Jet Substructure with Energy Correlators

Heavy Ion Physics in the EIC Era
INT

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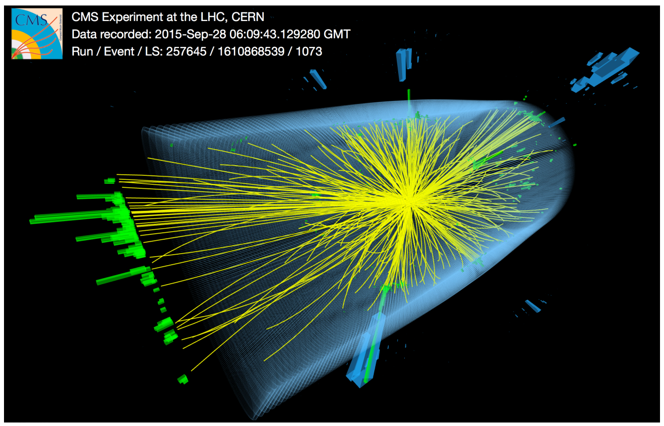
Yale

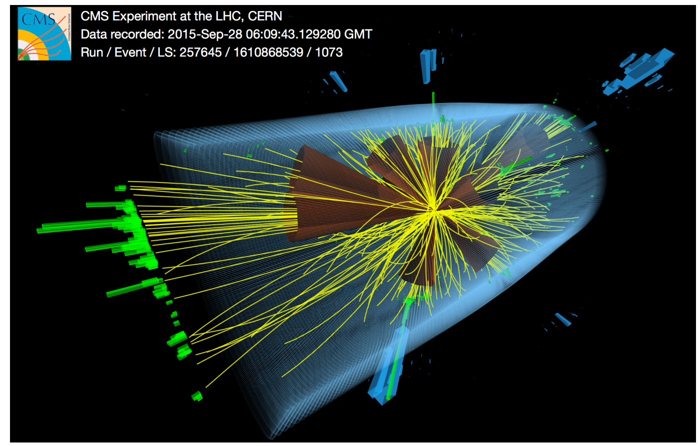
Based on arXiv: 2210.09311, 2205.03414 and work on progress with E.Craft, M. Gonzales, K.Lee and I.Moult

August 14, 2024

QCD at Colliders

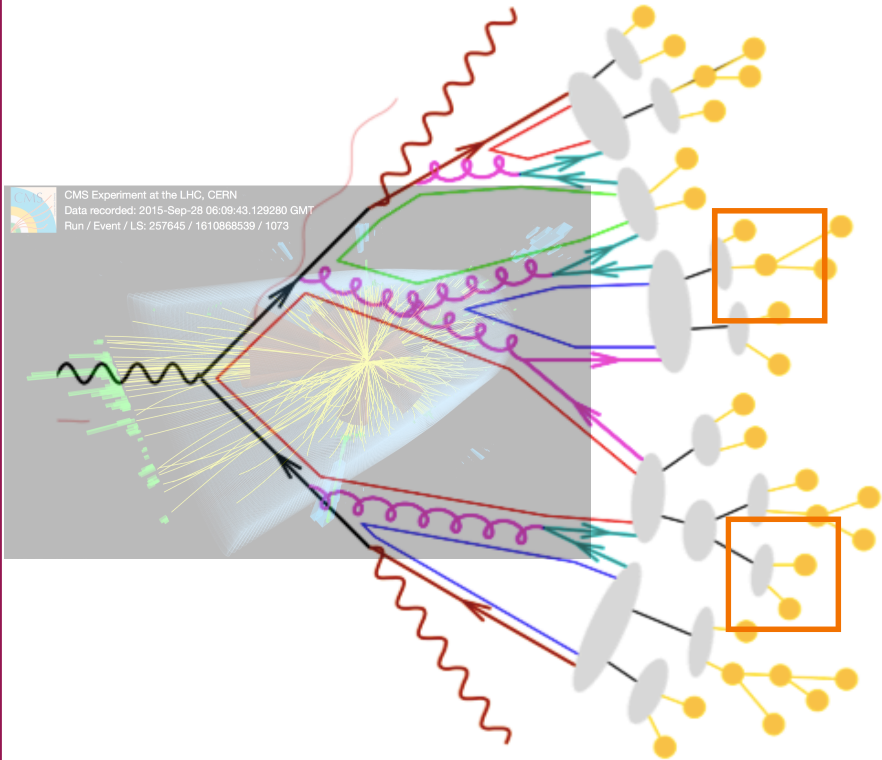
Jets are naturally emergent phenomena at colliders



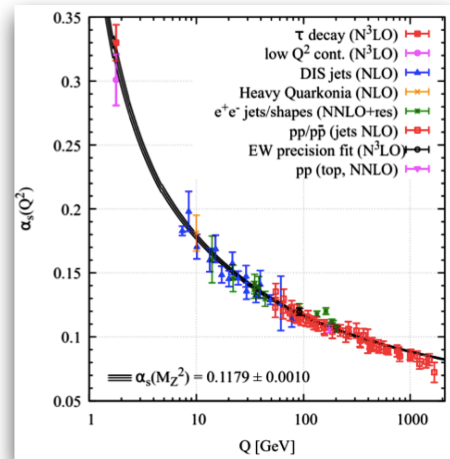


Jets are reconstructed using jet algorithms (anti-k_T)

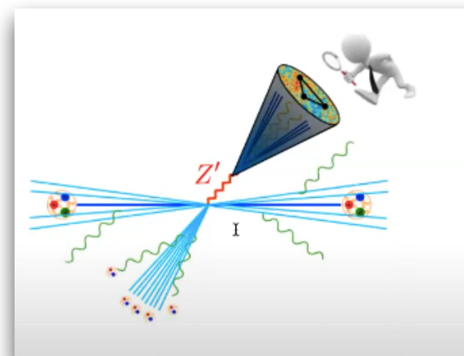
Cacciari, Salam 2006 Salam; Soyez 2007



QCD precision studies



New Physics searches

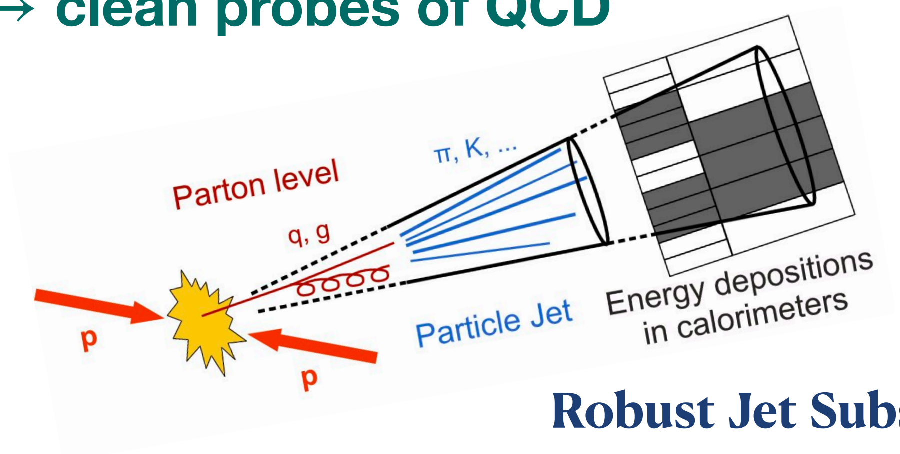


Jet Substructure

How can we learn about underlying physics from the reconstructed jets?

- Study the internal structure of a jet →theoretical analysis and measurements of kinematic properties

- Underlying Physics and intrinsic properties are imprinted in jet substructure → clean probes of QCD



Robust Jet Substructure Observables!

Event Shapes

- **Weighted cross-sections: distribution of outgoing particles/charges**

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) \omega(X) \,|\, \langle X \,|\, O(0) \,|\, 0 \rangle \,|^2$$

Local operator that creates the state $|X\rangle$ with momentum k_X

Event Shapes

- **Weighted cross-sections: distribution of outgoing particles/charges**

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Local operator that creates the state $|X\rangle$ with momentum k_X

Weight factor depends on the measurement.

Event Shapes

- **Weighted cross-sections: distribution of outgoing particles/charges**

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) E_{\{X\}} |\langle X | O(0) | 0 \rangle|^2$$

Local operator that creates the state $|X\rangle$ with momentum k_X

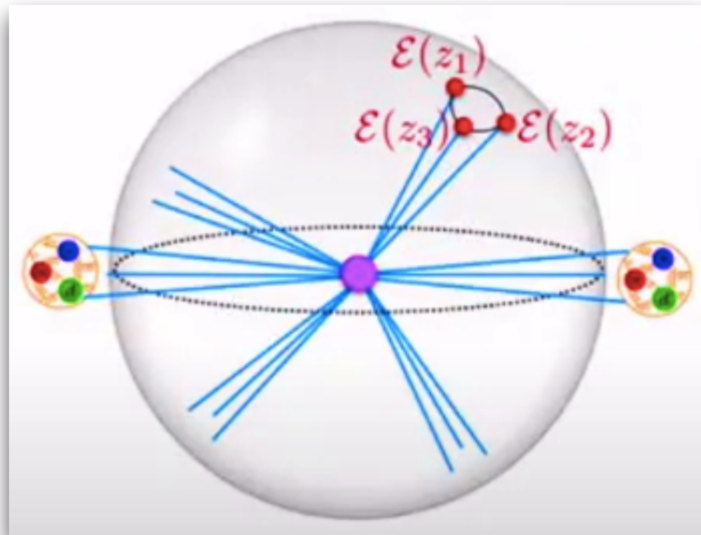
For $\omega(X)$ weighted energy this expression gives the distribution of energy inside the jet.

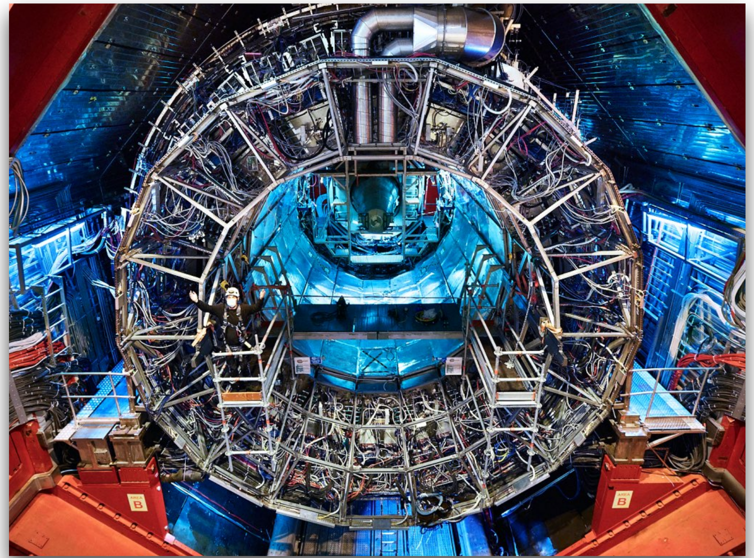
$E_{\{X\}}$ are different permutations for all $\{X\}$ final states

Reformulate such event shape distributions with correlation functions!

