Combining the multiple messengers of neutron star physics

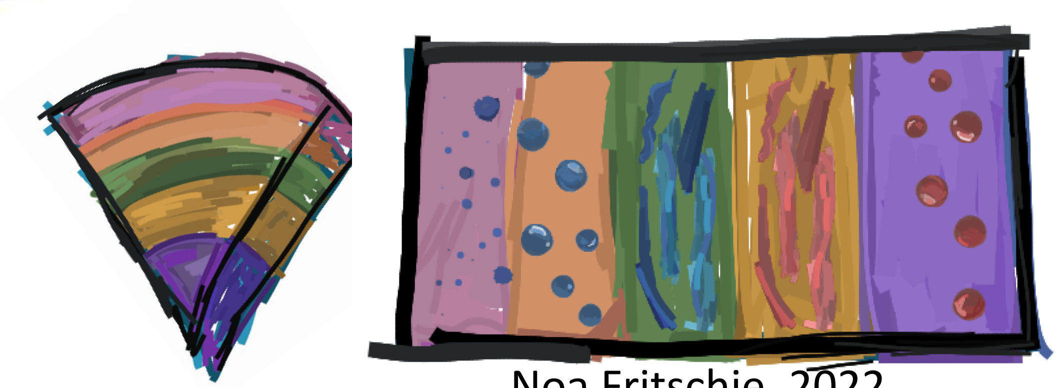
William G. Newton

The work presented in this talk would not be possible without an amazing team of undergraduates and Master’s students, including

Rebecca Preston, Amber Stinson, Lauren Balliet, Michael Ross, Gabriel Crocombe

Texas A&M University-Commerce

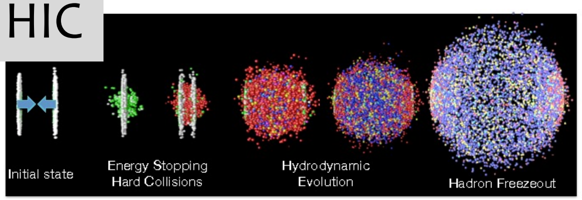
Duncan Neill, David Tsang – University of Bath



Noa Fritschie, 2022

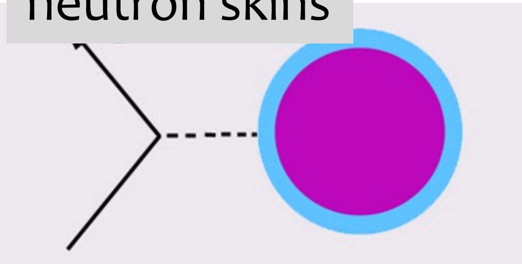
Nuclear structure/ dynamics

HIC

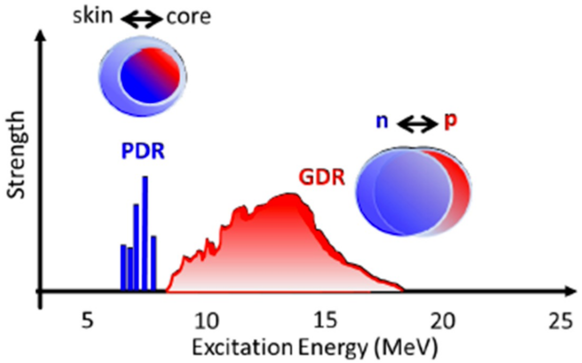


T.K.Nayak, arxiv:1201.4264

neutron skins

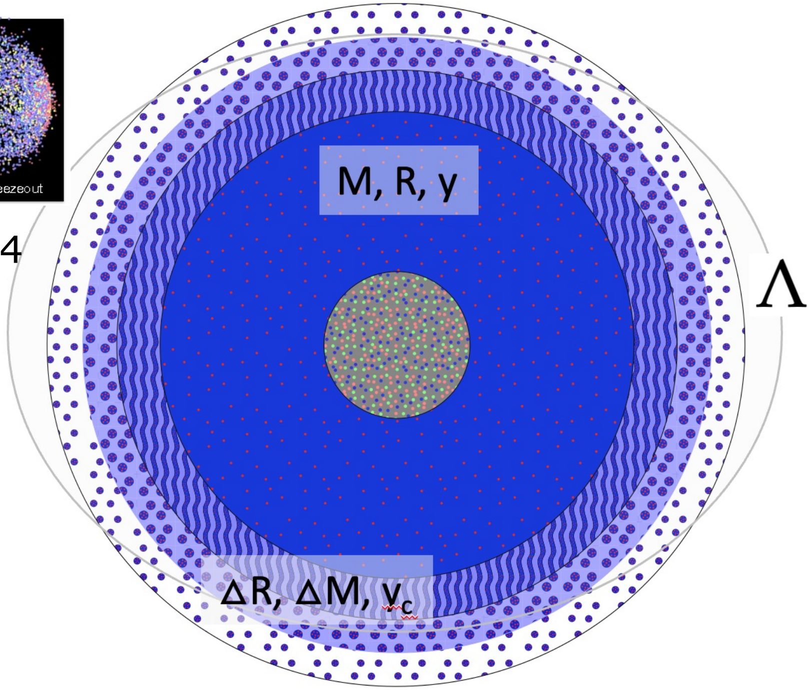


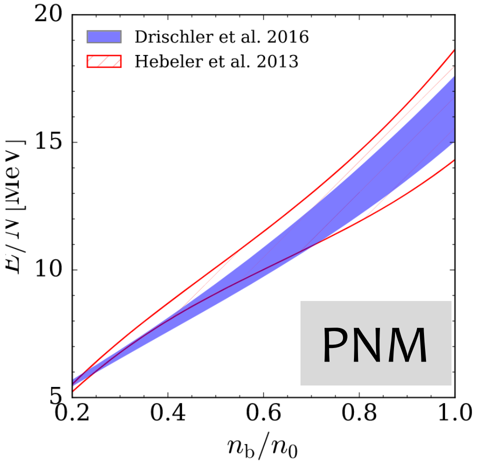
Abrahamyan+,
PRL 108, 112592 (2012)



Bracco, Lanza, Tamii,
PPNP 106, 360 (2019)

