Gluon Double-Spin Asymmetry at Small-x in Longitudinally Polarized Proton-Proton Collisions

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Y. Kovchegov and M. Li, JHEP **05** (2024) 177.

Outline

- Introduction and Motivation
- Gluon double-spin asymmetry at small-x in Gluon+Proton collisions
- Generalization of gluon double-spin asymmetry to Proton+Proton collisions
- K_T-factorization and including small-x helicity evolution.
- Summary

Origin of Nucleon Spin

The RHIC Spin Collaboration (2015)

Jaffe-Manohar spin sum rule for proton

$$S_q + L_q + S_G + L_G = \frac{1}{2}$$

Quark Spin

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$$S_q(Q^2 = 10\mathrm{GeV}^2) \approx [0.15,\ 0.20]$$

$$x \in [0.001, 0.7]$$

Gluon Spin

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

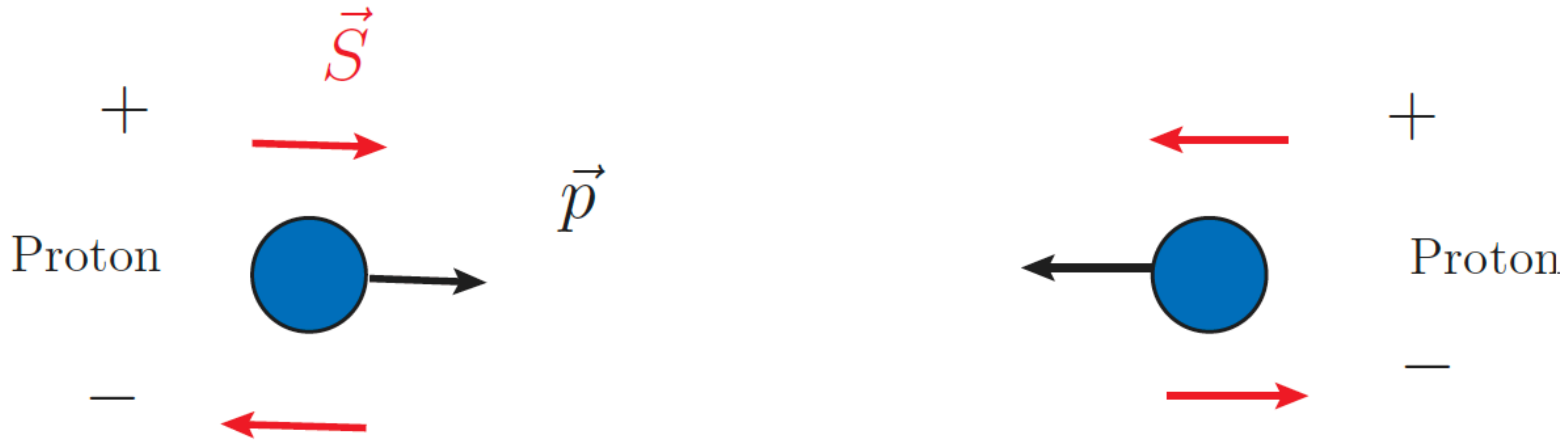
$$S_G(Q^2 = 10\mathrm{GeV}^2) \approx [0.13,\ 0.26]$$

$$x \in [0.05, 0.7]$$

Missing spin of the proton maybe in quark and gluon orbital angular momentum L_q and L_G and/or smaller values of x

Longitudinal Double-Spin Asymmetry

How to measure quark and gluon intrinsic spin inside a proton?



$$A_{LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

RHIC has measured A_{LL} for the productions of jets, dijets, π^0, $\pi^{\pm}$, direct photons…
at mid-rapidity, intermediate rapidity, forward rapidity…
at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ and $\sqrt{s_{NN}} = 510\,\mathrm{GeV}$

RHIC Spin Collaboration, arXiv: 2302.00605

Longitudinal Double-Spin Asymmetry

Longitudinal double-spin asymmetry is related to parton helicity distribution.

$$A_{\mathrm{LL}} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

Collinear Factorization (also parity invariance)

Babcock, Monsay and Sivers (1979),
De Florian, Sassot, Stratmann and Vogelsang (2008)(2014) ***(DSSV)***

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \, \Delta f_a(x_a, Q^2)\, \Delta f_b(x_b, Q^2)\, d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

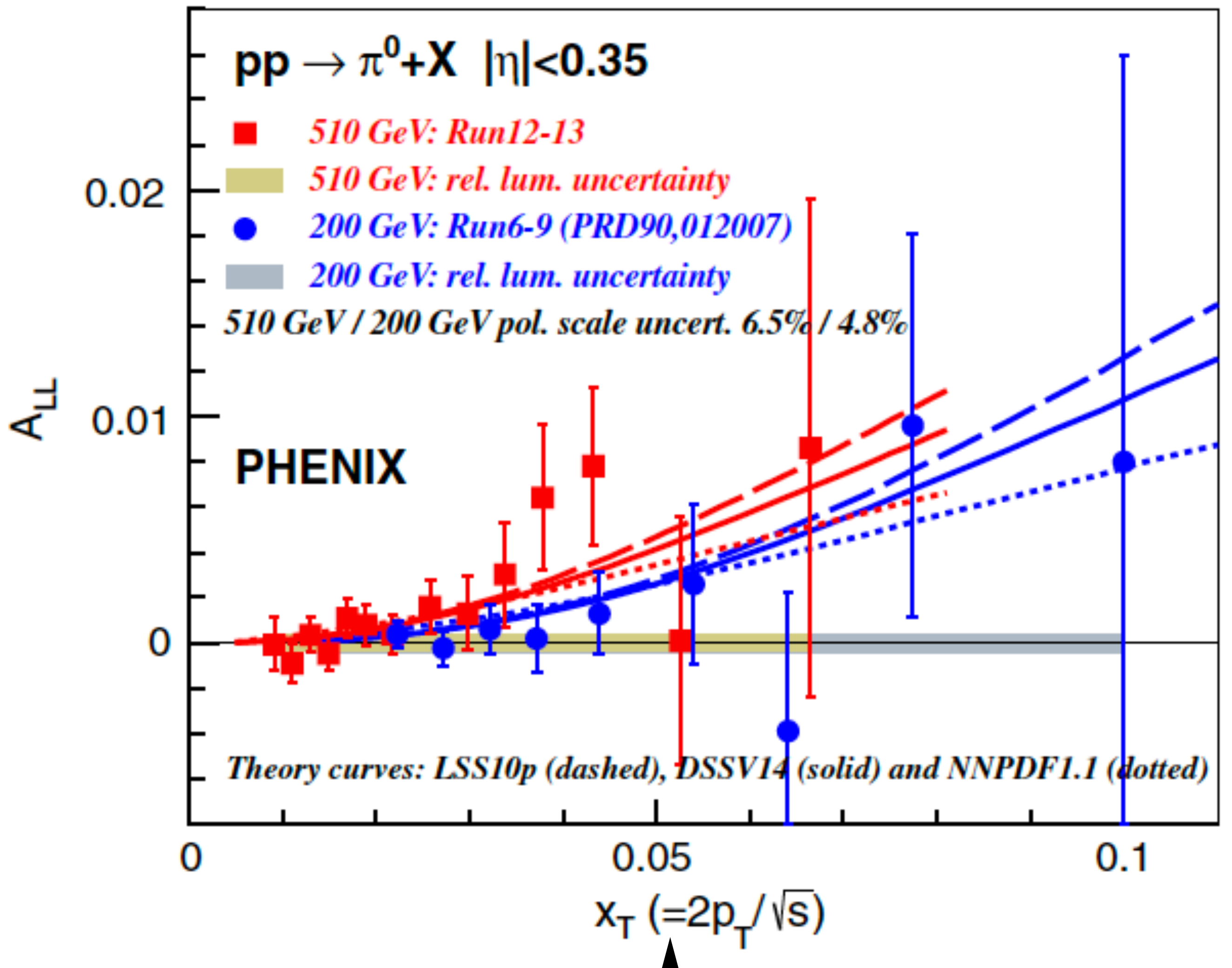
(Anti) quark and gluon helicity distribution $\Delta f_j(x, Q^2) \equiv f_j^+(x, Q^2) - f_j^-(x, Q^2)$

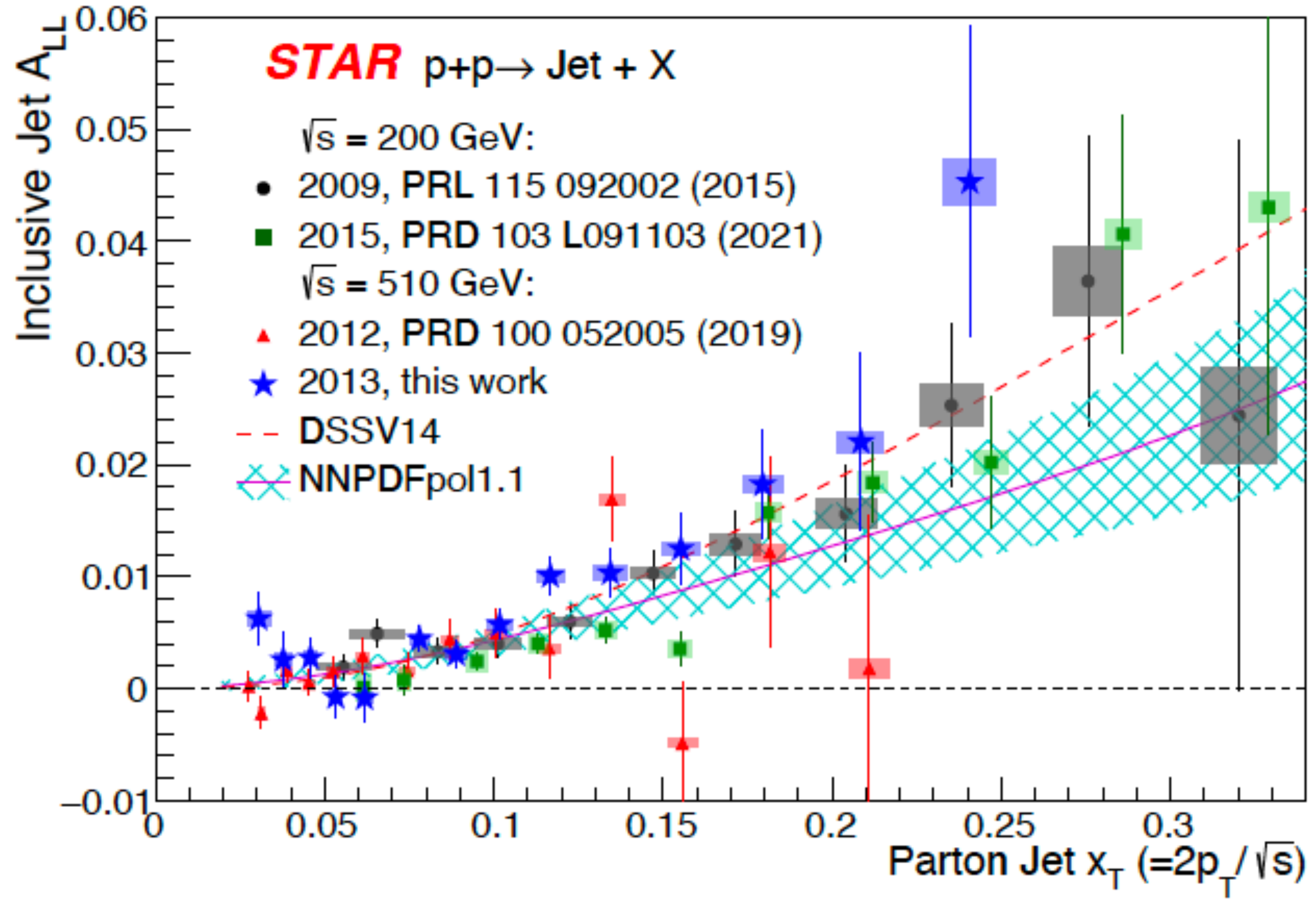
Partonic level double-spin asymmetry $d\Delta\hat{\sigma} = d\hat{\sigma}^{++} - d\hat{\sigma}^{+-}$

