ΔΔ Intermediate States in NN Scattering from a Large-N_c Perspective

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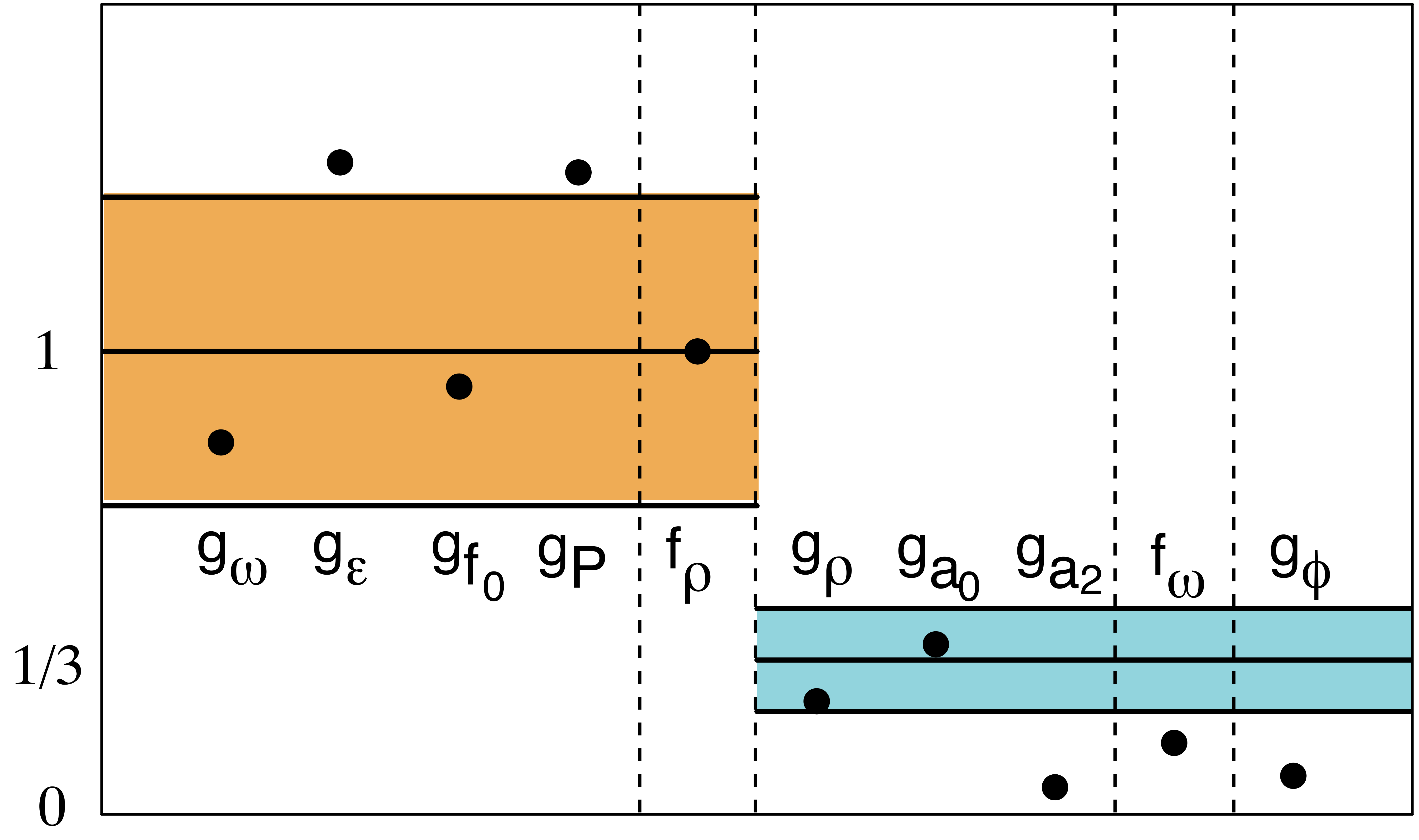
Large-N_c expansion and nucleon interactions

- Constraints on NN and 3N interactions from considering $N_c \to \infty$
- Largely in agreement with data where available
  - NN scattering
  - External currents
  - Neutrinoless double beta decay
  - Parity violation and time reversal invariance violation (?)

