R-ratio from Lattice QCD using Bayesian Reconstruction

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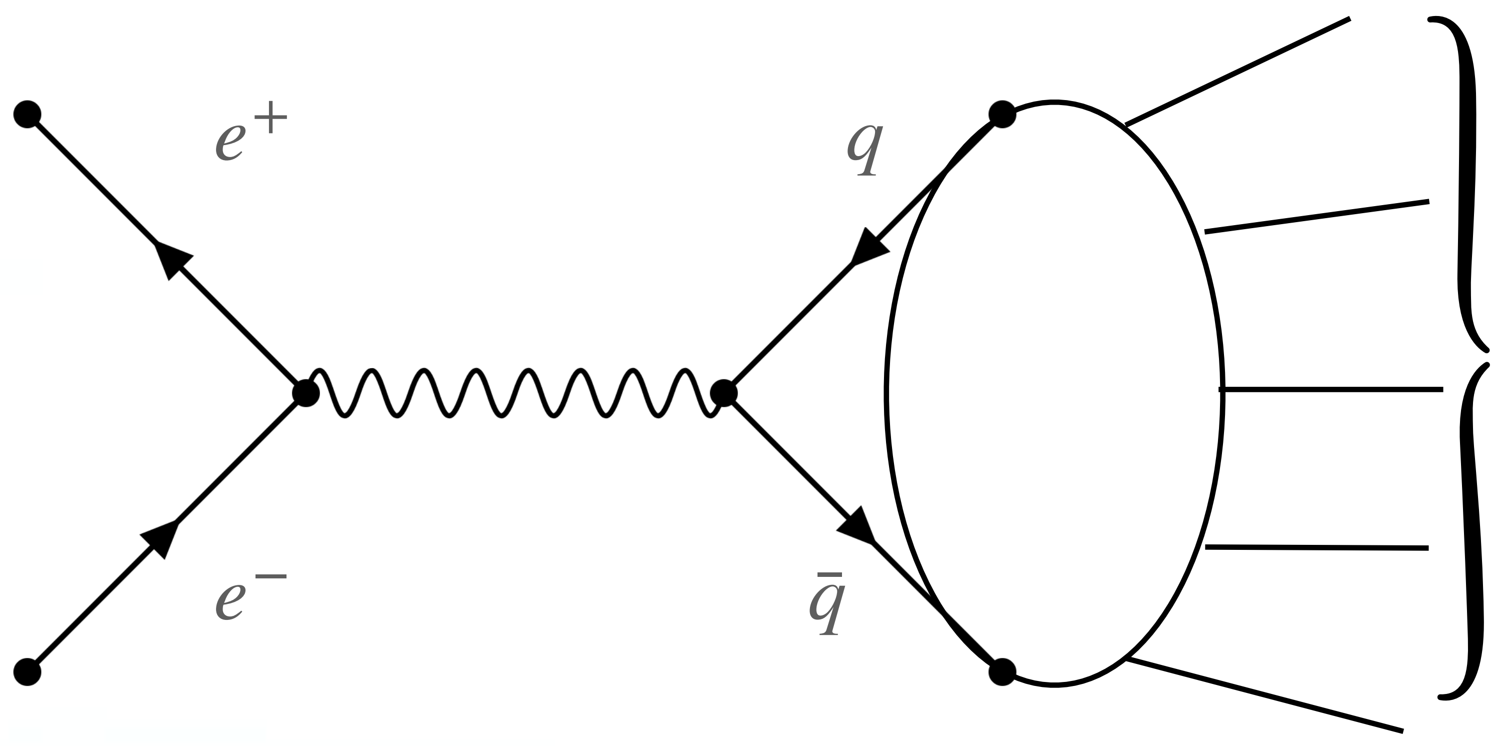
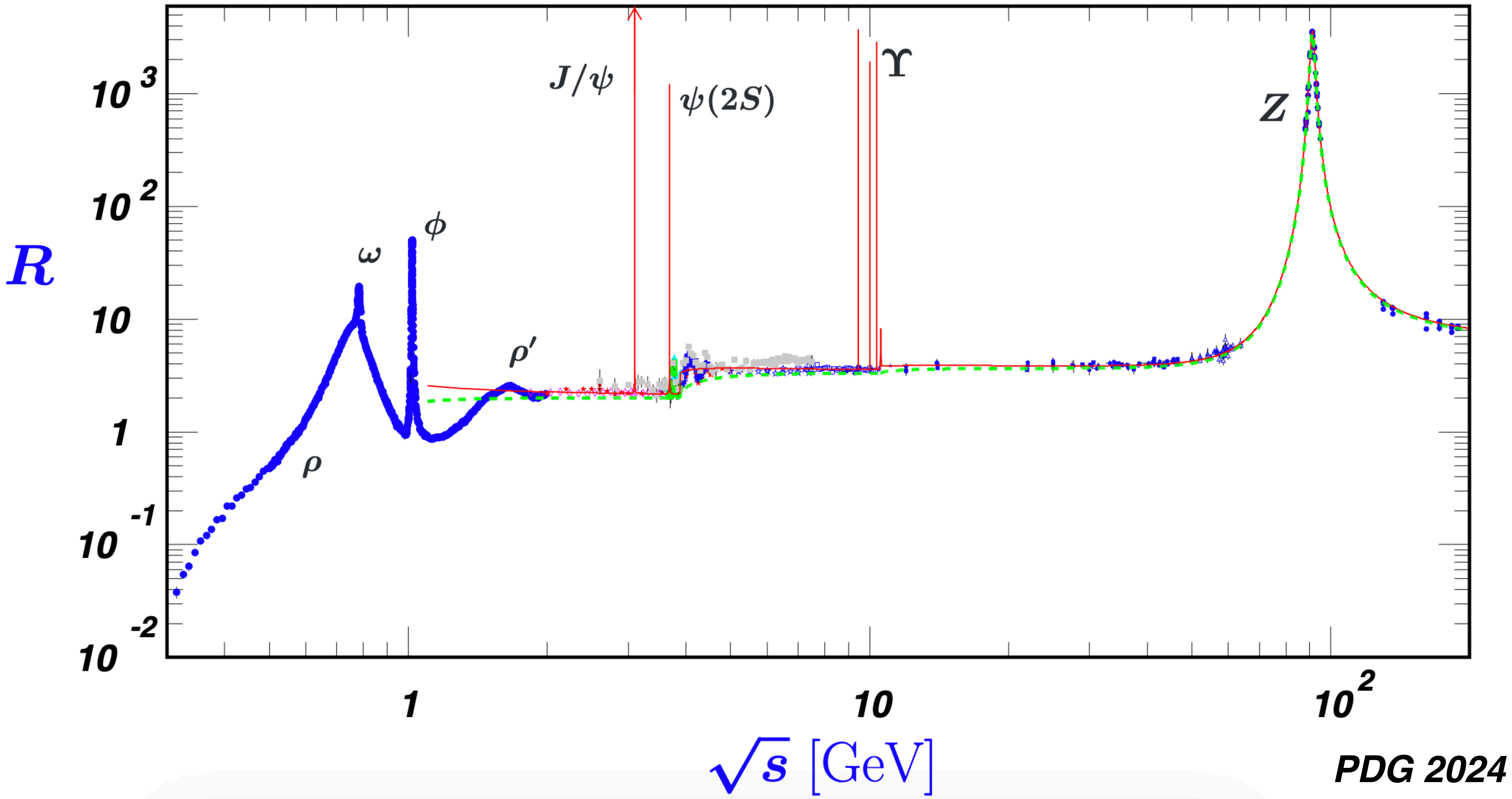
South China Normal University

07/10/2024 @ INT workshop “Inverse Problems and Uncertainty Quantification in Nuclear Physics”

The R ratio



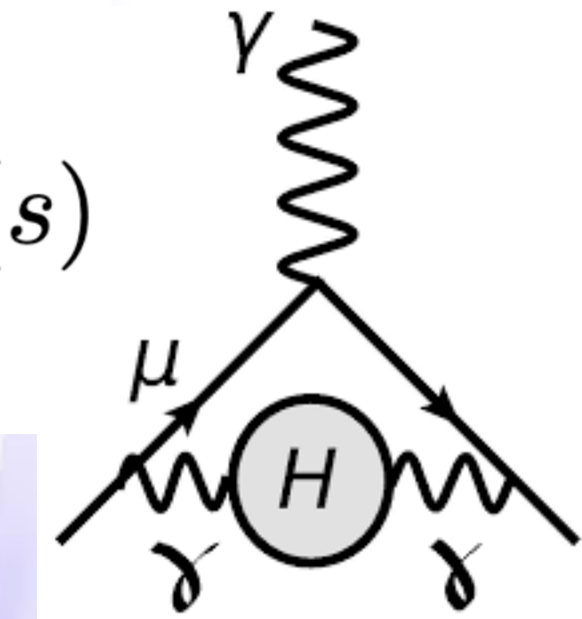
$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



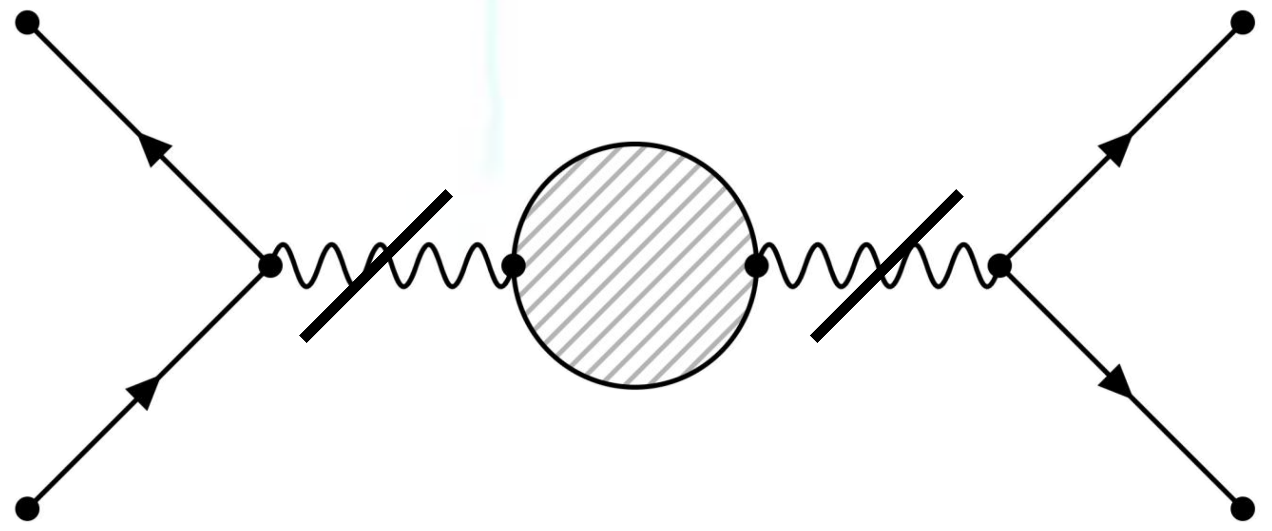
PDG 2024

$$R = N_c \sum_{i=1}^{n} Q_i^2 = \begin{cases} \frac{2}{3} N_c & i = u, d, s \\ \frac{10}{9} N_c & i = u, d, s, c \\ \frac{11}{9} N_c & i = u, d, s, c, b \end{cases}$$

$$a_\mu^{\text{Had}}[\text{LO}] = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R^{(0)}(s)$$



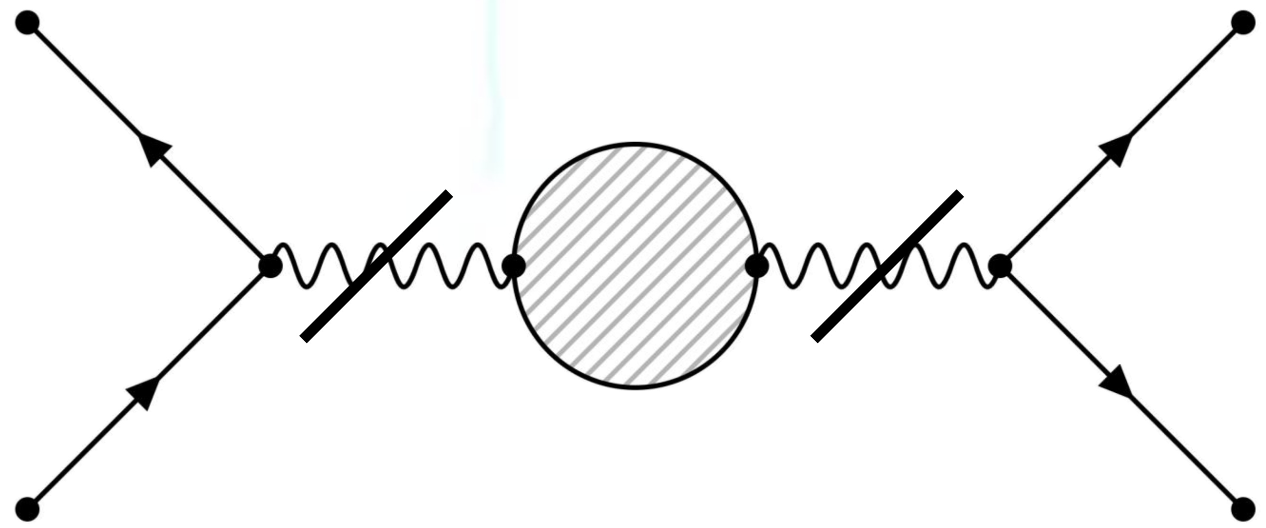
The R ratio from lattice QCD



$$C_2(t) = \left\langle J_\mu^{\mathrm{em}}(t)\ J_\mu^{\mathrm{em}}(0) \right\rangle = \int d\omega \rho(\omega) e^{-\omega t} \qquad R(\omega) = \frac{12\pi^2}{\omega^2}\rho(\omega)$$

$\rho(\omega)$ **from solving the inverse problem**

The R ratio from lattice QCD



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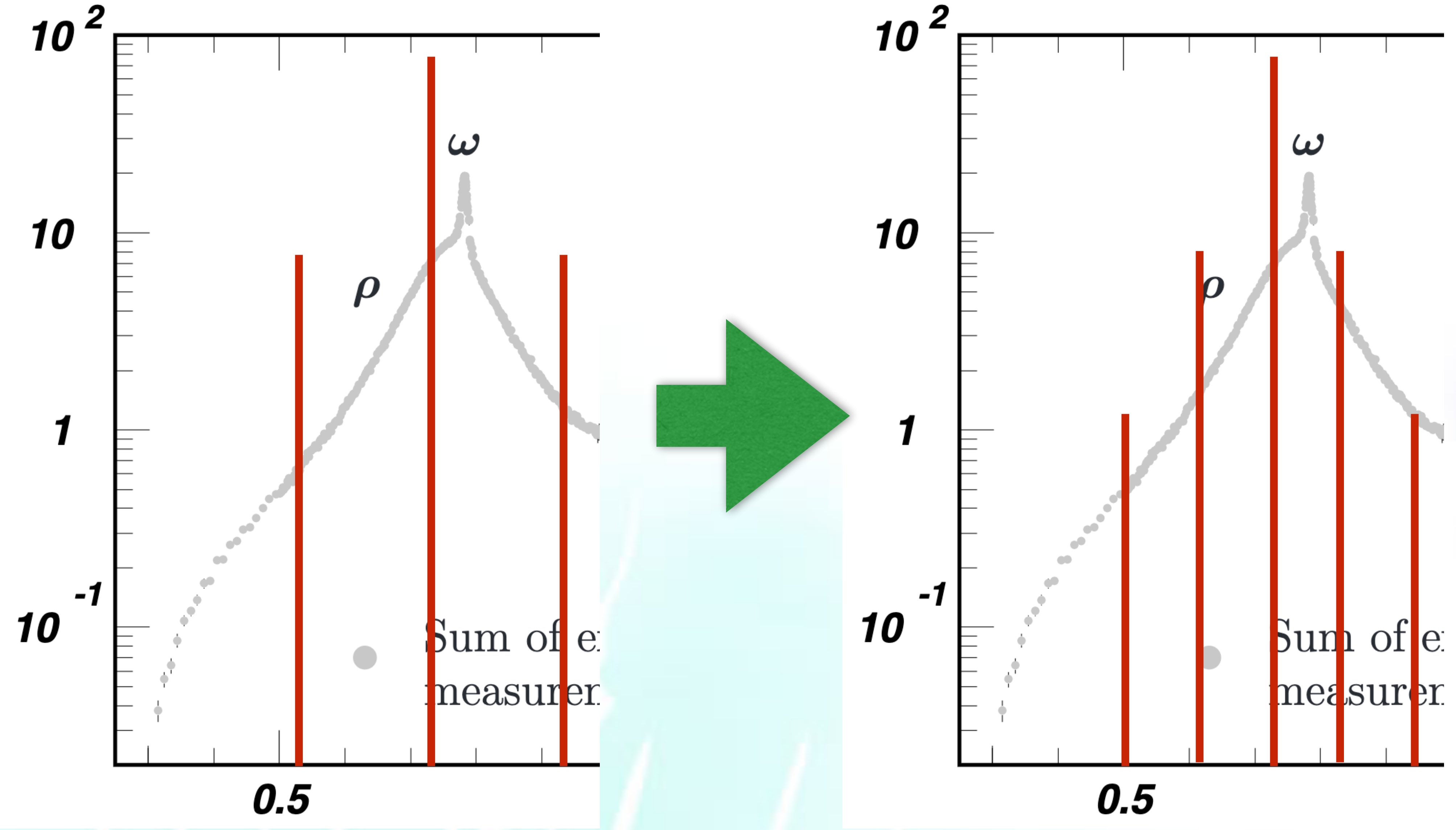
$\rho(\omega)$ **from solving the inverse problem**

$$\rho(\omega) = \sum_n A_n \delta(\omega, \omega_n)$$

Lattice finite-volume discrete spectrum!

