On the convergence of baryon chiral perturbation theory

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INT workshop on

Chiral EFT: New Perspectives

MH, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 192301

Siemens, Ruiz de Elvira, Epelbaum, MH, Krebs, Kubis, Meißner, PLB 770 (2017) 27

Gupta, Park, MH, Mereghetti, Yoon, Bhattacharya, PRL 127 (2021) 242002; PoS CD2021 (2024) 060

Hall et al., arXiv:2503.09891

MH, Mereghetti, Ruiz de Elvira, Siemens, Walker-Loud, work in progress

Disclaimers

- This talk is **not** about chiral EFT $\Rightarrow$ won't cover non-perturbative resummations
- This talk is **not** about strangeness $\Rightarrow$ won't cover $SU(3)$ uncertainties

- How ***SU*(2) heavy-baryon ChPT** is supposed to converge

$$\epsilon_\pi = \frac{M_\pi}{\Lambda_\chi} \simeq \frac{M_\pi}{4\pi F_\pi} \simeq \frac{M_\pi}{m_N} \simeq \frac{M_\pi}{M_\rho} \simeq 0.12 \ldots 0.18$$

- Known reasons this may not be accurate
  - **Chiral logarithms**: $\simeq M_\pi^2 \log M_\pi^2$
  - **Numerical enhancements**: factors π (or even 4π) in non-analytic loop functions
  - **Low-lying baryon resonances**: the $\Delta(1232)$
- Maybe less known reasons this may not be accurate
  - **(Anomalous) thresholds**: distorted analytic structure, e.g., $\operatorname{Im} F_1^v(t)\big|_{\text{HB}} \propto (t-4M_\pi^2)^{1/2}$
  - **Resonance-enhanced loop effects**: the $\Delta(1232)$ at higher orders

Chiral expansion of the nucleon mass and the pion–nucleon σ-term

Chiral expansion of m_N

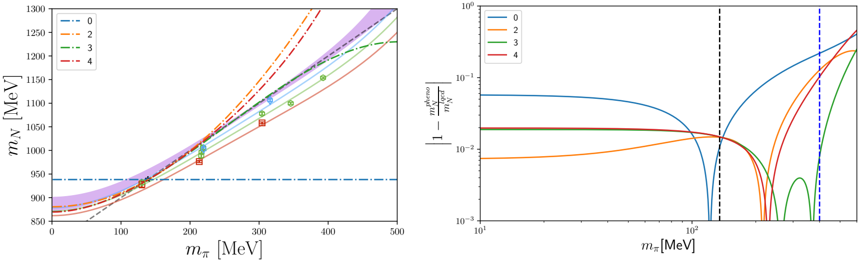
$$m_N = m_0 - 4c_1 M_\pi^2 - \frac{3g_A^2 M_\pi^3}{32\pi F_\pi^2} - \frac{3\left[g_A^2 + m_N(-8c_1 + c_2 + 4c_3)\right]}{32\pi^2 F_\pi^2 m_N} M_\pi^4 \log\frac{M_\pi}{m_N} + \left\{e_1 - \frac{3\left(2g_A^2 - c_2 m_N\right)}{128\pi^2 F_\pi^2 m_N}\right\} M_\pi^4 + \mathcal{O}\left(M_\pi^5\right)$$

$$\sigma_{\pi N} = M_\pi^2 \left\{1 - \frac{M_\pi^2}{32\pi^2 F_\pi^2}\left(\bar{l}_3 - 1\right)\right\} \frac{\partial m_N}{\partial M_\pi^2} = \cdots$$

- Let's look at the coefficients:
  - $\mathcal{O}(\epsilon_\pi^2)$: $-4c_1 m_N \simeq 2.8$
  - $\mathcal{O}(\epsilon_\pi^3)$: $-\frac{3\pi g_A^2}{2} \simeq -7.6$
  - $\mathcal{O}(\epsilon_\pi^4 \log \epsilon_\pi^2)$: $-\frac{3\left[g_A^2 + m_N(-8c_1 + c_2 + 4c_3)\right]}{4} \simeq 6.1$, driven by $m_N(-8c_1 + c_2 + 4c_3) \simeq -9.8$
  - $\mathcal{O}(\epsilon_\pi^4)$: $m_N^3 e_1 - \frac{3\left(2g_A^2 - c_2 m_N\right)}{8} \simeq 11.2$, driven by e_1 to match $\sigma_{\pi N}$

$\hookrightarrow$ apart from maybe $\mathcal{O}(\epsilon_\pi^2)$ none of them are $\mathcal{O}(1)$!