Large-N_c expectation vs Nijmegen



Kaplan, Manohar (1997)

NN scattering in the large-N_c expansion

- Baryon-baryon scattering amplitude $\sim O(N_c)$

- Large-N_c analysis applied to potential

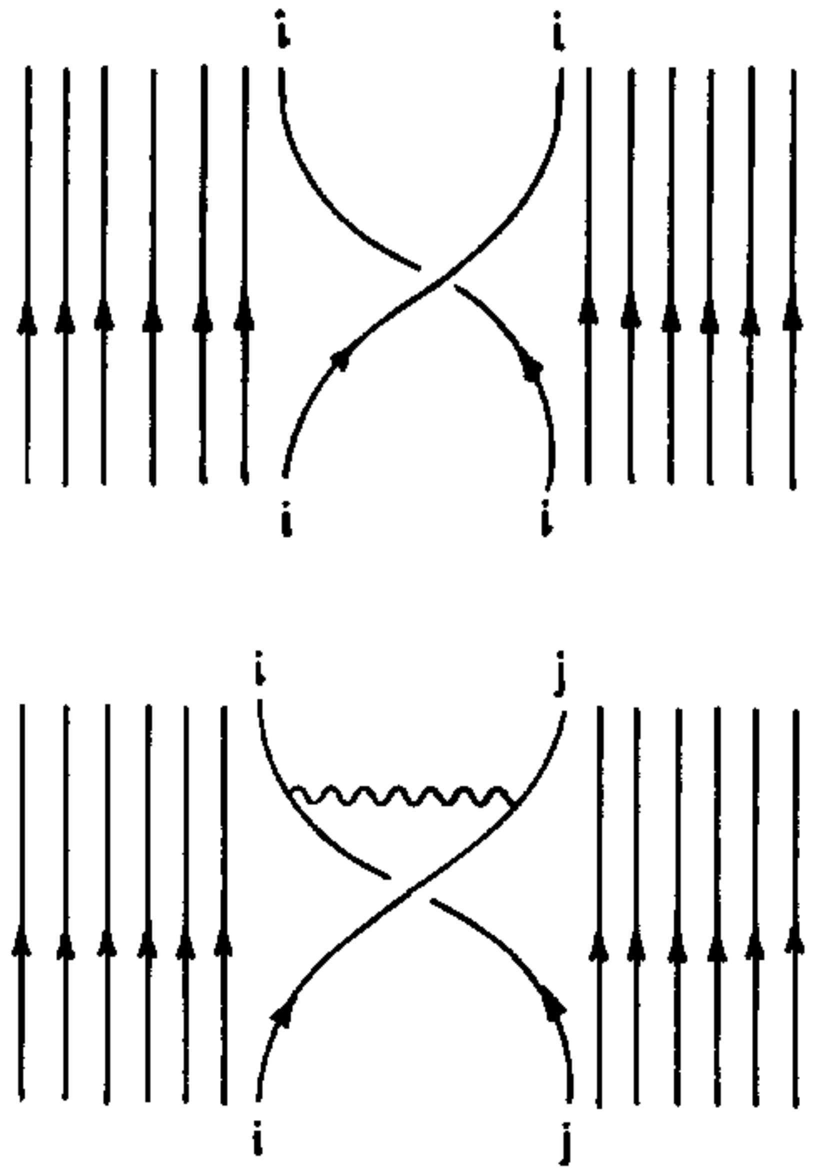
$$V(p_-, p_+) = \langle N(p_1'), N(p_2') \,|\, H \,|\, N(p_1), N(p_2) \rangle$$

- Effective Hamiltonian

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^u$$

- Building blocks

$$S^i = q^\dagger \frac{\sigma^i}{2} q, \quad I^a = q^\dagger \frac{\tau^a}{2} q, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$



Witten (1979); Dashen, Jenkins, Manohar (1994); Kaplan, Savage (1996); Kaplan, Manohar (1997)

Large-N_c scaling

- Leading-in-N_c matrix elements factorize

- Nucleon matrix elements

$$\langle N'|G^{ia}|N\rangle \sim \langle N'|1|N\rangle \sim O(N_c)$$
$$\langle N'|S^{i}|N\rangle \sim \langle N'|I^{a}|N\rangle \sim O(1)$$