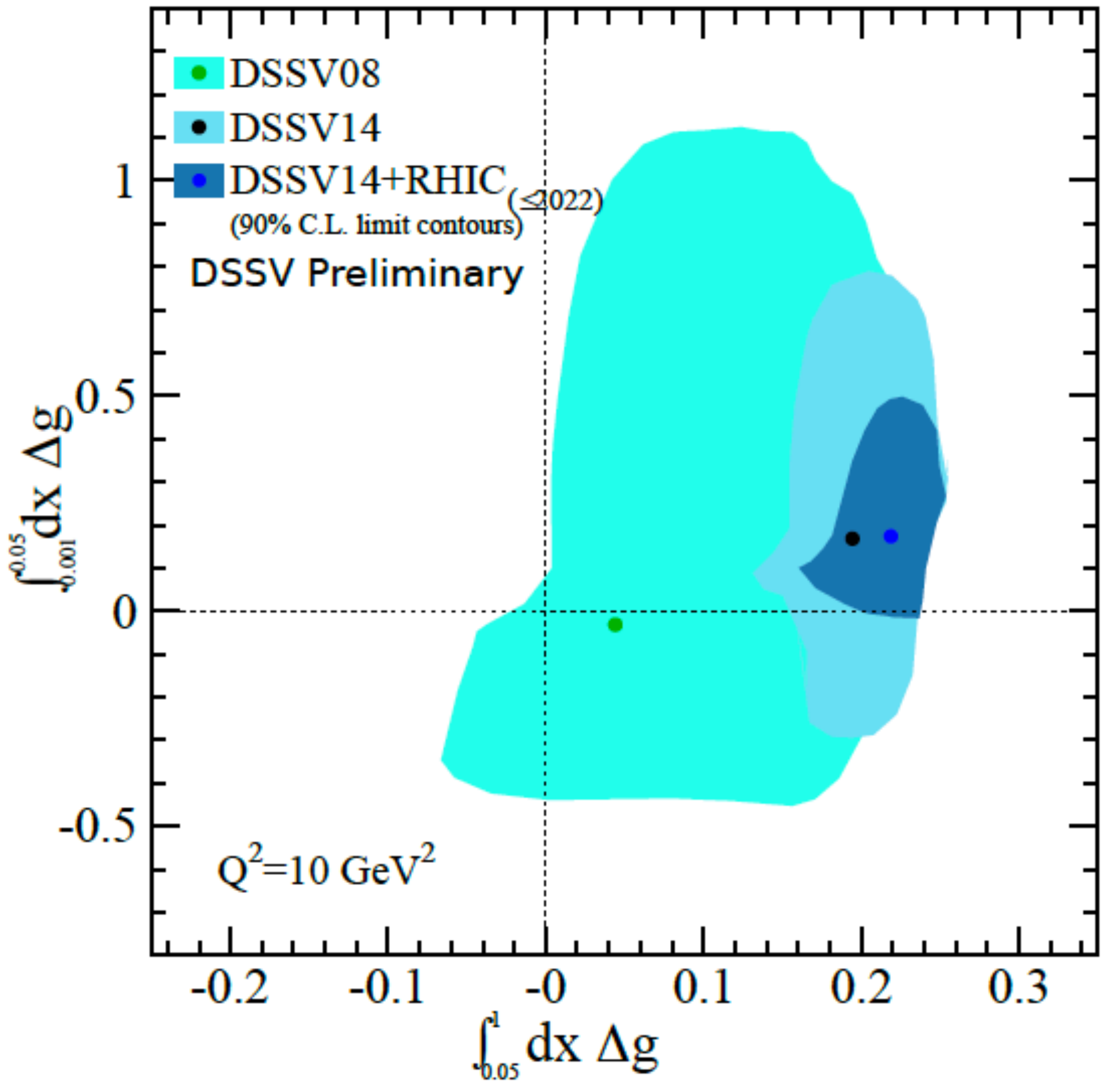
Longitudinal Double-Spin Asymmetry at small x

RHIC Spin Collaboration (2015, 2023)

A_{LL} **for inclusive neutral pion and inclusive jet productions at mid-rapidity**







5 GeV (12.5 GeV)

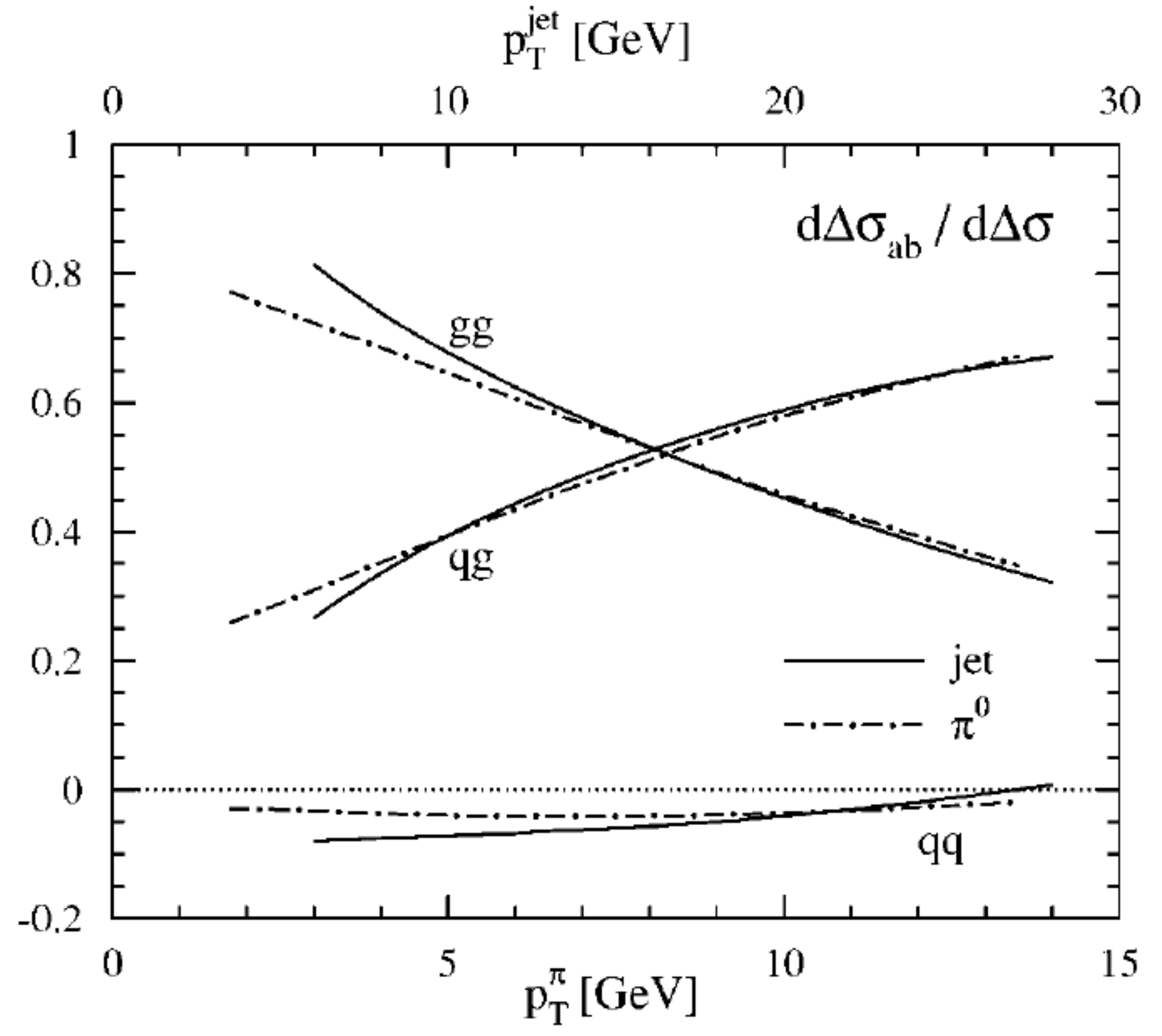
Low transverse momentum region, sensitive to small x gluons, collinear factorization probably breaks down.

very large uncertainty in constraining the small-x region of gluon helicity PDF using the collinear factorization formalism.

Transverse momentum dependent framework + Small-x helicity evolution equations, to describe A_{LL} in the low transverse momentum region and to constrain gluon helicity at smaller values of x.

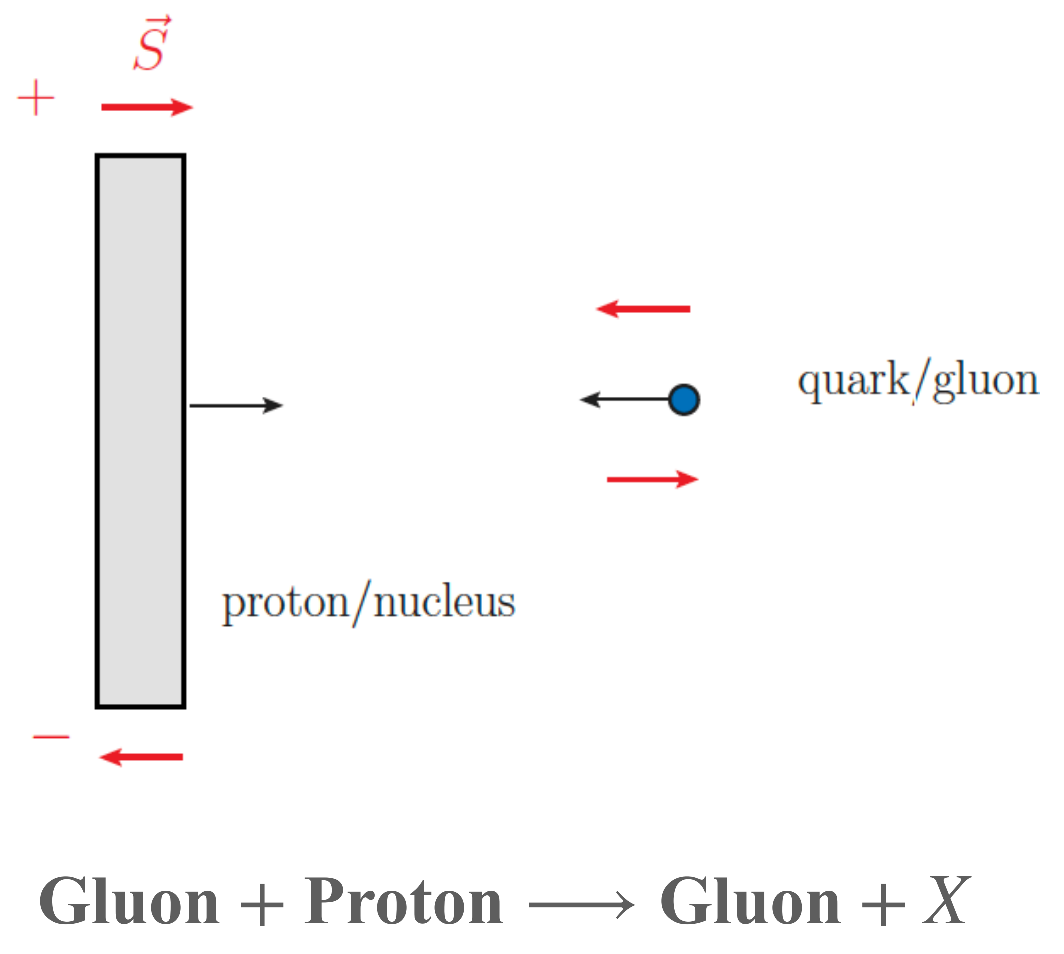
Gluon Double-Spin Asymmetry at Mid-Rapidity

