Critical fluctuations in heavy-ion collisions

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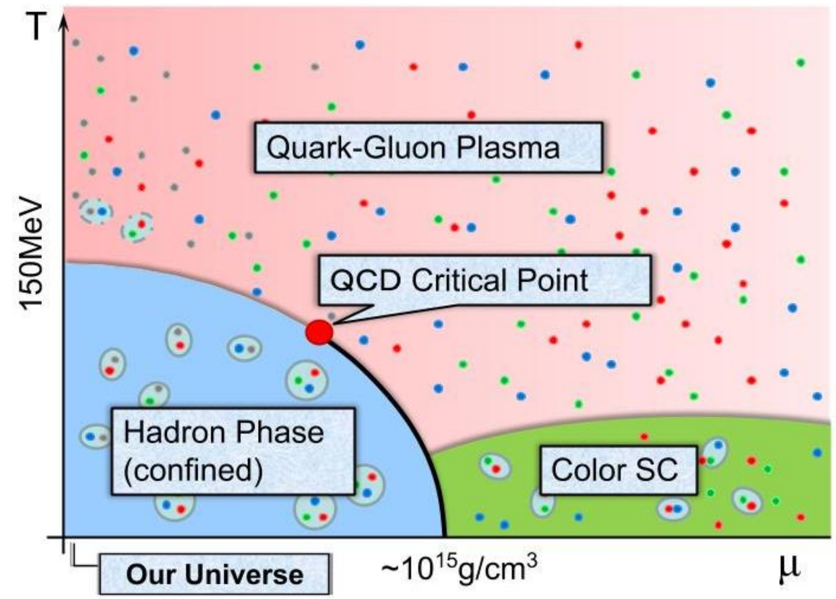
Institute for Nuclear Theory, Seattle
International Visitor Program &
“Heavy Ion Physics in the EIC Era”

Extreme QCD in heavy-ion collisions

Understanding the dynamics of the strong interaction under extreme conditions of temperature and density!

Important questions:

- Onset of deconfinement and chiral symmetry restoration?
- Properties of the strongly coupled QGP?
- Existence of a phase transition with critical end point?
- What are the dof in the core of compact stars?



Connect first-principle QCD calculations with experimental observables via a realistic dynamical modeling of heavy-ion collisions and astrophysical events**!**

How to observe the critical point in HIC

- At a critical point, the correlation length ξ diverges and so do the fluctuations.
- Observable in higher-order cumulants of net-baryon number.

$$\chi_1 = \frac{1}{VT^3}\langle N \rangle\,, \quad \chi_2 = \frac{1}{VT^3}\langle(\Delta N)^2\rangle\,, \quad \chi_3 = \frac{1}{VT^3}\langle(\Delta N)^3\rangle\,,$$

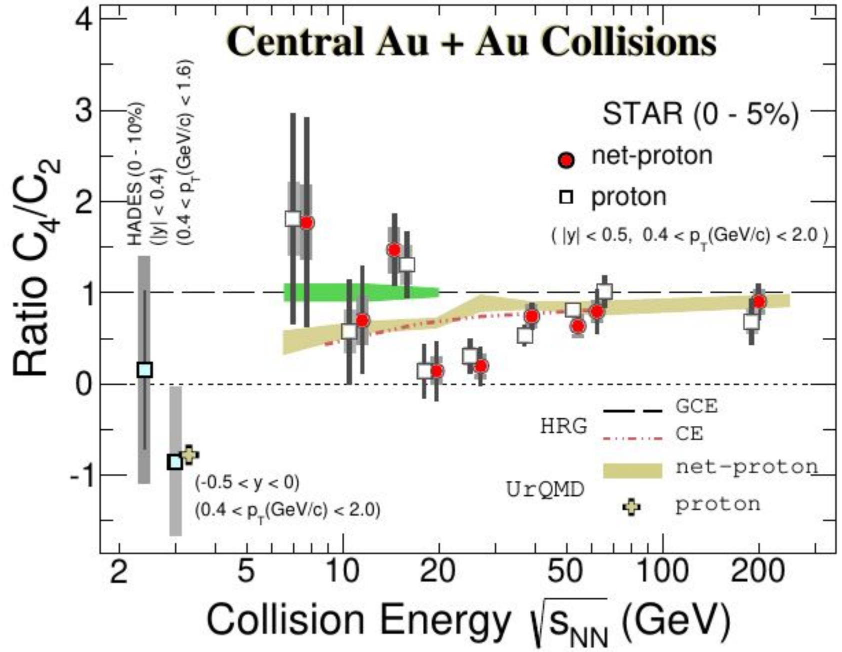
$$\chi_4 = \frac{1}{VT^3}\langle(\Delta N)^4\rangle_c \equiv \frac{1}{VT^3}\left(\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2\right)\,.$$

- To 0th order in V fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

variance Skewness Kurtosis

- At a CP the τ_{relax} diverges with ξ which leads to critical slowing down



STAR, PRL 128 (2022)

Interesting deviations from the baseline in the experimental data... are they due to the critical point of QCD?

Need a dynamical model!

Importance of dynamical modeling

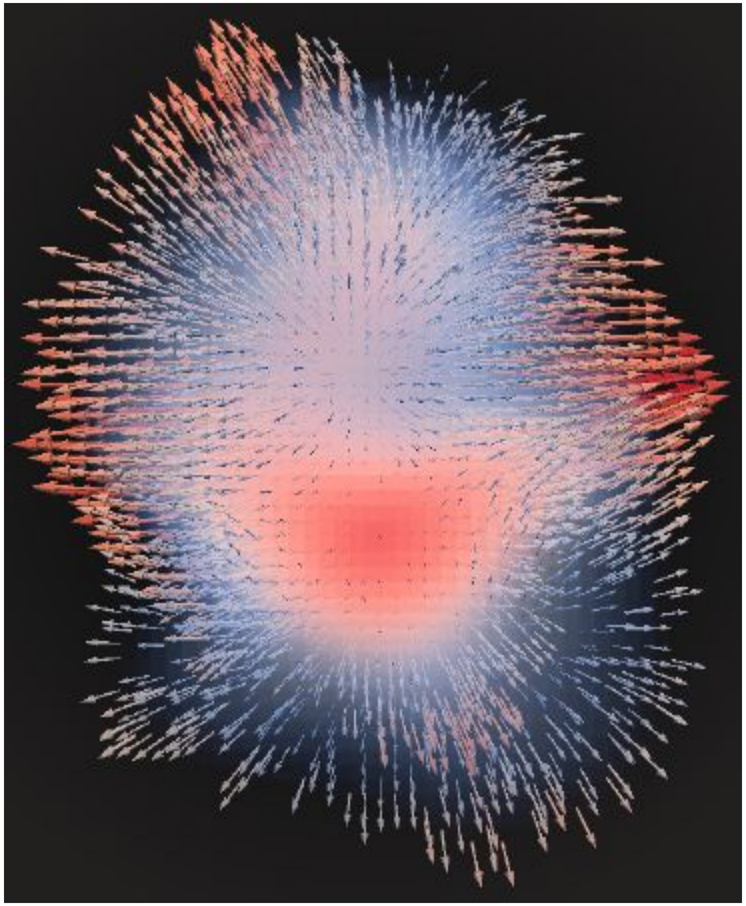
In a grand-canonical ensemble the system is...

- in thermal equilibrium (= long-lived)
- in equilibrium with a particle heat bath
- spatially infinite
- and static

Systems created in a heavy-ion collision are

- short-lived
- spatially small
- inhomogeneous
- and highly dynamical!

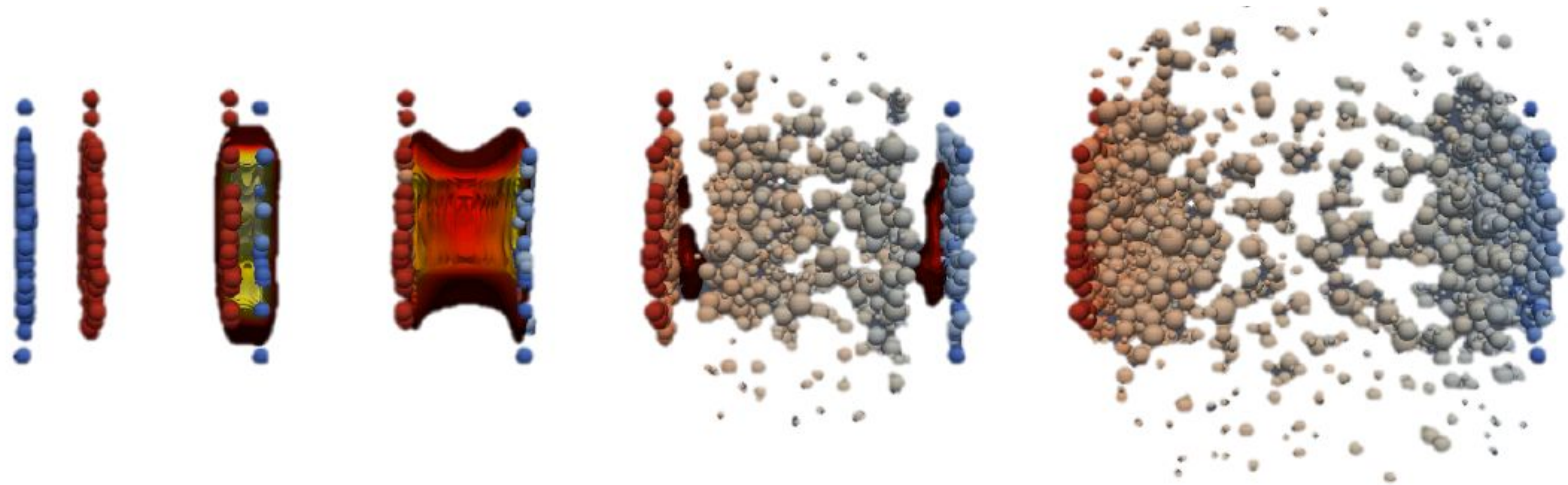
Solution: Develop dynamical models to describe the phase transition in heavy-ion collisions
Event-by-event dynamical modeling allows us in addition to study different particle species, experimental cuts, hadronic final interactions, etc.



madai.us

EMMI Rapid Reaction Task Force: M. Bluhm, A. Kalweit, MN et al. NPA 1003 (2020), 2001.08831

Fluctuations all along the way



- Initial state fluctuations due to quantum mechanical fluctuations and multiplicity fluctuations
- Thermal fluctuations, including the formation of critical fluctuations
- Fluctuations due to the hadronization process
- Fate of fluctuations in the hadronic phase
- Imperfect detection efficiency and finite acceptance

Approaches to fluid dynamical fluctuations

There are two main approaches of describing fluid dynamics with noise:

Hydro-kinetics

- Set of deterministic kinetic equations for n-point functions of fluid dynamical fields
- Renormalization (perturbatively) performed during the derivation
- Statistical average performed in the derivation of deterministic equations

A.Andreev, Sov. Phys. JETP 32 no. 5 (1971) and 48 no. 3 (1978); Y. Akamatsu et al., PRC 95 no. 1 (2017) and 97 no. 2 (2018); M. Stephanov et al., PRD 98 (2018); M. Martinez et al., PRC 99 no. 5 (2019); X. An et al., PRC 100 no. 2 (2019), PRL 127 (2021); L. Du et al., PRC 102 (2020); K. Rajagopal, NPA 1005 (2021)

Stochastic/fluctuating fluid dynamics

- Numerical implementation of the fluid dynamical equations with stochastic conservation law:

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + T^{\mu\nu}_{\text{viscous}} + S^{\mu\nu}_{\text{noise}}\,,$$
$$\partial_\mu J^{\mu} = 0, \quad J^{\mu} = J^{\mu}_{\text{ideal}} + J^{\mu}_{\text{viscous}} + I^{\mu}_{\text{noise}}\,.$$

- Sample discretized noise event-by-event
- Observables are calculated from statistical averaging over events.
- Can easily be integrated in standard event generators of HIC!
- Many challenges....

