Symmetry Energy and its Effect on Structure and Composition of the Neutron Stars: Implications for Model Inference

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Emanuel Hoque

Misba Afrin

Symmetry Energy: Two Definitions

Two definitions of symmetry energy widely used:

Definition-I:

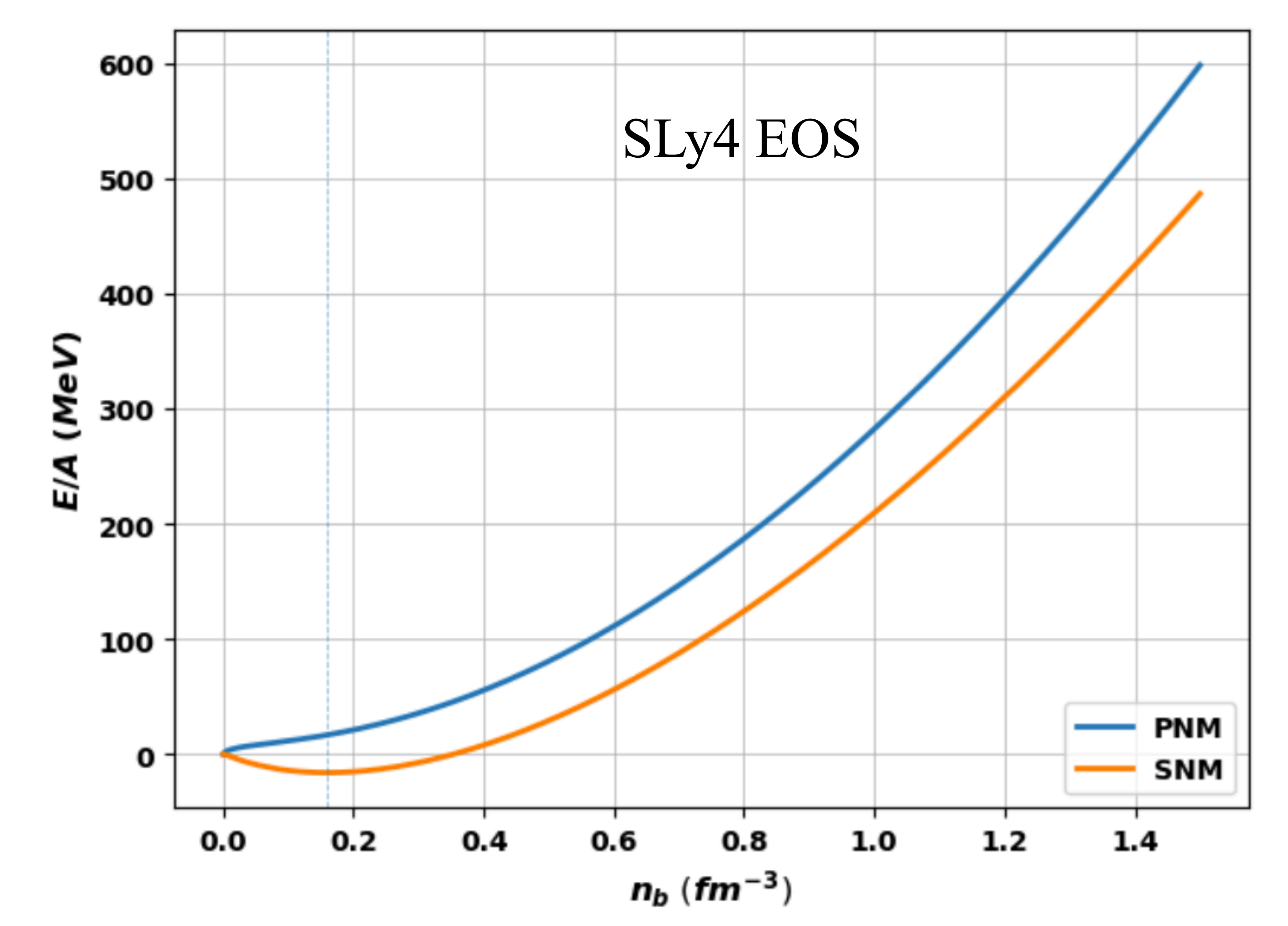
$$\frac{E}{A}(n_b, I=1) - \frac{E}{A}(n_b, I=0)$$

Definition-II:

$$\frac{1}{2}\frac{\partial^2[\frac{E}{A}(n_b)]}{\partial I^2}\bigg|_{I=0}$$

See:
Sun, Bhattiprolu & Lattimer (2023)

Q: Does it create any issue?



Binding energy per particle for symmetric nuclear matter (SNM) and pure neutron matter (PNM) over a wide range of baryon density relevant for neutron star

Skyrme-Type Effective Interaction

- We explore non-relativistic Skyrme-type effective interaction model:

Model parameters:
{t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, τ} →

$$
\begin{aligned}
V(\vec{r_1}, \vec{r_2}) = &\, t_0(1 + xP_\sigma)\delta(\vec{r}) \\
&+ \frac{1}{2} t_1 (1 + x_1 P_\sigma)[\vec{p'}^2 \delta(\vec{r}) + \delta(\vec{r})\vec{p}^2] \\
&+ t_2(1 + x_2 P_\sigma)\vec{p'}.\delta(\vec{r})\vec{p} \\
&+ \frac{1}{6} t_3 (1 + x_3 P_\sigma)[\rho(\vec{R})]^\iota \delta(\vec{r}) \\
&+ iW_0 \vec{\sigma}.[\vec{p'} \times \delta(\vec{r})\vec{p}]
\end{aligned}
$$

- Using this form, get the binding energy : $$< \psi | H | \psi >= \int \mathcal{H}(r) d^3 r$$

- The Hamiltonian: $$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \mathcal{H}_{fin} + \mathcal{H}_{so} + \mathcal{H}_{sg} + \mathcal{H}_{Coul}$$

Different Components of Skyrme-Model

$$\mathcal{H}_0 = \frac{1}{4} t_0 [(2 + x_0) n_b^2 - (2x_0 + 1)(n_p^2 + n_n^2)]$$

$$\mathcal{H}_3 = \frac{1}{24} t_3 n_b^{\iota} [(2 + x_3) n_b^2 - (2x_3 + 1)(n_p^2 + n_n^2)]$$

$$\mathcal{H}_{eff} = \frac{1}{8} [t_1 (2 + x_1) + t_2 (2 + x_2)] \tau n_b + \frac{1}{8} [t_2 (2x_2 + 1) - t_1 (2x_1 + 1)](\tau_p n_p + \tau_n n_n)$$

➡ Important terms

$$\mathcal{H}_{fin} = \frac{1}{32} [3t_1 (2 + x_1) - t_2 (2 + x_2)](\vec{\nabla} n_b)^2 - \frac{1}{32} [3t_1 (2x_1 + 1) + t_2 (2x_2 + 1)][(\vec{\nabla} n_p)^2 + (\vec{\nabla} n_n)^2]$$

$$\mathcal{H}_{so} = \frac{1}{2} W_0 [\vec{J}.\vec{\nabla} n_b + \vec{J}_p.\vec{\nabla} n_p + \vec{J}_n.\vec{\nabla} n_n]$$

$$\mathcal{H}_{sg} = -\frac{1}{16} (t_1 x_1 + t_2 x_2) \vec{J}^2 + \frac{1}{16} (t_1 - t_2)[\vec{J}_p^2 + \vec{J}_n^2]$$

➡ Unimportant for NS

KE of Isospin Asymmetric Matter

$$\mathcal{K} = \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} n_b^{\frac{5}{3}} F_{5/3}(I)$$

➡ kinetic energy-density term

$$F_N(I) = \frac{1}{2}[(1+I)^N + (1-I)^N]$$

➡ Isospin asymmetry factor

$$I = \frac{(n_n - n_p)}{(n_n + n_p)}$$

➡ Isospin asymmetry parameter

$$\Longrightarrow \quad n_n = \frac{1}{2}(1+I)n_b, \quad n_p = \frac{1}{2}(1-I)n_b$$

Total Energy for Isospin-Asymmetric Matter

- Energy per particle for isospin-asymmetric nuclear matter:

$$
\begin{aligned}
\frac{E}{A}(n_b, I) =& \frac{\epsilon_b}{n_b} \\
=& \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} n_b^{\frac{2}{3}} F_{5/3}(I) \\
&+ \frac{1}{8} t_0 n_b [2(x_0+2) - (2x_0+1)F_2(I)] \\
&+ \frac{1}{48} t_3 n_b^{\iota+1} [2(x_3+2) - (2x_3+1)F_2(I)] \\
&+ \frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} n_b^{\frac{5}{3}} \Big\{ [t_1(x_1+2) + t_2(x_2+2)] F_{5/3}(I) \\
&+ \frac{1}{2}[t_2(2x_2+1) - t_1(2x_1+1)] F_{8/3}(I) \Big\}
\end{aligned}
$$

- Taylor-series expansion of E/A in powers of Isospin-asymmetric param (I)

$$
\frac{E}{A}(n_b, I) = \frac{E}{A}(n_b, I=0) + a_s^{(2)}(n_b) I^2 + a_s^{(4)}(n_b) I^4 + a_s^{(6)}(n_b) I^6 + \ldots + a_s^{(2n)}(n_b) I^{2n} \ldots
$$

Isospin expansion coefficients

n = 1, i.e. I^2

$$a_s^{(2)} = \frac{1}{2}\frac{\partial^2[\frac{E}{A}(n_b)]}{\partial I^2}\Big|_{I=0}$$
$$= \frac{1}{3}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} n_b^{\frac{2}{3}} - \frac{1}{8}t_0(2x_0+1)n_b$$
$$-\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}(3t_1x_1 - t_2(4+5x_2))n_b^{\frac{5}{3}}$$
$$-\frac{1}{48}t_3(2x_3+1)n_b^{\iota+1}$$

n = 2, i.e. I^4

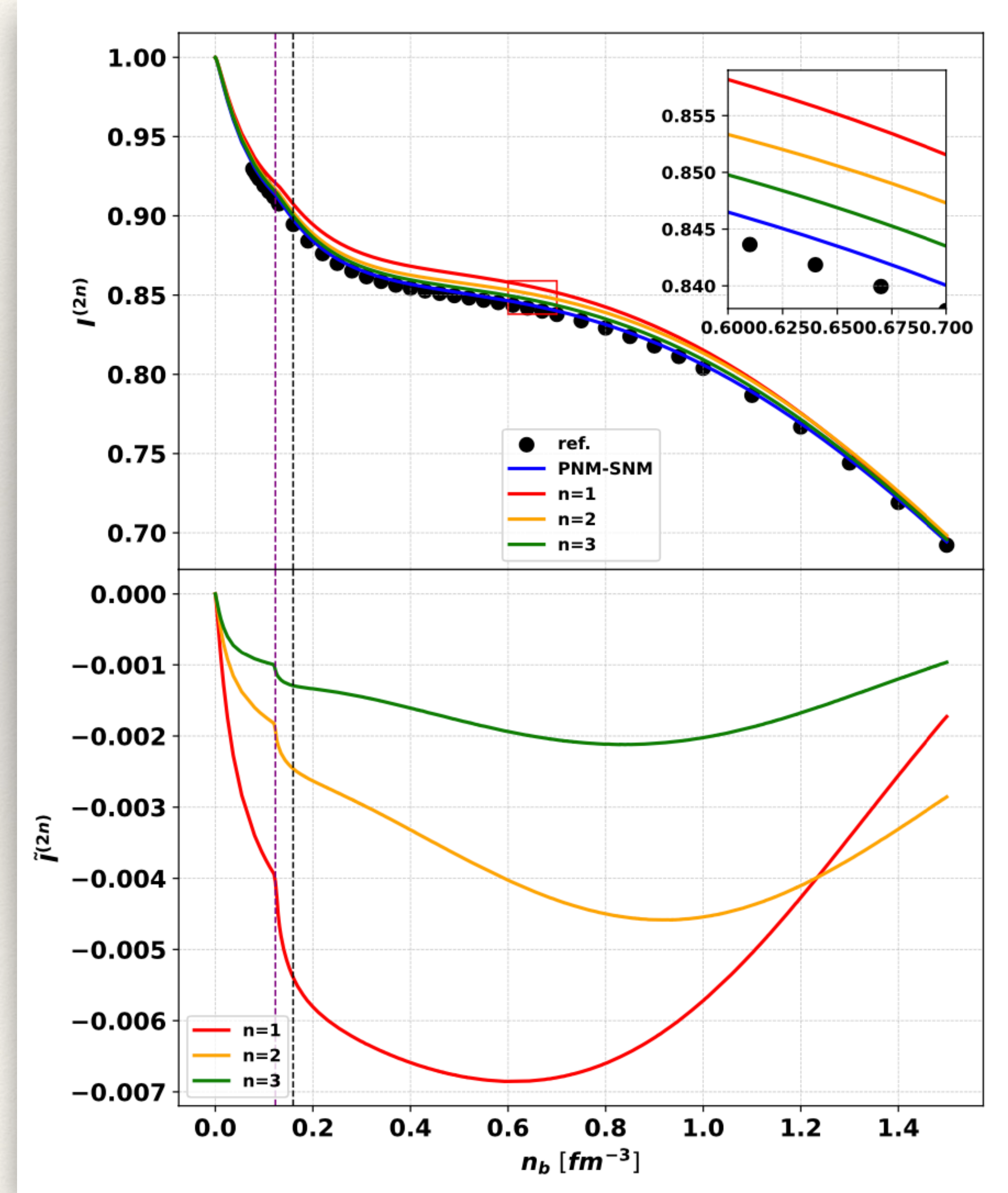
$$a_s^{(4)} = \frac{1}{4!}\frac{\partial^4[\frac{E}{A}(n_b)]}{\partial I^4}\Big|_{I=0}$$
$$= \frac{1}{81}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} n_b^{\frac{2}{3}}$$
$$+\frac{1}{648}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}(3t1(1+x_1) + t_2(1-x_2))n_b^{\frac{5}{3}}$$

n = 3, i.e. I^6

$$a_s^{(6)} = \frac{1}{6!}\frac{\partial^6[\frac{E}{A}(n_b)]}{\partial I^6}\Big|_{I=0}$$
$$= \frac{7}{2187}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} n_b^{\frac{2}{3}}$$
$$+\frac{7}{87480}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}}(3t1(4+3x_1) + t_2(8+x_2))n_b^{\frac{5}{3}}$$

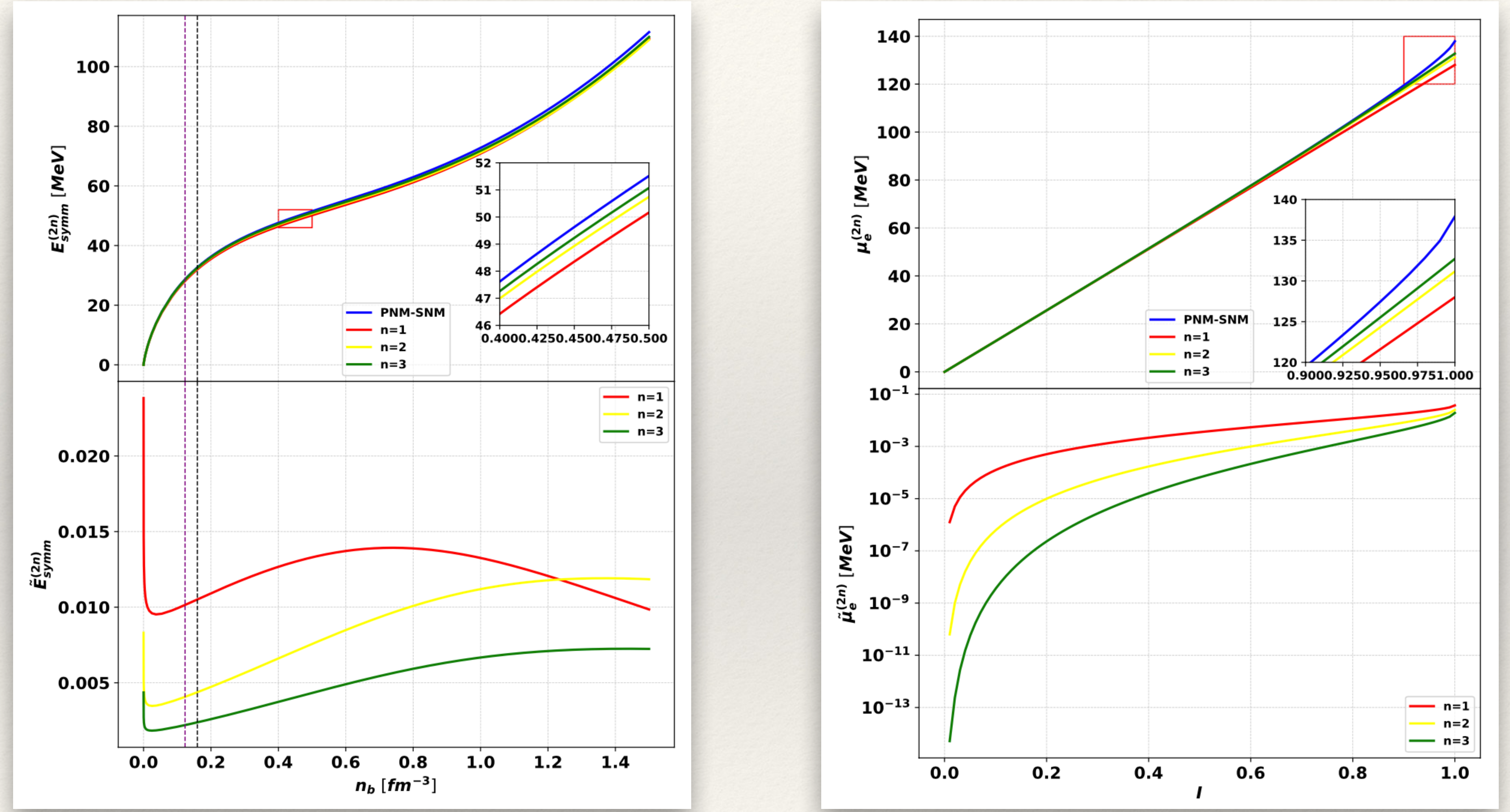
➡ We can analytically compute these coefficients

