2-loop RGEs of the LEFT ΔB sector

Workshop on Baryon number violation:
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Motivation

Running 1 GeV $\longrightarrow$ 10^{15} GeV

$\longrightarrow$ big scale separation

$\longrightarrow$ RGE effects are big

$\longrightarrow$ poor loop expansion

1-loop RGE resums $\alpha^n \log^n$ LL

2-loop RGE resums $\alpha^{n+1} \log^n$ NLL

Some 2-loop ΔB RGEs exist [Nihei, Arafune, 1994], but

- only evolves $\Lambda_{QCD} \longrightarrow \Lambda_{EW}$, not $\Lambda_{EW} \longrightarrow \Lambda_{GUT}$
- just QCD corrections
- lacking scheme definition

Talk Overview

- Chiral symmetry in HV
- EFTs, the LEFT and its ΔB sector
- 2-loop RGEs & scheme dependence quirks
- Evanescent operators
- Our scheme
- Calculational details: diagrams, integrals, tools
- Some preliminary results

Question

Take the LEFT BNV interaction

lepton

$$\mathcal{L} = \left(L^{S,LL}_{ddd}\right)_{prst}\, \epsilon_{\alpha\beta\gamma}\, \left(d^{\alpha T}_{Lp}\, C\, d^{\beta}_{Lr}\right)\left(\bar{e}_{Rs}\, d^{\gamma}_{Lt}\right)$$

Wilson coefficient   down-type quarks

$\longrightarrow$ annihilates 3 quarks, so $\Delta B = 1$.

Inspection of 1-loop RGE [Jenkins, Manohar, Stoffer, 2018] reveals

$$\dot{L}^{S,LL}_{ddd} = \mu \frac{d}{d\mu} L^{S,LL}_{ddd} = \ldots\, L^{S,LL}_{ddd} \quad \longrightarrow \text{ no terms } L^{S,LR}_{ddd} \text{ or } L^{S,RL}_{ddd}$$

some function of gauge couplings   different chiralities

Q1: Why?

Q2: Do we expect same behavior at two loops?

Spurion Chiral Symmetry

E.g. mass terms violate chiral symmetry $\quad \psi_{L,R} \to U_{L,R}\,\psi_{L,R}$

$$\mathcal{L}_{mass} = -\bar{\psi}_R M \psi_L - \bar{\psi}_L M^\dagger \psi_R$$

It can be restored by promoting parameters to spurions

$$M \to U_R M U_L^\dagger\ , \quad L_{eg} \to U_L L_{eg} U_R^\dagger$$

$\mu = \underbrace{0,1,2,3}_{\bar{\mu}},\underbrace{\ldots, D}_{\hat{\mu}}$

Restores symmetry, if it wasn't for

$$\bar{\psi}\, i\not{\partial}\, \psi = \bar{\psi}_L\, i\bar{\not{\partial}}\, \psi_L + \bar{\psi}_R\, i\bar{\not{\partial}}\, \psi_R + \underline{\bar{\psi}_L\, i\hat{\not{\partial}}\, \psi_R} + \underline{\bar{\psi}_R\, i\hat{\not{\partial}}\, \psi_L}$$

$\longrightarrow$ spurions symmetry-violating terms in intermediate steps

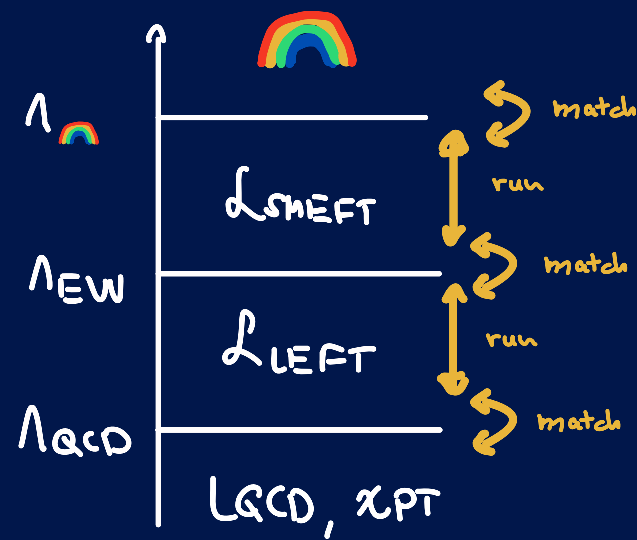
$\longrightarrow$ SB effects are local $\longrightarrow$ absorb in δL

