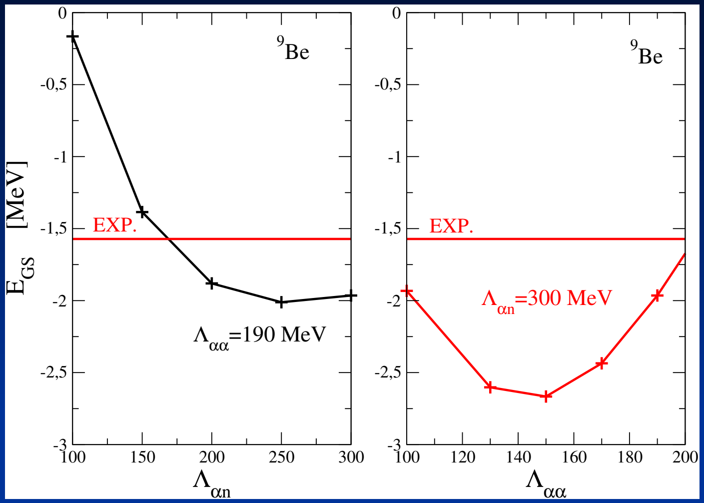

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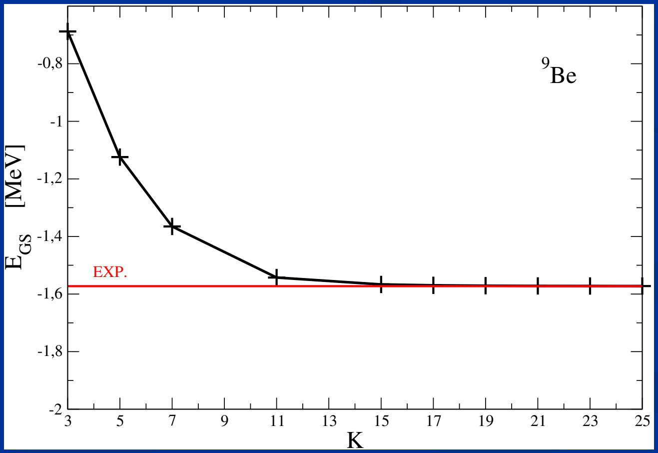
HH convergence in function of grand-angular quantum number K



^{9}Be photodisintegration

Only E1 transitions are considered, since ^{9}Be has $J^\pi = (3/2)^-$
one has $(1/2)^+$, $(3/2)^+$ and $(5/2)^+$ final states

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Current operator in LO resulting from minimal coupling in free Lagrangian in limit of vanishing photon momentum proportional to

$$e\,(\mathbf{p}_{\alpha_1,\perp} + \mathbf{p}_{\alpha_2,\perp})\,/\,m_{\alpha} \qquad \text{(convection current)}$$

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However, interaction Lagrangian is momentum dependent

$\Rightarrow$ existence of two- and more-body currents

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Siegert theorem: use continuity equation to replace current by charge operator

Which partial waves are leading order?

Certainly the resonant $\alpha\alpha$ s-wave and αn p-wave,
but also the nonresonant αn s-wave ?

In a ^{6}He bound-state calculation in Halo EFT, (^{6}He as αnn state)
αn s-wave interaction was not considered to be LO
(C. Ji, C. Elster, and D. R. Phillips, PRC 90, 044004 (2014)).

In fact we find only a mild effect of this interaction on the ^{9}Be ground state

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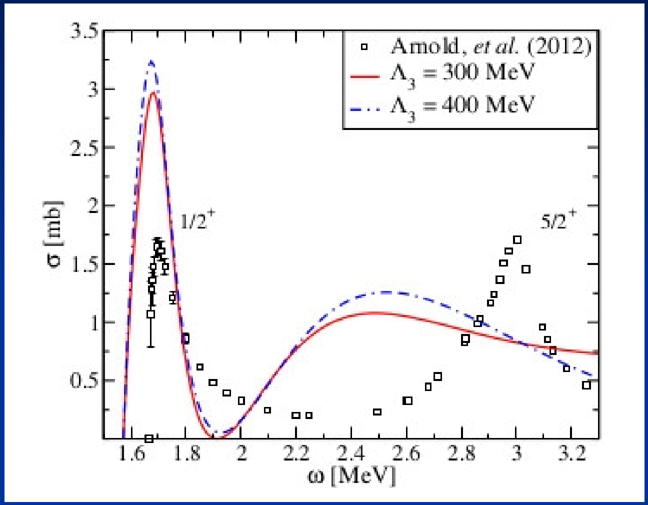
But different for final $\alpha\alpha$n states

Some results for ^{9}Be photodisintegration to 1/2+ final state



Cross section without αn s-wave

Much too large!