Energy Correlators

Energy Correlators describe the calorimeter cells at infinity on the celestial sphere



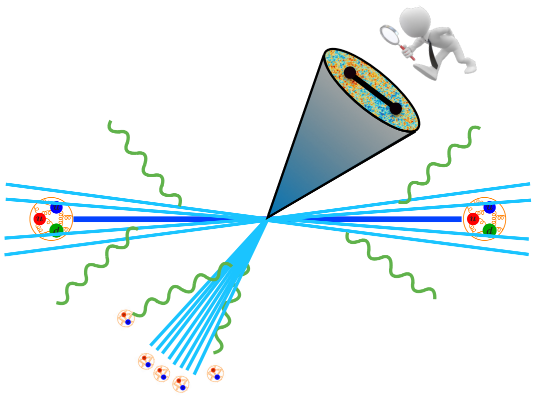


$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$

Energy Flow Inside the Jet

- Distribution of energy inside the jet is described by correlation functions of the energy flow operators ⇒ Energy Correlators.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})$$

Defined from first principles in QFT!

- Any physics dynamics will be imprinted in the energy distributions inside the jet.

Energy Flow Inside the Jet

Correlation functions of the energy flow operators $\left\langle \varepsilon\left(\vec{n}_1\right)\cdots\varepsilon\left(\vec{n}_n\right)\right\rangle$ characterize the final state hadrons in QCD

Energy Correlations in electron - Positron Annihilation: Testing QCD

C.Louis Basham (Washington U., Seattle), Lowell S. Brown (Washington U., Seattle), Stephen D. Ellis (Washington U., Seattle), Sherwin T. Love (Washington U., Seattle)

Aug, 1978

13 pages

Published in: *Phys.Rev.Lett.* 41 (1978) 1585

DOI: 10.1103/PhysRevLett.41.1585

Report number: RLO-1388-759

View in: OSTI Information Bridge Server, ADS Abstract Service

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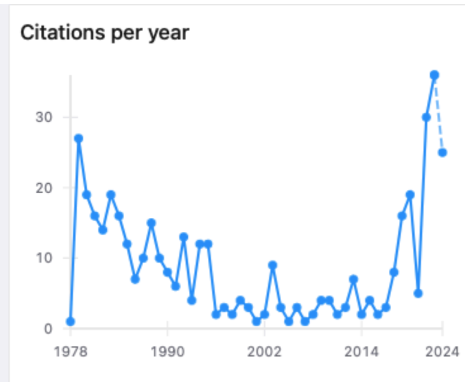

Abstract: (APS)

An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process e+e−→hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order 1W2) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

Theoretical and Phenomenological Motivation

- Transition from TeV to GeV scales is a high multiplicity regime to study QCD with jet substructure
- Use correlation functions to characterize the theory
- At colliders $\sqrt{s} \gg m \rightarrow$ Conformal Field Theory (CFT)

Motivation:

- Can we relate the asymptotic data at colliders to underlying properties of the theory
    - Coupling constants, transport coefficients, particle spectrum....?
    - What is the space of observables at null infinity?

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Energy Correlators from CFT to Jet-Substructure

Observables in CFT are used to describe data at hadron colliders

→take full advantage of the progress in formal field theory

Conformal collider physics:
Energy and charge correlations

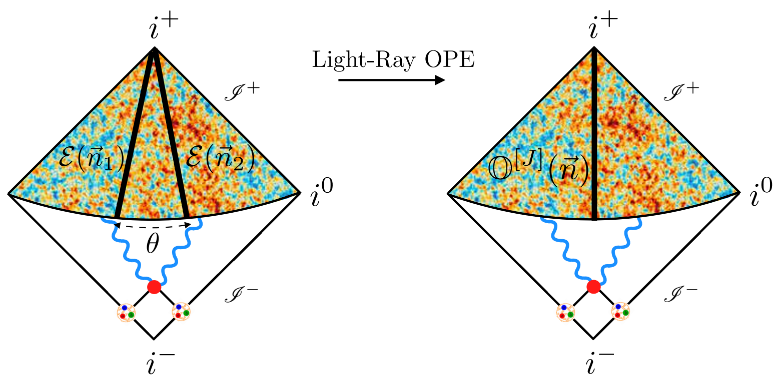
Diego M. Hofman[a] and Juan Maldacena[b]

[a] *Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA*

[b] *School of Natural Sciences, Institute for Advanced Study*
Princeton, NJ 08540, USA

Energy Correlators from CFT to Jet-Substructure

- **Energy correlators inside high energy jets at the LHC ⇒small angle limit**



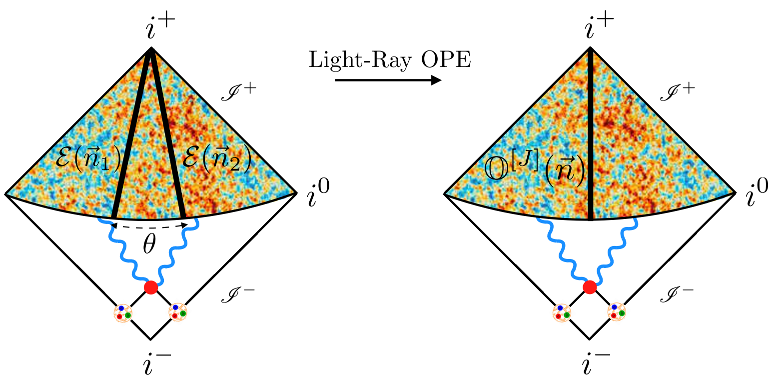
- **Energy correlators admit an Operator Product Expansion (OPE):**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

Scaling Behavior

- **Energy correlators inside high energy jets at the LHC $\Rightarrow$small angle limit**



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$\Rightarrow$ Use LHC jets to test the leading QCD operators in this expansion

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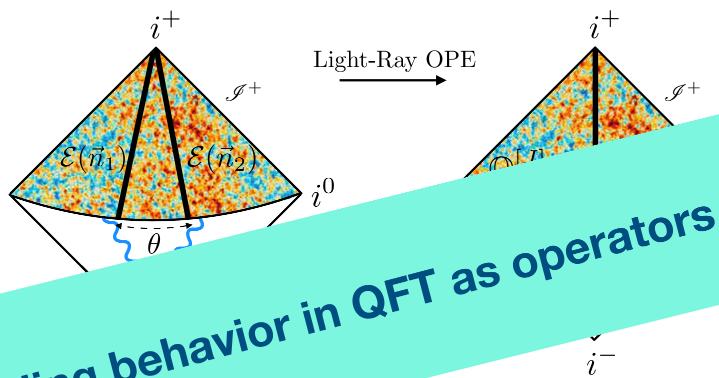

Universal scaling behavior in QFT as operators are brought together!

- **Energy ... an Operator Product Expansion (OPE):**

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⇒ Use LHC jets to test the leading QCD operators in this expansion

Scaling Behavior

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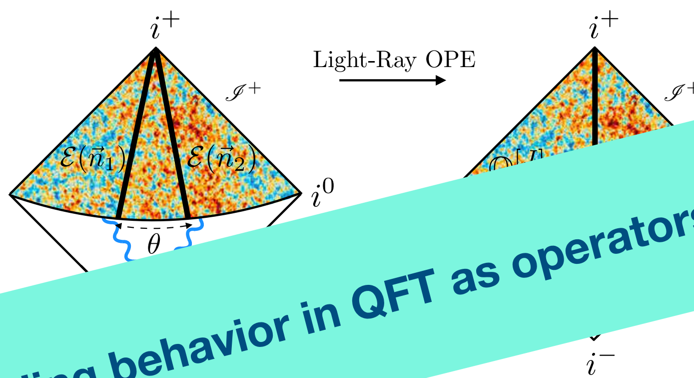

Universal scaling behavior in QFT as operators are ...ght together!

- **Energy ... an Operator Product Expa...**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

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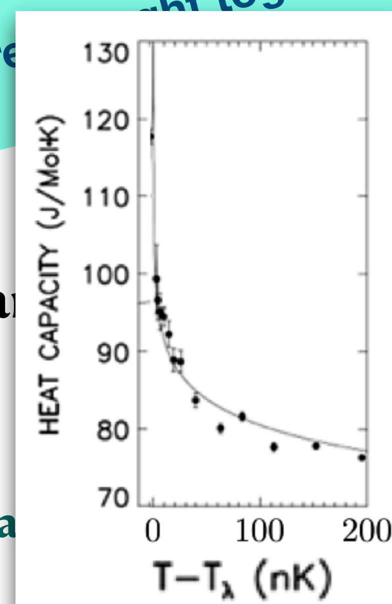
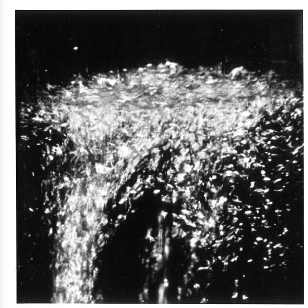
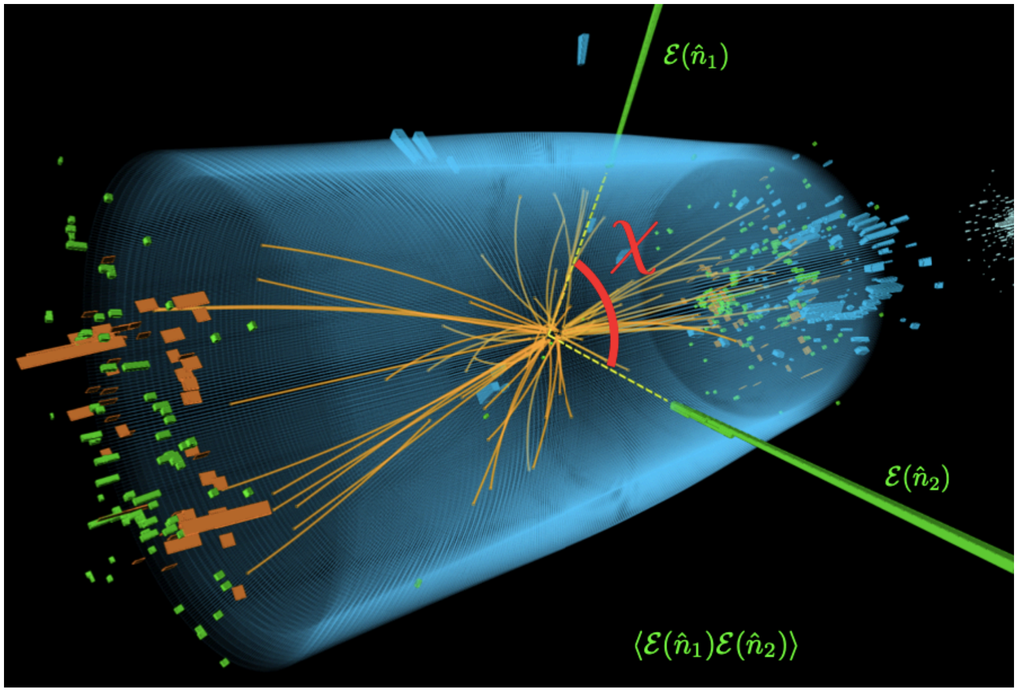