- Momenta (in t-channel)

$$p_- = (p_1' - p_2') - (p_1 - p_2) \qquad \sim O(1)$$
$$p_+ = (p_1' - p_2') + (p_1 - p_2) \sim 1/M_N \sim O(1/N_c)$$

- Coefficients (excluding momenta)

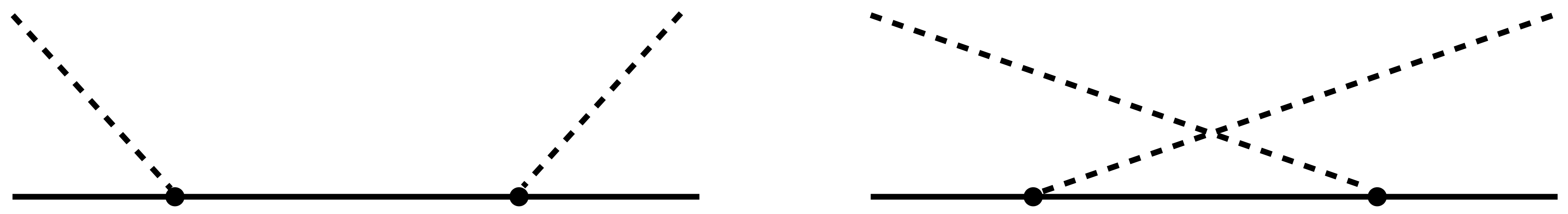
$$\tilde{v}_{stu} \sim 1$$

Dashen, Jenkins, Manohar (1994,95); Kaplan, Savage (1996); Kaplan, Manohar (1997)

Why care about the Δ?

- Intermediate Δ states ignored

- Large-N_c scaling based on (contracted) SU(4) spin-flavor symmetry $u\uparrow, u\downarrow, d\uparrow, d\downarrow$ in baryon spectrum



- Requires degenerate baryons with $I = J = \frac{1}{2}, \frac{3}{2}, \ldots,$ (i.e., $M_\Delta - M_N \to 0$)

- Meson-exchange of NN potential picture: Δ intermediate states needed for consistency

Given importance of Δ states - why the reasonable agreement?

Dashen, Jenkins, Manohar (1994), Banerjee, Cohen, Gelman (2002)

Pionless (and Δ-less) EFT and the large-N_c expansion

- LO S-wave interactions

$$\mathscr{L} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

- Spin-isospin structure of operators

$$(N^\dagger N)(N^\dagger N) \sim 1_1 \cdot 1_2 \qquad (N^\dagger \sigma^i N)(N^\dagger \sigma^i N) \sim \hat{S}_1 \cdot \hat{S}_2$$