Lepage plot for the nucleon mass



Courtesy of André Walker-Loud, INT program "New Physics Searches at the Precision Frontier"

- Comparison of ChPT with lattice data, **LECs predicted from phenomenology**
- Despite the enhancements, works well up to N^{2}LO
- Agreement becomes worse at N^{3}LO, could be lattice-pheno tension in $\sigma_{\pi N}$, but also large coefficient of $M_\pi^4 \log M_\pi$

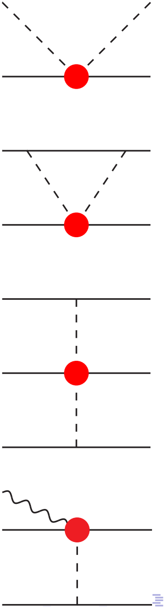
  $\hookrightarrow$ look at low-energy constants (LECs) next

Low-energy constants of baryon ChPT

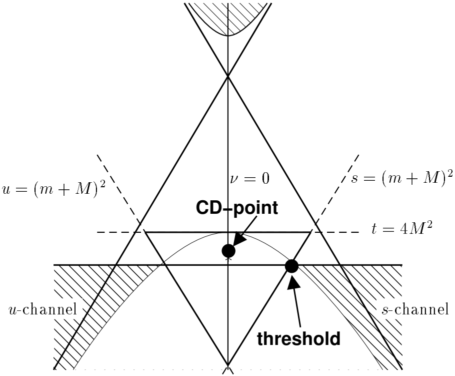
- EFT idea: fix LECs in the simplest process, predict more complicated ones
  - LO meson ChPT: $F, M \to F_\pi, M_\pi$
  - LO baryon ChPT: $m, g \to m_N, g_A$
  - NLO baryon ChPT: c_i
- c_i determine **long-range part** of
  - NN potential
  - $3N$ force
  - Axial current
- c_i incorporate the $\Delta(1232)$, to LO in M_π and $\Delta = m_N - m_\Delta$

$$c_1^\Delta = 0 \qquad c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9\Delta} = 3.0\,\text{GeV}^{-1}$$

- Maybe not even that large, but **pion loops compound the issue**



The Mandelstam plane for pion–nucleon scattering



$$\pi(p) + N(q) \to \pi(p') + N(q')$$

$$s = (p+q)^2 \quad t = (p-p')^2 \quad u = (p-q')^2$$

$$s + t + u = 2m_N^2 + 2M_\pi^2$$

$$\nu = \frac{s-u}{4m_N}$$

- **Subthreshold expansion** around $(\nu, t) = (0,0) \leftrightarrow (s,t) = (m_N^2 + M_\pi^2, 0)$
- **Threshold expansion** around $(\nu, t) = (M_\pi, 0) \leftrightarrow (s,t) = ((m_N + M_\pi)^2, 0)$
- *NN* potential largely sensitive to $(\nu, t) = (-\frac{M_\pi^2}{m_N}, 0) \leftrightarrow (s,t) = (m_N^2 - M_\pi^2, 0)$ Kaiser 2001

Low-energy constants: subthreshold matching

	NLO	N^2LO	N^3LO	N^3LO^{NN}
c_1	$-0.74(2)$	$-1.07(2)$	$-1.11(3)$	$-1.10(3)$
c_2	$1.81(3)$	$3.20(3)$	$3.13(3)$	$3.57(4)$
c_3	$-3.61(5)$	$-5.32(5)$	$-5.61(6)$	$-5.54(6)$
c_4	$2.17(3)$	$3.56(3)$	$4.26(4)$	$4.17(4)$
$\bar{d}_1 + \bar{d}_2$	—	$1.04(6)$	$7.42(8)$	$6.18(8)$
$\bar{d}_3$	—	$-0.48(2)$	$-10.46(10)$	$-8.91(9)$
$\bar{d}_5$	—	$0.14(5)$	$0.59(5)$	$0.86(5)$
$\bar{d}_{14} - \bar{d}_{15}$	—	$-1.90(6)$	$-13.02(12)$	$-12.18(12)$