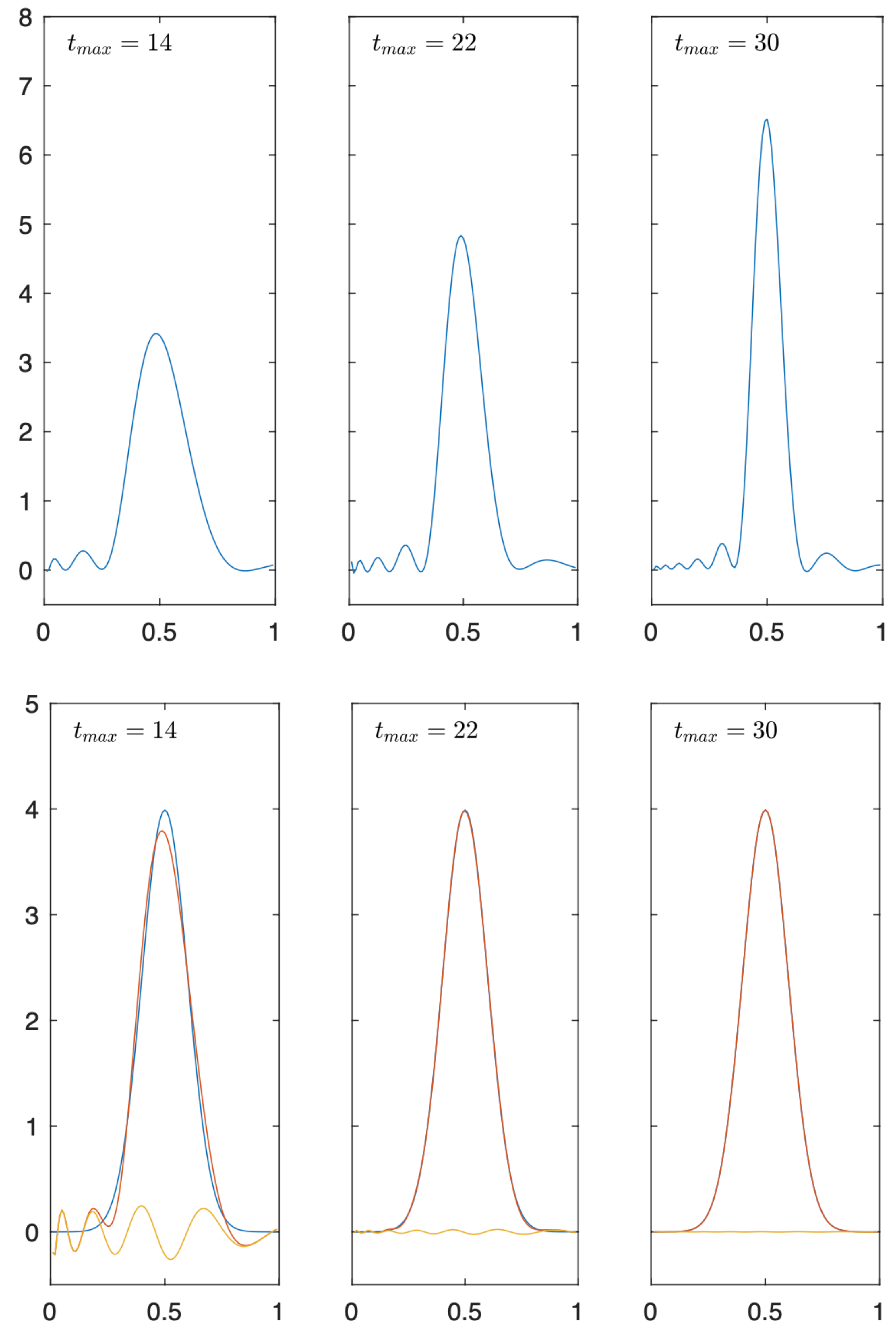


$$\rho^S(\omega, L, \Delta) = \int d\omega' \mathcal{S}(\omega, \omega') \rho(\omega', L)$$

$$\rho(\omega) = \lim_{\Delta \to 0} \lim_{L \to \infty} \rho^S(\omega, L, \Delta)$$

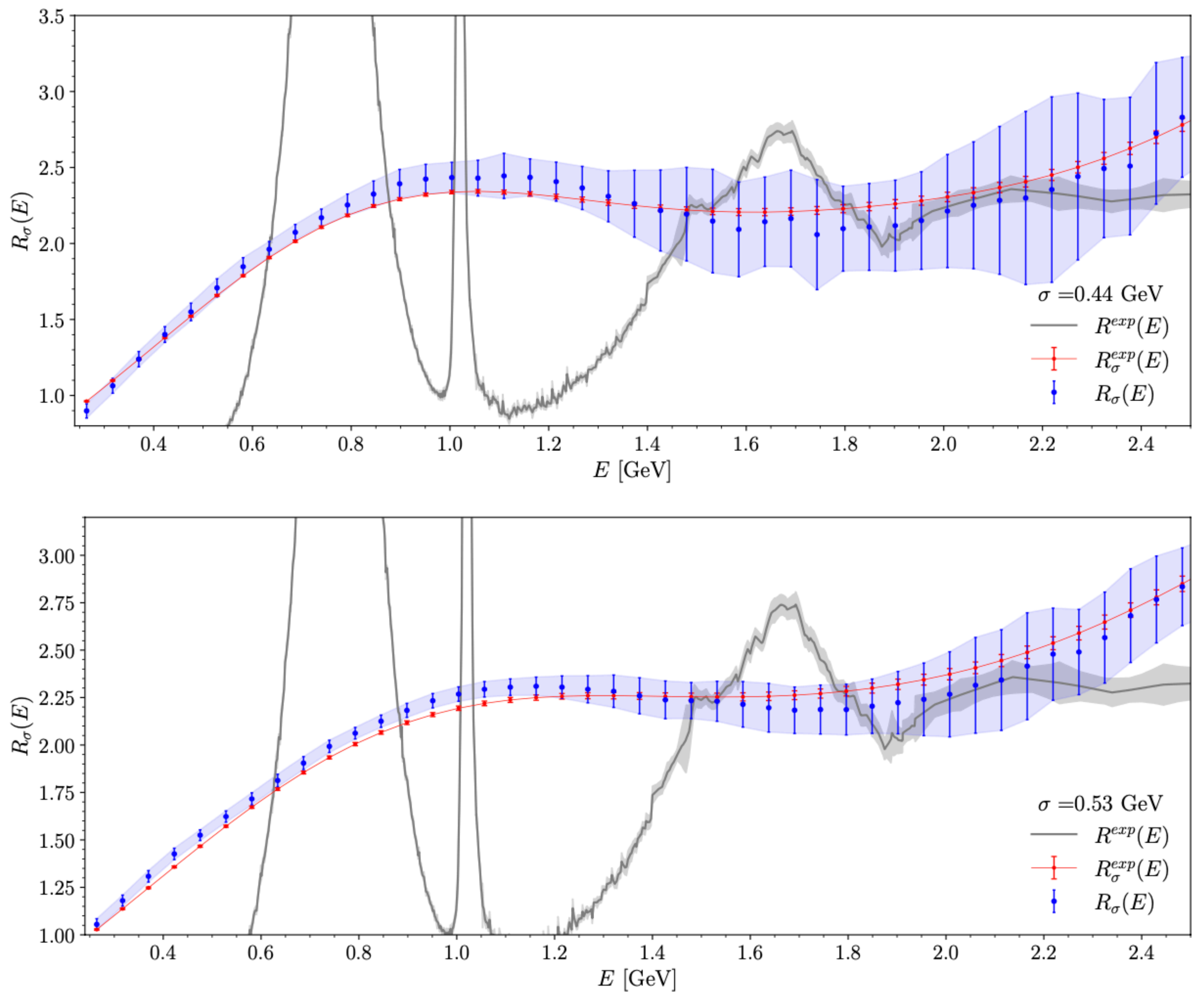
M. T. Hansen et al., Phys. Rev. D 96, 094513 (2017)

The R ratio from lattice QCD



$$\rho(\omega) = \lim_{\Delta \to 0} \lim_{L \to \infty} \rho^S(\omega, L, \Delta)$$

$$\rho^S(\omega, \Delta) = \lim_{L \to \infty} \rho^S(\omega, L, \Delta) \leftrightarrow \rho_P^S(\omega, \Delta)$$



M. T. Hansen et al., Phys. Rev. D 99, 094508 (2019)

C. Alexandrou et al., Phys. Rev. Lett. 130, 241901 (2023)

Our setup

Label	L/T	Mpi (MeV)	a (fm)	L (fm)
48I	48/96	139	0.11406	5.47
64I	64/128	139	0.08365	5.35
24D	24/64	139	0.1940	4.656
32D	32/64	139	0.1940	6.208
48D	48/96	139	0.1940	9.312

R. Arthur et al., PRD87, 094514 (2013)
T. Blum et al., PRD93, 074505 (2016)
P. Boyle et al., PRD 93, 054502 (2016)

Overlap fermions on RBC/UKQCD domain wall gauge ensembles at the **physical point with different lattice spacings and volumes**

High-precision current-current correlation functions for both *u/d* and *s*

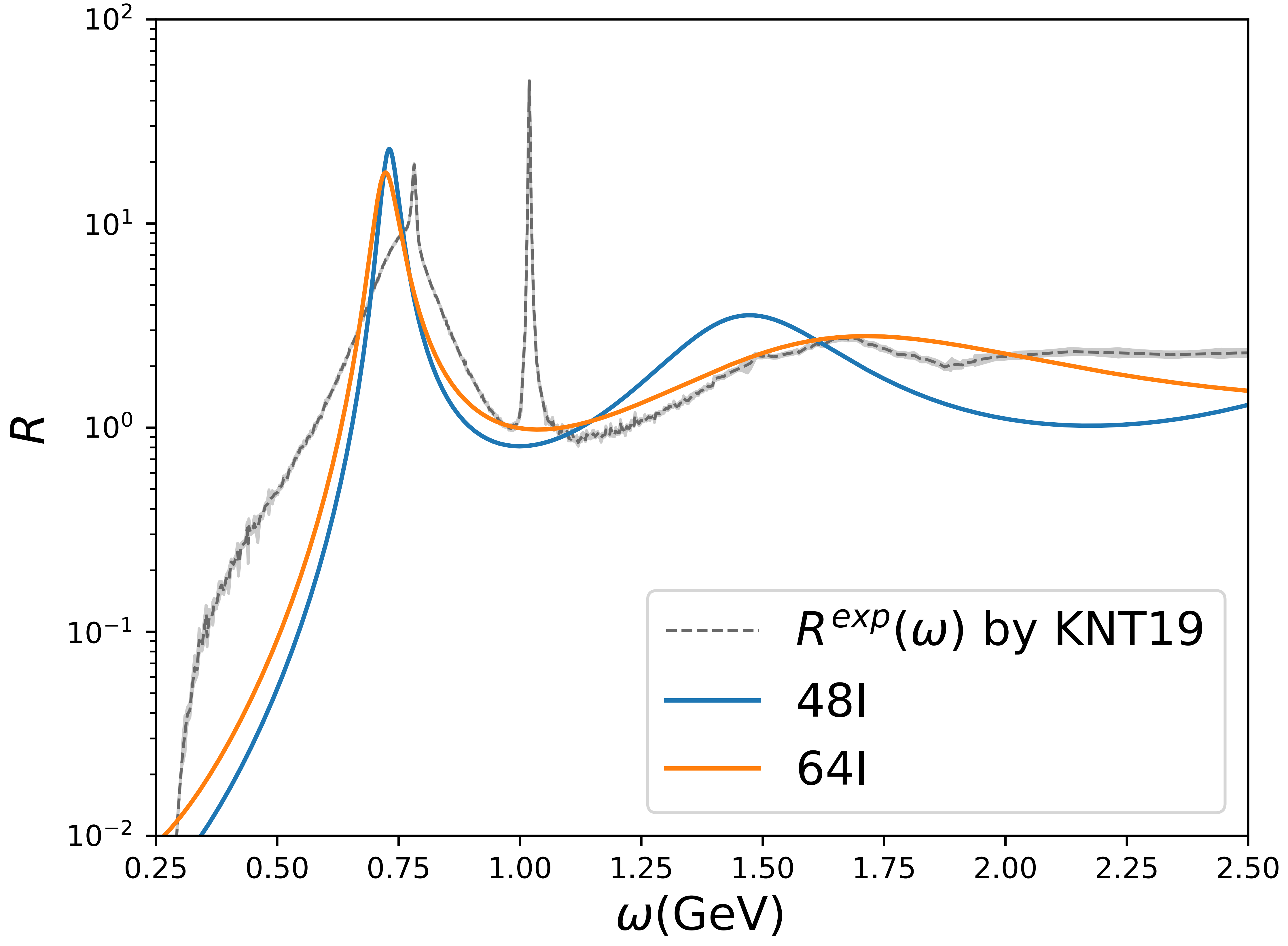
Gen Wang et al., Phys. Rev. D 107, 034513 (2023)