- Large-N_c scaling of LECs

$$C_S \sim O(N_c) \qquad C_T \sim O(1/N_c)$$

- In large-N_c limit

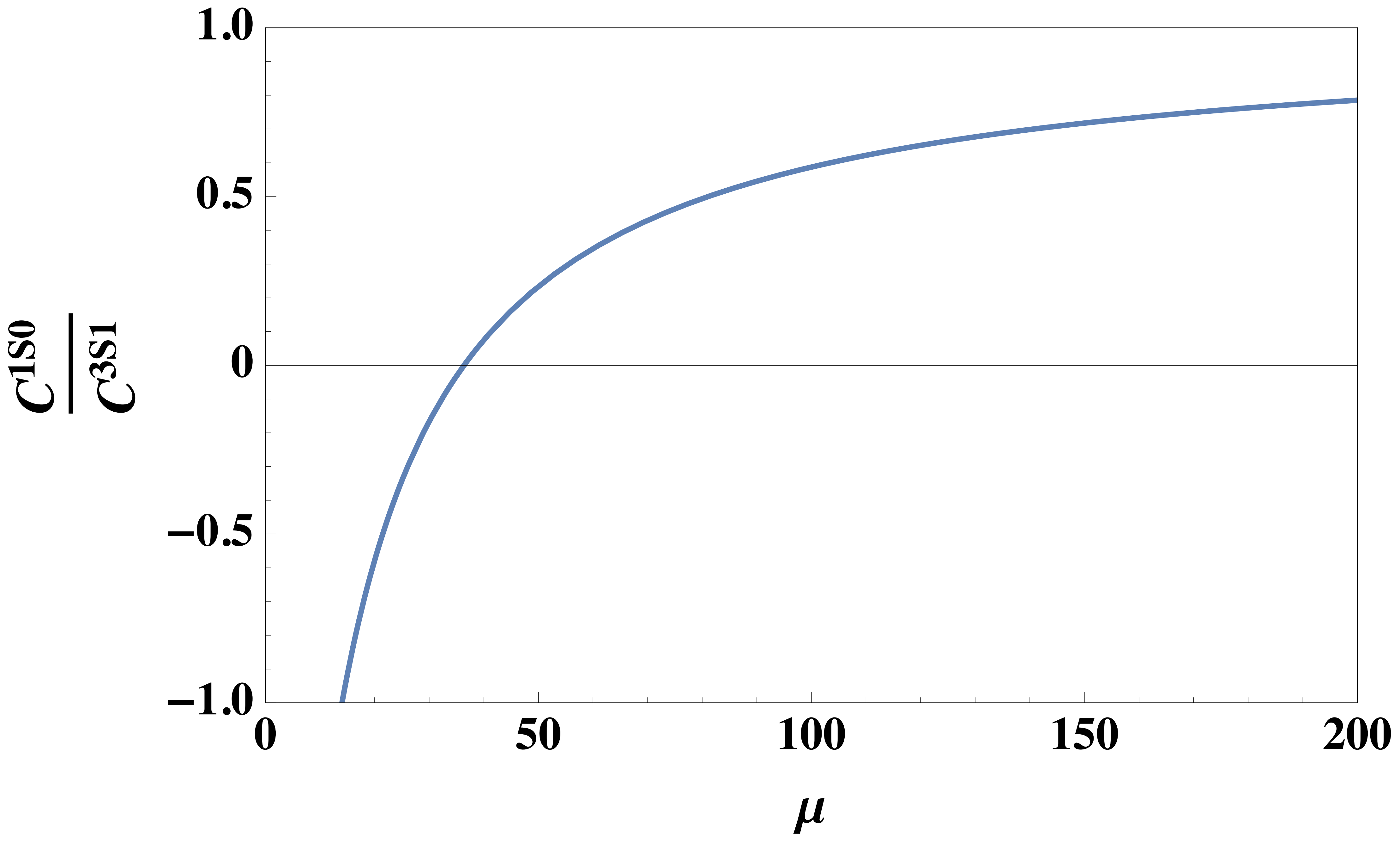
$$C^{(^1S_0)} = C^{(^3S_1)}$$

Kaplan, Savage (1996)

Renormalization-point dependence

- Unnaturally large scattering lengths dominate for small μ

- Agreement with large-N_c expected errors for $\mu \gtrsim m_\pi$ (in PDS)



Kaplan, Savage (1996)

LO SU(4)-symmetric Lagrangian

- N and Δ ground states in same SU(4) multiplet ψ^{ABC}

- LO Lagrangian in large-N_c limit is SU(4) symmetric

$$\mathscr{L} = -\,\tilde{a}\left(\psi^{\dagger}_{ABC}\psi^{ABC}\right)^2 - \tilde{b}\,\psi^{\dagger}_{ABC}\psi^{ABD}\psi^{\dagger}_{EFD}\psi^{EFC}$$

- Describes $NN, N\Delta$, and $\Delta\Delta$ S-wave interactions

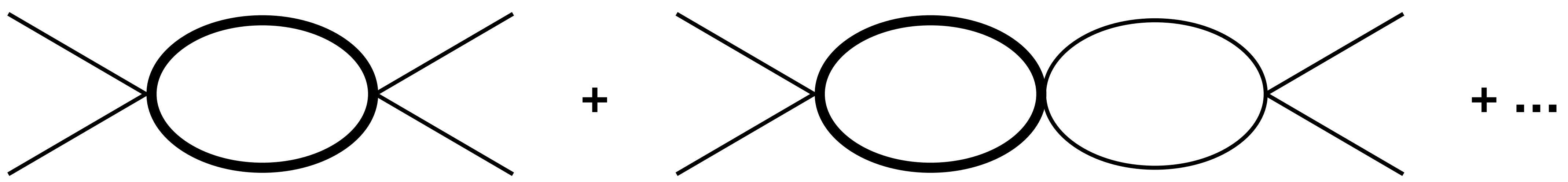
- SU(4)-symmetric Lagrangian contains two independent LECs