	N^3LO	N^3LO^{NN}
$\bar{e}_{14}$	$0.89(4)$	$1.18(4)$
$\bar{e}_{15}$	$-0.97(6)$	$-2.33(6)$
$\bar{e}_{16}$	$-2.61(3)$	$-0.23(3)$
$\bar{e}_{17}$	$0.01(6)$	$-0.18(6)$
$\bar{e}_{18}$	$-4.20(5)$	$-3.24(5)$

- One-to-one correspondence between **subthreshold parameters** and LECs
  - Can solve for LECs analytically
  - Maximal distance from threshold singularities
  - πN amplitude can be expanded as a polynomial
  - Subthreshold parameters from Roy–Steiner equations, input from pionic atoms
  - Uncertainties negligible compared to chiral expansion

Low-energy constants: subthreshold matching

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- **Sizable shifts due to pion loops** at subleading orders that need not cancel in observables, e.g., only partial compensation for $3N$ force and axial current

$$\delta c_1 = -\frac{g_A^2 M_\pi}{64\pi F_\pi^2} \simeq -0.13\,\text{GeV}^{-1} \qquad \delta c_3 = -\delta c_4 = \frac{g_A^4 M_\pi}{16\pi F_\pi^2} = 0.85\,\text{GeV}^{-1}$$

A brutal example: the axial charge of the nucleon

Chiral expansion of g_A

$$g_A = g_0\bigg[1 - \frac{M_\pi^2}{16\pi^2 F_\pi^2}\Big((1+2g_0^2)\log\frac{M_\pi^2}{\mu^2} + g_0^2\Big) + \frac{4M_\pi^2 d_{16}^r(\mu)}{g_0} + \frac{M_\pi^3}{24\pi F_\pi^2 m_N}\Big(3(1+g_0^2) + 4m_N(2c_4 - c_3)\Big)\bigg]$$

- Let's again look at the coefficients:
  - $\mathcal{O}(\epsilon_\pi^2)$: $-g_A^2 \simeq -1.6$
  - $\mathcal{O}(\epsilon_\pi^2 \log \epsilon_\pi^2)$: $-(1+2g_A^2) \simeq -4.2$
  - $\mathcal{O}(\epsilon_\pi^3)$: $\frac{2\pi\left[3(1+g_A^2)+4m_N(2c_4-c_3)\right]}{3} \simeq 127.5$, driven by $m_N(2c_4 - c_3) \simeq 13.3 \times$ factor $\frac{8\pi}{3}$
- g_A is maybe the worst case imaginable, but in general large c_i in loops have the potential to wreak havoc on convergence
- Will come back to g_A later, next: look into convergence in πN scattering, from which c_i are extracted

Pion–nucleon scattering: chiral expansion

- **Low-energy theorems** Weinberg 1966

$$a^-_{0+} = \frac{M_\pi m_N}{8\pi F_\pi^2 (m_N + M_\pi)} \sim 79.4 \times 10^{-3} M_\pi^{-1} \qquad a^+_{0+} = 0$$

- **Chiral suppression of isoscalar channel**
    - More sensitive to LECs
    - Fewer non-trivial orders
    - Expect worse chiral convergence
    - Related to πN σ-term via Cheng–Dashen low-energy theorem
- To **test the chiral expansion**, including role of $1/m_N$ corrections:
    1. Fix LECs at subthreshold point
    2. Calculate threshold parameters for HB-πN, HB-NN, and covariant formulation
    3. Compare to phenomenology