Goal: A_{LL} at small-x for Gluon production at mid-rapidity

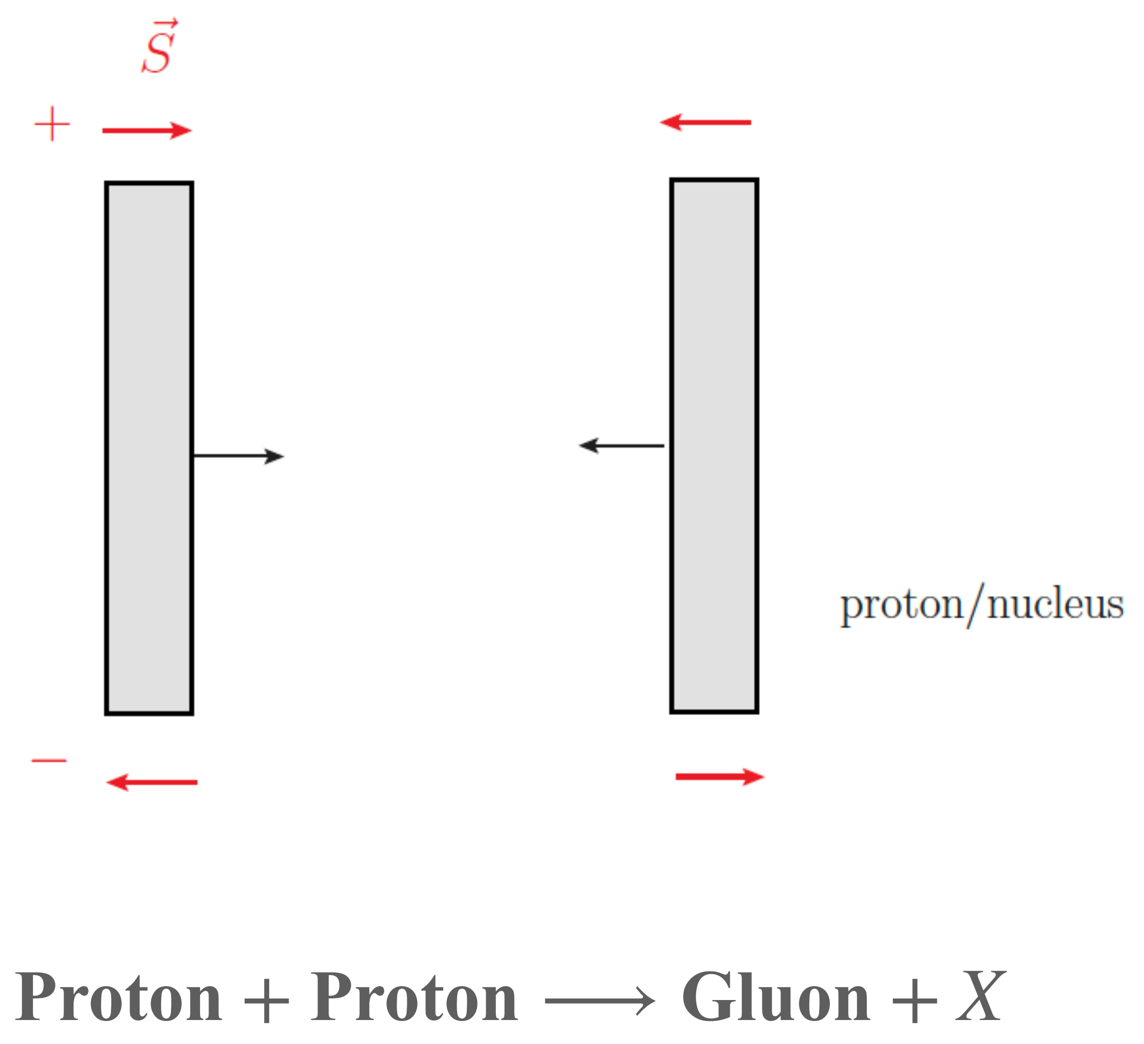


Jager, Stratmann and Vogelsang (2004)

For low transverse momentum, the gg channel dominates.



Generalization



$$A_{LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

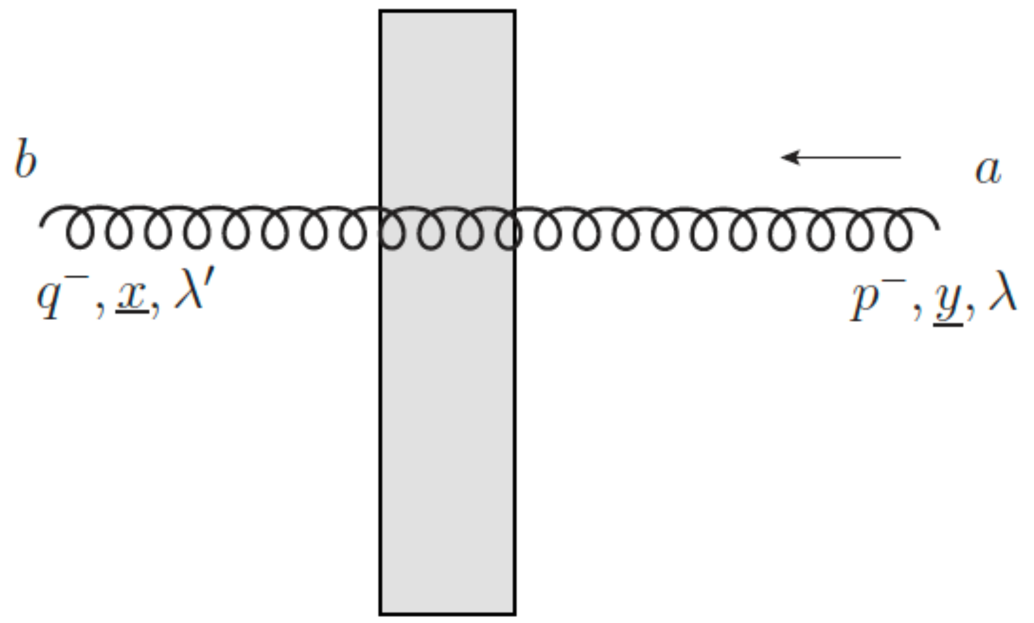
What we calculate

The leading order unpolarized gluon production at small-x has already been calculated.

Kovchegov and Mueller (1998), Kopeliovich, Tarasov and Schafer(1999), Dumitru and McLerran (2002)

High Energy Scatterings at Subeikonal Order

The eikonal order interaction with the proton is insensitive to the spin structure of the proton.



$$M^{g\to g}\Big|_{\rm eikonal} = U^{ba}_{\underline{x}}\delta^{(2)}(\underline{x}-\underline{y})(2\pi)2p^{-}\delta(p^{-}-q^{-})\delta_{\lambda\lambda'}$$

$$U_{\underline{x}}[x_f^-, x_i^-] = \mathcal{P}\exp\left[ig\int_{x_i^-}^{x_f^-} dx^- \mathcal{A}^+(0^+, x^-, \underline{x})\right]$$

We need subeikonal order interaction with the proton at high collisional energies.

The shockwave picture of high energy scatterings: proton is treated as background gluon and quark fields

Bjorken, Kogut and Soper (1971)

Eikonality expansion = Expansion around infinite boost

$$\begin{aligned} S_{\rm fi} =& \langle\phi_{\rm f}|e^{i\omega\hat{K}^3}\mathcal{P}\exp\left\{-i\int_{-\infty}^{+\infty}dz^- V_{\rm I}(z^-)\right\}e^{-i\omega\hat{K}^3}|\phi_{\rm i}\rangle \\ =& \langle\phi_{\rm f}|\mathcal{P}\exp\left\{-i\int_{-\infty}^{+\infty}dz^- e^{i\omega\hat{K}^3}V_{\rm I}(z^-)e^{-i\omega\hat{K}^3}\right\}|\phi_{\rm i}\rangle. \end{aligned}$$

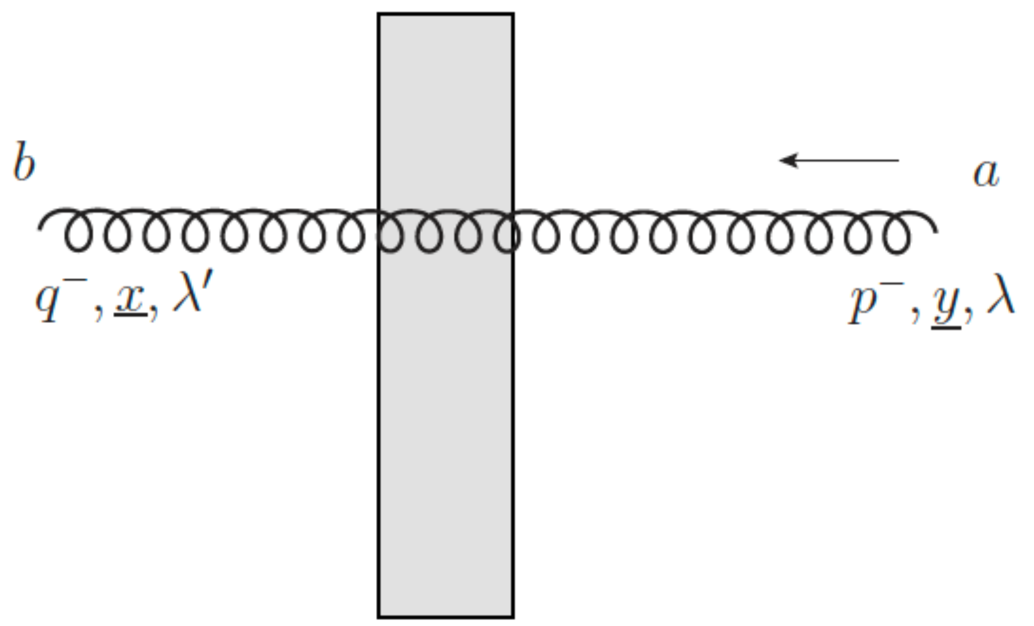
1. Light-cone Hamiltonian in the background fields.