Fluctuating Dissipative Fluid Dynamics

The correlators of the thermal noise terms in the energy momentum tensor and the conserved currents :

$$\langle S^{\mu\nu}(x_1) S^{\alpha\beta}(x_2)\rangle = 2T \left[\begin{array}{c} \eta\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right) \\ + \left(\zeta - \frac{2}{3}\eta\right)\Delta^{\mu\nu}\Delta^{\alpha\beta} \end{array} \right] \delta^{(4)}(x_1 - x_2).$$

$$\langle I^{\mu}(x_1) I^{\nu}(x_2)\rangle = 2T\sigma\Delta^{\mu\nu}\delta^{(4)}(x_1 - x_2)\,.$$

Several issues arise from the discretization of the Dirac delta function in the noise

- Stochastic noise introduces a lattice spacing dependence.
- Correction terms due to renormalization become large for small lattice spacings.
- Large noise contributions can locally lead to negative energy densities.
- Large gradients introduced by the uncorrelated noise is a problem for PDE solvers.

EMMI Rapid Reaction Task Force: M. Bluhm, A. Kalweit, MN et al. NPA 1003 (2020), 2001.08831

Fluctuating Dissipative Fluid Dynamics

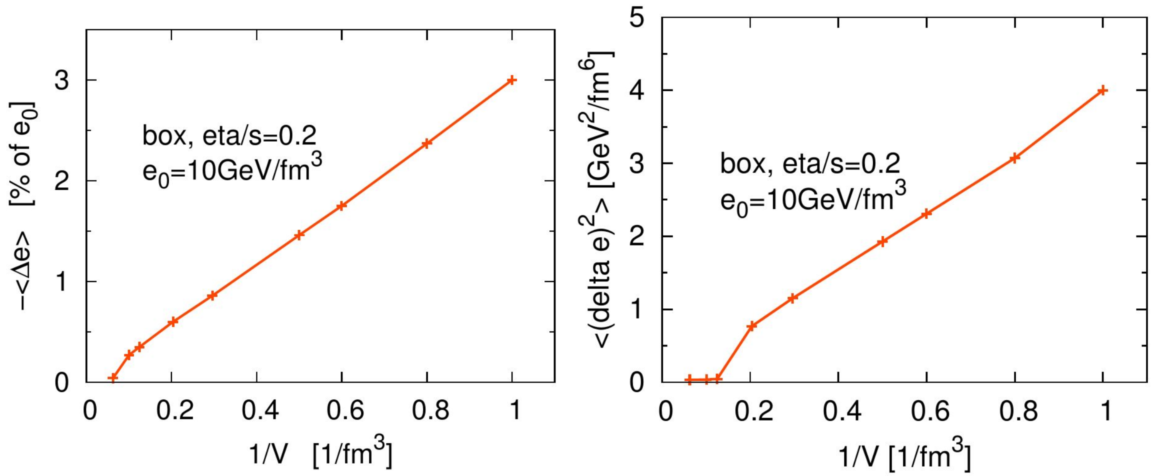
First implementations of FDFD have shown: need to limit the resolution scale; simulate noise down to a particular filter length scale, for which:

$$l_{\text{grid}} < l_{\text{filter}} \lesssim l_{\text{noise}} \ll l_{\text{hydro}}$$

Murase et al.: noise is smeared by Gaussians with widths of 1-1.5 fm (choice not discussed), large enhancement of flow observed. K. Murase et al, NPA 956 (2016);

Nahrgang et al.: noise is coarse-grained over distances of approx. 1fm, lattice spacing dependence of the energy density and its fluctuations observed.

M. Nahrgang et al, Acta Phys. Polon. 10 (2017);



Singh et al.: high-mode Fourier filter with a coarse graining scale of > 1fm, multiplicities and flow are little affected by the inclusion of fluctuations

M. Singh se et al, NPA 982 (2019);

Renormalizing critical dynamics in model A of Hohenberg-Halperin in 3 dimensions

Cassol-Seewald et al. 0711.1866
Farakos et al. 9412091, 9404201
Gleiser, Ramos 9311278

Stochastic relaxation equation of chiral order parameter

$$\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi + \eta\frac{\partial\varphi}{\partial t} + \frac{\partial V_{\text{eff}}}{\partial\varphi} = \xi$$

Ginzburg-Landau effective potential, ϵ encodes phase transition

$$V_{\text{eff}}(\varphi) = \frac{1}{2}\epsilon\varphi^2 + \frac{1}{4}\lambda\varphi^4$$

White thermal noise

$$\langle\xi(\vec{x},t)\rangle = 0 \quad \text{and} \quad \langle\xi(\vec{x},t)\xi(\vec{x}',t')\rangle = 2\eta T\;\delta(\vec{x}-\vec{x}')\delta(t-t')$$

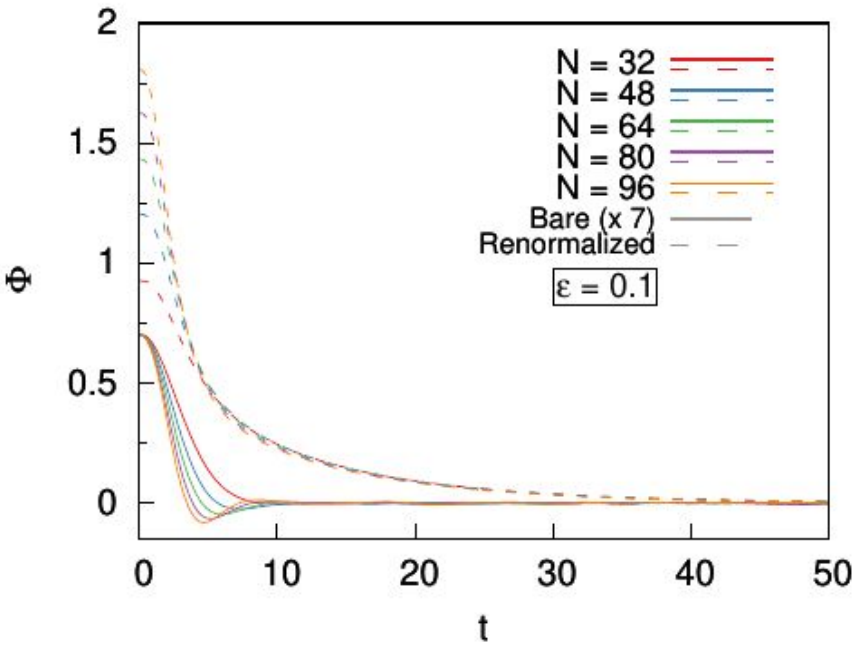
Counterterm for mass renormalization

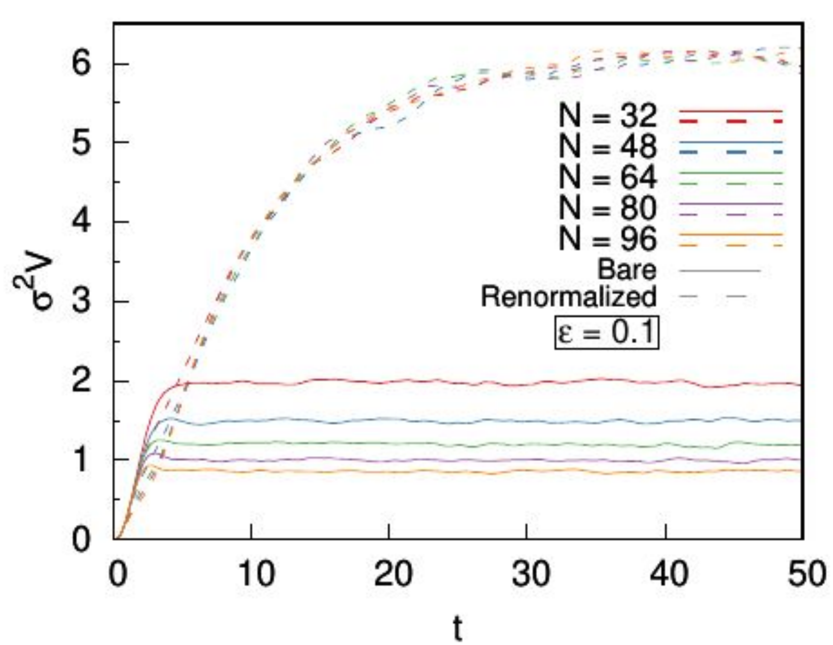
$$V_{\text{CT}} = \left\{ -\frac{3\lambda\Sigma}{4\pi}\frac{T}{\mathrm{d}x} + \frac{3}{8}\left(\frac{\lambda T}{\pi}\right)^2\left[\ln\left(\frac{6}{M\mathrm{d}x}\right) + \zeta\right]\right\}\frac{\varphi^2}{2}$$

Renormalizing critical dynamics in model A

Time evolution near a critical point of

mean & variance





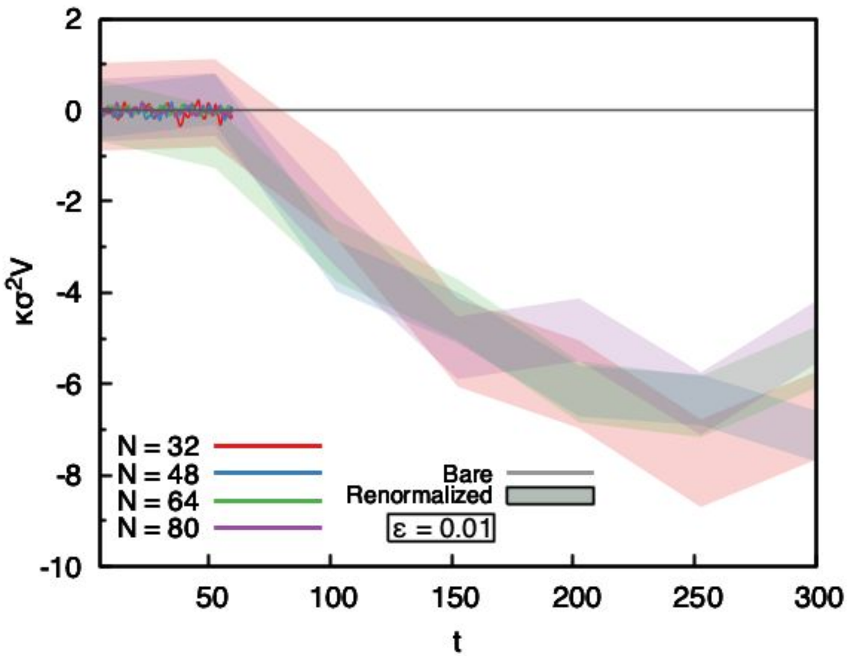
Restoration of lattice spacing independence after including the counterterm

N. Attieh, M. Bluhm, N. Touroux, M. Kitazawa, T. Sami, MN, in preparation

Renormalizing critical dynamics in model A

Time evolution near a critical point of

kurtosis



A nonzero kurtosis is observed after inclusion of the counterterm. Lattice spacing dependence or independence cannot be resolved within the available statistics.