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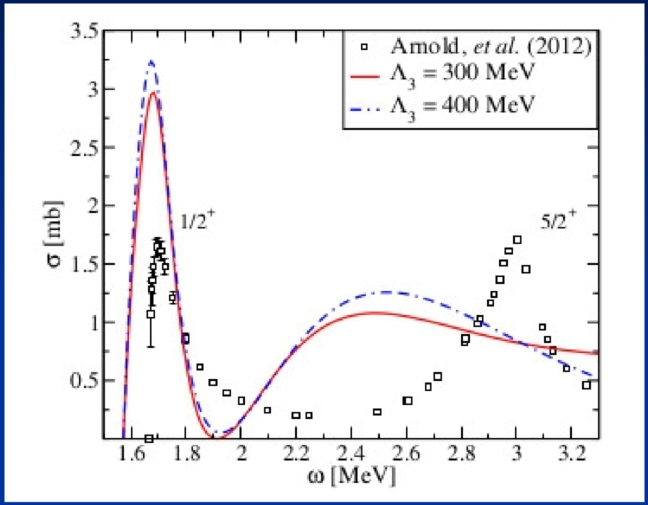

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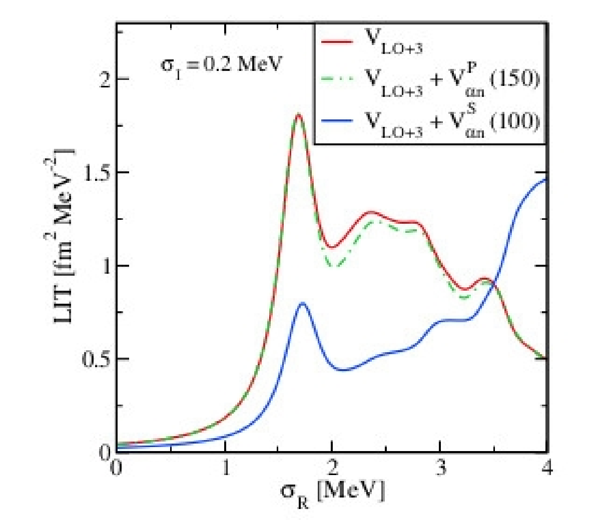

LIT: Blue line with inclusion of αn s-wave interaction

Strong reduction

Conclusion: αn s-wave has to be of leading order as low-energy continuum state

Confirmation comes from the Halo-EFT calculation

(P. Bedaque, H.-W. Hammer, U. van Kolck, PLB 569, 159 (2003))

Conclusion: αn s-wave has to be leading order as low-energy continuum state

Back to the projection technique for αn s-wave potential $V(p,p') + \Gamma \phi(p)\phi(p')$

To get rid of the deep ^{5}He bound state a value for Γ of 15 MeV is sufficient, but in principle Γ should go to infinity.

In fact to get a parameter independent result for ^{9}Be and the $\alpha\alpha$n continuum one has to increase Γ considerably (^{9}Be: Γ = 500 MeV)

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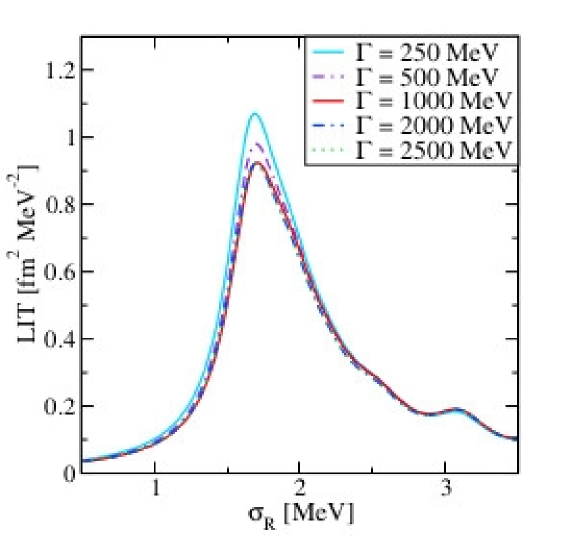
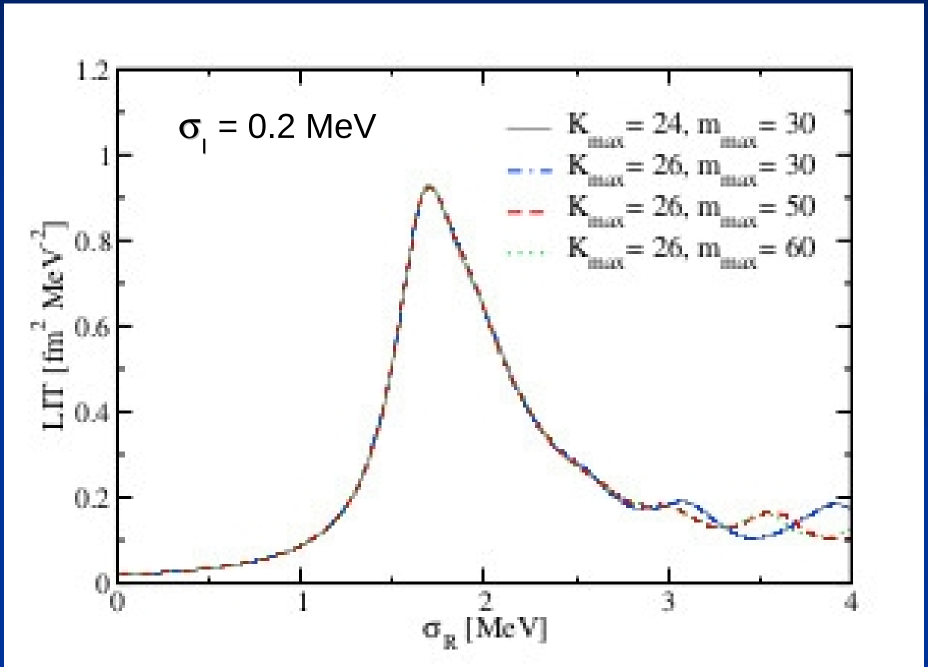
LIT with various values for the parameter Γ