Isospin Asymmetry Parameter at Different Order



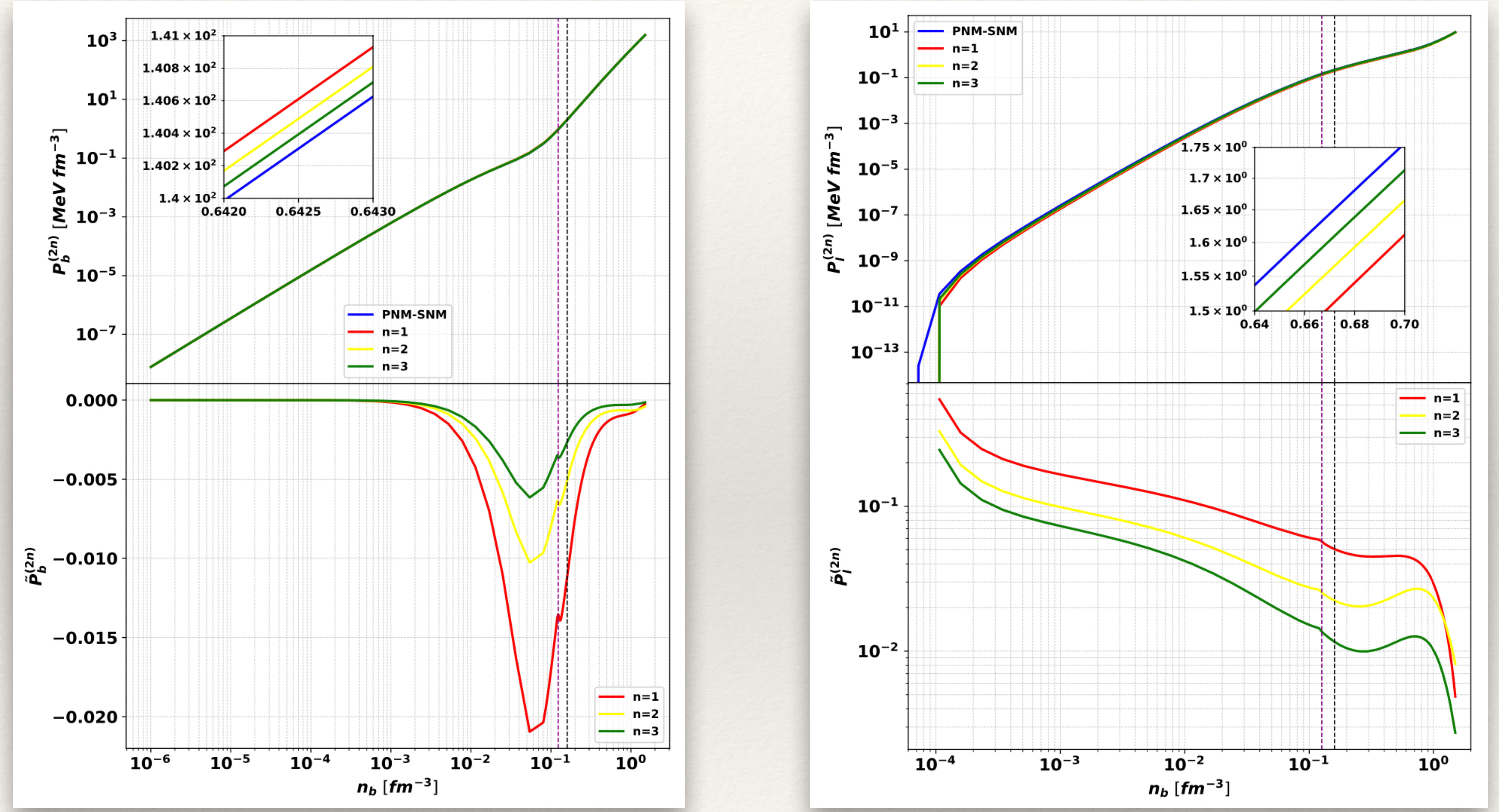
- (*Top-panel*):Isospin asymmetry parameter (I) at different order of symmetry energy correction for n=1, 2, 3 as well as exact value of it (for SLy4) have been computed over the density-range relevant for NS
- It has been compared to the values (see black dots) from original Douchin & Hansel (A&A, 2001) paper
- (*Bottom-panel*) Fractional difference in Isospin asymmetry parameter (I) at different order of symmetry energy correction for n = 1, 2, 3 as well as exact value of it have been shown

Symmetry Energy & Chemical Potential at Different Orders



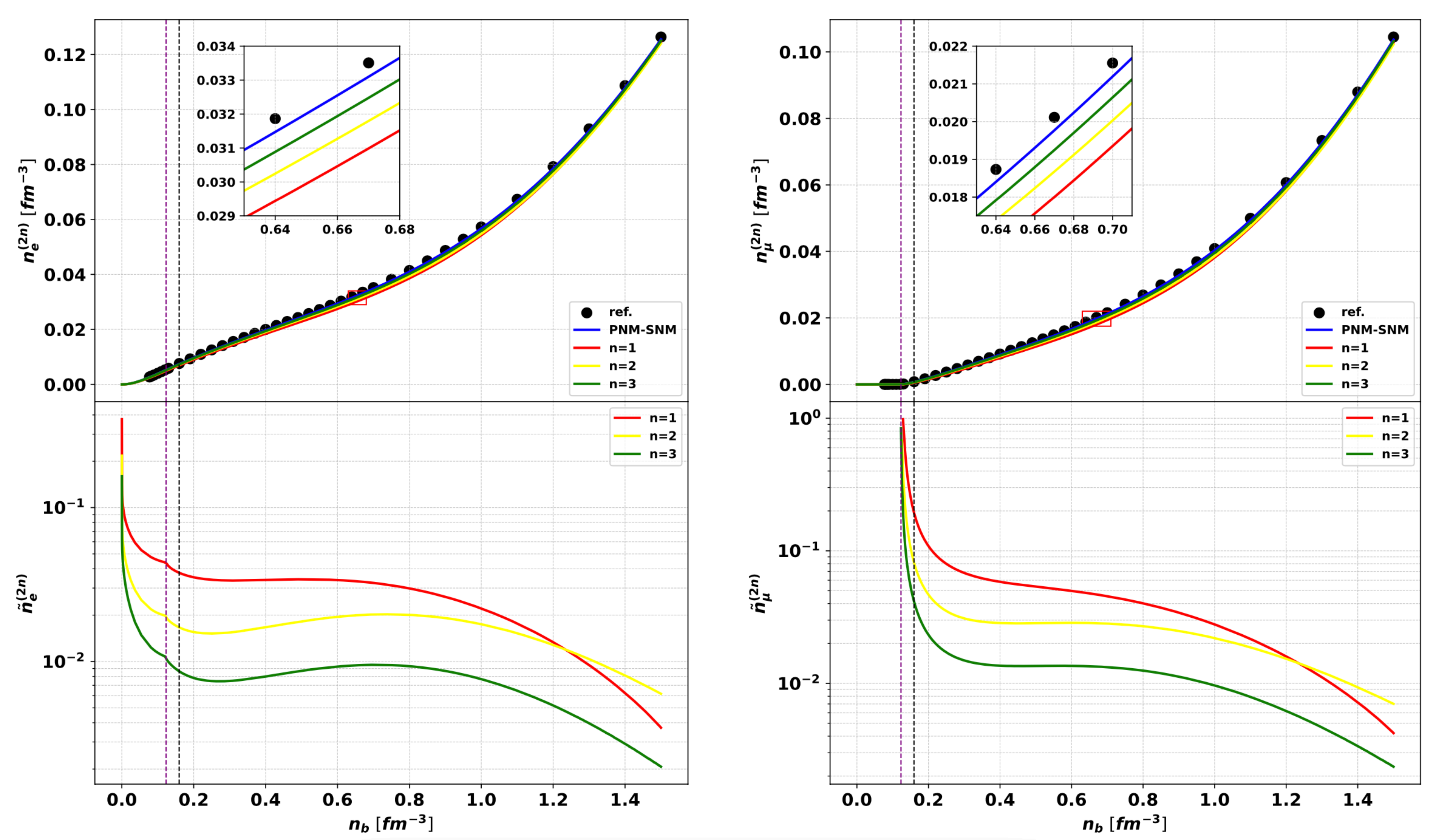
Symmetry energy (left-panel) and lepton chemical potential (right-panel) for SLy4 at different orders of `n'

Baryon & Lepton Pressure at Different Orders



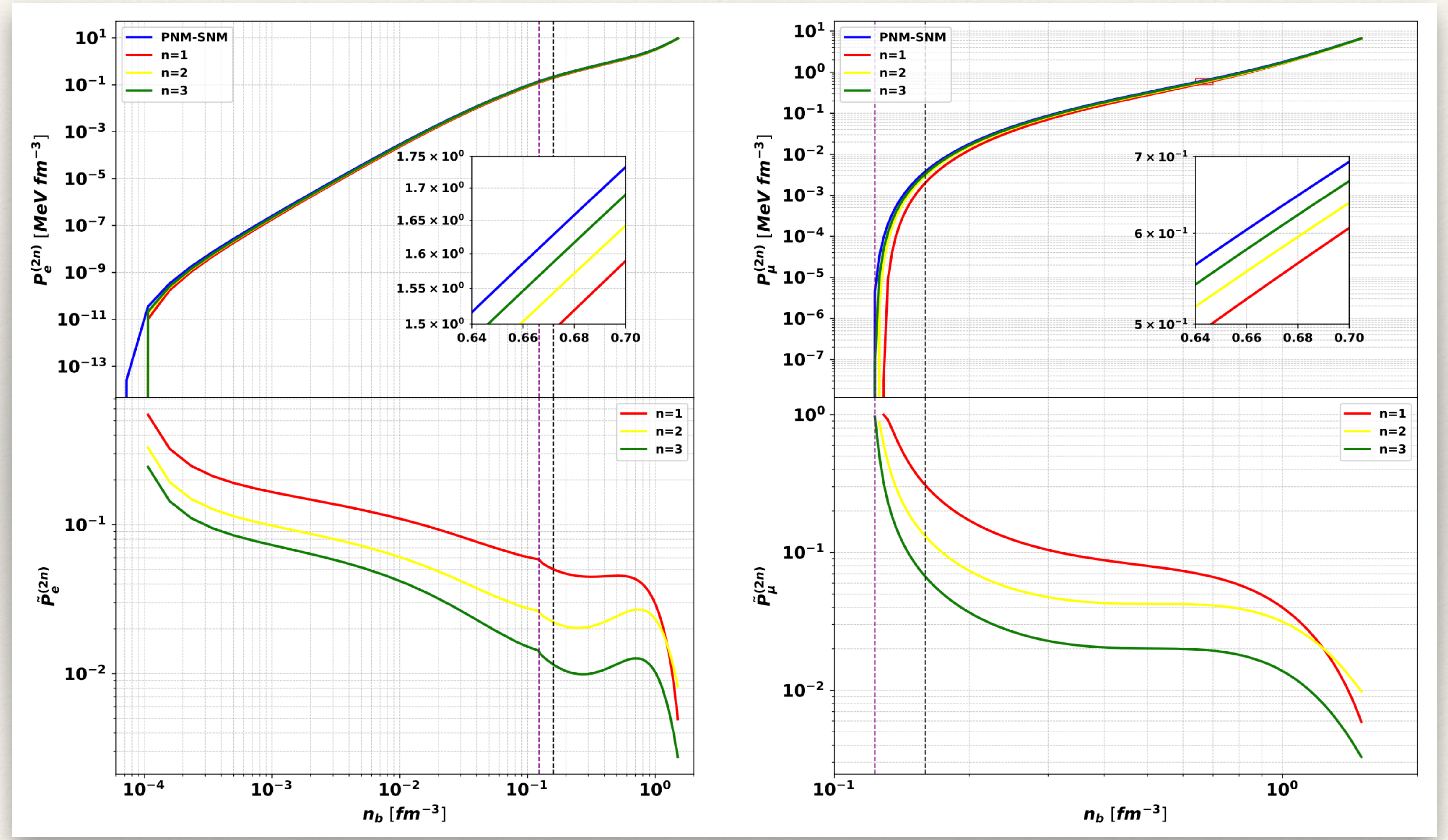
Baryonic Pressure (left-panel) and lepton chemical potential (right-panel) for SLy4 at different orders of `n'

Electron & Muon Density at Different Orders



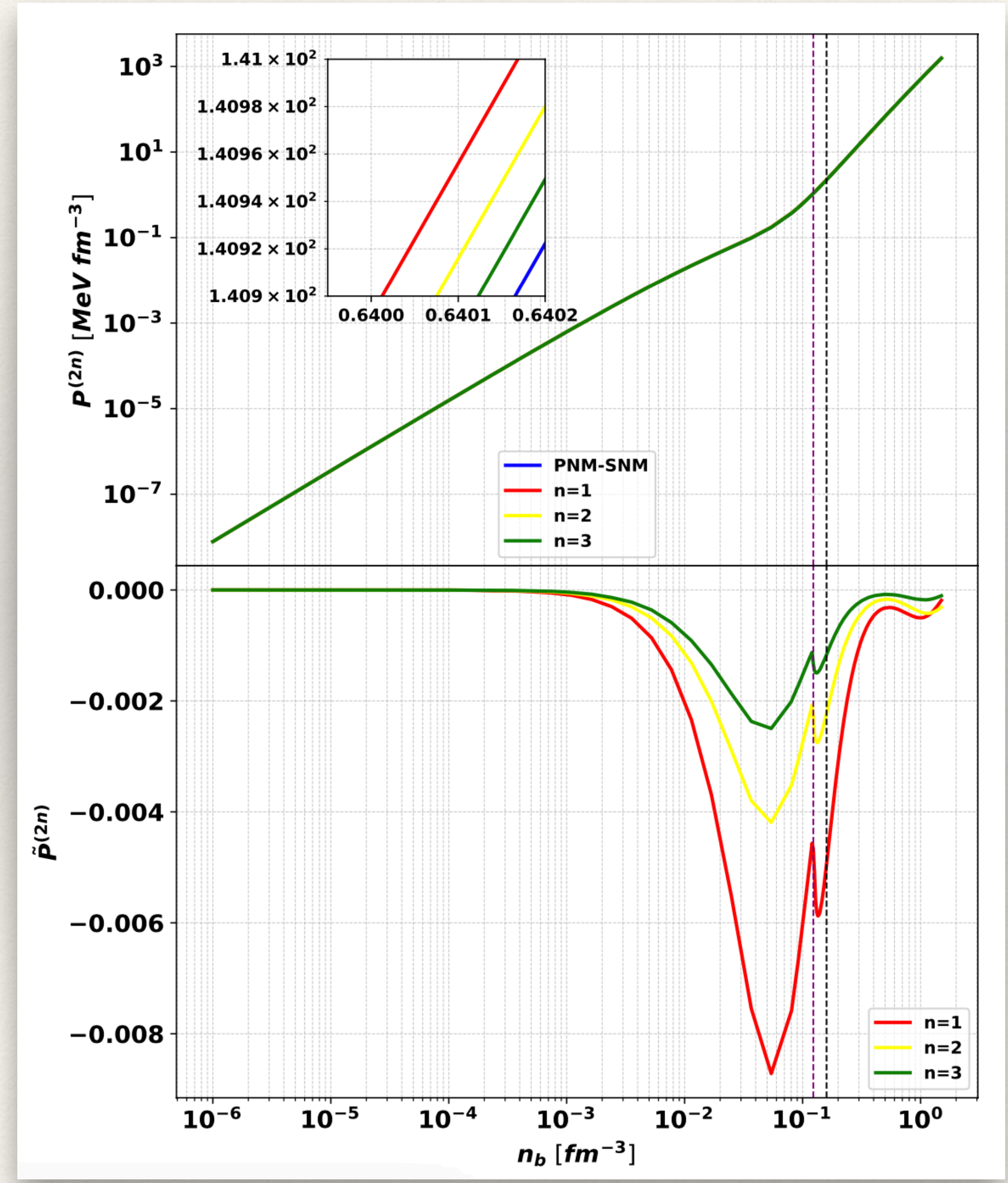
Electron (left-panel) and muon (right-panel) number density for SLy4 at different orders of `n'