N. Attieh, M. Bluhm, N. Touroux, M. Kitazawa, T. Sami, MN, in preparation

Net-baryon diffusion: model B in 1+1 dimensions

- In the long-time, equilibrium limit the net-baryon density is the slowest mode near the CP.
- For baryonic matter that decouples from the energy flow of the system (model B of Hohenberg, Halperin), the diffusive dynamics follows the minimization of the free energy F

$$\partial_t n_B(t,x) = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}[n_B]}{\delta n_B} \right) + \nabla \mathbf{J}(\mathbf{t},\mathbf{x})$$

with the stochastic current (Gaussian, white noise)

$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x), \quad \kappa = \frac{Dn_c}{T}$$

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

- Apply the evolution in a 1+1 dimensional, boost-invariant Bjorken expansion

Fluctuations in an expanding background, e.g. J. Kapusta et al, PRC 85 (2012); Y. Akamatsu et al. PRC 95 (2017), M. Martinez et al, PRC 99 (2019)

- The nonlinear stochastic diffusion equation transforms as:

$$\partial_\tau n_B = \frac{Dn_c}{\tau \chi_2(\tau)} \partial_y^2 n_B - \frac{Dn_c K(\tau)}{\tau} \partial_y^4 n_B + \frac{Dn_c}{6\,\tau \chi_4(\tau)} \partial_y^2 {n_B}^3 - \partial_y \xi \,.$$

In Gauss limit: M. Sakaida et al PRC 95 (2017); nonlinear (only critical): M. Kitazawa, G.Pihan, N. Touroux, M. Bluhm, MN NPA 1005 (2021)

Singular and regular susceptibilities

- Parametrize the susceptibilities $\chi_2(\tau)$ and $\chi_4(\tau)$ with a regular part using the argument in

M. Asakawa, U. Heinz, B. Müller, PRL 85 (2000)

$$\chi_n(\tau) = \frac{\langle \Delta N_B^n \rangle}{S}\Big|_{\mathrm{QGP/HRG}} = \frac{\chi_\mathrm{B}^n}{s/T^3}\Big|_{\mathrm{QGP/HRG}}$$

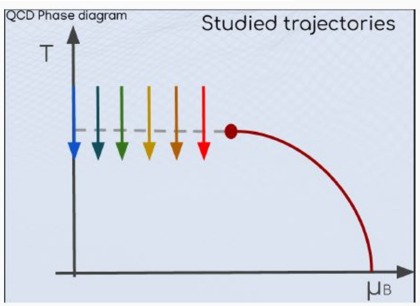
With χ_B^n and the entropy fixed to lattice results at T=280 MeV for the QGP and T=130 MeV for the HRG, matched via tanh function.

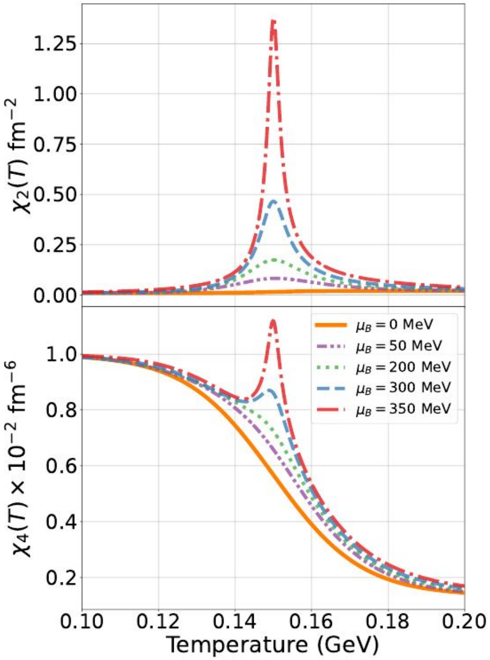
- Couple with the singular contribution (3D Ising) via

$$\chi_n(T) = \chi_n^{sing}(T) + \chi_n^{reg}(T)$$

- Match to the coefficients in the expansion of the free energy density functional.

$$\chi_n(\tau) = \tau \left(\frac{\delta^n \mathcal{F}}{\delta n_B^n}\Big|_{\Delta n_B=0} \right)^{-1}$$





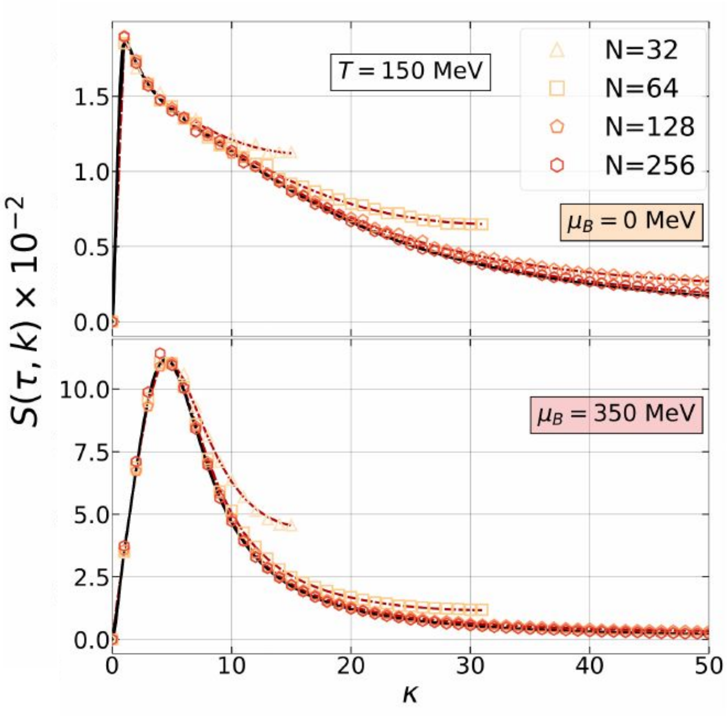
- Investigate several trajectories in the QCD phase diagram.

G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

Validating the linear model

only here
$\lambda_4 = 0$!

Structure factor Sk



G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

Important step for all fluctuating codes:

validation of the appropriate linear model:

- Structure factor and equal-time correlation function are well reproduced
- Approach to continuum as resolution is increased.
- Lower wavenumbers well described with the maximal resolution chosen for this work.
- Enhancement of fluctuations with low wavenumbers at T_c = 150 MeV.
- Discretization and baryon conservation effects under control.

Anticorrelations as a signal for the critical point

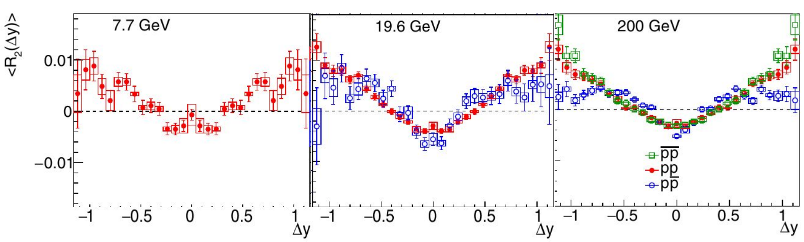
$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M}$ $\qquad$ $\frac{\chi_3}{\chi_2} = S\sigma$ $\qquad$ $\frac{\chi_4}{\chi_2} = \kappa\sigma^2$

variance $\qquad$ Skewness $\qquad$ Kurtosis

Higher-order moments are more sensitive
- to the divergence of the correlation length
- and to any other noncritical aspect of HIC...

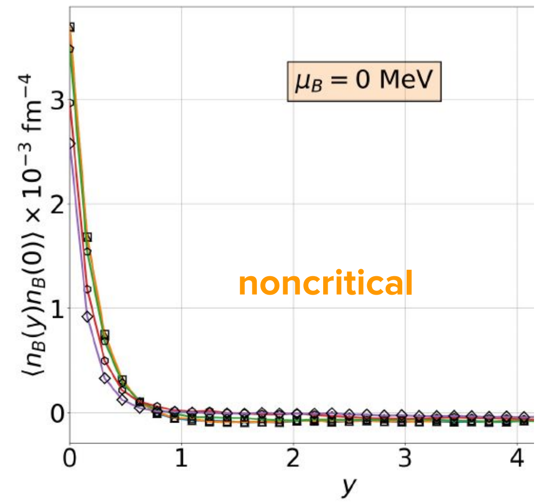
Here: dynamical fluctuations of net-baryon density

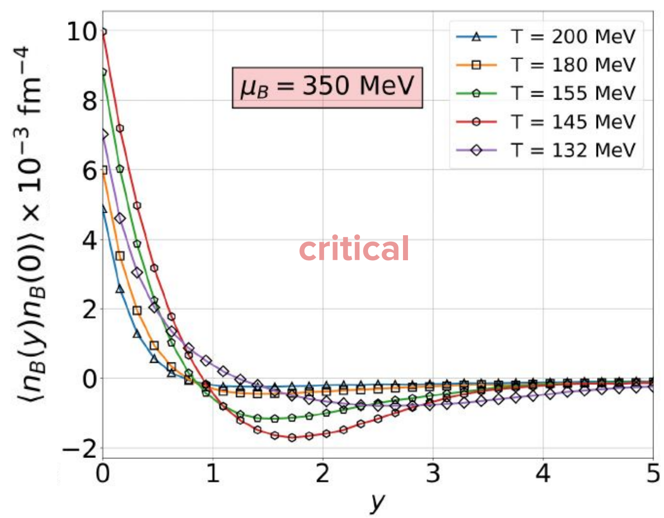
- Large fluctuations are balanced by large anti-correlations (net-baryon conservation)
- Due to the dynamics these anti-correlations cannot diffuse fast enough
- Approaching Tc they are visible at y ∼ 1-2
- At lower T the minimum becomes smaller and moves to larger y
- → Possible detection depends crucially on T_{FO}
- Interesting experimental data:
  (STAR AuAu, 30-40% most central, 0.4<pT<2GeV)