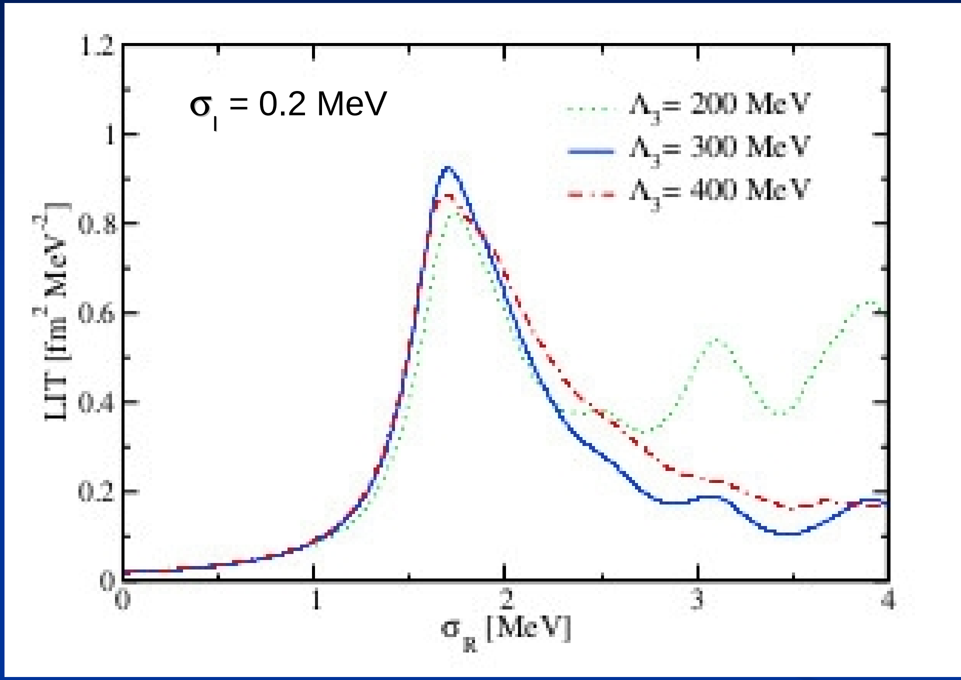


Convergence checks for LIT via increase of number of HH basis functions



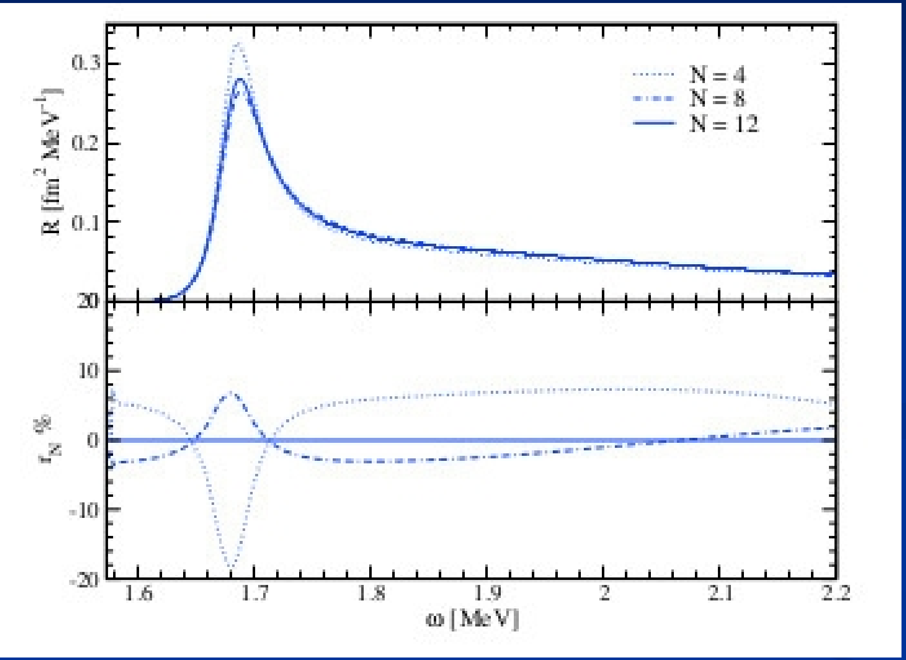
Transition to 1/2+

Dependence on cut Λ_3 of three-body potential