Electron & Muon Pressure at Different Orders



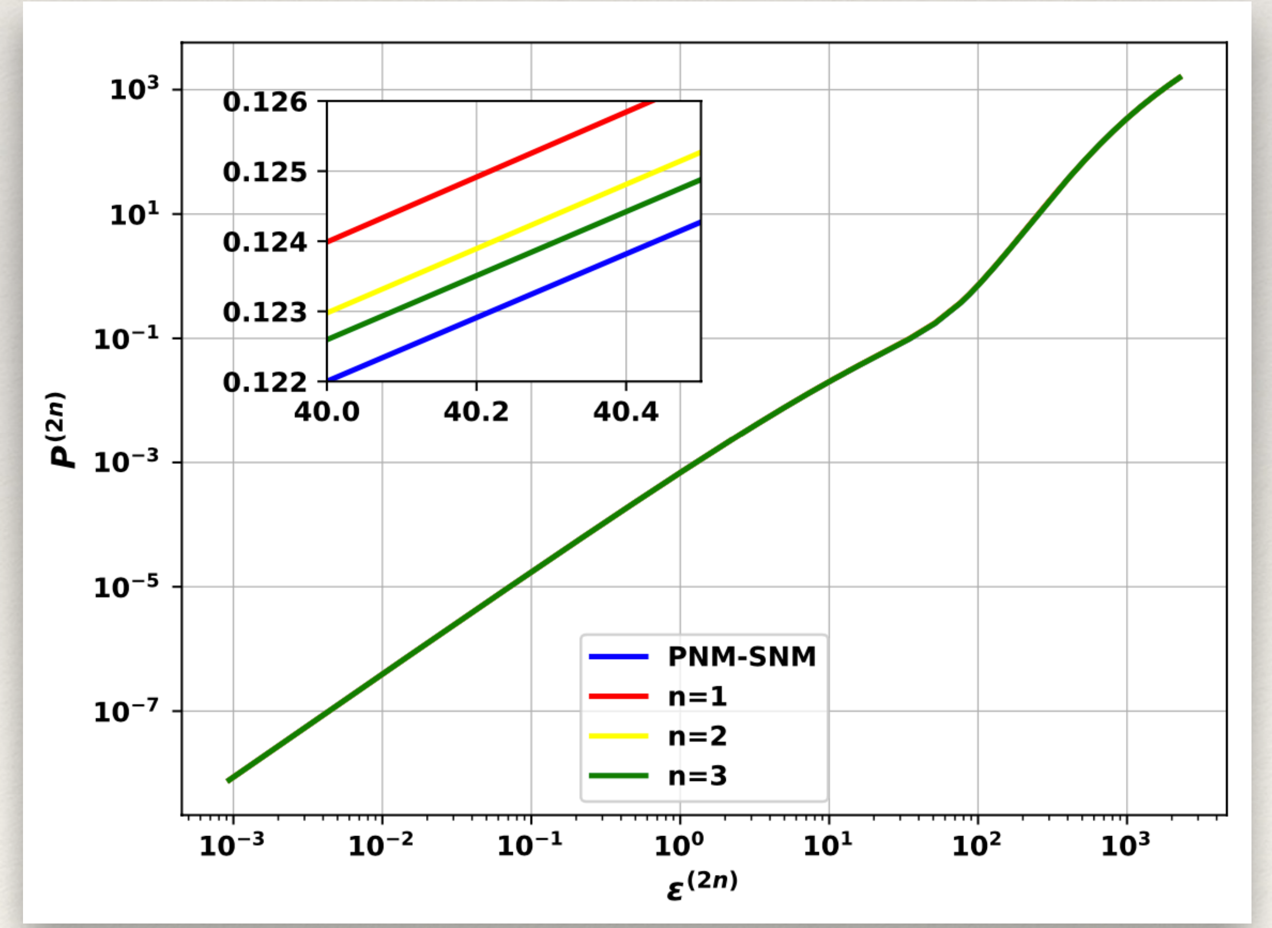
Electron (left-panel) and muon (right-panel) pressure for SLy4 at different orders of `n'

Beta-Equilibrium EOS at Different Orders

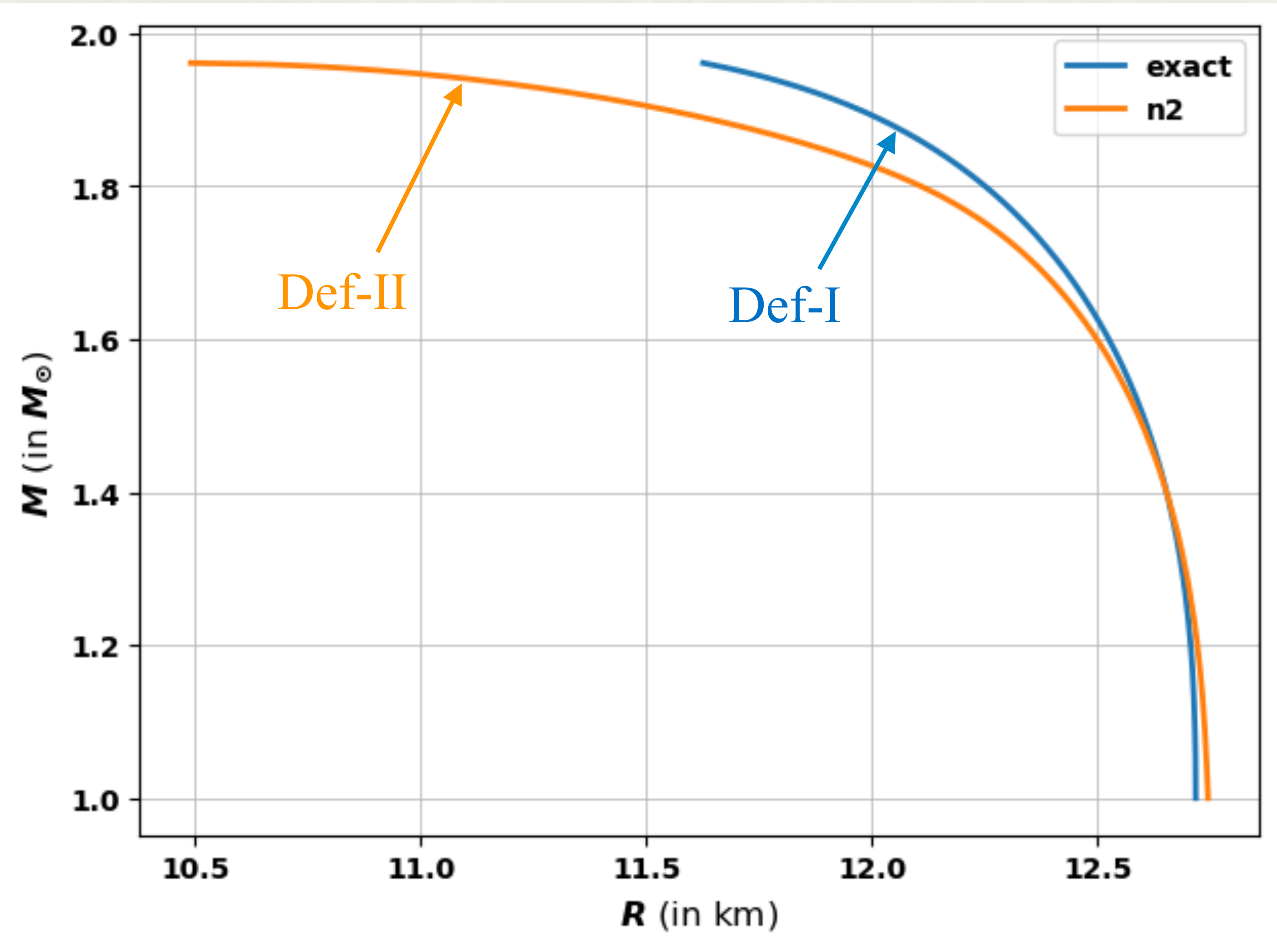


Pressure of the β-equilibrium matter

Finally, β-equilibrium SLy4 EOS at different orders



M-R Relation at Different Orders



Mass-radius curves for the two different orders of symmetry energy correction

A pair of M-R curves for one typical set of Skyrme parameters

The M-R curves are generally stiffer for the exact E_{sym} case than $E_{sym}^{(2n)}$ case for massive stars

It is exactly the opposite for lighter mass stars

Around 1.4 solar mass star they crossover!

Statistical Population Analysis Over The Skyrme Parameter Space

How About the Other Skyrme Parameters?

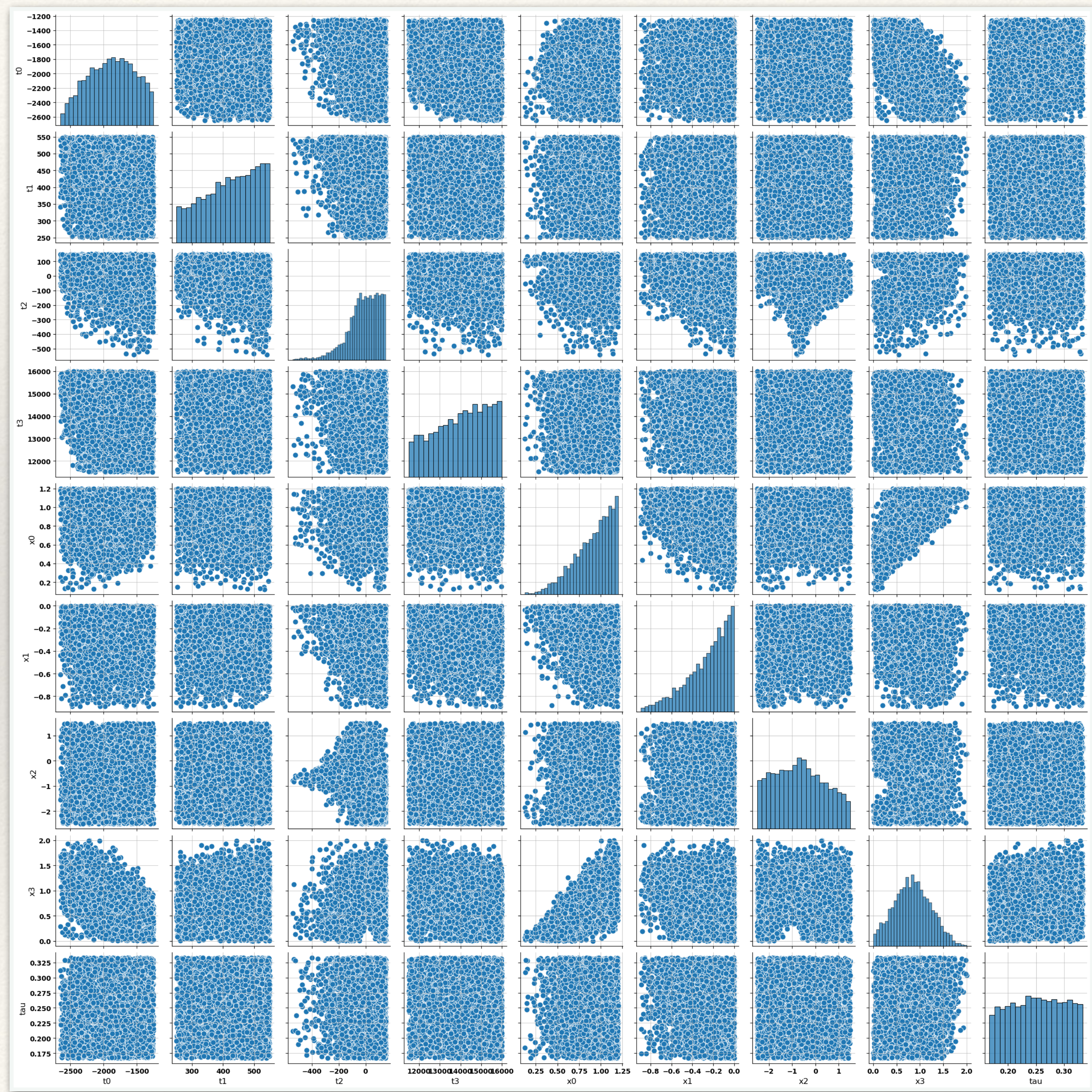
- **We thoroughly sample all the Skyrme-interaction parameters**

Start with ~ 2M Skyrme params!

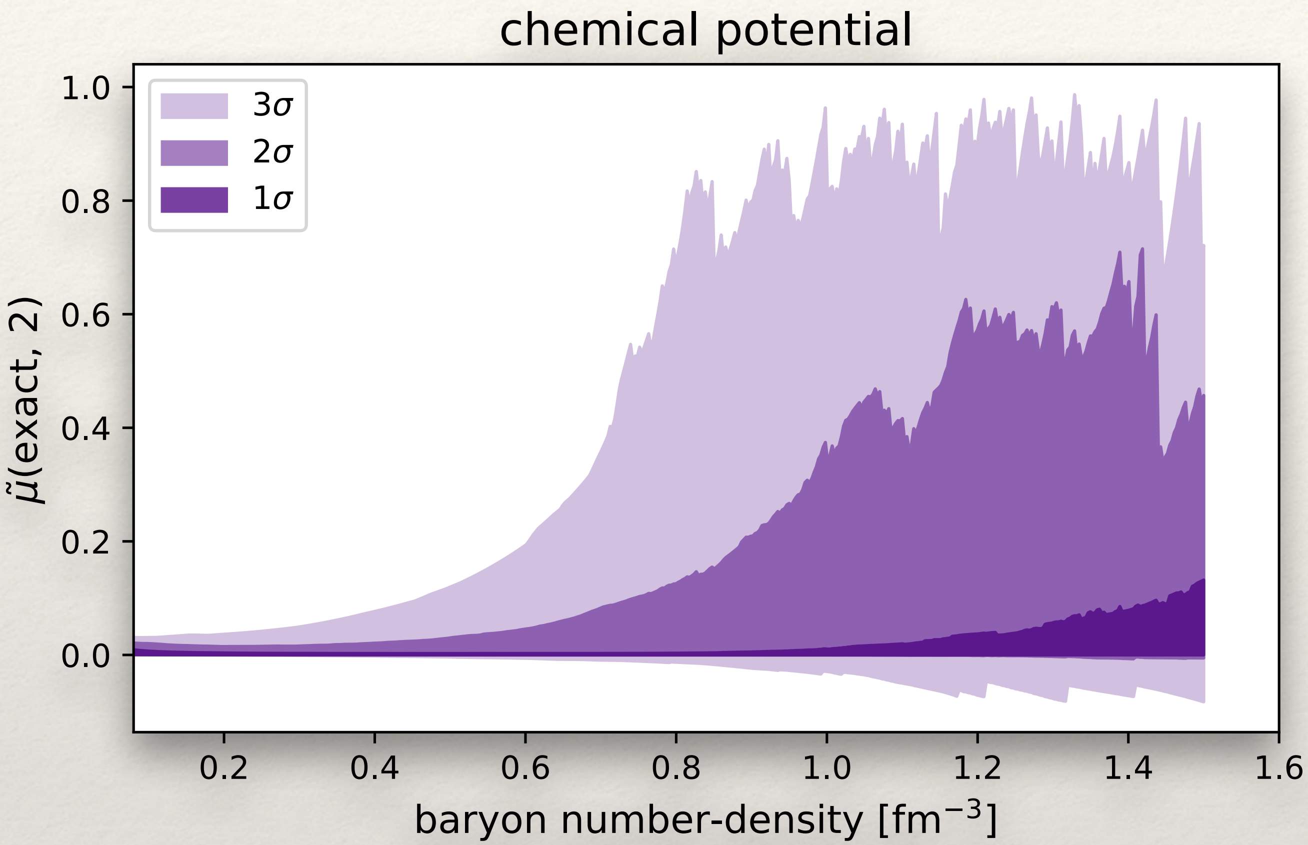
We apply filter with four criteria:

- thermodynamic stability
- causality
- +ve semi-definiteness of symmetry energy
- Bounds on saturation density $0.14 < n_{b0} < 0.17$, and κ
- $M_{TOV} > 2\ M_{sun}$

About ~ 5k Skyrme EOS (~0.25%) survive!

