The Sins of the Priors: Inverse Problems and Statistical Estimation

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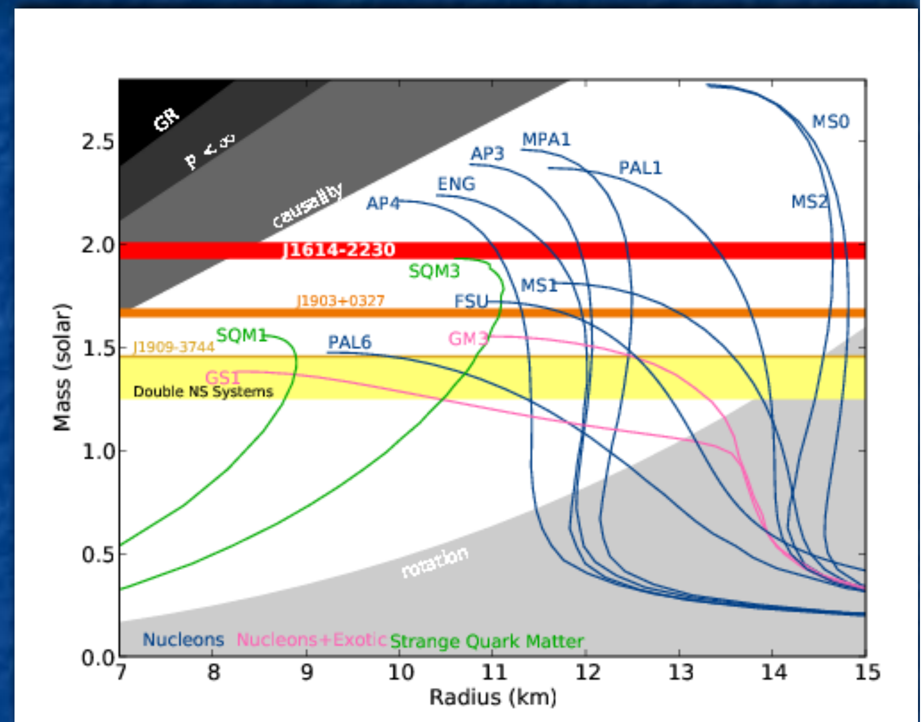
Plan of Talk

- To be provocative: why we shouldn't talk about inverse problems
- How much do priors matter in our inferences about the properties of dense matter?

Dense Matter to an Astronomer

- T=0 (10^9 K << 10^{12} K)
- Equilibrium
- Equation of state, not composition
- Thus P=P(ρ) (or P(n), ...)
- Example: polytrope
  $P=K\rho^\Gamma$
  Piecewise polytrope
- Or smooth slope distribution, e.g., spectral



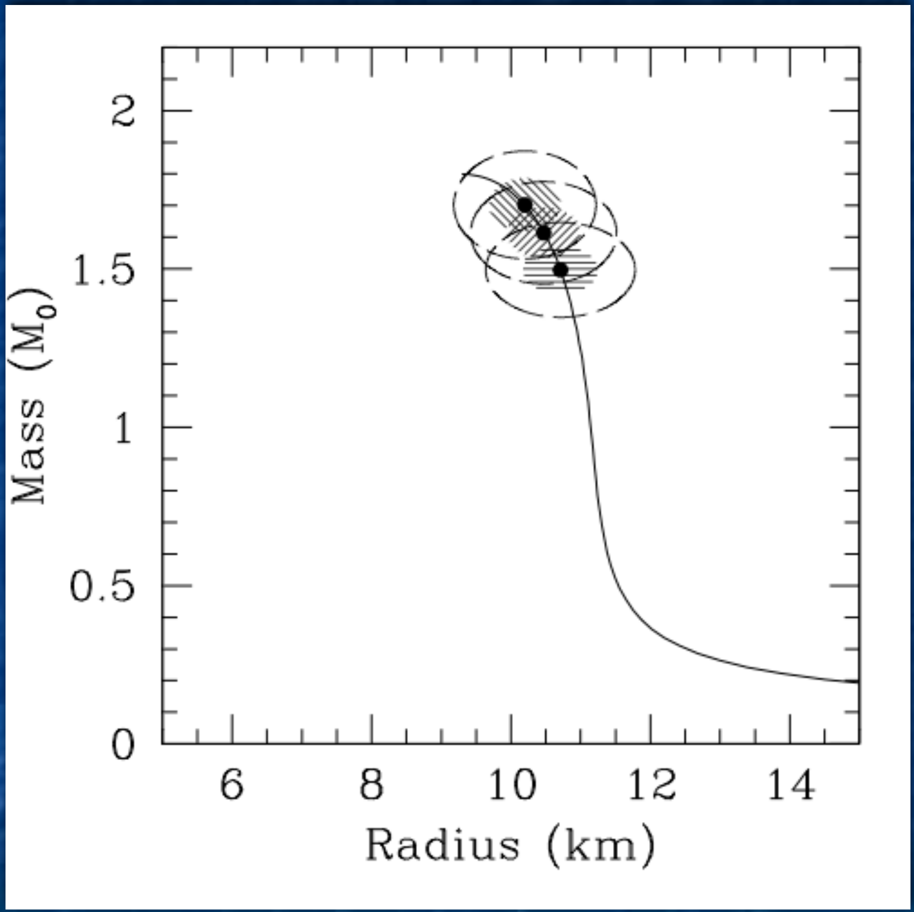
Demorest et al. 2010

Inverse Temptations

- There is a strong temptation to look at the path from measurements to constraints on dense matter as a mathematical problem: just invert the system
- For example, if we have N measurements and an N-parameter model, we just invert. The more measurements the better
- What could be simpler?