Multi-messenger Nuclear & Astro Physics

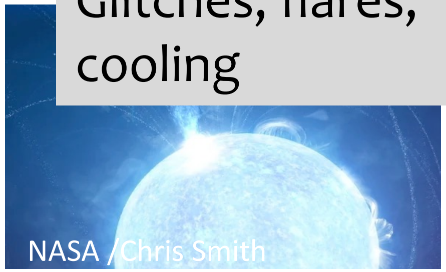


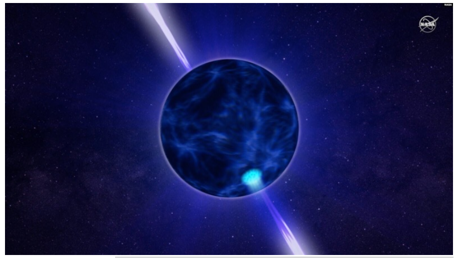


Haensel, Fortin JPhysG 2017

Neutron star structure/ dynamics

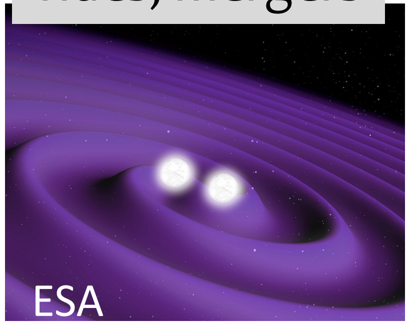
Glitches, flares, cooling





Hot spots
Oscillations,
Crust cooling

Tides, mergers



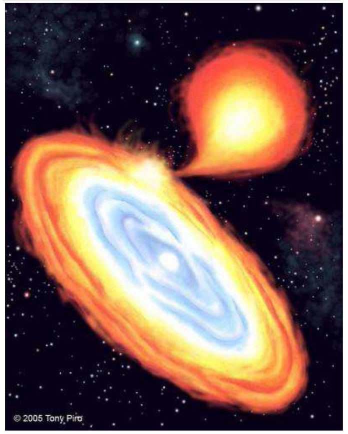


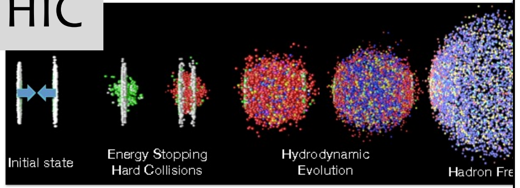
Figure: Artist's impression of a LMXB - credit Tony Piro, 2005.

Nuclear structure/ dynamics

Multi-messenger Nuclear & Astro Physics

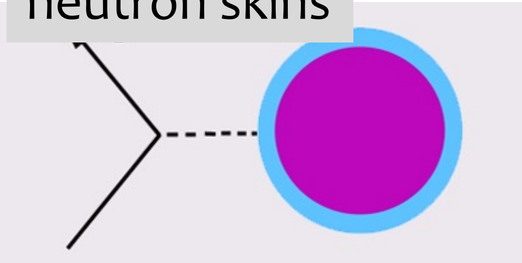
Neutron star structure/ dynamics

HIC



T.K.Nayak, arxiv:1201.426

neutron skins



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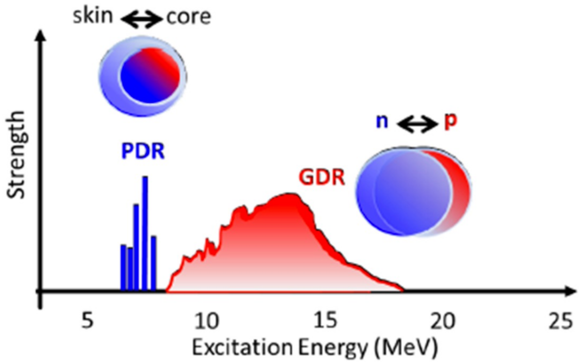

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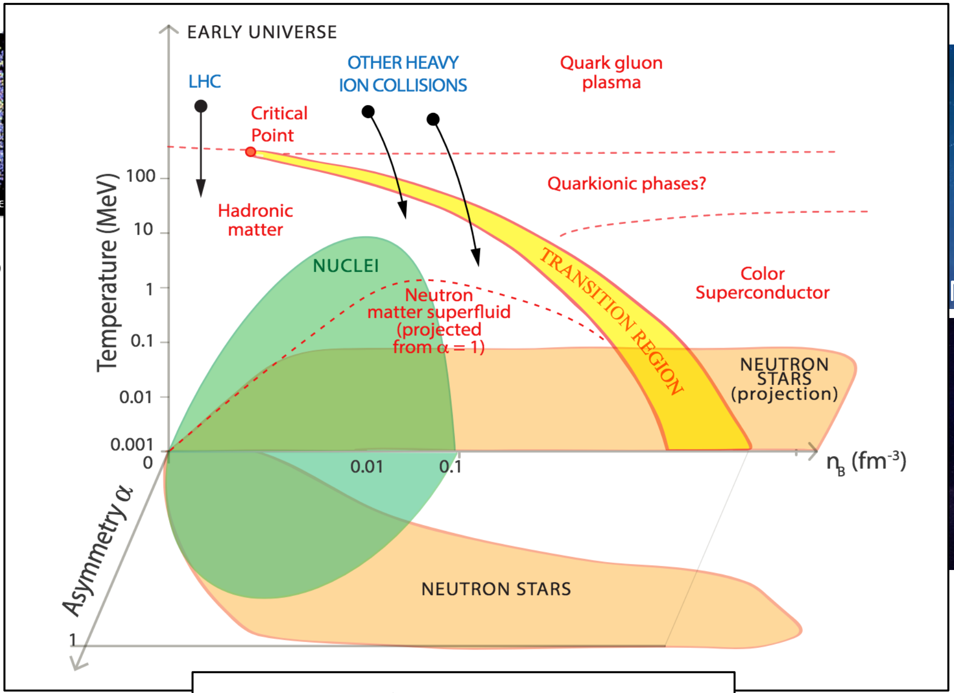
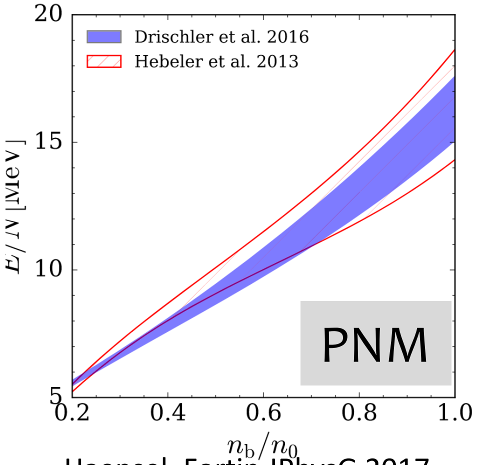


Watts et al arxiv:1501.00042



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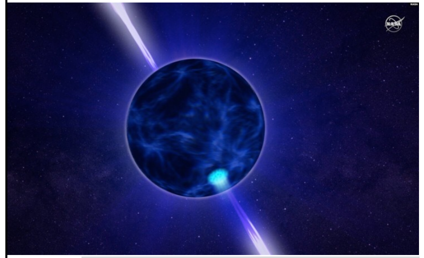
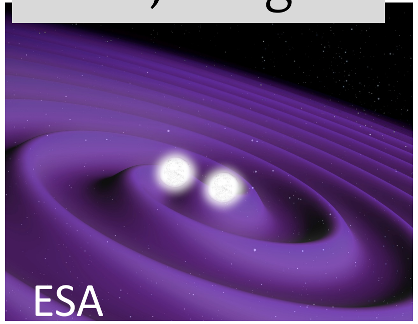
Glitches, flares, cooling



Tides, mergers



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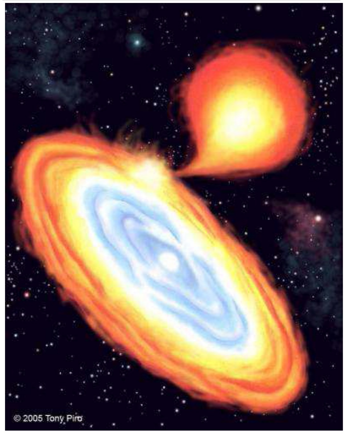