$\longrightarrow$ obtain SCS scheme

Effective Field Theories

- 1-loop RGEs

  [Jenkins, Manohar, Trott, 2013] [Jenkins, Manohar, Stoffer, 2017]
  [Fuentes et al., 2010] [Aebischer et al., 2018]



- tree + 1-loop matchings

  [Jenkins, Manohar, Stoffer, 2017]
  [Dekens, Stoffer, 2019]

- next: dim-8 and 2-loop

  as Λ increases dim-8 effects decrease
  ↳ 2-loop becomes more important

dim-6 2-loop dominating over dim-8 1-loop

Naively (LEFT) : $\frac{\alpha_s}{4\pi} \approx \frac{2}{100}$ vs. $\frac{p^2}{\Lambda^2} \approx \left(\frac{1}{100}\right)^2$

Effective Field Theories

- old results for 2-loop EFT RGEs

  [Bijnens, Colangelo, Ecker, 1999] functional

  [Nihei, Arafune, 1994] diagrammatic

- more recent

  [Jenkins, Manohar, LN, Pagès] geometric

  [Born, Fuentes-Martín, Kvedaraitė, Thomsen] functional

  [LN, Stoffer, 2025] LEFT dim-5 2-loop RGE diagrammatic

- soon

  [LN, Stoffer, exp. soon] LEFT ΔB 2-loop RGE diagrammatic

  [LN, Stoffer, exp. 2025] LEFT dim-6 2-loop RGE diagrammatic

The LEFT ΔB sector

$\Delta B = \Delta L = 1 + \text{h.c.}$	
$\mathcal{O}^{S,LL}_{udd}$	$\epsilon_{\alpha\beta\gamma}(u^{\alpha T}_{Lp} C d^{\beta}_{Lr})(d^{\gamma T}_{Ls} C \nu_{Lt})$
$\mathcal{O}^{S,LL}_{duu}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Lp} C u^{\beta}_{Lr})(u^{\gamma T}_{Ls} C e_{Lt})$
$\mathcal{O}^{S,LR}_{uud}$	$\epsilon_{\alpha\beta\gamma}(u^{\alpha T}_{Lp} C u^{\beta}_{Lr})(d^{\gamma T}_{Rs} C e_{Rt})$
$\mathcal{O}^{S,LR}_{duu}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Lp} C u^{\beta}_{Lr})(u^{\gamma T}_{Rs} C e_{Rt})$
$\mathcal{O}^{S,RL}_{uud}$	$\epsilon_{\alpha\beta\gamma}(u^{\alpha T}_{Rp} C u^{\beta}_{Rr})(d^{\gamma T}_{Ls} C e_{Lt})$
$\mathcal{O}^{S,RL}_{duu}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Rp} C u^{\beta}_{Rr})(u^{\gamma T}_{Ls} C e_{Lt})$
$\mathcal{O}^{S,RL}_{dud}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Rp} C u^{\beta}_{Rr})(d^{\gamma T}_{Ls} C \nu_{Lt})$
$\mathcal{O}^{S,RL}_{ddu}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Rp} C d^{\beta}_{Rr})(u^{\gamma T}_{Ls} C \nu_{Lt})$
$\mathcal{O}^{S,RR}_{duu}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Rp} C u^{\beta}_{Rr})(u^{\gamma T}_{Rs} C e_{Rt})$

$\Delta B = -\Delta L = 1 + \text{h.c.}$	
$\mathcal{O}^{S,LL}_{ddd}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Lp} C d^{\beta}_{Lr})(\bar{e}_{Rs} d^{\gamma}_{Lt})$
$\mathcal{O}^{S,LR}_{udd}$	$\epsilon_{\alpha\beta\gamma}(u^{\alpha T}_{Lp} C d^{\beta}_{Lr})(\bar{\nu}_{Ls} d^{\gamma}_{Rt})$
$\mathcal{O}^{S,LR}_{ddu}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Lp} C d^{\beta}_{Lr})(\bar{\nu}_{Ls} u^{\gamma}_{Rt})$
$\mathcal{O}^{S,LR}_{ddd}$	$\epsilon_{\alpha\beta\gamma}(d^{\alpha T}_{Lp} C d^{\beta}_{Lr})(\bar{e}_{Ls} d^{\gamma}_{Rt})$
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2-loop RGE: Ingredients