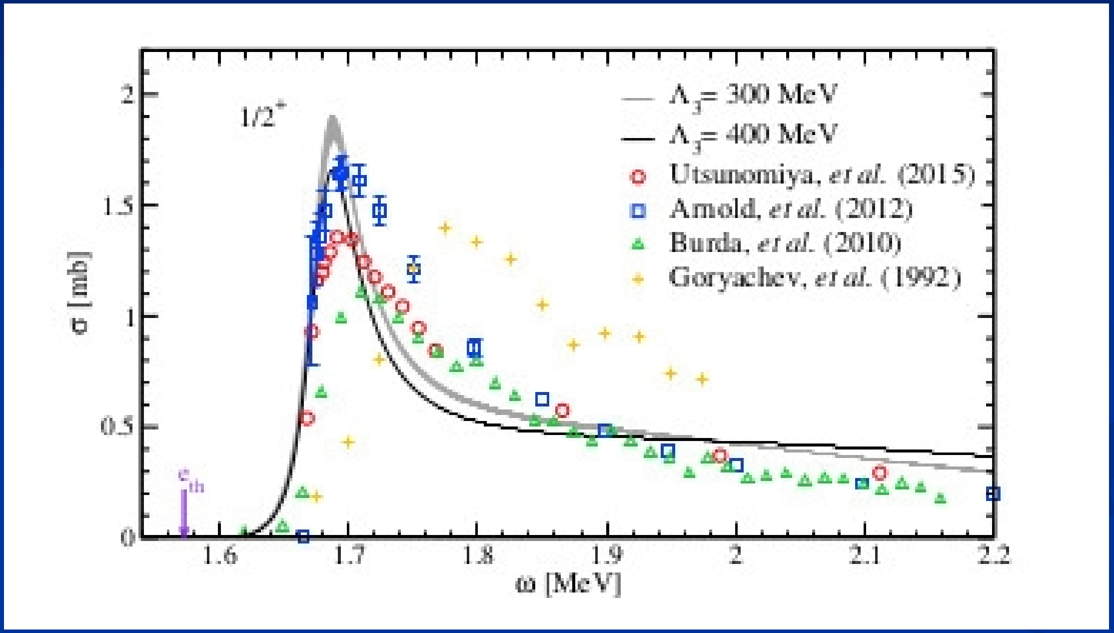
Transition to 1/2+

Inversion of LIT



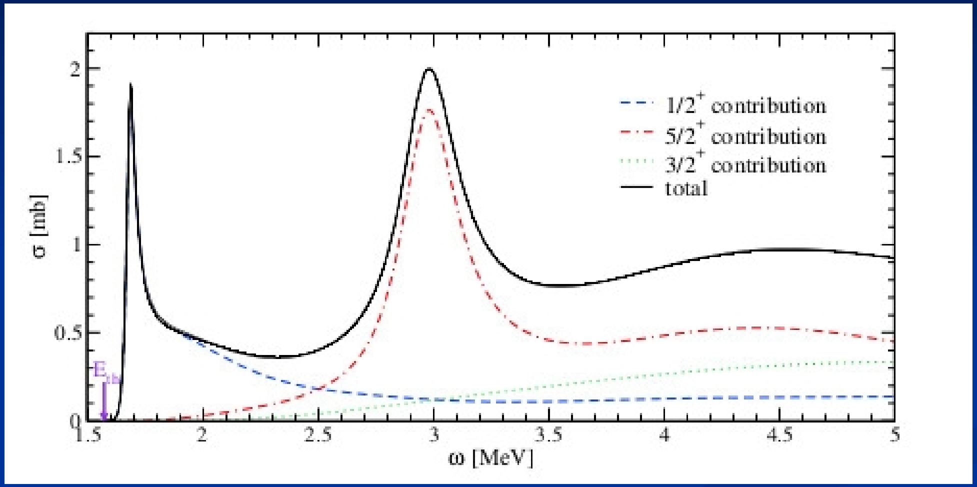
Transition to 1/2+

Comparison to experimental data

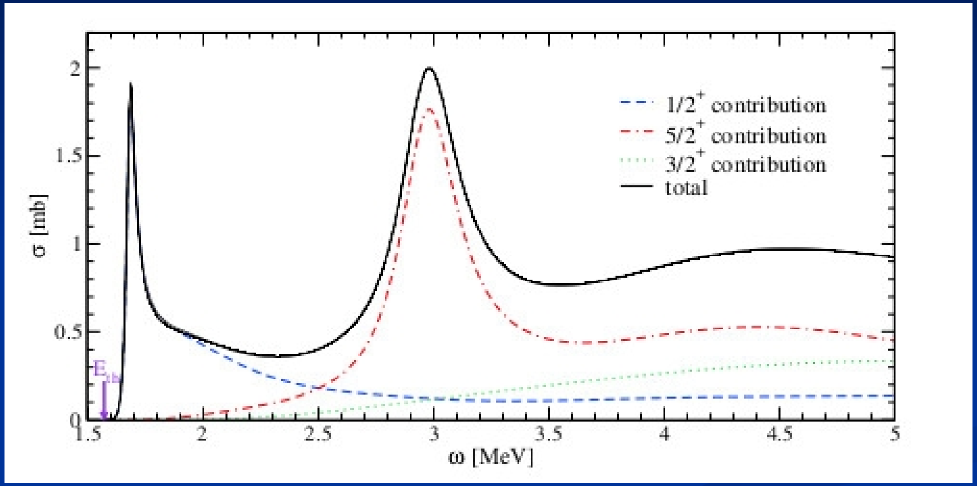