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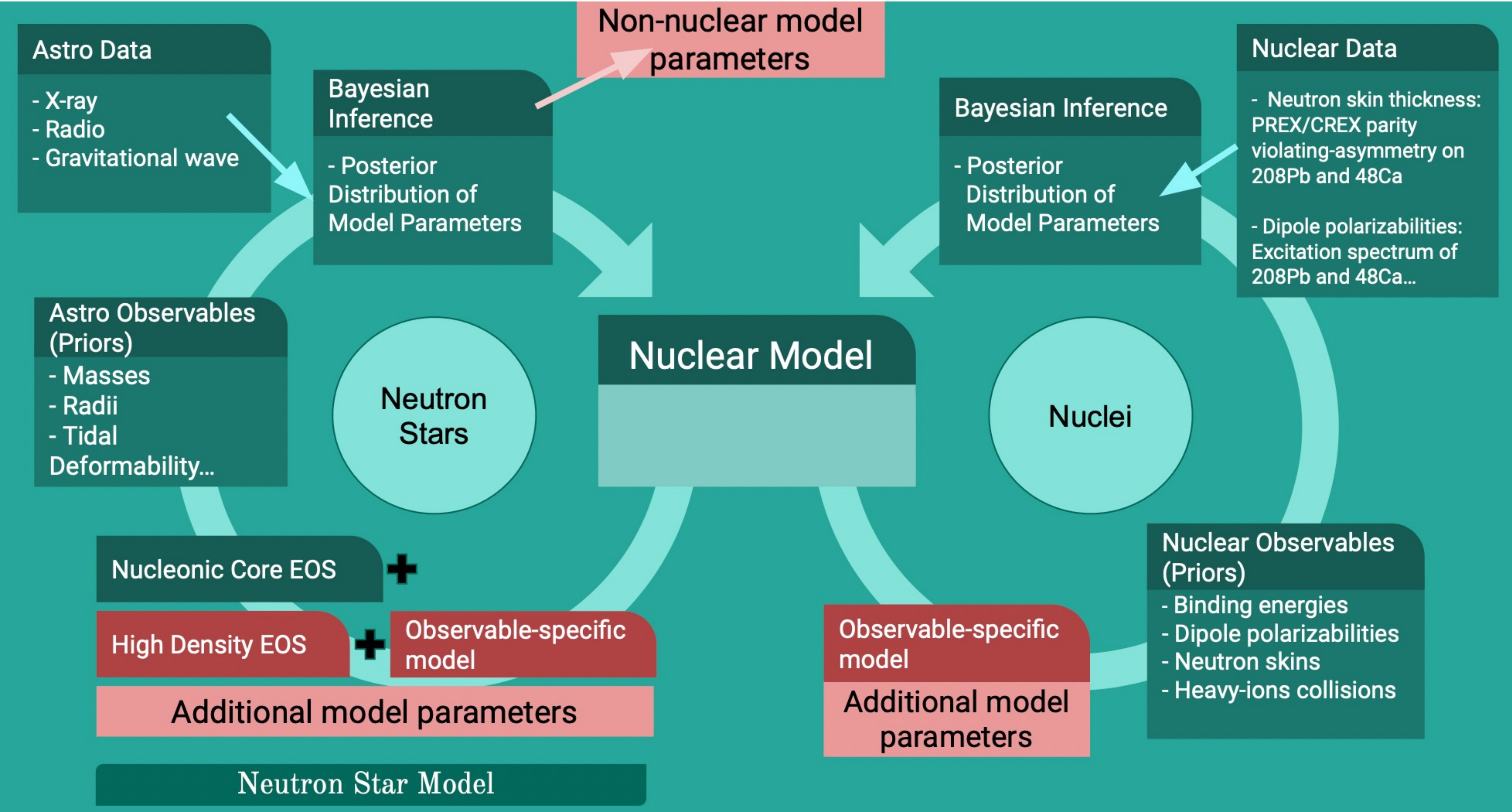


Astro Data

- X-ray
- Radio
- Gravitational wave

Bayesian Inference

- Posterior Distribution of Model Parameters

Non-nuclear model parameters

Bayesian Inference

- Posterior Distribution of Model Parameters

Nuclear Data

- Neutron skin thickness: PREX/CREX parity violating-asymmetry on 208Pb and 48Ca
- Dipole polarizabilities: Excitation spectrum of 208Pb and 48Ca...

Astro Observables (Priors)

- Masses
- Radii
- Tidal Deformability...