evanescents $K_i\,E_i$

Using $\mu \frac{d}{d\mu} L_i^{\text{bare}} = 0$ and graph theory:

$$\dot{L}_i = \mu\frac{d}{d\mu} L_i = \underbrace{\left\{ 2 L_i^{(1,1)} \right\}_1}_{\text{1-loop RGE}} + \underbrace{\left\{ 4 L_i^{(2,1)} - 2 L_j^{(1,0)} \frac{\partial L_i^{(1,1)}}{\partial L_j} - 2 L_j^{(1,1)} \frac{\partial L_i^{(1,0)}}{\partial L_j} - 2 K_j^{(1,1)} \frac{\partial L_i^{(1,0)}}{\partial K_j} \right\}_2}_{\text{2-loop RGE}}$$

where $L_i^{\text{bare}} = \mu^{n_i \epsilon}\left(L_i + \sum_{\ell,n} \frac{1}{(16\pi^2)^\ell} \frac{L_i^{(\ell,n)}}{\epsilon^n} \right)$, $\quad \ell = 1, \dots$, $n = 0, \dots, n$ $\qquad \{X\}_\ell := \frac{1}{(16\pi^2)^\ell} X$

Thus we need:

- $L_i^{(1,1)}$ $\longrightarrow$ known from 1-loop RGE
- $L_i^{(1,0)}, K_i^{(1,1)}$ $\longrightarrow$ [LN, Stoffer, 2023]
- $L_i^{(2,1)}$ $\longrightarrow$ in the works
- Define a scheme !

ℓ-loop $\overline{\text{MS}}$:

$\dot{L}_i^{(\ell)} = 2\ell\, L_i^{(\ell,1)}$

Scheme Dependence

HV Scheme:

dimensional split

$$\mu = \underbrace{0,1,2,3}_{\bar{\mu}},\underbrace{\dots,D}_{\hat{\mu}}$$

$$\{\gamma_5,\bar{\gamma}_\mu\} = 0$$

$$[\gamma_5,\hat{\gamma}_\mu] = 0$$

Through $L^{(2,1)}$, $L^{(1,0)}$ RGE depends on:

(1) γ_5 prescription. We use 't Hooft - Veltman (HV)

(2) The precise definition of the EFT basis

- physical sector: $\bar{\Psi}\sigma^{\mu\nu}\Psi F_{\mu\nu}$ vs. $\bar{\Psi}\overline{\sigma^{\mu\nu}}\Psi\overline{F_{\mu\nu}}$
- evanescent sector: $(\bar{\Psi}\hat{\gamma}^\mu\hat{\gamma}^\nu\Psi)(\bar{\Psi}\hat{\gamma}_\mu\hat{\gamma}_\nu\Psi)$ vs. $(\bar{\Psi}\hat{\sigma}^{\mu\nu}\Psi)(\bar{\Psi}\overline{\sigma_{\mu\nu}}\Psi)$
- class II (EOM) operators: shouldn't matter

(3) Renormalization scheme:

$\overline{MS}$ vs. finite renormalizations

compensate evanescents

we do $L^{(1,0)} = L^{(1,0)}_{\chi S} + L^{(1,0)}_{ev.}$

restore chiral symm.

[Dugan, Grinstein, 1991]

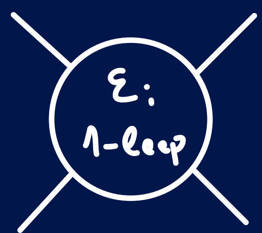
Evanescent operators

$\mathcal{E}_i$ created in renormalization. Starting with just $\mathcal{O}_i$

$$= \frac{1}{\epsilon}\,\mathcal{O}_j + \frac{1}{\epsilon}\,\mathcal{E}_j + \text{fin.}$$

$\longrightarrow$ need $K_i \mathcal{E}_i$ to renormalize theory

But for $L_i^{(2,1)}$ and RGE formula we also need



$$= \underbrace{\frac{\epsilon}{\epsilon}\,\mathcal{O}_i}_{\text{local, finite}} + \underbrace{\frac{1}{\epsilon}\,\mathcal{E}_i}_{\text{not needed}}$$