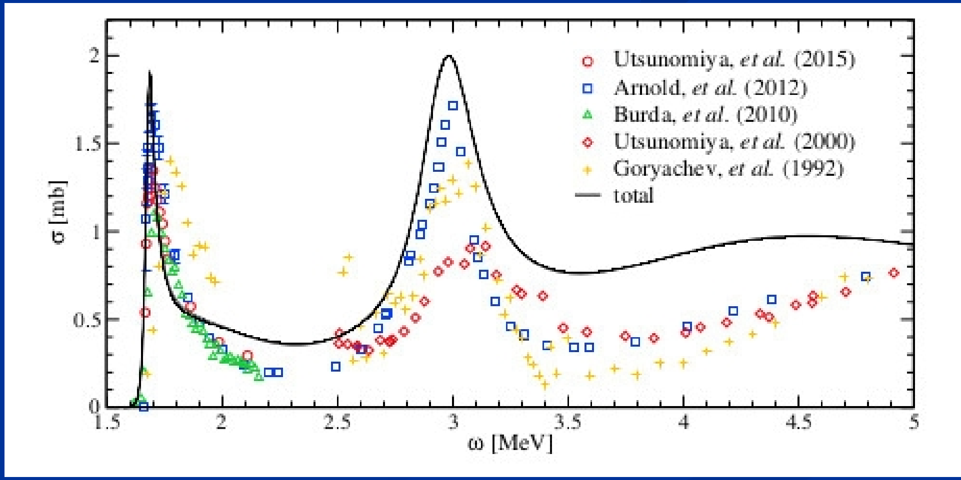
Transition to 1/2+

Inclusion of transitions to 3/2+ and 5/2+

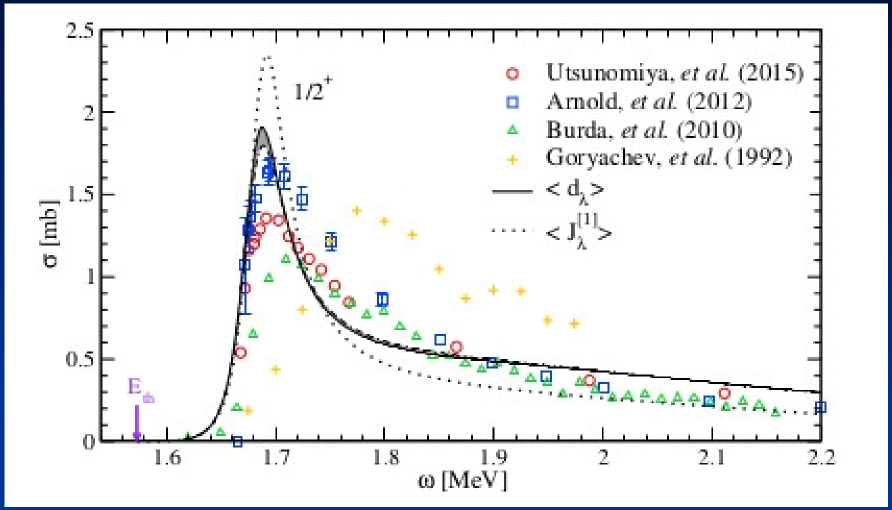


Inclusion of transitions to 3/2+ and 5/2+