Neutron Stars

Nuclear Model

Nuclei

Nucleonic Core EOS +

High Density EOS + Observable-specific model

Additional model parameters

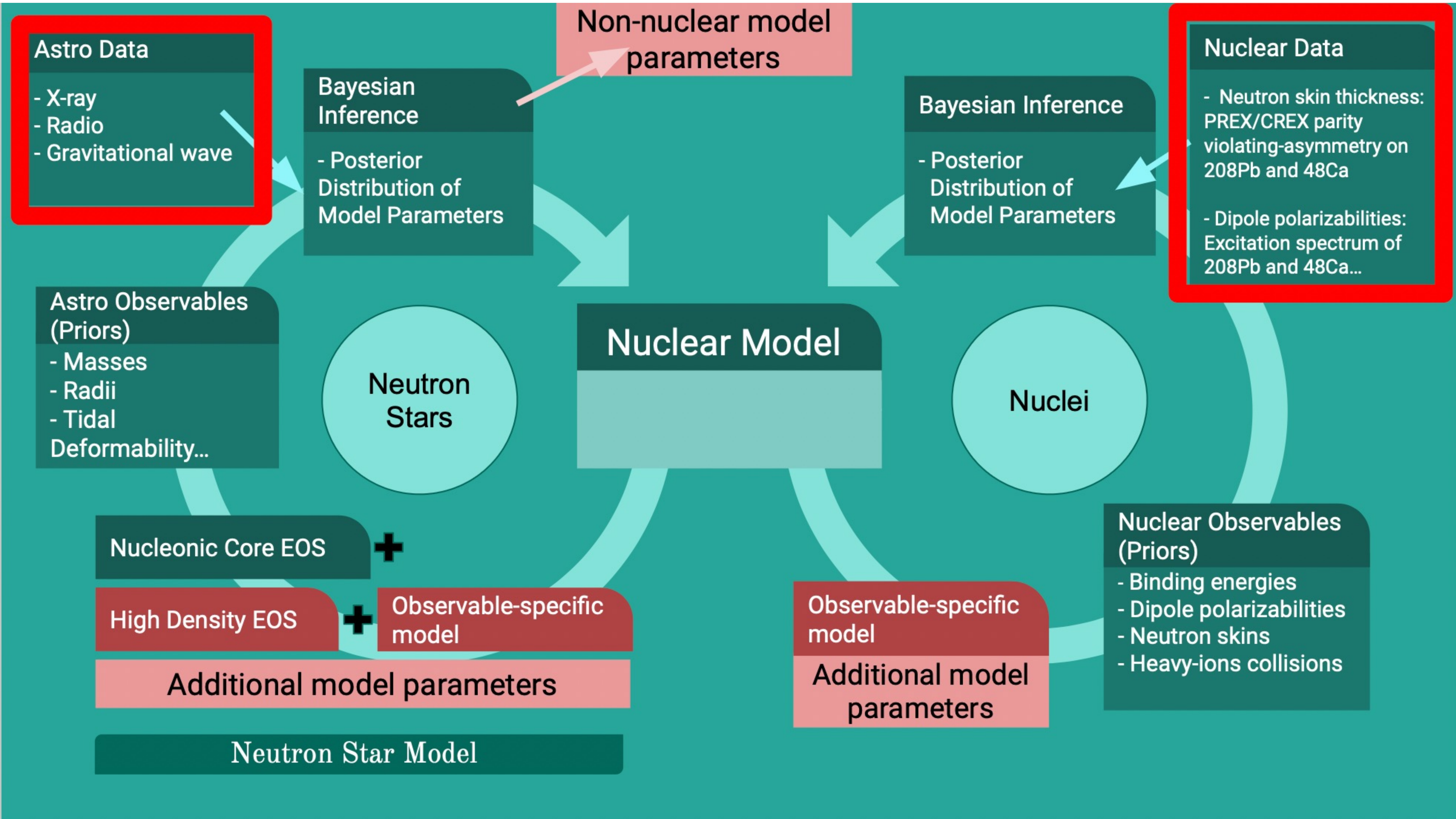
Neutron Star Model

Observable-specific model

Additional model parameters

Nuclear Observables (Priors)

- Binding energies
- Dipole polarizabilities
- Neutron skins
- Heavy-ions collisions



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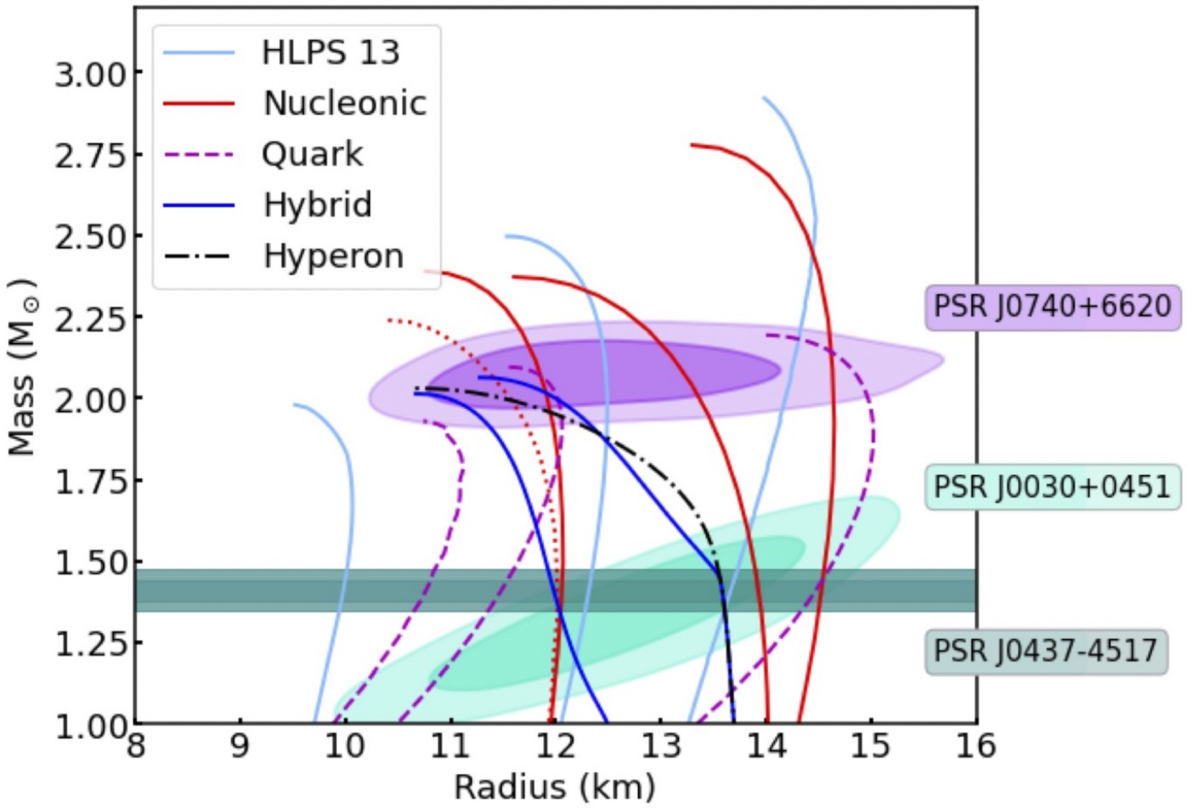
Neutron Star Model

Observable-specific model

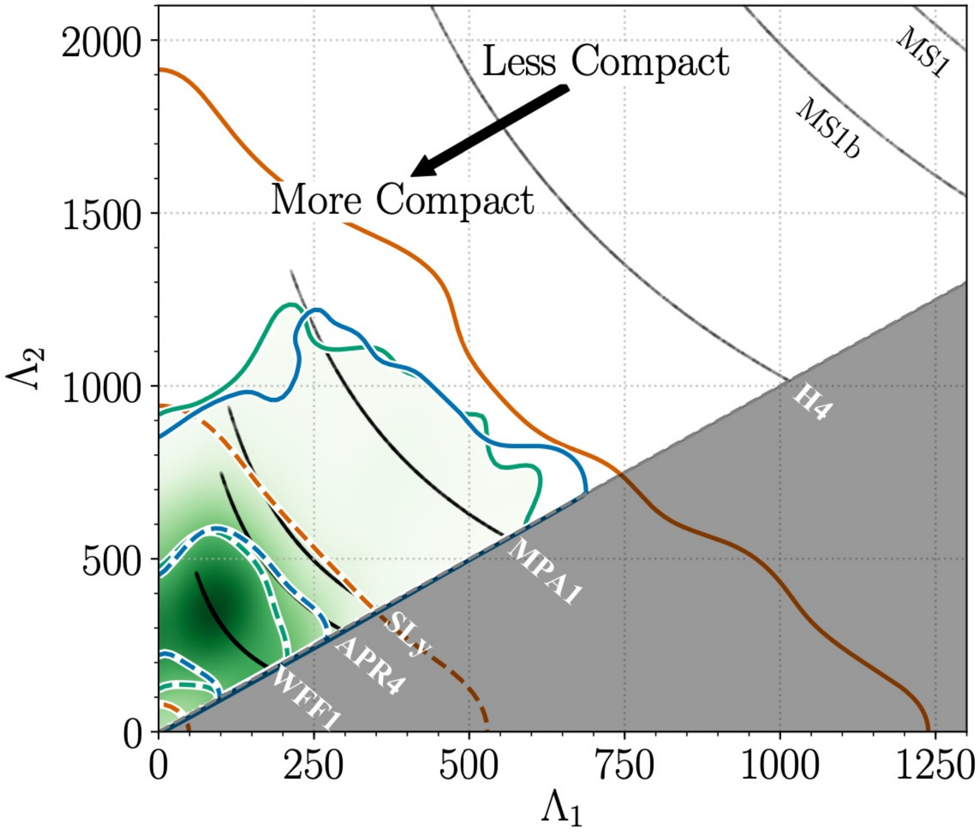
Additional model parameters

Data: Neutron star mass/radii (e.g. NICER)