Example inversion proposal

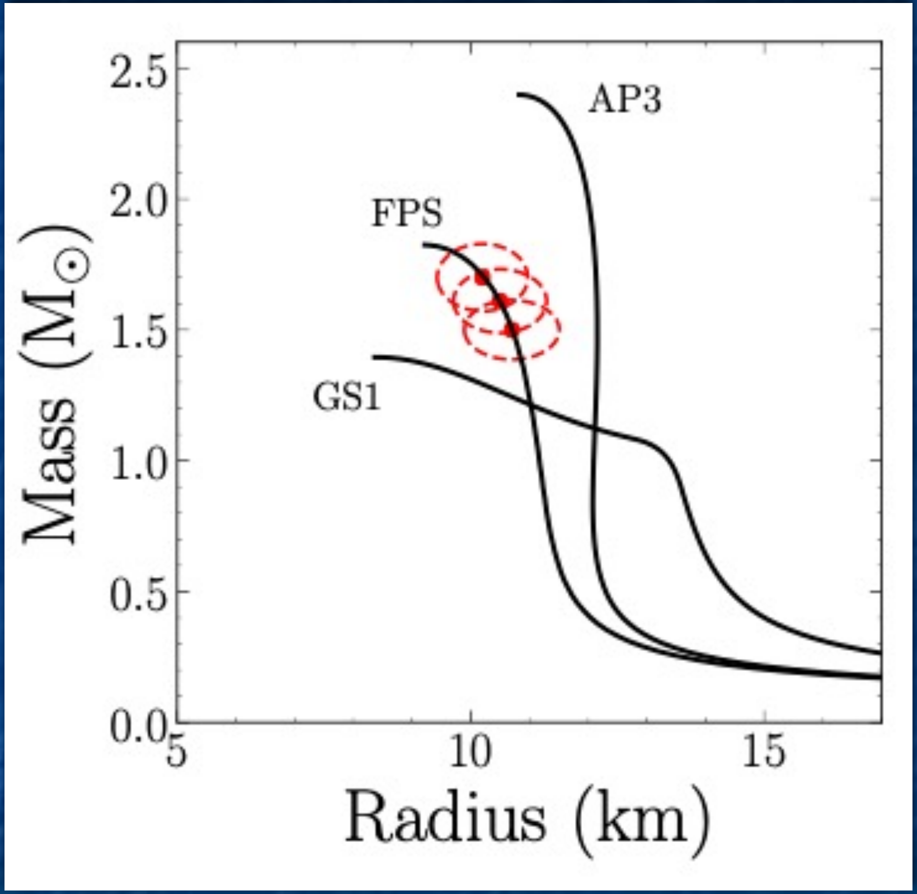
- The idea is that we have a parameterized EOS model with three parameters. Three measurements will determine those parameters.
- Simple and clean, right? Just use Jacobians



Özel and Psaltis 2009

Problems with inversion

- Several issues:
- The inversion is usually singular (crossing points)
- Some NS might not reach highest density
- Want more measurements than parameters! Otherwise we don't test the model



Raaijmakers+ 2018

What should we do instead?

- It may be less exciting (assuming you get excited by mathematical inversion...), but the right approach is statistical estimation
- Standard Bayesian analysis: start with priors, update with likelihood of data
- But this brings out an issue: how dependent are we on our priors?

Priors for EOS inference

- People have their own preferences…
- Our group uses three frameworks:
  Piecewise polytrope (many groups)
  Spectral decomposition (e.g., Lindblom+)
  Gaussian processes (e.g., Landry+Essick)
- Many others are possible
- A point from Andrew Steiner: flat in one domain (e.g., M,R) != flat in another (P,ρ)