Effect of many-body currents



1/2+

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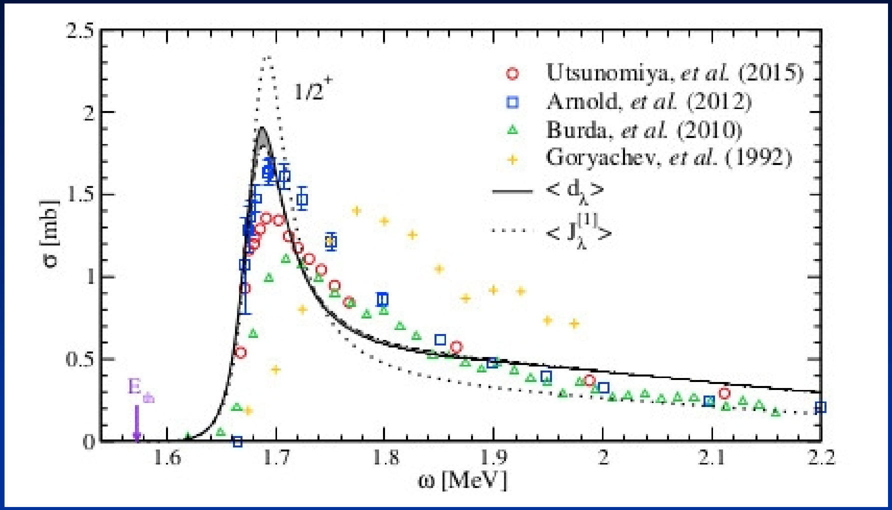
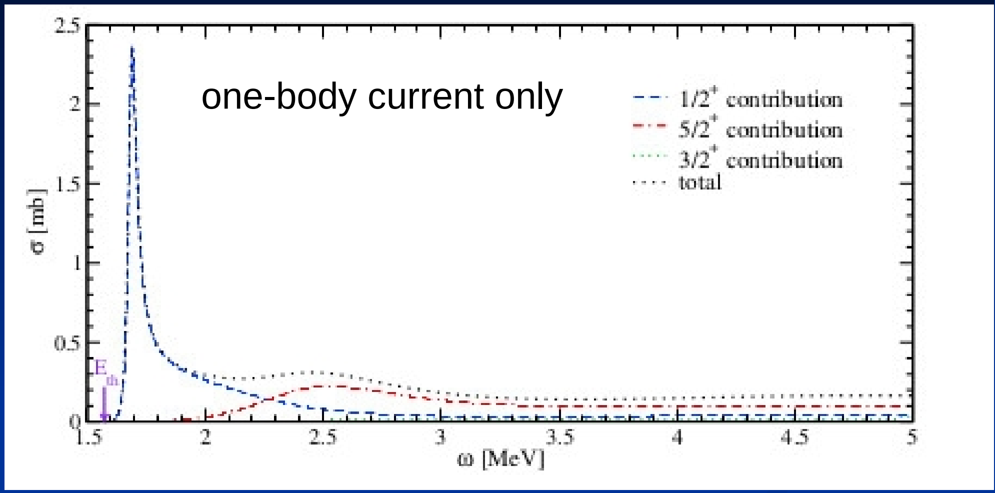
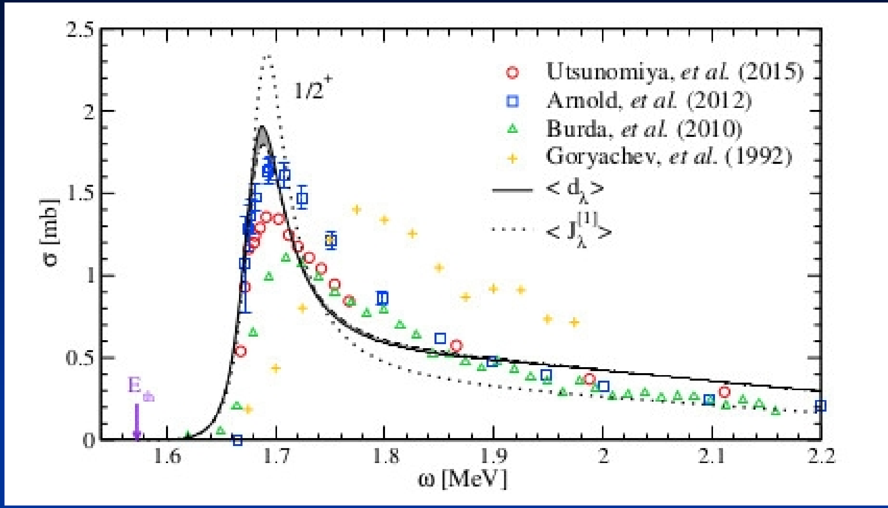