STAR Collab, PRC 101 (2021)

correlation function in rapidity





G. Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, PRC 107 (2023)

Word of caution: not yet an apple-to-to apple comparison possible

Non-monotonic kurtosis as a signal for the critical point

- Monotonic increase in the variance
- Non-monotonic Kurtosis only for the trajectories with critical point.
- This non-monotonic behavior of the kurtosis survives the rapid expansion for a diffusion length D = 1 fm

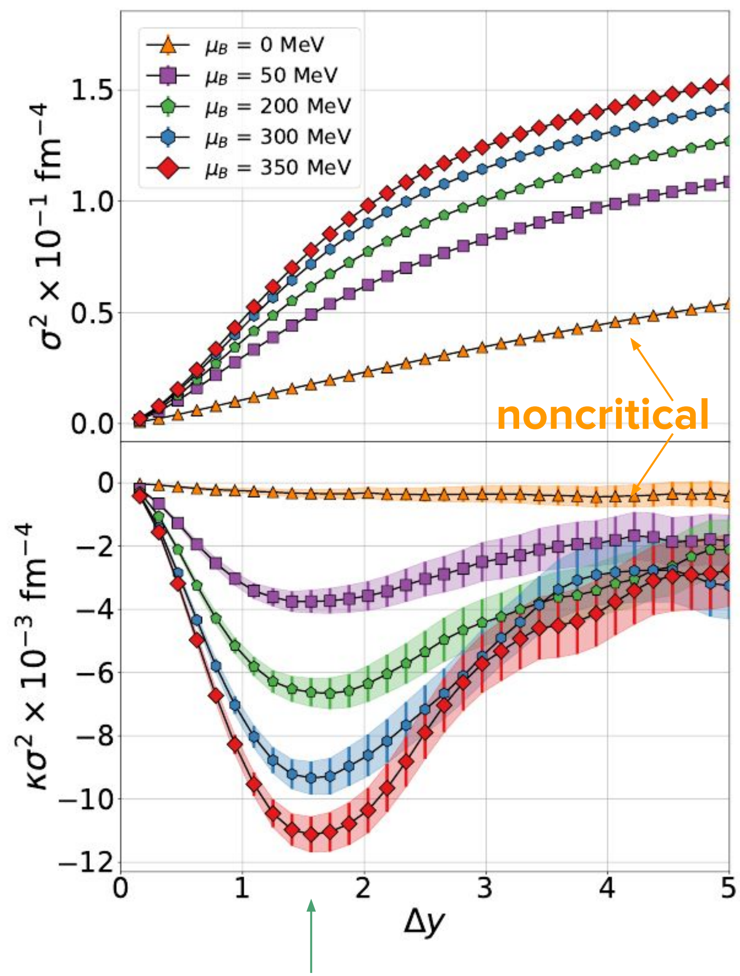
→ strong indication for the presence of the critical point.

- For increasing D the minimum moves to larger distances in rapidity.

→ Essential for the experiment to cover a wide range in rapidity to see the non-monotonicity.

- Interesting experimental data:
(STAR AuAu, 0-5% most central, 0.4<pT<2GeV, |y|<ymax)

rapidity dependence of fluctuation observables



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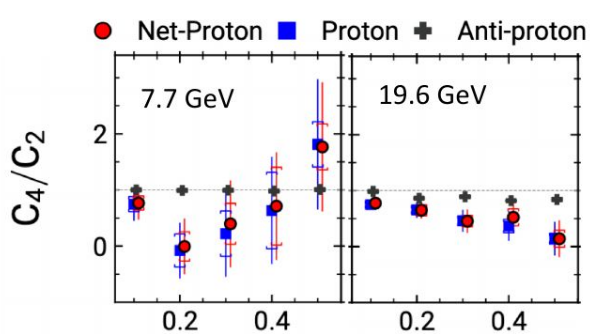
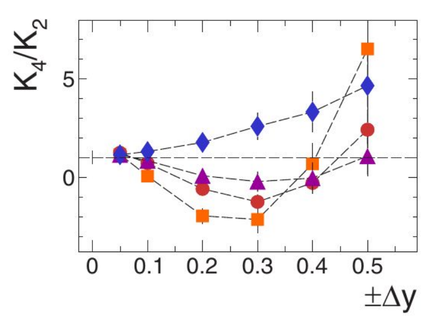


STAR Collab, PRC 104 (2021)



HADES Collab, PRC 102 (2020)

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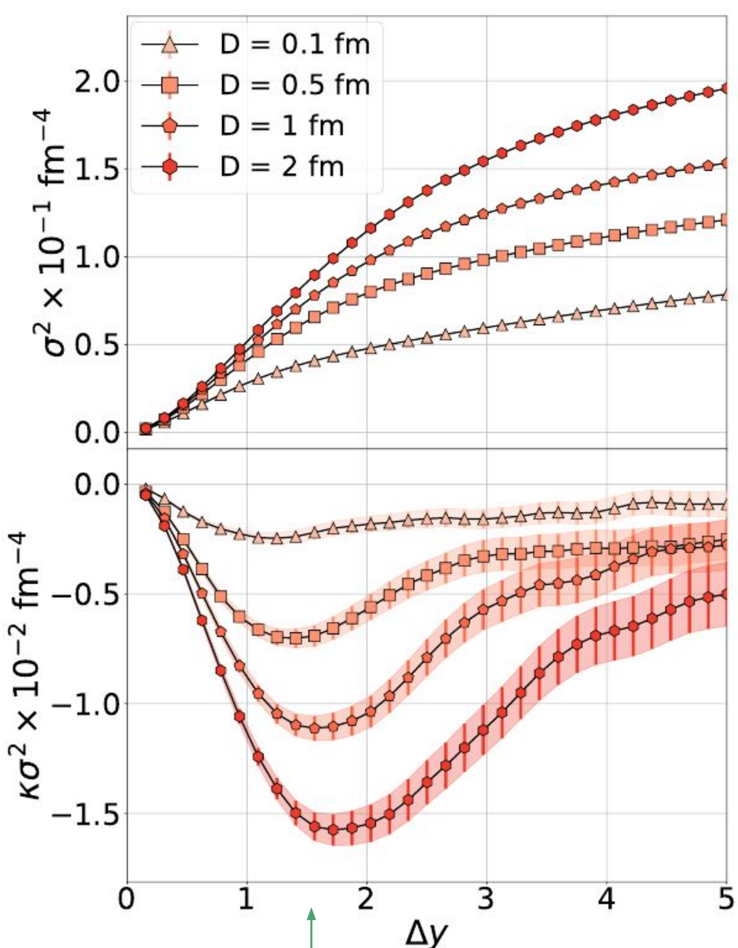

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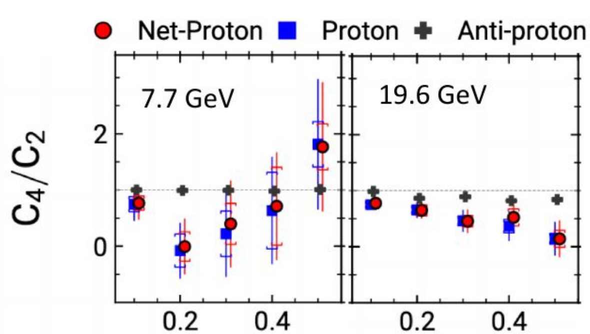
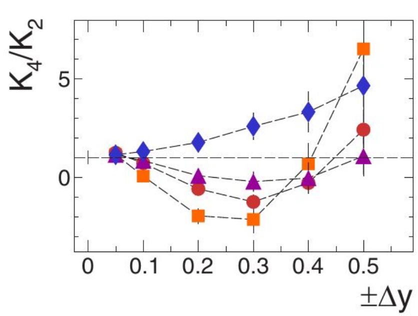
STAR iTPC



STAR Collab, PRC 104 (2021)



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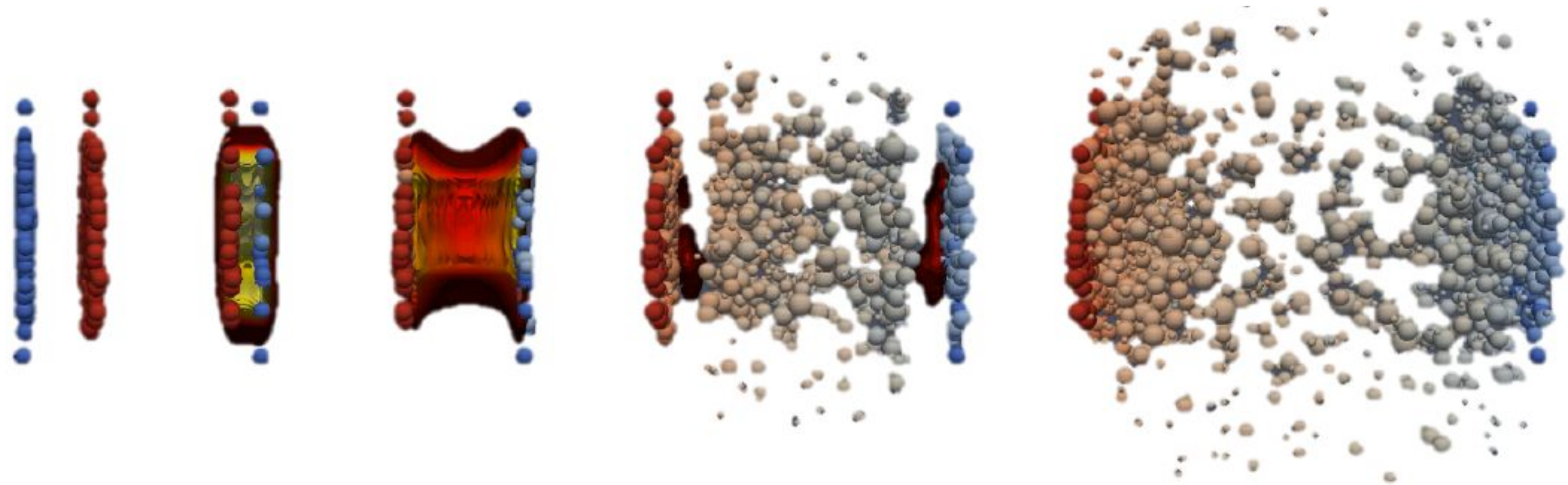
Fluctuations all along the way