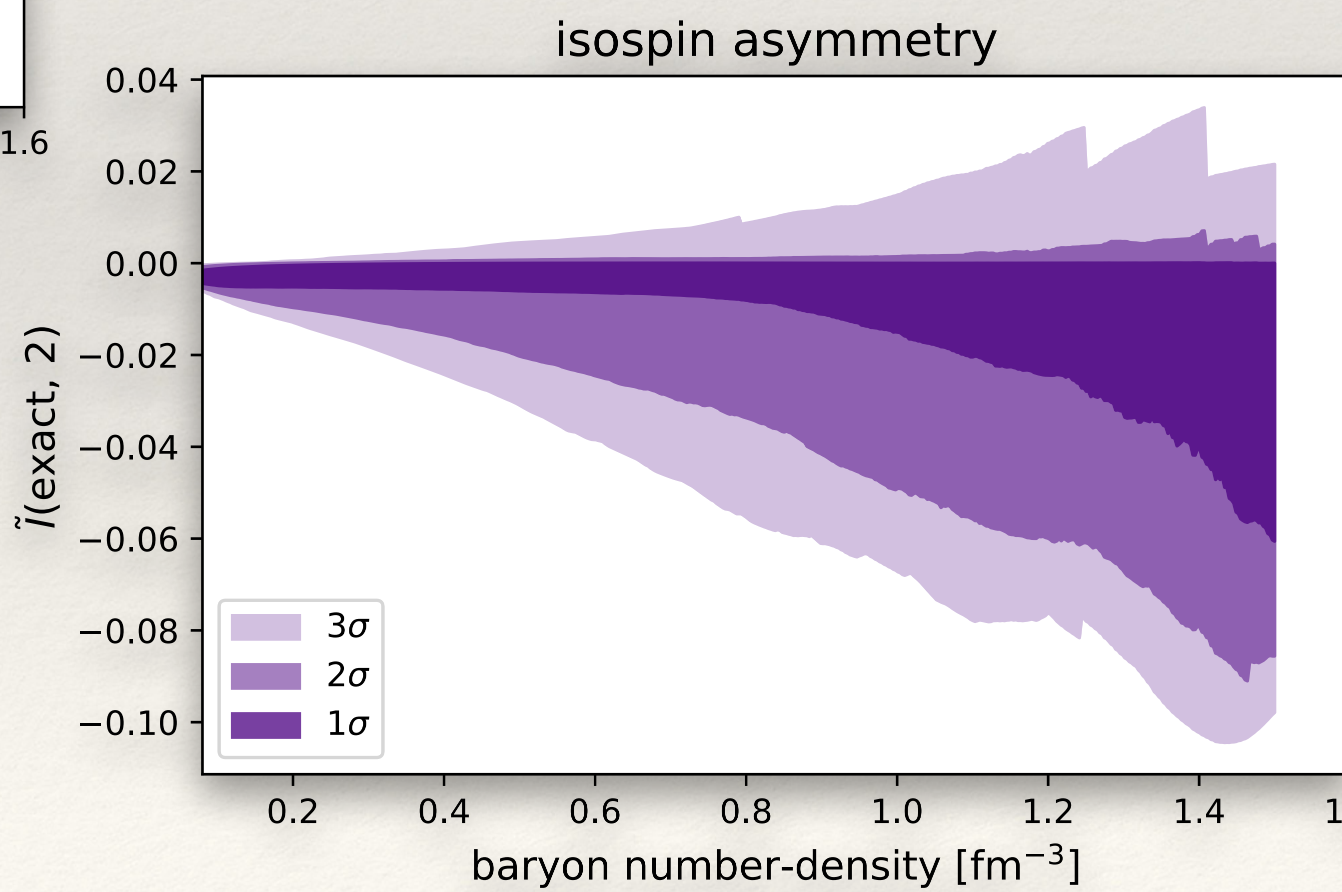


Chemical Potential & Isospin Parameter



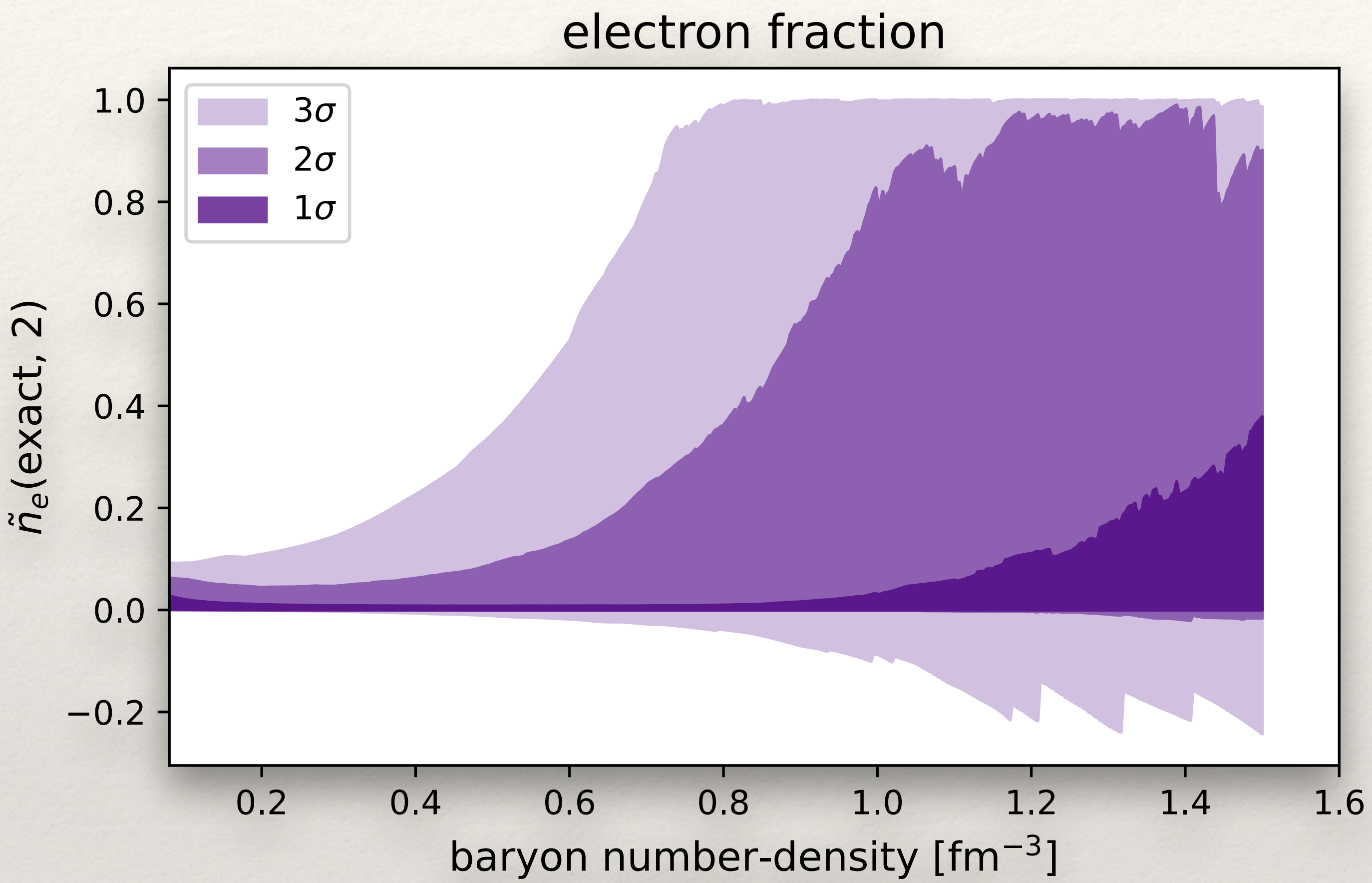
$$\tilde{I} = \frac{I^{(ext)} - I^{(2)}}{I^{(ext)} + I^{(2)}}$$

$$\tilde{\mu}_l = \frac{\mu_l^{(ext)} - \mu_l^{(2)}}{\mu_l^{(ext)} + \mu_l^{(2)}}$$



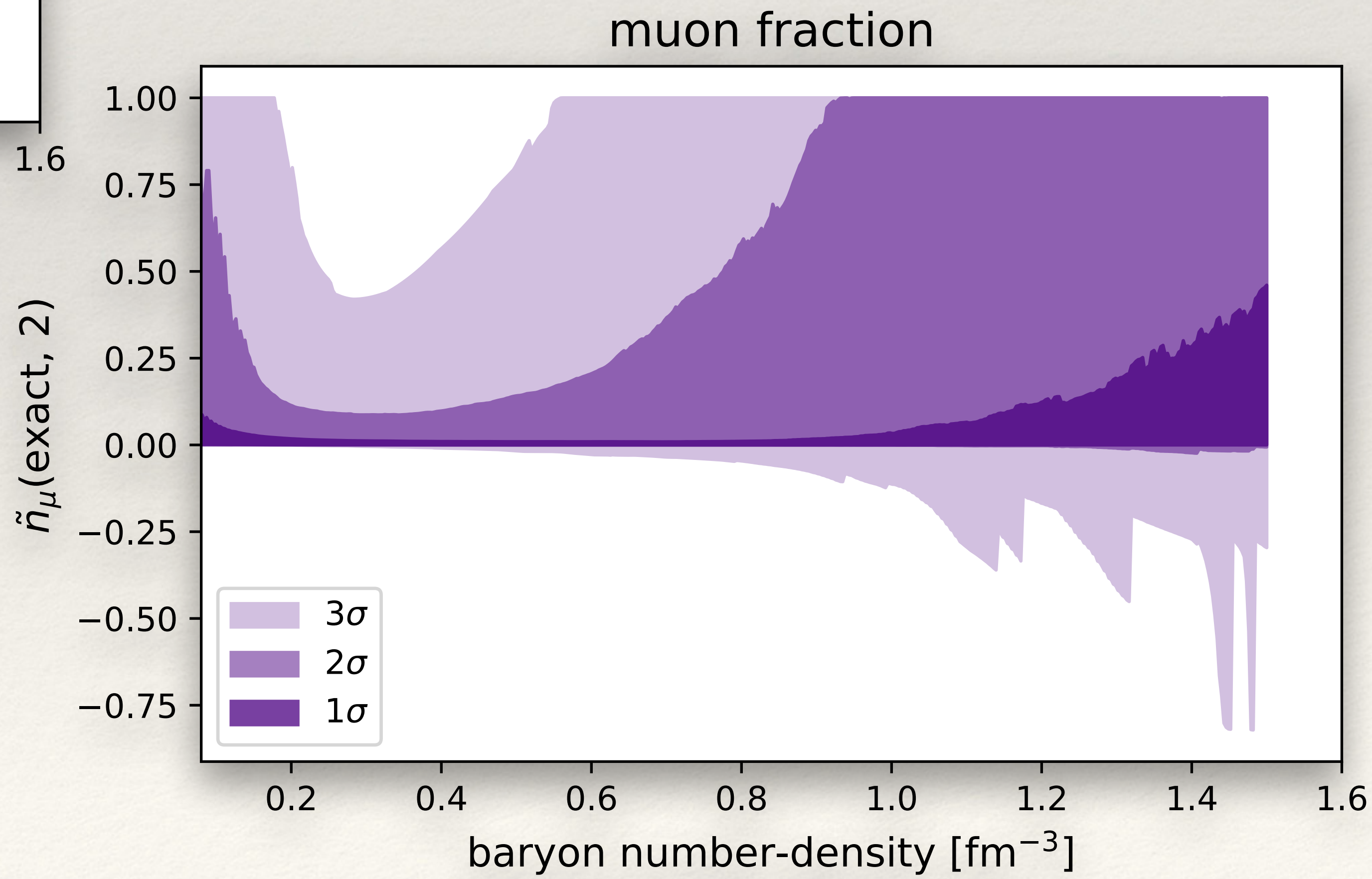
E. Hoque, AM (in prep.)

Electron & Muon Number Densities

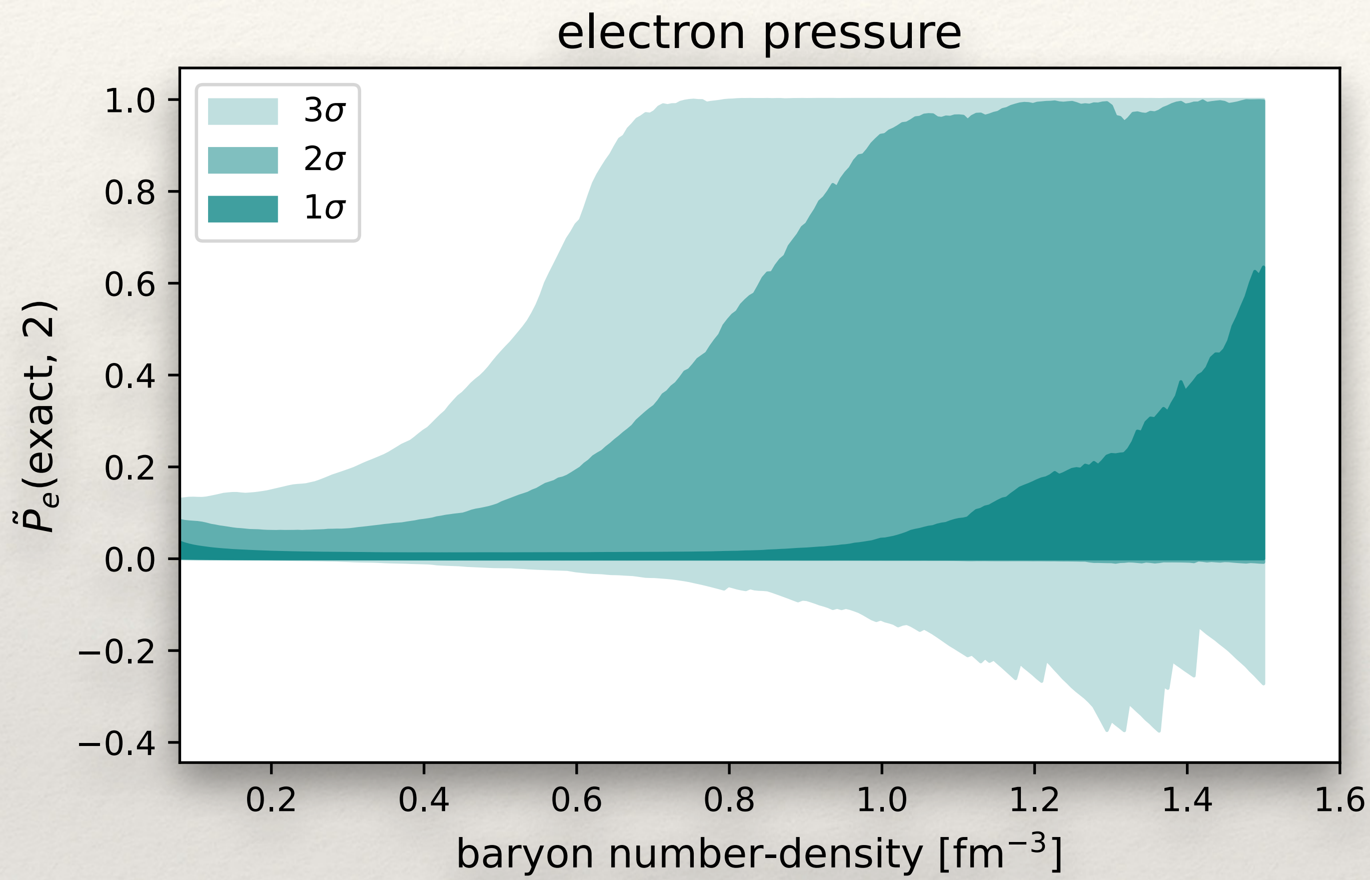


$$\tilde{n}_e = \frac{n_e^{(ext)} - n_e^{(2)}}{n_e^{(ext)} + n_e^{(2)}}$$

$$\tilde{n}_\mu = \frac{n_\mu^{(ext)} - n_\mu^{(2)}}{n_\mu^{(ext)} + n_\mu^{(2)}}$$