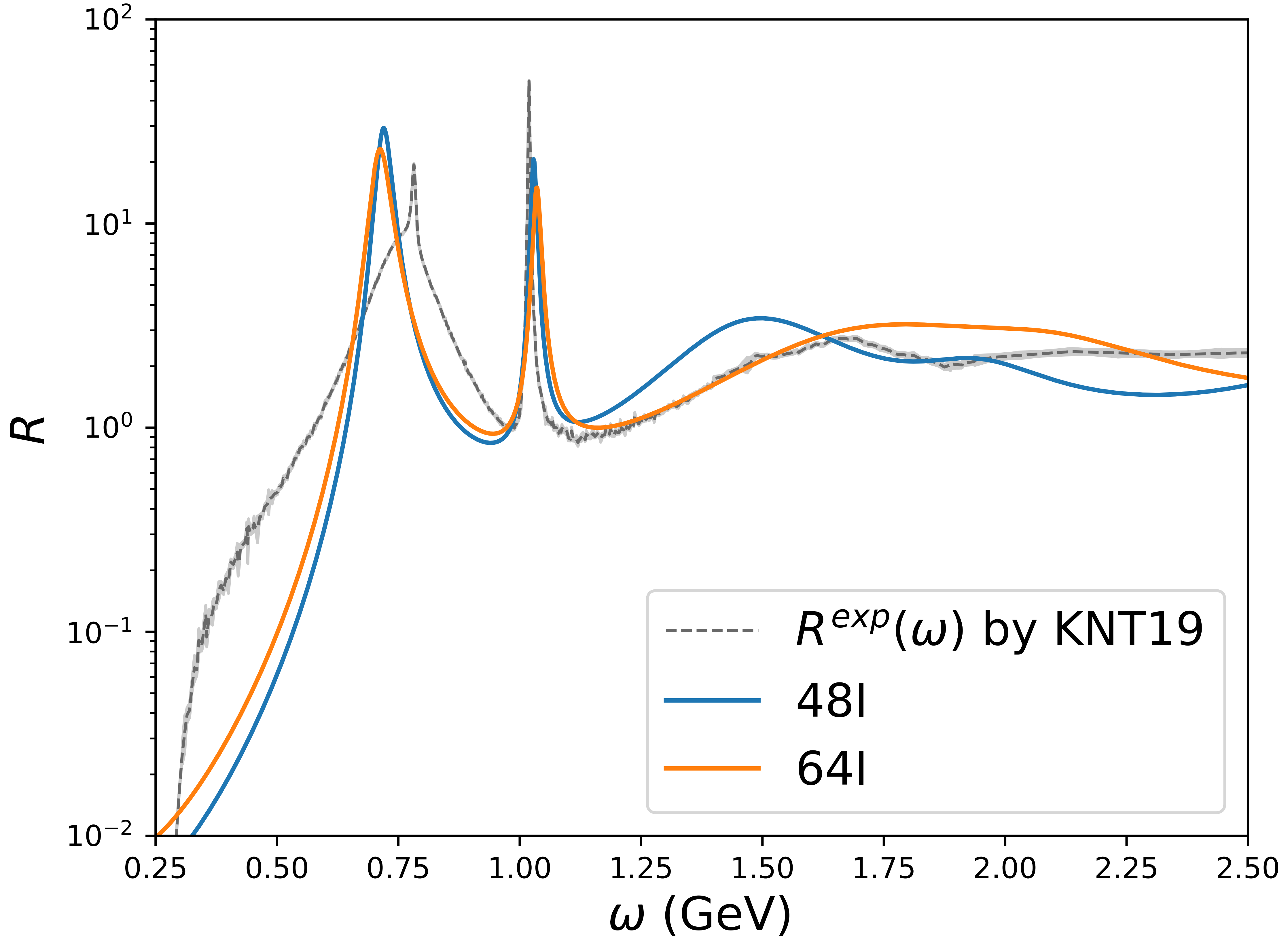
Bayesian reconstruction (BR) algorithm for solving the inverse problem. High resolution so the smearing can be applied afterwards

Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

Comprehensive systematic uncertainty study

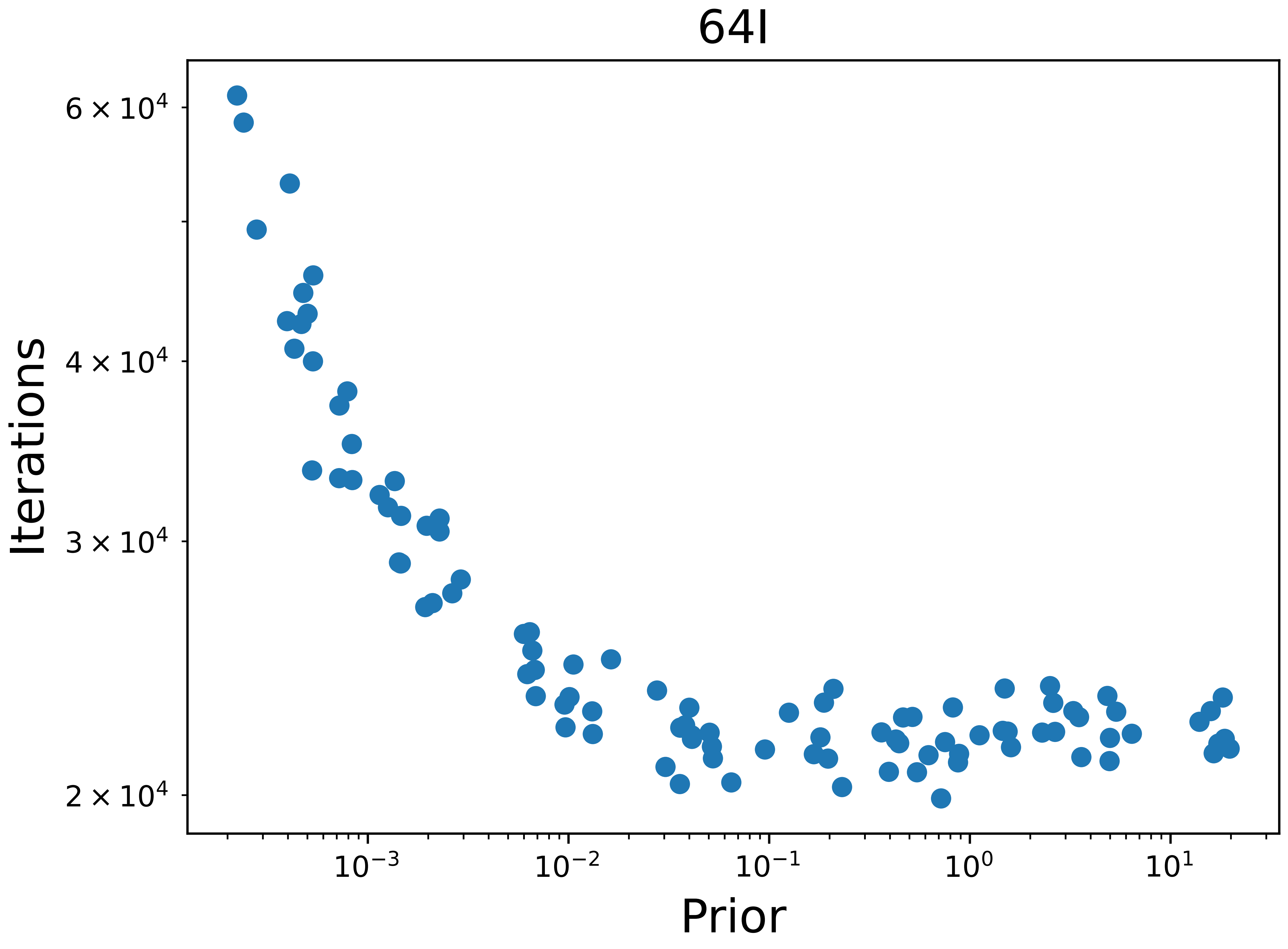
Results without smearing

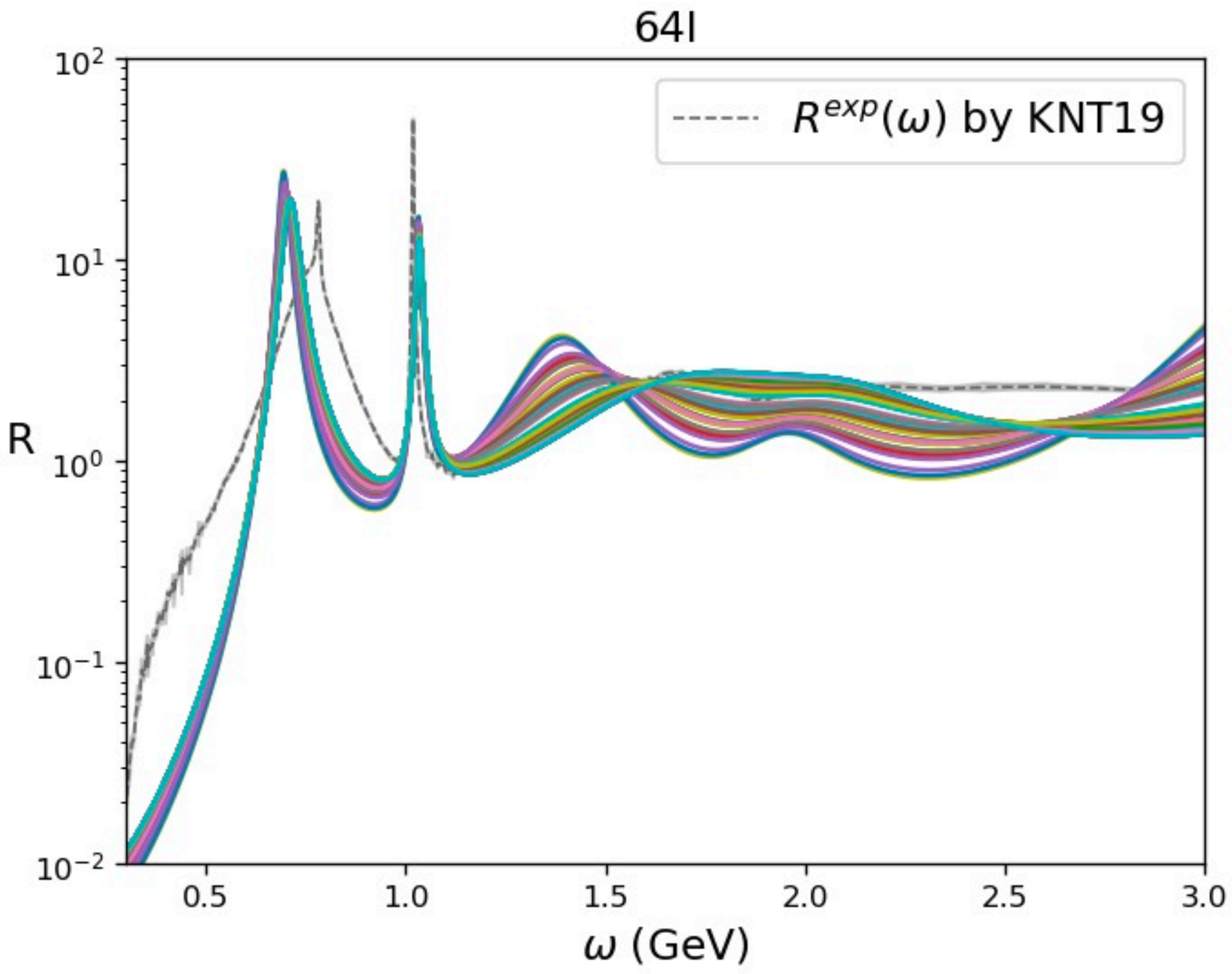




Additional ϕ peek by applying separate BR on light and strange correlators

Prior dependence



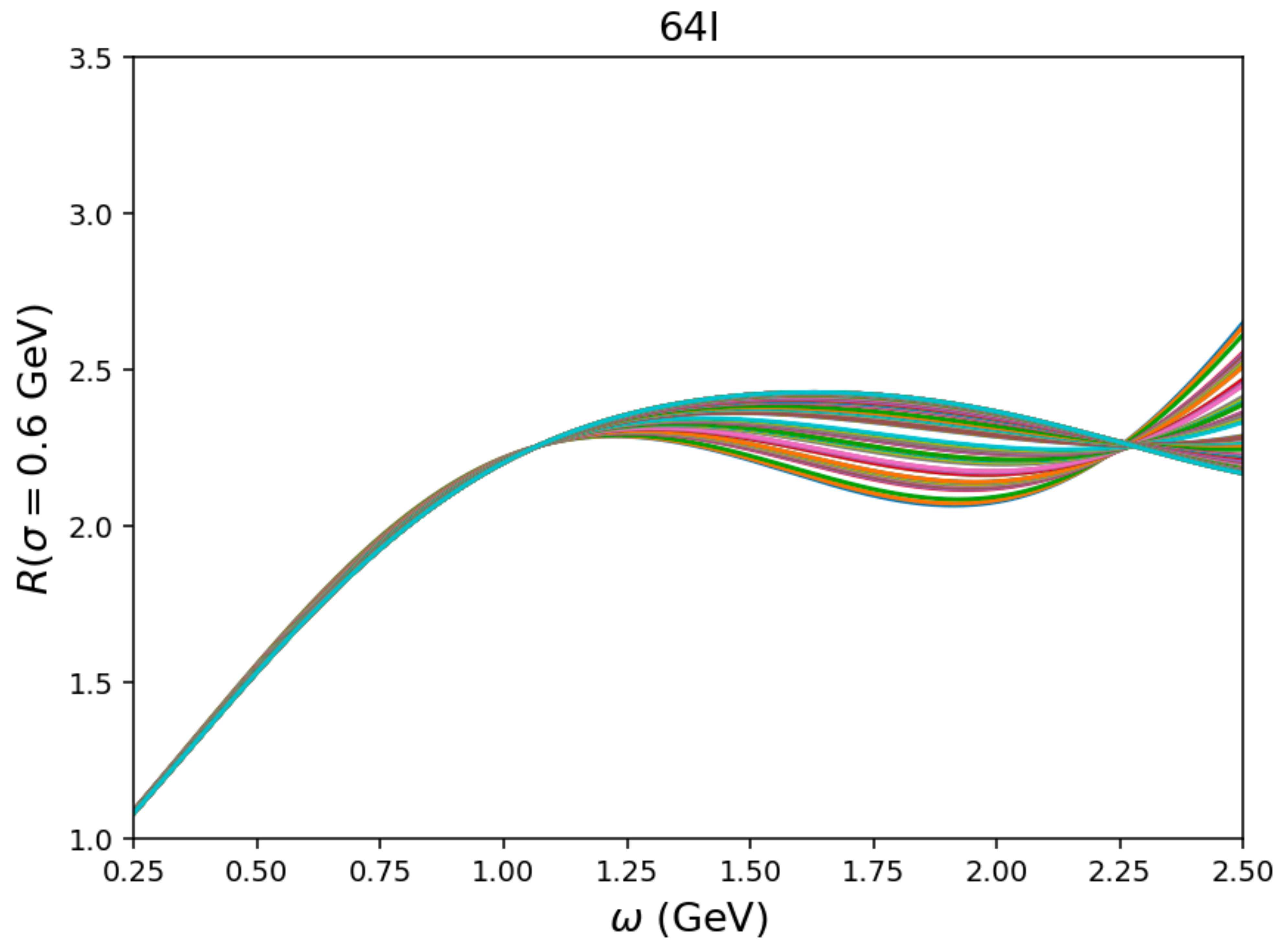


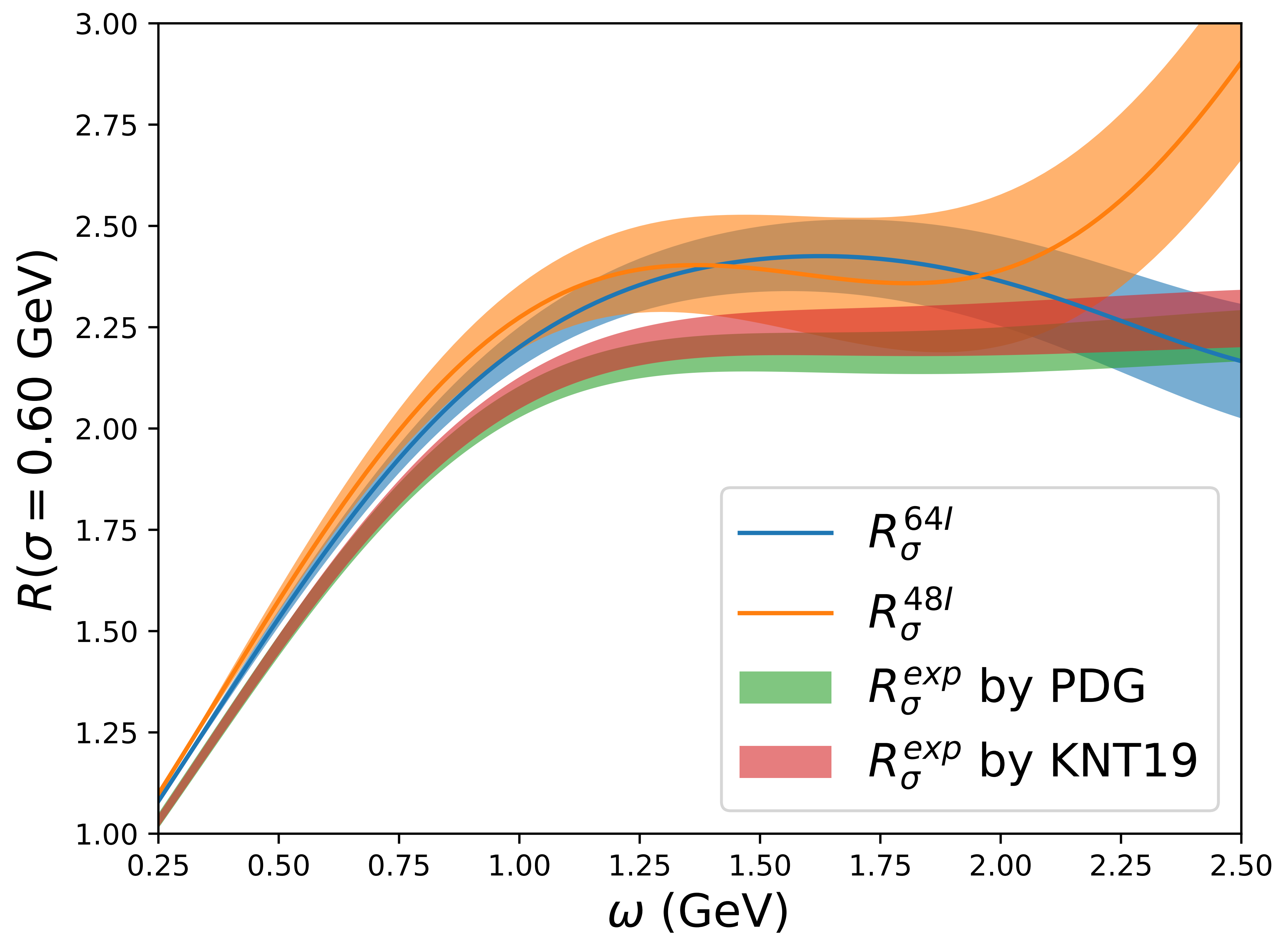
Constant priors ranging from $2\times10^{-4}\sim2\times10^{1}$

Smearing and the prior uncertainty

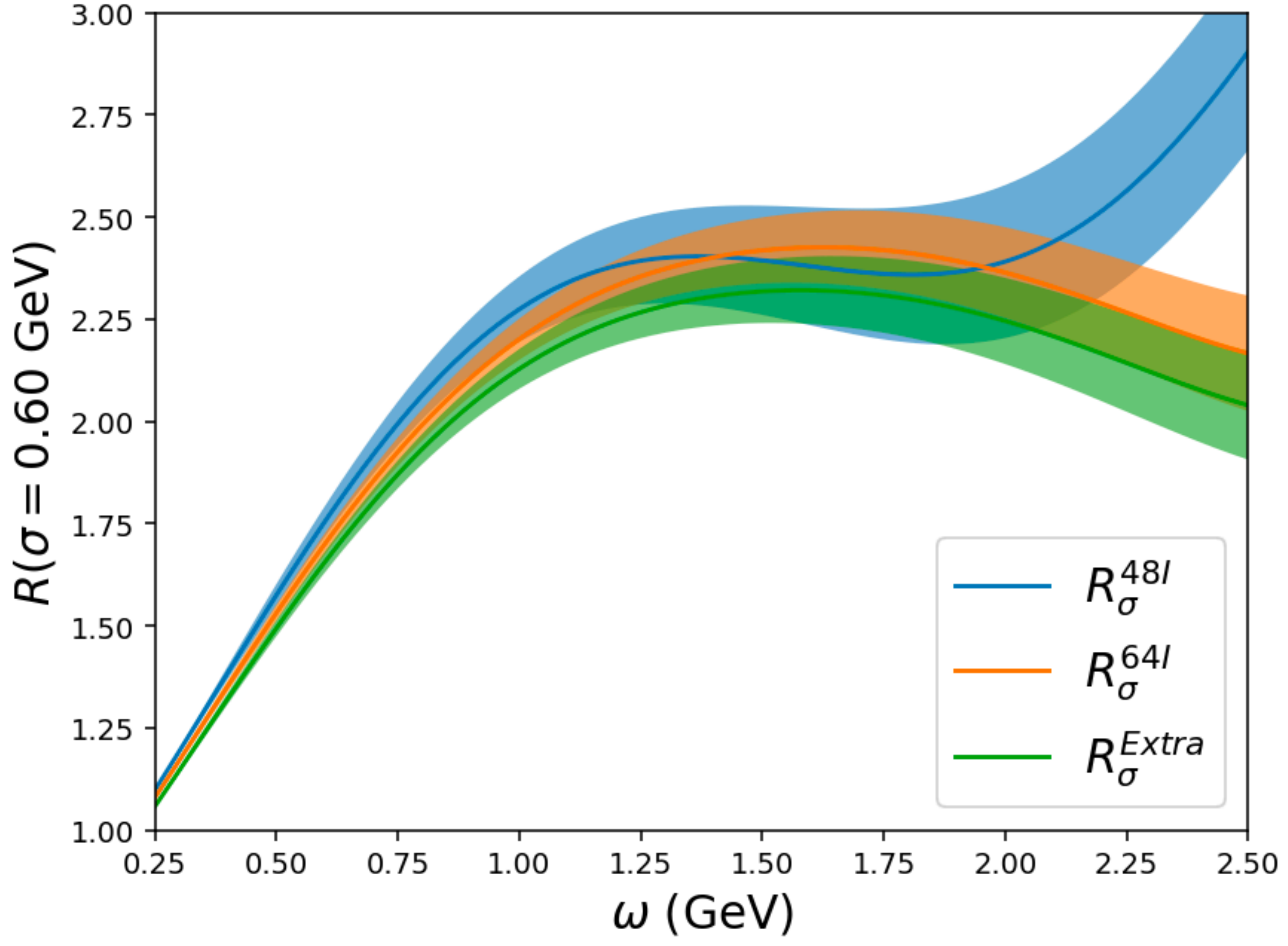
$$\rho^S(\omega, L, \Delta) = \int d\omega' \mathcal{S}_\Delta(\omega, \omega')\rho(\omega', L)$$

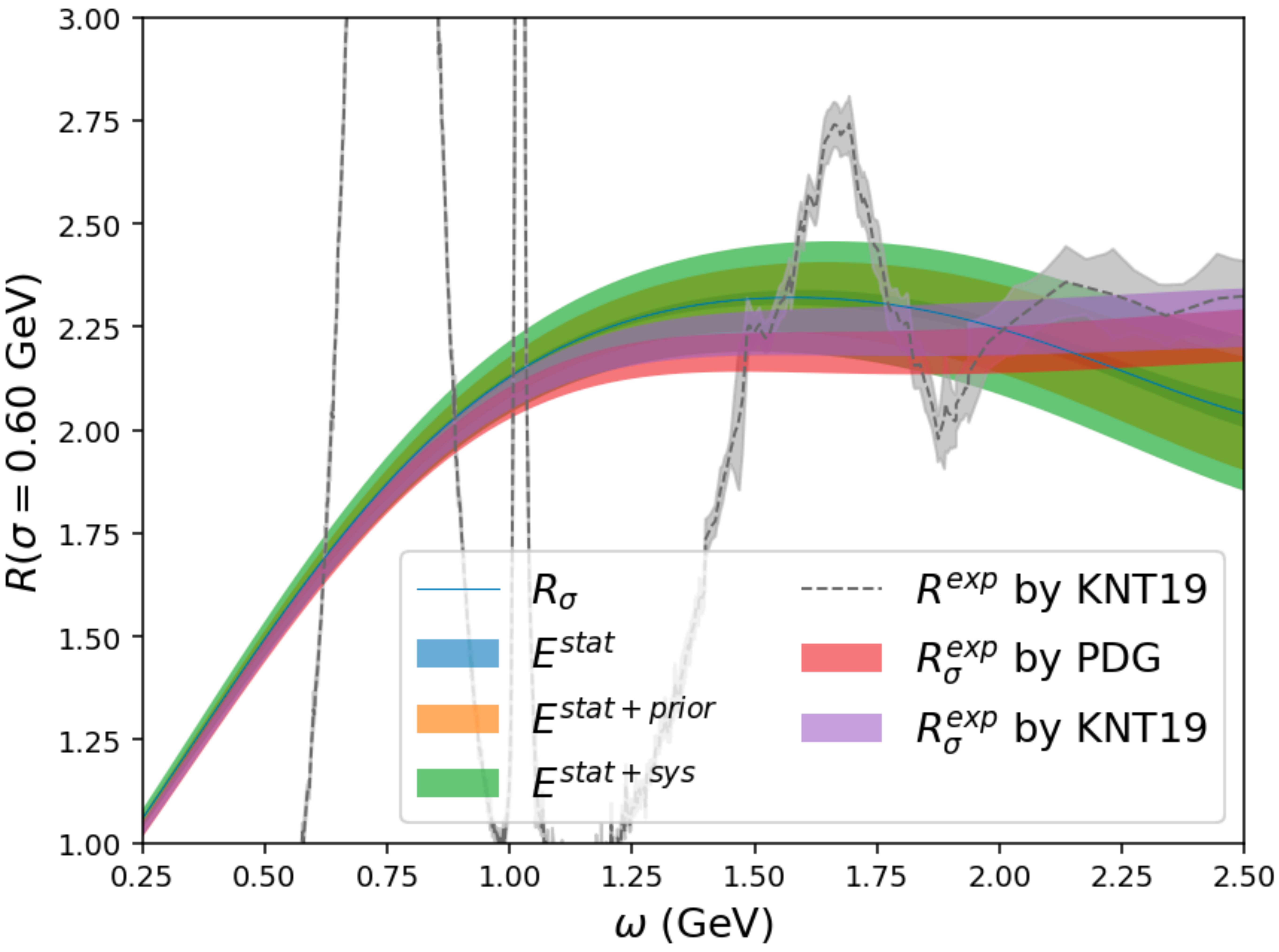
$$\mathcal{S}_\Delta(\omega, \omega') \sim \exp\left(-\frac{(\omega - \omega')^2}{2\Delta^2}\right)$$