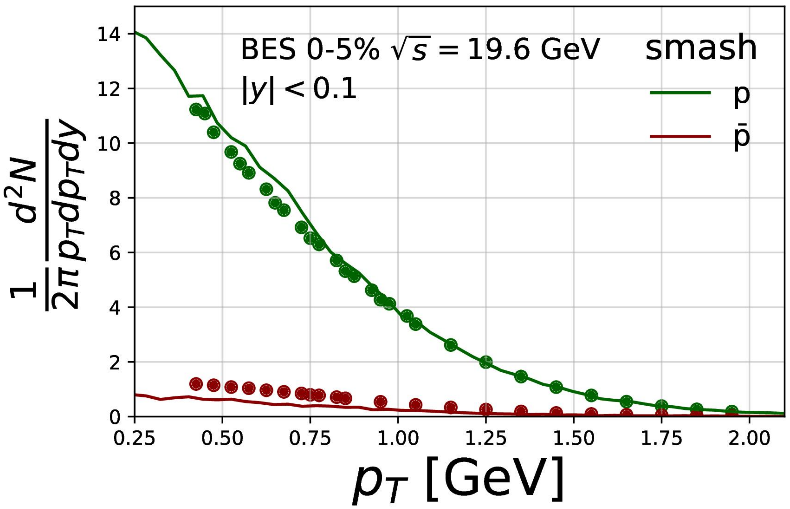
- Initial state fluctuations due to quantum mechanical fluctuations and multiplicity fluctuations
- Thermal fluctuations, including the formation of critical fluctuations
- Fluctuations due to the hadronization process
- Fate of fluctuations in the hadronic phase
- Imperfect detection efficiency and finite acceptance

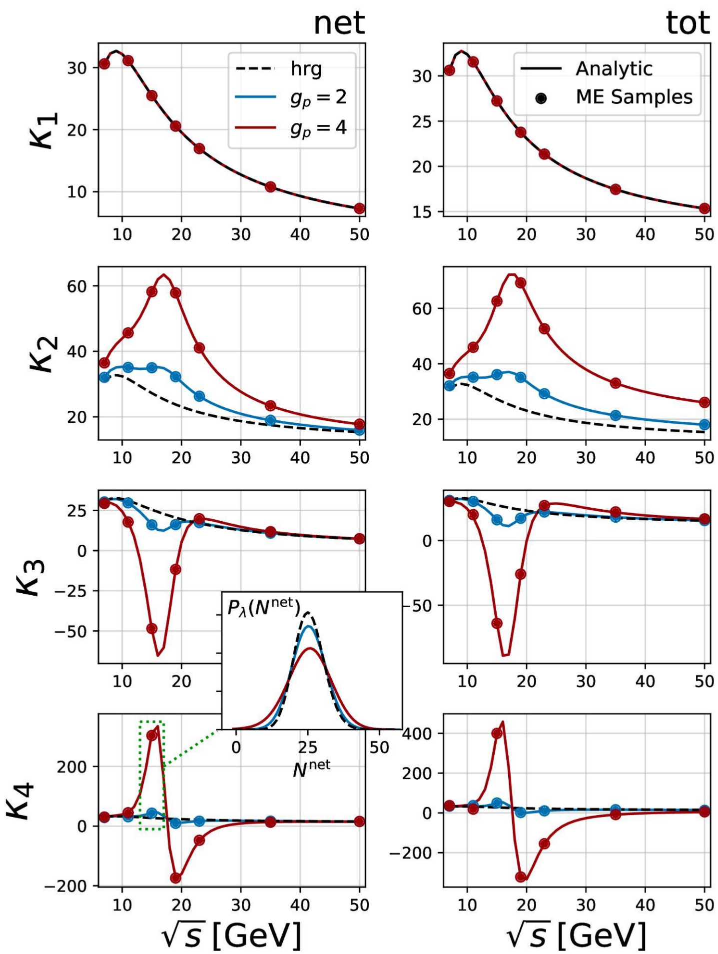
Fate of critical fluctuations in the hadronic phase

- Calculate up to 4th order cumulants of critical fluctuations from an 3d Ising model mapping to QCD and couple it to HRG cumulants (g_c=2,4).
- Reconstruct the particle distributions from the cumulants + maximum entropy constraint
- Assume simple geometry at particlization: uniform spatial distribution in a sphere (R=9fm)

  Momentum distribution $f_{i,k} = e^{-u \cdot k_i/T}$

  velocity $\vec{u}(r) = \vec{e}_r u_0 r/R$ with u_0=0.5



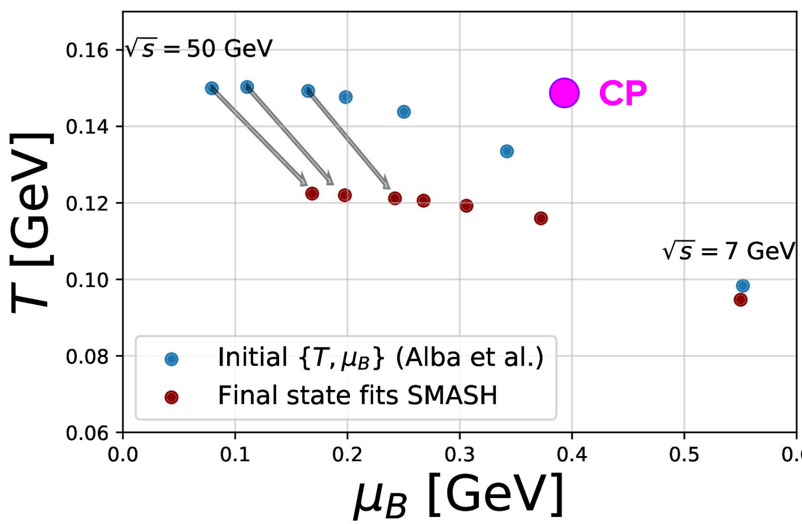


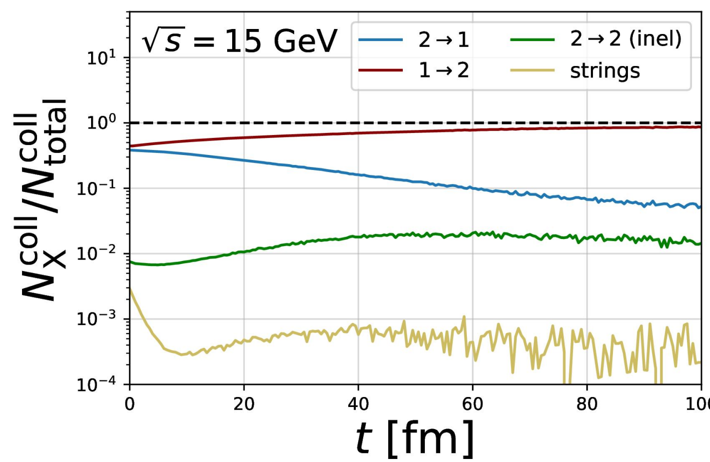
J. Hammelmann,, M. Bluhm, MN, H. Elfner, 2310.06636

Fate of critical fluctuations in the hadronic phase

- Apply smash (https://smash-transport.github.io) to the final hadronic interactions of the initialized particles.
- Resonance decay and regeneration are the dominant processes during the hadronic expansion.

Particle	Mass $[\mathrm{GeV}/c^2]$	Degenercy
π	0.138	3
ρ	0.776	6
K	0.494	4
$K^\star(892)$	0.892	8
N	0.938	8
Δ	1.232	32
Λ	1.116	2
Σ	1.189	12





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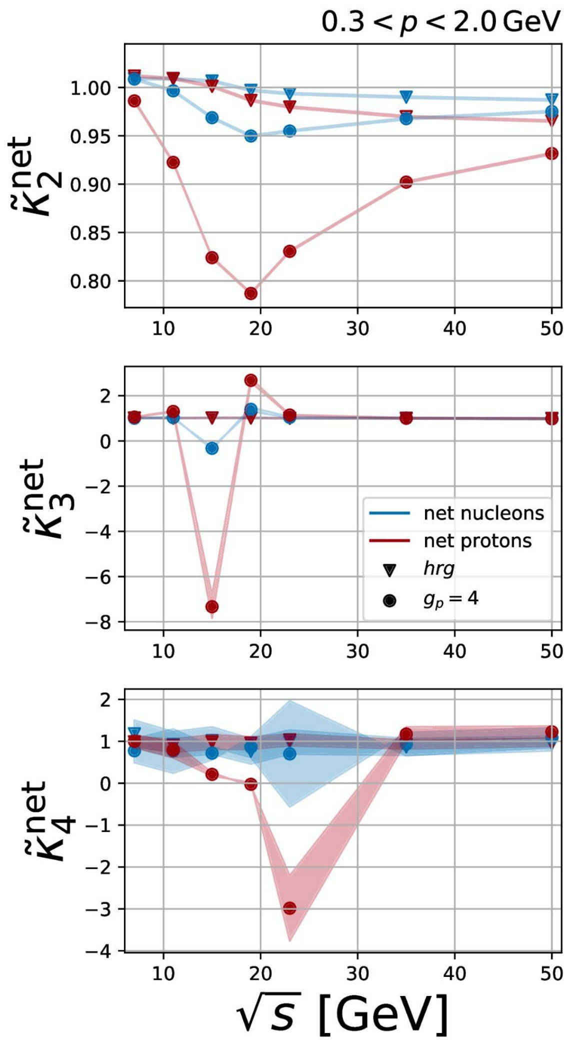
Impact of resonance dynamics vs decay

- Compare the dynamical effect of the resonance decay and regeneration compared to only resonance decay:

$$\tilde{\kappa}_n = \frac{\kappa_n^{\text{dynamical}}}{\kappa_n^{\text{decays}}}$$

- Net-proton cumulants are strongly impacted by the hadron dynamics compared to the net-nucleons -> importance of isospin randomization processes.