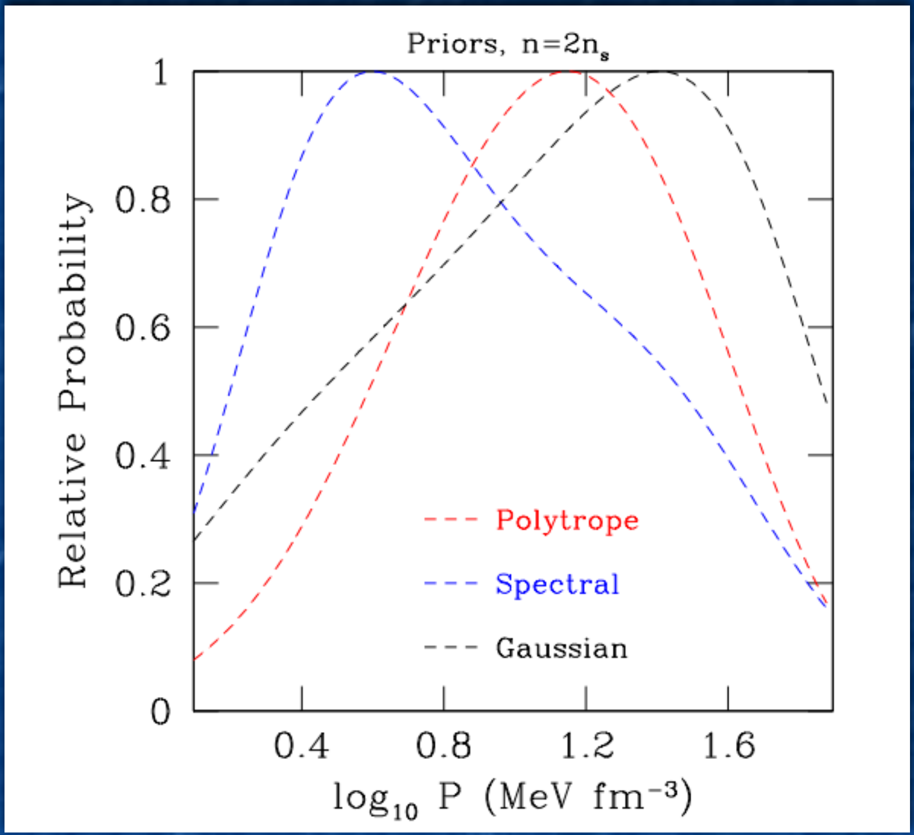
Constraints that we apply

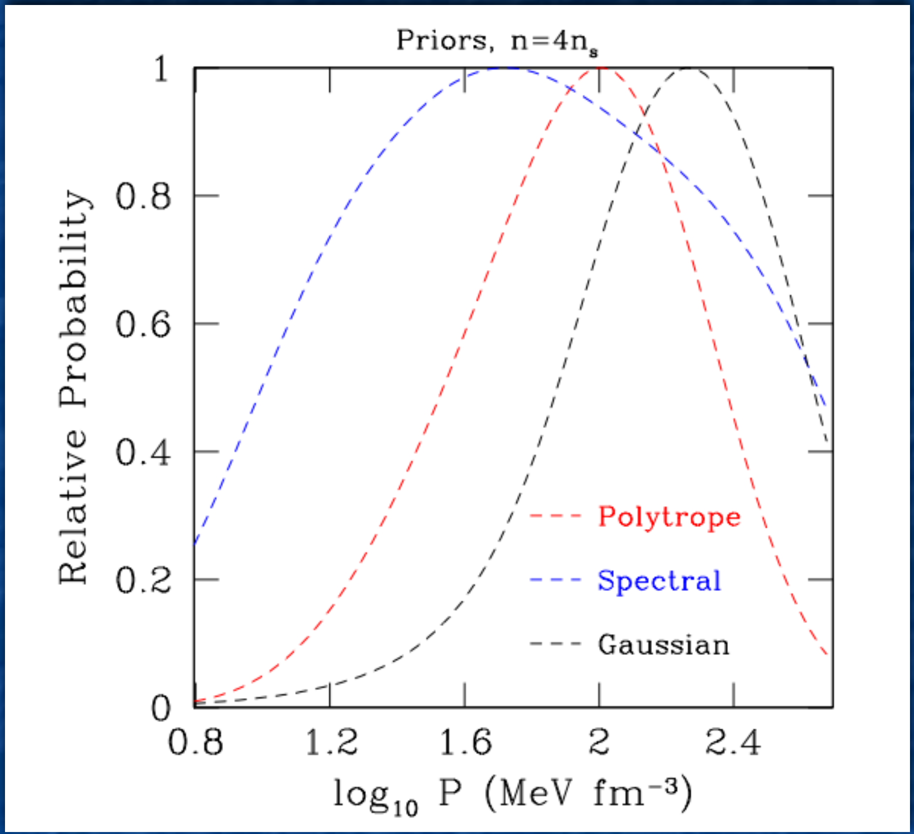
- Symmetry energy at n_{sat}=0.16 fm^{-3}
  Roughly approximated by $e/n-m_nc^2$+16 MeV
  Assume E_{Sym}=32+−2 MeV at n_{sat}
- Masses of three ~2 M_{sun} NS
- Tidal deformability from GW170817
- Mass-radius of J0030 from NICER
- Mass-radius of J0740 from NICER
- Formalism: Miller, Chirenti, Lamb 2020

A diversion on divergences

- Common to use K-L divergence, P|Q: $D_{KL}(P|Q)=\sum P_i \log(P_i/Q_i)$, i is over data set
- Asymmetric! Useful for progressive constraints but not as much for distributions on the same footing, e.g., different priors
- Quote both directions for prior comparison
- Shannon entropy: $S=-\sum p_i \log(p_i)$
  Use entropy over set of 100,000 EOS
  Normalize: S=1 if uniform, =0 for only 1 EOS