Chiral expansion of the πN scattering lengths

	a^-_{0+} $[10^{-3}M_\pi^{-1}]$			a^+_{0+} $[10^{-3}M_\pi^{-1}]$		
	HB-NN	HB-πN	covariant	HB-NN	HB-πN	covariant
LO	79.4	79.4	79.4	0	0	0
NLO	79.4	79.4	80.1	-14.2	-24.0	-24.1
N^{2}LO	92.2	92.9	89.9	0.5	0.5	-14.8
N^{3}LO	68.5	58.6	83.8	-1.5	-8.0	-5.7
Pionic atoms		85.4(9)			$-0.9(1.4)$	

- Expansion of a^+_{0+} about as good as could be expected, but a^-_{0+} terrible at N^{3}LO
- Why does the scheme make such a big difference? No anomalous thresholds, just normal threshold expansion

Role of the $\Delta(1232)$: low-energy constants

- At N^{3}LO: 1-loop diagrams with c_i insertions

$$g_A^2(c_3 - c_4) \simeq -15.9\,\text{GeV}^{-1}$$

$\hookrightarrow$ suggests to **include $\Delta(1232)$ as an explicit degree of freedom**

- For full 1-loop amplitudes need (ϵ counting: $m_\Delta - m_N = \mathcal{O}(M_\pi)$)

LO:	$h_A = 1.40(5)$	$\pi N\Delta$
N^{2}LO:	$g_1 = 2.32(26)$	$\pi\Delta\Delta$
N^{3}LO:	$b_4 + b_5 = \pm 5\,\text{GeV}^{-1}$	$\pi N\Delta$
	$b_4 - b_5 = \pm 5\,\text{GeV}^{-1}$	$\pi N\Delta$

- Phenomenology for h_A (Δ width), large-N_c for the others

Role of the $\Delta(1232)$: chiral expansion of a_{0+}^-

a_{0+}^- $[10^{-3} M_\pi^{-1}]$	HB-NN		HB-πN		covariant	
	Δ-less	Δ-ful	Δ-less	Δ-ful	Δ-less	Δ-ful
NLO	79.4	79.4(0)	79.4	79.4(0)	80.1	81.9(1)
N^2LO	92.2	92.7(1.0)	92.9	90.5(9)	89.9	81.7(1.2)
N^3LO	68.5	96.3(2.0)	58.6	69.1(1.2)	83.8	83.4(1.0)
Pionic atoms			85.4(9)			

- Including the $\Delta(1232)$ indeed helps
- Still large differences between the schemes, covariant formulation clearly superior
- Overall: HB-NN a little better than HB-πN
- How can $1/m_N$ corrections make such a huge difference?

$\Delta(1232)$ and relativistic corrections

	HB-NN		HB-πN		covariant		RS
N^{3}LO	Q^4	ϵ^4	Q^4	ϵ^4	Q^4	ϵ^4	
a^+_{0+} $[M_\pi^{-1}10^{-3}]$	-1.5	$-1.5(8.5)$	-8.0	$1.4(7.5)$	-5.7	$-0.7(6.6)$	$-0.9(1.4)$
a^-_{0+} $[M_\pi^{-1}10^{-3}]$	68.5	$96.3(2.0)$	58.6	$69.1(1.2)$	83.8	$83.4(1.0)$	$85.4(9)$
a^+_{1+} $[M_\pi^{-3}10^{-3}]$	134.3	$136.2(8.2)$	132.1	$135.8(7.9)$	128.0	$132.7(7.6)$	$131.2(1.7)$
a^-_{1+} $[M_\pi^{-3}10^{-3}]$	-80.9	$-80.0(3.0)$	-90.1	$-86.5(3.1)$	-78.1	$-81.1(2.1)$	$-80.3(1.1)$
a^+_{1-} $[M_\pi^{-3}10^{-3}]$	-55.7	$-47.2(5.0)$	-73.7	$-56.6(4.6)$	-53.5	$-51.4(4.9)$	$-50.9(1.9)$
a^-_{1-} $[M_\pi^{-3}10^{-3}]$	-10.0	$-6.0(2.9)$	-23.7	$-15.2(2.8)$	-11.8	$-10.3(3.9)$	$-9.9(1.2)$
b^+_{0+} $[M_\pi^{-3}10^{-3}]$	-42.2	$-30.8(7.9)$	-44.5	$-30.6(7.3)$	-54.7	$-33.8(6.6)$	$-45.0(1.0)$
b^-_{0+} $[M_\pi^{-3}10^{-3}]$	-31.6	$7.6(2.3)$	-65.2	$-35.0(2.3)$	2.3	$2.8(2.8)$	$4.9(8)$