M. Bluhm, MN, S. Bass, T. Schaefer, Eur.Phys.J.C 77 (2017); MN, M. Bluhm, P. Alba, R. Bellwied, C. Ratti Eur.Phys.J.C 75 (2015); M. Kitazawa, M. Asakawa, Phys.Rev.C 85 (2012)



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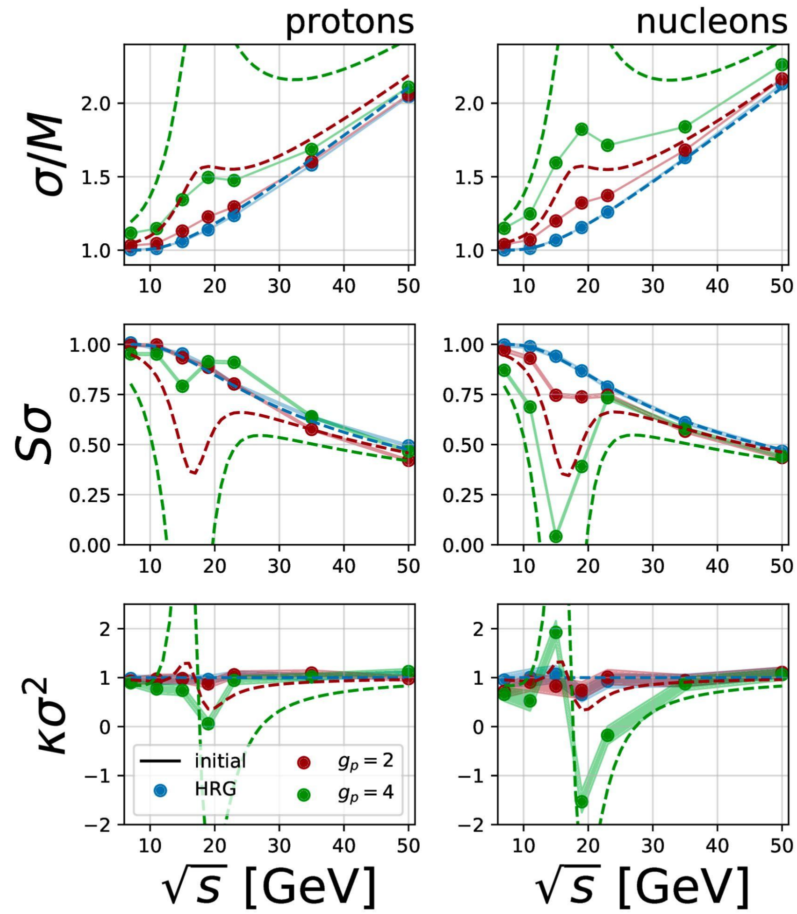
Final proton and nucleon cumulant ratios

- For g_c = 2, no critical signal is seen in the net-proton variance and skewness, a very small signal in the kurtosis survives.
- For g_c = 4, the net-proton variance shows critical features -> not compatible with experiment.
- The nucleon critical signal is significantly more pronounced than for protons only.
- Signal depends strongly on the rapidity acceptance and can even change sign in the kurtosis.



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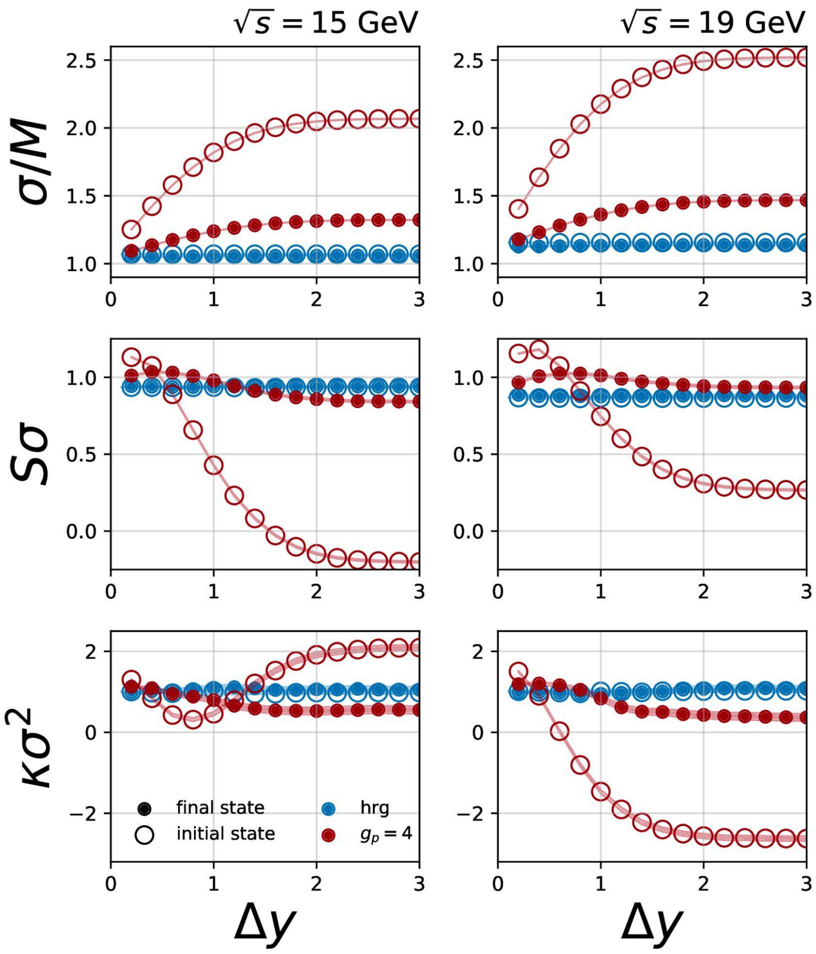

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Workshop at Subatech, Nantes, October 28-31, 2024

Hydrodynamics and related observables in heavy-ion collisions

https://indico.in2p3.fr/event/32630/

Broad topics:

- hydrodynamic fluctuations
- finite baryon density
- spin-hydrodynamics
- anisotropic hydrodynamics
- stable first-order hydrodynamics
- jets and heavy-flavor dynamics
- rapid hydrodynamization
- numerical developments

Conclusions

Treating the dynamics of fluctuations near the Critical Point is important for quantitative statements about its existence based on heavy-ion collision data!

- Net-baryon fluctuations are strongly impacted by the expansion dynamics.
- Anticorrelations of baryons can signal the CP.
- Non-monotonic dependence of the kurtosis on the rapidity window near the CP.
- Resonance decay and regeneration strongly affects the critical fluctuations.

Currently explored:

- Renormalizing (chiral) fluid dynamics with N. Attieh
- Implementing (renormalized) fluctuating fluid Dynamics with J. Sterba, I. Karpenko, B. Tomasik

APPENDIX

Importance of fluctuations for transport coefficients

$\eta \sim \int d^3x dt \, \langle T^{ij}(x,t) T^{ij}(0,0) \rangle$

Included in fluid dynamics

NOT included in fluid dynamics

- symmetrized correlator:

$$G_S^{xyxy}(\omega, \mathbf{0}) = \int \mathrm{d}^3x\mathrm{d}t\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \left\langle \frac{1}{2}\{T^{xy}(t,\mathbf{x}), T^{xy}(0,\mathbf{0})\} \right\rangle$$

- for the shear-shear contribution $\Rightarrow$

$$G_{R,\mathrm{shear-shear}}^{xyxy}(\omega, \mathbf{0}) = -\frac{7T}{90\pi^2}\Lambda^3 - i\omega\frac{7T}{60\pi^2}\frac{\Lambda}{\gamma_\eta} + (i+1)\omega^{3/2}\frac{7T}{90\pi^2}\frac{1}{\gamma_\eta^{3/2}}$$

cutoff-dependent fluctuation contribution to the pressure

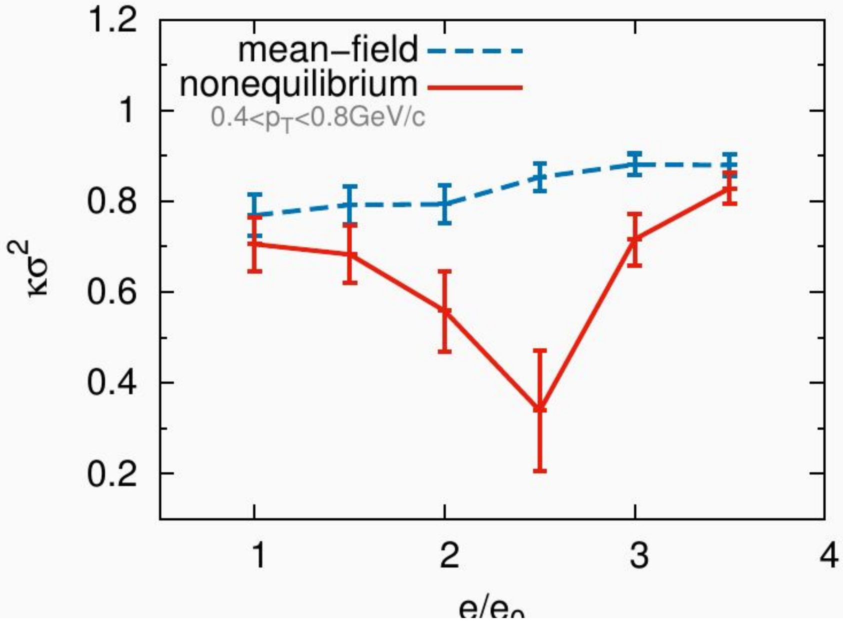
cutoff-dependent correction to η

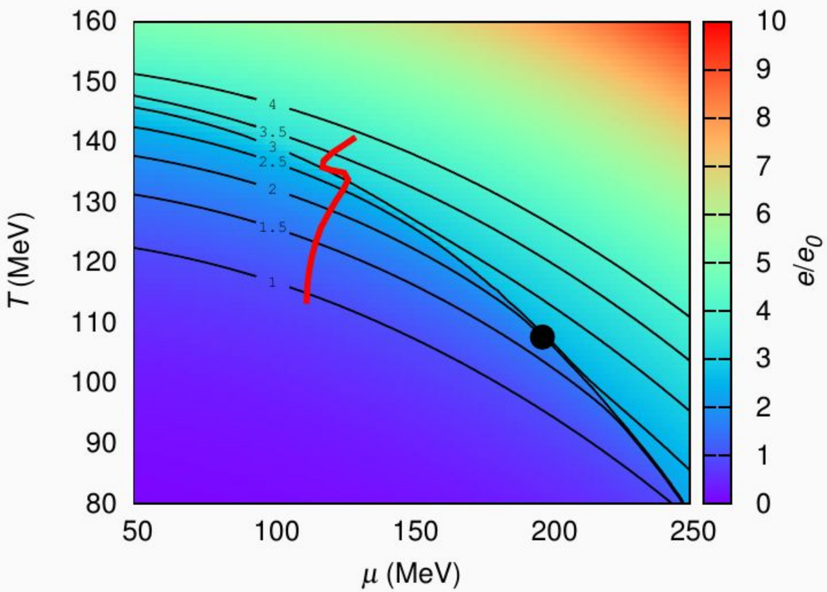
frequency-dependent contribution to η and τ_π

P. Kovtun et al, PRD84 (2011); C.Chafin, T. Schäfer, PRA87 (2013); P. Romatschke et al, PRA87 (2013)

Net-proton fluctuations near the critical point

- UrQMD initial conditions rescaled to the EoS of the effective model.
- From densities to particles via Cooper-Frye particlization.
- At particlization: densities of the sigma field coupled to the FD densities.