Priors at $2n_{sat}$ and $4n_{sat}$



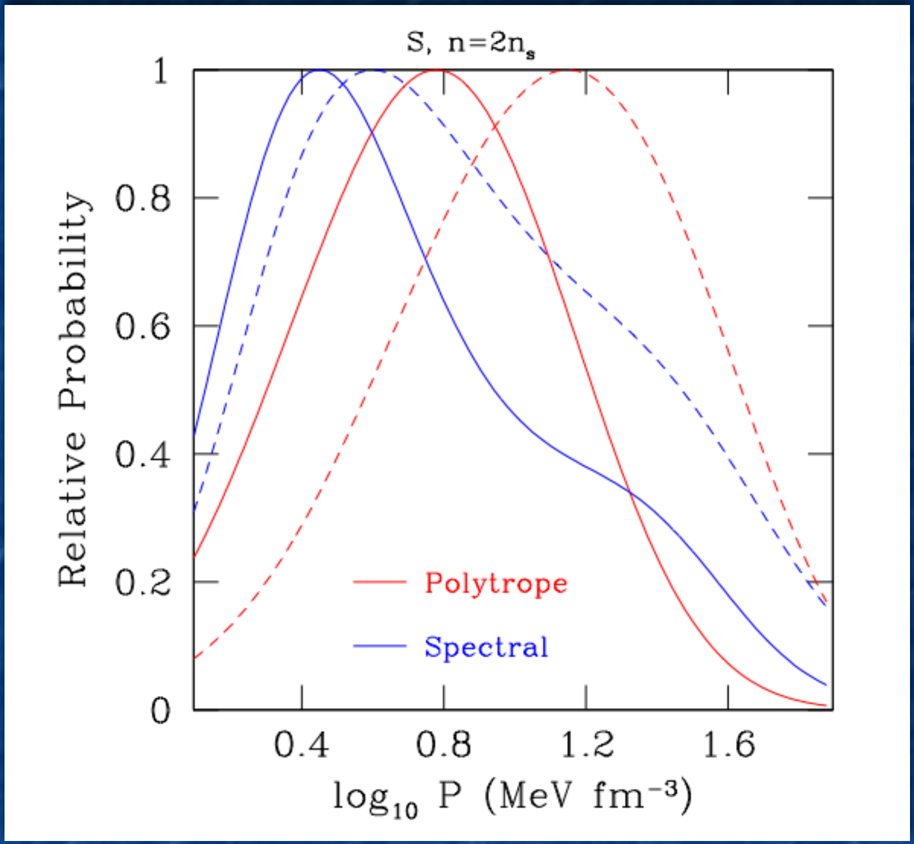


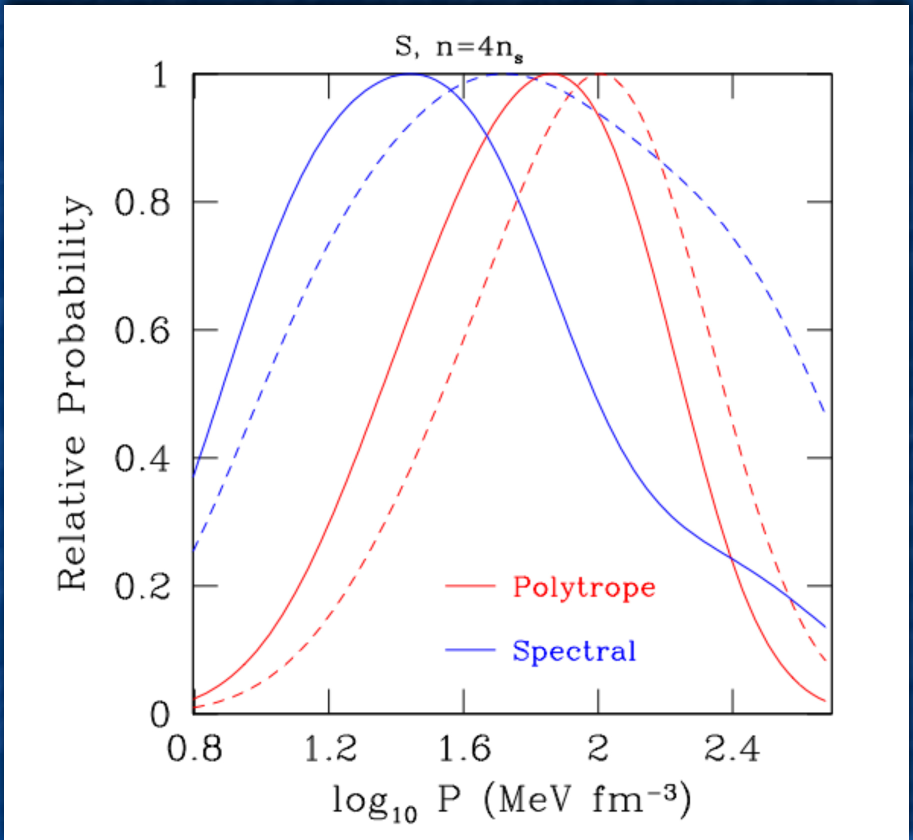
$D_{KL}(P,S)$=0.15 or 0.17

$D_{KL}(P,S)$=0.17 or 0.23

Entropy = 1 in EOS set, by normalization

+Symmetry Energy

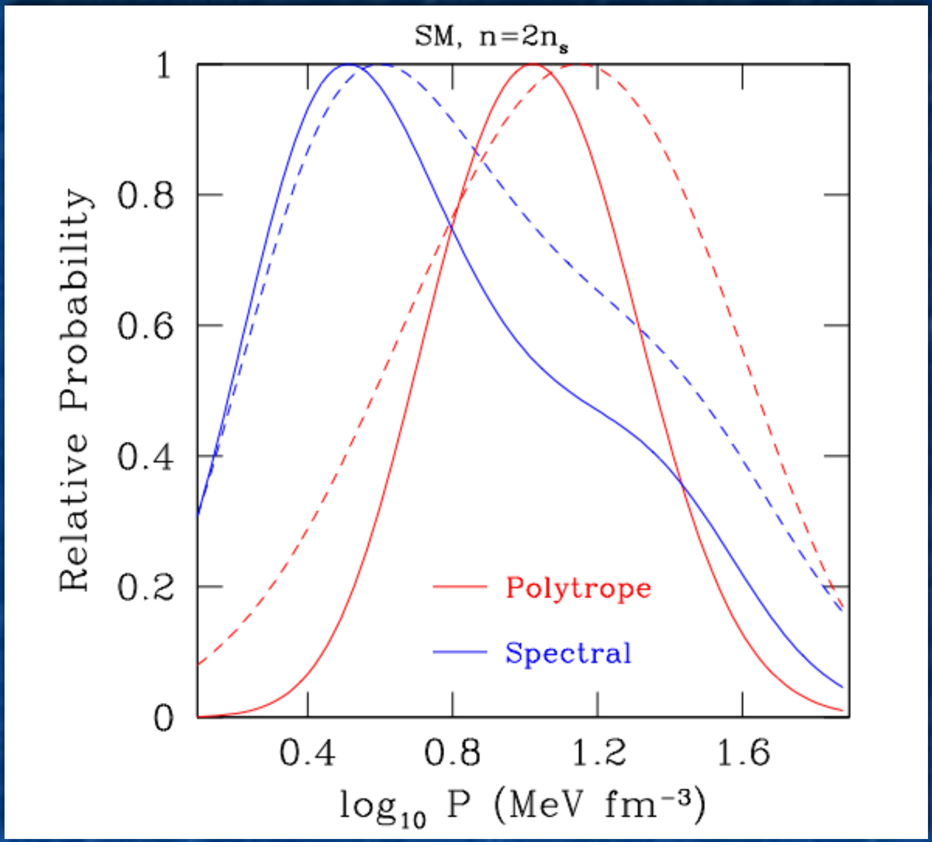


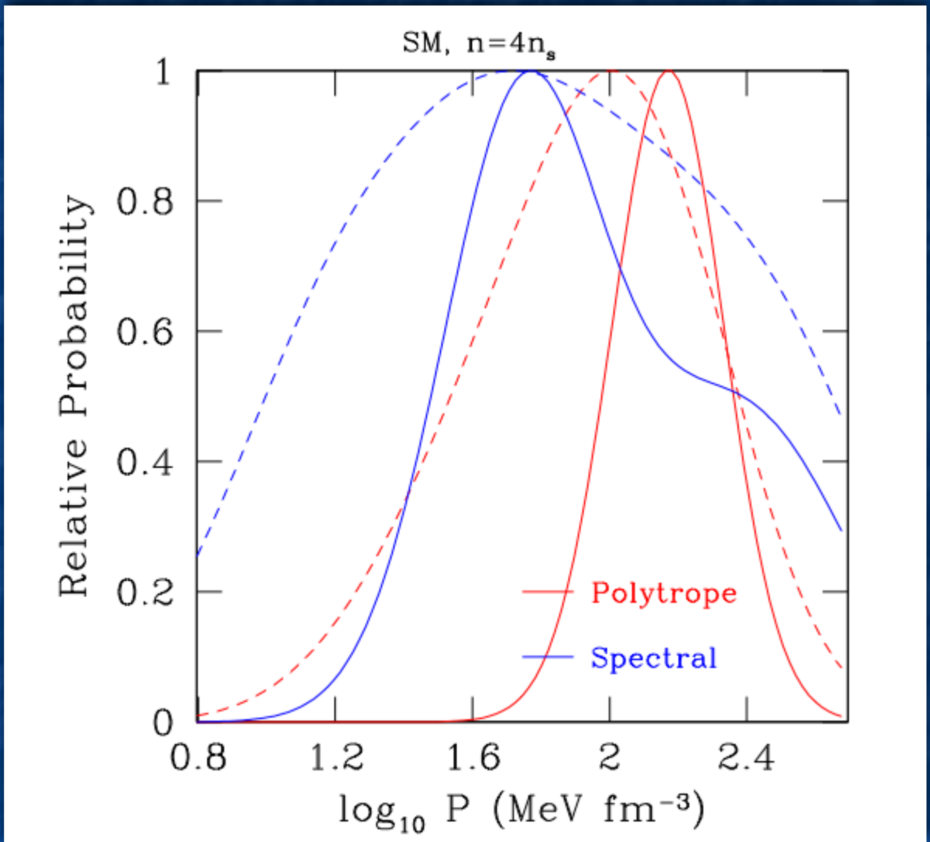