Energy Correlators for Hadronic Final States at the LHC

Energy Correlator as an Observable

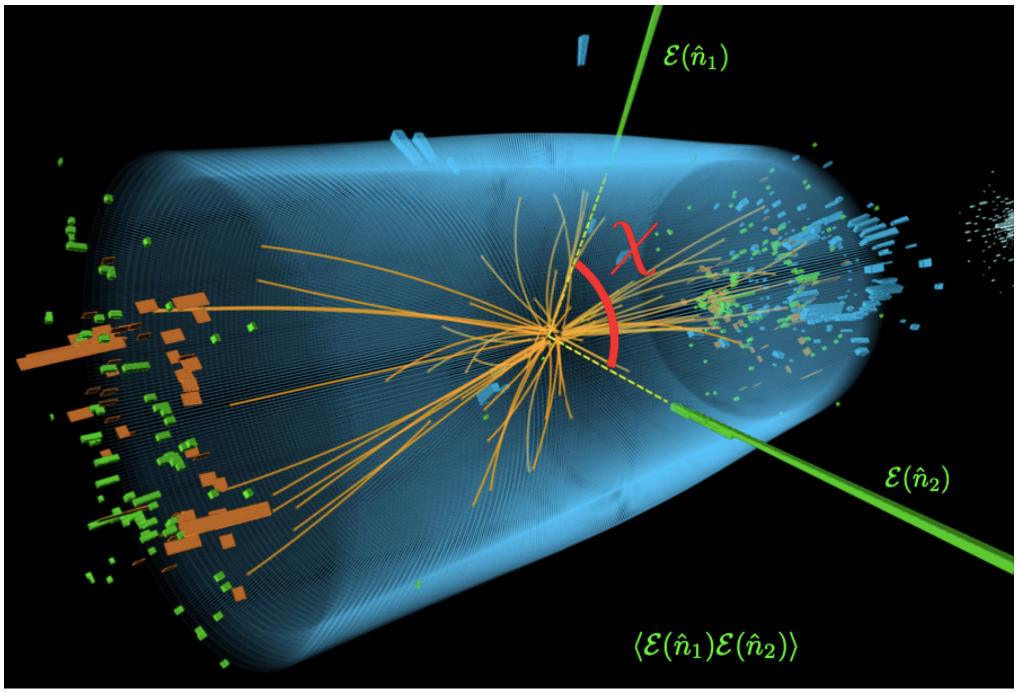


Energy weighted cross sections

$$\text{EEC} = \sum_{i,j} d\sigma \frac{2E_iE_j}{Q^2\sigma_{\text{tot}}} \delta(\cos\theta_{ij} - \cos\chi)$$

Studied first in e^+e^- by [Basham, Brown, Ellis, Love]

Energy Correlator as an Observable

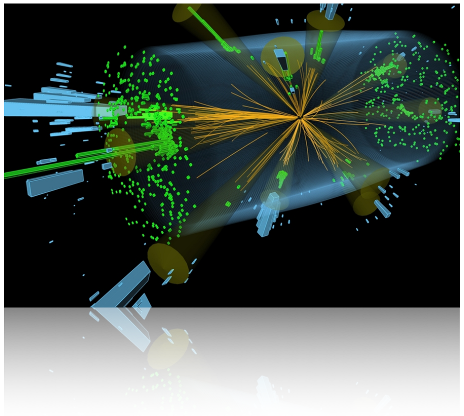


$$\text{EEC} = \sum_{i,j} d\sigma \frac{2E_i E_j}{Q^2 \sigma_{\text{tot}}} \delta(\cos\theta_{ij} - \cos\chi)$$

- **Generally one can study EEC for any angle** χ
- **Most interesting phenomenological case:** $\chi \to 0$ **and** $\chi \to \pi$
- **Here we study** $\chi \to 0$ **case.**

Factorization theorem

Can compute any higher point correlators on massive quarks at LHC at NLL



Describes the production of the collinear source

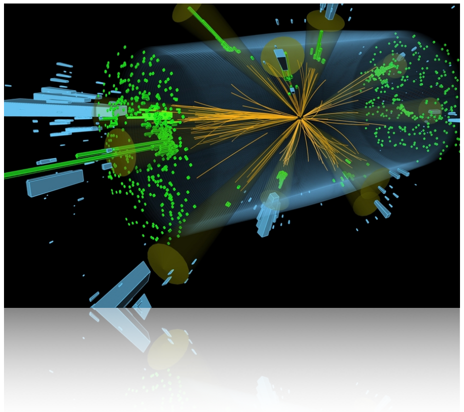
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Describes the dependance on the observable

[Craft, Lee, BM, Moult]

Factorization theorem

- **Can compute any higher point correlators on massive quarks at LHC at NLL**



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Describes the dependance on the observable

Three scales:

$\mu_H \sim p_T$

$\mu_J \sim p_T R$

m_Q

- **Factorization theorem derived within soft-collinear effective theory.**

[Bauer, Fleming, Pirjol, Stewart]

Factorization theorem

$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

$$\vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) = \begin{Bmatrix} \vec{J}_g^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_q^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_Q^{[N]}\left(R_L, x, m_Q, \mu\right) \end{Bmatrix}$$

$$\vec{H}\left(x, p_T^2, \mu\right) = \begin{Bmatrix} H_g\left(x, p_T^2, \mu\right) \\ H_q\left(x, p_T^2, \mu\right) \\ H_Q\left(x, p_T^2, \mu\right) = H_q\left(x, p_T^2, \mu\right) \end{Bmatrix}$$

Factorization theorem

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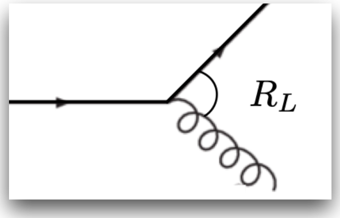

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Heavy Quark Jet Function

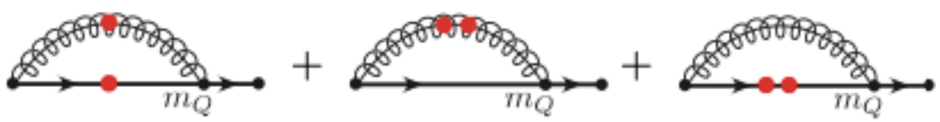
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

$$W_{n_i}^{(A)}(x) = P\,\exp\left[ig_A\, t_A^a \int_{-\infty}^{0} ds\, \bar{n}_i \cdot A_{n_i}^a(x + s\bar{n}_i)\right]$$

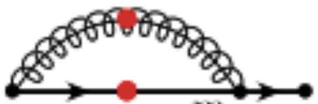
$$J_Q^{[N]}\left(R_L, m_Q\right) = \sum_X \sum_{i_1, i_2, \ldots, i_N \in X} \left\langle 0 \left| \bar{\chi}_n \right| X \right\rangle \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{p_T^N} \Theta\left(\max\left\{\theta_{ij}\right\} < R_L\right) \left\langle X \left| \chi_n \right| 0 \right\rangle$$

$$\chi_{n_i}(x) = \frac{\not{n}_i \not{\bar{n}}_i}{4} W_{n_i}^{\dagger}(x)\, \psi(x)$$

At $\mathcal{O}(\alpha_s)$ the jet function is describes by the one-loop $1 \rightarrow 2$ splitting of a quark weighted by the energy of each particle in the loop



Heavy Quark Jet Function



$$\delta = \frac{im_Q}{p_T R_L}$$

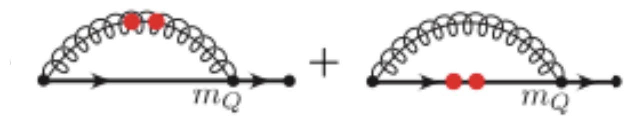
$$J_Q^{[N]}(R_L, m_Q)|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi} \int dx \frac{2(1-(1-x)^N - x^N)\left[2x^3 + (1+x^2)(x+\delta)(x+\bar{\delta}) \ln \frac{\delta\bar{\delta}}{(x+\delta)(x+\bar{\delta})}\right]}{(-1+x)(x+\delta)(x+\bar{\delta})}$$

$$J_Q^{[2]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\delta^4 - 4\delta^3 + 2\delta^2 - 3\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(9\delta^2 + \frac{31}{6}\right)\right\} + c.c\,,$$

$$J_Q^{[3]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{3}{2}\delta^4 - 6\delta^3 + 3\delta^2 - \frac{9}{2}\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(\frac{27}{2}\delta^2 + \frac{31}{4}\right)\right\} + c.c\,,$$

$$J_Q^{[4]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{2}{3}\delta^6 - \frac{16}{5}\delta^5 - \delta^4 - \frac{20}{3}\delta^3 + 4\delta^2 - \frac{83}{15}\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(\frac{106}{15}\delta^4 + \frac{74}{5}\delta^2 + \frac{1417}{150}\right)\right\} + c.c\,,$$

Heavy Quark Jet Function



$$J_Q^{[2]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-3\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{M^2}\right]-\frac{49}{6}\right\},$$

$$J_Q^{[3]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{9}{2}\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{M^2}\right]-\frac{47}{4}\right\},$$

$$J_Q^{[4]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{83}{15}\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{M^2}\right]-\frac{6611}{450}\right\},$$

$$\delta = \frac{i m_Q}{p_T R_L}$$

- **Mass regulates IR divergences!**
- **The remaining** $\frac{1}{\epsilon}$ **poles are UV poles regulated by renormalization.**

Comparison with massless jet functions

Two-point energy-energy correlator (EEC)

The mass should not affect the UV behavior of the jet function.

This can be seen from comparing the UV poles with the light quark jet function.

$$J_Q^{EEC}(z, M, \mu) = \delta(z)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\rm UV}} + \ln\frac{\mu^2}{M^2}\right) - \frac{19}{6}\right]\right) + \text{finite terms}$$

$$z = \frac{1 - \cos\theta_{ij}}{2}$$

$$J_q^{EEC} = \delta(z) + \frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\frac{1}{\epsilon_{\rm UV}} - \frac{37}{3}\right) + 3\frac{Q^2}{\mu^2}\mathscr{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$

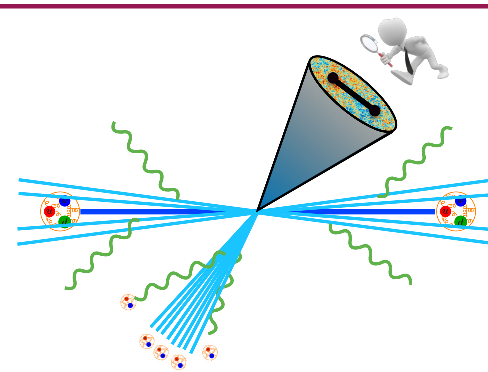
$$\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} = \begin{pmatrix} \frac{25}{6}C_F & -\frac{7}{15}n_f \\ -\frac{7}{6}C_F & \frac{14}{5}C_A + \frac{2}{3}n_f \end{pmatrix}$$

Heavy quark jet function

Result

$$J_Q^{EEC}(z,M,\mu) = \delta(z)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[\left(-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\right)\left(\frac{1}{\epsilon_{\mathrm{UV}}} + \ln\frac{\mu^2}{M^2}\right) - \frac{19}{6}\right]\right)$$

$$+\frac{\alpha_s C_F}{\pi}\frac{1}{z}\left[\frac{3}{4} - \frac{5}{2}\delta^2 - \frac{\delta^4}{1+\delta^2} + 3\delta^3 \arctan\left(\frac{1}{\delta}\right) + \frac{1}{2}\delta^2\left(1-\delta^2\right)\ln\frac{\delta^2}{1+\delta^2}\right]$$



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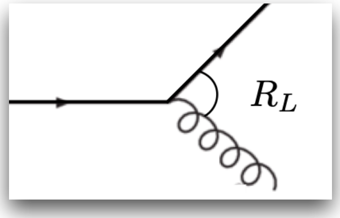
[Craft, Lee, BM, Moult]