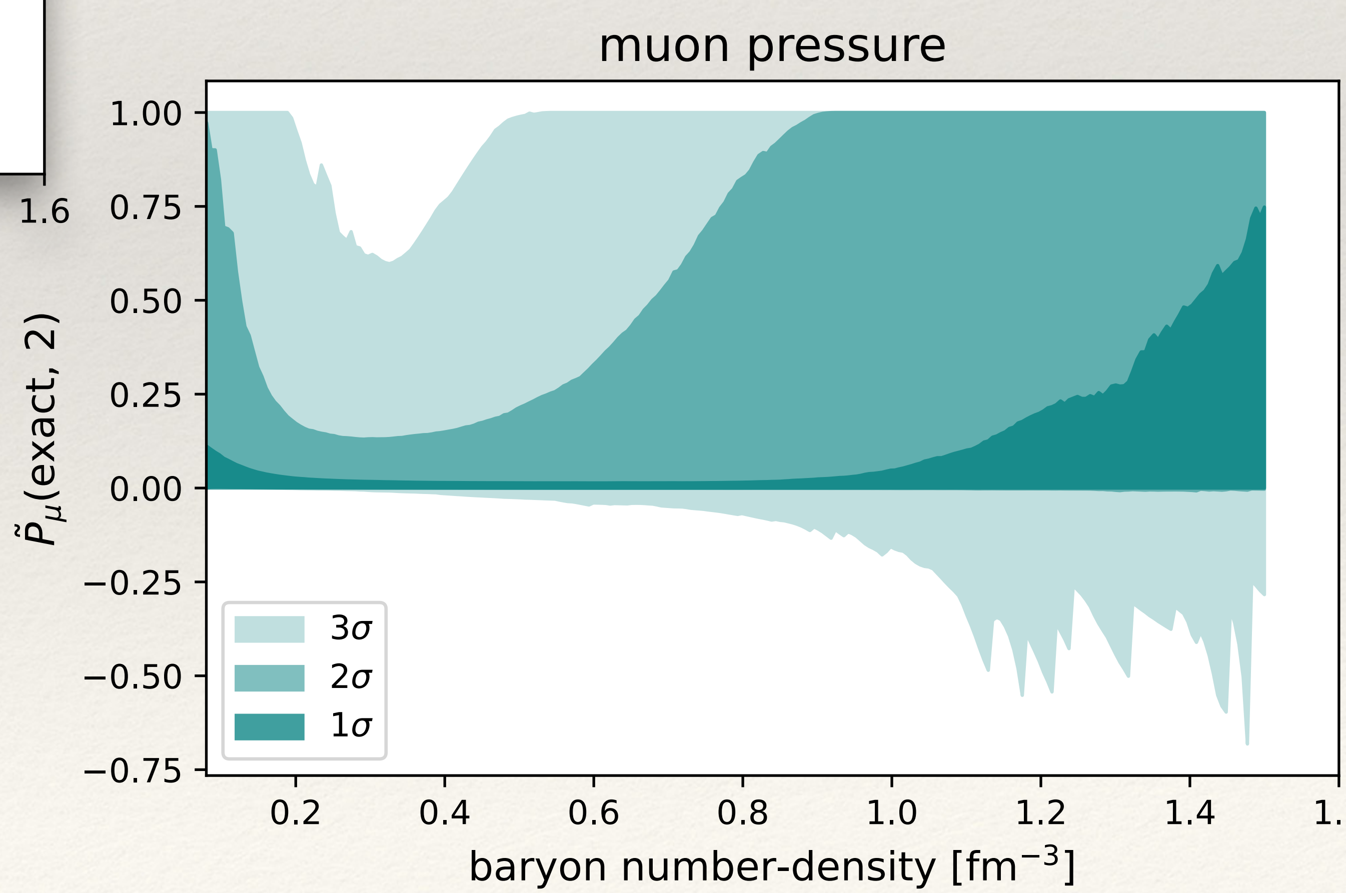


Electron & Muon Pressure: Statistical Analysis

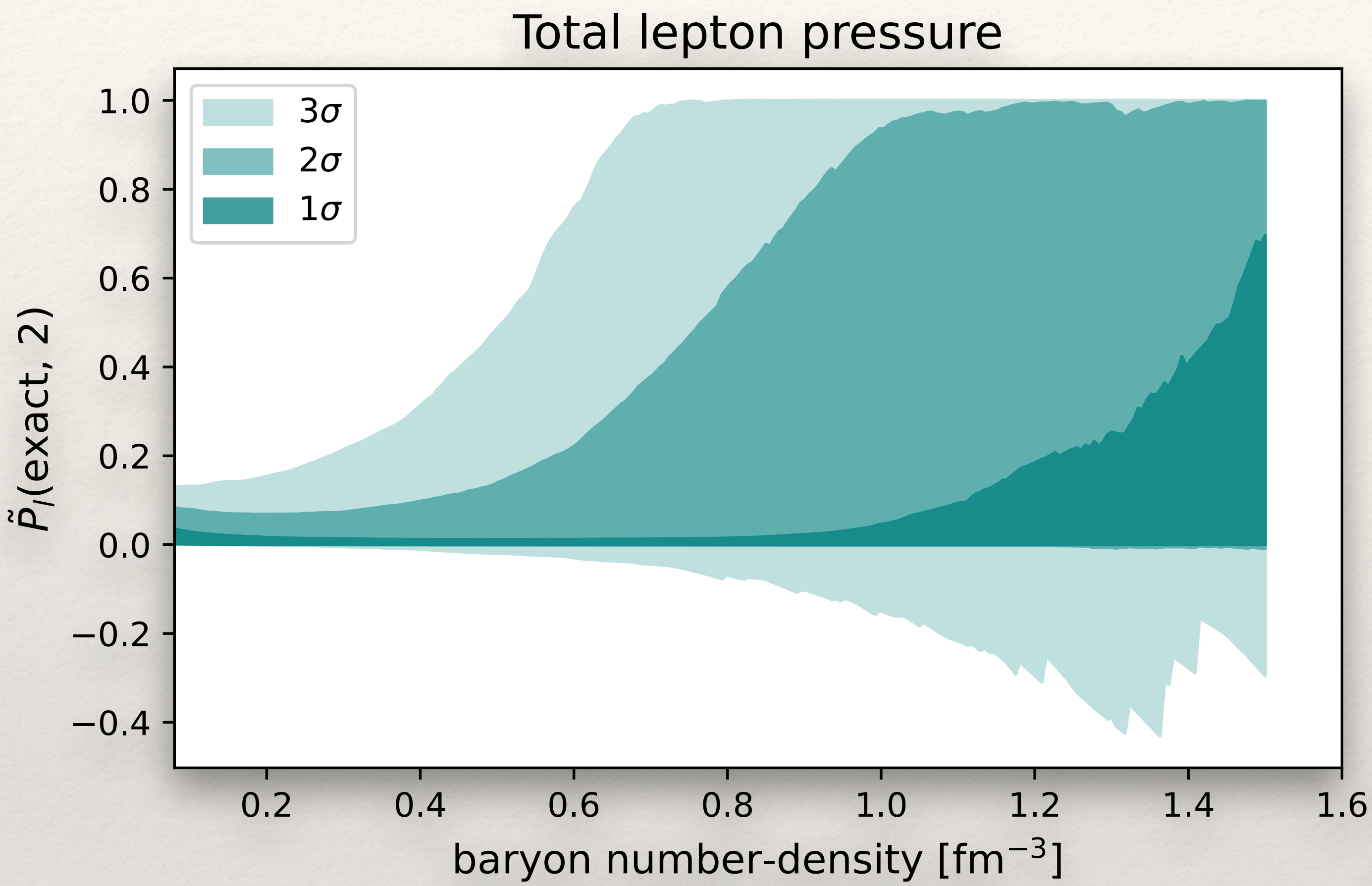


$$\tilde{P}_e = \frac{P_e^{(ext)} - P_e^{(2)}}{P_e^{(ext)} + P_e^{(2)}}$$

$$\tilde{P}_\mu = \frac{P_\mu^{(ext)} - P_\mu^{(2)}}{P_\mu^{(ext)} + P_\mu^{(2)}}$$

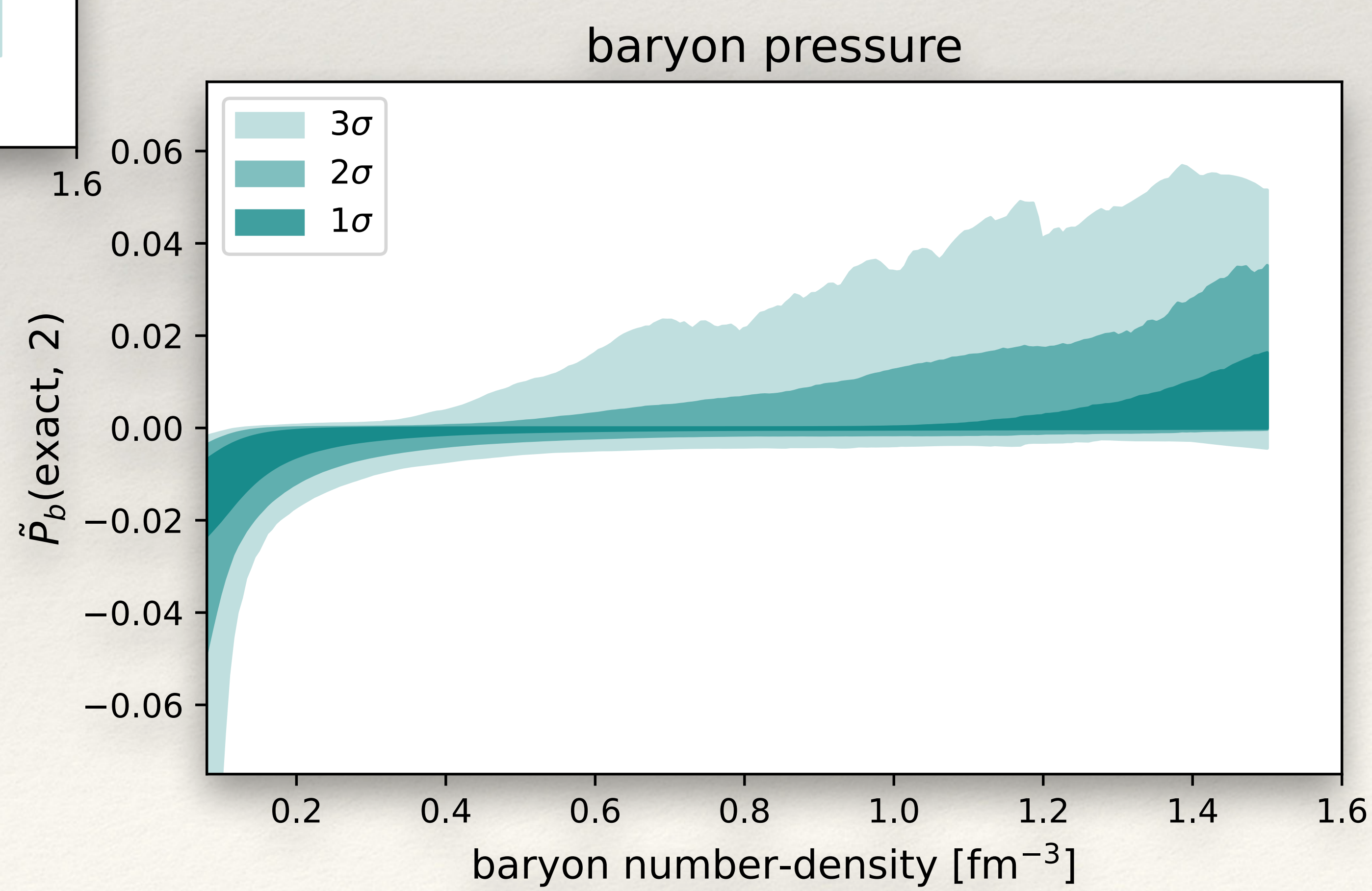


Lepton & Baryon Pressure: Statistical Analysis



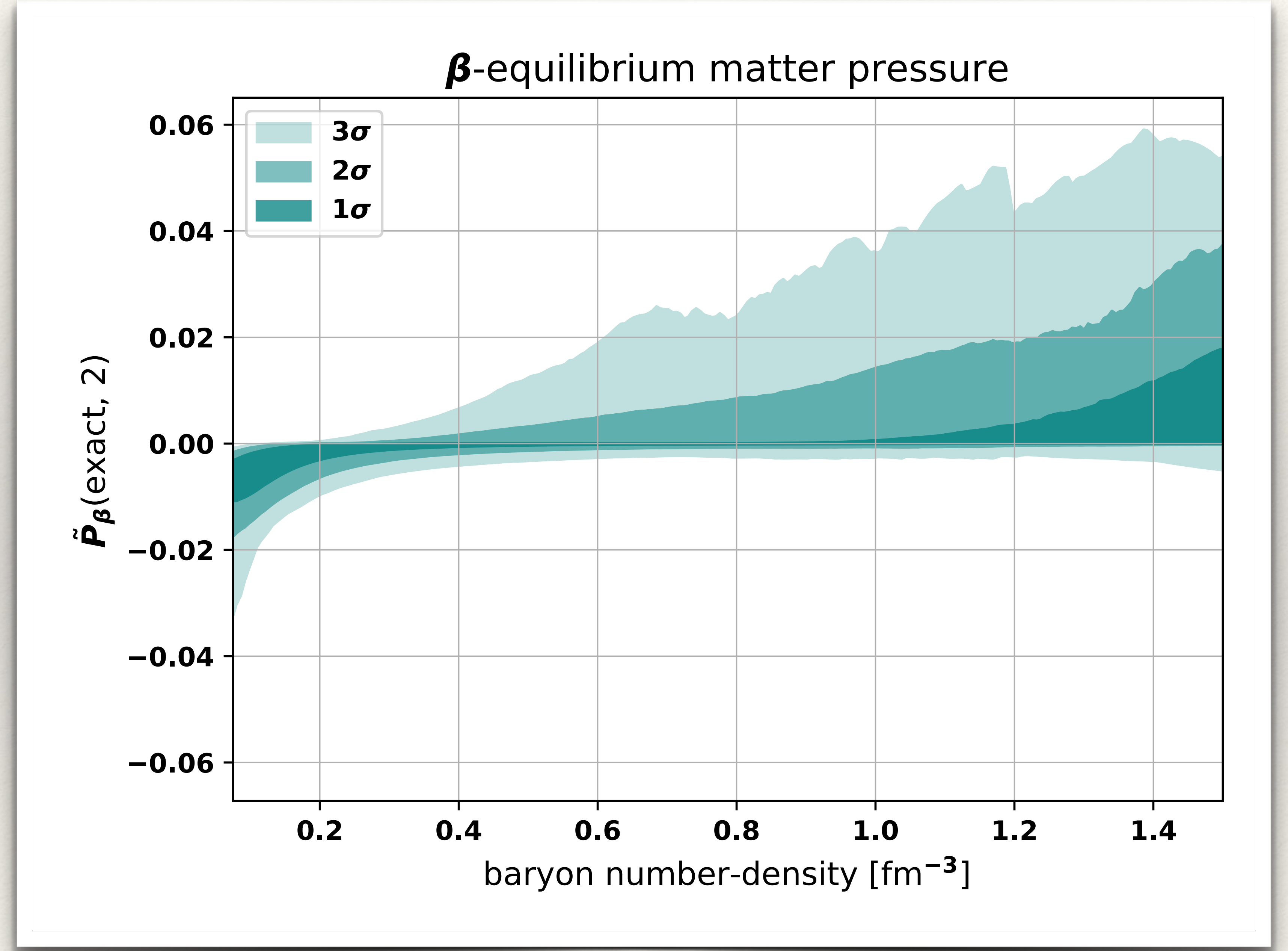
$$\tilde{P}_l = \frac{P_l^{(ext)} - P_l^{(2)}}{P_l^{(ext)} + P_l^{(2)}}$$