⇒ relationships between $NN, N\Delta$, and $\Delta\Delta$ LECs in different S waves

Kaplan, Savage (1996)

ΔΔ intermediate states in NN scattering

- Resummation of intermediate $\Delta\Delta$ states in S-wave NN scattering



$\Rightarrow$ effective NN parameters

$$\tilde{C}_E(p) = C_{NN} + \frac{(C_{N\Delta})^2 I_\Delta(p)}{1 - C_{\Delta\Delta} I_\Delta(p)}$$

where I_Δ: Δ loop integral

Savage (1996)

Relating $^1S_0$ and $^3S_1$ channels

- 20-dim representation in terms of N and Δ

$$\Psi^{ABC} \to \Psi^{abc}_{\alpha\beta\gamma} = \Delta^{abc}_{\alpha\beta\gamma} + \frac{1}{\sqrt{18}}\left(N^a_\alpha \epsilon^{bc}\epsilon_{\beta\gamma} + N^b_\beta \epsilon^{ac}\epsilon_{\alpha\gamma} + N^c_\gamma \epsilon^{ab}\right)$$

- Additional invariance under spin-isospin exchange ("flip" F) $a \leftrightarrow \alpha, \ldots$

- Under F:

$$\mathcal{O}^{(^1S_0)}_{NN} \overset{F}{\longleftrightarrow} \mathcal{O}^{(^3S_1)}_{NN}, \quad \mathcal{O}^{(^1S_0)}_{\Delta N} \overset{F}{\longleftrightarrow} \mathcal{O}^{(^3S_1)}_{\Delta N}, \quad \mathcal{O}^{(^1S_0)}_{\Delta\Delta} \overset{F}{\longleftrightarrow} \mathcal{O}^{(^3S_1)}_{\Delta\Delta}$$

- LECs in Δ-full theory:

$$C^{(^1S_0)}_{NN} = C^{(^3S_1)}_{NN}, \quad C^{(^1S_0)}_{N\Delta} = C^{(^3S_1)}_{N\Delta}, \quad C^{(^1S_0)}_{\Delta\Delta} = C^{(^3S_1)}_{\Delta\Delta}$$

Kaplan, Savage (1996); Richardson, MRS, Springer (2024)

Relating $^1S_0$ and $^3S_1$ channels

- LO amplitudes

$$\mathscr{A}^{(^1S_0)}_{NN,\mathrm{LO}} = \mathscr{A}^{(^3S_1)}_{NN,\mathrm{LO}}$$

- Matching amplitudes to Δ-less theory:

$$C^{(^1S_0)}_{NN,\not\Delta} = C^{(^3S_1)}_{NN,\not\Delta}$$

Same result as applying large-N_c scaling rules to Δ-less theory directly

- Does not imply $C^{(S)}_{NN} = C^{(S)}_{NN,\not\Delta}$

- Also holds for two-derivative S-wave interactions

Kaplan, Savage (1996); Richardson, MRS, Springer (2024)

Other partial waves

- For $L \geq 1$ NN interactions in Δ-less theory perturbative $\rightarrow$ assume also holds for Δ interactions

$\Rightarrow$ Δ contributes at higher order

- Exception: interactions mixing S and higher partial waves (S-D, parity violation, etc)
  - Additional mixing operators with Δ fields
  - Can again resum $\Delta\Delta$ intermediate states
  - Example:

$$\tilde{C}_{NN}^{(S-P)}(p) \equiv \left[C_{NN}^{(S-P)} + C_{N\Delta}^{(S-P)} \frac{C_{\Delta N}^{(S)} I_{\Delta}}{1 - C_{\Delta\Delta}^{(S)} I_N} \right]$$

Richardson, MRS, Springer (2024)

SU(4) constraints on LECs

- SU(4) symmetric Lagrangian depends on two LECs $\tilde{a}$ and $\tilde{b}$

$\Rightarrow$ Constraints on $NN, N\Delta, \Delta\Delta$ LECs

- NN:

$$C_{NN}^{(^1S_0)} = 2\left(\tilde{a} - \frac{\tilde{b}}{27}\right) = C_{NN}^{(^3S_1)}$$

- $N\Delta$:

$$C_{\Delta N}^{(^1S_0)} = \frac{8\sqrt{5}}{27}\tilde{b} = C_{\Delta N}^{(^3S_1)}$$