[LN, Stoffer, 2023]

And must respect $\mathcal{E}_i$ in 2-loop diagrams

$$\mu = \underbrace{0,1,2,3}_{\bar\mu},\underbrace{\ldots,d}_{\hat\mu}$$

Evanescent operators in ΔB

In HV scheme

Operators with explicit $\hat\gamma_\mu$:

$$E_1 = \epsilon_{\ldots}(\ldots\hat\gamma^\mu\bar\gamma^\nu\ldots)(\ldots\hat\gamma_\mu\bar\gamma_\nu\ldots)$$

$$E_2 = \epsilon_{\ldots}(\ldots\hat\sigma^{\mu\nu}\ldots)(\ldots\hat\sigma_{\mu\nu}\ldots)$$

$$\vdots$$

using σ shifts $\mathcal{O}$:

Fierzing does not hold, even for $\bar\gamma_\mu$

$$[\bar\sigma^{\mu\nu}P_L] \otimes (\bar\sigma_{\mu\nu}P_L) - 8(P_L] \otimes [P_L) + 4(P_L) \otimes [P_L] \neq 0$$

$\longrightarrow$ gives rise to Fierz-evanescent operators

$$E_1^{\text{Fierz}} = \epsilon_{\ldots}\Big\{(\ldots_p\bar\sigma^{\mu\nu}\ldots_r)(\ldots_s\bar\sigma_{\mu\nu}\ldots_t) + 8(\ldots_p\ldots_s)(\ldots_r\ldots_t) - 4(\ldots_p\ldots_r)(\ldots_s\ldots_t)\Big\}$$

In NDR scheme

- Fierz-evanescents

$$E^{\text{Fierz}} = \epsilon_{\ldots}\Big\{(\ldots_p\sigma^{\mu\nu}\ldots_r)(\ldots_s\sigma_{\mu\nu}\ldots_t) + 8(\ldots_p\ldots_s)(\ldots_r\ldots_t) - 4(\ldots_p\ldots_r)(\ldots_s\ldots_t)\Big\}$$

- Chisholm-evanescents

$$E^{\text{Chis}} = \epsilon_{\ldots}\Big\{(\ldots_p\gamma^\mu\gamma^\nu\gamma^\rho\ldots_r)(\ldots_s\gamma_\mu\gamma_\nu\gamma_\rho\ldots_t) + 4(4-\epsilon)(\ldots_p\gamma^\mu\ldots_r)(\ldots_s\gamma_\mu\ldots_t)\Big\}$$

Our Scheme

Choosing scheme means:

(A) choosing Operator basis

choice does not affect RGE

$$\longrightarrow \mathcal{L}_{\text{LEFT}} = \underbrace{\mathcal{L}_{\text{QED+QCD}}}_{d\text{-dim}} + \underbrace{\sum_i L_i \mathcal{O}_i}_{4\text{-dim}} + \underbrace{\sum_i L_i^{\text{red}} \mathcal{O}_i^{\text{red}}}_{\text{redundant}} + \underbrace{\sum_i K_i E_i}_{\text{evanescent}}$$

(B) renormalization scheme

$$\longrightarrow L^{\text{bare}} = \mu^{4\epsilon}\Big(L^{\text{ren}} + \underbrace{\delta L^{\text{div}}}_{\text{mandatory}} + \underbrace{\delta L^{\text{finite}}_{\text{ev}} + \delta L^{\text{finite}}_{\text{χS}}}_{\text{choice}} \Big)$$

(C) algebraic prescription in $D = 4 - 2\epsilon$

$\longrightarrow$ HV scheme for γ_5, $\epsilon_{\mu\nu\rho\sigma}$

Our Scheme

The upshot is

- Green's functions are free of K_i and CSB terms
- Same for RGEs, matching relations !