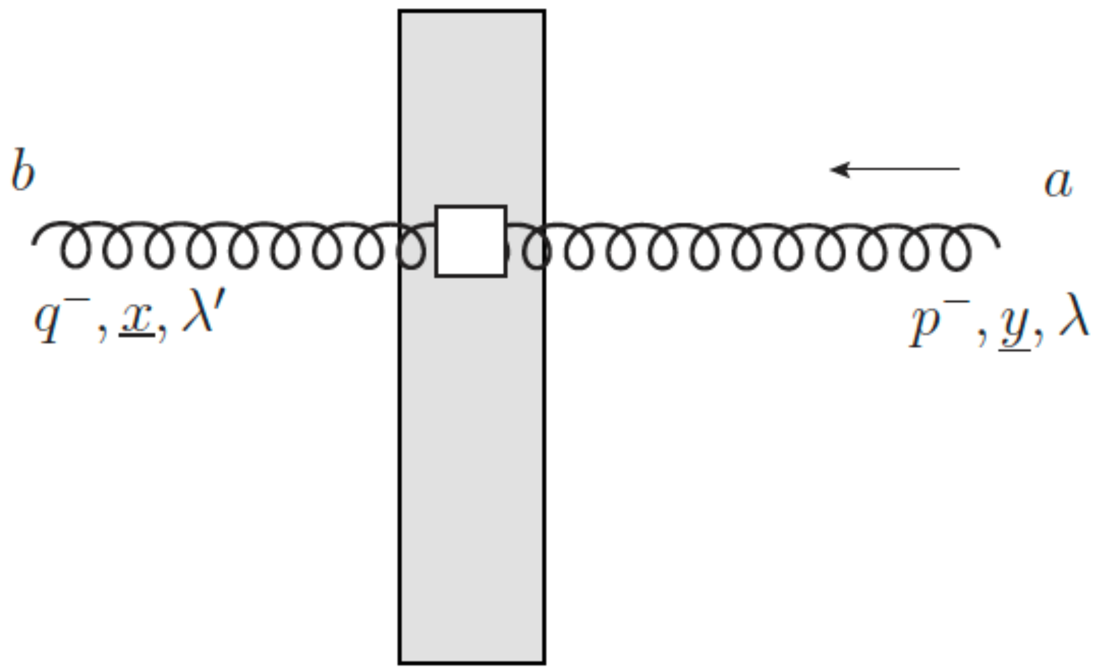
2. Boosting the background fields, expanding in powers of $\xi = e^{-\omega}$.

3. Small-x effective Hamiltonian up to linear order in ξ (subeikonal order).

M. Li, JHEP 07 (2023) 158.

Wilson Lines at Sub-eikonal Order





$$M^{g\to g} =(2\pi)2p^-\delta(p^- - q^-)\delta_{\lambda\lambda'}\left[\delta^{(2)}(\underline{x}-\underline{y})U_{\underline{x}} + \lambda\delta^{(2)}(\underline{x}-\underline{y})U_{\underline{x}}^{G[1]} + U_{\underline{x},\underline{y}}^{G[2]}\right]^{ba}$$
$$+\delta_{\lambda\lambda'}\delta^{(2)}(\underline{x}-\underline{y})(2\pi)(p^- + q^-)\delta'(p^- - q^-)\left(U_{\underline{x}}^{G[3]}\right)^{ba} + \mathcal{O}(1/s^2)$$

M. Li, JHEP 07 (2023) 158.

Chirilli (2019), Kovchegov et al. (2022), Altinoluk and Beuf (2022)

$$U_{\underline{x}}^{G[1]} = \frac{2igP^+}{s}\int_{-\infty}^{+\infty} dx^- U_{\underline{x}}[\infty, x^-]\mathcal{F}^{12}(x^-, \underline{x})U_{\underline{x}}[x^-, -\infty],$$

Longitudinal Chromomagnetic Field

$$U_{\underline{x},\underline{y}}^{G[2]} = -\frac{iP^+}{s}\int_{-\infty}^{+\infty} dz^- d^2z\, U_{\underline{x}}[\infty, z^-]\delta^{(2)}(\underline{x}-\underline{z})\overleftarrow{\mathcal{D}}(z^-, \underline{z})\overrightarrow{\mathcal{D}}(z^-, \underline{z})\delta^{(2)}(\underline{y}-\underline{z})U_{\underline{y}}[z^-, -\infty],$$

$$\Rightarrow\quad U_{\underline{x}}^{i,G[2]} = \frac{igP^+}{s}\int_{-\infty}^{+\infty} dx^- x^- U_{\underline{x}}[\infty, x^-]\mathcal{F}^{+i}(x^-, \underline{x})U_{\underline{x}}[x^-, -\infty],$$

Transverse Chromoelectric Field

$$U_{\underline{x}}^{G[3]} = -g\int_{-\infty}^{+\infty} dx^- U_{\underline{x}}[\infty, x^-]\mathcal{F}^{+-}(x^-, \underline{x})U_{\underline{x}}[x^-, -\infty].$$

Longitudinal Chromoelectric Field

Subeikonal order classical gluon fields were obtained by solving classical Yang-Mills equations at subeikonal order.

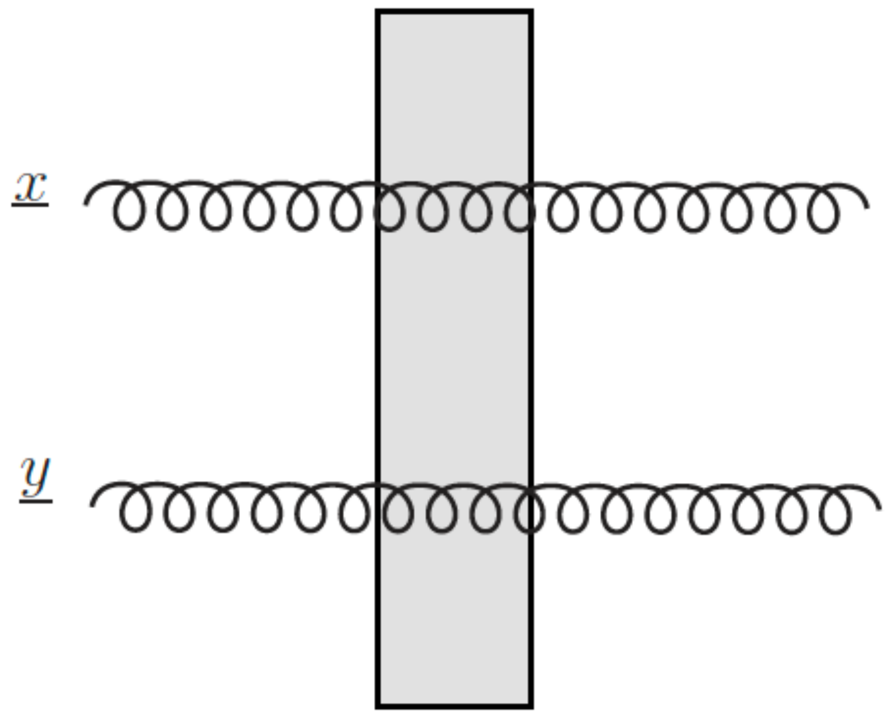
$$\mathcal{A}^+ = \mathcal{A}^+_{\text{eik}} + \mathcal{A}^+_{\text{sub}},\quad \mathcal{A}^i = \mathcal{A}^i_{\text{sub}}.$$

$$J^+ = J^+_{\text{eik}} + J^+_{\text{sub}},\quad J^i = J^i_{\text{sub}}.$$

M. Li, PRL 133 (2024) 2, 021902.

Polarized Wilson Line Correlators

Unpolarized gluon dipole correlator



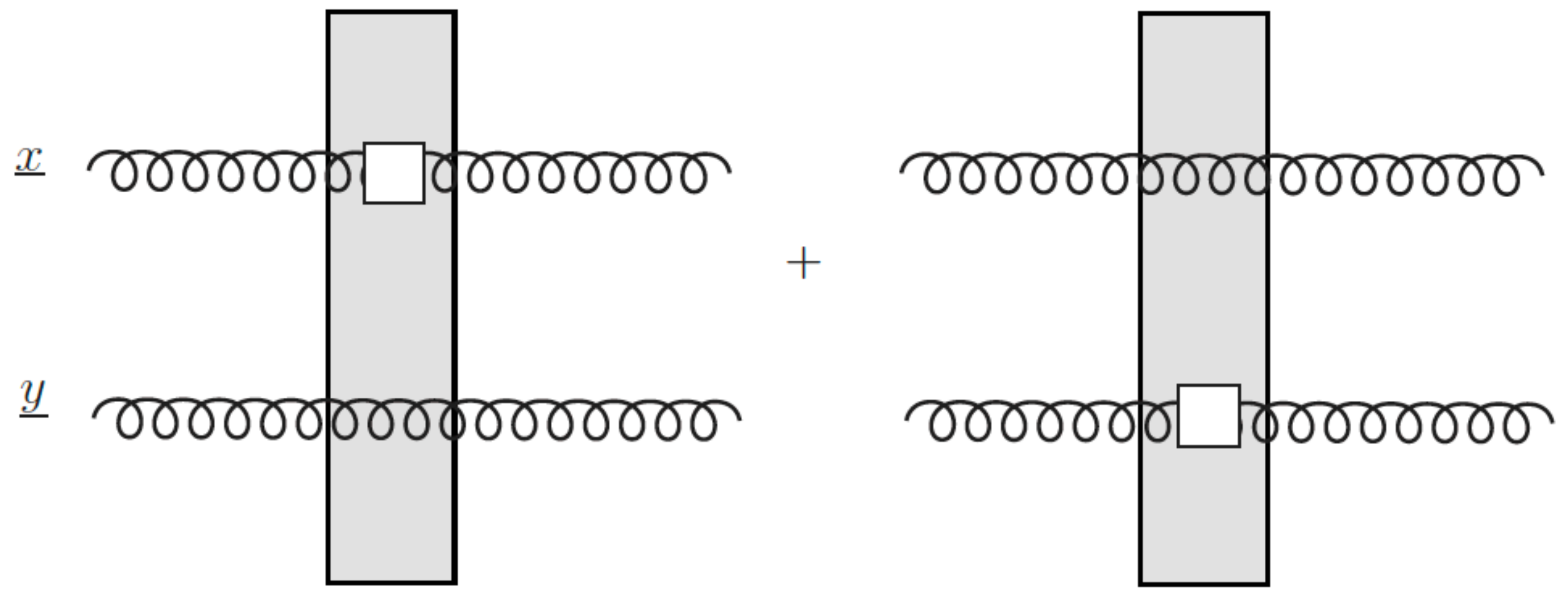
$$D_{\underline{x},\underline{y}} \equiv \frac{1}{(N_c^2-1)} \left\langle \mathrm{Tr}\left[U_{\underline{x}} U_{\underline{y}}^{\dagger}\right]\right\rangle$$

Averaging under Two-Gluon-Exchange Approximation

$$D_{\underline{x},\underline{y}} \simeq 1 - \frac{1}{2}\frac{\pi\alpha_s^2 N_c}{C_F}|\underline{x}-\underline{y}|^2 \ln\frac{1}{\Lambda^2|\underline{x}-\underline{y}|^2} + \ldots$$