Kaplan, Savage (1996); Richardson, MRS, Springer (2024)

- $\Delta\Delta$:

$$C_{\Delta\Delta}^{(0,1)} = 2\left(\tilde{a} + \frac{\tilde{b}}{27}\right) = C_{\Delta\Delta}^{(1,0)} \qquad C_{\Delta\Delta}^{(1,2)} = 2\left(\tilde{a} - \frac{\tilde{b}}{27}\right) = C_{\Delta\Delta}^{(2,1)}$$

$$C_{\Delta\Delta}^{(0,3)} = 2\left(\tilde{a} - \frac{\tilde{b}}{3}\right) = C_{\Delta\Delta}^{(3,0)} \qquad C_{\Delta\Delta}^{(3,2)} = 2\left(\tilde{a} + \frac{\tilde{b}}{3}\right) = C_{\Delta\Delta}^{(2,3)}$$

- Equality NN LECs ⇒ information from NN not sufficient to determine $\tilde{a}$ and $\tilde{b}$

- Combine NN with $\Delta\Delta$ scattering (e.g., $S = 3, I = 0$) → lattice [$d^{\star}(2380)$]?

Gongyo et al. (2020)

Unitary limit

- Take unitary limit for NN scattering
- Amplitudes in other channels μ-dependent unless $\tilde{b}(\mu) = 0$
- Amplitudes in other channels also in unitary limit
- For $\tilde{b}(\mu) = 0$:

$$C_{\Delta N}^{(^1S_0)} = 0 = C_{\Delta N}^{(^3S_1)}$$

Decoupling of $\Delta\Delta$ sector from NN

- $\tilde{C}_{NN}^{(S-D)}/\tilde{C}_{NN}^{(S)} \rightarrow C_{NN,\not\Delta}^{(S-D)}/C_{NN,\not\Delta}^{(S)}$, $\tilde{C}_{NN}^{(S-P)}/\tilde{C}_{NN}^{(S)} \rightarrow C_{NN,\not\Delta}^{(S-P)}/C_{NN,\not\Delta}^{(S)}$

$\tilde{b}(\mu) = 0$

- Unitary limit not required, also consequence of finite $a_{NN} = a_{\Delta\Delta}^{(3,0)}$ (cf Wigner)

- No effect on large-N_c expectations for NN S-wave interactions:

$$C_{NN}^{(^1S_0)} = C_{NN}^{(^3S_1)} \text{ for any value of of } \tilde{b}$$

- $C_{\Delta N} = 0$ and all $C_{NN} = C_{\Delta\Delta} = 2\tilde{a}$

Including strangeness

- Extension to three flavors ⇒ SU(6) symmetry in large-N_c limit

- Octet and decuplet in 56-dim representation

- Constraints on Savage-Wise (octet-octet) coefficients $c_1, \ldots, c_6$
  ⇒ only c_5 dependent on $\tilde{a}$

- Lattice calculations → suppression of $\tilde{b}$?

- If $\tilde{b} = 0$ ⇒ SU(16) symmetry in octet-octet interactions

- Also corresponds to suppression of spin entanglement in octet-octet scattering

Kaplan, Savage (1996); NPLQCD (2017, 2021); Klco et al (2018)

Including strangeness

- Extend to decuplet interactions

$$C_{\Omega\Omega}^{(^1S_0)} = C_{\Delta\Delta}^{(0,3)} = C_{\Delta\Delta}^{(3,0)}$$

- HAL QCD: $\Omega\Omega$ scattering in $^1S_0$ channel close to unitary limit

$$\tilde{b}(\mu) = 0$$

- SU(56) symmetry

Richardson, MRS, Springer (2024); Gongyo et al (2018)

Conclusions

- Δ plays important role in baryon sector in large-N_c limit
- Large-N_c applied to NN interactions without Δ $\rightarrow$ reasonable agreement
- In pionless EFT in large-N_c limit:
  - S waves: ratios of LECs identical in theories with and without Δ
  - Higher partial waves: perturbative, Δ contributes at higher order
  - Potential issue: S-D mixing, parity-violating S-P interactions

Conclusions

- SU(4) constraints on baryon-baryon LECs
- Unitary limit ⇒ decoupling of Δ sector
- Similar conclusions for three flavor SU(6)