Example results in $\overline{MS}$:

$$\dot{e} = \Big\{ 4e^5 n_f + 48 e^4 \big(\mathrm{tr}(L_{e\gamma} M_e^\dagger) + \mathrm{tr}(L_{e\gamma}^\dagger M_e) \big) - 224 e^4 \big(\mathrm{tr}(L_{e\gamma} M_e) + \mathrm{tr}(L_{e\gamma}^\dagger M_e^\dagger) \big) \Big\}_2$$

$$\dot{L}^{S,LL}_{\substack{duu \\ prst}} = \Big\{ \frac{274}{9} g^4 L^{S,LL}_{\substack{duu \\ prst}} - \frac{176}{3} g^4 L^{S,RL}_{\substack{duu \\ prst}} + \frac{352}{3} g^4 L^{S,RL}_{\substack{uud \\ rspt}} + \text{terms with } e \Big\}_2$$

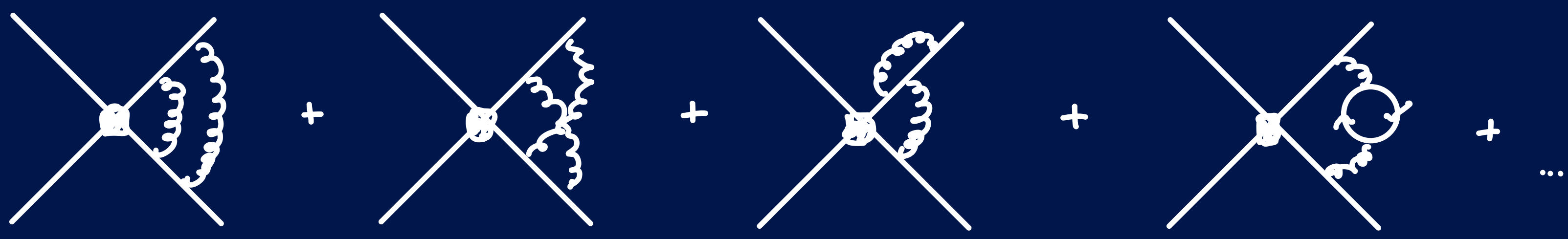
Example results in our scheme :

$$\dot{e}^{(2)} = \Big\{ 4e^5 n_f - 80 e^4 \big(\mathrm{tr}(L_{e\gamma} M_e) + \mathrm{tr}(L_{e\gamma}^\dagger M_e^\dagger) \big) \Big\}_2$$

$$\dot{L}^{S,LL}_{\substack{duu \\ prst}} = \Big\{ 34 g^4 L^{S,LL}_{\substack{duu \\ prst}} \Big\}_2$$

2-loop diagrams

At dimension 6 have just QED + QCD to ΔB insertion



no γ_5 problem

No γ_5 problems in ΔB. Have just QED + QCD UP traces.

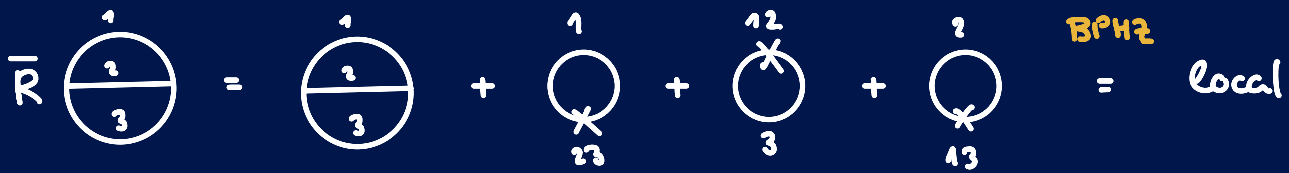
$\longrightarrow$ calculation can be done in NDR without ambiguities

$\longrightarrow$ we provide results both in NDR & HV

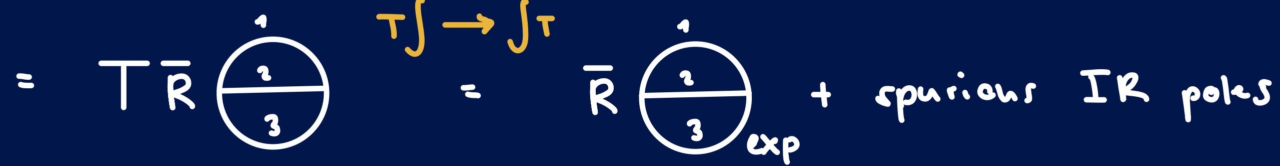
$\longrightarrow$ allows cross-check in relations between observables