Massive jets

Massive Energy Correlator Jet Function

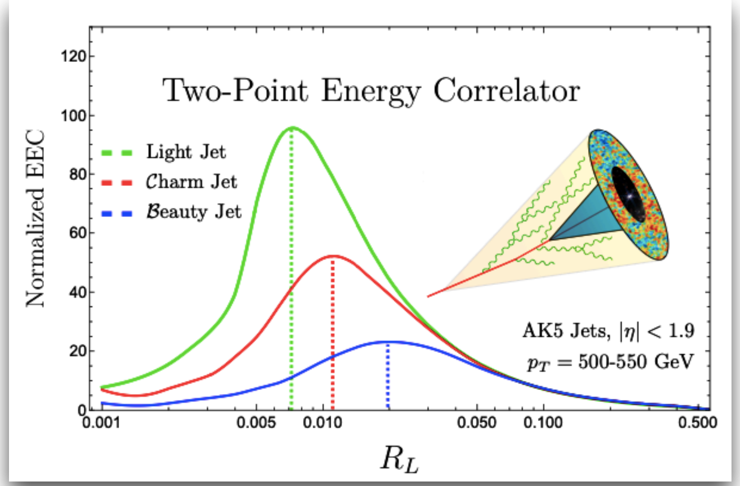
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

Hard function



Virtuality $\sim p_T R_L + m_Q^2$

- **Formation time changes with the mass of the quark.**
- **Can clearly see this from the two-point EEC.**

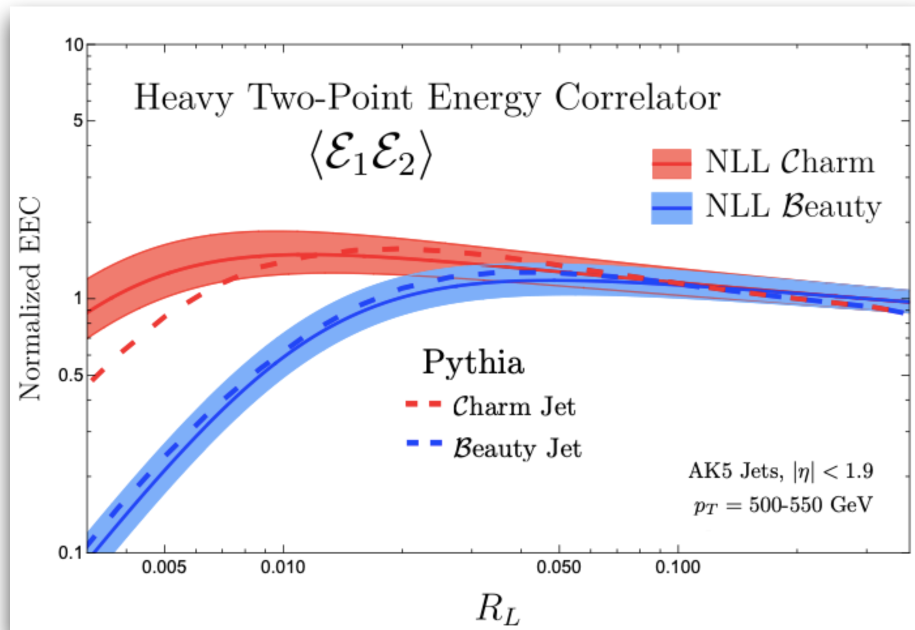


[Craft, Lee, BM, Moult]

Massive two point correlator

A massive jet substructure observable at NLL

- **Scaling behaviour identical to massless case for larger scales.**
- **A turn-over for $R_L \to m_Q/p_T$**
- **The change in the slope is perturbative effect contrary to massless jets:** $R_L \to \Lambda_{QCD}/p_T$
- **The turn-over region is of interest for improving heavy quark description in parton shower.**



[Craft, Lee, BM, Moult]

Dead-cone effect in QCD

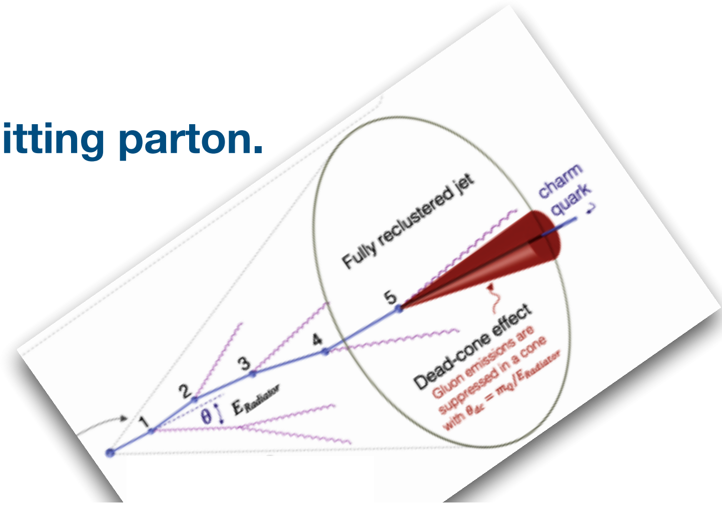
Fundamental phenomena

- **Parton-shower pattern depends on the mass of the emitting parton.**
- **Angular suppression** $\propto \frac{M}{E}$.

Observable used for the observation of the dead-cone effect in LHC data

$$R(\theta) = \frac{1}{N^{\mathrm{D^0\,jets}}} \frac{\mathrm{d}n^{\mathrm{D^0\,jets}}}{\mathrm{d}\ln(1/\theta)} \Big/ \frac{1}{N^{\mathrm{inclusive\,jets}}} \frac{\mathrm{d}n^{\mathrm{inclusive\,jets}}}{\mathrm{d}\ln(1/\theta)} \Bigg|_{k_{\mathrm{T}}, E_{\mathrm{Radiator}}}$$

- **Can we observe the dead-cone with EEC?**



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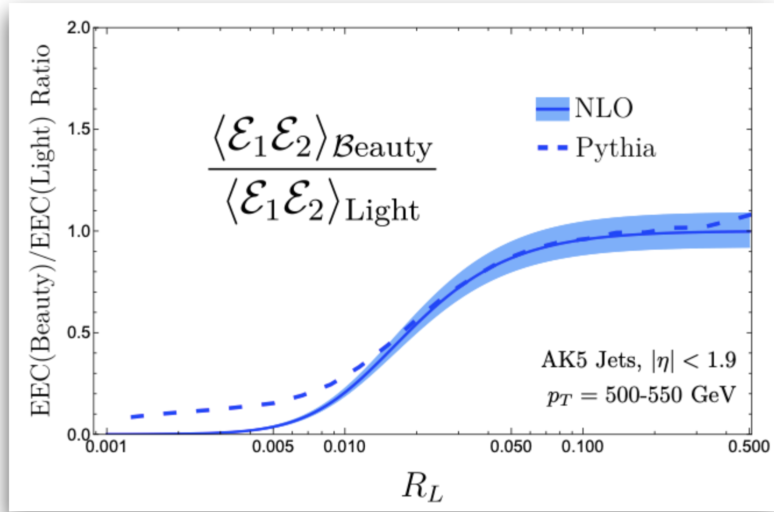
Direct observation of the dead-cone effect in quantum chromodynamics

ALICE Collaboration

Intrinsic mass effects

Dead-cone effect

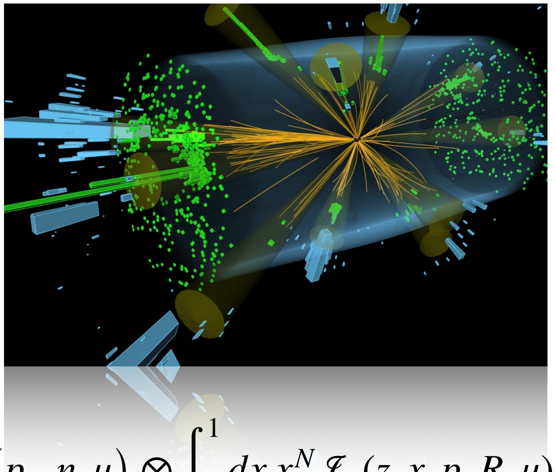


[Craft, Lee, BM, Moult]

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**
- **A transition region related to the quark mass: perturbatively calculable.**
- **Excellent agreement with MC.**
- **Small angle suppression can be interpreted as a dead-cone effect.**

Energy Correlators for light quarks

Factorization Formula



$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \mathscr{H}_i \left(p_T, \eta, \mu\right) \otimes \int_0^1 dx\, x^N \mathscr{J}_{ij}(z, x, p_T R, \mu)\, J_j^{[N]}(z, x, \mu)$$

Can calculate any higher point correlator at the LHC

$$z = \frac{1 - \cos\theta_{ij}}{2}$$

$$x = \frac{2E_i}{Q}$$

Hard function: includes pdfs

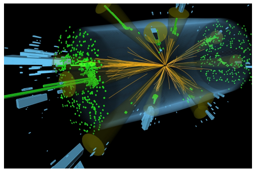
Matching coefficient, jet algorithm

Energy correlator jet function

[Lee, BM, Moult]

Two-point energy correlator

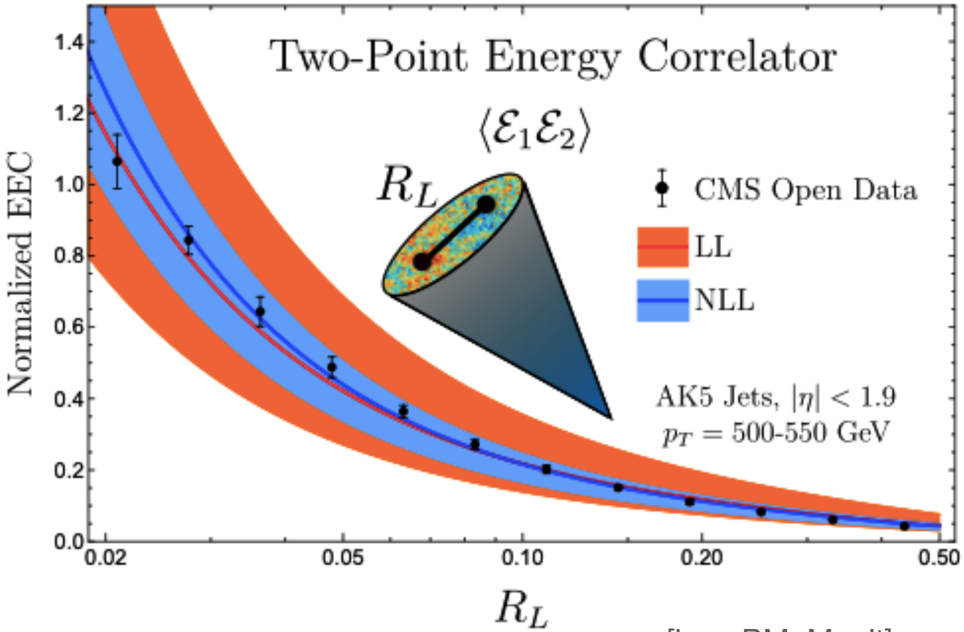
The simplest jet substructure observable



- The complicated LHC environment is described by a simple observable!
- Probe the OPE structure of $\langle \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\rangle$

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi\rangle \sim \sum \theta^{\gamma_i}\mathcal{O}_i(\vec{n}_1)$$

- A jet substructure observable that can test quantum scaling behavior of operators.