C. Herold, MN, Y. Yan and C. Kobdaj, PRC 93 (2016) no.2

- No non-monotonic behavior in pure mean-field equilibrium calculations.
- Clear signal for criticality in net-proton fluctuations at transition energy density!
- Overall decreasing trend probably due to net-baryon number conservation

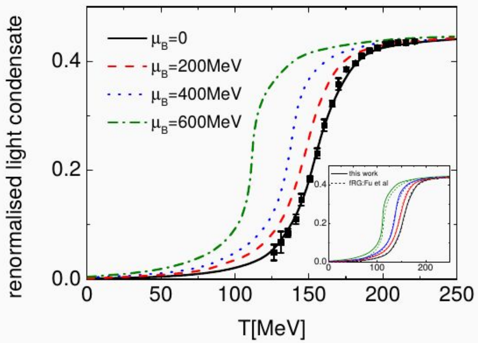
MN et al, EPJC72 (2012), 0903.2911

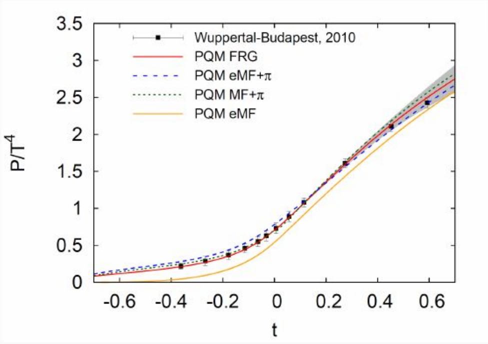
NchiFD + FRG ⟶ QCD assisted transport

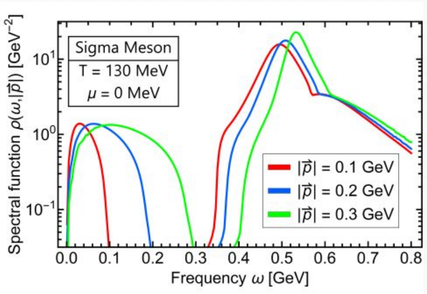
- Include effective potential beyond mean field, momentum dependent equilibrium sigma spectral function ⇒ linear response regime of QCD.

First-principle approach to QCD from the Functional Renormalization Group (FRG)

Cyrol, Mitter, Pawlowski, Strodthoff PRD97 (2018)

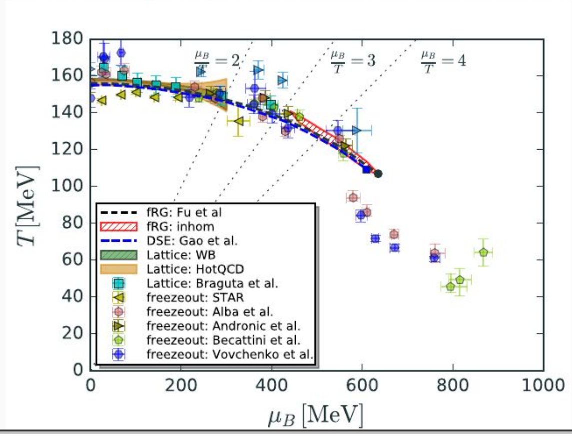






F. Gao, J. Pawlowski, 2010.13705; T. Herbst et al, PLB731 (2014); T. Herbst PRD88 (2013); F. Rennecke, J. Pawlowski, N. Wink

- Excellent description of phase structure at vanishing chemical potential.
- Phase structure qualitatively similar to the conjectured QCD phase diagram.
- Obtain spectral functions from analytical continuation.



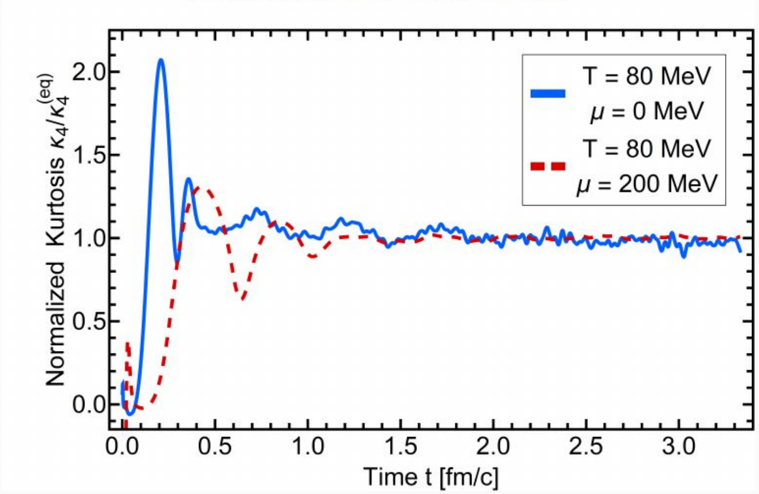
9 / 2

NchiFD + FRG ⟶ QCD assisted transport

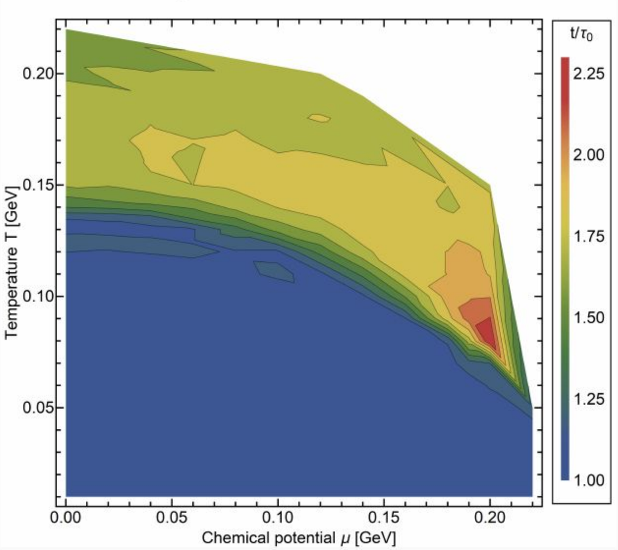
M. Bluhm et al., NPA982 (2019)

Transport equation: $\frac{\delta\Gamma}{\delta\sigma} = \xi$, where $\{\Re\Gamma^{(2)}_{\sigma}(\omega,\vec{p}), \Im\Gamma^{(2)}_{\sigma}(\omega,\vec{p}), U\} \in \Gamma$

Normalized Kurtosis:



Equilibration time:



- Critical end point and the phase structure are clearly identifiable.
- Critical slowing down in the vicinity of the critical point, but no dramatic enhancement of τ_{relax} in a dynamic setup!

Diffusive dynamics of net-baryon density

- In the long-time, equilibrium limit the net-baryon density is the slowest mode near the CP.
- For baryonic matter that decouples from the energy flow of the system (model B of Hohenberg, Halperin), the diffusive dynamics follows the minimization of the free energy F

$$\partial_t n_B(t,x) = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}[n_B]}{\delta n_B} \right) + \nabla \mathbf{J}(\mathbf{t}, \mathbf{x})$$

with the stochastic current (Gaussian, white noise)

$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x), \quad \kappa = \frac{Dn_c}{T}$$

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

First: static box with fixed temperature

Critical energy density from 3d Ising Model

Ginzburg-Landau

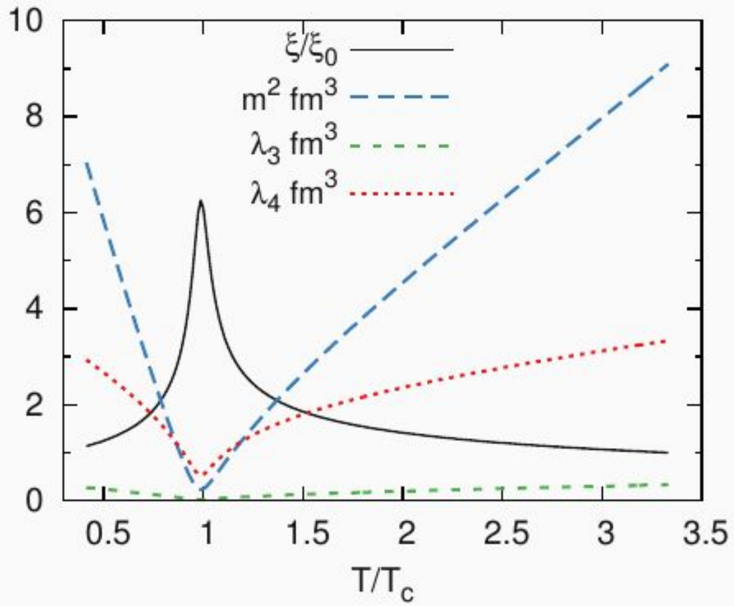
$$\mathcal{F}[n_B] = T\int d^3r \left(\frac{m^2}{2n_c^2}\Delta n_B^2 + \frac{K}{2n_c^2}(\nabla\Delta n_B)^2 + \frac{\lambda_3}{3n_c^3}\Delta n_B^3 + \frac{\lambda_4}{4n_c^4}\Delta n_B^4 + \frac{\lambda_6}{6n_c^6}\Delta n_B^6 \right)$$

The couplings depend on temperature via the correlation length $\xi(T)$:

Gauss + surface

$$m^2 = 1/(\xi_0\xi^2)$$
$$K = \tilde{K}/\xi_0$$
$$\lambda_3 = n_c\,\tilde{\lambda}_3\,(\xi/\xi_0)^{-3/2}$$
$$\lambda_4 = n_c\,\tilde{\lambda}_4\,(\xi/\xi_0)^{-1}$$
$$\lambda_6 = n_c\,\tilde{\lambda}_6$$

M. Tsypin PRL 73 (1994); PRB 55 (1997)



in this Fig.: $\tilde{\lambda}_3 = 1,\ \tilde{\lambda}_4 = 10$

parameter choice: $\Delta n_B = n_B - n_c$

$\xi_0 \sim 0.5$ fm, $T_c = 0.15$ GeV, $n_c = 1/3$ fm^{-3}

$K = 1/\xi_0$ (surface tension)

$\tilde{\lambda}_3,\ \tilde{\lambda}_4,\ \tilde{\lambda}_6$ (universal, but mapping to QCD)