D_{KL} from prior: P=0.39; S=0.06 D_{KL} from prior; P=0.08; S=0.13

Entropy in EOS set is 0.92 for P, 0.97 for S

+High Masses



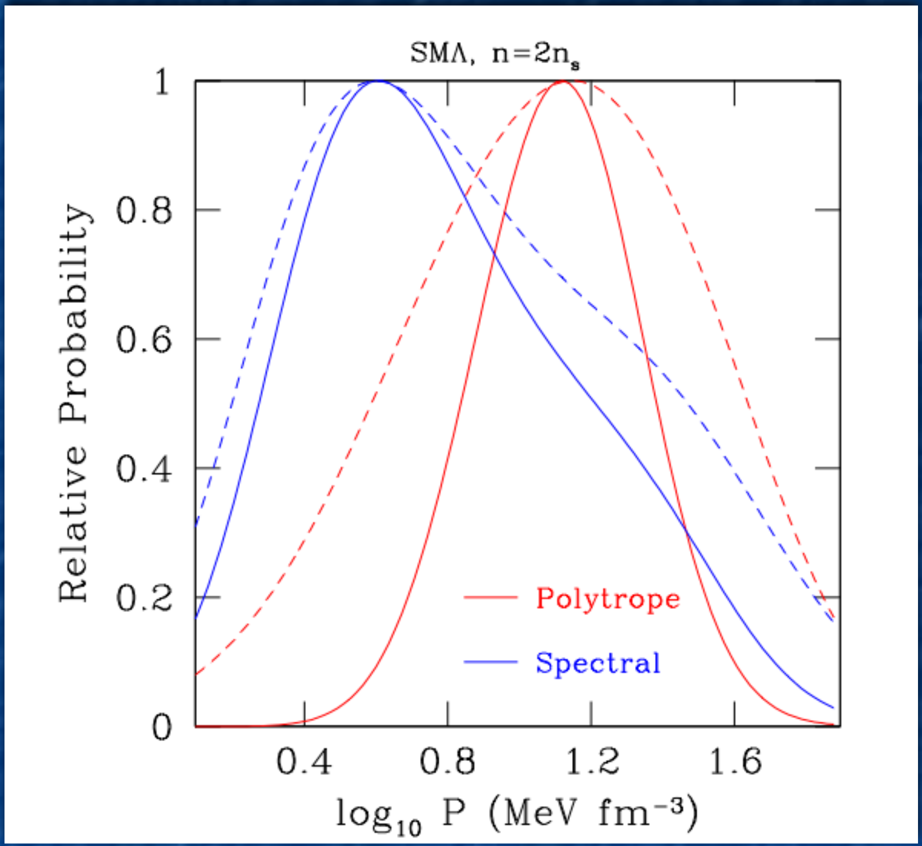


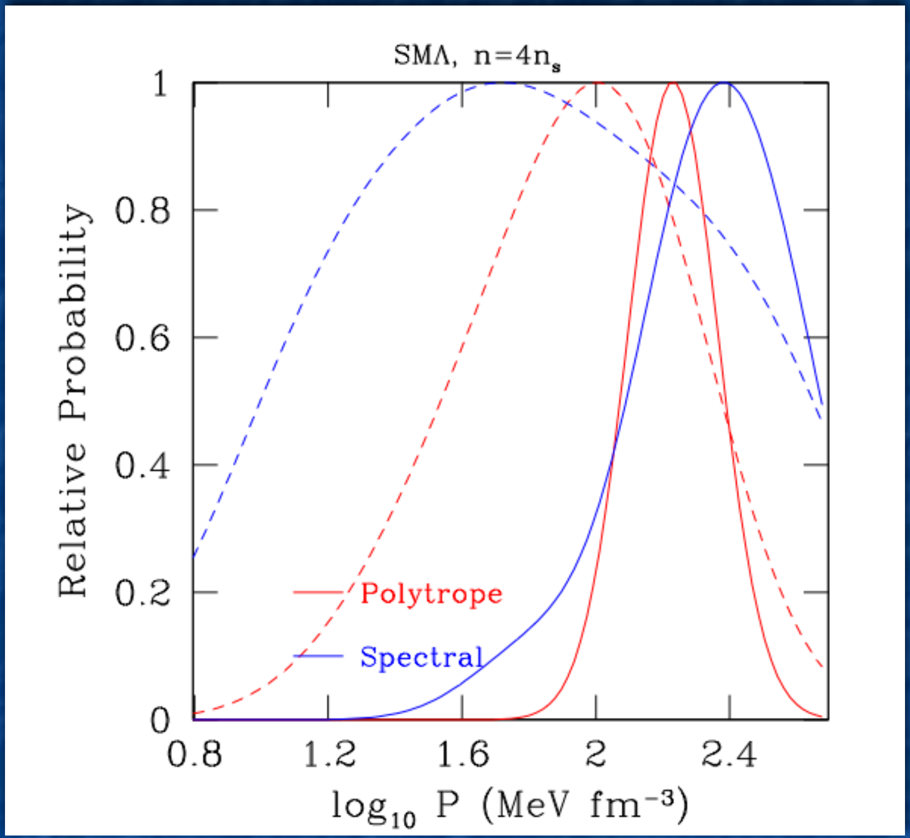
D_{KL} from prior: P=0.21; S=0.03

D_{KL} from prior; P=1.94, S=0.45

Entropy in EOS set is 0.83 for P, 0.96 for S

+Tidal Deformability

