Shear viscosity of the parton matter and application to the AMPT model

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Outline

1) Motivation
2) Isotropic versus forward-angle scatterings
3) Comparison of η and η/s from different methods
4) Application to parton matter in the AMPT model
5) Conclusions

Mostly based on
Noah MacKay & ZWL,
Eur Phys J C 82, 918 (2022)

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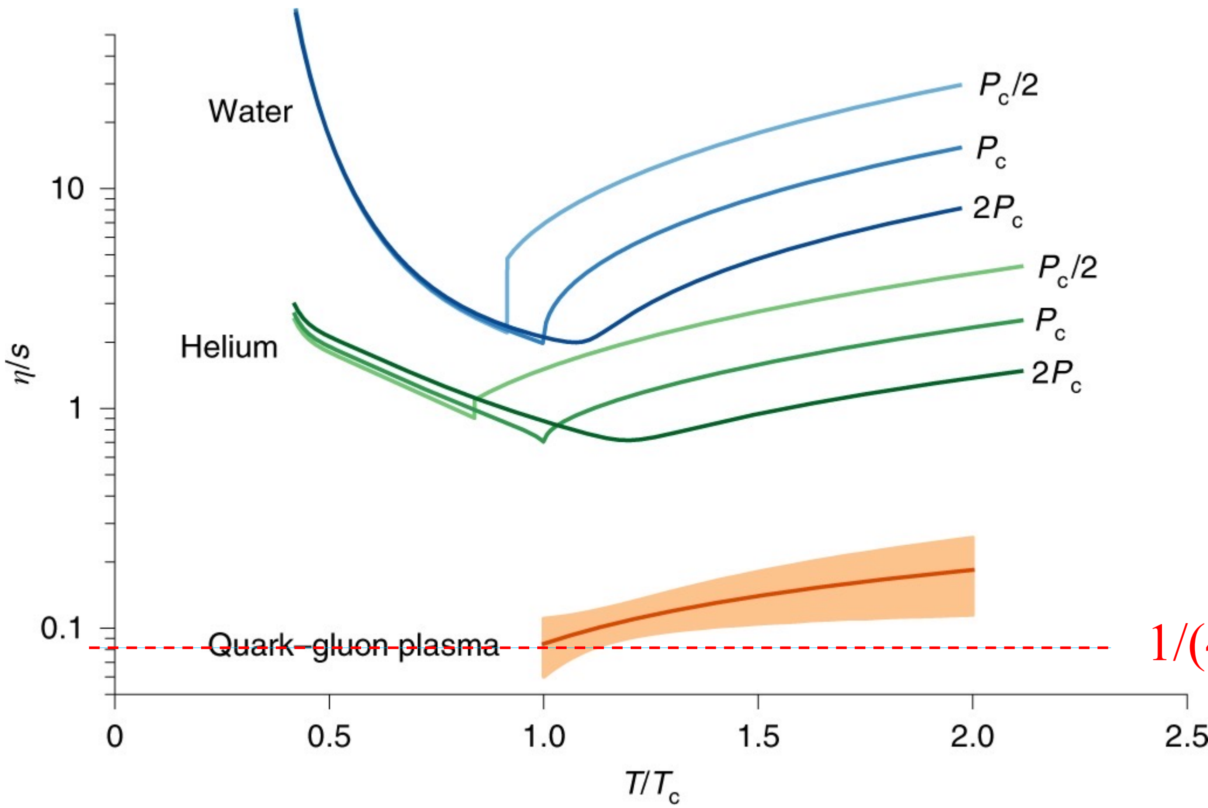
1) Motivation

Shear viscosity η is an important property of the quark–gluon plasma:

$$T^{\mu\nu} = eu^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

η *or* η/s: is an input function to viscous hydrodynamics;
is generated by interactions in kinetic theory: relation?

η/s can be extracted from data/model comparisons:



Bernhard, Moreland & Bass, Nature Phys (2019)

$1/(4\pi)$ bound from AdS/CFT:

Kovtun, Son & Starinets, Phys Rev Lett (2005)

2) Isotropic versus forward-angle two-body scatterings

Here, we only consider a massless parton matter with Boltzmann statistics in thermal equilibrium under 2-to-2 elastic scatterings.

- Isotropic scattering: $$\frac{d\sigma}{d\Omega} = constant = \frac{\sigma}{4\pi}$$

- Forward-angle scattering:
  As the example, we take the parton cross section used in AMPT/ZPC/MPC:

$$\frac{d\sigma}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2}(1+a)\frac{1}{(\hat{t}-\mu^2)^2}$$

$a \equiv \frac{\mu^2}{\hat{s}}$ is added to obtain a $\hat{s}$-independent cross section: $\sigma = \frac{9\pi\alpha_s^2}{2\mu^2}$

This is based on the pQCD gg-gg cross section:

$$\frac{d\sigma}{d\hat{t}} \propto 3 - \frac{\hat{t}\,\hat{u}}{\hat{s}^2} - \frac{\hat{s}\,\hat{u}}{\hat{t}^2} - \frac{\hat{s}\,\hat{t}}{\hat{u}^2}$$ + screening mass μ

2) Isotropic versus forward-angle two-body scatterings

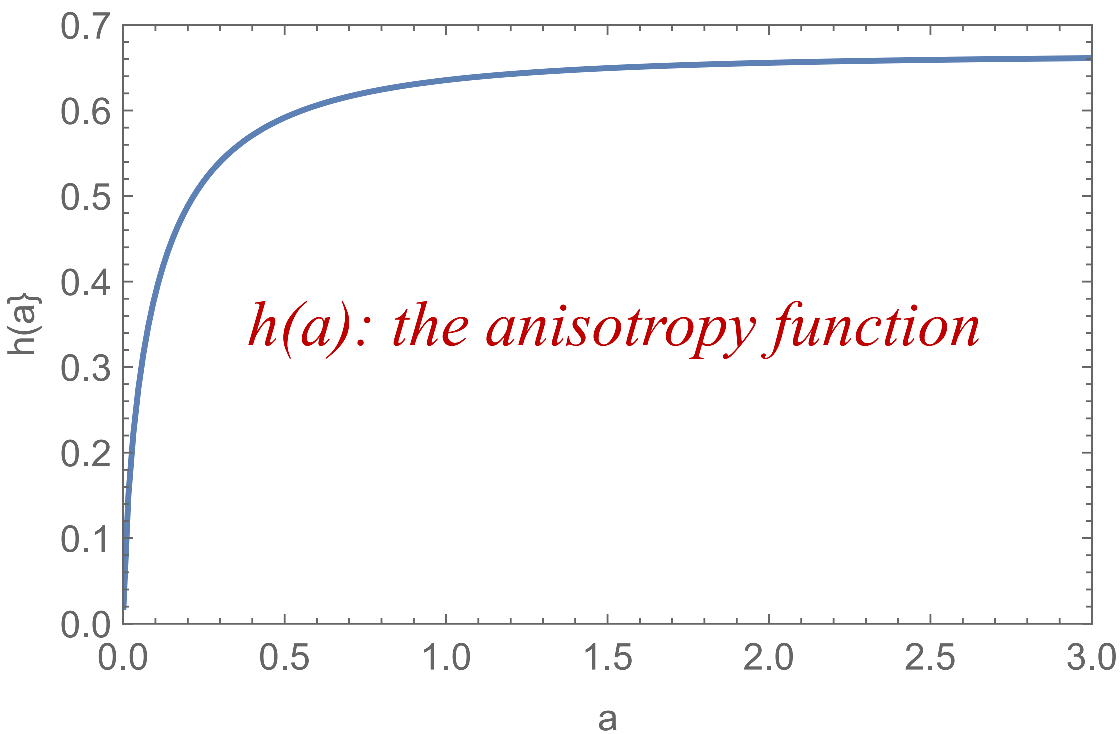
Transport cross section σ_{tr} often appears in shear viscosity expressions:

$$\sigma_{tr} \equiv \int d\sigma \sin^2\theta_{cm}$$

θ_{cm}: scattering angle in the 2-parton CM frame

Molnar & Gyulassy, Nucl Phys A (2002)

- Isotropic scattering: $\sigma_{tr} = \frac{2}{3}\sigma$
- Forward-angle scattering: $\sigma_{tr} = 4a(1+a)\left[(1+2a)\,ln\left(1+\frac{1}{a}\right) - 2\right]\sigma \equiv h(a)\,\sigma$



$a \equiv \frac{\mu^2}{\hat{s}}$

$\ll 1$ very forward, *h(a)*→0: *scatterings are less effective*

$\gg 1$ ~isotropic, *h(a)*→2/3

2) Isotropic versus forward-angle two-body scatterings

Thermal average:

even if σ is a constant, σ_{tr} is not since it depends $a \equiv \frac{\mu^2}{\hat{s}}$.

For a parton matter in thermal equilibrium at temperature T,
the thermal average (for Boltzmann statistics) is Kolb & Raby, Phys Rev D (1983)

$$\langle \sigma_{tr} \rangle = \frac{\sigma}{32} \int_0^\infty du \, [\, u^4 K_1(u) + 2u^3 K_2(u)] \, h\left(\frac{w^2}{u^2}\right)$$
$$\equiv \sigma \, h_0(w)$$

K_n: Bessel functions

$w \equiv \frac{\mu}{T}, u \equiv \frac{\sqrt{\hat{s}}}{T}$

$h_0(w)$ is just an average of the anisotropy function *h(a)*,

$h_0(w) \rightarrow 0$ for $w \ll 1$

$h_0(w) \rightarrow 2/3$ for $w \gg 1$

3) Comparison of η and η/s from different methods

Analytical:

- Israel–Stewart (IS) method: $\eta^{IS} = \frac{6T}{5\sigma}$ for isotropic scatt.