Continuum extrapolation



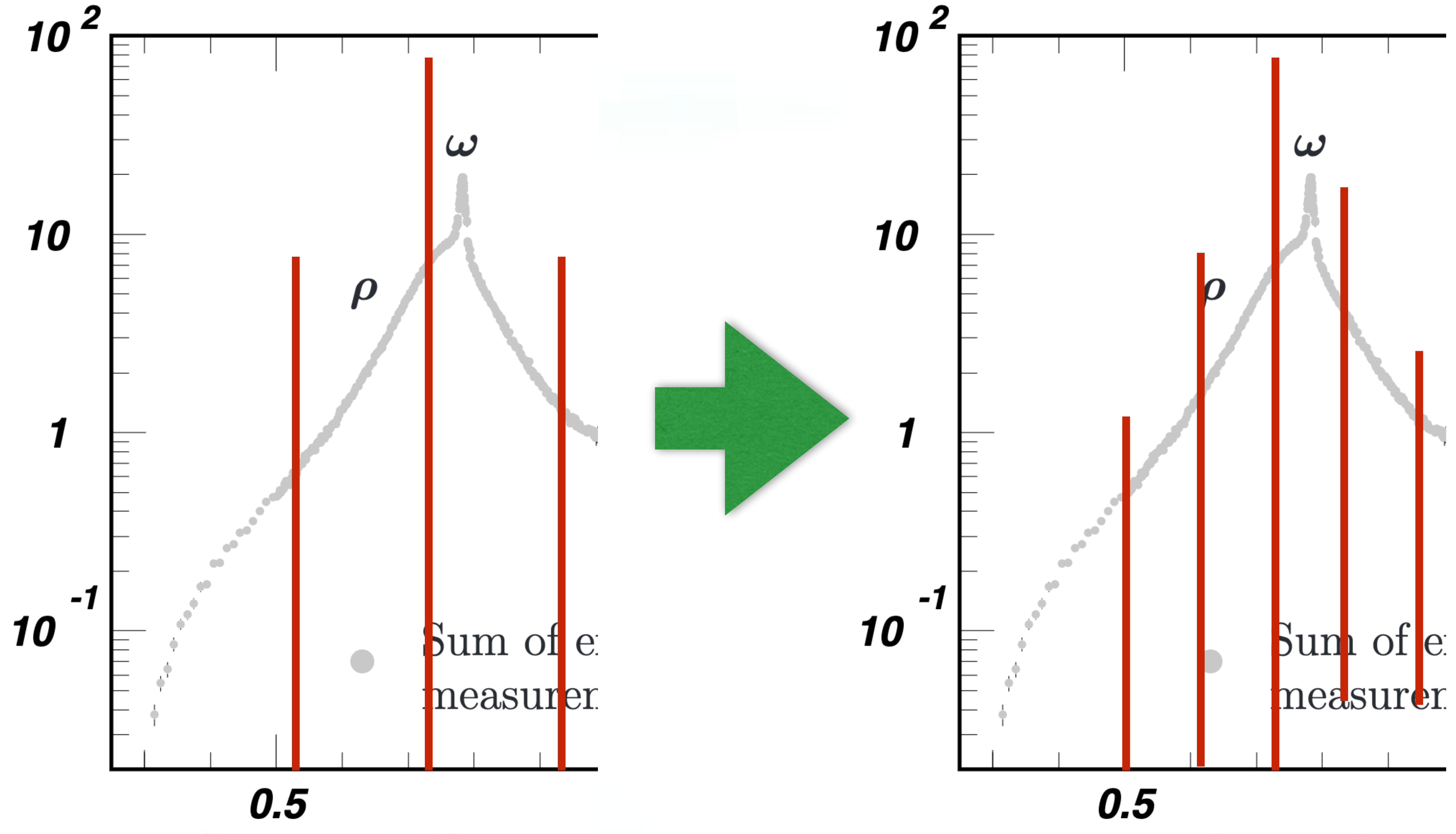


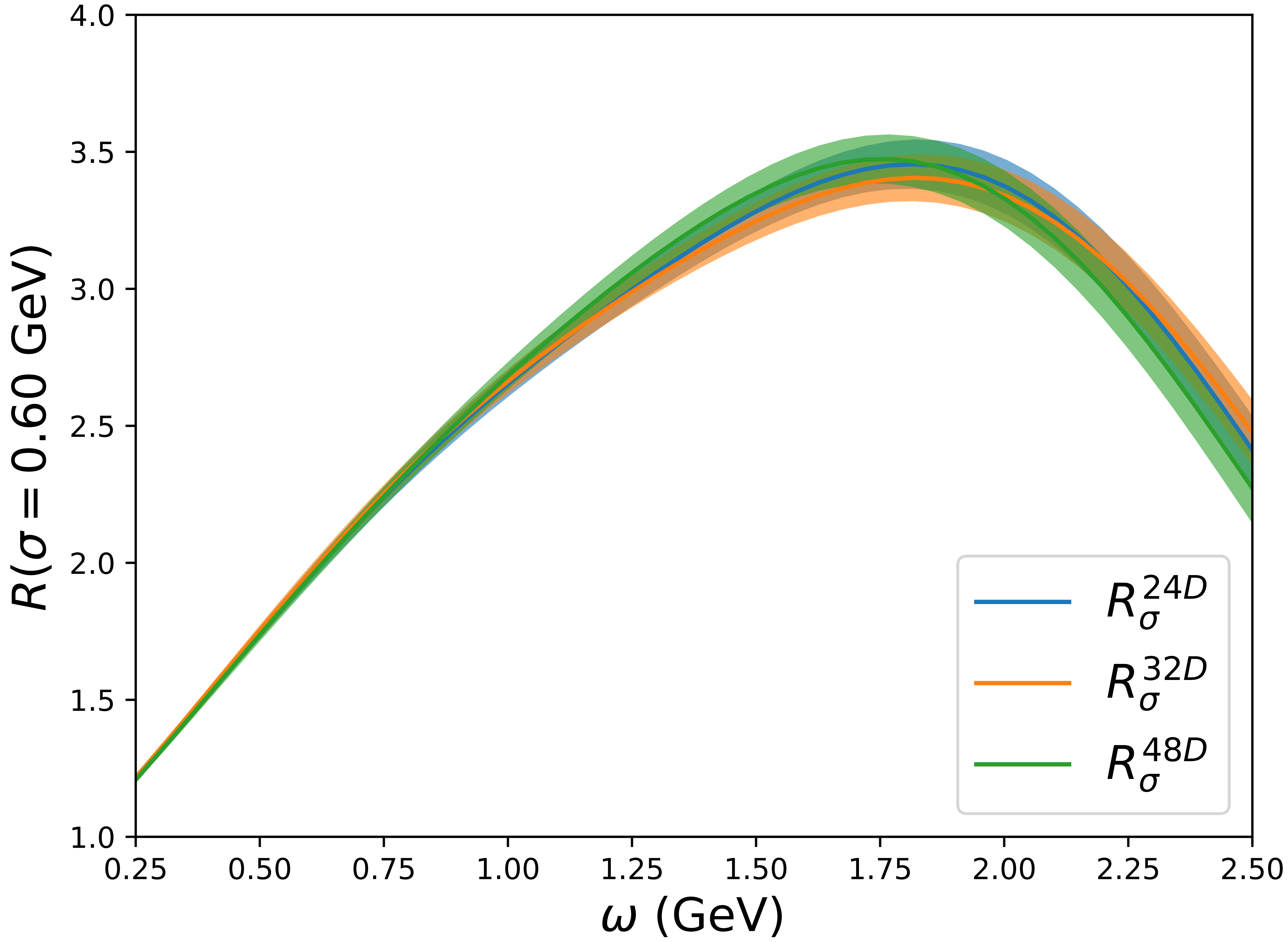
The systematic uncertainty of the continuum extrapolation is estimated to be the difference between the extrapolated results and the results of the finest lattice

Volume dependence

$$\rho(\omega) = \sum_n A_n \delta(\omega, \omega_n)$$

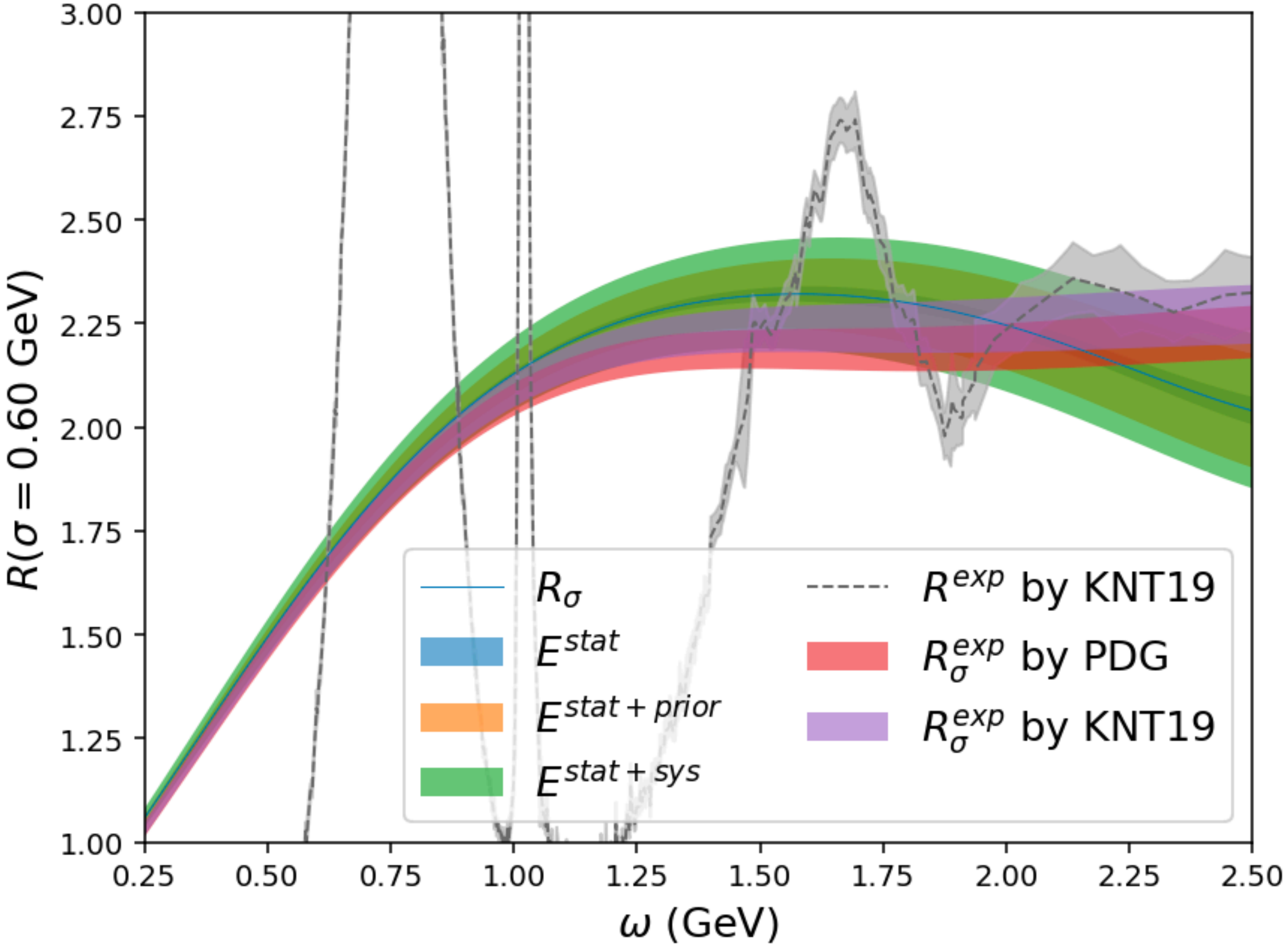
Lattice finite-volume discrete spectrum!

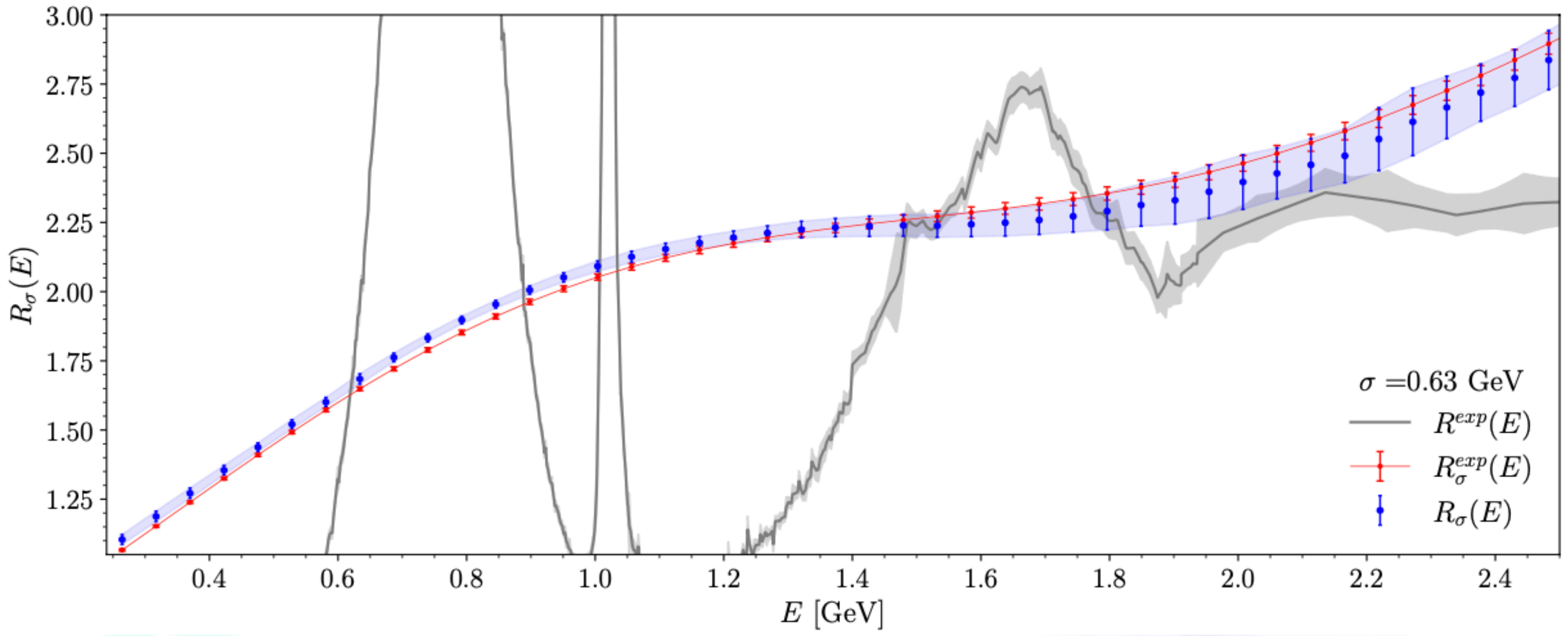




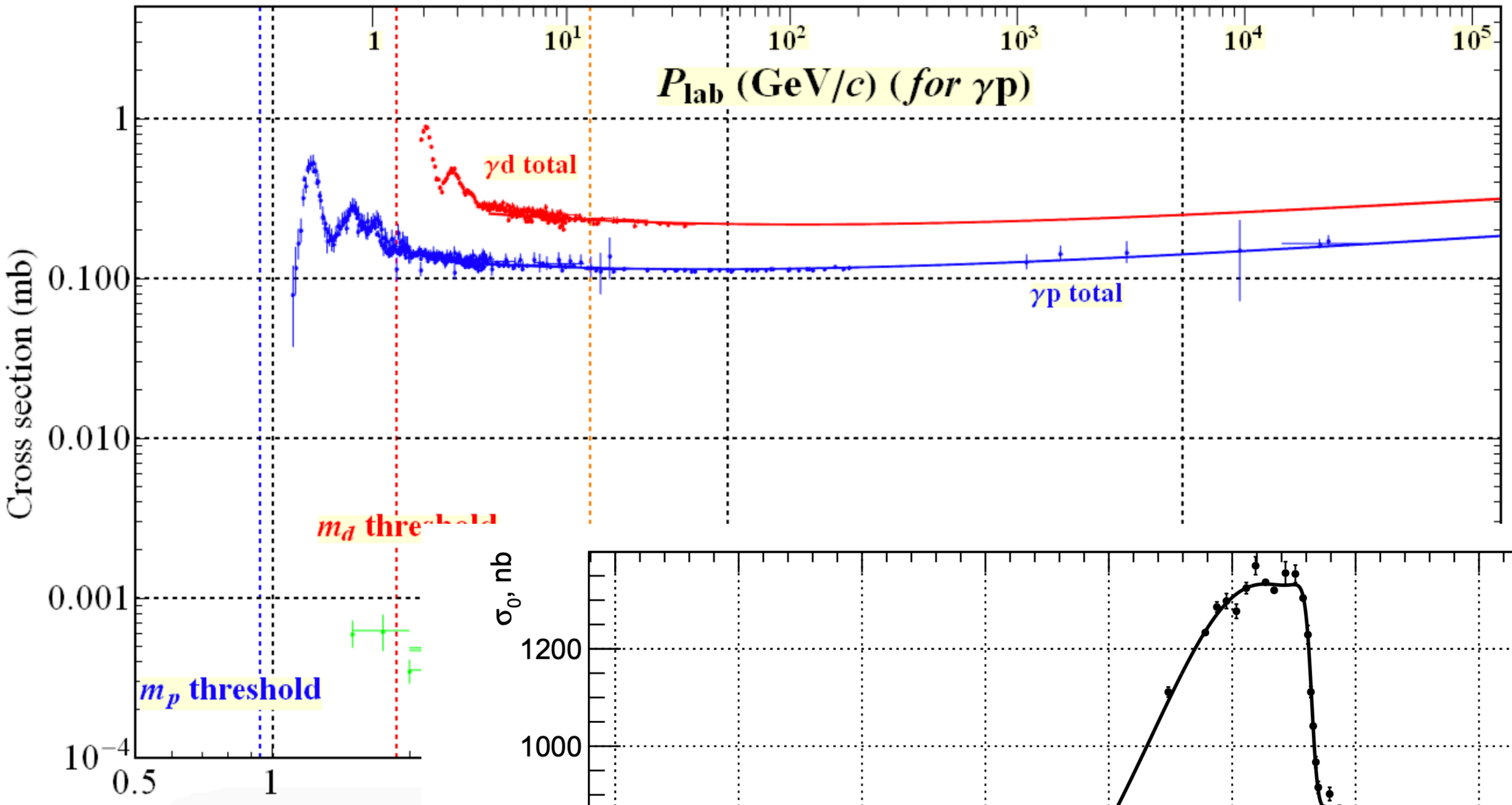
$$\rho^S(\omega, \Delta) = \lim_{L\to\infty} \rho^S(\omega, L, \Delta) \leftrightarrow \rho_P^S(\omega, \Delta)$$