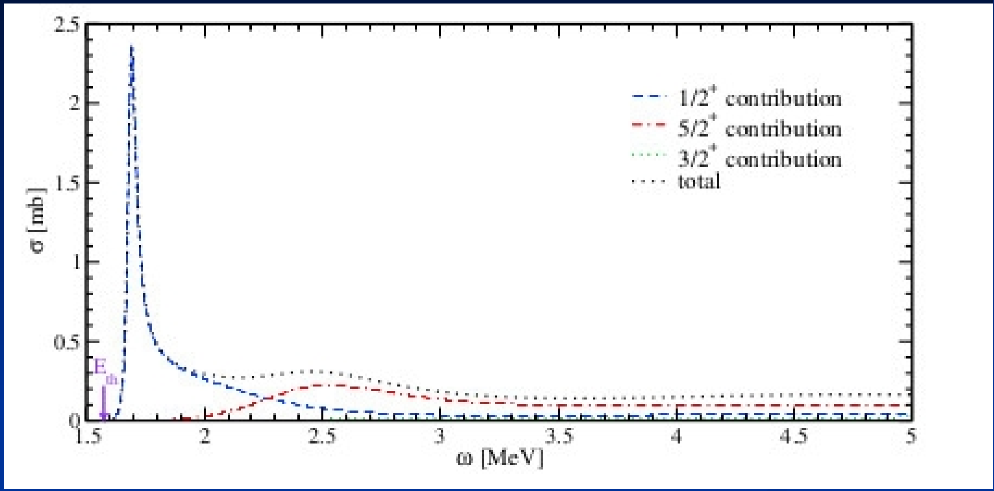


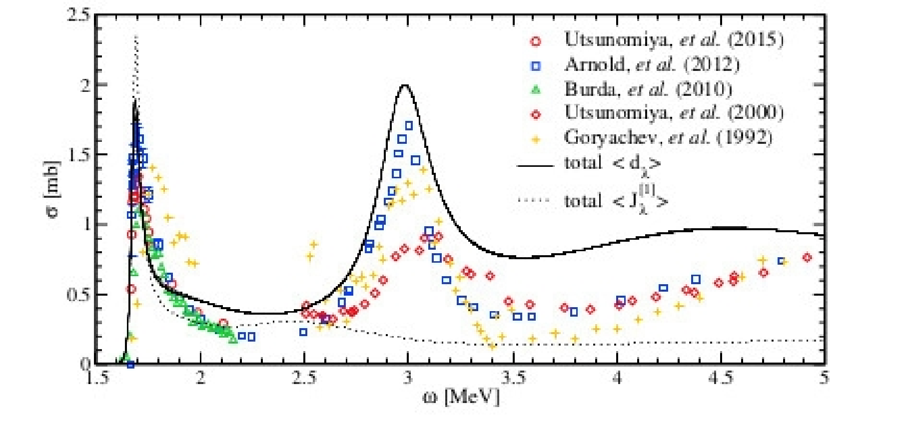
1/2+

1/2+, 3/2+, 5/2+

Effect of many-body currents







^{6}He as αnn cluster

In collaboration with

Edna Pinilla, Pierre Descouvemont, Giuseppina Orlandini

(arXiv:2409.03074)

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αn interaction as before

nn interaction for $^{1}S_{0}$ derived in the same way as before (via a_0 and r_0)

However, here we use an HH expansion in coordinate space. Coordinate space two-body potentials V(r,r') are determined from Fourier transforms of the momentum space V(p,p') potentials.

Correctness has been checked by calculating the corresponding phase shifts

Three-body force given here by: $V_3 = \lambda_3 \exp\left[-\rho^2/\rho_0^{\ 2}\right]$ with hyperradius ρ and cutoff ρ_0

E1 strength calculated with electric dipole operator (Siegert theorem)

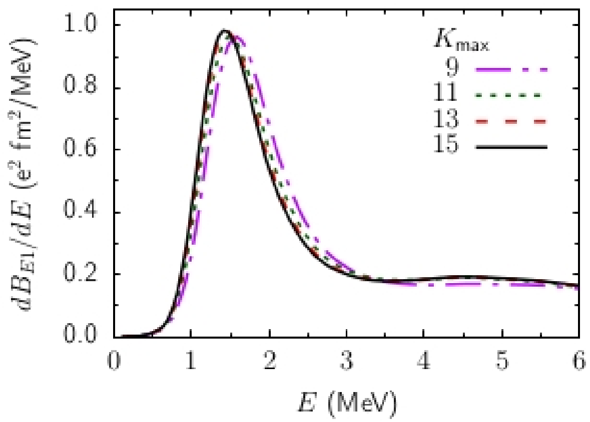


FIG. 3. Convergence of the E1 strength distribution of ^{6}He with K_{max}.