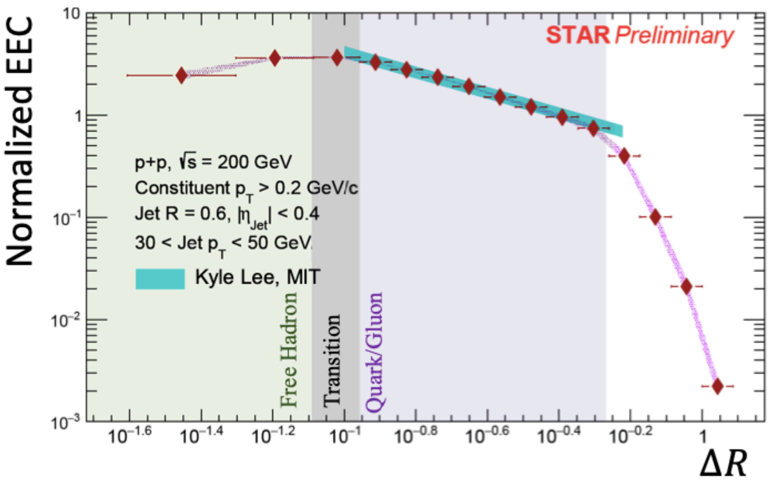


[Lee, BM, Moult]

Experimental results

Talk by N.Sahoo and A.Tamis at
HARD PROBES-March 2023

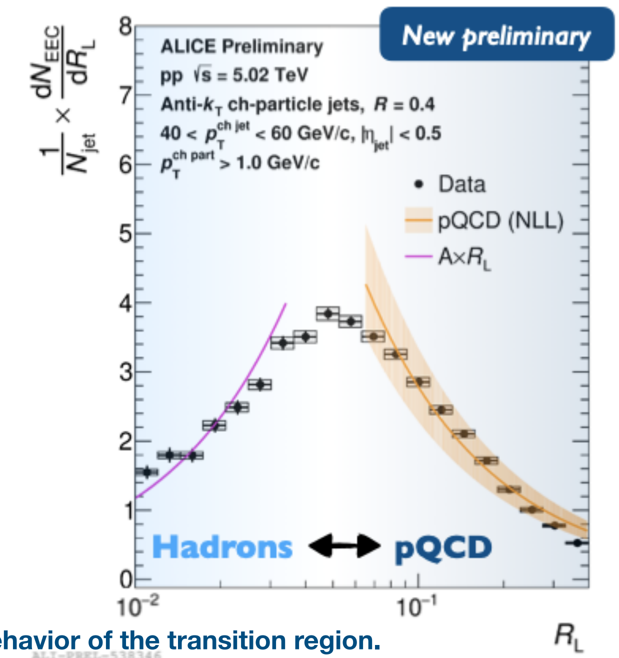
- STAR collaboration $\sqrt{s} = 200GeV$



$$\text{Normalized EEC} = \frac{1}{\sum_{\text{jets}} \sum_{i \neq j} \frac{E_i E_j}{p_{T,\text{jet}}^2}} \frac{d\left(\sum_{\text{jets}} \sum_{i \neq j} \frac{E_i E_j}{p_{T,\text{jet}}^2}\right)}{d\,(\Delta R)}$$

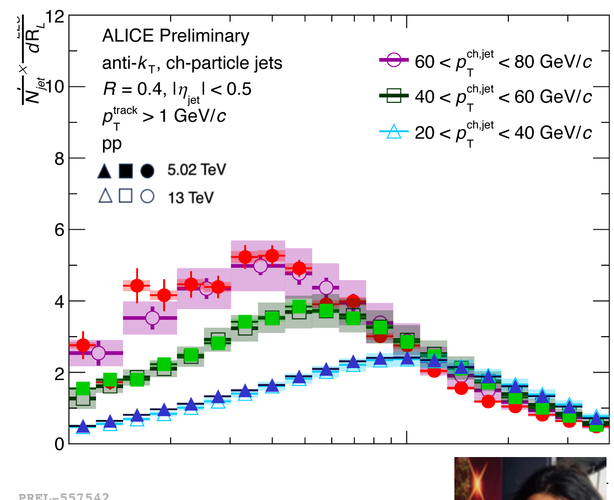
Direct observation of the transition from free hadrons to quarks/gluons at a universal scaling!

- ALICE collaboration $\sqrt{s} = 5TeV,\ 20GeV,\ 40GeV,\ 60GeV$



Universal behavior of the transition region.

Preliminary results from Ananya Rai



Talk by J.Mulligan and R.Cruz-Torres
at HARD PROBES-March 2023

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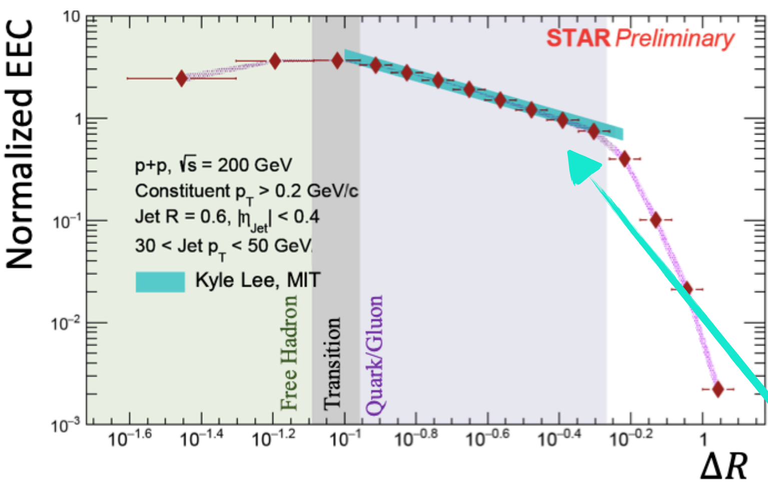

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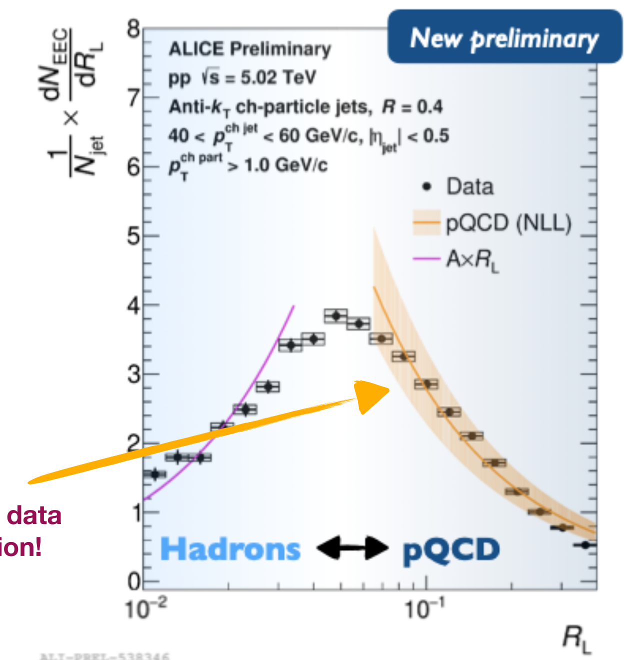

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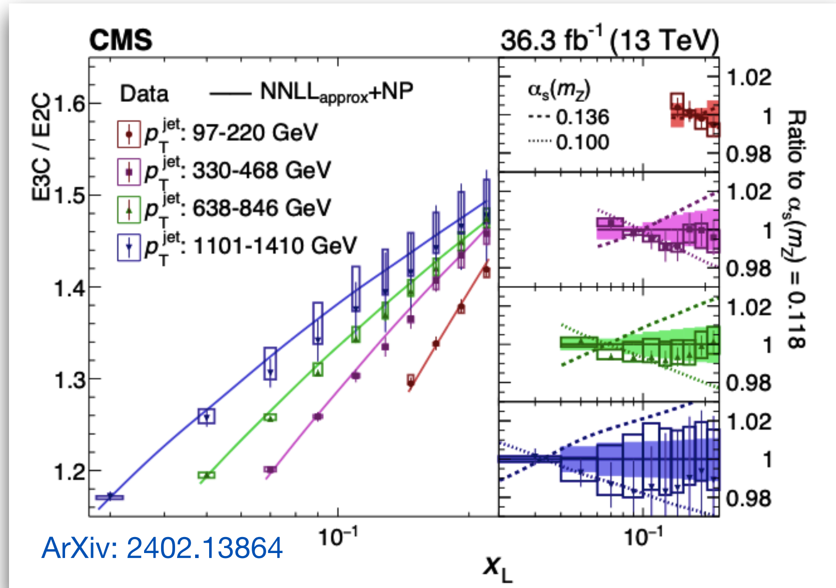
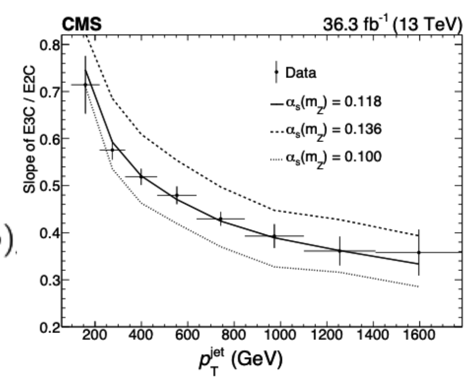


Extraction
of the strong coupling

$$\alpha_S(m_Z) = 0.1229^{+0.0014}_{-0.0012}\,(\text{stat})^{+0.0030}_{-0.0033}\,(\text{theo})^{+0.0023}_{-0.0036}\,(\text{exp})$$



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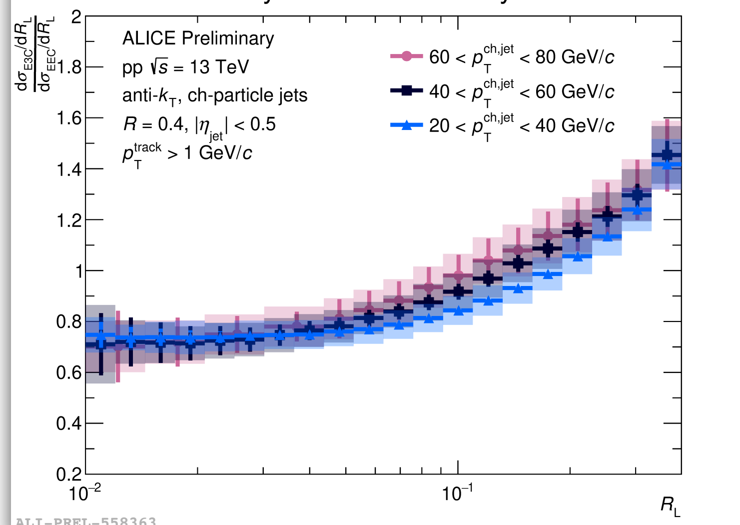

Experimental results

an and R.Cruz-Torres
OBES-March 2023

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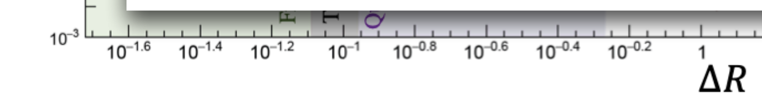