  Huovinen & Molnar, Phys Rev C (2009)

- Navier–Stokes (NS) method: $\eta^{NS} \approx 1.2654 \frac{T}{\sigma}$ for isotropic scatt.

  de Groot, van Leeuwen & Weert book (1980)

- Relaxation time approximation (RTA) & modified version (RTA*):

$$\eta^{RTA} = \frac{4T}{5\sigma} \qquad \eta^{RTA*} = \frac{6T}{5\sigma}$$

for isotropic scatt.

Anderson & Witting, Physica (1974) Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

- Chapman–Enskog (CE) method: $\eta^{CE} = \frac{T\,\gamma_0^2}{10\,c_{00}}, \quad \ldots$

  Wiranata & Prakash, Phys Rev C (2012)

Numerical:

- Green–Kubo relation: $\eta = \frac{V}{T}\int_0^\infty dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$

τ: relaxation time extracted from correlation <…>(t)

3) Comparison of η and η/s from different methods

Analytical:

- Israel–Stewart (IS) method: $\eta^{IS} = \frac{4T}{5\langle\sigma_{tr}\rangle} = \frac{4T}{5\sigma\, h_0(w)}$
- Navier–Stokes (NS) method: $\eta^{NS} \approx 0.8436 \frac{T}{\langle\sigma_{tr}\rangle}$
- Relaxation time approximation (modified version RTA*):

$$\eta^{RTA*} = \frac{4T}{5\langle\sigma_{tr} v_{rel}\rangle}$$

Huovinen & Molnar, Phys Rev C (2009)

generalized to anisotropic scatt. using σ_{tr} instead of σ

Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

- Chapman–Enskog (CE) method: $\eta^{CE} = \frac{T\,\gamma_0^2}{10\, c_{00}}, \quad \ldots$

Wiranata & Prakash, Phys Rev C (2012)

Numerical:

- Green–Kubo relation: $\eta = \frac{V}{T}\int_0^\infty dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$

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3) Comparison of η and η/s from different methods

Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)
MacKay & ZWL, Eur Phys J C (2022)

More on analytical methods:

- Relaxation time approximation (modified version RTA*):

$$\eta^{RTA*} = \frac{4T}{5\langle\sigma_{tr} v_{rel}\rangle} \quad \rightarrow \frac{4T}{5\sigma h_1(w)}$$

$$\langle\sigma_{tr} v_{rel}\rangle = \frac{8z}{K_2^2(z)} \int_1^\infty dy\, y^2 (y^2-1) K_1(2zy) \int d\sigma \sin^2\theta_{cm}$$

in general for massive partons ($z \equiv m/T$)

$$\rightarrow \frac{\sigma}{16} \int_0^\infty du\, u^4 K_1(u)\, h\left(\frac{w^2}{u^2}\right) \equiv \sigma h_1(w)$$

for massless partons & AMPT $\frac{d\sigma}{d\hat{t}}$

- Chapman–Enskog (CE) method: $\eta^{CE} = \dfrac{T\gamma_0^2}{10\, c_{00}} \quad \rightarrow \dfrac{4T}{5\sigma h_2(w)}$

$$\frac{8c_{00}}{\gamma_0^2} = \frac{32z^3}{25K_3^2(z)} \int_1^\infty dy (y^2-1)^3 \left[\left(y^2 + \frac{1}{3z^2}\right) K_3(2zy) - \frac{y}{z} K_2(2zy)\right] \int d\sigma \sin^2\theta_{cm}$$

in general

$$\rightarrow \frac{\sigma}{6400} \int_0^\infty du\, u^6 \left[\left(\frac{u^2}{4} + \frac{1}{3}\right) K_3(u) - \frac{u}{2} K_2(u)\right] h\left(\frac{w^2}{u^2}\right) \equiv \sigma h_2(w)$$

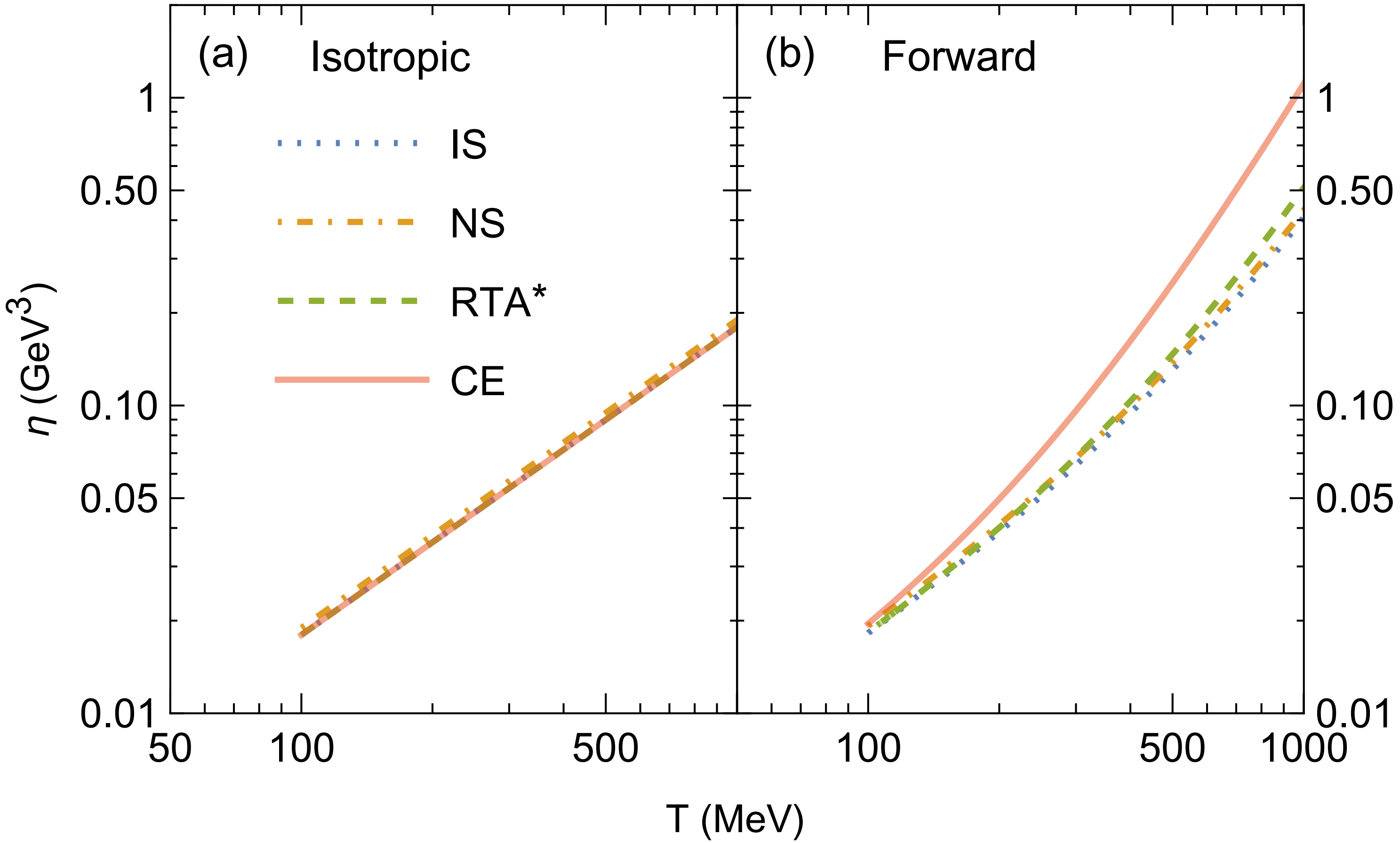
for massless partons & AMPT $\frac{d\sigma}{d\hat{t}}$

$h_1(w)$ & $h_2(w)$ are different averages of the anisotropy function *h(a)*

3) Comparison of η and η/s from different methods

Analytical results of η

for massless gluons & σ=2.6 mb (or μ~0.7GeV at $\alpha_s \approx$0.47):



- For isotropic scatterings:

IS=RTA*=CE
≈NS (~5% higher)

- For forward scatterings:

IS≈RTA*≈NS < CE *mostly*

T<<μ → almost isotropic

3) Comparison of η and η/s from different methods

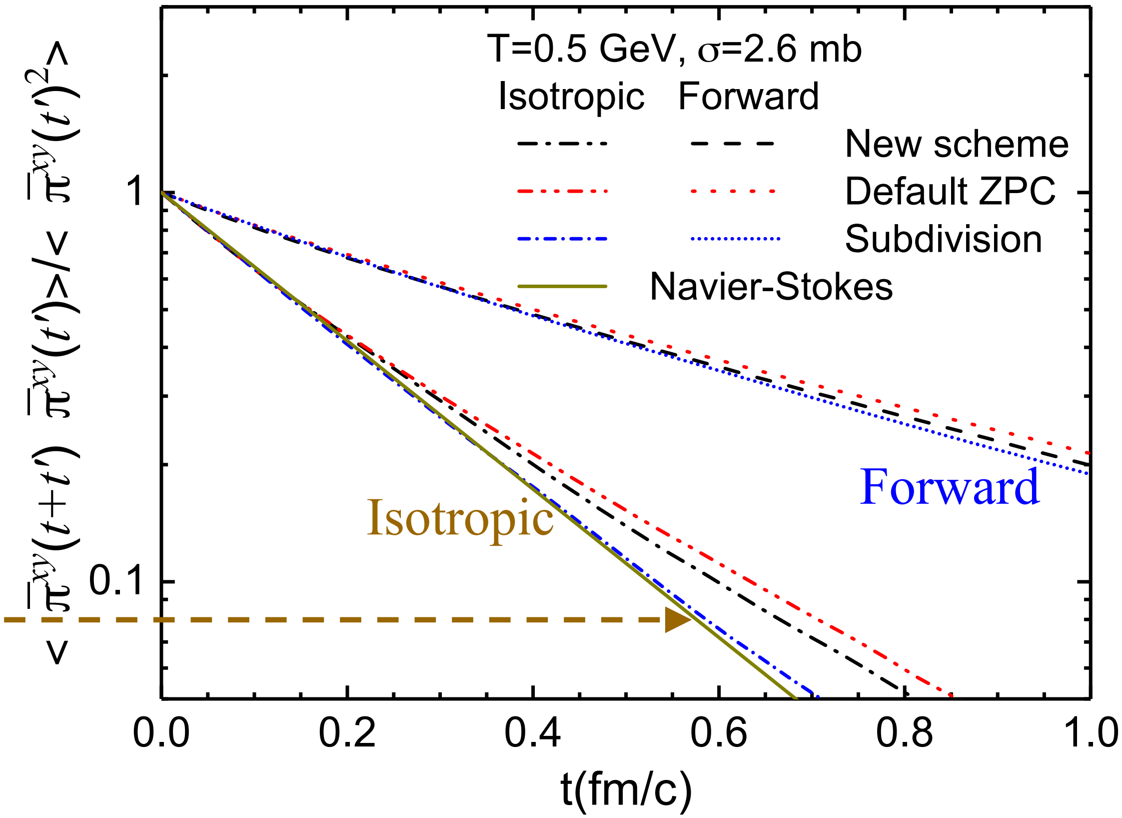
Q: which analytical result of η is accurate?

A: compare with numerical results from Green-Kubo:

$$\eta = \frac{V}{T}\int_0^{\infty} dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$$

With ZPC parton cascade, we have calculated η of gluons in a box with Green-Kubo relation for 3 cases: *new collision scheme, default ZPC collision scheme, parton subdivision.*

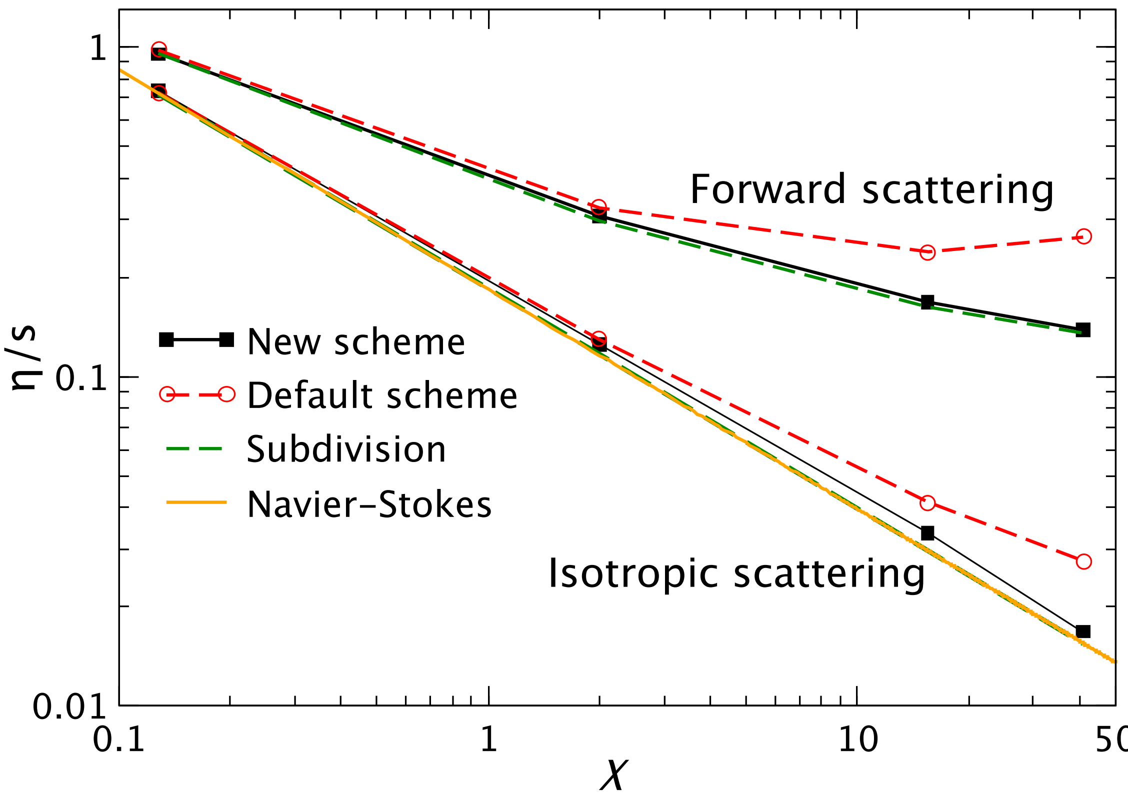
Subdivision (*with factor $l=10^6$*): results should be accurate (*no causality violation from cascade*), agree well with the NS expectation for isotropic scatterings.



Zhao, Ma, Ma & ZWL, Phys Rev C (2020)

3) Comparison of η and η/s from different methods

We have extracted η/s of gluons in a box versus χ with the Green-Kubo relation for the 3 cases:



Zhao, Ma, Ma & ZWL, Phys Rev C (2020)

- χ (opacity parameter)

≡ radius of interaction / mean free path

$$\chi = \sqrt{\frac{\sigma}{\pi}}/\lambda = n\sqrt{\frac{\sigma^3}{\pi}}$$

Zhang, Gyulassy & Pang, Phys Rev C (1998)

- For fixed α_s, η/s is only a function of χ.

For example:

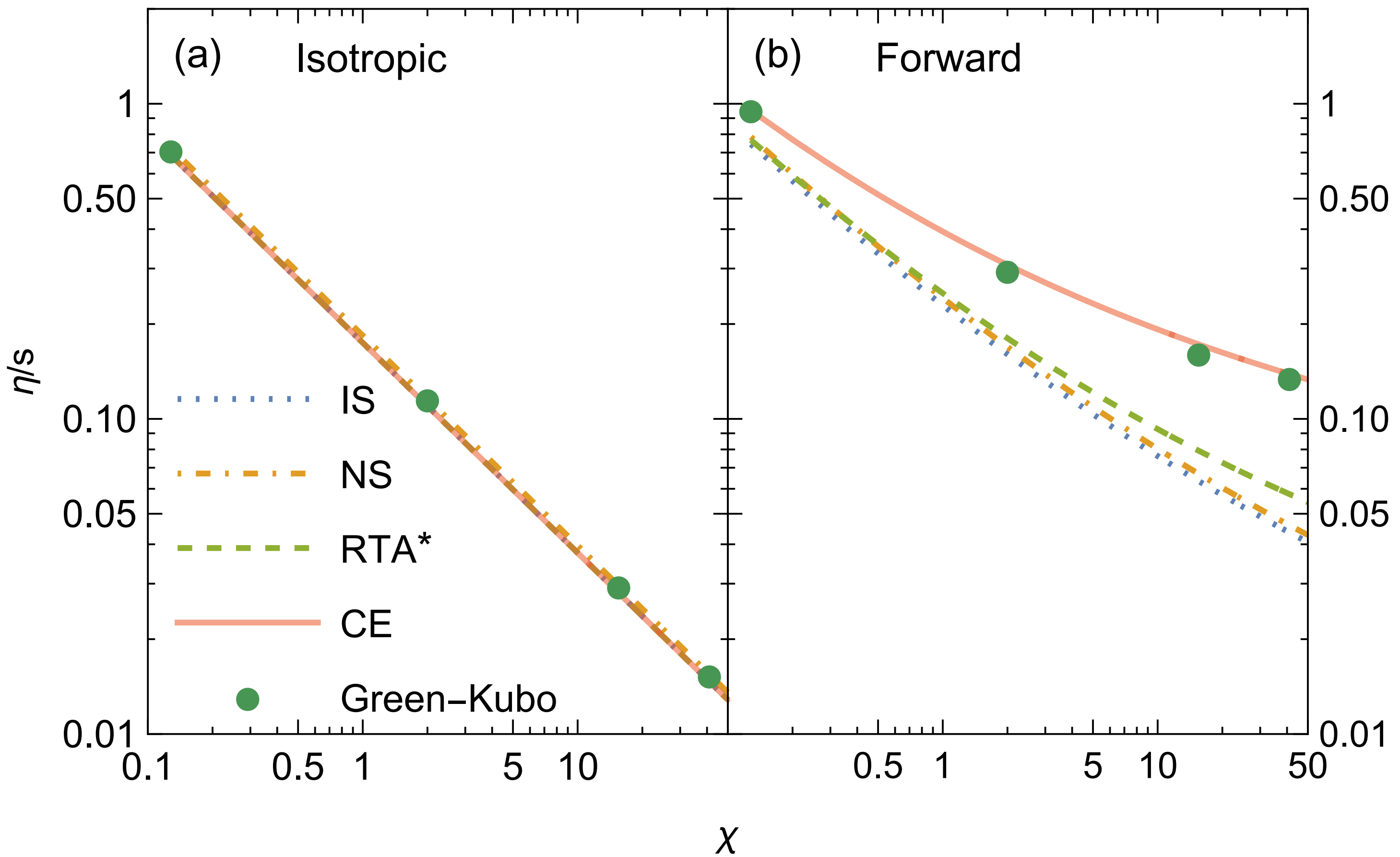
$$\left(\frac{\eta}{s}\right)^{NS} \simeq \frac{0.4633}{d_g^{1/3}\chi^{2/3}} = \frac{0.1839}{\chi^{2/3}}$$

for gluons (d_g=16) under isotropic scatterings

3) Comparison of η and η/s from different methods

Compare 4 analytical methods with subdivision Green-Kubo results for η/s versus χ:

MacKay & ZWL, Eur Phys J C (2022)



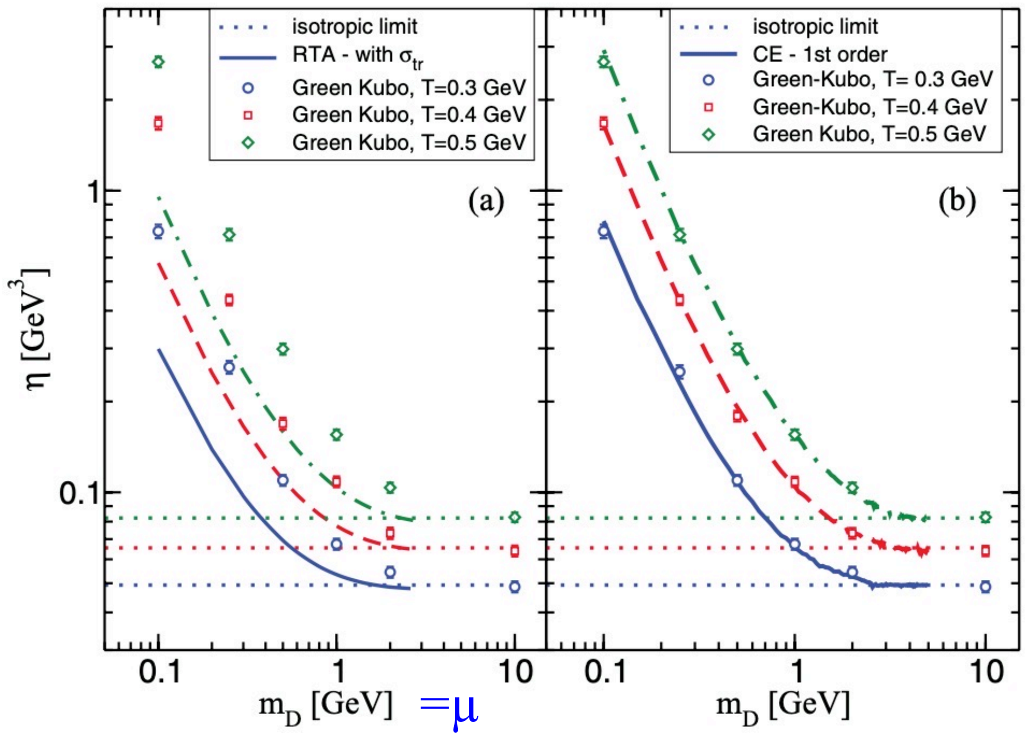
- For isotropic scatterings: all methods agree well.
- For anisotropic scatterings: CE agrees well with Green-Kubo; but the other analytical methods are not accurate.

3) Comparison of η and η/s from different methods

The fact that Green-Kubo agrees with CE (but not with RTA*) has been shown in

Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

despite two typos in the η formulae.



- Relaxation time approximation (modified version RTA*):

$$\langle h(a)\, v_{\rm rel}\rangle = \frac{8z}{K_2^2(z)}\int_1^\infty dy\, y^2\,(y^2-1)\, h(2zy\,\bar a)\, K_1(2zy) = f(z,\bar a), \quad (35)$$

$$\eta^*_{\rm RTA} = 0.8\,\frac{1}{f(z,\frac{T}{m_D})}\,\frac{T}{\sigma_{\rm tot}}, \quad (36)$$

- Chapman–Enskog (CE) method:

$$[\eta_s]^I_{CE} = 0.8\,\frac{1}{g(z,\bar a)}\,\frac{T}{\sigma_{\rm tot}}, \quad (37)$$

with

$$g(z,\bar a) = \frac{32}{25}\,\frac{z}{K_3^2(z)}\int_1^\infty dy\,(y^2-1)^3\, h(2zy\,\bar a) \times [(z^2y^2+1/3)K_3(2zy) - zyK_2(2zy)]. \quad (38)$$

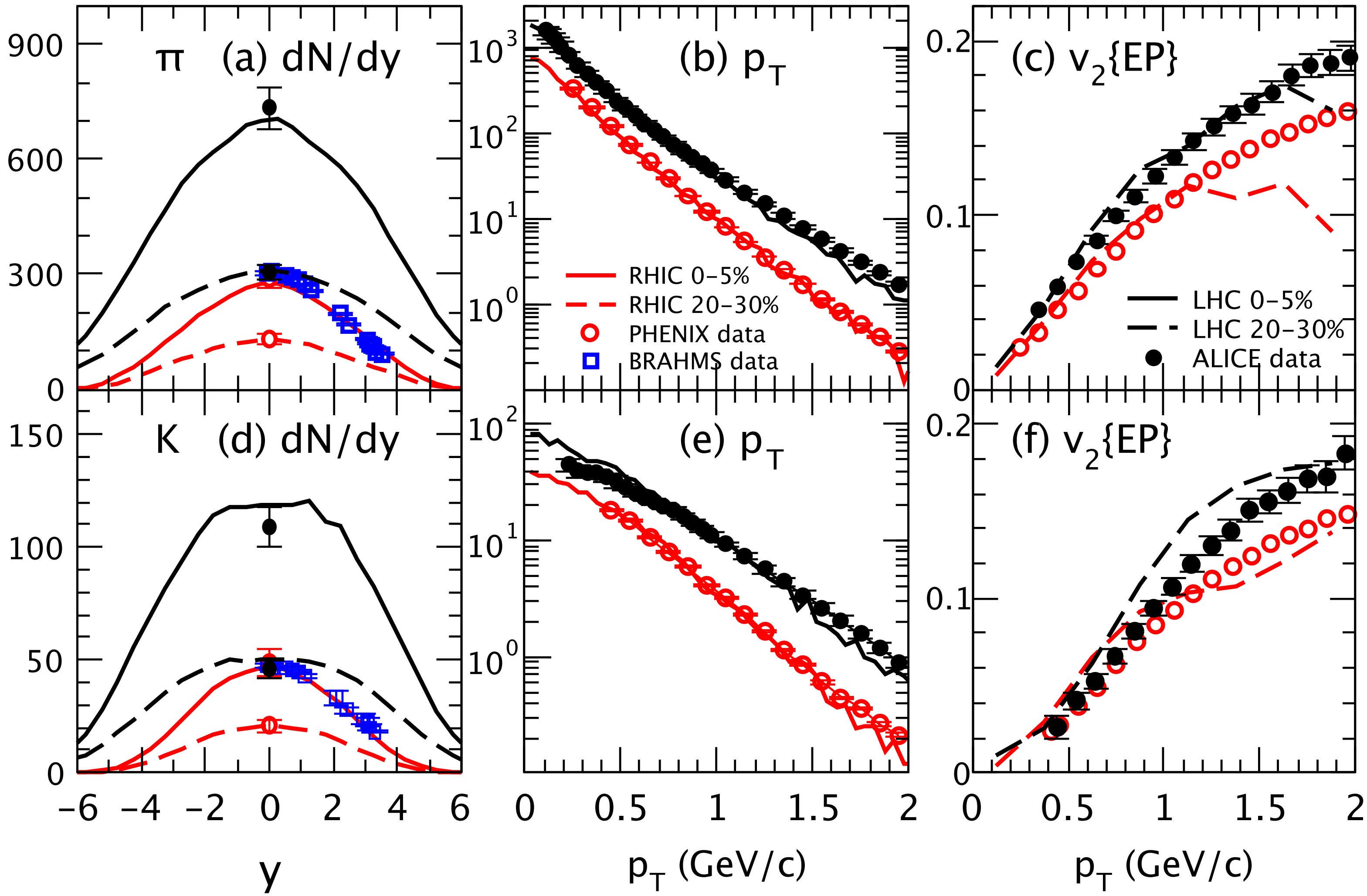
should be $h(1/(2zy\bar a)^2)$

4) Application to parton matter in the AMPT model

We now apply the Chapman–Enskog (CE) method to study η and η/s of the parton matter in the string melting AMPT model for A+A.