Quadratically approaches 1 as $\underline{y} \to \underline{x}$

Chromo-electromagnetically polarized gluon dipole correlators



$$G^{\mathrm{adj}}_{\underline{x},\underline{y}}(s) \equiv \frac{1}{2(N_c^2-1)} \left\langle\!\!\left\langle \mathrm{Tr}\left[U^{\mathrm{G}[1]}_{\underline{x}}\, U^{\dagger}_{\underline{y}}\right] + \mathrm{Tr}\left[U_{\underline{x}}\, U^{\mathrm{G}[1]\,\dagger}_{\underline{y}}\right]\right\rangle\!\!\right\rangle$$

$$G^{i,\mathrm{adj}}_{\underline{x},\underline{y}}(s) \equiv \frac{1}{2(N_c^2-1)} \left\langle\!\!\left\langle \mathrm{Tr}\left[U^{i,\mathrm{G}[2]}_{\underline{x}}\, U^{\dagger}_{\underline{y}}\right] - \mathrm{Tr}\left[U_{\underline{x}}\, U^{i,\mathrm{G}[2]\,\dagger}_{\underline{y}}\right]\right\rangle\!\!\right\rangle$$

$$G^{\mathrm{adj}}_{\underline{x},\underline{y}} \simeq \lambda'\, 2\frac{\pi\alpha_s^2 N_c}{C_F} \ln s|\underline{x}-\underline{y}|^2 + \ldots$$

$$G^{i,\mathrm{adj}}_{\underline{x},\underline{y}} \simeq \lambda' \frac{\pi\alpha_s^2 N_c}{C_F}\epsilon^{ij}(\underline{x}-\underline{y})^j \ln\frac{1}{\Lambda^2|\underline{x}-\underline{y}|^2} + \ldots$$

Logarithmically/linearly approaches 0 as $\underline{y} \to \underline{x}$

Relating to Gluon TMDs at Small-x

Polarized Wilson line correlators are related to the small-x limit of various gluon helicity TMDs.

$$\Gamma^{\mu\nu;\rho\sigma}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P,S|\mathrm{Tr}\left[F^{\mu\nu}(0)\,\mathcal{U}^{[+]}(0,\xi)\,F^{\rho\sigma}(\xi)\,\mathcal{U}^{[-]}(\xi,0)\right]|P,S\rangle$$

Mulders and Rodrigues (2001)

$\mu\nu;\rho\sigma = +i;+j$

$$\int dk^- \,\Gamma^{+i;+j}(k,P,S_L) = \frac{i}{4} x P^+ S_L \epsilon^{ij} g_{1L}^G(x,k_T^2)$$

$x \to 0$

$$g_{1L}^G(x,k_T^2) = -\frac{N_c}{\alpha_s 4\pi^4} i\epsilon^{ij}\underline{k}^i \int d^2\xi d^2\zeta e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} G^j_{\underline{\xi},\underline{\zeta}}(s)$$

Dipole Gluon helicity TMD

Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

$\mu\nu;\rho\sigma = ij;l+$

$$\int dk^- \,\Gamma^{ij;l+}(k;P,S_L) = -\frac{i}{4} S_L\,\epsilon^{ij}\,k^l\,\Delta H_{3L}^{\perp}(x,k_T^2)$$

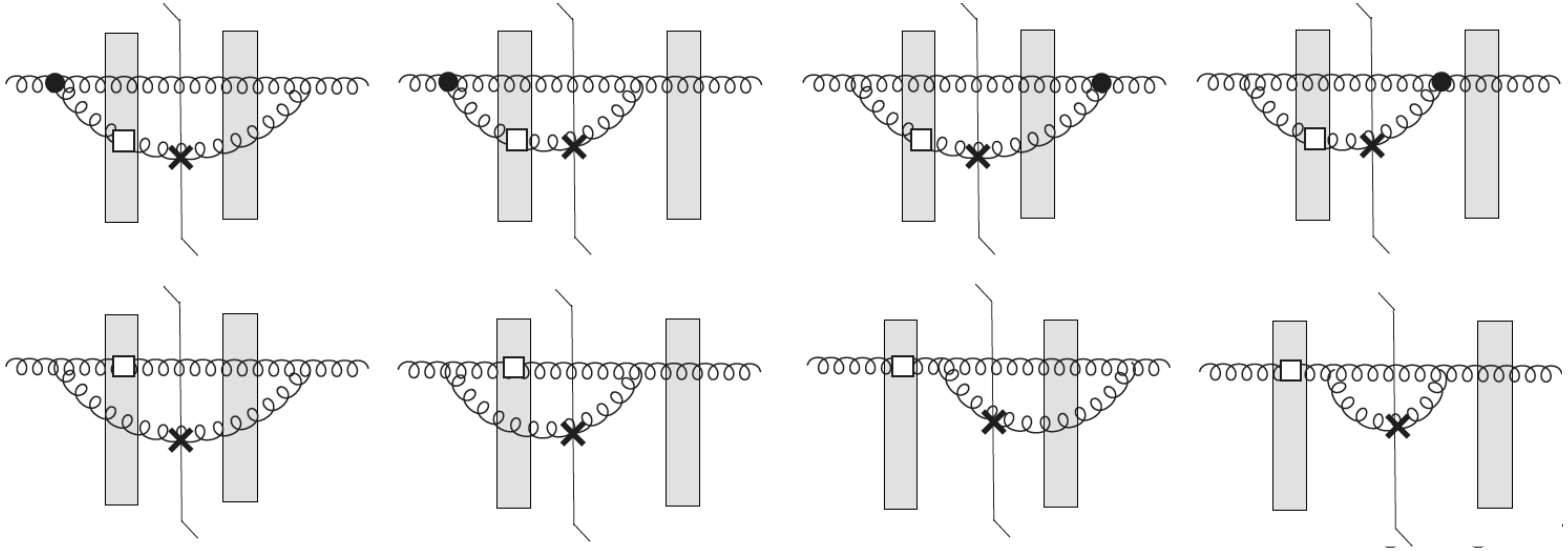
$x \to 0$

$$\Delta H_{3L}^{\perp}(x,k_T^2) = \frac{N_c}{\alpha_s 4\pi^4} \int d^2\xi d^2\zeta\, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} G_{\underline{\xi},\underline{\zeta}}(s)$$

Twist-3 gluon helicity-flip TMD

Gluon + Proton $\longrightarrow$ Gluon + X

The calculation is performed in transverse coordinate space.



Black dot: Subeikonal order gluon splitting wavefunction.

Gluon momentum $p_2 = (0^+, p_2^-, \underline{0})$

Proton momentum $p_1 = (p_1^+, 0^-, \underline{0})$

$$\beta = \frac{k^-}{p_2^-}, \quad \alpha = \frac{k^+}{p_1^+}$$

impact parameter

$$\int d^2b\, G^{\text{adj}}_{\underline{b},\underline{b}-\underline{x}}(\beta s) = G^{\text{adj}}(x_\perp^2, \beta s),$$

$$\int d^2b\, G^{i,\text{adj}}_{\underline{b},\underline{b}-\underline{x}}(\beta s) = x^i\, G_1^{\text{adj}}(x_\perp^2, \beta s) + \epsilon^{ij}\, x^j\, G_2^{\text{adj}}(x_\perp^2, \beta s).$$

$$\frac{d\sigma(\lambda)}{d^2k_T dy} = \lambda \frac{\alpha_s N_c}{\pi^4} \frac{1}{s} \int d^2x\, d^2y\, d^2b e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} \left\{ \frac{\underline{x}-\underline{b}}{|\underline{x}-\underline{b}|^2} \cdot \frac{\underline{y}-\underline{b}}{|\underline{y}-\underline{b}|^2} \left[\left(G^{\text{adj}}_{\underline{x},\underline{y}}(\beta s) - G^{\text{adj}}_{\underline{x},\underline{b}}(\beta s) \right) - \frac{1}{4} \left(G^{\text{adj}}_{\underline{b},\underline{y}}(\beta s) + G^{\text{adj}}_{\underline{b},\underline{x}}(\beta s) \right) \right] - 2i\, k^i \frac{\underline{x}-\underline{b}}{|\underline{x}-\underline{b}|^2} \times \frac{\underline{y}-\underline{b}}{|\underline{y}-\underline{b}|^2} G^{i\,\text{adj}}_{\underline{x},\underline{b}}(\beta s) \right\}$$

$$\frac{d\sigma(\lambda)}{d^2k_T\, dy} = \lambda \frac{2\alpha_s N_c}{\pi^3} \frac{1}{s} \int d^2x e^{-i\underline{k}\cdot\underline{x}} \left[\ln\left(\frac{1}{x_\perp \Lambda} \right) G^{\text{adj}}(x_\perp^2, \beta s) - i \frac{\underline{x}}{|\underline{x}|^2} \cdot \frac{\underline{k}}{|\underline{k}|^2} \left(\frac{3}{2}\, G^{\text{adj}}(x_\perp^2, \beta s) + 2\, G_2^{\text{adj}}(x_\perp^2, \beta s) \right) \right]$$

Sanity Check: Leading Perturbative Result

We calculated A_{LL} for gluon production in *Gluon+Proton* collisions:

$$\frac{d\sigma(\lambda)}{d^2k_T\,dy} = \lambda \frac{2\alpha_s N_c}{\pi^3}\frac{1}{s}\int d^2x e^{-i\underline{k}\cdot\underline{x}}\left[\ln\left(\frac{1}{x_\perp \Lambda}\right) G^{\text{adj}}(x_\perp^2, \beta s) - i\frac{\underline{x}}{|\underline{x}|^2}\cdot\frac{\underline{k}}{|\underline{k}|^2}\left(\frac{3}{2}G^{\text{adj}}(x_\perp^2,\beta s) + 2\,G_2^{\text{adj}}(x_\perp^2,\beta s)\right)\right]$$