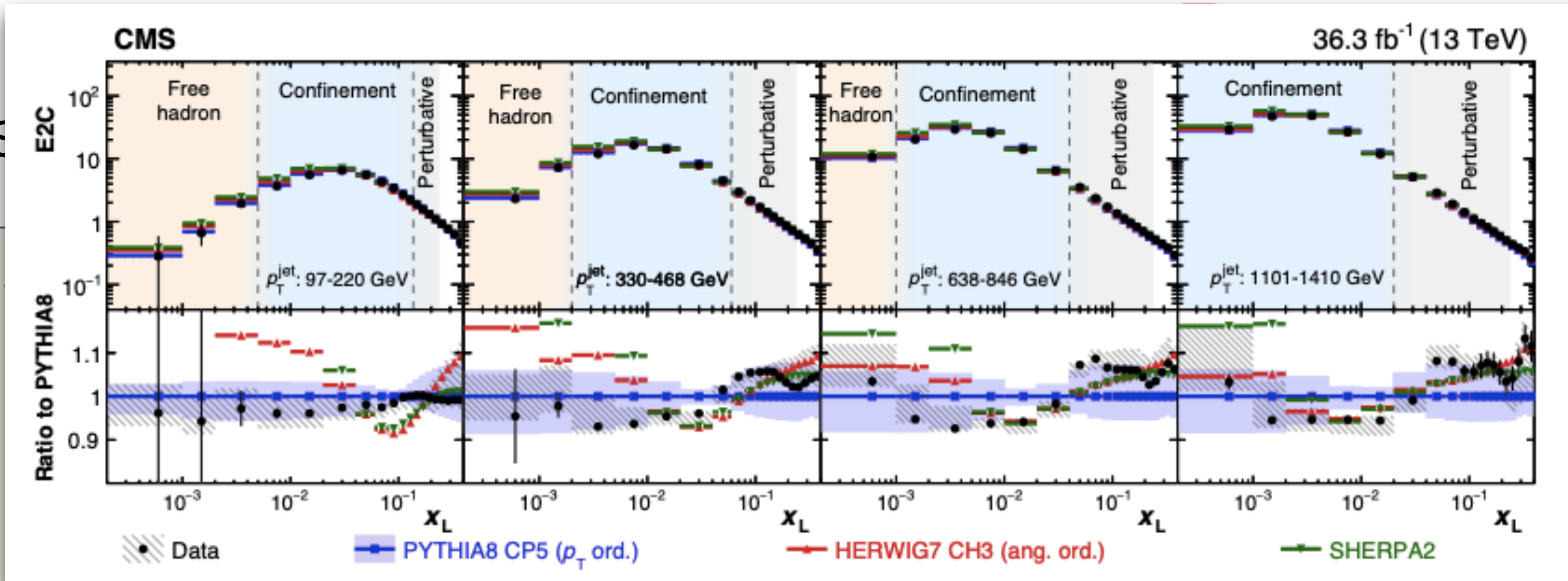

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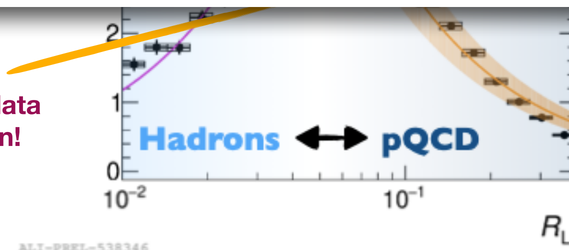
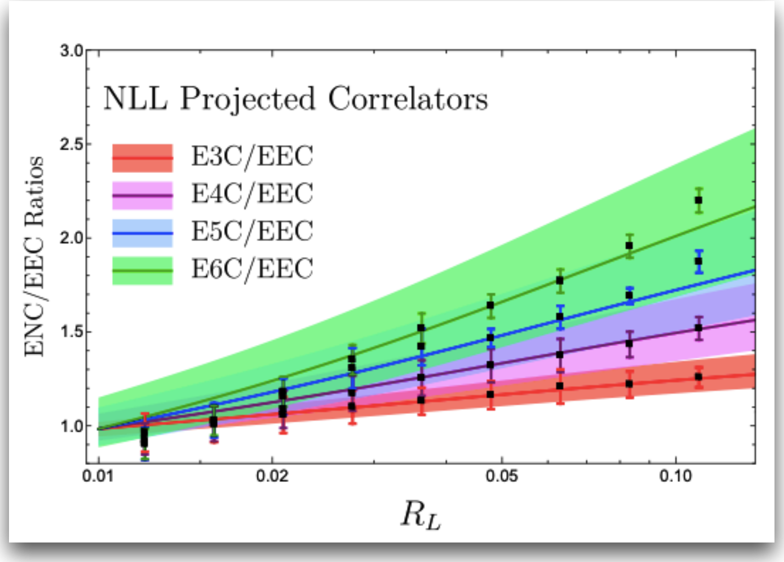


Universal behavior of the transition region.

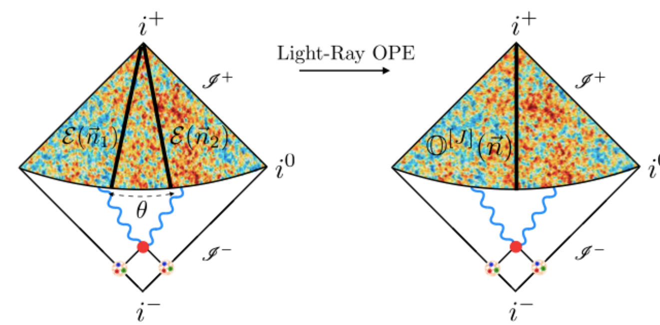
Higher point correlators



The light-ray OPE

- The leading scaling behavior at the LHC is described by the leading terms in the OPE: **twist two light-ray operators**.
- Light-ray OPE is a rigorous and convergent expansion in CFT.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle = \sum c_i \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$



$$\langle \varepsilon\left(\vec{n}_1\right) \varepsilon\left(\vec{n}_2\right) \cdots \varepsilon\left(\vec{n}_k\right) \rangle = \frac{1}{R_L^2} \left\{ f_q^{[k]}\left(u_i, v_i\right) \mathbb{O}_q^{[k+1]}\left(\vec{n}_1\right) + f_g^{[k]}\left(u_i, v_i\right) \mathbb{O}_g^{[k+1]}\left(\vec{n}_1\right) \right\} + \mathcal{O}\left(R_L^0\right)$$

$$u_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_3}}{x_{i_1 i_3} x_{i_2 i_4}}\right)^2 \qquad v_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_4}}{x_{i_1 i_4} x_{i_2 i_3}}\right)^2$$

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ \left(iD^+\right)^{J-1} \psi,$$

$$\vec{\mathbb{O}}^{[J]} = \left(\mathbb{O}_q^{[J]}, \mathbb{O}_g^{[J]}\right)^T = \lim_{r \to \infty} r^2 \int_0^\infty dt \vec{\mathcal{O}}^{[J]}(t, r\vec{n})$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \left(iD^+\right)^{J-2} F_a^{\mu+}$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

Leading twist light-ray OPE

Control scaling at leading power

- **Twist-2 operators in QCD are characterized by a spin J and transverse spin j=0,2.**
- **They can be transformed to a twist-2 light-ray operator vector parametrized by J**

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+ \left(iD^+\right)^{J-1}\psi,$$

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$$\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}\left(iD^+\right)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

$$\xrightarrow{\lim_{r\to\infty} r^2\int_0^\infty dt}$$

$$\vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

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Unpolarized

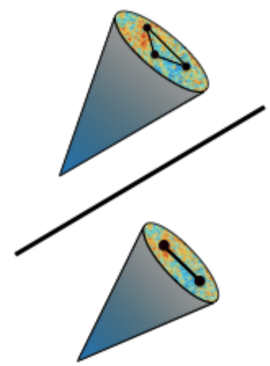
Polarized

Unpolarized Scaling

LHC scenario

- **Probe the unpolarized spin $j = 0$ operators**
- **The leading scaling behavior is determined by the anomalous dimension $\gamma(N+1)$ for an operator of spin $N+1$.**

$\rightarrow$ can isolate the anomalous dimensions!

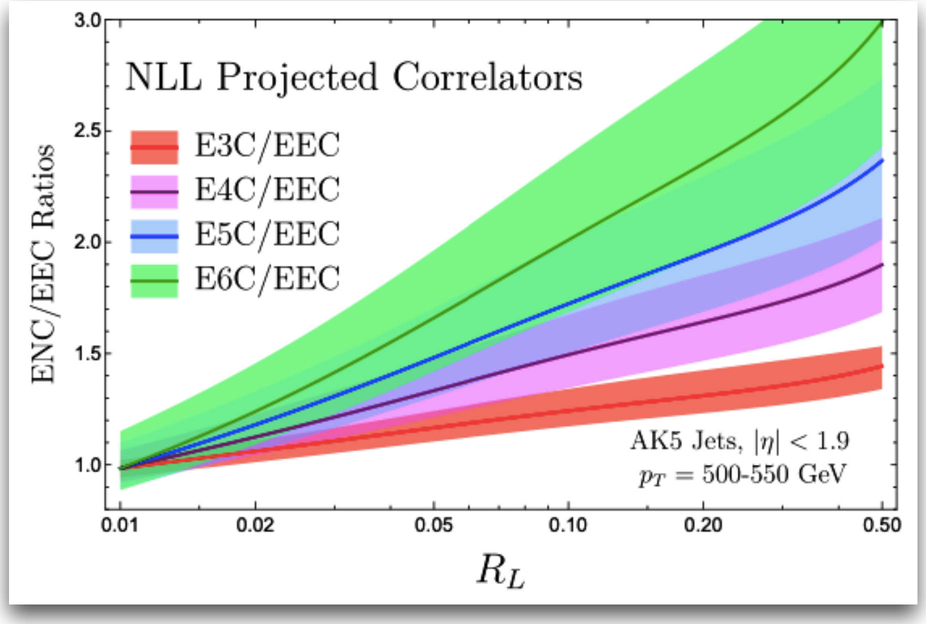


$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$

The jet spectrum

Higher-point correlators

- **Asymptotic energy flux directly probes the spectrum of (twist-2) lightray operators at the quantum level**
- **Ratio of the higher-point correlators with the two-point isolates anomalous scaling!**
- **The anomalous scaling behavior depends on N (slope increases with N)**
- **First hand probe of the anomalous dimensions of QCD operators.**



[Lee, BM, Moult]
[Chen, Moult, Zhang, Zhu]

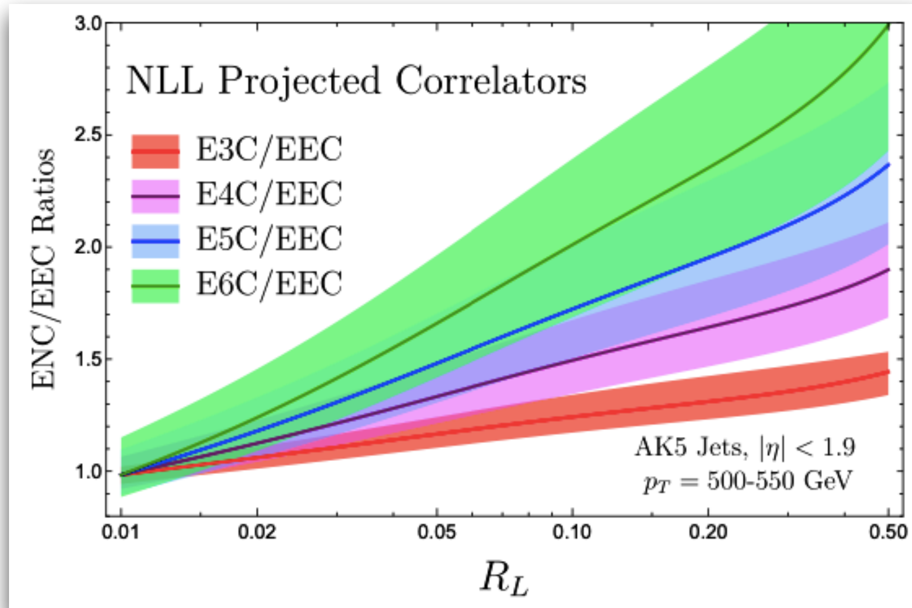
The jet spectrum

Higher-point correlators

- **Non-perturbative effects cancel in the ratio**
- **A clean measurement of strong coupling**

$$\theta^{\gamma} \to \exp(\frac{\hat{\gamma}}{2\beta_0} \ln \frac{\alpha_s(\theta Q)}{\alpha_s(Q)})$$