Riley arxiv:1912.05702, arxiv:2105.06980
Miller et al arxiv:2105.06979, arxiv:1912.05705
Raajimakers et al arxiv: 1912.05703, 2105.06981

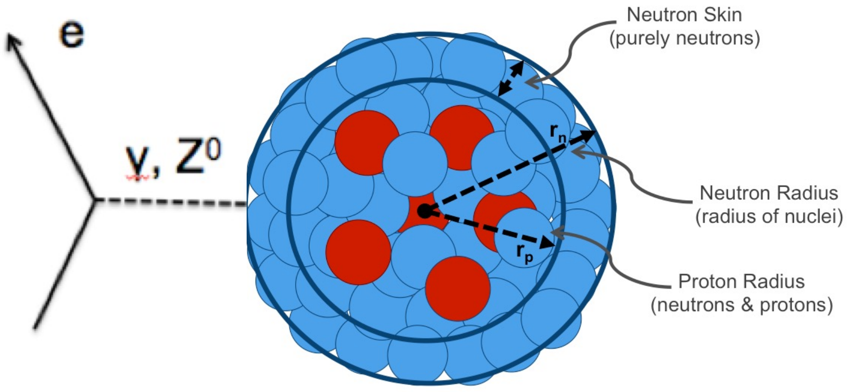
Data: Tidal Deformability



LIGO/Virgo arxiv:1805.11581

Data: Neutron Skins

PREX, CREX



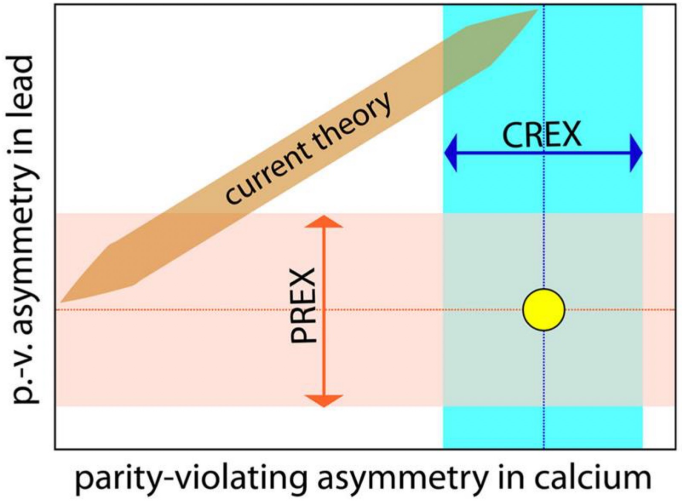
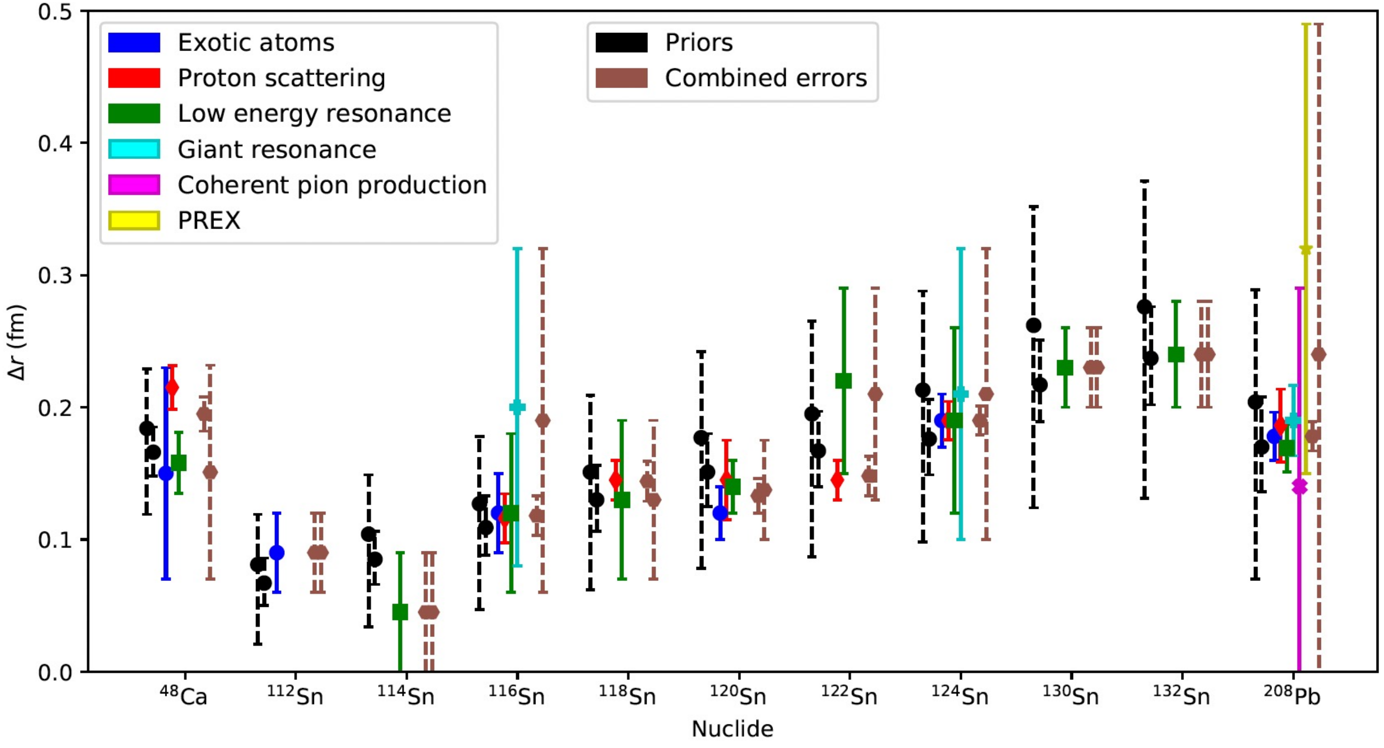