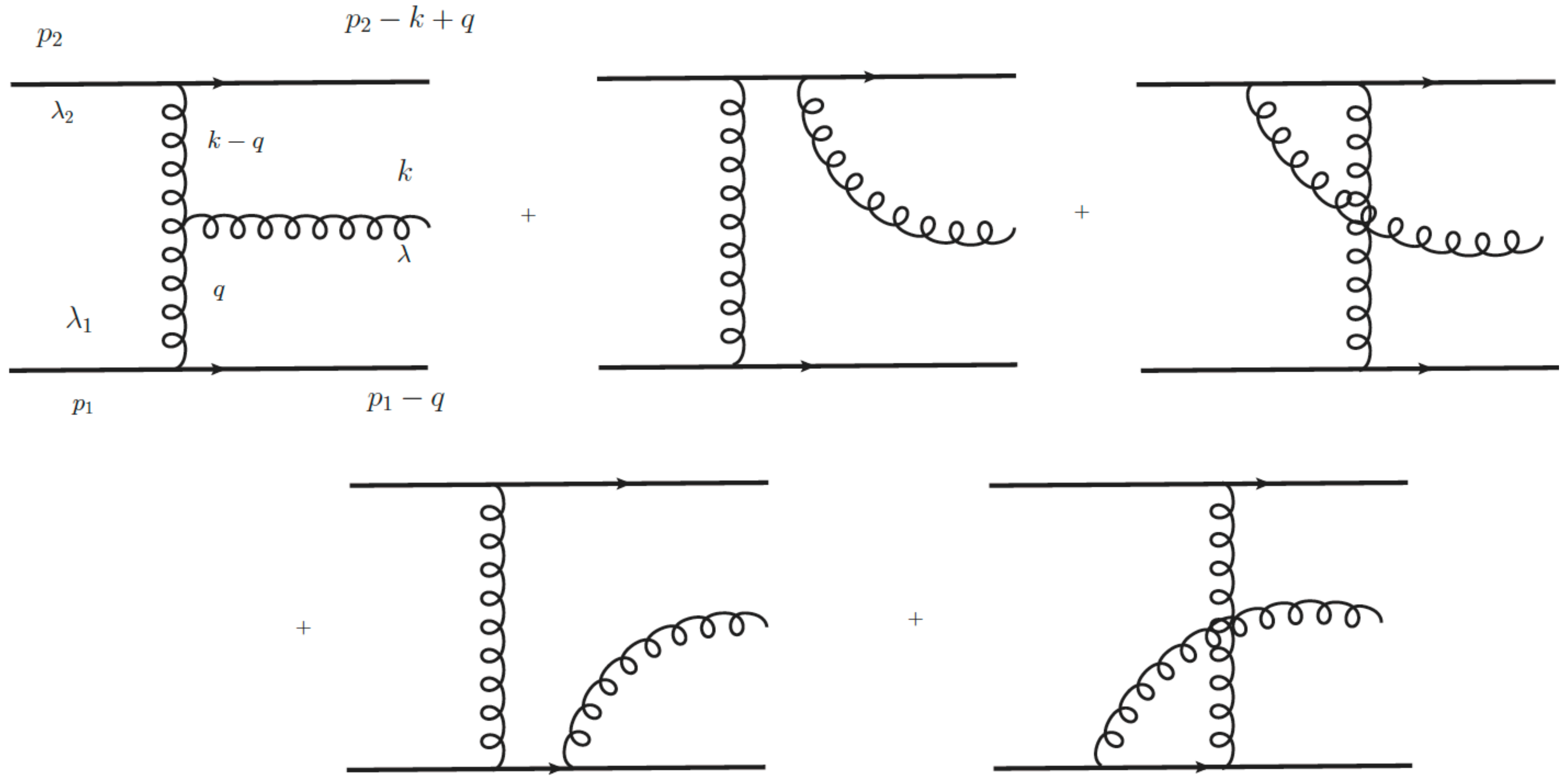
How to obtain A_{LL} for gluon production in *Gluon+Gluon* collisions?

Cougoulic and Kovchegov (2020)

Use the Born level expressions:

$$G^{\text{adj}\,(0)}(x_\perp^2, \beta s) = 2\,\alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln(\beta s\, x_\perp^2),$$

$$G_2^{\text{adj}\,(0)}(x_\perp^2, \beta s) = \alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln\left(\frac{1}{x_\perp^2\,\Lambda^2}\right).$$



$$\frac{d\sigma_{LO}^{GG\to GGG}}{d^2k_T\,dy} = \frac{8\,\alpha_s^3}{\pi}\,\frac{N_c}{s\,k_T^2}\left\{3\,\ln\frac{k_T^2}{\Lambda^2} + \ln\left(\frac{\min\{\alpha,\beta\}\,s}{\Lambda^2}\right)\right\}$$

$$\frac{d\sigma_{LO,\,\text{unpolarized}}^{GG\to GGG}}{d^2k_T\,dy} = \frac{4\,\alpha_s^3\,N_c^2}{\pi\,C_F}\,\frac{1}{k_T^4}\,\ln\frac{k_T^2}{\Lambda^2}$$

$$A_{LL} \sim \frac{k_T^2}{s}$$

Quadratically dependent on k_T at large external transverse momentum

Proton + Proton $\longrightarrow$ Gluon + X

We calculated A_{LL} for gluon production in *Gluon+Proton* collisions:

$$\frac{d\sigma(\lambda)}{d^2k_T\,dy} = \lambda \frac{2\alpha_s N_c}{\pi^3}\frac{1}{s}\int d^2x e^{-i\underline{k}\cdot\underline{x}}\left[\ln\left(\frac{1}{x_\perp\Lambda}\right) G_T^{\text{adj}}(x_\perp^2,\beta s) - i\frac{\underline{x}}{|\underline{x}|^2}\cdot\frac{\underline{k}}{|\underline{k}|^2}\left(\frac{3}{2}G_T^{\text{adj}}(x_\perp^2,\beta s) + 2\,G_{2T}^{\text{adj}}(x_\perp^2,\beta s)\right)\right]$$

It is projectile-target asymmetric!

Inspired by the case for unpolarized gluon production, Kovchegov and Tuchin(2002)

How to obtain A_{LL} for gluon production in *Proton+Proton* collisions?

$$\ln\left(\frac{1}{x_\perp\Lambda}\right) \sim G_{2P}^{\text{adj}\,(0)}(x_\perp^2,\alpha s)$$

$$\frac{x^i}{|\underline{x}|^2} \sim c_1\,\partial^i G_P^{\text{adj}(0)}(x_\perp^2,\alpha s) + c_2\,\partial^i G_{2P}^{\text{adj}(0)}(x_\perp^2,\alpha s)$$

Born level expressions for projectile proton:

$$G_P^{\text{adj}\,(0)}(x_\perp^2,\alpha s) = 2\,\alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln(\alpha s\, x_\perp^2),$$
$$G_{2P}^{\text{adj}\,(0)}(x_\perp^2,\alpha s) = \alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln\left(\frac{1}{x_\perp^2\Lambda^2}\right).$$

Step 0: Integration by parts.

$$\int d^2x e^{-i\underline{k}\cdot\underline{x}}\left[\ln\left(\frac{1}{x_\perp\Lambda}\right) - 2i\,\frac{\underline{x}}{|\underline{x}|^2}\cdot\frac{\underline{k}}{|\underline{k}|^2}\right] G^{\text{adj}}(x_\perp^2,\beta s)$$
$$= -\int d^2x\, e^{-i\underline{k}\cdot\underline{x}}\ln\left(\frac{1}{x_\perp\Lambda}\right)\frac{1}{k_T^2}\nabla_\perp^2 G^{\text{adj}}(x_\perp^2,\beta s).$$

$$G_P^{\text{adj}\,(0)} \longrightarrow G_P^{\text{adj}},\quad G_{2P}^{\text{adj}\,(0)} \longrightarrow G_{2P}^{\text{adj}}$$

The final expression should be projectile and target symmetric ($T \leftrightarrow P$).

k_T-Factorization

The A_{LL} for gluon production in *Proton+Proton* collisions:

$$\frac{d\sigma}{d^2k_T\,dy} = \frac{C_F}{\alpha_s\,\pi^4}\frac{1}{s\,k_T^2}\int d^2x\, e^{-i\underline{k}\cdot\underline{x}} \left(G_P^{\text{adj}}(x_\perp^2,\alpha s) \quad G_{2P}^{\text{adj}}(x_\perp^2,\alpha s)\right) \begin{pmatrix} \frac{1}{4}\overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp & \overleftarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp \\ \overrightarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp & 0 \end{pmatrix} \begin{pmatrix} G_T^{\text{adj}}(x_\perp^2,\beta s) \\ G_{2T}^{\text{adj}}(x_\perp^2,\beta s) \end{pmatrix}$$

In momentum space:

$$\frac{d\sigma}{d^2k_T\,dy} = -\frac{C_F}{\alpha_s\,\pi^4}\frac{1}{s\,k_T^2}\int\frac{d^2q}{(2\pi)^2}\left(G_P^{\text{adj}}(q_T^2,\alpha s) \quad G_{2P}^{\text{adj}}(q_T^2,\alpha s)\right)\begin{pmatrix} \frac{1}{4}\underline{q}\cdot(\underline{k}-\underline{q}) & \underline{q}\cdot\underline{k} \\ \underline{k}\cdot(\underline{k}-\underline{q}) & 0 \end{pmatrix}\begin{pmatrix} G_T^{\text{adj}}((\underline{k}-\underline{q})^2,\beta s) \\ G_{2T}^{\text{adj}}((\underline{k}-\underline{q})^2,\beta s) \end{pmatrix}$$

In terms of dipole gluon helicity TMD and twist-3 helicity-flip TMD:

$$\frac{d\sigma}{d^2k_T\,dy} = -\frac{32\pi^4\,\alpha_s}{N_c}\frac{1}{s\,k_T^2}\int\frac{d^2q}{(2\pi)^2}\left(\Delta H_{3L}^{\perp,P}(q_T^2,\tfrac{k_T^2}{\alpha s}) \quad g_{1L}^{G,P}(q_T^2,\tfrac{k_T^2}{\alpha s})\right)\begin{pmatrix} \underline{q}\cdot(\underline{k}-\underline{q}) & \underline{q}\cdot\underline{k} \\ \underline{k}\cdot(\underline{k}-\underline{q}) & 0 \end{pmatrix}\begin{pmatrix} \Delta H_{3L}^{\perp,T}((\underline{k}-\underline{q})^2,\tfrac{k_T^2}{\beta s}) \\ g_{1L}^{G,T}((\underline{k}-\underline{q})^2,\tfrac{k_T^2}{\beta s}) \end{pmatrix}$$

This equation is only applicable in the small-x regime.

$$\alpha s = 2p_2^-k^+ = \sqrt{2}p_2^-k_T\,e^{-y},$$
$$\beta s = 2p_1^+k^- = \sqrt{2}p_1^+k_T\,e^{y}$$

k_T-Factorization

The A_{LL} for gluon production in *Proton+Proton* collisions:

$$\frac{d\sigma}{d^2k_T\,dy} = -\frac{32\pi^4\,\alpha_s}{N_c}\frac{1}{s\,k_T^2}\int\frac{d^2q}{(2\pi)^2}\begin{pmatrix}\Delta H_{3L}^{\perp,P}(q_T^2,\frac{k_T^2}{\alpha s}) & g_{1L}^{G,P}(q_T^2,\frac{k_T^2}{\alpha s})\end{pmatrix}\begin{pmatrix}\underline{q}\cdot(\underline{k}-\underline{q}) & \underline{q}\cdot\underline{k}\\ \underline{k}\cdot(\underline{k}-\underline{q}) & 0\end{pmatrix}\begin{pmatrix}\Delta H_{3L}^{\perp,T}((\underline{k}-\underline{q})^2,\frac{k_T^2}{\beta s})\\ g_{1L}^{G,T}((\underline{k}-\underline{q})^2,\frac{k_T^2}{\beta s})\end{pmatrix}$$

1. $\Delta H_{3,L}^{\perp}(k_T^2,s)$ is a pure TMD effect that doesn't contribute to gluon helicity PDF.