$$\tilde{P}_b = \frac{P_b^{(ext)} - P_b^{(2)}}{P_b^{(ext)} + P_b^{(2)}}$$



E. Hoque, AM (in prep.)

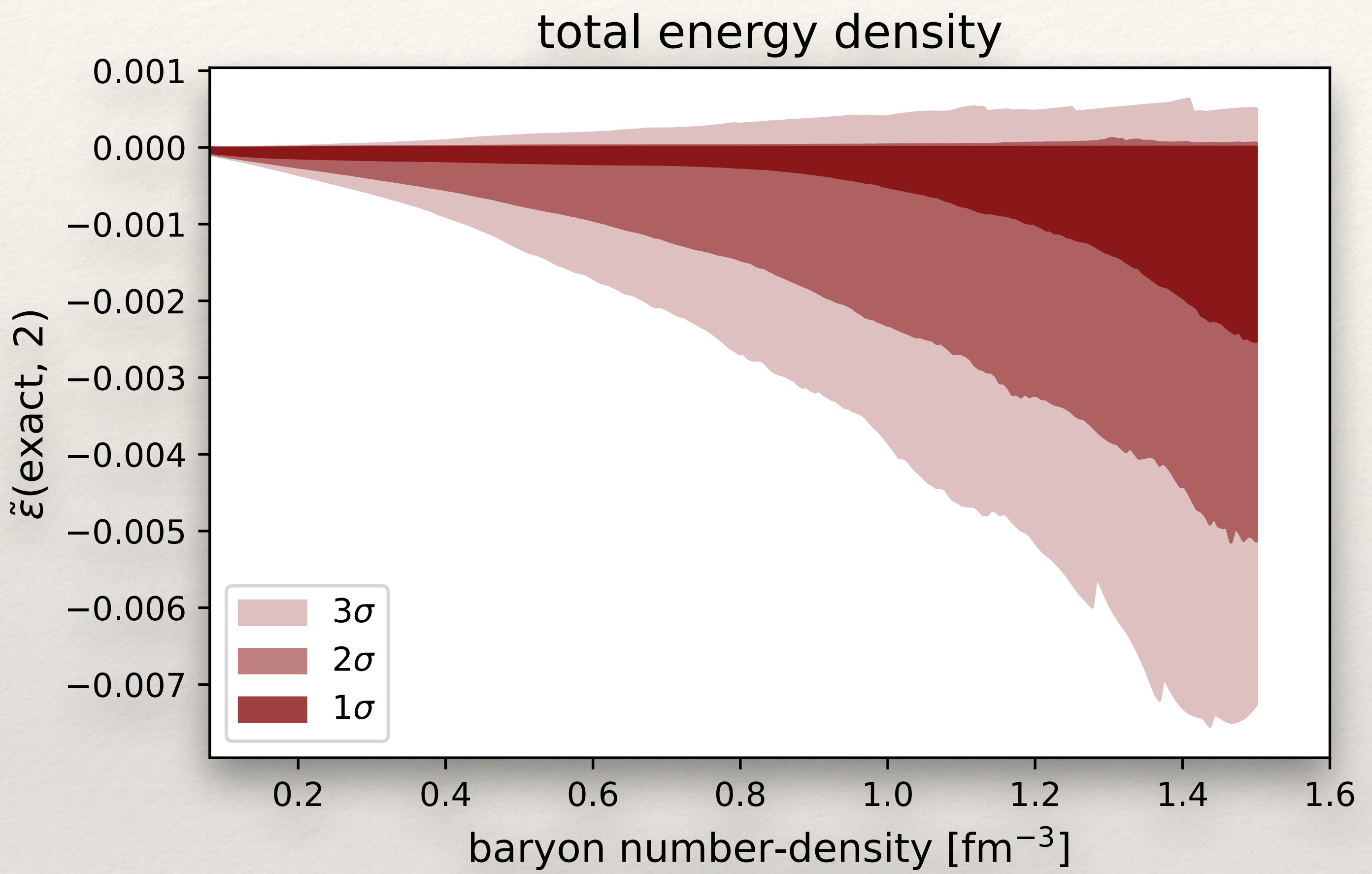
Beta-Equilibrium Pressure: Statistical Analysis



$$\tilde{P}_\beta = \frac{P_\beta^{(ext)} - P_\beta^{(2)}}{P_\beta^{(ext)} + P_\beta^{(2)}}$$

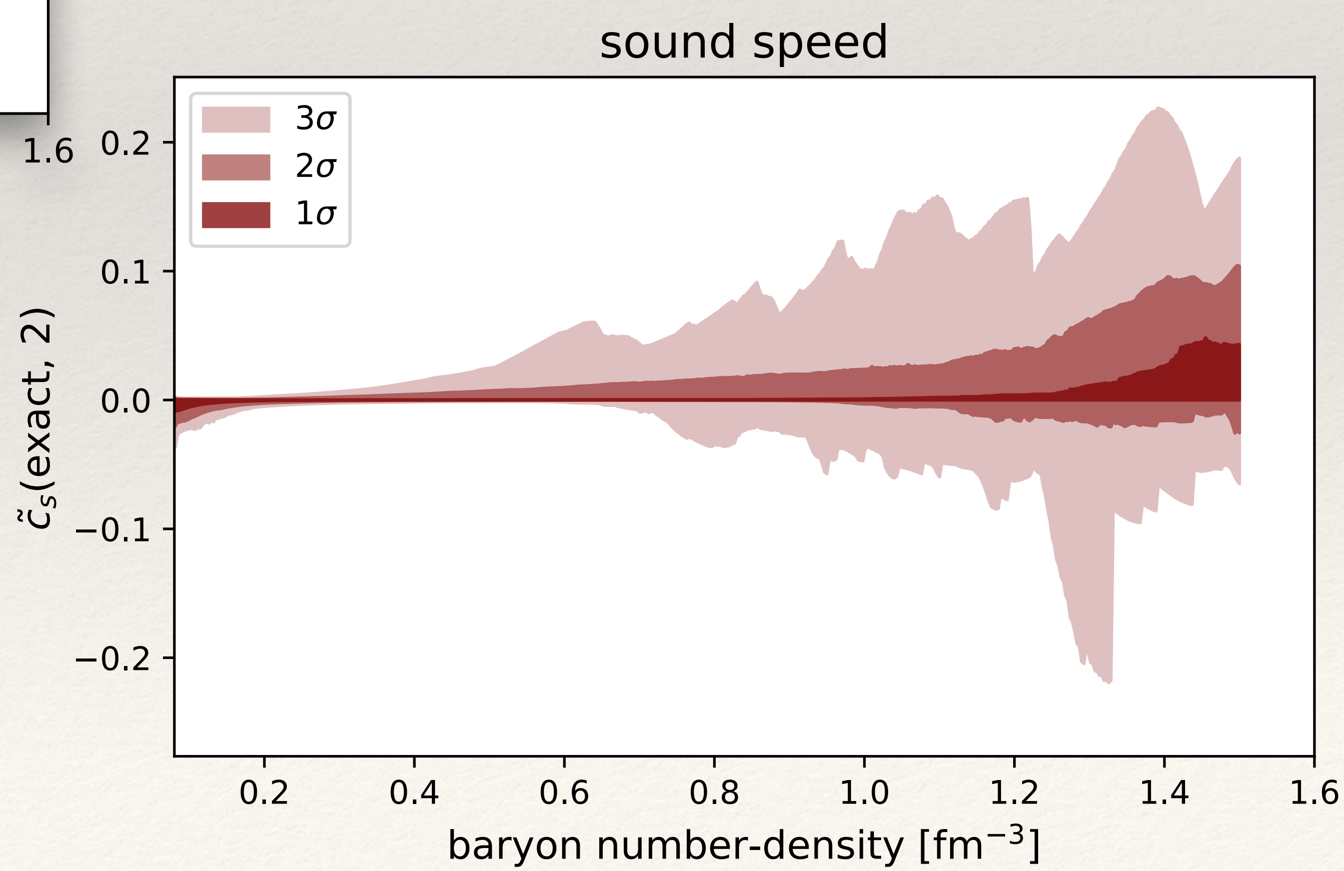
Finally, pressure of the β-equilibrium matter

EOS Property: Statistical Analysis



$$\tilde{\epsilon} = \frac{\epsilon^{(ext)} - \epsilon^{(2)}}{\epsilon^{(ext)} + \epsilon^{(2)}}$$

$$\tilde{C}_s = \frac{C_s^{(ext)} - C_s^{(2)}}{C_s^{(ext)} + C_s^{(2)}}$$

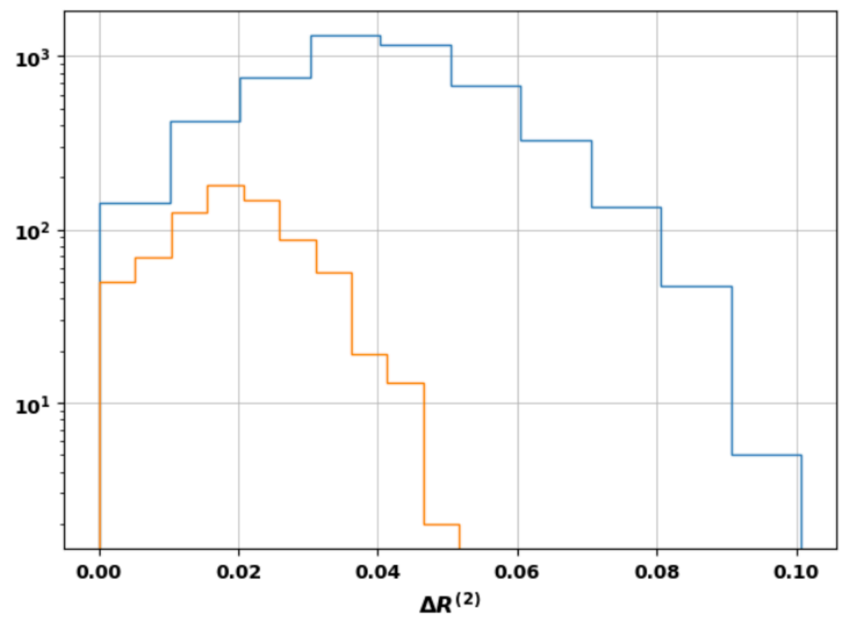


What About Neutron Star Observables?

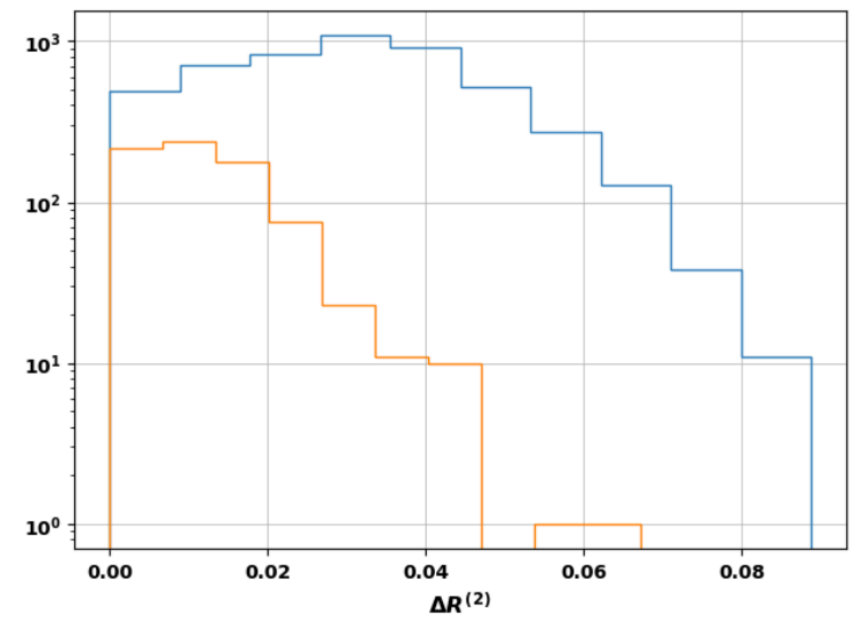
Differences in Mass, Radius, Tidal deformability parameters for different Symmetry Energy definitions

M-R Discrepancy: Population Studies

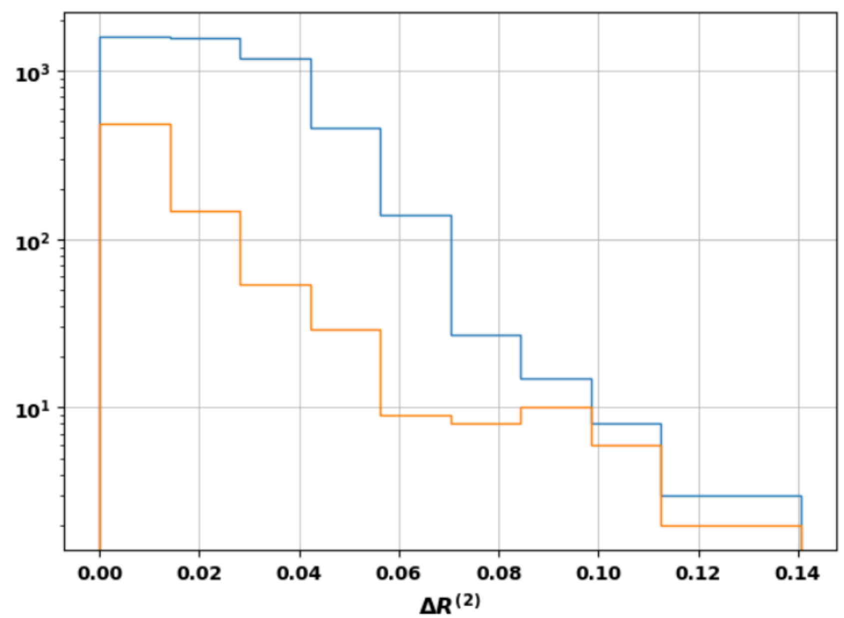
- Difference in radii for two E_{sym} definitions
- Population with wide range of NS masses



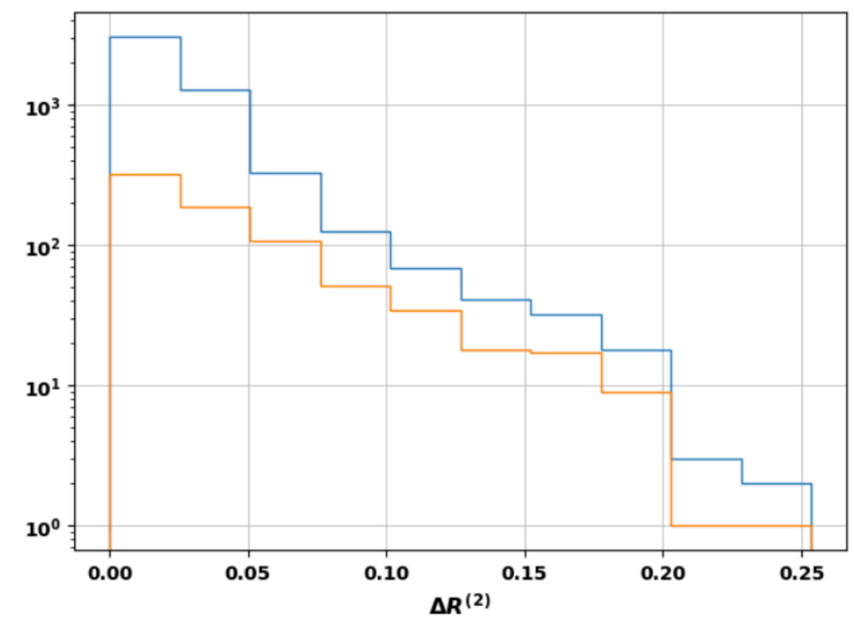
(a) Radius difference distribution of the sampled points at $1.2M_\odot$