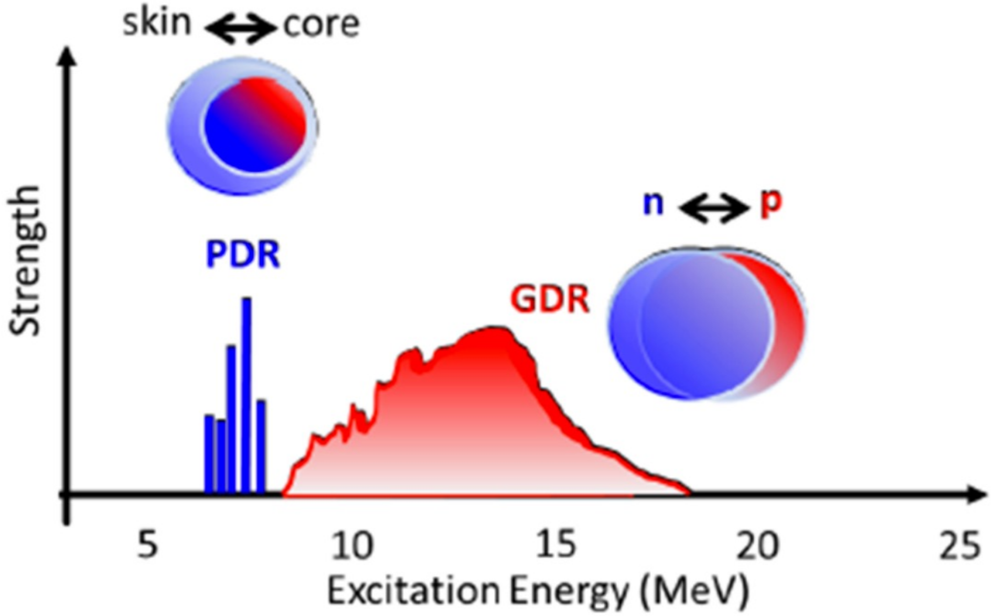
Image: Witold Nazarewicz

Other Probes



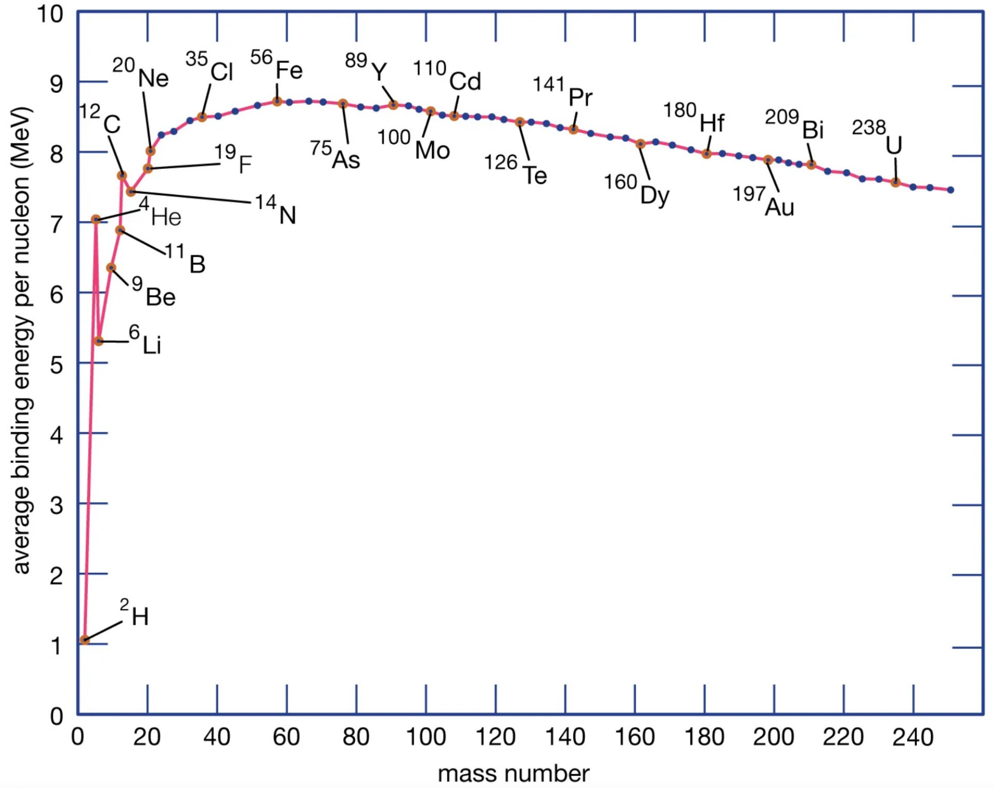
Newton, Crocombe arxiv:2008.00042

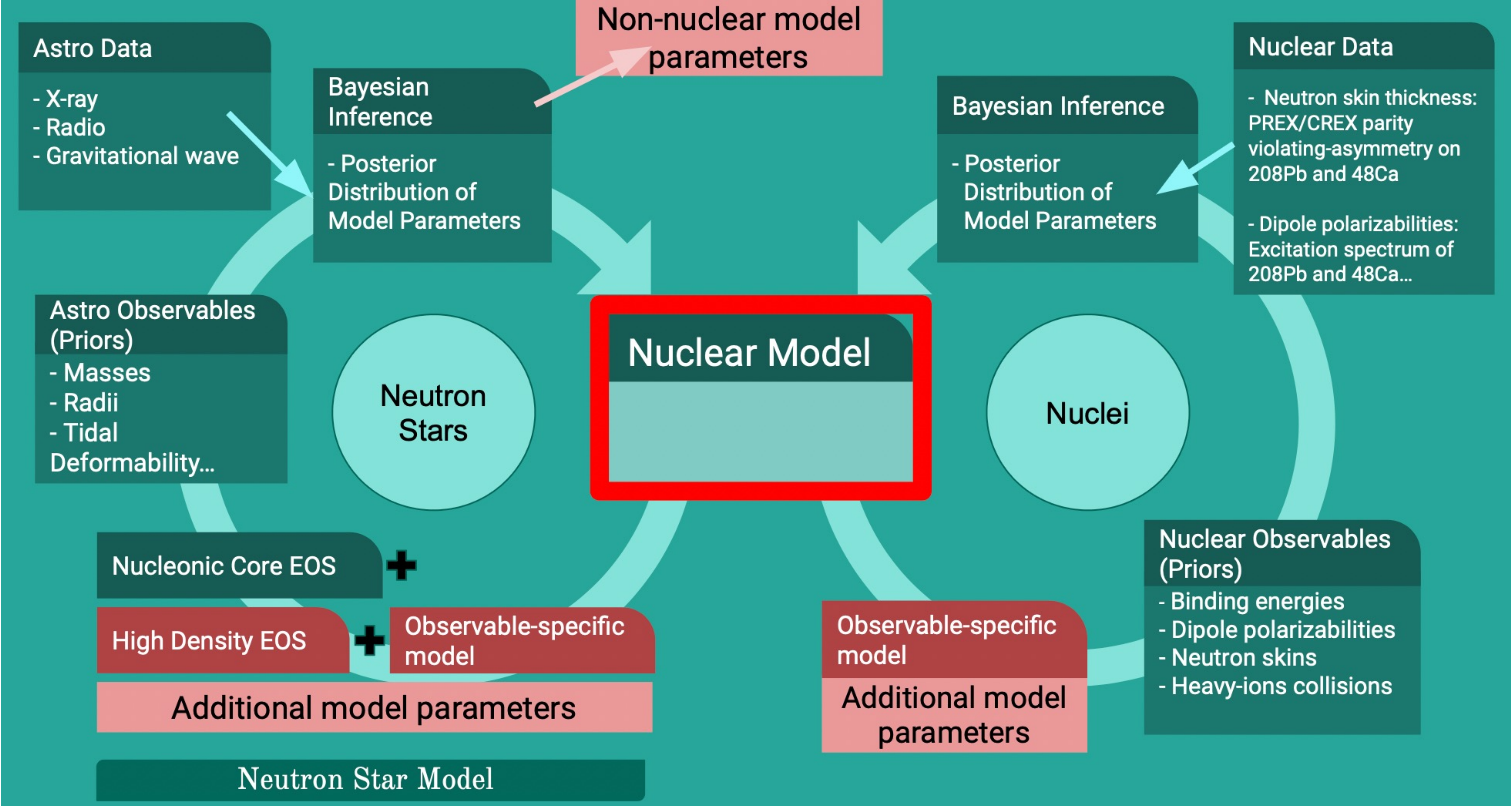
Data: Dipole Polarizability, Nuclear Masses