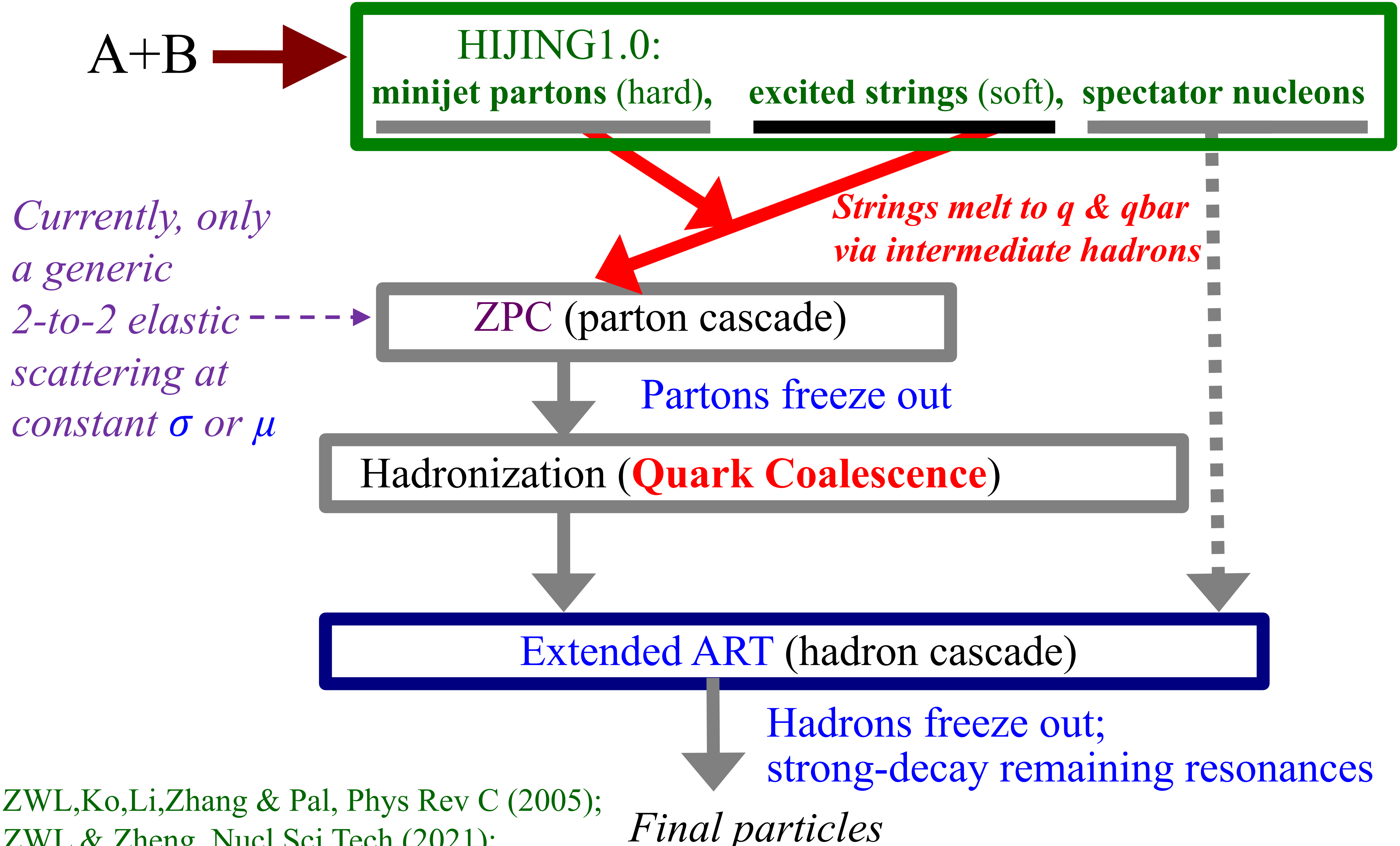
The kinetic-theory based AMPT model can reasonably describe the bulk matter observables at low p_T in A+A collisions:

ZWL, Phys Rev C (2014)



4) Application to parton matter in the AMPT model

Structure of the String Melting version of AMPT:



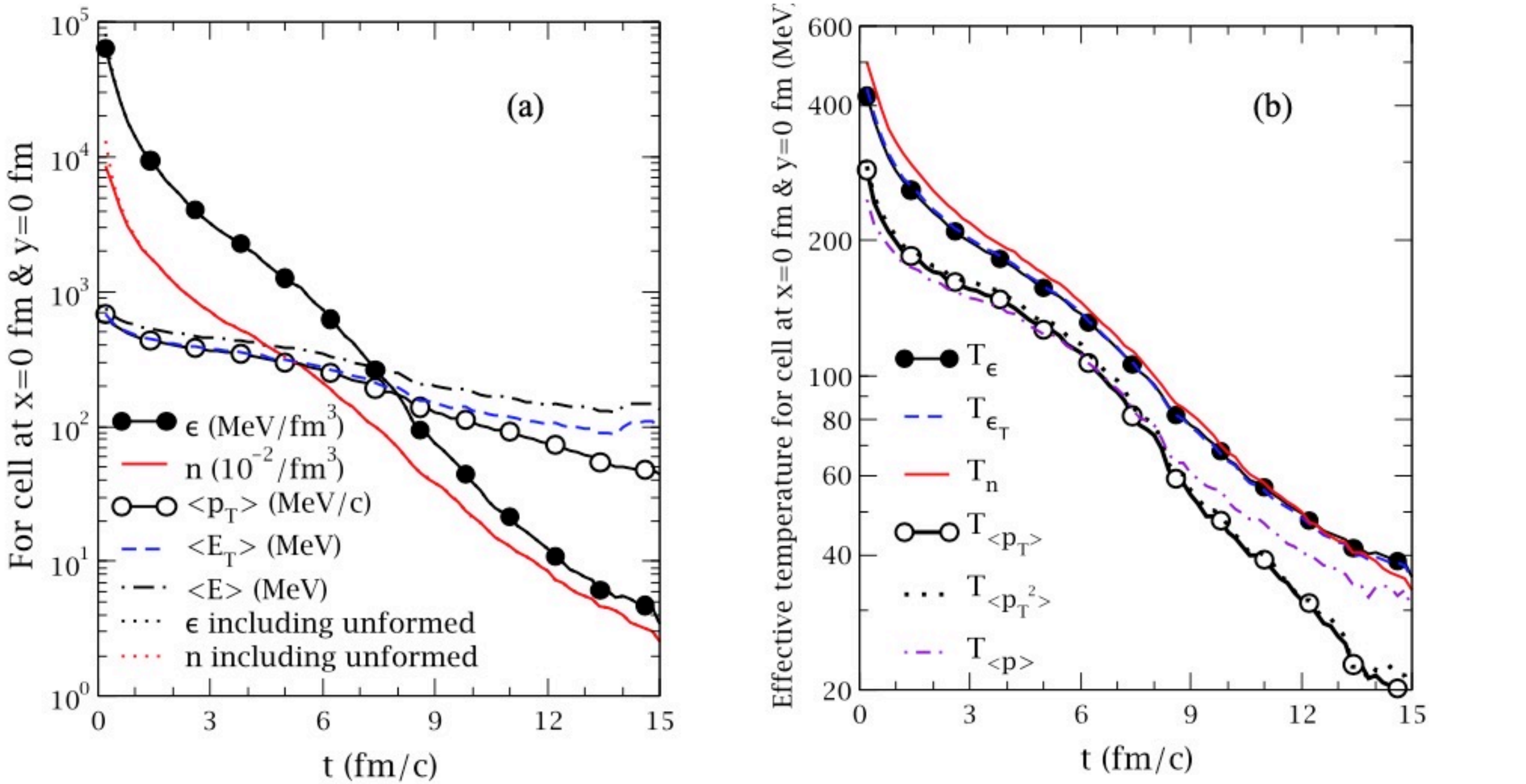
ZWL,Ko,Li,Zhang & Pal, Phys Rev C (2005);
ZWL & Zheng, Nucl Sci Tech (2021);
https://myweb.ecu.edu/linz/ampt/

4) Application to parton matter in the AMPT model

For parton matter in the center cell,
we have extracted the effective temperatures.

ZWL, Phys Rev C (2014)

For example, central Au+Au at 200A GeV:



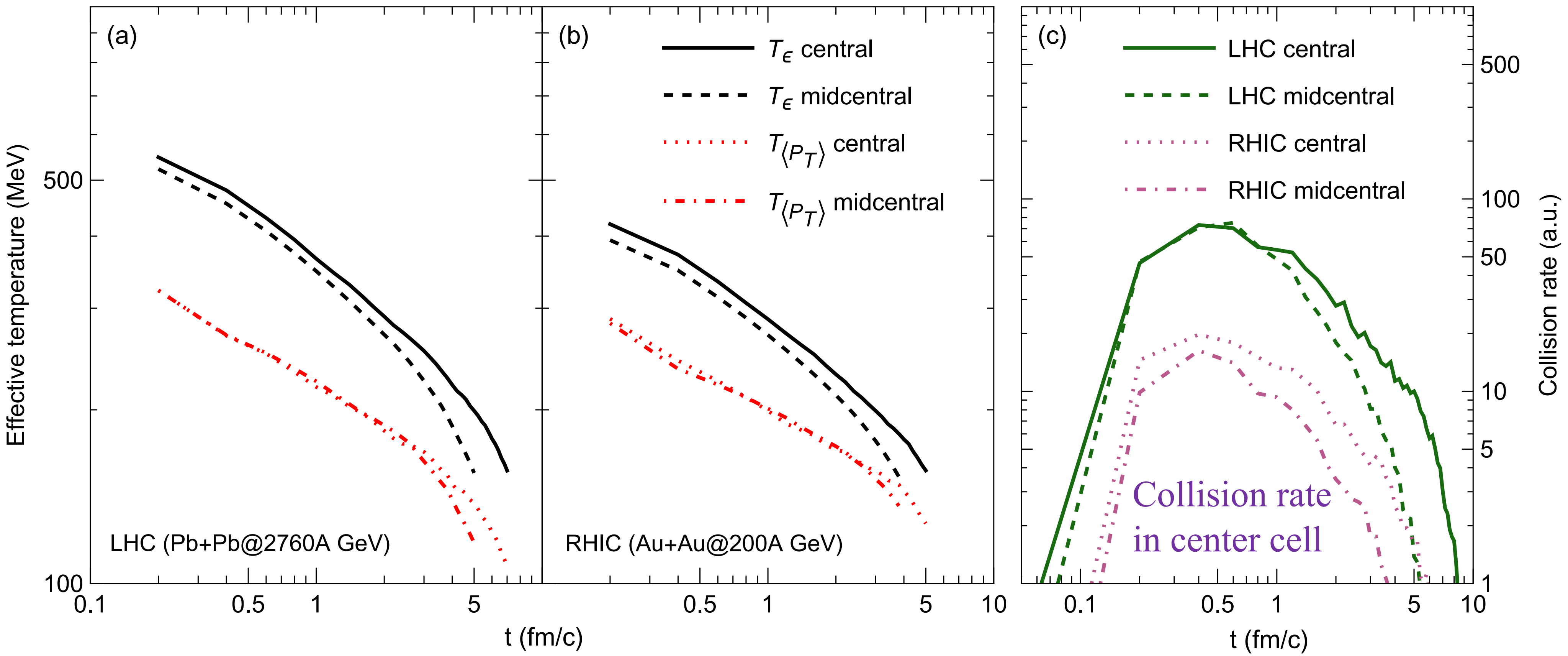
using $\varepsilon = \frac{3g_B}{\pi^2} T_\varepsilon^4, \quad T_{\langle p_T \rangle} = \frac{4}{3\pi}\langle p_T \rangle, \quad \ldots$

$T_{\langle p_T \rangle} < T_\varepsilon$ → the parton matter is not in chemical equilibrium.

4) Application to parton matter in the AMPT model

We have extracted effective temperatures $T_{\langle p_T \rangle}$ & T_ε of the center cell for 4 different collision systems:

ZWL, Phys Rev C (2014)



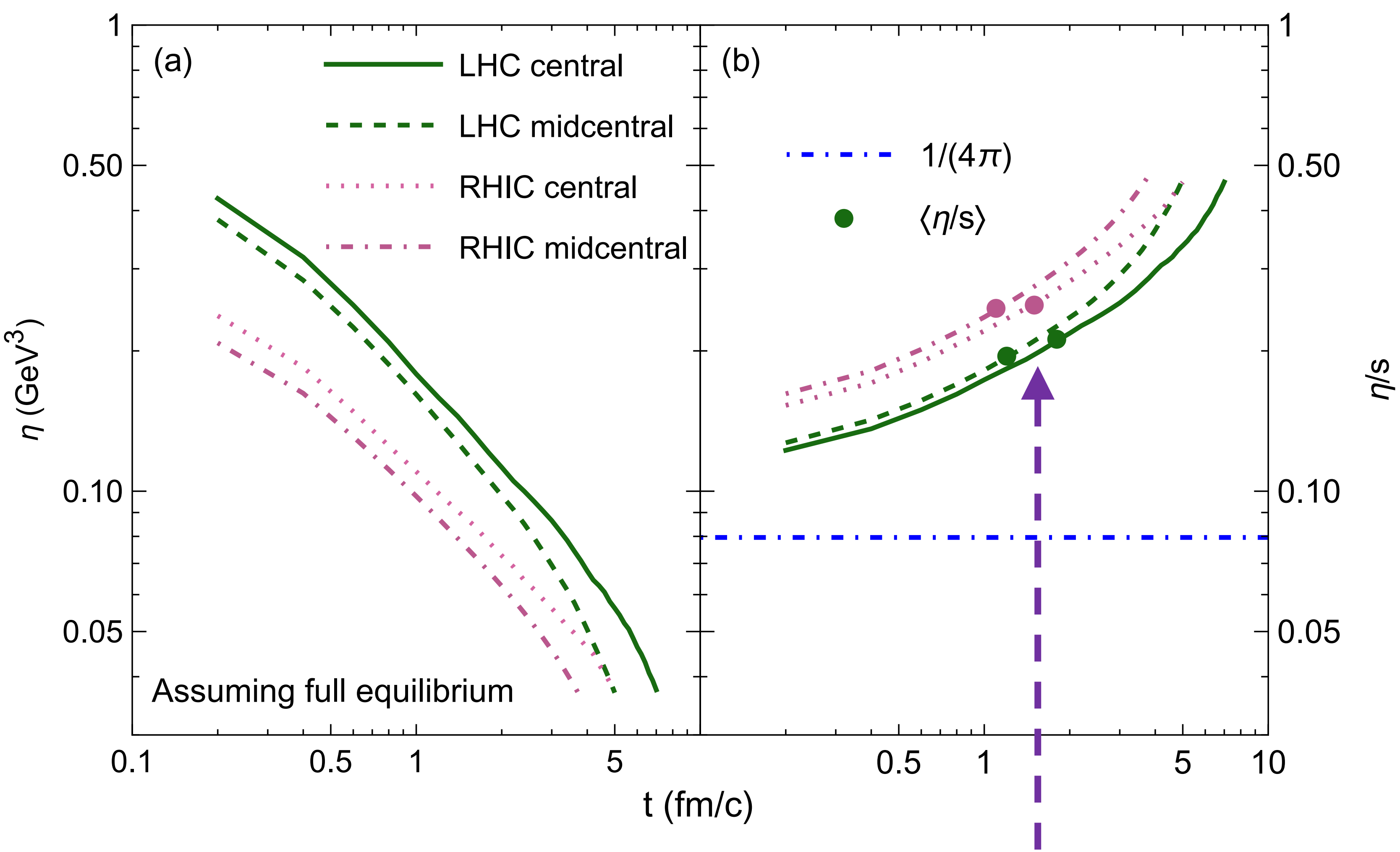
MacKay & ZWL, Eur Phys J C (2022)

We use these temperatures to calculate η and η/s of the center cell

(a volume around mid-pseudorapidity with 1 fm² in transverse area)

4) Application to parton matter in the AMPT model

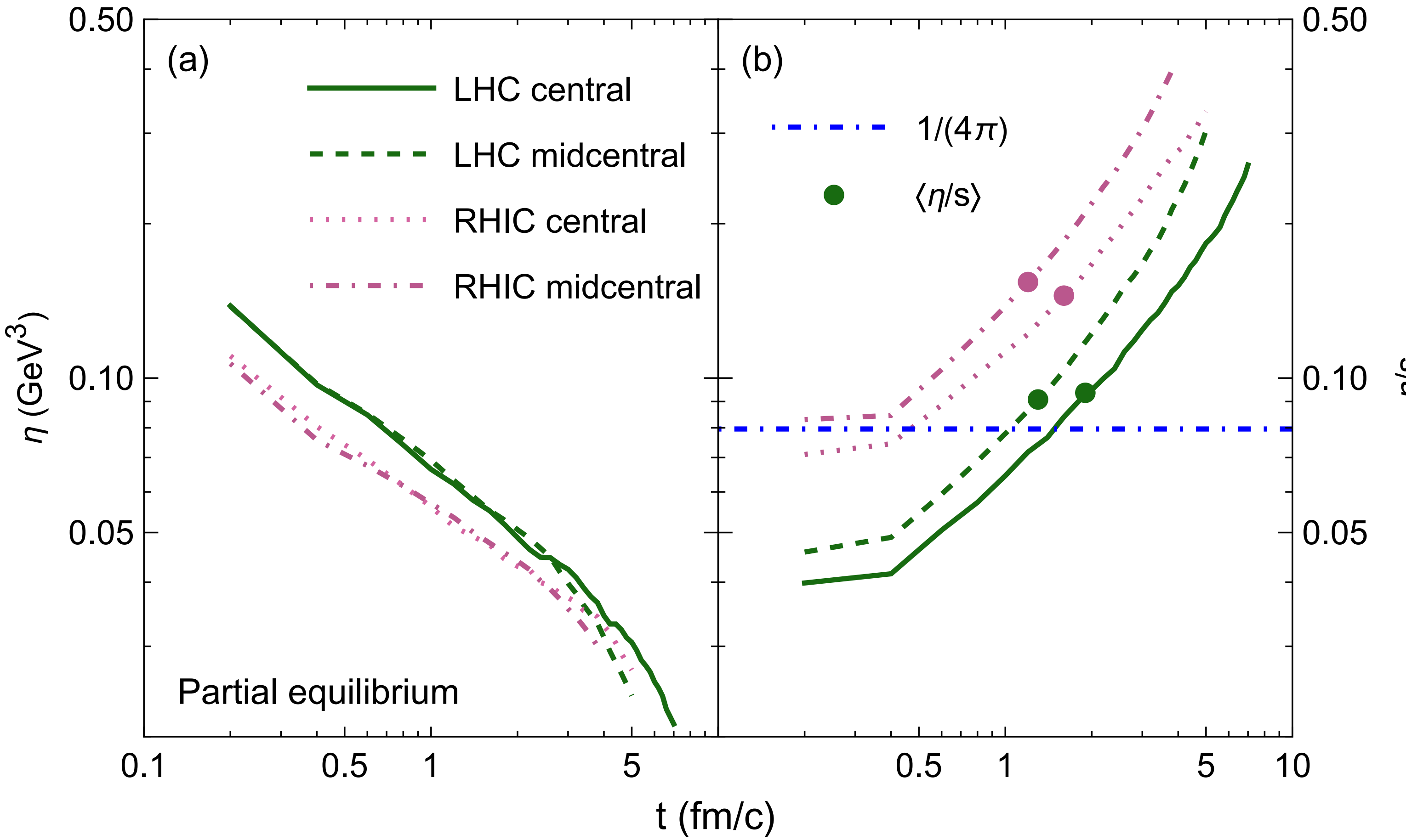
A) When treating the matter as a QGP in full equilibrium (N_f=3), we use temperature T_ε to calculate both η and s:



- Time-averaged value weighted by collision rates: $<\eta/s>$ is quite small.
- Temperature dependence of η/s is "wrong", due to constant σ

4) Application to parton matter in the AMPT model

B) When treating the matter as a QGP in partial chemical equilibrium, we use temperature $T_{\langle p_T \rangle}$ to calculate η but use T_ε to calculate s, since η is determined by momentum transfer but not density:



$<\eta/s>$ is even smaller

- η & η/s are lower in partial equilibrium due to $T_{\langle p_T \rangle} < T_\varepsilon$: lower T (*at constant* μ) makes scattering more isotropic and effective

4) Application to parton matter in the AMPT model

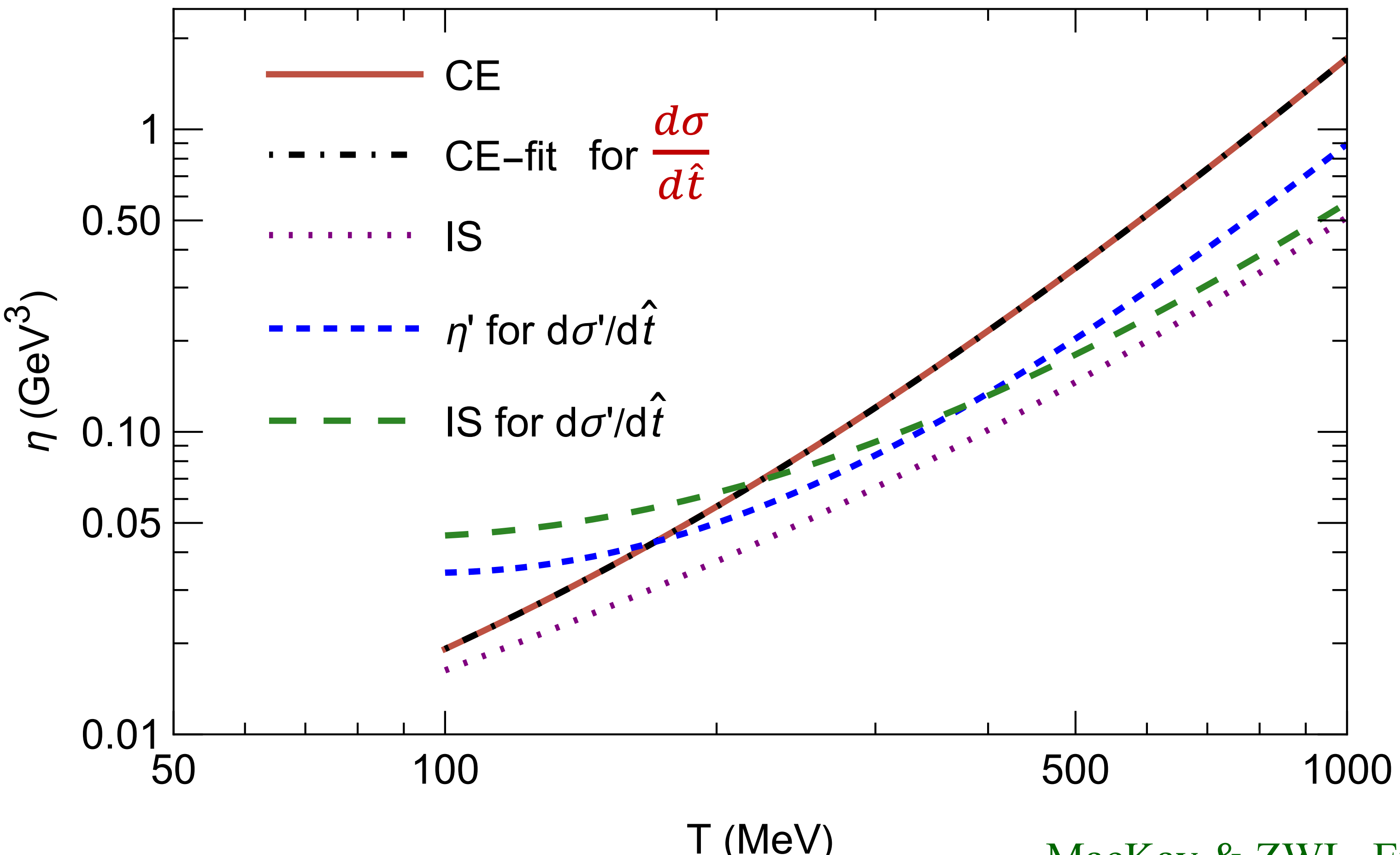
- Our results will improve previous calculations of η for parton matter, such as

$$\eta' = \frac{4\,T^3}{5\pi\alpha_s^2\left[\left(1+\frac{\mu^2}{9T^2}\right)\ln\left(1+\frac{18T^2}{\mu^2}\right)-2\right]}$$

in Magdy et al., Eur Phys J C (2021)

based on IS, using $\frac{d\sigma'}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2}\frac{1}{(\hat{t}-\mu^2)^2}$

& approximating $\hat{s}$ with $\langle\hat{s}\rangle = 18T^2$



Note: AMPT uses

$$\frac{d\sigma}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2}\frac{1+a}{(\hat{t}-\mu^2)^2}$$

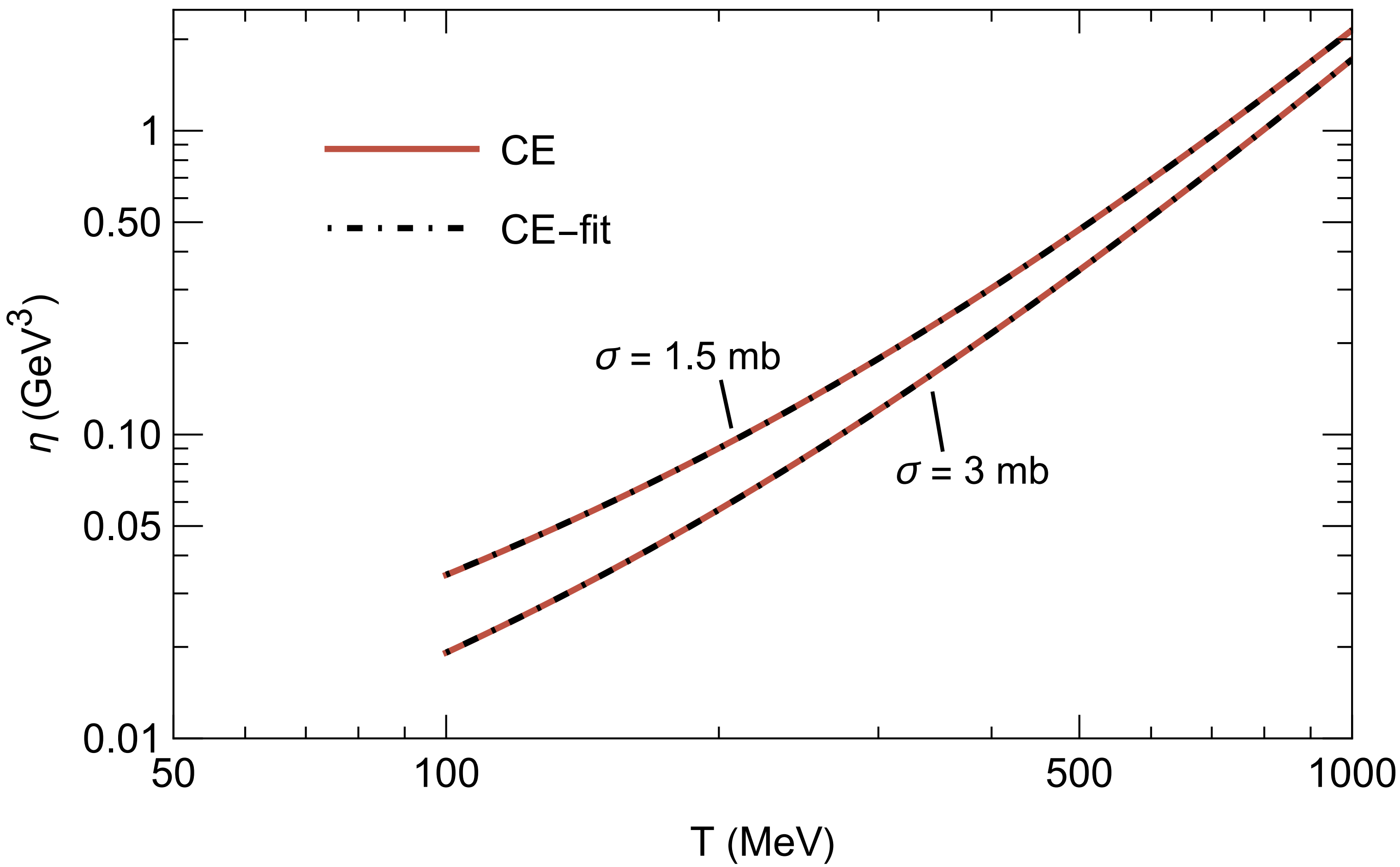
MacKay & ZWL, Eur Phys J C (2022)

4) Application to parton matter in the AMPT model

- The preferred σ value for high energy A+A collisions from AMPT is smaller (3mb → 1.5mb) when the new quark coalescence model is used.