(b) Radius difference distribution of the sampled points at $1.4M_\odot$

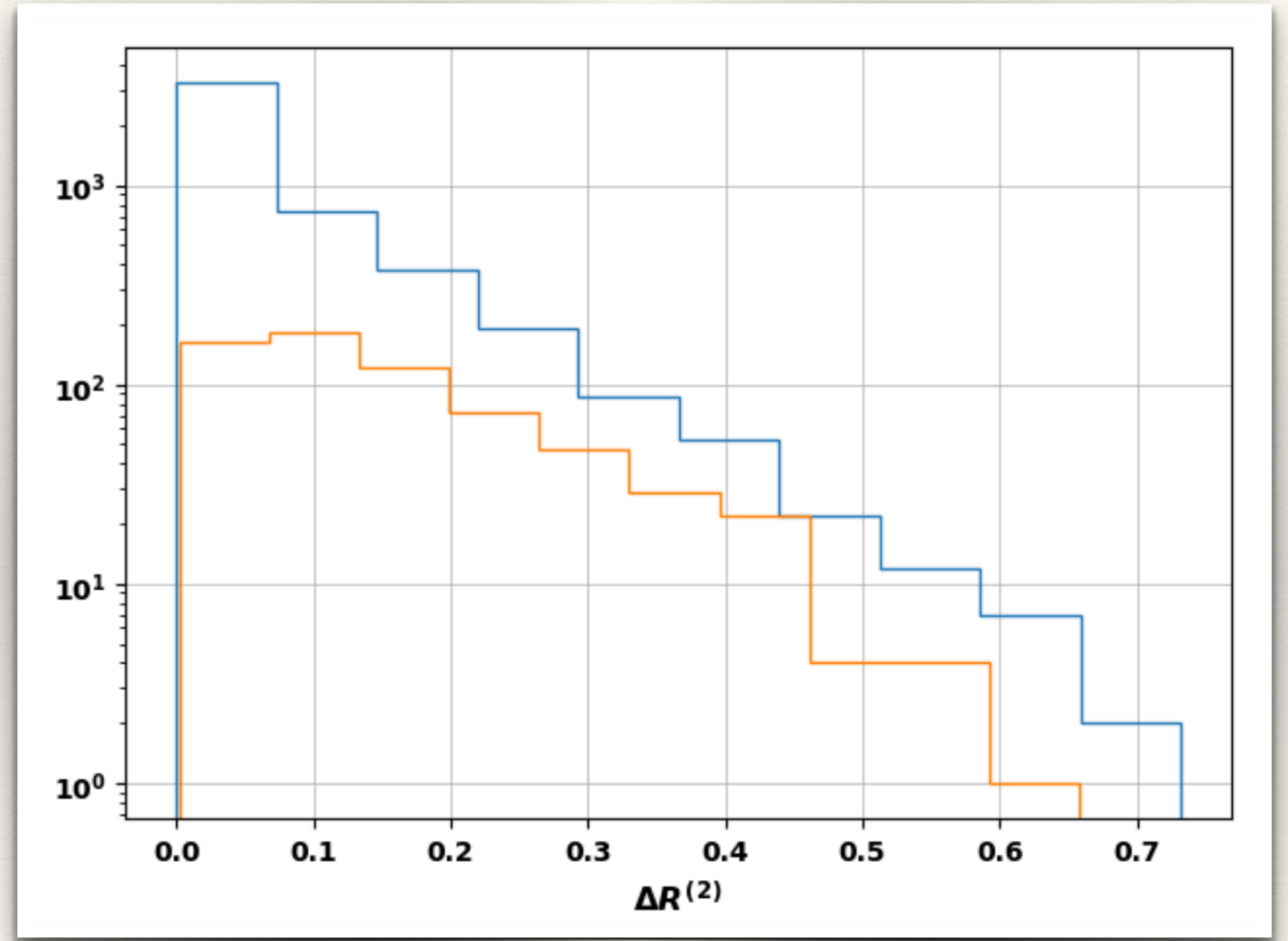


(c) Radius difference distribution of the sampled points at $1.6M_\odot$



(d) Radius difference distribution of the sampled points at $1.8M_\odot$

M-R Discrepancy: Population Studies-II



Difference in radii for 2 M_{sun} neutron star

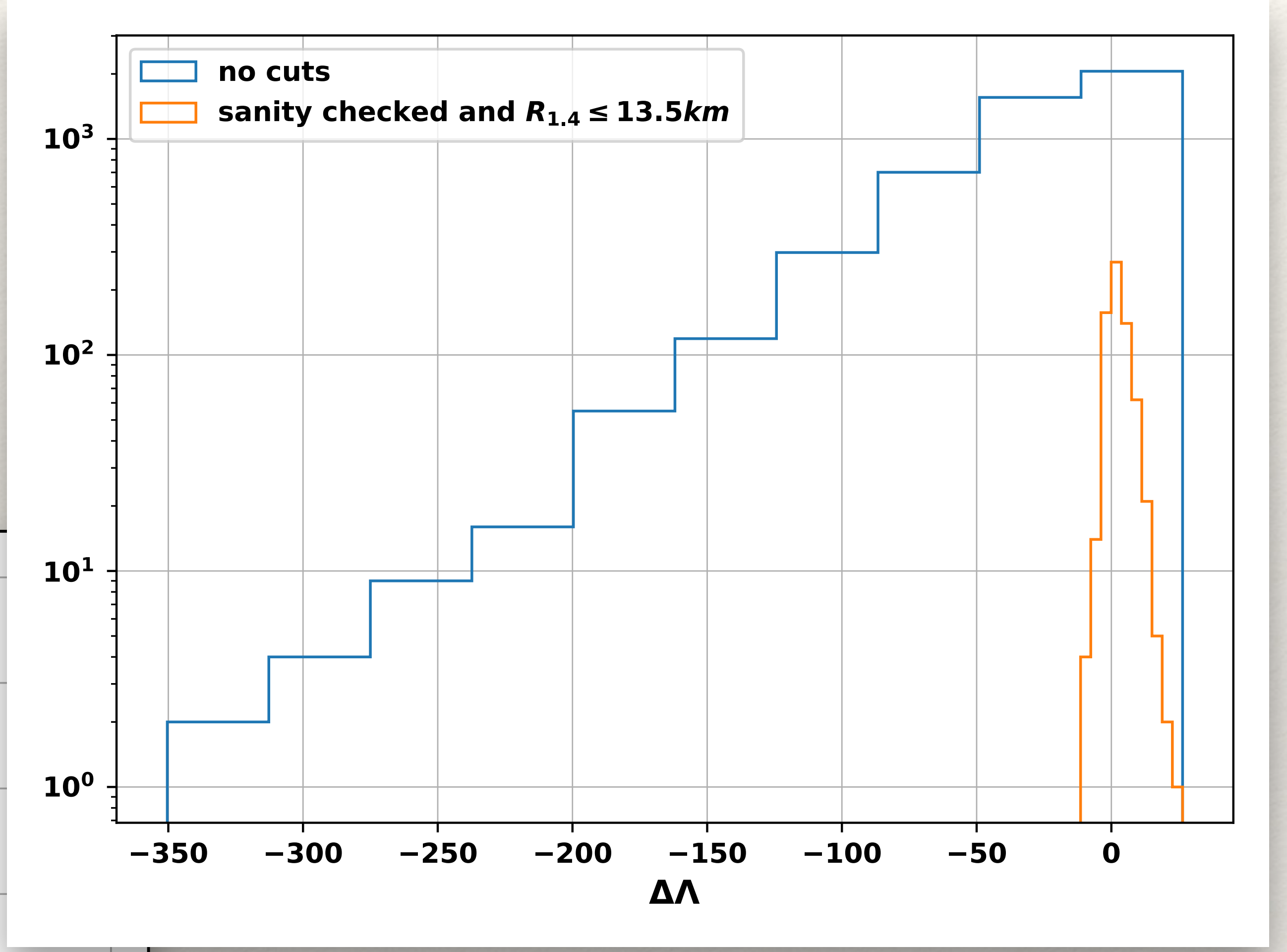
- Differences in radii are more prominent for massive NS
- For 2 M_{sun} it can exceed 0.5 km!

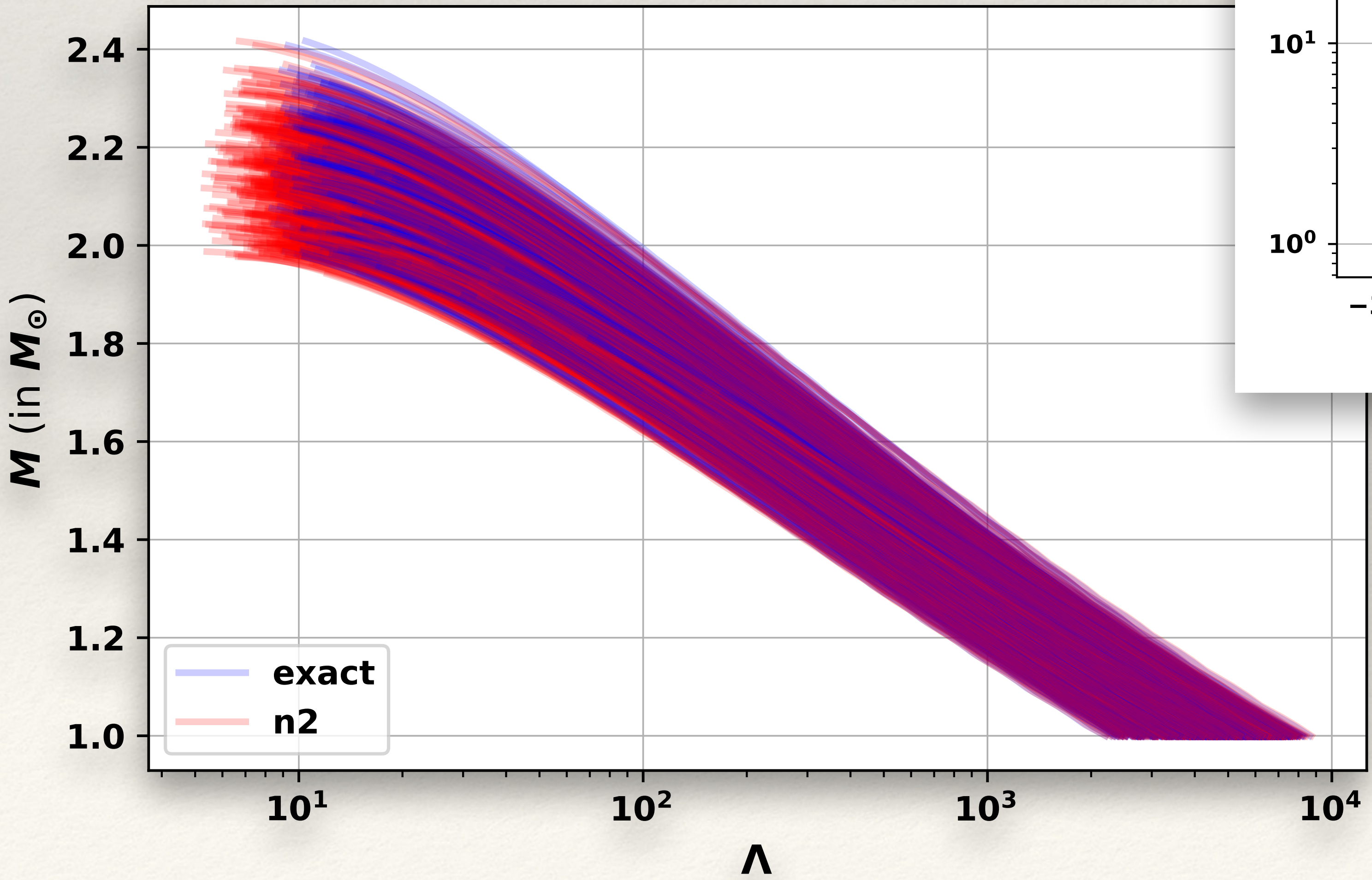
➡ **Immediate question: whether it is a measurable effect?**

E. Hoque, AM (WIP)

M-Λ Discrepancy: Population Studies

- Fractional difference in tidal deformability (Λ) is higher for more massive star
- Absolute difference in Λ is higher for the lighter star