- **Can be observed at the high energies at the LHC at high precision**



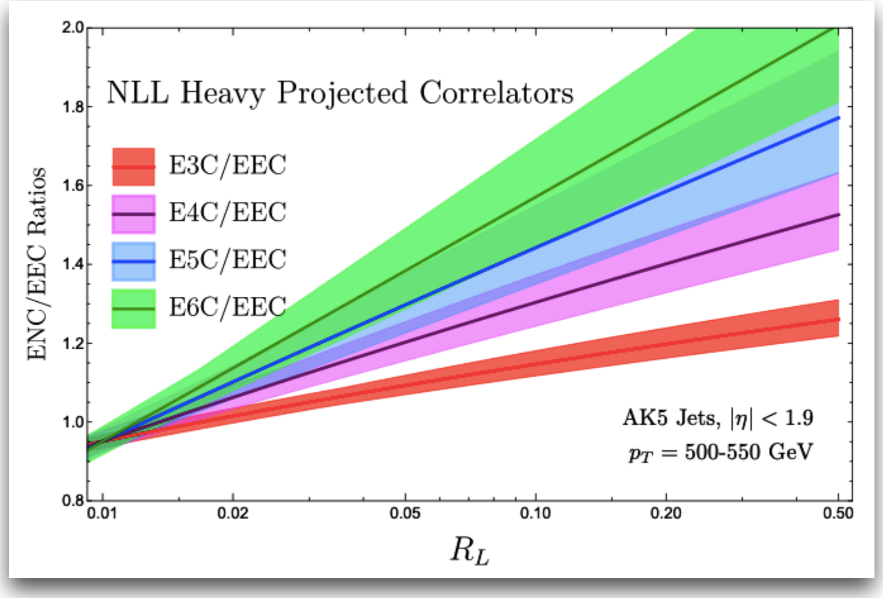
[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

Heavy Projected Energy Correlators

Resolve the UV scaling behaviour

- Ratios of higher point correlators with the two point EEC are independent of IR effects, including quark mass.
- The exact behavior as the massless case.
- Non-trivial cross check of the factorization theorem!
- Anomalous dimensions should not be affected by the IR physics.

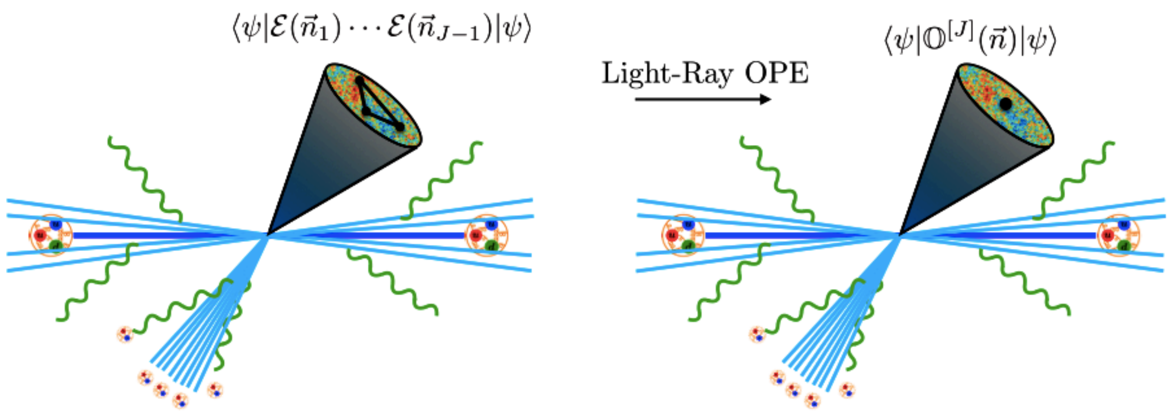


[Craft, Lee, BM, Moult]

Jet substructure from first principles!

- Energy correlator is a jet substructure observable defined from first principles in QFT

⇒ No ambiguity between what is measured and the theory calculation.



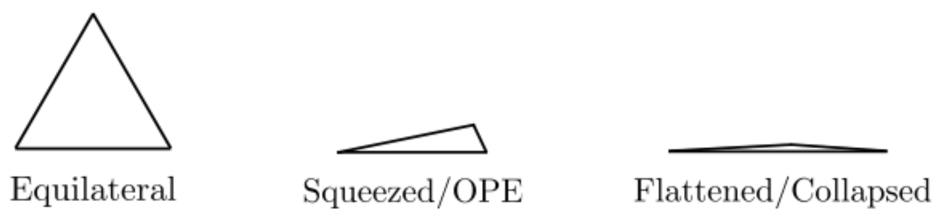
- Formalism can be applied for any conserved charge for LHC processes.
- No jet grooming or pruning is needed to extract the final results, pure QFT calculation!
- Not sensitive to soft and wide angle radiations.

Higher point correlators in $1 \to 3$ splittings

Shape dependance

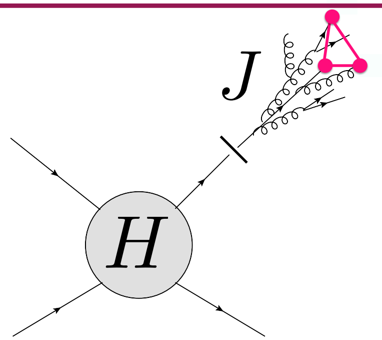
- Different “shapes” for higher point functions are distinguished by different physics.
- It is interesting to probe such shape dependance.
- First time calculated in the context of cosmological three point function.

[Maldacena and Pimentel]



Higher Point Splittings

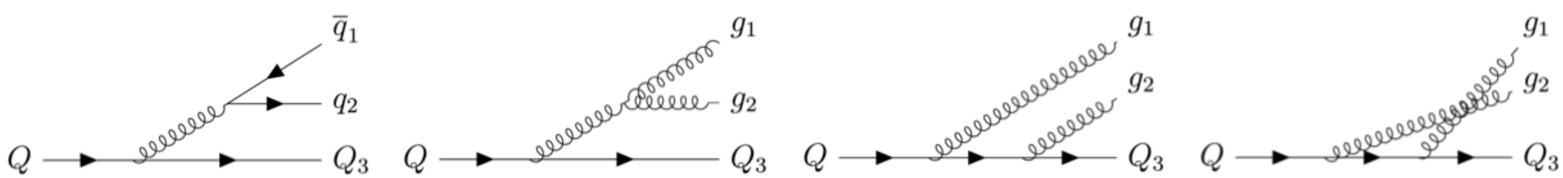
$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dz} = \frac{\int d^4x e^{iq\cdot x}\left\langle \mathcal{O}(x)\varepsilon\left(\vec{n}_1\right)\varepsilon\left(\vec{n}_2\right)\varepsilon\left(\vec{n}_3\right)\mathcal{O}^\dagger(0)\right\rangle}{\int d^4x e^{iq\cdot x}\left\langle \mathcal{O}(x)\mathcal{O}^\dagger(0)\right\rangle}$$



$$\frac{1}{\sigma_{\rm tot}}\frac{d^3\Sigma}{dx_1 dx_2 dx_3} = \sum_{i,j,k}\int \frac{E_iE_jE_k}{Q^3} d\sigma\delta\left(x_1 - \frac{1-\cos\theta_{ij}}{2}\right)\delta\left(x_2 - \frac{1-\cos\theta_{jk}}{2}\right)\delta\left(x_3 - \frac{1-\cos\theta_{kl}}{2}\right).$$

- **Presence of a mass term: analytic integration becomes more complex.**
- **At the same time more interesting analytic structures to study.**

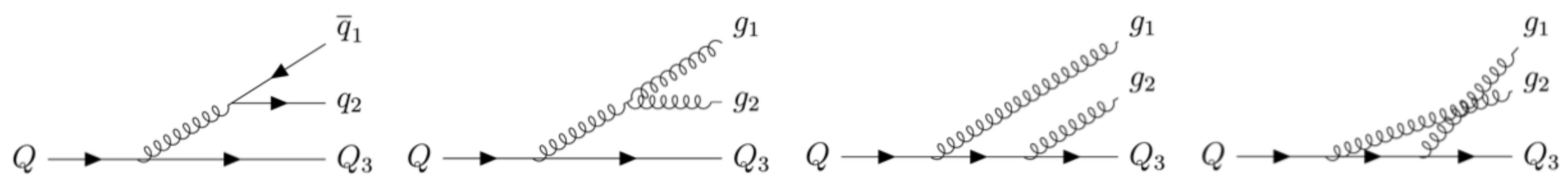
Higher Point Splittings



- Compute first the massive $1 \to 3$ QCD splitting functions
- There are four distinct denominator dependencies on the kinematics

$$\frac{1}{s_{ij}s_{ik}}, \frac{1}{s_{ij}s_{jk}}, \frac{1}{s_{ik}s_{jk}}, \frac{1}{s_{ijk}} \qquad s_{ijk} = (p_i + p_j + p_k)^2$$

Higher Point Splittings



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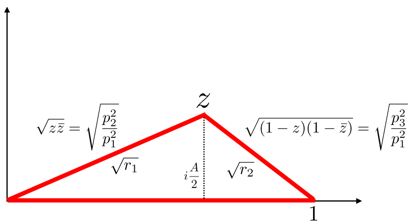
Source of non-trivial elliptic structure

Other cases are multi-polylogs!

[Dhani, Rodrigo, Sborlini]

[Craft, Gonzales, Lee, BM, Moult]

Higher Point Splittings



- The integrant can be written in the compact form

$$G\left(z, \bar{z}, x_L, m/p\right) = \int \frac{d\xi_1 d\xi_2 d\xi_3 \left(\xi_1 \xi_2 \xi_3\right)^2}{\left[\xi_1 \xi_2 + \xi_1 \xi_3 (1-z)(1-\bar{z}) + \xi_2 \xi_3 z\bar{z} + \frac{m^2}{4p^2 x_L}\left(\frac{\xi_1+\xi_2}{\xi_3}\right)\right]^2} P_{ijk} \delta\left(1 - \sum_i \xi_i\right)$$

$$\frac{1-\cos\theta_{23}}{1-\cos\theta_{12}} = z\bar{z}$$

- Integration of any two of the energy fractions reveals a square root of a quartic polynomial in the third variable in the denominator

$$\frac{1-\cos\theta_{23}}{1-\cos\theta_{13}} = (1-z)(1-\bar{z})$$

$$\xi_i = \frac{E_i}{p}$$

$$P_4(\xi_3; z, \bar{z}, \alpha) = x_L^2[(z-\bar{z})^2\, \xi_3^4 + 2\,(2z\bar{z} - z - \bar{z})\, \xi_3^3 + (1-\alpha)\, \xi_3^2 + \alpha\, \xi_3]$$

Higher Point Splittings

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Elliptic Integrals!

Elliptic Structure

- The final result for the three-point massive EEC takes the compact form:

$$\int_0^1 d\xi_3 \left[R_1(\xi_3) + \frac{R_2(\xi_3)}{\sqrt{P_4}} + \sum_i \left(R_3^i(\xi_3) + \frac{R_4^i(\xi_3)}{\sqrt{P_4}} \right) \log \left(R_5^i(\xi_3) + \frac{R_6^i(\xi_3)}{\sqrt{P_4}} \right) \right]$$

- Can use a basis of kernels to rewrite the result in terms of eMPLs.
- **Similar analogy to the higher loop massive amplitudes structure, but for a jet observable**

Elliptic Structure

- Can use a basis of kernels to rewrite the result in terms of eMPLs.

$$\int_0^1 d\xi_3 \left[R_1(\xi_3) + \frac{R_2(\xi_3)}{\sqrt{P_4}} + \sum_i \left(R_3^i(\xi_3) + \frac{R_4^i(\xi_3)}{\sqrt{P_4}}\right) \log\left(R_5^i(\xi_3) + \frac{R_6^i(\xi_3)}{\sqrt{P_4}}\right)\right]$$

- Similar analogy to the higher loop massive amplitudes structure, but for a jet observable.