Bracco, Lanza, Tamii,
PPNP 106, 360 (2019)

e.g. proton scattering





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Non-nuclear model parameters

Astro Observables (Priors)

- Masses
- Radii
- Tidal Deformability…

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High Density EOS + Observable-specific model

Additional model parameters

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Nuclei

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- Binding energies
- Dipole polarizabilities
- Neutron skins
- Heavy-ions collisions

Observable-specific model

Additional model parameters

The nuclear symmetry energy: parameterizing our ignorance in a physically meaningful way

$$E_{\rm sym}(\rho) \;=\; E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3$$

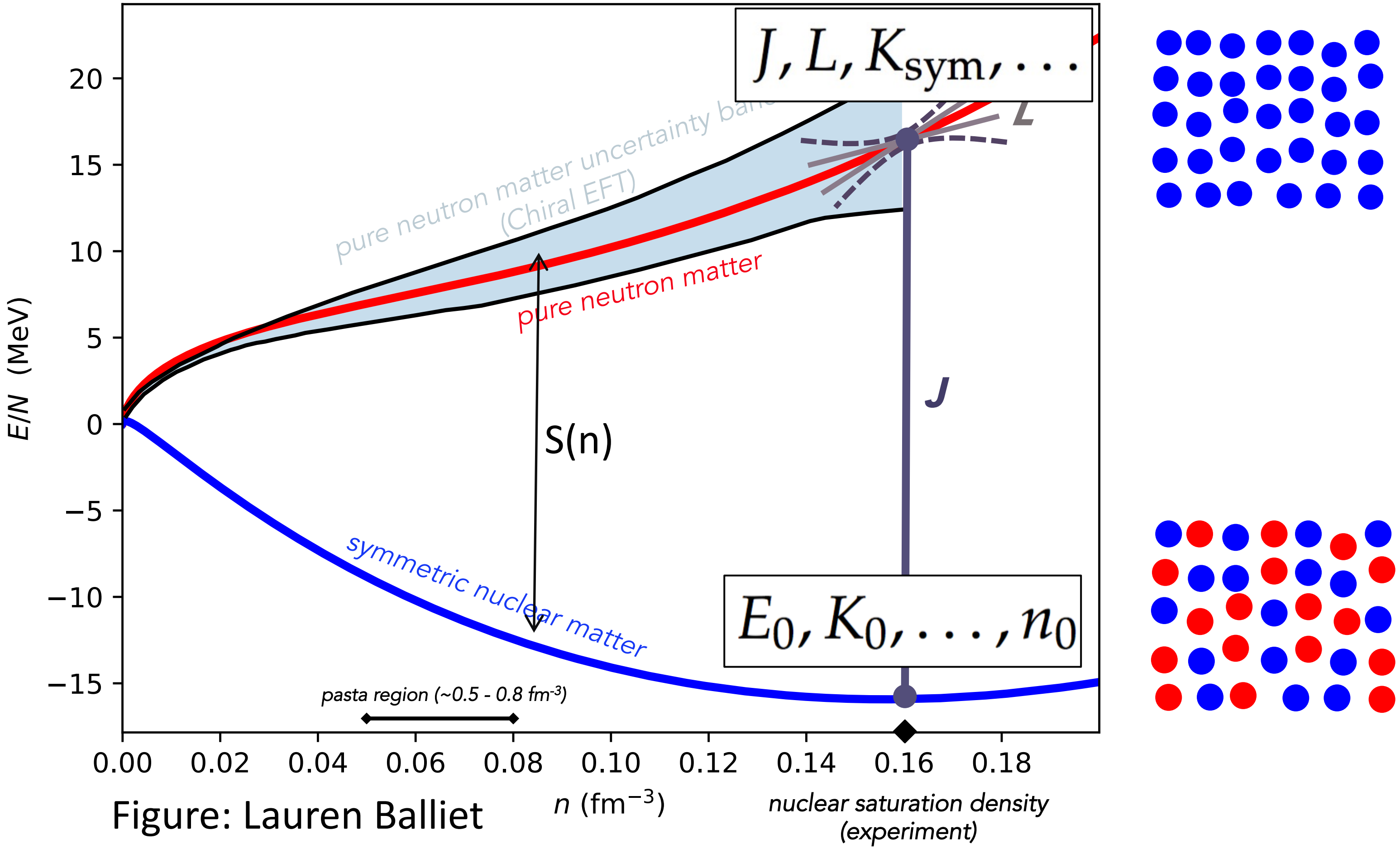


Figure: Lauren Balliet

$$E_0(\rho) \;=\; E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3,$$

Li, arxiv:2105.04629

Our choice of model: Skyrme-Hartree-Fock

Density Functional Theory (e.g. Skyrme)

$$\mathcal{H}_\delta = \frac{1}{4} t_0 \rho^2 [(2 + x_0) - (2x_0 + 1)(y_p^2 + y_n^2)]$$

Local interaction

$$\mathcal{H}_\rho = \frac{1}{4} t_3 \rho^{2+\alpha_3} [(2 + x_3) - (2x_3 + 1)(y_p^2 + y_n^2)] + \frac{1}{4} t_4 \rho^{2+\alpha_4} [(2 + x_4) - (2x_4 + 1)(y_p^2 + y_n^2)]$$

Density dependent

$$\mathcal{H}_{\mathrm{eff}} = \frac{1}{8} \rho [t_1(2 + x_1) + t_2(2 + x_2)] \tau + \frac{1}{8} \rho [t_1(2x_1 + 1) + t_2(2x_2 + 1)](\tau_p y_p + \tau_n y_n)$$