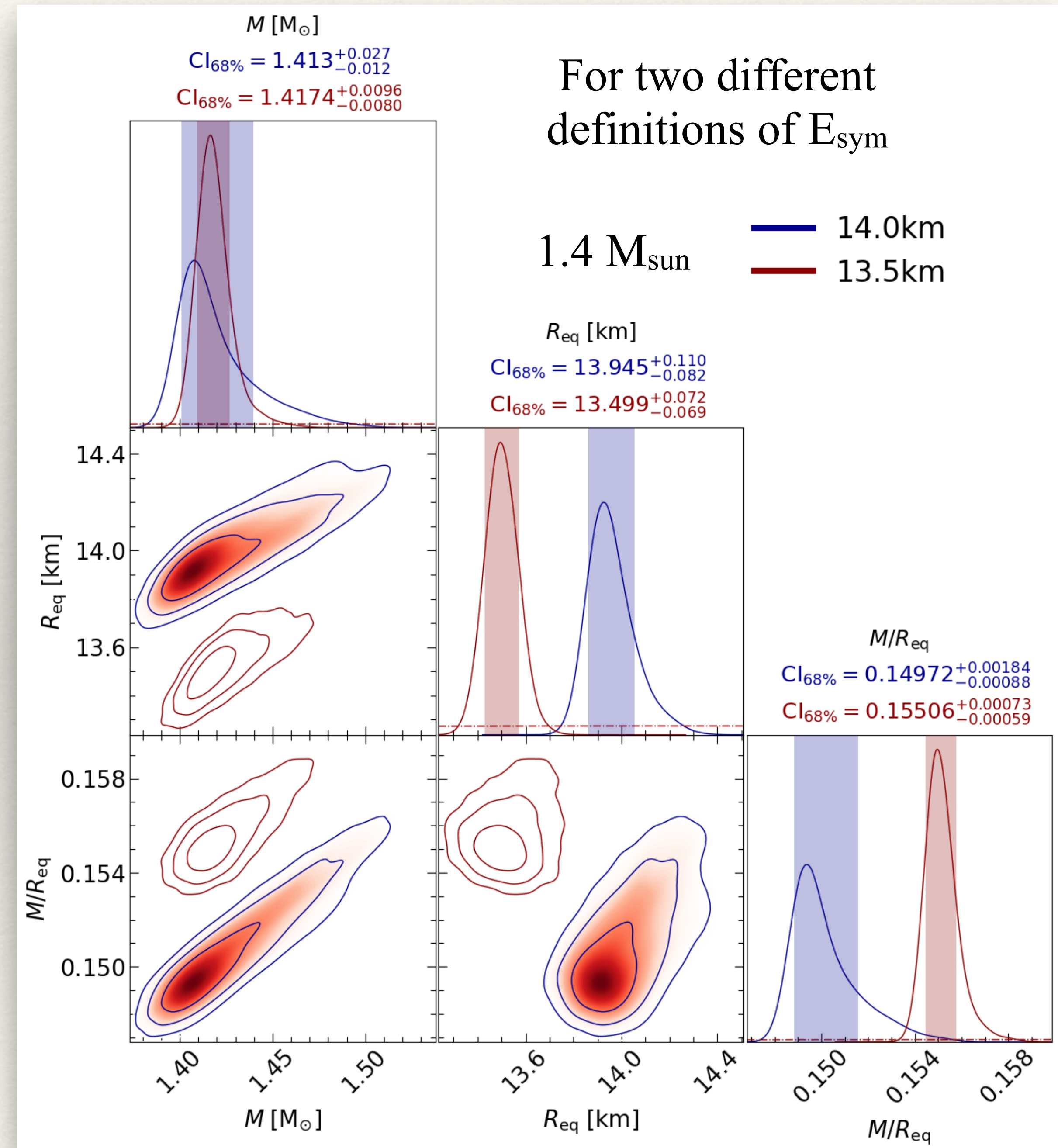


- GW-detectors are sensitive to absolute values

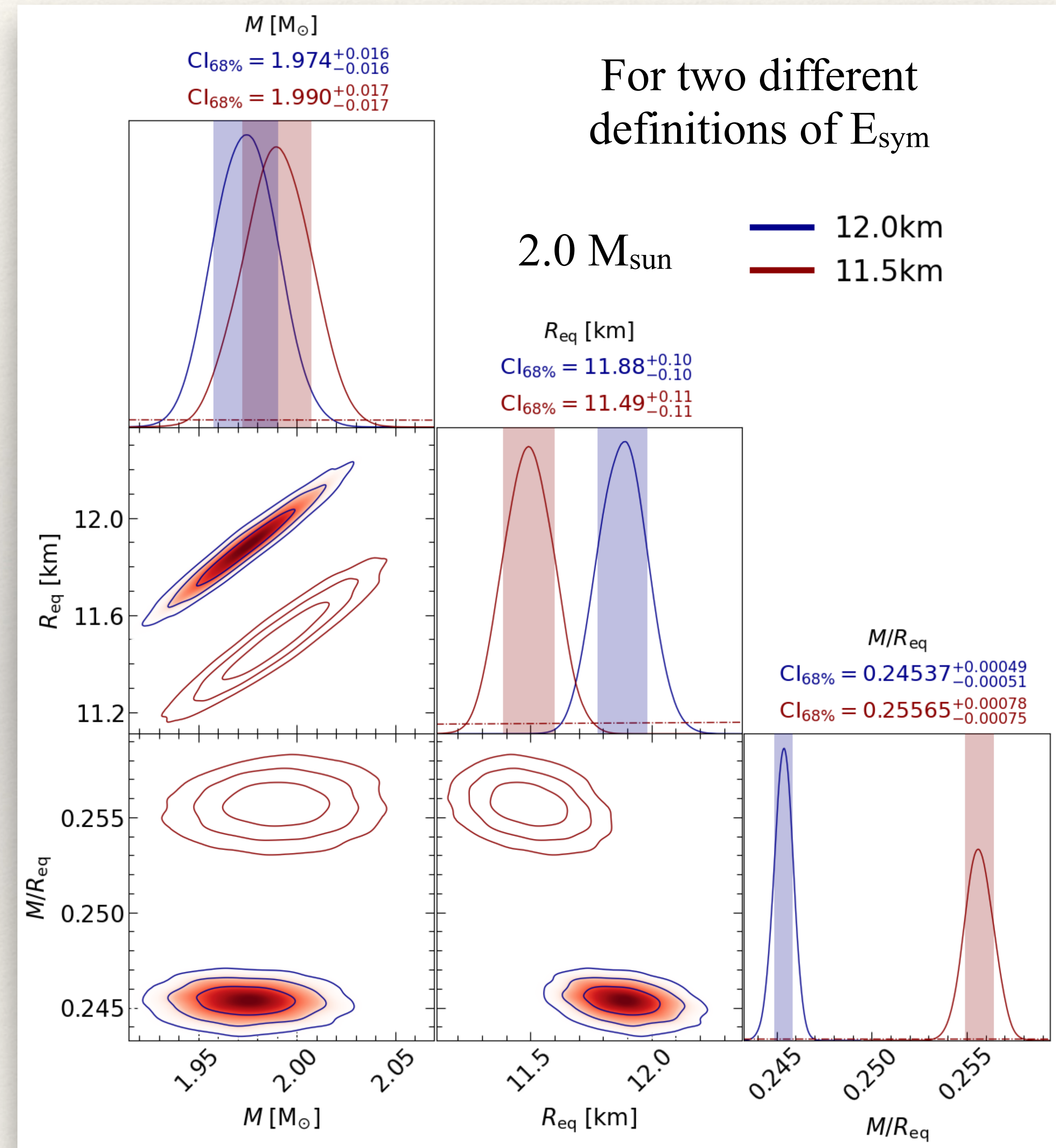
➡ **Can LVK/3G detectors measure this difference?**

Implications For (Semi-)Realistic Observations Of Neutron Stars

M-R Estimates From X-Ray Simulations-I



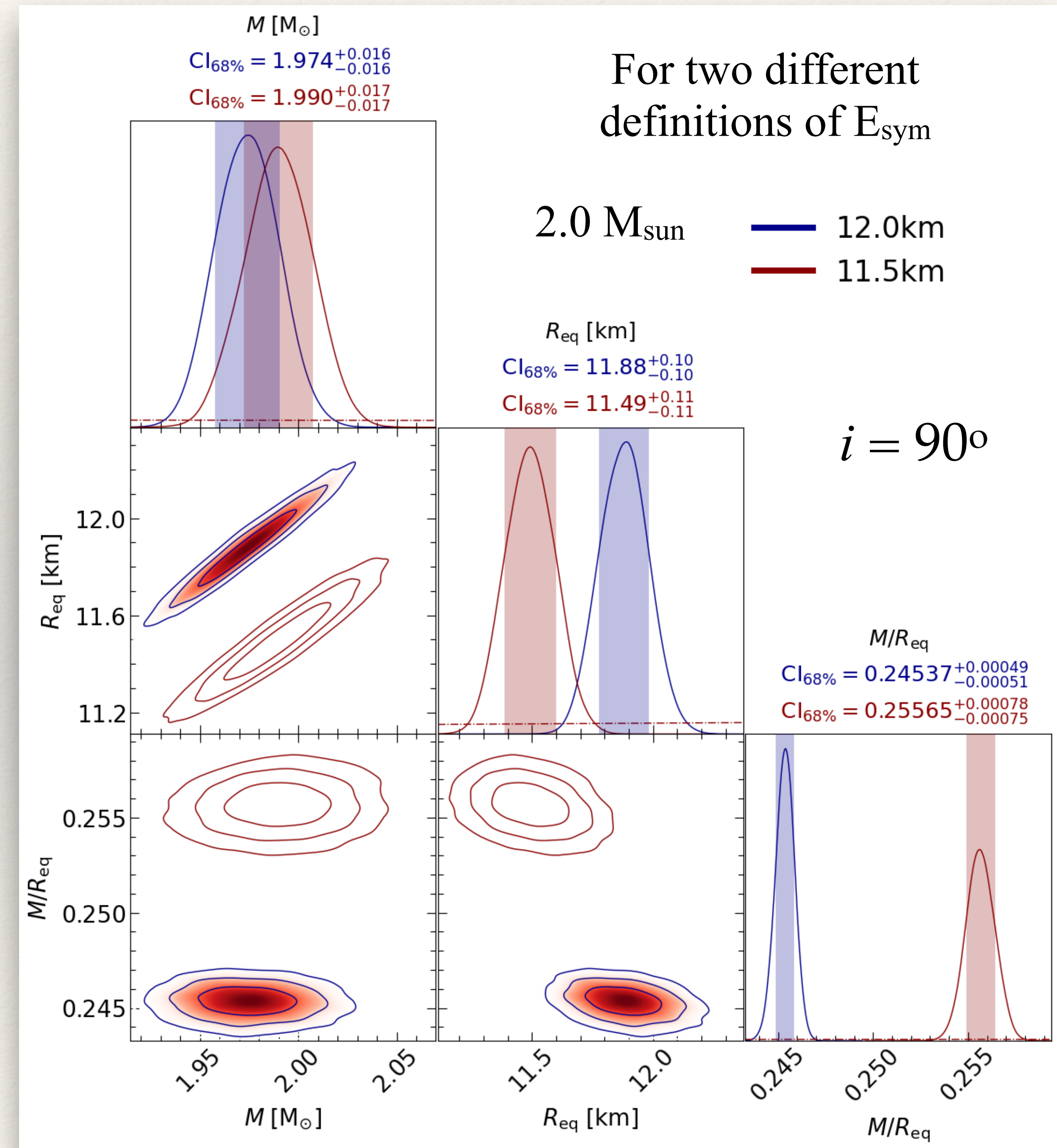
Stiffer EOS

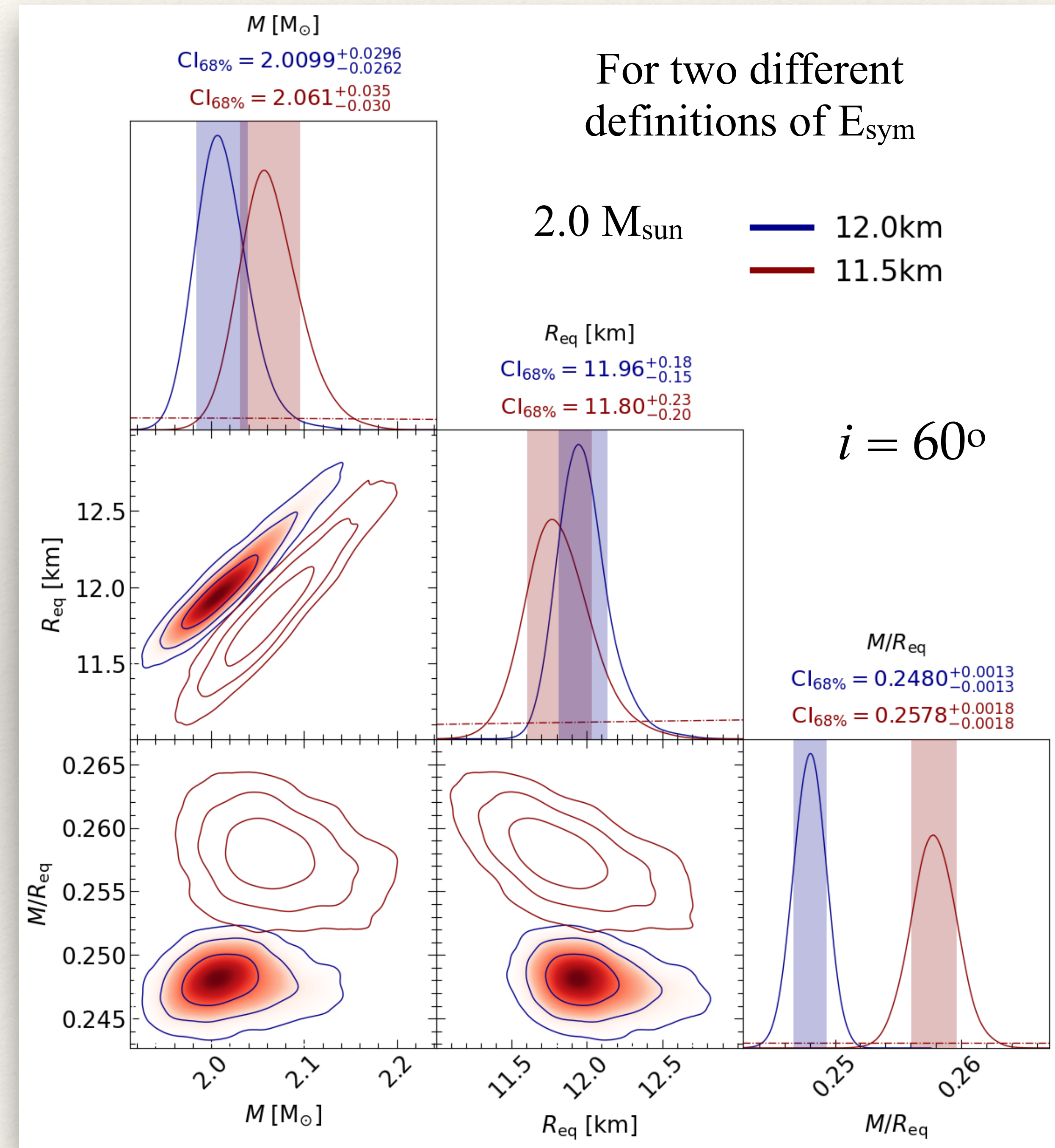


Softer EOS

Afrin, **AM**+ (WIP)

M-R Estimates From X-Ray Simulations-II





high-inclination angle

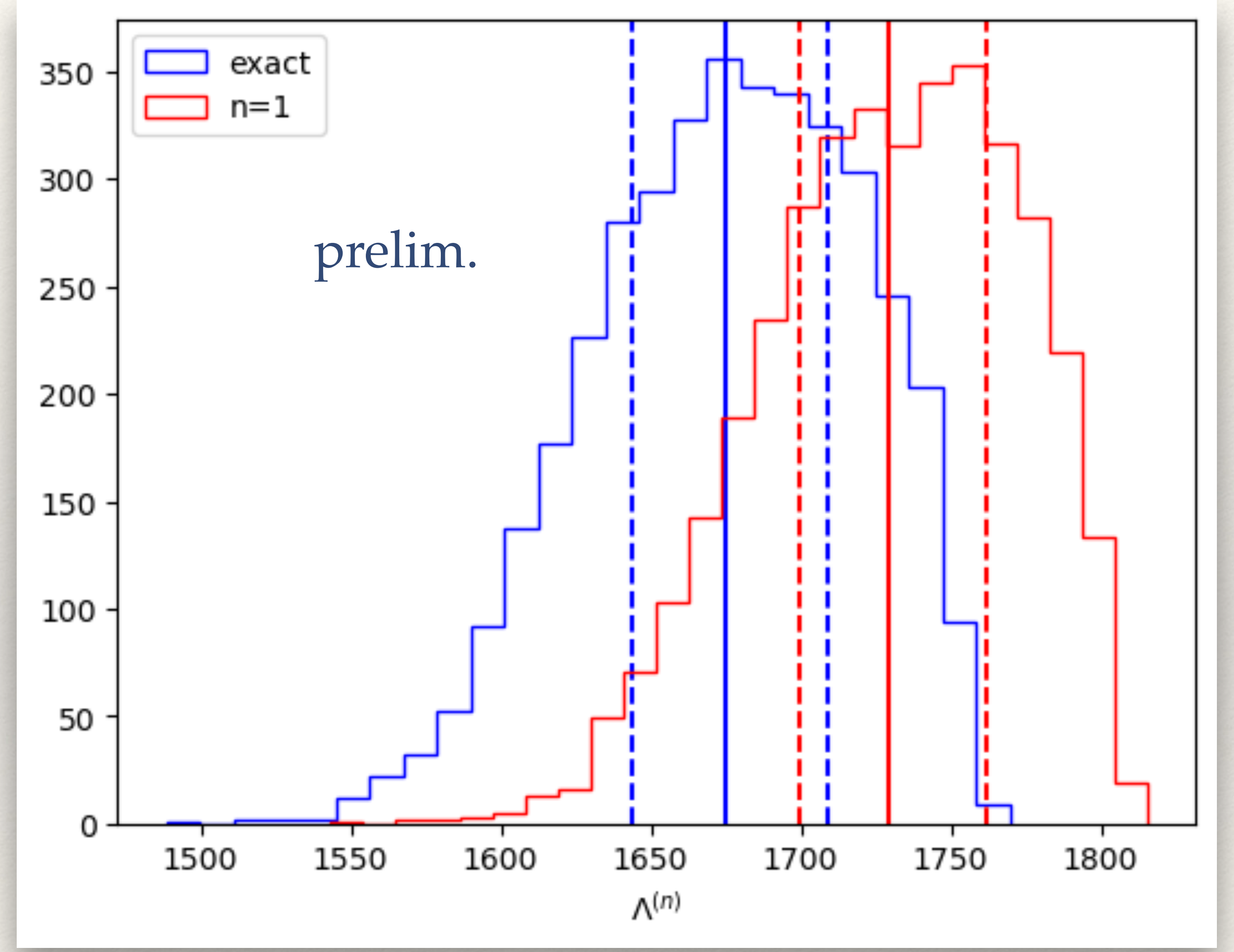
moderate-inclination angle

M-Λ Estimates From GW Observations

- We performed Bayesian PE of BNS-signal with synthetic data to gauge the effect of different E_{sym}

E. Hoque, AM (WIP)

- Injection parameters:
  - $m_1 = m_2 = 1.2\ M_{sun}$ (equal mass)
  - $\chi_1 = 0.04$ $\chi_2 = 0.01$ (slow spin)
  - $D_L = 40$ Mpc (**GW170817-like**)
  - GW waveform: IMRPhenomPv2_NRTidal
  - **GW noise-curve: adv-LIGO @ design sensitivity**



Discrepancy in inferred tidal deformability (Λ) parameter for different definitions of E_{sym}

Implications to (Global) Model Inference

- One of our primary goal is to develop a unified model for the dense matter interactions, the one that can explain finite nuclei to neutron stars
- However, difference in definitions for symmetry energy seems to be an issue
- It can induce systematic (elusive) biases while trying to perform statistical inference from unified models/joint analysis
- In particular, **lepton number density and pressures can have significant deviations** between two different definitions of symmetry energy
  - **Transport properties**, e.g., thermal electrical conductivity, viscosity, d-URCA & m-URCA rates, dynamical/ν-driven wind, etc. will be significantly affected
- M-R estimation from **NICER** and similar future instruments can infer a **systematic bias** in the microscopic model for the NS EOS
- Although the effect in tidal deformability is small, it could still be detectable and **important enough for future 3G detectors**, e.g., **CE & ET**

Is there any good reason to over look it?

IMHO: We need to make an effort to resolve this issue!

Thank you!

Questions & Comments