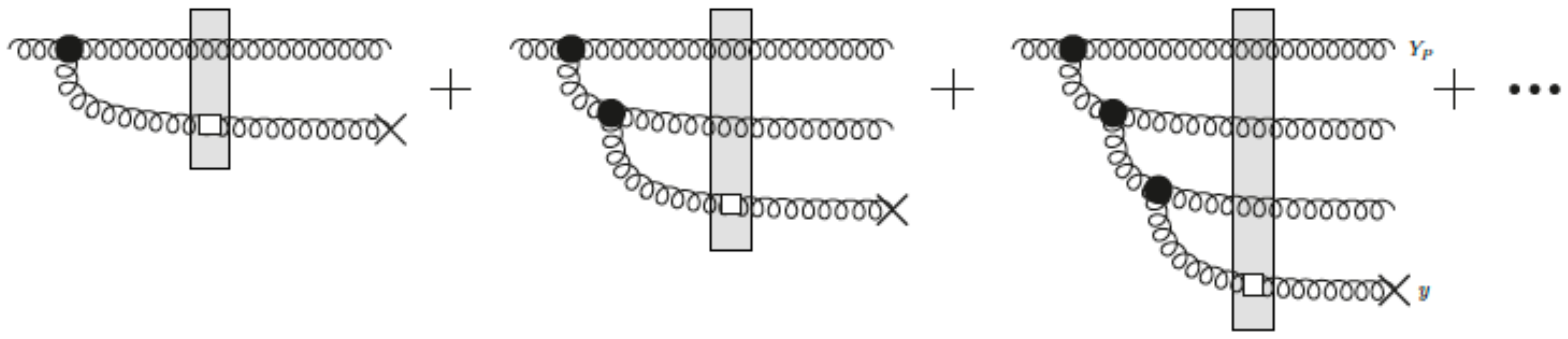
$$\int_0^{Q^2} d^2k\Delta H_{3L}^{\perp}(k_T^2,s)\approx 0.$$

2. Collinear limit? From $2\to 3$ process to $2\to 2$ process?

3. Solving $\Delta H_{3L}^{\perp}(k_T^2,x)$ and $g_{1L}^{G}(k_T^2,x)$ from the small-x helicity evolution equations.

Including Small-x Helicity Evolutions

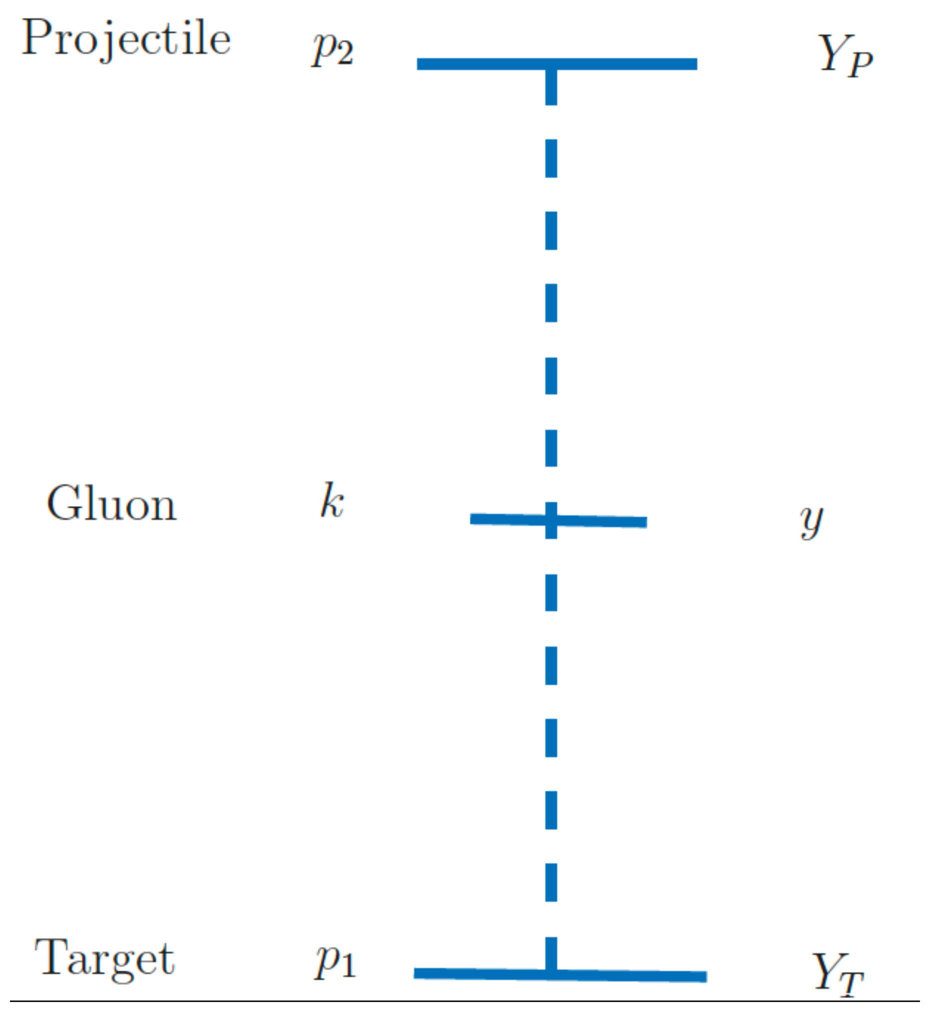
$$\frac{d\sigma}{d^2k_T\,dy} = \frac{C_F}{\alpha_s\,\pi^4}\frac{1}{s\,k_T^2}\int d^2x\,e^{-i\underline{k}\cdot\underline{x}}\left(G_P^{\text{adj}}(x_\perp^2, Y_P - y) \quad G_{2P}^{\text{adj}}(x_\perp^2, Y_P - y)\right)\begin{pmatrix}\frac{1}{4}\overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp & \overleftarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp \\ \overrightarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp & 0\end{pmatrix}\begin{pmatrix}G_T^{\text{adj}}(x_\perp^2, y - Y_T) \\ G_{2T}^{\text{adj}}(x_\perp^2, y - Y_T)\end{pmatrix}$$



In the double-logarithmic approximation:

$$\alpha_s \ll 1, \qquad \ln\frac{1}{x} \gg 1. \quad \longrightarrow \quad \alpha_s \ln\frac{1}{x} \ll 1, \qquad \alpha_s \ln^2\frac{1}{x} \sim 1.$$

single-logarithmic terms can be discarded.



When $\alpha_s(Y_P - y)^2 \sim 1$, including small-x helicity evolution on the projectile side.

When $\alpha_s(y - Y_T)^2 \sim 1$, including small-x helicity evolution on the target side.

The small-x helicity evolution equations under double-logarithmic approximation, which close at large-N_c, have been derived.

Kovchegov, Pitonyak and Sievert (2015-2019)
Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

Including Small-x Helicity Evolutions

$$\frac{d\sigma}{d^2k_T\, dy} = \frac{C_F}{\alpha_s\, \pi^4}\frac{1}{s\, k_T^2}\int d^2x\, e^{-i\underline{k}\cdot\underline{x}} \left(G_P^{\rm adj}(x_\perp^2, Y_P - y) \quad G_{2P}^{\rm adj}(x_\perp^2, Y_P - y)\right) \begin{pmatrix} \frac{1}{4}\overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp & \overleftarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp \\ \overrightarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp & 0 \end{pmatrix} \begin{pmatrix} G_T^{\rm adj}(x_\perp^2, y - Y_T) \\ G_{2T}^{\rm adj}(x_\perp^2, y - Y_T) \end{pmatrix}$$

In the double-logarithmic approximation, large-N_c and dilute limit:

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi}\int_{\frac{1}{sx_{10}^2}}^{z}\frac{dz'}{z'}\int_{\frac{1}{z's}}^{x_{10}^2}\frac{dx_{21}^2}{x_{21}^2}\Big[\Gamma(x_{10}^2, x_{21}^2, z's) + 3G(x_{21}^2, z's) + 2G_2(x_{21}^2, z's) + 2\Gamma_2(x_{10}^2, x_{21}^2, z's)\Big],$$

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi}\int_{\frac{1}{sx_{10}^2}}^{z'}\frac{dz''}{z''}\int_{\frac{1}{z''s}}^{\min[x_{10}^2, x_{21}^2\frac{z'}{z''}]}\frac{dx_{32}^2}{x_{32}^2}\Big[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3G(x_{32}^2, z''s) + 2G_2(x_{32}^2, z''s) + 2\Gamma_2(x_{10}^2, x_{32}^2, z''s)\Big],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi}\int_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'}x_{10}^2, \frac{1}{\Lambda^2}]}\frac{dx_{21}^2}{x_{21}^2}\Big[G(x_{21}^2, z's) + 2G_2(x_{21}^2, z's)\Big],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi}\int_{\frac{\Lambda^2}{s}}^{z'\frac{x_{21}^2}{x_{10}^2}}\frac{dz''}{z''}\int_{\max[x_{10}^2, \frac{1}{z''s}]}^{\min[\frac{z'}{z''}x_{21}^2, \frac{1}{\Lambda^2}]}\frac{dx_{32}^2}{x_{32}^2}\Big[G(x_{32}^2, z''s) + 2G_2(x_{32}^2, z''s)\Big].$$

Kovchegov, Pitonyak and Sievert (2015-2019)
Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

$$G^{\rm adj} = 4G, \quad G_2^{\rm adj} = 2G_2\,.$$

Γ and Γ_2 have the same operator definition as G and G_2, respectively. But they have different life time ordering constraints.

Can be thought of as spin-dependent BFKL for polarized Wilson line dipoles at large-N_c.

Conclusions

- We derived the first-ever transverse momentum dependent small-x expression for double-spin asymmetry of gluon production at mid-rapidity in longitudinally polarized proton-proton collisions.

- In the pure glue case, the expression contains dipole gluon helicity TMDs and twist-3 helicity-flip TMDs from both the projectile and the target in a projectile-target symmetric form.

- The expression exhibits k_T- factorization. Together with the small-x helicity evolution equations under double-logarithmic approximation, it can be used to constrain gluon helicity distribution at small-x using experimental data from RHIC on A_{LL} for inclusive jet and neutral pion productions. (ongoing work by Nicholas Baldonado and Matthew Sievert within the JAM collaboration)

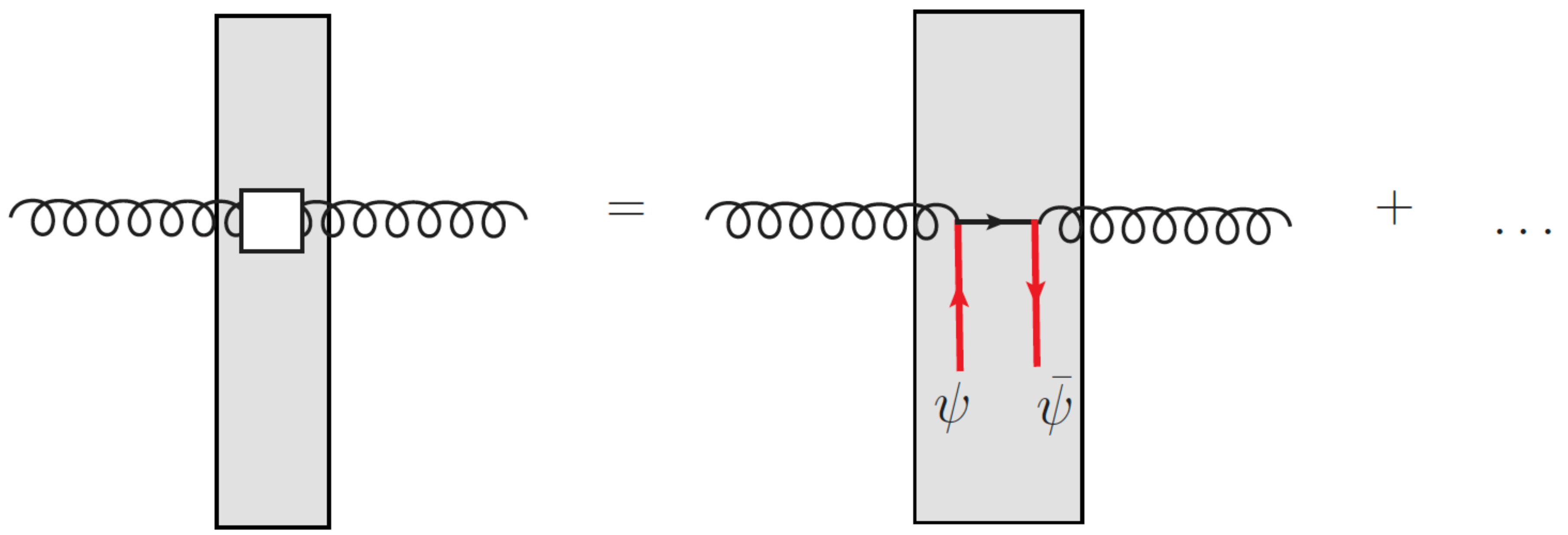
- Including the qg and qq channels for phenomenological applications.