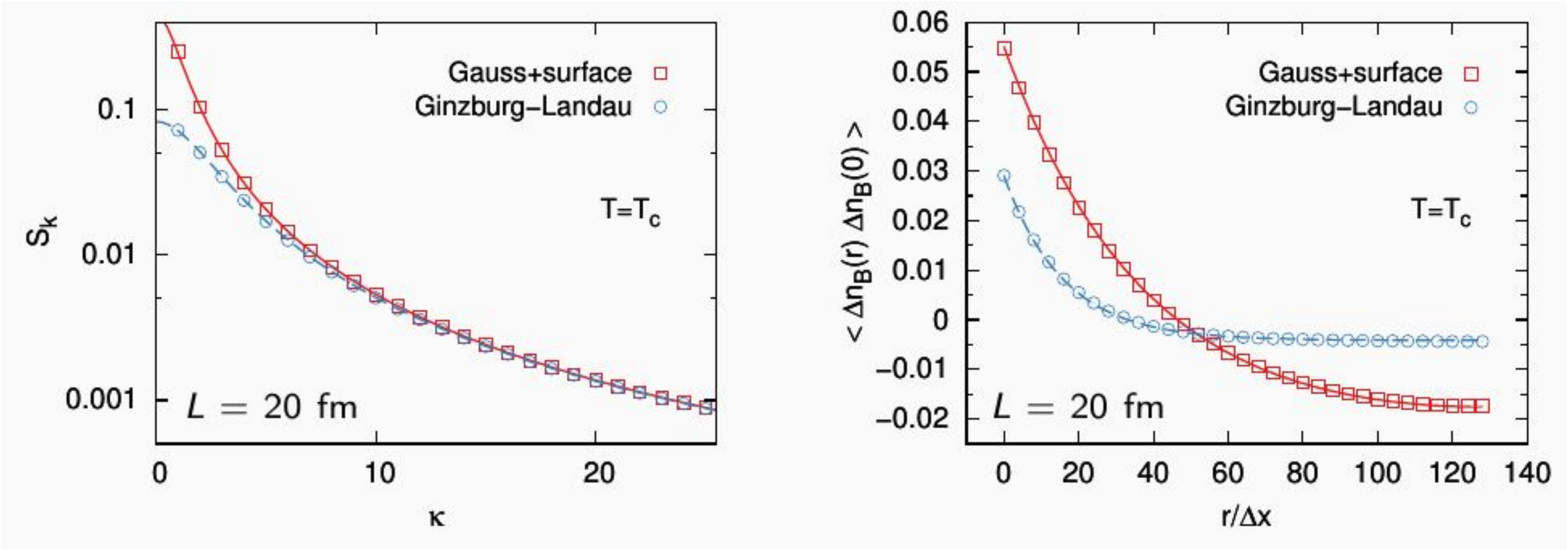
MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Studied in a static and cooling box of QGP

Validated in the equilibrium limit for the Gauss + surface model:

- Structure factor and equal-time correlation function are well reproduced
- Approaches continuum as resolution is increased
- Baryon conservation effects under control

Important step for all fluctuating codes!



MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Scaling of equilibrium cumulants

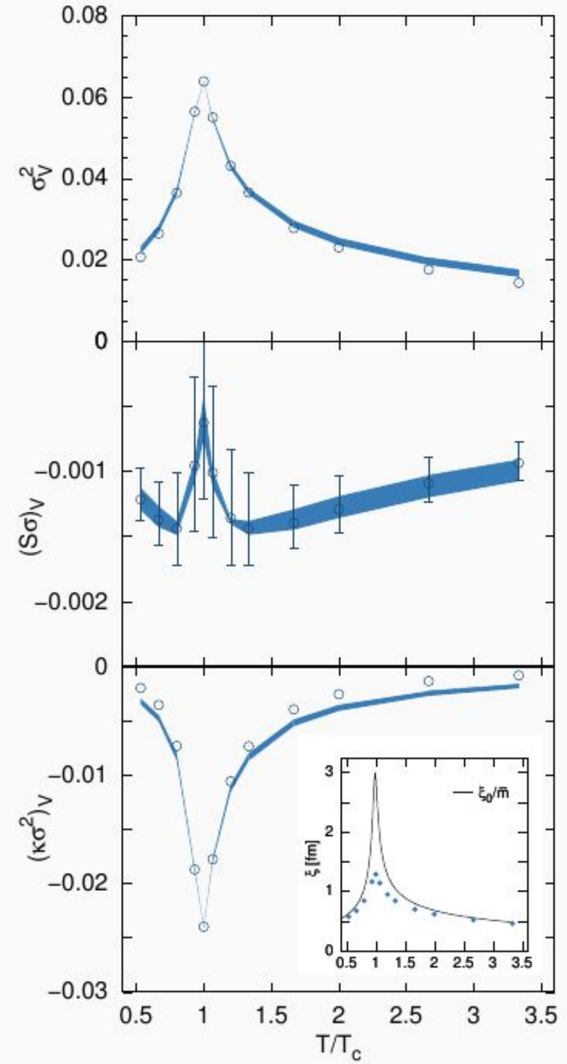
- Expected scaling in an infinite system ($\xi \ll V$): M. Stephanov PRL 102 (2009)

$$\sigma_V^2 \propto \xi^2, \quad (S\sigma)_V \propto \xi^{2.5}, \quad (\kappa\sigma^2)_V \propto \xi^5$$

- Here, a finite system with exact baryon conservation ($\xi \lesssim V$)! Can be systematically studied in $\xi/V \Rightarrow$ affects equilibrium scaling!

- E.g. for the skewness terms $\propto \lambda_3\lambda_4$ and $\propto \lambda_3\lambda_6$ contribute with opposite sign.

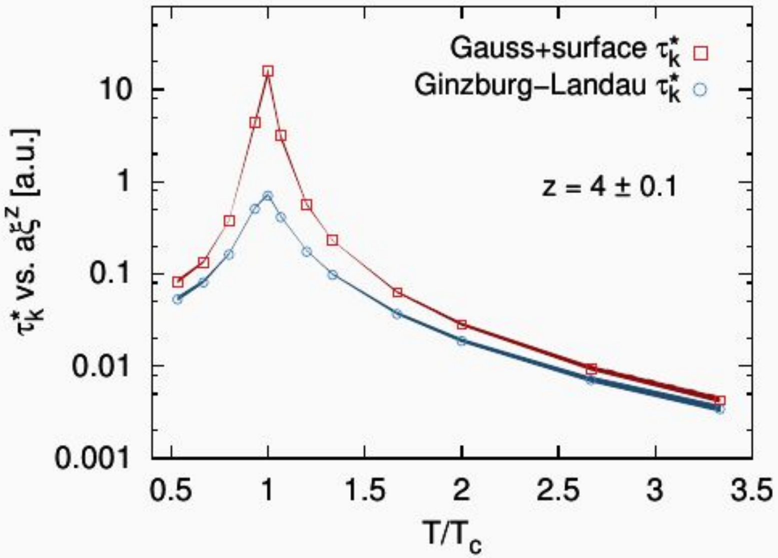
$$\sigma_V^2 \propto \xi^{1.3\pm0.05}$$
$$(S\sigma)_V \propto -\#\xi^{1.47\pm0.05} + \#\xi^{2.4\pm0.05}$$
$$(\kappa\sigma^2)_V \propto \xi^{2.5\pm0.1}$$



MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Dynamical critical scaling

- Dynamical structure factor for Gaussian model in continuum:
  $S(k,t) = S(k)\exp(-t/\tau_k)$ with $\tau_k^{-1} = \frac{Dm^2}{n_c}\left(1 + \frac{K}{m^2}k^2\right)k^2$
- Analyze ξ-dependence of relaxation time for modes with $k^* = 1/\xi$:



for both models: $\tau_k^* = a\,\xi^z$ with

$$z = 4 \pm 0.1$$

$$a = \frac{n_c \xi_0}{D(1+\tilde{K})}$$

$\Rightarrow$ Simulations reproduce scaling of model B!

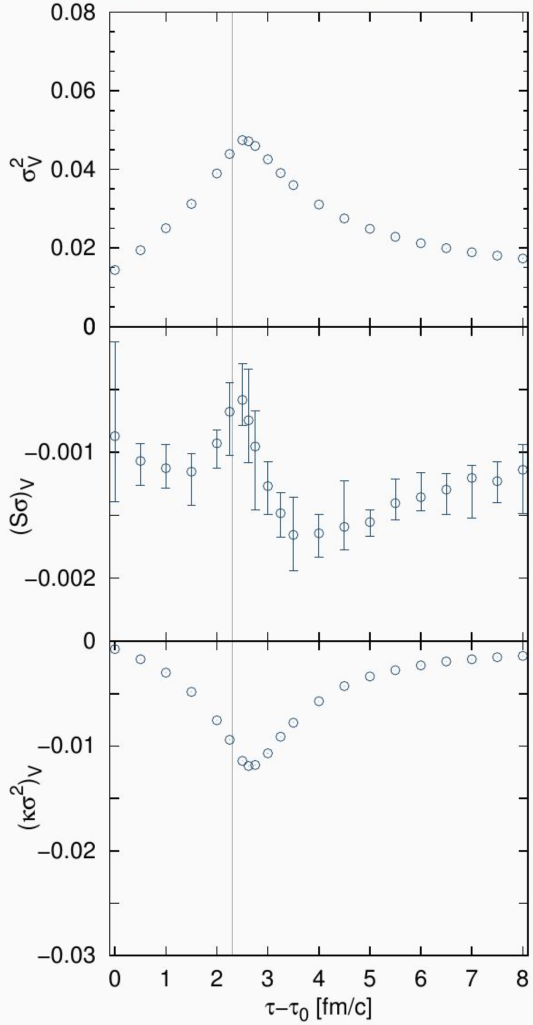
For the full dynamics of a HIC, couple to fluctuations in $T^{\mu\nu}$ → model H

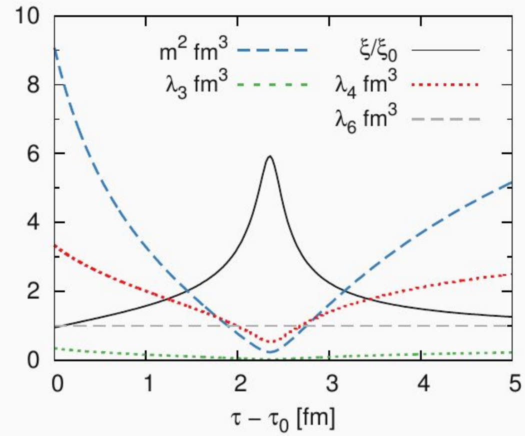
According to Hohenberg, Halperin, Rev. Mod. Phys. 49 (1977)

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

Time evolution of critical fluctuations

For a Bjorken-like temperature evolution:





- shift of extrema for variance/kurtosis (retardation effects) to later times corresponding to $T(\tau) < T_c$
- |extremal values| in dyn simulations < equilibrium values (nonequilibrium effects):

$$(\sigma_V^2)_{\rm dyn}^{\rm max} \approx 0.75\,(\sigma_V^2)_{\rm eq}^{\rm max}$$
$$((\kappa\sigma^2)_V)_{\rm dyn}^{\rm min} \approx 0.5\,(\kappa\sigma_V^2)_{\rm eq}^{\rm min}$$

- expected behavior with varying D and c_s^2 (expansion rate)

MN and M. Bluhm, PRD 99 (2019) and PRD 102 (2020)

The role of numerics and computational resources

Time for two heavy nuclei to collide and produce particles:

~ 10^{-23} seconds

Time for a simulation of two colliding heavy nuclei and particle production:

~ 1 hour

Example: even with a Gaussian Process Emulator the Bayesian analysis of the temperature dependence of bulk and shear viscosity costs 100 Mio CPU hours.

This assumes O(10^4) events per point, fluctuation studies easily require O(10^8) events per point...