→ increase of η & η/s

He & ZWL, Phys Rev C (2017); Eur Phys J A (2020)



from Noah MacKay

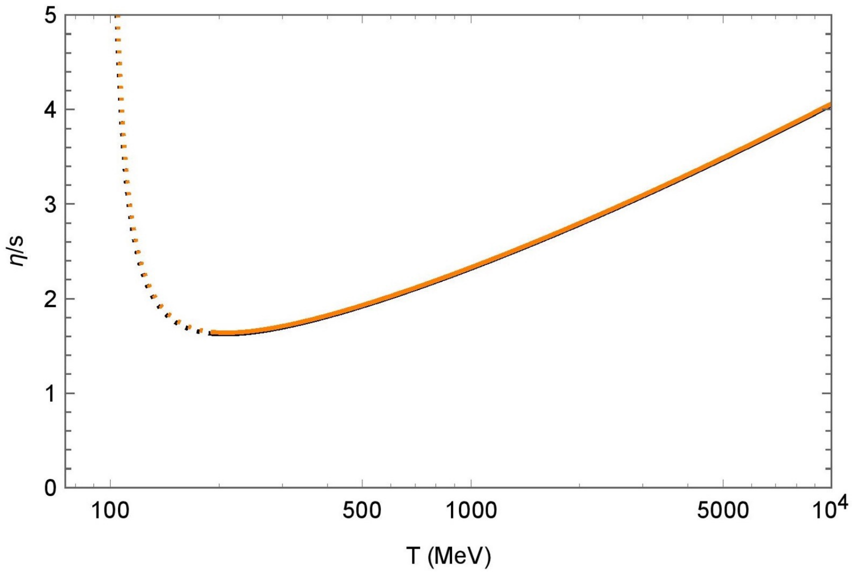
- CE-fit: we can use a fit function to calculate $\eta^{CE} = \frac{4T}{5\sigma h_2(w)}$ for forward scattering, to avoid the integral in $h_2(w)$

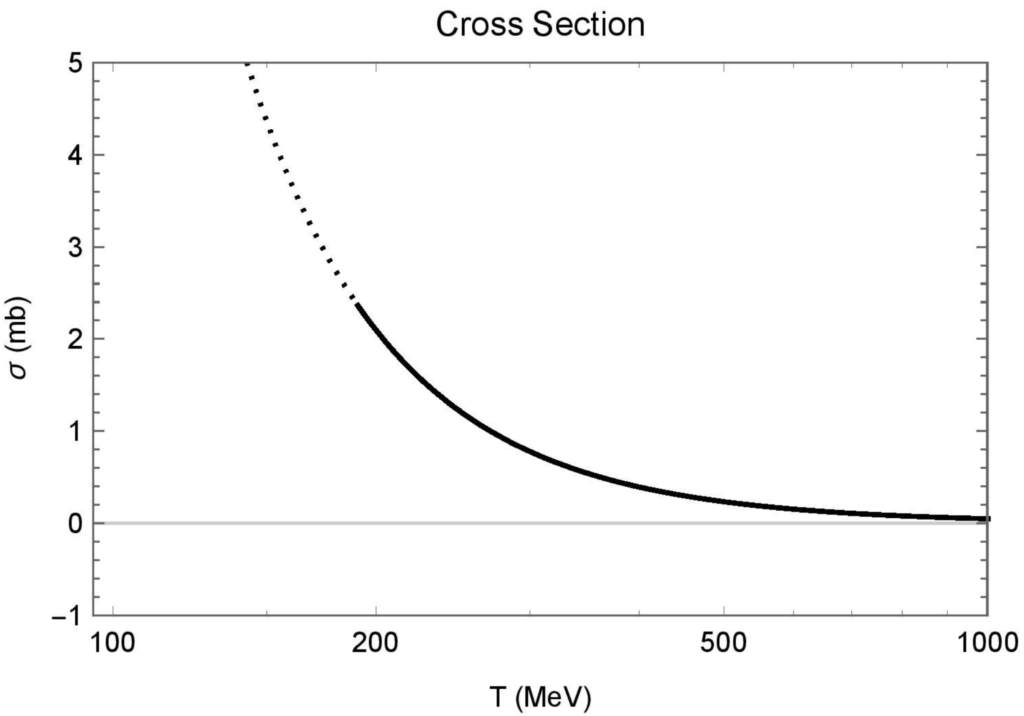
4) Application to parton matter in the AMPT model

pQCD
with $\mu \propto gT$:

$$\frac{\eta}{s} \approx \frac{5.12}{g^4 \ln(2.42/g)}$$

Arnold, Moore & Yaffe, JHEP (2003);
Csernai, Kapusta & McLerran, Phys Rev Lett (2006)





If we use $\mu \propto gT$

→ $\sigma \propto 1/\mu^2$ will be larger at lower T

→ $\eta/s \propto T/\sigma$ will have the expected T- and t-dependences

→ a direction to improve ZPC/AMPT

4) Application to parton matter in the AMPT model

$ab \leftrightarrow cd$	$\left\|\mathcal{M}^{ab}_{cd}\right\|^2/g^4$
$q_1q_2 \leftrightarrow q_1q_2$, $q_1\bar{q}_2 \leftrightarrow q_1\bar{q}_2$, $\bar{q}_1q_2 \leftrightarrow \bar{q}_1q_2$, $\bar{q}_1\bar{q}_2 \leftrightarrow \bar{q}_1\bar{q}_2$	$8\,\frac{d_{\rm F}^2\,C_{\rm F}^2}{d_{\rm A}}\left(\frac{s^2+u^2}{\underline{t^2}}\right)$
$q_1q_1 \leftrightarrow q_1q_1$, $\bar{q}_1\bar{q}_1 \leftrightarrow \bar{q}_1\bar{q}_1$	$8\,\frac{d_{\rm F}^2\,C_{\rm F}^2}{d_{\rm A}}\left(\frac{s^2+u^2}{\underline{t^2}}+\frac{s^2+t^2}{\underline{u^2}}\right)+16\,d_{\rm F}\,C_{\rm F}\left(C_{\rm F}-\frac{C_{\rm A}}{2}\right)\frac{s^2}{tu}$
$q_1\bar{q}_1 \leftrightarrow q_1\bar{q}_1$	$8\,\frac{d_{\rm F}^2\,C_{\rm F}^2}{d_{\rm A}}\left(\frac{s^2+u^2}{\underline{t^2}}+\frac{t^2+u^2}{s^2}\right)+16\,d_{\rm F}\,C_{\rm F}\left(C_{\rm F}-\frac{C_{\rm A}}{2}\right)\frac{u^2}{st}$
$q_1\bar{q}_1 \leftrightarrow q_2\bar{q}_2$	$8\,\frac{d_{\rm F}^2\,C_{\rm F}^2}{d_{\rm A}}\left(\frac{t^2+u^2}{s^2}\right)$
$q_1\bar{q}_1 \leftrightarrow g\,g$	$8\,d_{\rm F}\,C_{\rm F}^2\left(\frac{u}{\underline{t}}+\frac{t}{\underline{u}}\right)-8\,d_{\rm F}\,C_{\rm F}\,C_{\rm A}\left(\frac{t^2+u^2}{s^2}\right)$
$q_1\,g \leftrightarrow q_1\,g$, $\bar{q}_1\,g \leftrightarrow \bar{q}_1\,g$	$-8\,d_{\rm F}\,C_{\rm F}^2\left(\frac{u}{s}+\frac{s}{\underline{u}}\right)+8\,d_{\rm F}\,C_{\rm F}\,C_{\rm A}\left(\frac{s^2+u^2}{\underline{t^2}}\right)$
$g\,g \leftrightarrow g\,g$	$16\,d_{\rm A}\,C_{\rm A}^2\left(3-\frac{su}{\underline{t^2}}-\frac{st}{\underline{u^2}}-\frac{tu}{s^2}\right)$

Arnold, Moore & Yaffe, JHEP (2003)

- Improve ZPC/AMPT
by treating all 2-to-2 scatterings with AMY-like cross sections and $\mu \propto gT$
$\rightarrow$ toward numerical solution of finite-T QCD kinetic transport

- Wiranata, Koch, Prakash & Wang, Phys. Rev. C 88 (2013)
extended the CE formula to a mixture of hadrons under elastic scatterings.

- Still need to extend the CE formula to include 2-to-2 inelastic scatterings

Conclusions

- The Chapman–Enskog (CE) method gives accurate expression of η for parton matter under anisotropic 2-to-2 scatterings
- Applying the CE method, $<\eta/s>$ for parton matter in the center cell of high energy A+A collisions from the AMPT model is very small at (1-3)/(4π)
- T-dependence or time-dependence of η/s in AMPT is opposite to pQCD expectation, because of the constant σ or screening mass μ
- This problem can be resolved by adopting $\mu \propto gT$; this improvement will lead to a better ZPC/AMPT as a dynamical model for non-equilibrium studies