Backup

Including Quarks (work in progress)

Scattering amplitudes depend on background (anti) quark fields.

$$(U_{\underline{x},\underline{y};\lambda',\lambda})^{ba} \equiv (U_{\underline{x}})^{ba}\delta^{(2)}(\underline{x}-\underline{y})\delta_{\lambda,\lambda'} + \lambda\delta_{\lambda,\lambda'}\left(U^{G[1]}_{\underline{x}} + U^{q[1]}_{\underline{x}}\right)^{ba}\delta^{(2)}(\underline{x}-\underline{y}) + \delta_{\lambda,\lambda'}\left(U^{G[2]}_{\underline{x},\underline{y}} + U^{q[2]}_{\underline{x}}\delta^{(2)}(\underline{x}-\underline{y})\right)^{ba}$$

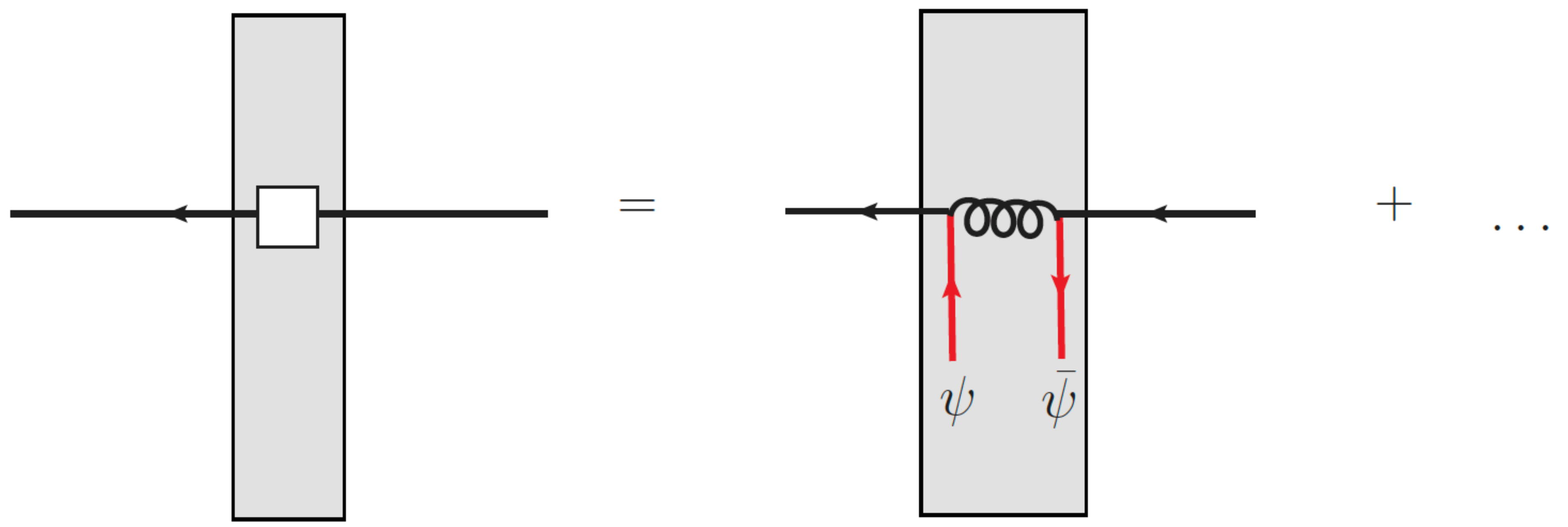


$$(U^{q[1]}_{\underline{x}})^{ba} = \frac{g^2 p_1^+}{s}\int_{-\infty}^{\infty} dx_1^- \int_{x_1^-}^{\infty} dx_2^- \,(U_{\underline{x}}[\infty,x_2^-])^{bb'}\bar{\psi}(x_2^-,\underline{x})t^{b'}V_{\underline{x}}[x_2^-,x_1^-]\frac{\gamma^+\gamma^5}{2}t^{a'}\psi(x_1^-,\underline{x})(U_{\underline{x}}[x_1^-,-\infty])^{a'a} + c.c.$$

$$(U^{q[2]}_{\underline{x}})^{ba} = -\frac{g^2 p_1^+}{s}\int_{-\infty}^{\infty} dx_1^- \int_{x_1^-}^{\infty} dx_2^- \,(U_{\underline{x}}[\infty,x_2^-])^{bb'}\bar{\psi}(x_2^-,\underline{x})t^{b'}\,V_{\underline{x}}[x_2^-,x_1^-]\frac{\gamma^+}{2}t^{a'}\,\psi(x_1^-,\underline{x})(U_{\underline{x}}[x_1^-,-\infty])^{a'a} - c.c.$$

We also need the subeikonal order quark Wilson lines:

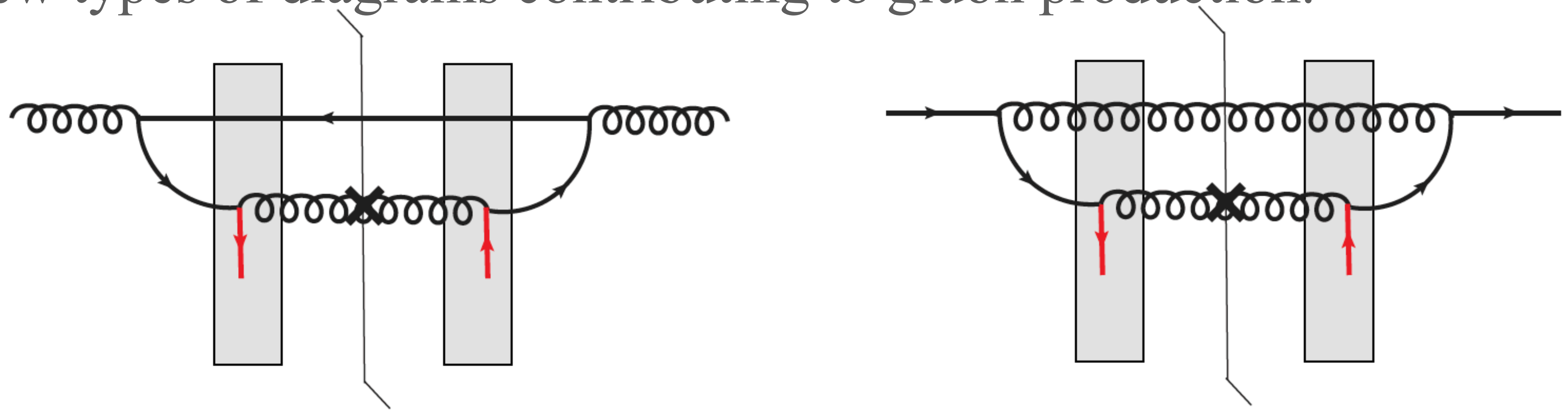
$$(V_{\underline{x},\underline{y};\sigma',\sigma})^{ij} \equiv (V_{\underline{x}})^{ij}\delta^{(2)}(\underline{x}-\underline{y})\delta_{\sigma,\sigma'} + \sigma\delta_{\sigma,\sigma'}\left(V^{G[1]}_{\underline{x}} + V^{q[1]}_{\underline{x}}\right)^{ij}\delta^{(2)}(\underline{x}-\underline{y}) + \delta_{\sigma,\sigma'}\left(V^{G[2]}_{\underline{x},\underline{y}} + V^{q[2]}_{\underline{x}}\delta^{(2)}(\underline{x}-\underline{y})\right)^{ij}$$



Quark initiated channels: Quark + Proton $\longrightarrow$ Gluon + X

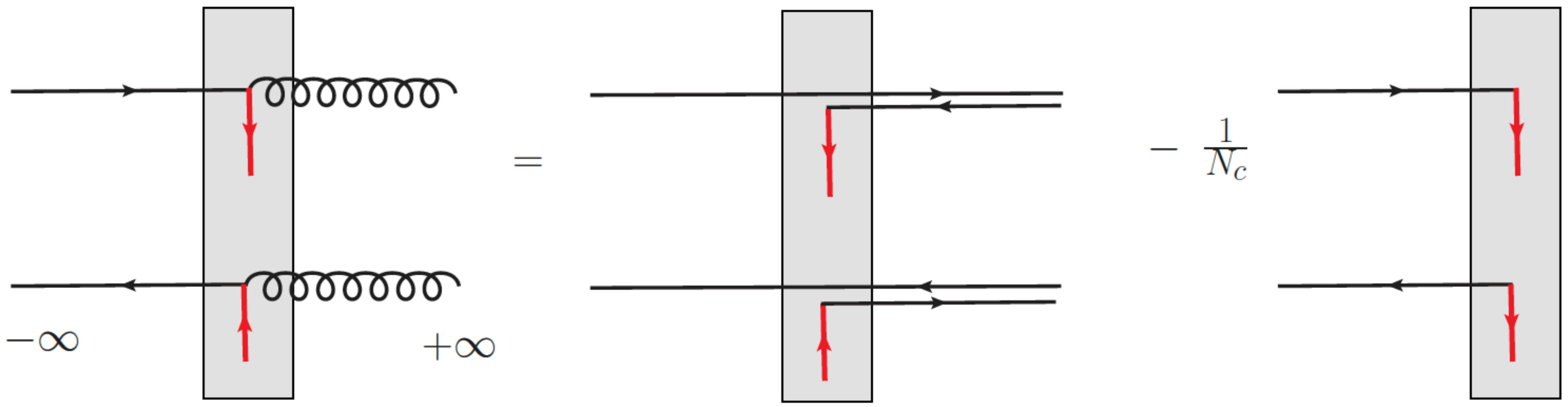
Including Quarks (work in progress)

New types of diagrams contributing to gluon production.



Altinoluk, Armesto and Beuf (2023)

$$\hat{O}(\underline{x},\underline{y}) = \frac{g^2 P^+}{s} \int_{-\infty}^{+\infty} dx^- \int_{-\infty}^{+\infty} dy^- \, U_{\underline{y}}^{ce}[+\infty, y^-] \, \bar{\psi}(y^-, \underline{y}) \left(t^e V_{\underline{y}}[y^-, -\infty] V_{\underline{x}}^{\dagger}[x^-, -\infty] t^d \right) \left[\frac{\gamma^- \gamma^5}{2} \right] \psi(x^-, \underline{x}) \, U_{\underline{x}}^{cd}[+\infty, x^-] + c.c.$$



The new operator is related to the small-x limit of quark helicity TMD.

Chirilli (2021)

We have extended the small-x helicity evolution equations to include the quark-gluon (gluon-quark) transition operators in the large N_c & N_f limit.

Borden, Kovchegov and Li, arXiv:2406.11647.