- Similar conclusions as before
  - Including the $\Delta(1232)$ definitely helps (size of LECs reduced to more natural values)
  - Still large variation among schemes
  - Covariant best, HB-NN somewhat better than HB-πN

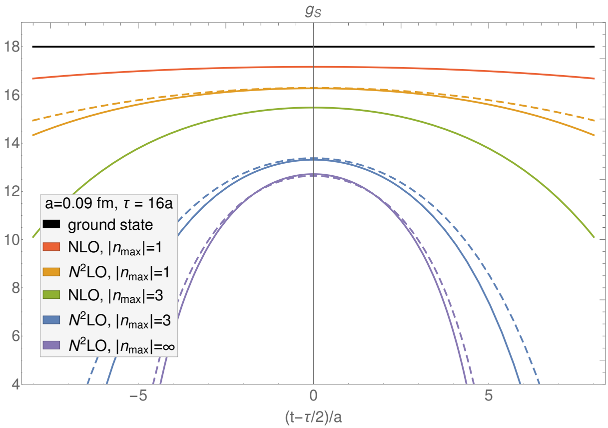
Covariant vs. heavy-baryon ChPT

- The observation that covariant ChPT performs better is not new, e.g. chiral extrapolation of lattice data for baryon masses Martin Camalich et al. 2010, Ren et al. 2014, …
- In nuclear structure, relativistic corrections typically small, how can they make such a big effect in the single-baryon sector?
- Possible origins (from analytic expressions):
  - **Non-analytic functions in M_π^2**: $\arctan \frac{M_\pi}{m_N}$, factors of π
    $\hookrightarrow$ not nearly enough to overcome $M_\pi/m_N \simeq 0.15$
  - **Higher-order logs**: $\log \frac{M_\pi^2}{m_N^2} \simeq -3.8$

$$\log \frac{M_\pi^2}{m_N^2} = \Bigg[\underbrace{32\pi^2\bar{\lambda} + \log \frac{M_\pi^2}{\mu^2}}_{\text{pion tadpole, infrared singular}} \Bigg] - \Bigg[\underbrace{32\pi^2\bar{\lambda} + \log \frac{m_N^2}{\mu^2}}_{\text{nucleon tadpole, infrared regular}} \Bigg]$$

  - In HB nucleon tadpoles absent, pion tadpoles absorbed by the renormalization
    $\hookrightarrow$ LECs at higher orders will receive large contributions from these logs
  - In covariant formulation such logs are included explicitly, but the scale is arbitrary beyond 1-loop: **power-counting argument?**

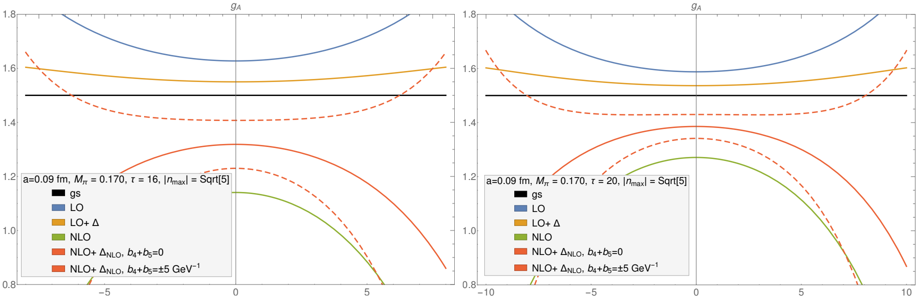
Excited-state contamination for $\sigma_{\pi N}$



- Sizable c_i effects could help explain tension between lattice and pheno for $\sigma_{\pi N}$
- Dashed lines N^2LO + leading $\Delta(1232)$ effect