R ratio with all systematic uncertainties





Outlook



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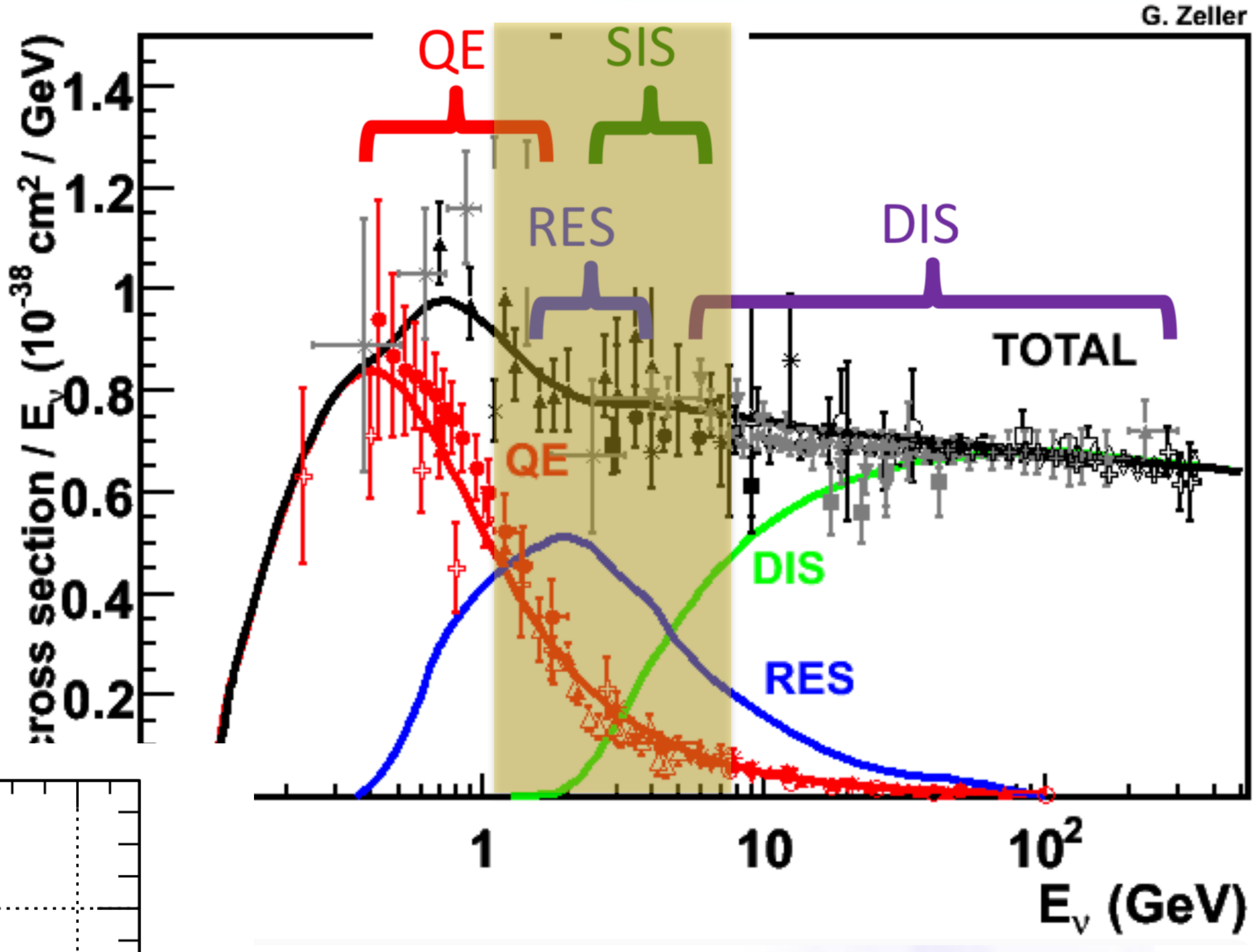
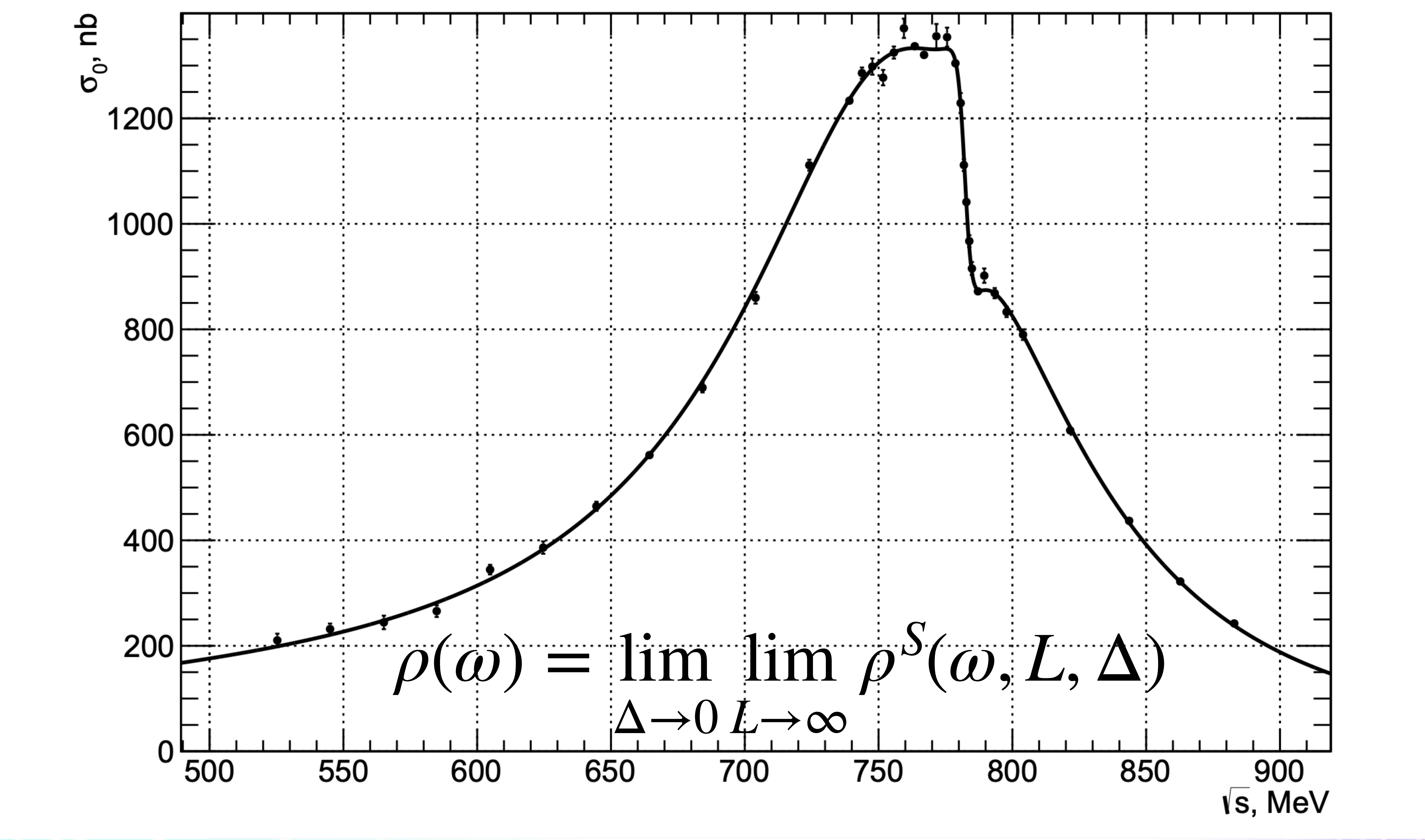


J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)



$$\rho(\omega) = \lim_{\Delta \to 0} \lim_{L \to \infty} \rho^S(\omega, L, \Delta)$$

SND collaboration, JHEP01, 113 (2021)

Summary

The BR method is used for reconstructing the R-ratio from lattice correlators. With proper smearing, the lattice results match the (smeared) experimental data very well.

The systematic uncertainties are carefully estimated.

It demonstrates that this is a feasible prescription to treat the problem of handling resonances and multi-particle states with lattice QCD.

It paves the way for further lattice calculations of many other interesting quantities such as the hadronic tensor.

Summary

The BR method is used for reconstructing the R-ratio from lattice correlators.
With proper smearing, the lattice results match the (smeared) experimental data very well.

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Thank you

Bayesian Reconstruction

$$P[\rho \,|\, D, \alpha, m] \propto e^{Q(\rho)}$$

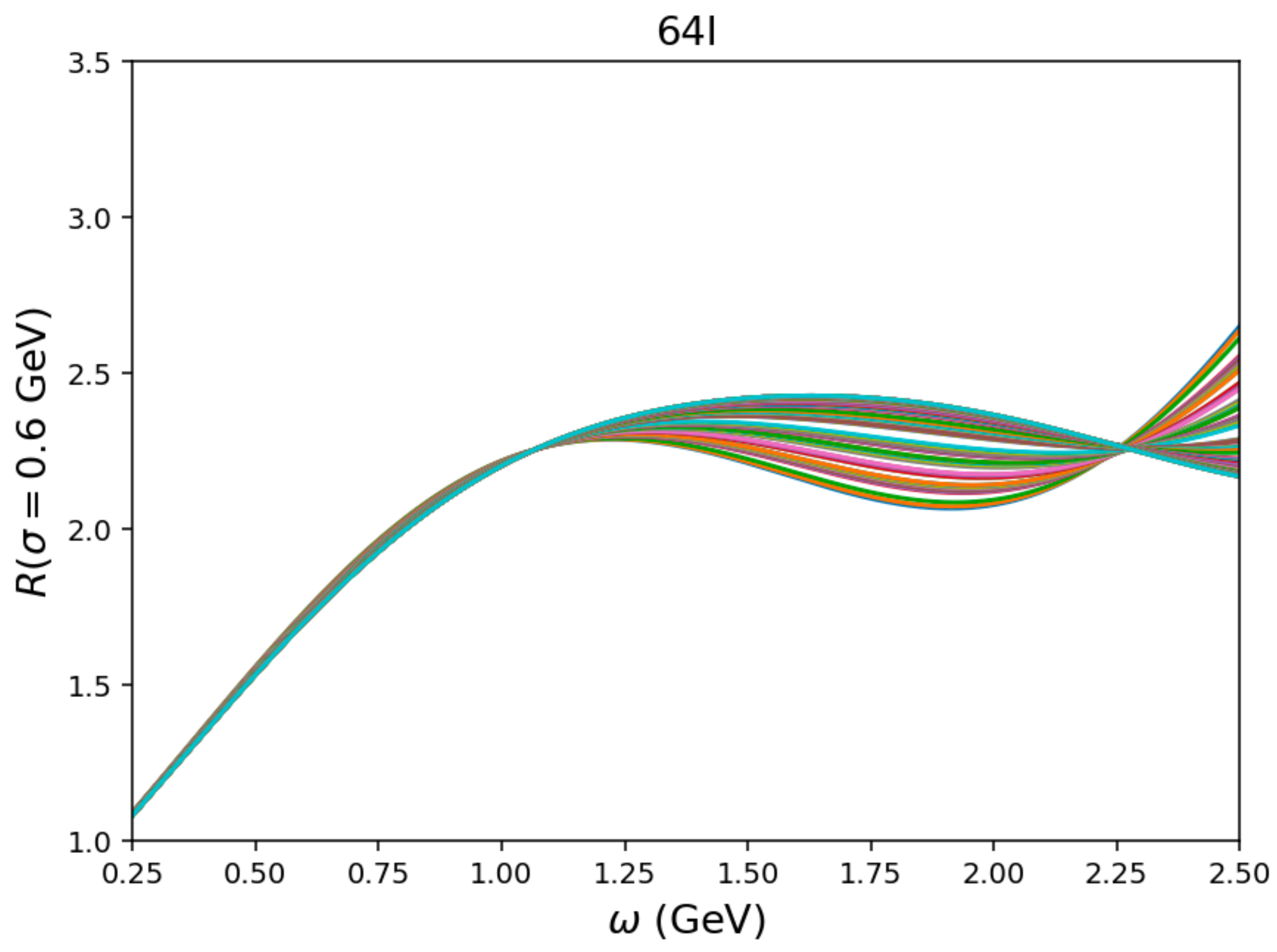
$$Q = \alpha S - L - \gamma(L - N_\tau)^2$$

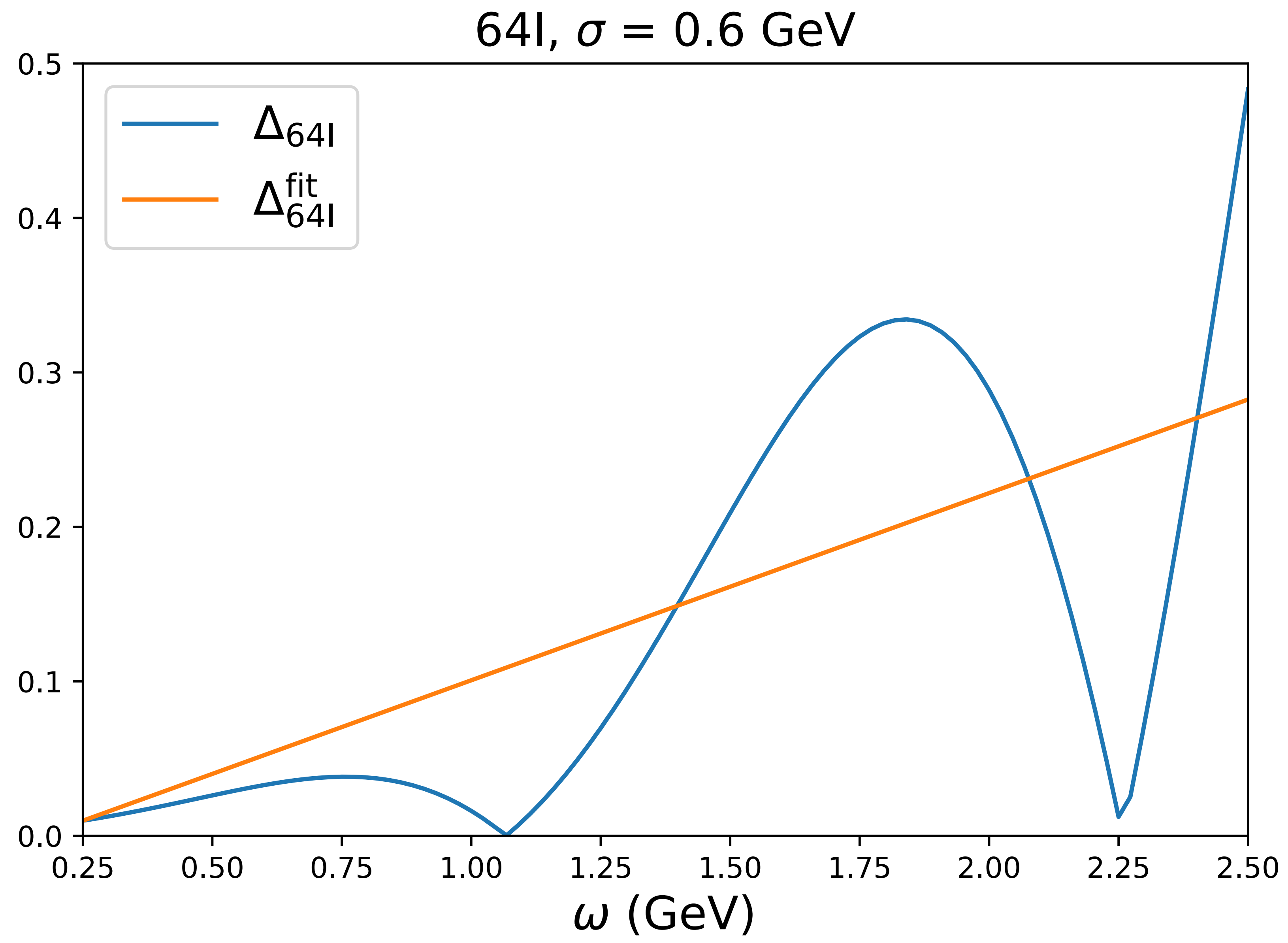
$$S = \sum_{\omega} \left[1 - \frac{\rho(\omega)}{m(\omega)} + \log\left(\frac{\rho(\omega)}{m(\omega)} \right) \right] \Delta\omega$$