- **Kernel Basis:**

$$\int \frac{1}{\sqrt{P4}}, \quad \int \frac{\xi}{\sqrt{P4}}, \quad \int \frac{\xi^2}{\sqrt{P_4}}, \quad \int \frac{1}{(\xi - p)\sqrt{P_4}}$$

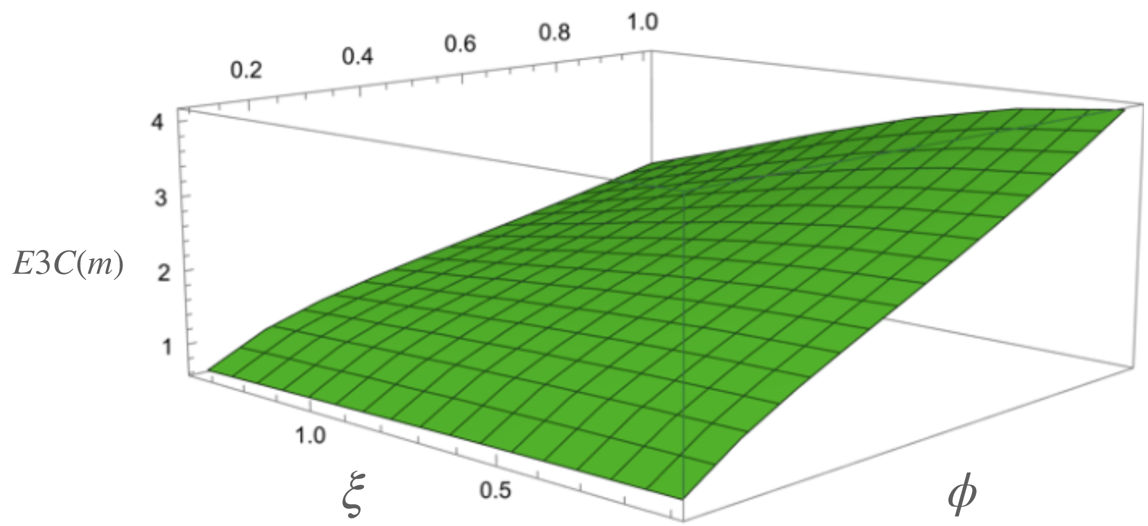
$$\Psi_1(c, x, \vec{a}) = \frac{1}{x-c},$$
$$\Psi_{-1}(c, x, \vec{a}) = \frac{y_c}{y(x-c)} + Z_4(c, \vec{a})\,\frac{c_4}{y},$$
$$\Psi_1(\infty, x, \vec{a}) = -Z_4(x, \vec{a})\,\frac{c_4}{y},$$
$$\Psi_{-1}(\infty, x, \vec{a}) = \frac{x}{y} - \frac{1}{y}\left[a_1 + 2c_4\, G_*(\vec{a})\right],$$

[Broedel, Duhr, Dulat, Penante, Tancredi]

Numerical results

Non-Gaussianity for the massive case

- **Probe effects of the splitting functions for massive particles.**
- **Small angle: little difference between $1 \to 2$ and $1 \to 3$ splitting**
- **Non-Gaussianity more pronounced for larger ξ**

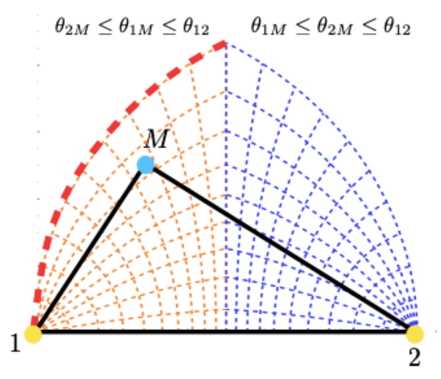


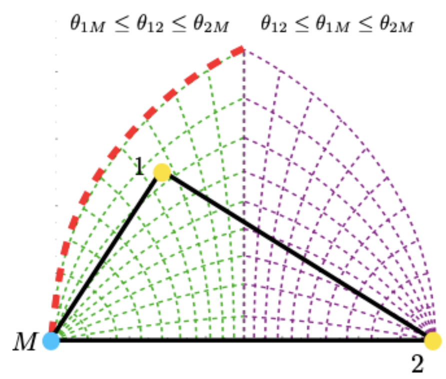
Isolating Mass Effects at Higher Points

- **Different types of detector combinations**

$$\langle \mathcal{E}\mathcal{E}\mathcal{E}_M \rangle, \quad \langle \mathcal{E}\mathcal{E}_M\mathcal{E}_M \rangle, \quad \langle \mathcal{E}_M\mathcal{E}_M\mathcal{E}_M \rangle, \quad \langle \mathcal{E}_{M'}\mathcal{E}_{M'}\mathcal{E}_M \rangle.$$

- **We isolate the case with two identical detectors and one massive one**



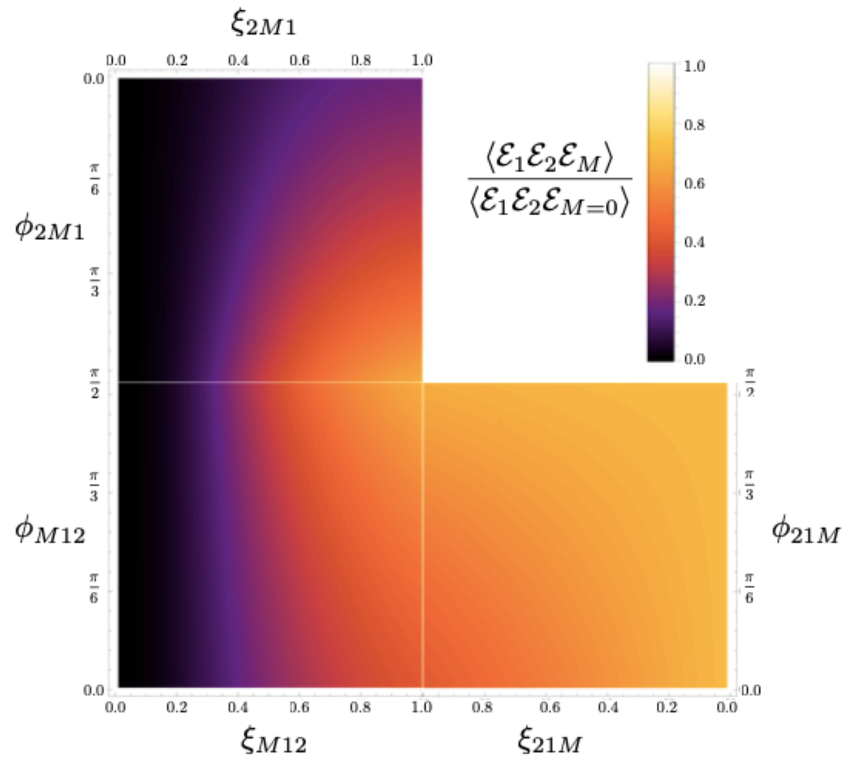


Numerical results

Dead-cone in the triple collinear splitting

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**

$$\xi = \frac{\theta_s}{\theta_m}, \quad \phi = \arcsin\sqrt{1 - \frac{(\theta_l - \theta_m)^2}{\theta_s^2}},$$

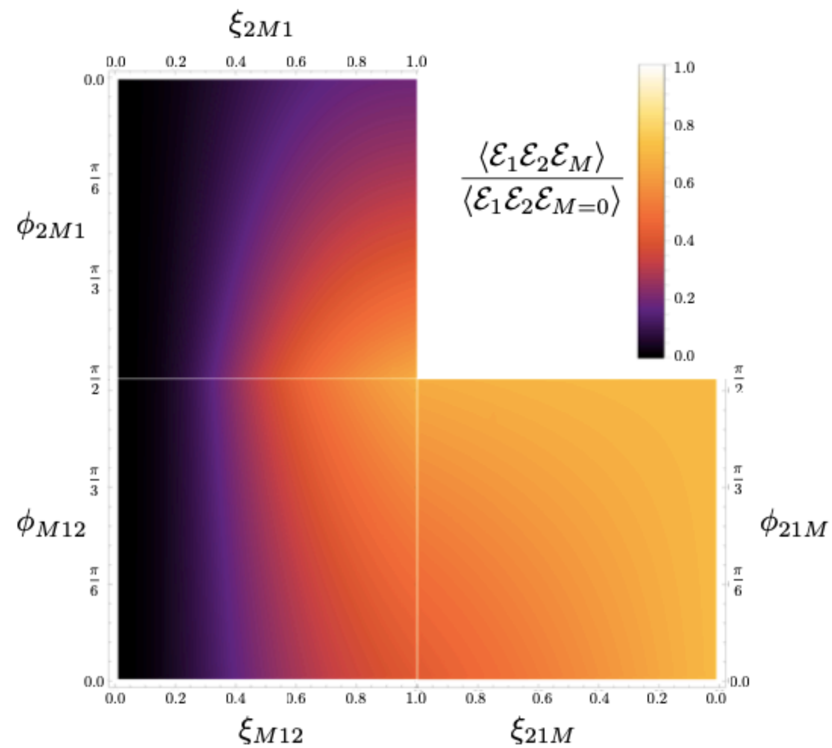


[Work in Progress: Craft, Gonzalez, Lee, BM, Moult]

Numerical results

Dead-cone in the triple collinear splitting

- Ratios of the massive and massless EEC isolate mass (IR) effects.
- A transition region related to the quark mass at small angles is also visible for the 3-point function.
- Small angle suppression can be interpreted as a dead-cone effect.



[Work in progress: Craft, Gonzalez, Lee, BM, Moult]

Future Prospects

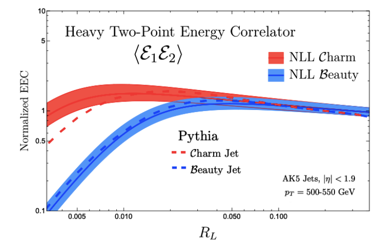
- **Jet modeling in MC simulations: heavy flavours**
- **Precision in parton showers: “reference resummation” for testing DGLAP finite moments.**
- **Understand properties of the QGP: multi-scale problem too, global properties of plasma.**

[Andres, Dominguez, Kunnawalkam Elayawalli, Holguin, Marquet, Moult,…]

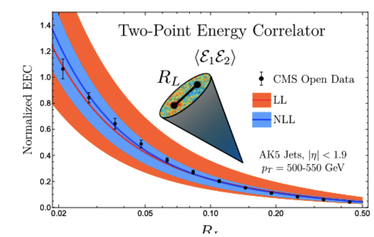
Conclusions

- **Factorization formula for calculating EEC for jet substructure at the LHC.**

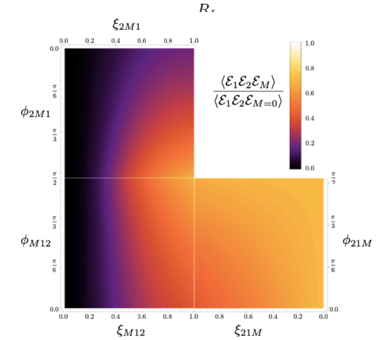
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$



- **Intrinsic mass effects of strongly interacting elementary particles.**
- **Observation of a scale dependance in asymptotic data using EEC**



- **Higher-point correlators can be calculated for LHC and probe anomalous scaling dimension of QCD operators.**
- **Experimental Results confirm the theory calculation.**
- **Exciting applications in different systems in QCD and Heavy Ions**



Thank You!