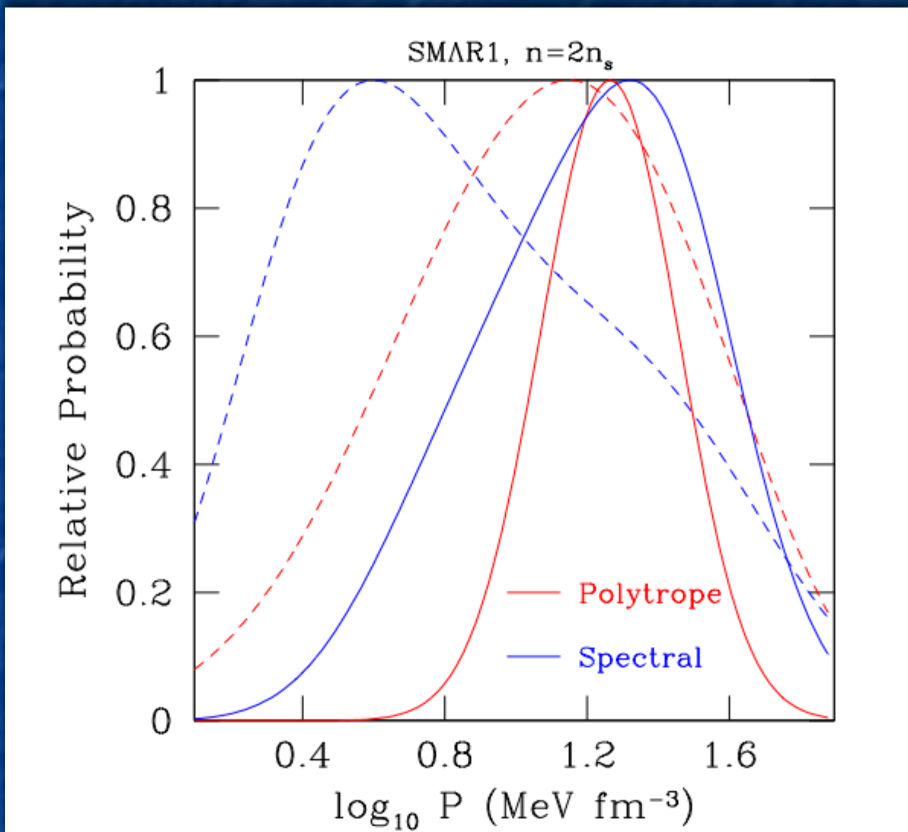


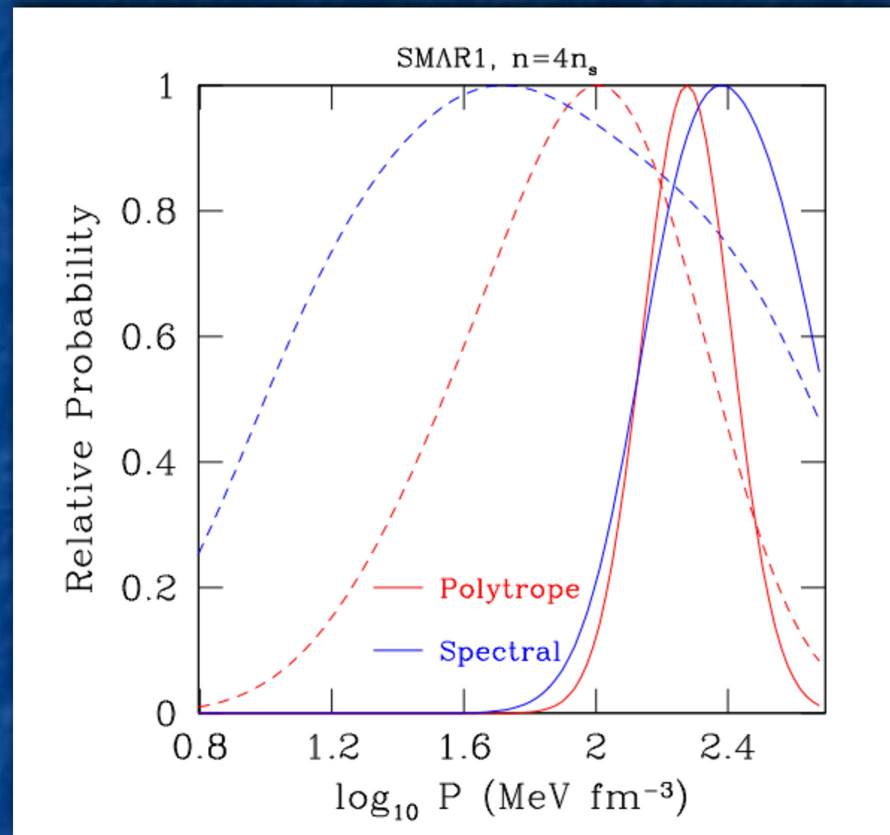


D_{KL} from prior: P=0.46; S=0.04 D_{KL} from prior; P=3.98, S=2.16

Entropy in EOS set is 0.82 for P, 0.95 for S

+Radius of J0030

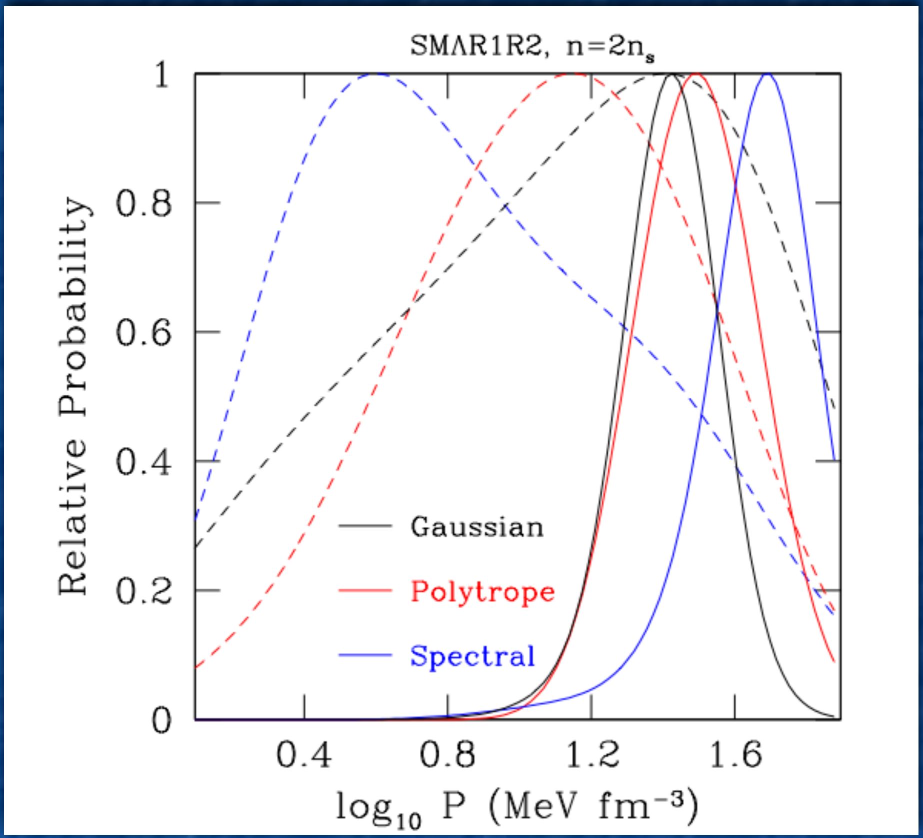


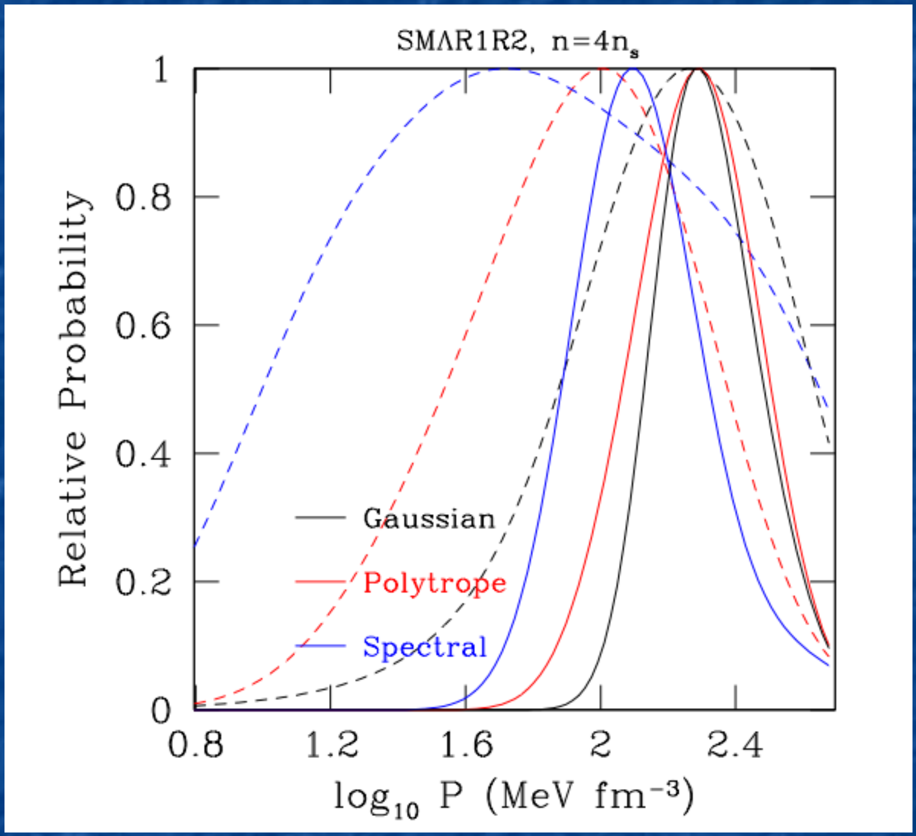


D_{KL} from prior: P=1.27; S=0.60 D_{KL} from prior; P=5.59, S=6.43

Entropy in EOS set is 0.81 for P, 0.91 for S

+Radius of J0740





D_{KL} from prior: P=4.77; S=4.27 D_{KL} from prior; P=2.78, S=4.19

Entropy in EOS set is 0.77 for P, 0.70 for S

What does all this tell us?

- Priors can have complicated effects
- If the likelihood pushes us to a tail of the prior, those effects can be amplified
- Polytrope-Spectral K-L divergence is greater in posterior than in prior!
- But this is mainly because the posterior is substantially tighter

Conclusions

- Residual differences between posteriors can be larger than we might think
- But it doesn't mean that we're not making progress!
- It does mean that we need to be careful with our priors