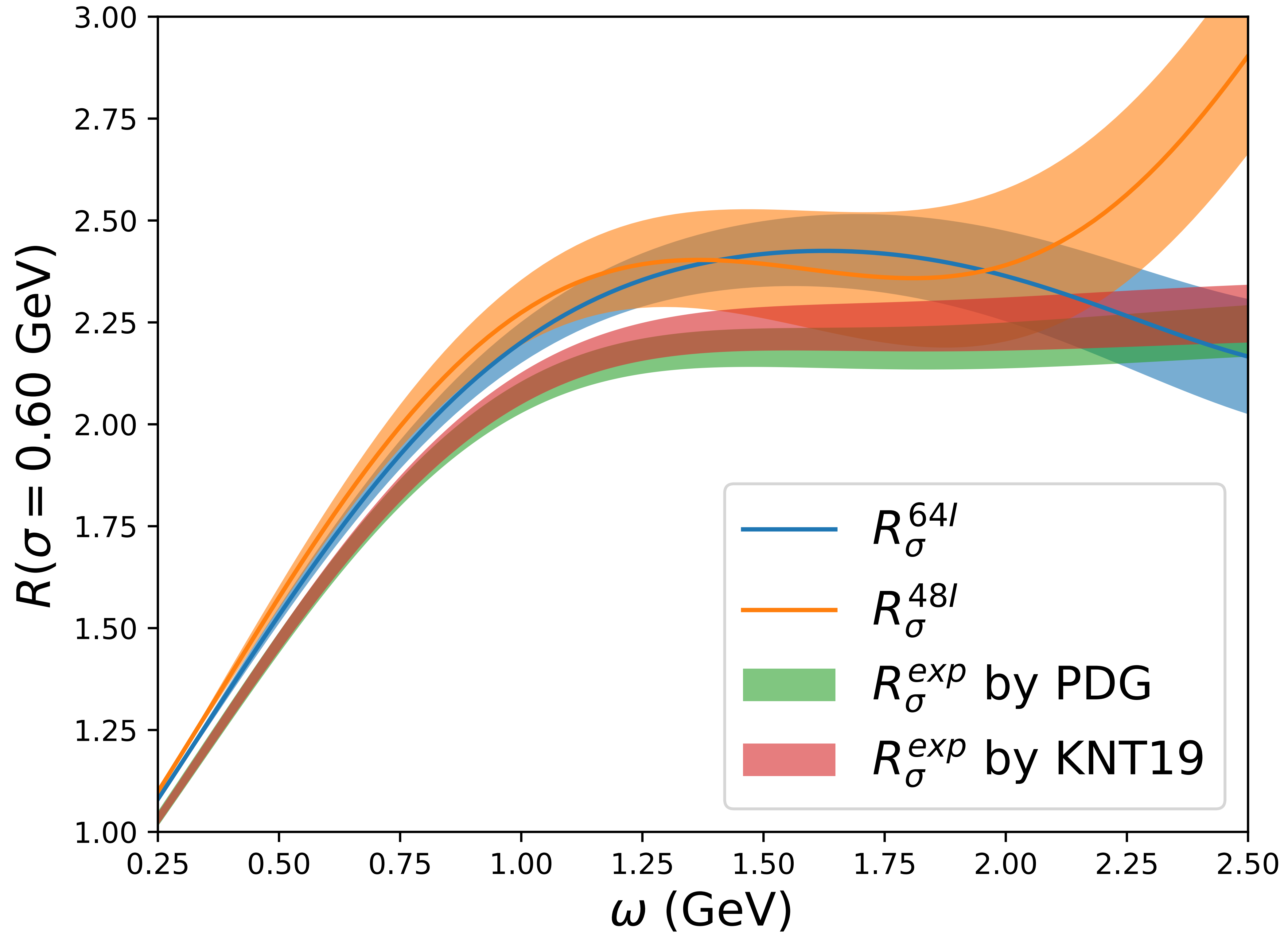
$$P[\rho \,|\, D, m] = \frac{P[D \,|\, \rho, I]}{P[D \,|\, m]} \int d\alpha P[\alpha \,|\, D, m]$$

- Hyper parameter α is integrated over
- Maximum search is in the entire parameter space($O(10^3)$)
- High precision architecture (e.g.,512-bit floating point number).

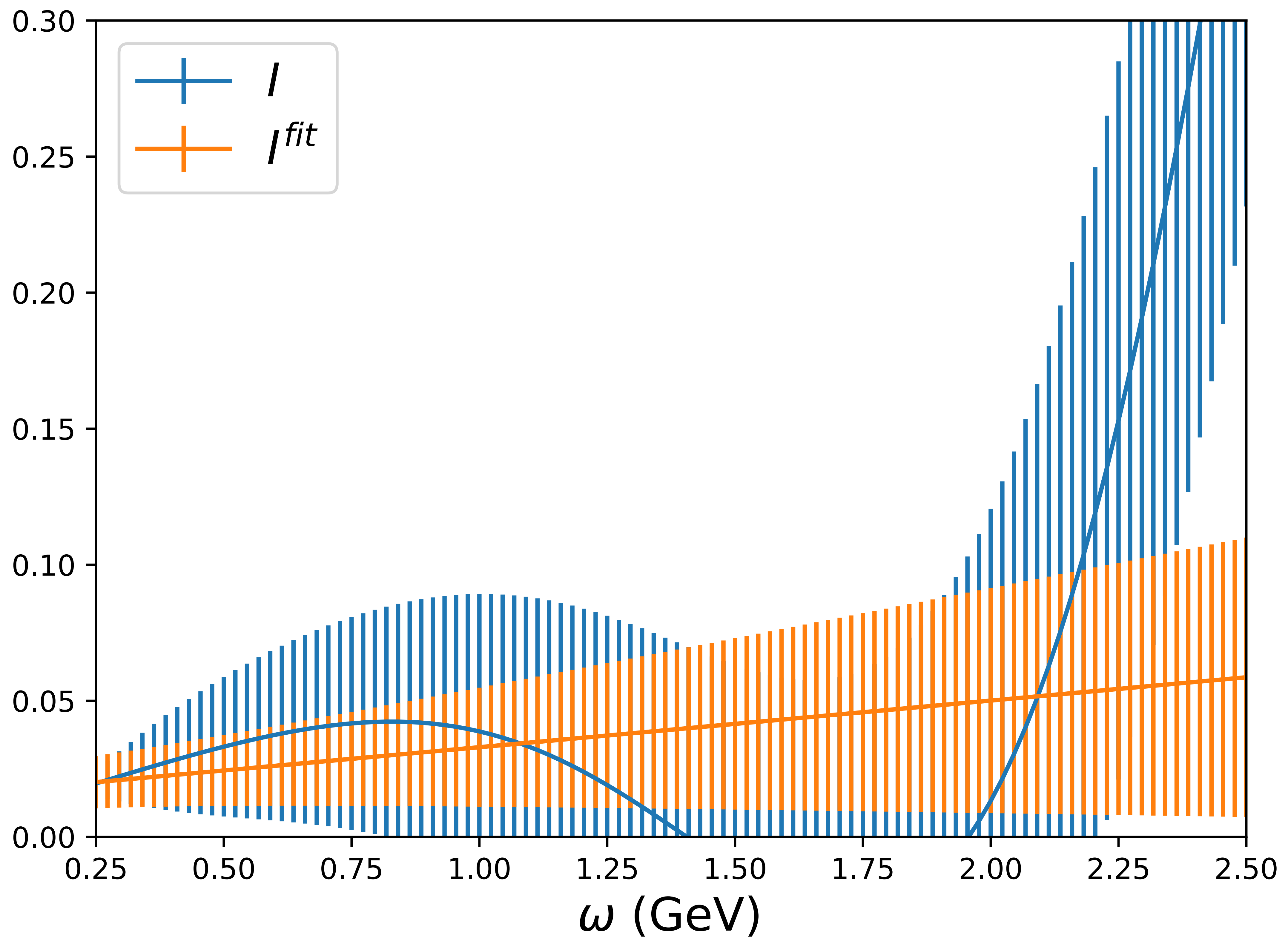
Outlook

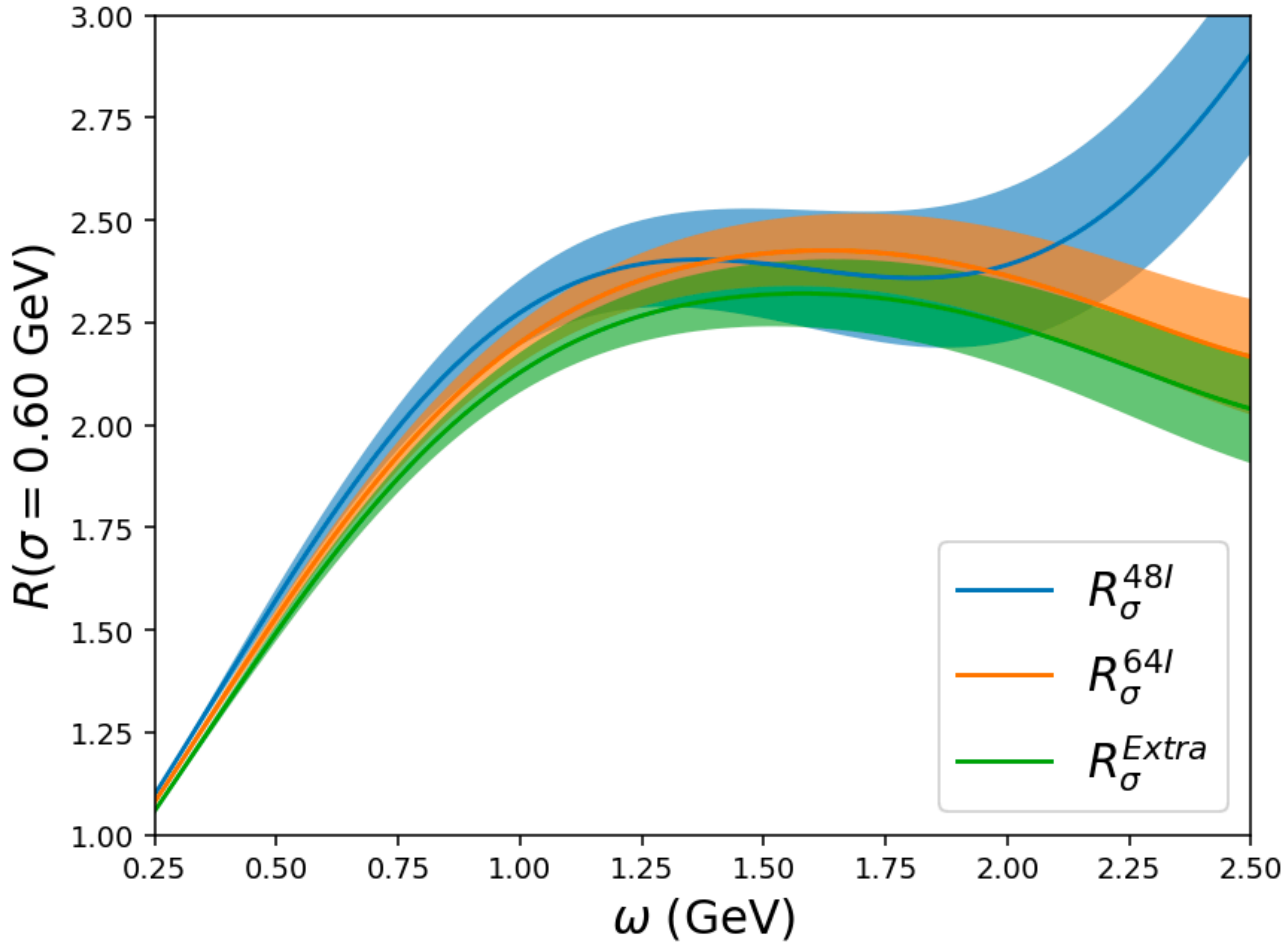

64I
R(σ = 0.6 GeV)
ω (GeV)

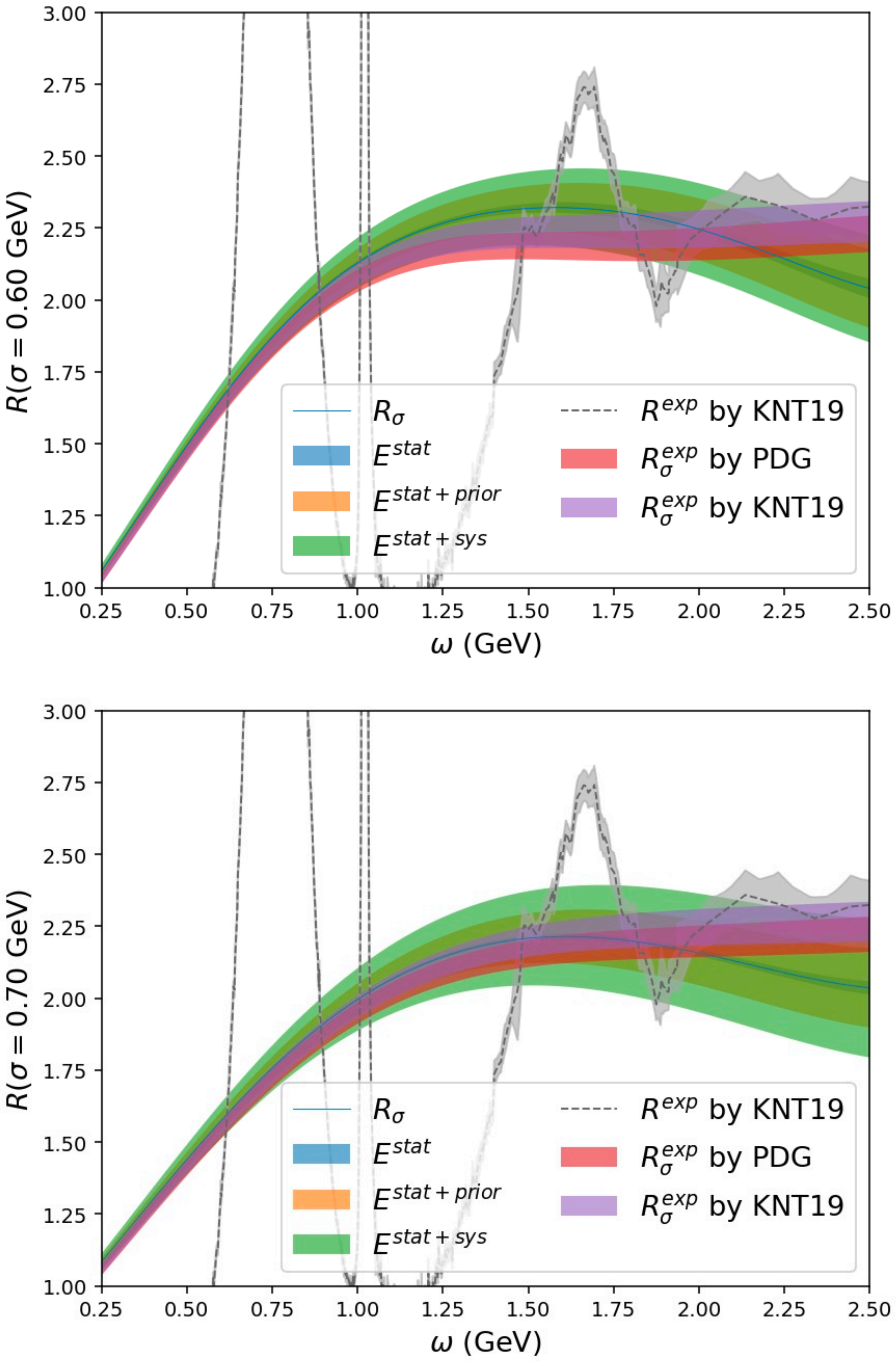

64I, σ = 0.6 GeV
Δ64I
Δ fit 64I
ω (GeV)