  ↪ chiral convergence looks very stable

Excited-state contamination for g_A



- LO result is larger than ground state

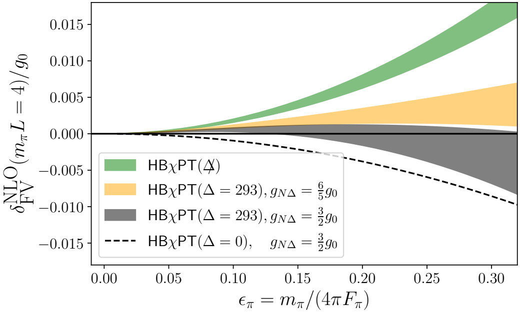
  ↪ excited-state contamination would have wrong sign

- NLO corrections do flip sign

  ↪ again large c_i effects

- Results less stable than for $\sigma_{\pi N}$ when considering explicit Δ degrees of freedom

A recent study of finite-volume corrections in g_A



Hall et al., arXiv:2503.09891

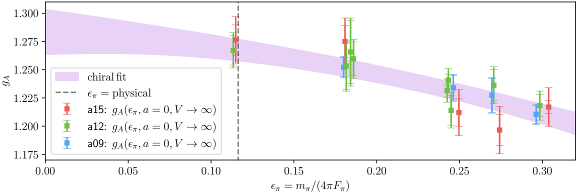
- **Finite-volume corrections** to g_A change sign when including the $\Delta(1232)$
- (At least) part of the reason: coefficient of chiral log vanishes for $N_c \to \infty$

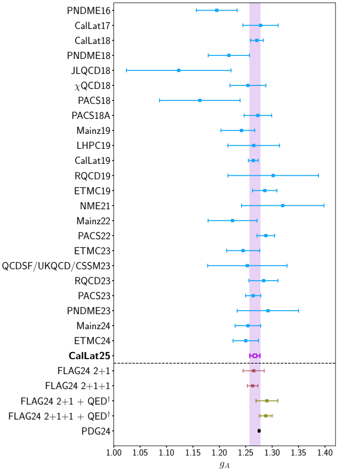
$$\lim_{N_c\to\infty}\left[g_A^3 + \frac{2}{9}g_A h_A^2 - \frac{50}{81}g_1 h_A^2\right] = 0$$

$\hookrightarrow$ finite-volume corrections inherit a similar pattern

- Paper includes finite-volume corrections up to N^{2}LO

A recent study of finite-volume corrections in g_A





- Main point: finite-volume corrections to become (serious) concern for sub-percent calculations of g_A
- Update for $g_A^{\text{QCD}} = 1.2674(96)$
- Chiral fit gives $2c_4 - c_3 = (0.66\text{–}0.70)\ \text{GeV}^{-1}$

  $\hookrightarrow$ factor 20 smaller than phenomenological value
- **For g_A, chiral expansions without the $\Delta(1232)$ look doomed to fail**

Conclusions

- **Pitfalls in the convergence of $SU(2)$ HB ChPT**
  - Large numerical coefficients ("π enhancement")
  - Large c_i in loops
  - Severity depends very much on case at hand, m_N maybe OK, g_A a disaster
- Full 1-loop calculation of πN scattering, including $\Delta(1232)$
  - Strict HB formulation not accurate enough to cover the whole low-energy region, **cannot connect subthreshold and physical region**
  - Including the $\Delta(1232)$ helps, but in addition/instead a **covariant formulation improves convergence**
  - Possibly related to **higher-order chiral logs**, but no power-counting argument
- Comparison to lattice QCD for m_N, $\sigma_{\pi N}$, and g_A
  - **Excited-state contamination** from ChPT: convergence for m_N, $\sigma_{\pi N}$ looks fine, g_A again much more unstable
  - **Finite-volume corrections** for g_A: large-N_c limit suggests significant cancellations
  - Absurd value for $2c_4 - c_3$ from chiral fit to lattice data for g_A
  - Comprehensive analysis of m_N and g_A (in analogy to πN) in progress, becoming possible with improved lattice data