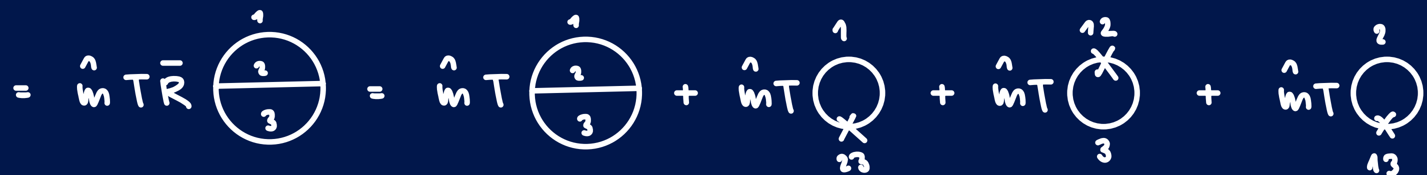
Extraction of UV divergences

- Taylor Expansion T leaves UV poles unchanged
- Introduction of dummy mass $\hat{m}$ regulates away IR poles



$\underline{\ i\ }$ has loop momentum k_i; $k_3 = k_1 + k_2$





Result has
$$\frac{\cdots}{(k_1^2-m^2)^a\,(k_2^2-m^2)^b\,(k_3^2-m^2)^c} \xrightarrow[\text{IBP}]{\text{tensor red.}} \frac{1}{(k_1^2-m^2)(k_2^2-m^2)(k_3^2-m^2)}$$

known

[Chetyrkin, Misiak, Muenz, 1997]

Steps in the calculation

(1) Generate diagrams

→ roughly 1.5k ΔB diagrams

QGraf

(2) R Operation → subgraphs and CT diagrams

(3) Apply Feynman Rules

Mathematica

(4) Color algebra

(5) Dirac Algebra

(6) Tensor Reduction

(7) Dirac Algebra

(8) Evaluation of Integrals

Form and Symbolica

(9) Absorb $L_i^{(2,1)}$

(10) Field Redefinition

(11) Checks + RGE Calculation

Mathematica

Conclusions

- We define the LEFT in HV, including ε:
- We propose a scheme, restoring CS and compensating ε:
- We derive the 2-loop RGE of the LEFT in HV
- ΔB sector also in NDR
- Results are part of an effort towards NLL accuracy

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Next steps

- Do the same for SMEFT ?
- NLL EFT analysis in ΔB ? What else is needed ?

?

Backup

Scheme Dependence

Theory with one parameter g

$$\dot g = b_0 g^3 + b_1 g^5 + b_2 g^7 + \dots$$

Now change scheme

$$g = \tilde g + a_1 \tilde g^3 + a_2 \tilde g^5 + \dots \xrightarrow{\text{invert}} \tilde g = g - a_1 g^3 + (3a_1 - a_2) g^5 + \dots$$

and compute RGE of $\tilde g$

$$\dot{\tilde g} = -3a_1 g^2 \dot g + 5(3a_1 - a_2) g^4 \dot g = \dots \text{ miracle } \dots = b_0 \tilde g^3 + b_1 \tilde g^5 + O(\tilde g^7)$$

$\longrightarrow$ 2-loop RGE is scheme independent ! (but not 3-loop)

With many couplings g_i

$$\dot g_i = A_i g_i^3 + B_{ij} g_i^3 g_j^2 \,, \quad g_i = \tilde g_i + X_{ij} \tilde g_i \tilde g_j^2$$

$$\dot{\tilde g}_i = \dots = A_i \tilde g_i^3 + B_{ij} \tilde g_i^3 \tilde g_j^2 + \underbrace{2X_{ij}\left(A_i \tilde g_i^3 \tilde g_j^2 - A_j \tilde g_i \tilde g_j^4\right)}_{\text{generally} \neq 0}$$

🙁

Global vs. local Renormalization

Global Renormalization $\longrightarrow$ Operation level: Green's function

$$(2\text{-loop}) + (1\text{-loop}) \times CT_s = \frac{\text{local}}{\epsilon} + \frac{\text{local}}{\epsilon}$$

\+ few counterterm diagrams

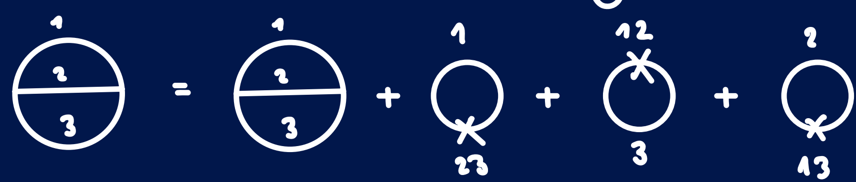
\+ checks on $\mathcal{L}_{CT}$

gauge-variant EOM

\− Requires $\mathcal{L}_{CT}$ at 1 loop, including class II_b (unless ... ?)