R(σ = 0.60 GeV)
ω (GeV)
R σ 64I
R σ 48I
R σ exp by PDG
R σ exp by KNT19

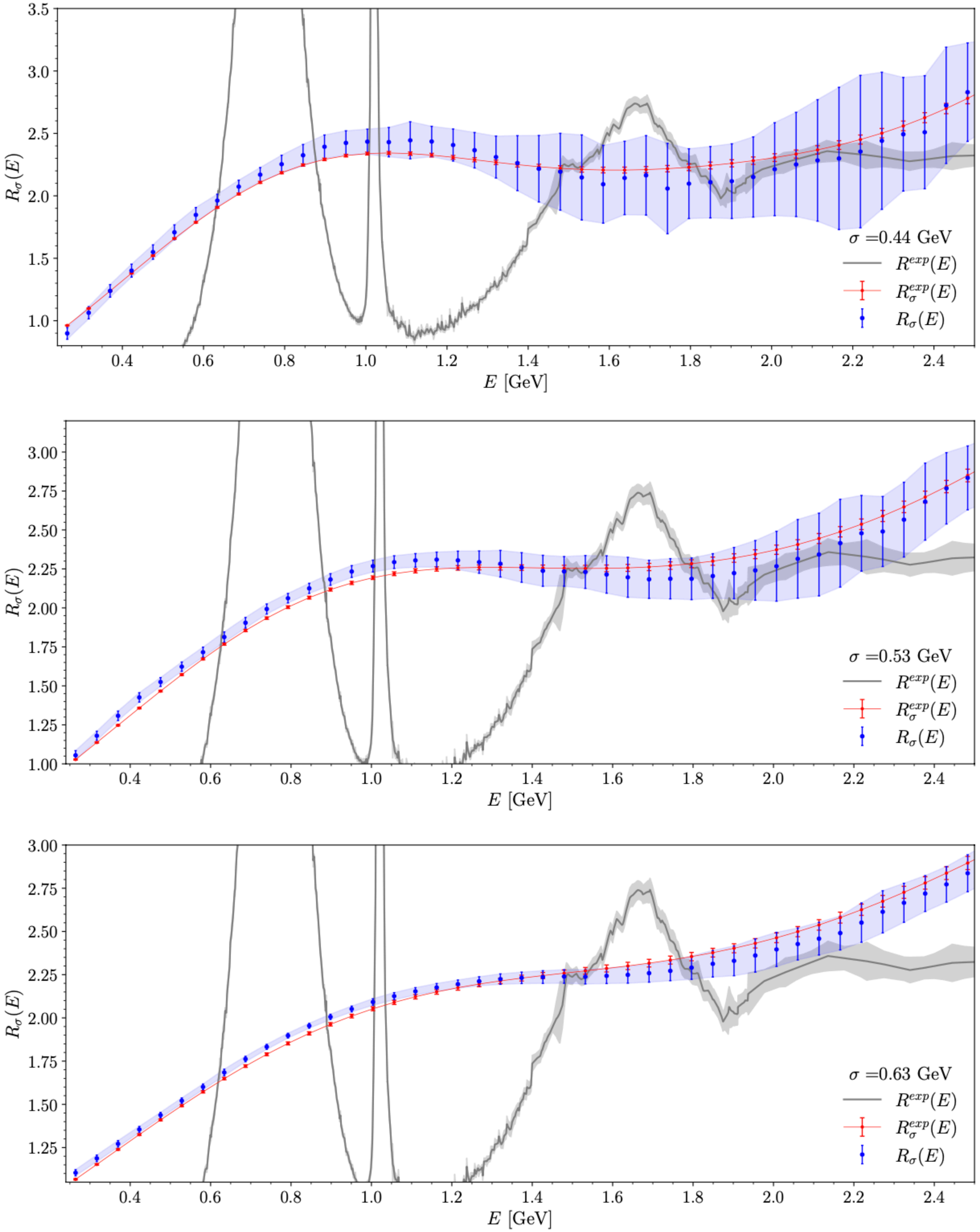
BR and significance



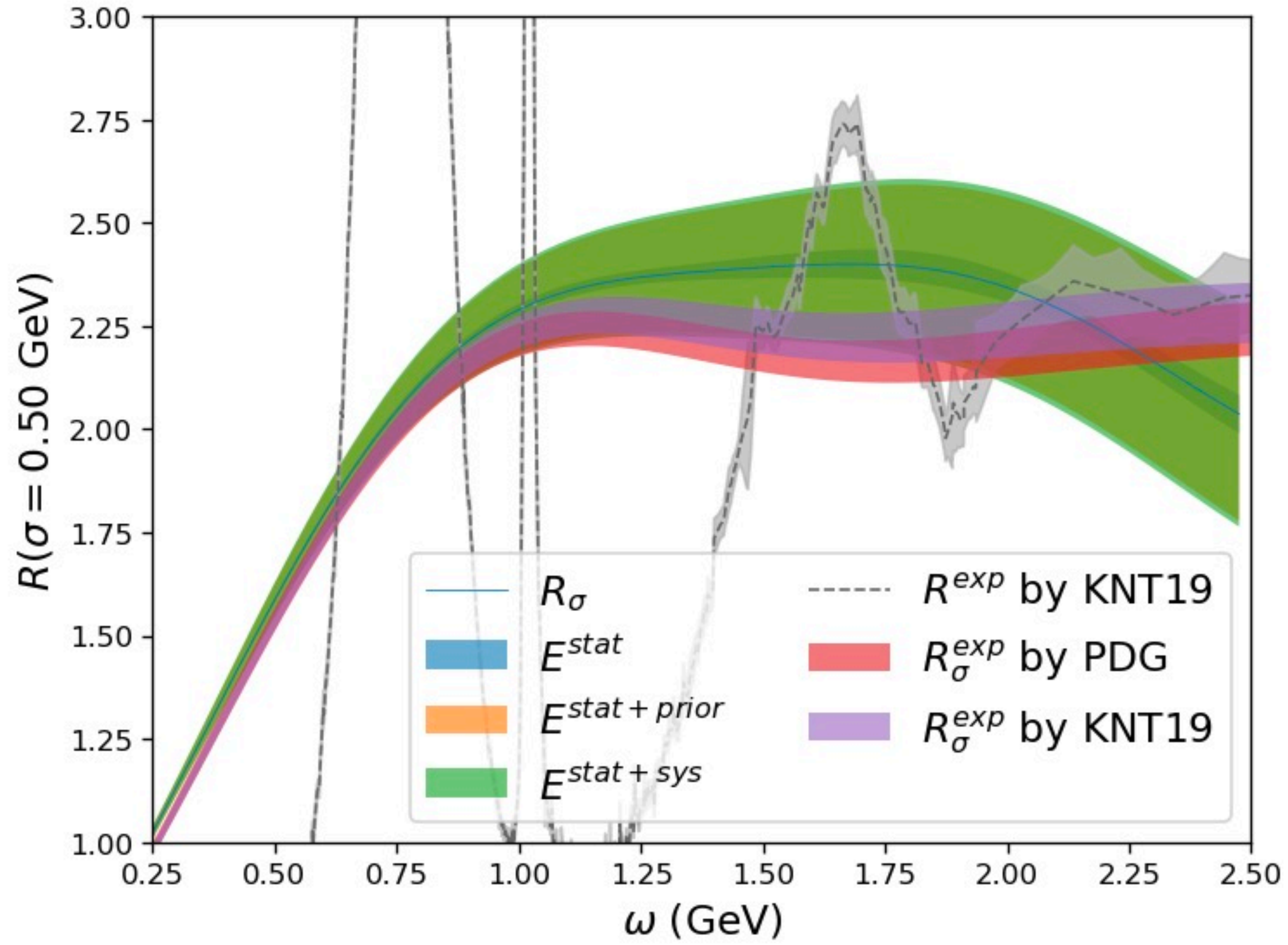




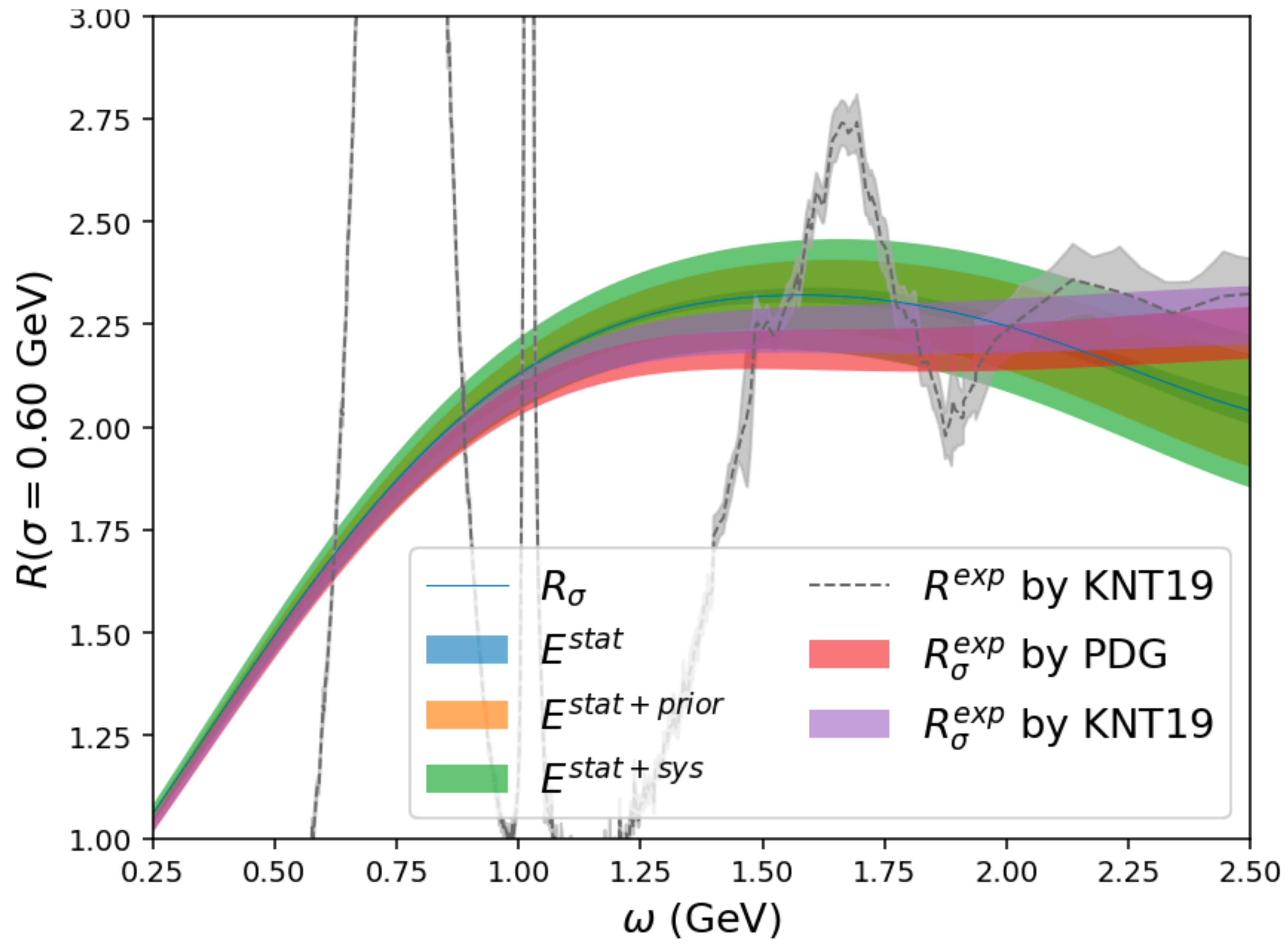
$R(\sigma = 0.60\ \mathrm{GeV})$
ω (GeV)
R_σ
E^{stat}
$E^{stat+prior}$
$E^{stat+sys}$
R^{exp} by KNT19
R^{exp}_σ by PDG
R^{exp}_σ by KNT19
$R(\sigma = 0.70\ \mathrm{GeV})$
ω (GeV)
R_σ
E^{stat}
$E^{stat+prior}$
$E^{stat+sys}$
R^{exp} by KNT19
R^{exp}_σ by PDG
R^{exp}_σ by KNT19



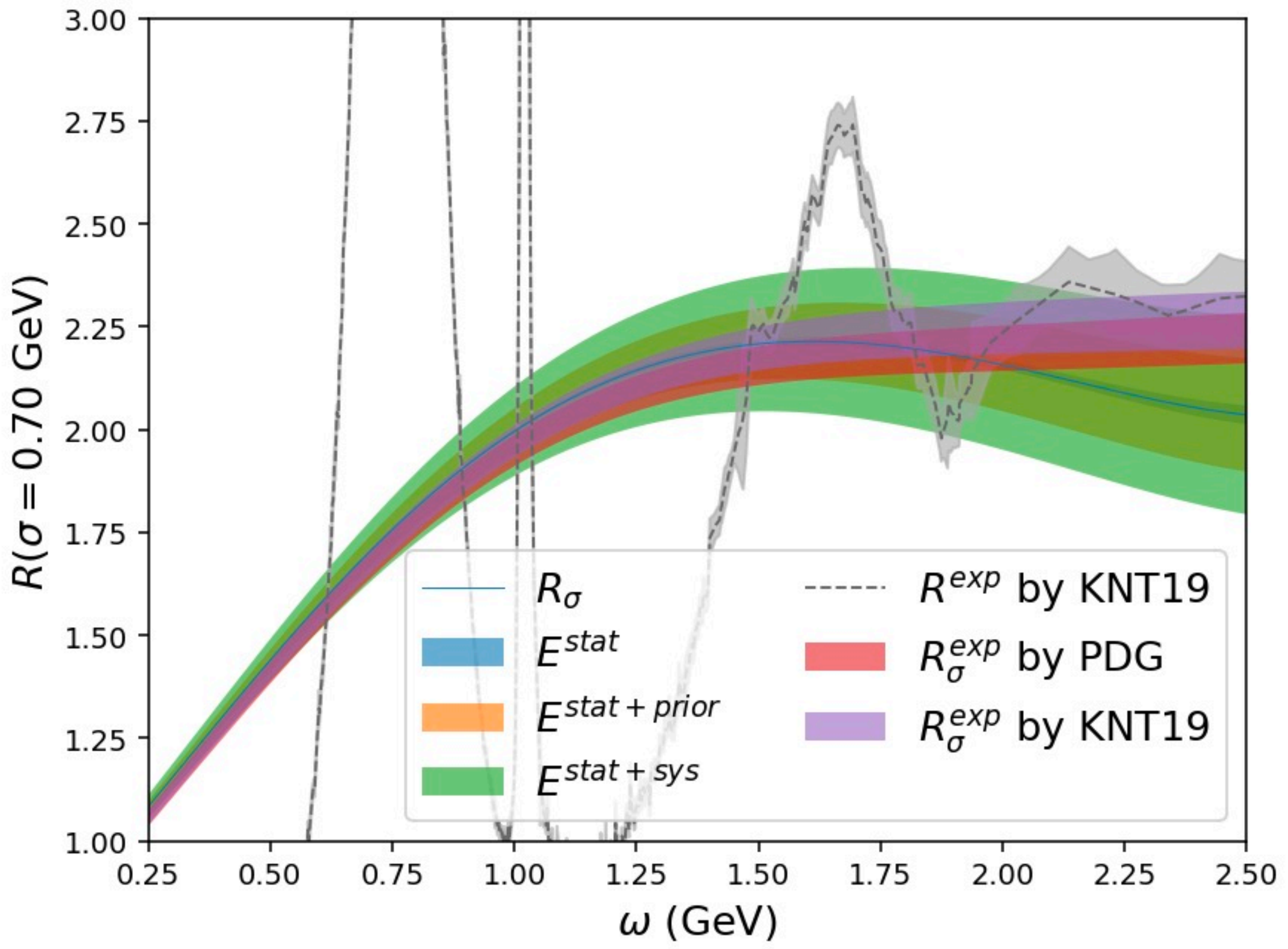
$R_\sigma(E)$
E [GeV]
σ =0.44 GeV
$R^{exp}(E)$
$R^{exp}_\sigma(E)$
$R_\sigma(E)$
$R_\sigma(E)$
E [GeV]
σ =0.53 GeV
$R^{exp}(E)$
$R^{exp}_\sigma(E)$
$R_\sigma(E)$
$R_\sigma(E)$
E [GeV]
σ =0.63 GeV
$R^{exp}(E)$
$R^{exp}_\sigma(E)$
$R_\sigma(E)$



$R(\sigma = 0.50$ GeV)
ω (GeV)
R_σ
E^{stat}
$E^{stat+prior}$
$E^{stat+sys}$
R^{exp} by KNT19
R_σ^{exp} by PDG
R_σ^{exp} by KNT19



$R(\sigma = 0.60$ GeV)
ω (GeV)
R_σ
E^{stat}
$E^{stat+prior}$
$E^{stat+sys}$
R^{exp} by KNT19
R_σ^{exp} by PDG
R_σ^{exp} by KNT19



$R(\sigma = 0.70$ GeV)
ω (GeV)
R_σ
E^{stat}
$E^{stat+prior}$
$E^{stat+sys}$
R^{exp} by KNT19
R_σ^{exp} by PDG
R_σ^{exp} by KNT19