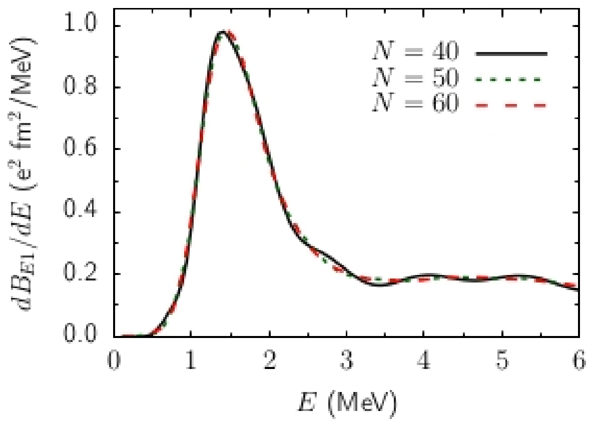
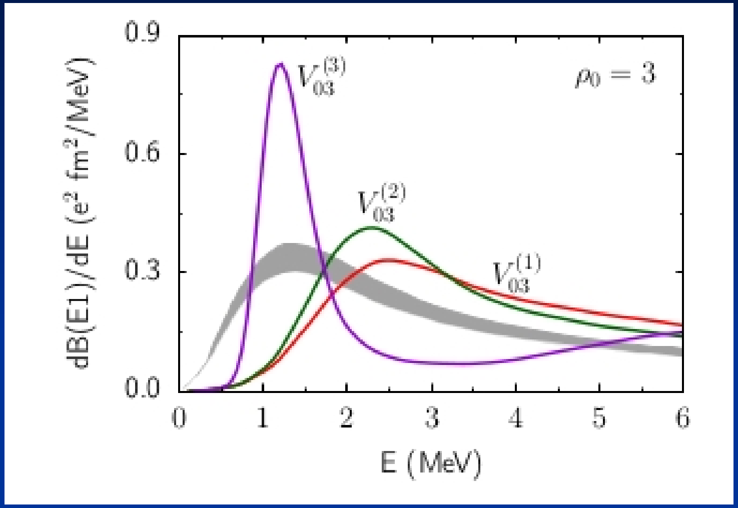


FIG. 4. Convergence of the E1 strength distribution of ^{6}He with the number of the Lagrange-Laguerre basis functions N.

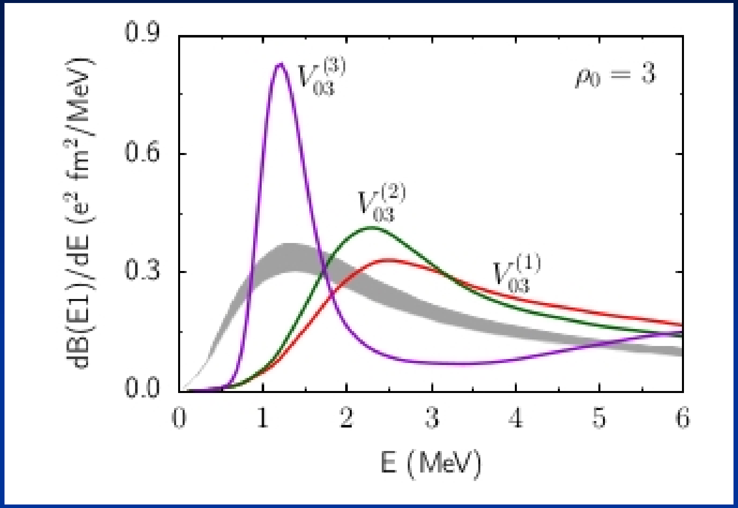
Convergence check of HH expansion



Various 3-body potential strengths for $J^{\pi} = 1^{-}$

Same as bound state: red curve

Stronger attractions for green and violett curves



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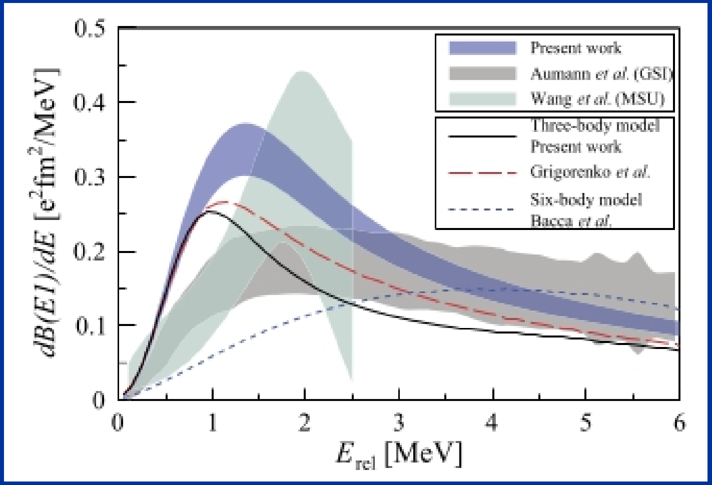

Outlook

Similar treatment for the low-energy photodisintegration of other nuclei?

^{12}C ($\alpha\alpha\alpha$ state): Calculation of E2 transition from carbon state 2+ to Hoyle state
some initial steps have been taken

^{16}O(4α state)

^{10}Be