Local Renormalization $\longrightarrow$ Operation level: diagram

Local R Operation $\bar{R}$



\+ $\mathcal{L}_{CT}$ automatically generated and inserted

\+ Individual CT-subtr. diagrams are local

\− fewer checks on $\mathcal{L}_{CT}$

Backup: Extraction of UV divergences

- $\hat{m}$ deforms theory by $\Delta\mathcal{L}^{cT} \sim m$. Result doesn't change. $\Delta\mathcal{L}^{cT}$ automatic
- $\hat{m}, T$ must act consistently across terms
- For T order compute mass dimension of (sub-) graph
- For $\hat{m}$

$\longrightarrow$ act at a precisely defined step of calculation

otherwise dangerous ambiguity in cancellation of k:

$$\frac{k^2}{k^2} = 1 \quad \text{vs.} \quad \frac{k^2}{k^2 - m^2} = 1 + \frac{m^2}{k^2 - m^2}$$

$\longrightarrow$ need prescription for fermionic propagators

our choice

$$\frac{i}{\not{k}} \xrightarrow{\hat{m}} \frac{i(\not{k} + m)}{k^2 - m^2} \qquad \text{vs.} \qquad \frac{i}{\not{k}} \xrightarrow{\hat{m}} \frac{i\not{k}}{k^2 - m^2}$$

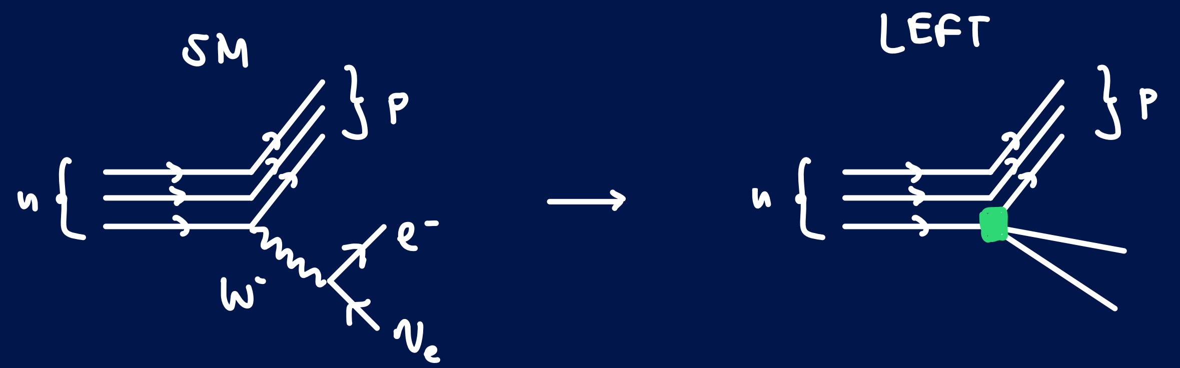
The LEFT

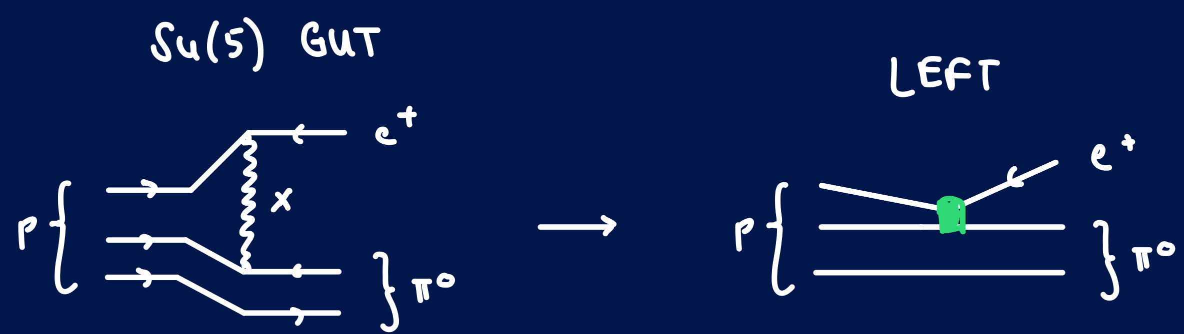
An EFT for $E \ll \Lambda_{EW}$. Integrated out t, h, W, Z:

$$\mathcal{L}_{LEFT} = \mathcal{L}_{QED+QCD} + \sum_i L_i \mathcal{O}_i$$

ΔB interactions at dim. 6

SM and BSM effects captured by $L_i \mathcal{O}_i$





Backup: R operation in our scheme

R := Renormalization Operator in $\overline{MS}$

R_f := Renormalization Operator in our scheme

R_f  $=$  $+$  $\cdot\, \delta Z^{(1)} + \dots + \times \cdot \delta Z^{(2)}$

$=$  $+$  $\cdot \left(\delta Z_d^{(1)} + \delta Z_f^{(1)}\right) + \times \cdot \left(\delta Z^{(2)} + \left(\delta Z_d^{(1)} + \delta Z_f^{(1)}\right)^2\right)$

$= R$  $+$  $\cdot\, \delta Z_f^{(1)} + 2 \cdot Z_d^{(1)} Z_f^{(1)} + \text{finite}$

divergent part

$\longrightarrow$ obtain correction globally from tree-level

Backup: The LEFT in HV

dipole operators:

$$L_{e\gamma}\, \bar{e}_L \bar{\sigma}^{\mu\nu} e_R F_{\mu\nu} + \text{h.c.}$$

$$L_{u\gamma}\, \bar{u}_L \bar{\sigma}^{\mu\nu} u_R F_{\mu\nu} + \text{h.c.}$$

$$L_{uG}\, \bar{u}_L \bar{\sigma}^{\mu\nu} T^A u_R G^A_{\mu\nu} + \text{h.c.}$$

$$\vdots$$

3 - gluon operators:

$$L_G\, f^{ABC}\, \overline{G^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}}$$

$$L_{\tilde{G}}\, f^{ABC}\, \overline{\tilde{G}^A_{\mu\nu} G^B_{\nu\rho} G^C_{\rho\mu}}$$

ψ^4 operators $\Delta L = \Delta B = 0$:

$$L^{V,LL}_{ee} (\bar{e}_L \bar{\gamma}^\mu e_L)(\bar{e}_L \bar{\gamma}^\mu e_L)$$

$$L^{V,LR}_{ee} (\bar{e}_L \bar{\gamma}^\mu e_L)(\bar{e}_R \bar{\gamma}^\mu e_R)$$

$$\vdots$$

ψ^4 operators $\Delta L, \Delta B \neq 0$:

$$L^{S,LL}_{\nu e} (\bar{\nu}_L^T C \nu_L)(\bar{e}_R e_L)$$

$$L^{T,LL}_{\nu e} (\bar{\nu}_L^T C \bar{\sigma}^{\mu\nu} \nu_L)(\bar{e}_R \bar{\sigma}_{\mu\nu} e_L)$$

$$\vdots$$

Backup: Facts about evanescents

1. Coefficients of evanescents can start at tree-level

But have in basis $\kappa \bar{\Psi} i \hat{\not{\partial}} \Psi$

$\longrightarrow$ needs loop or EFT suppression

to have well-defined perturbative expansion

2. Evanescents are not $\mathcal{O}(\epsilon)$

They have rank $\sim \epsilon$.

Double insertion effects on $\mathcal{O}_1$ produces ϵ, not ϵ^2.

Backup: γ_5 issues

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \longrightarrow \{\gamma^5, \gamma^\mu\} = 0 \quad \text{in} \quad D = 4$$

In $D \neq 4$

$$\{\gamma^5, \gamma^\mu\} = 0 \quad + \text{cyclicity} \longrightarrow \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 0$$

But in $D = 4$ must find

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 4i\,\epsilon^{\mu\nu\rho\sigma}$$

$\longrightarrow$ spoils analytic continuation

[Jegerlehner, 2000]