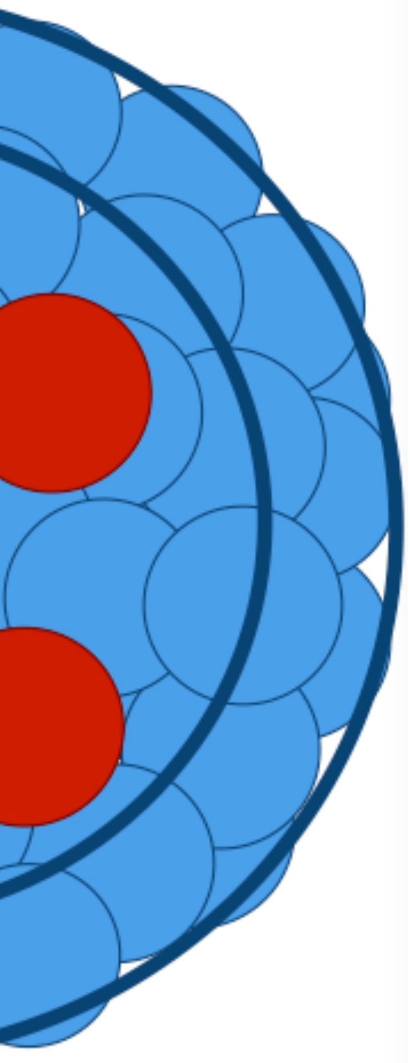
3 body

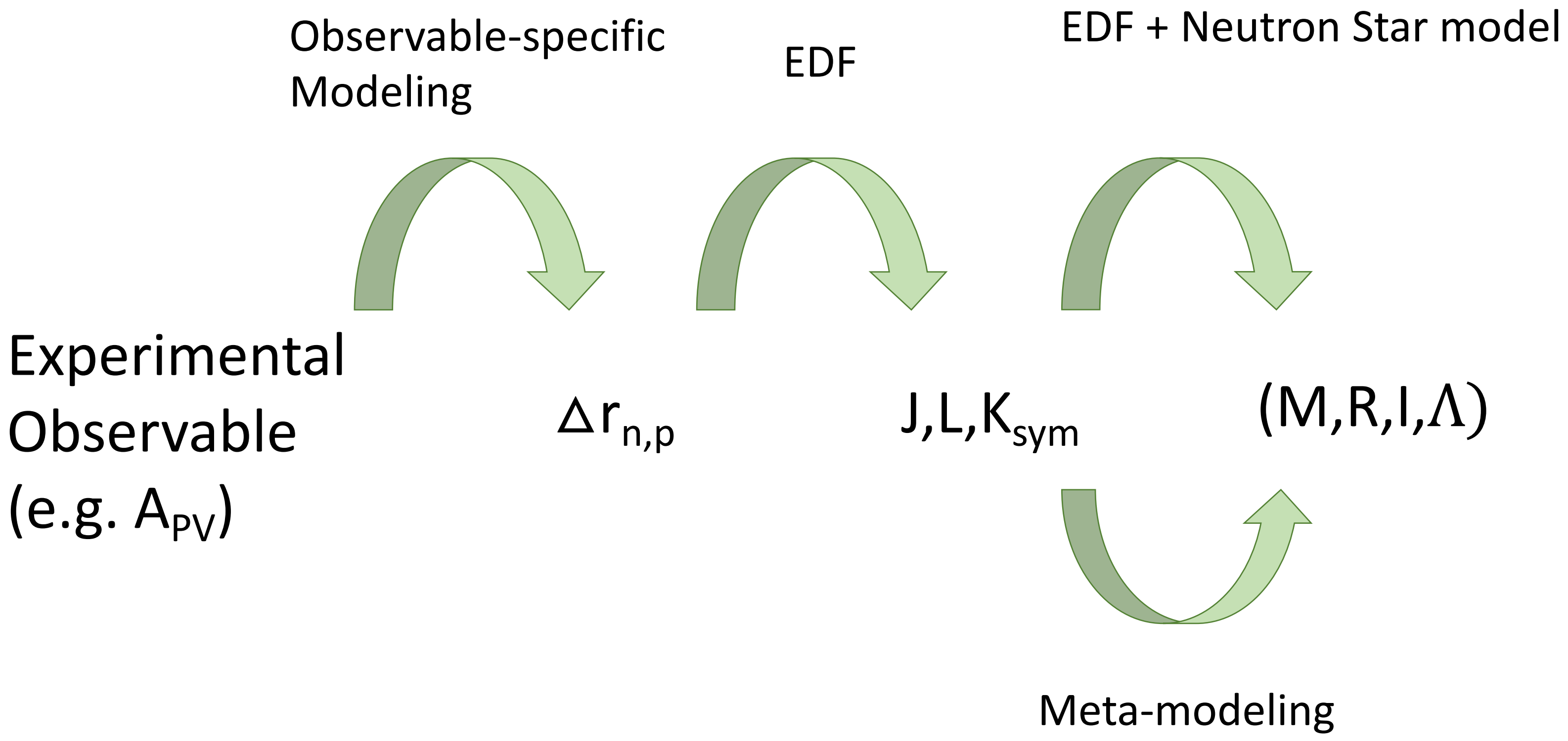
$$\mathcal{H}_{\mathrm{grad}} = \frac{1}{32} (\nabla \rho)^2 [3t_1(2 + x_1) - t_2(2 + x_2)] - \frac{1}{32} [3t_1(2x_1 + 1) + t_2(2x_2 + 1)][(\nabla \rho_p)^2 + (\nabla \rho_n)^2)$$

Gradient…

Used in a variational principle on total energy leads to coupled
Schrödinger-like equations for the wavefunctions.
Solutions converge to ground state (Hohenberg-Kohn theorem)

From skins to stars





Observable-specific Modeling

EDF

EDF + Neutron Star model

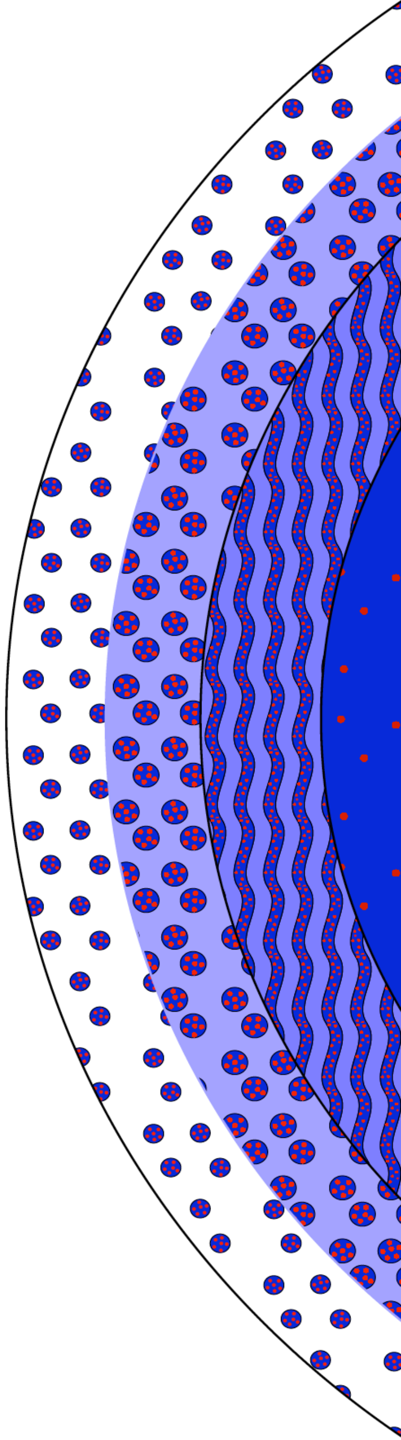
Experimental Observable (e.g. A_{PV})

$\Delta r_{n,p}$

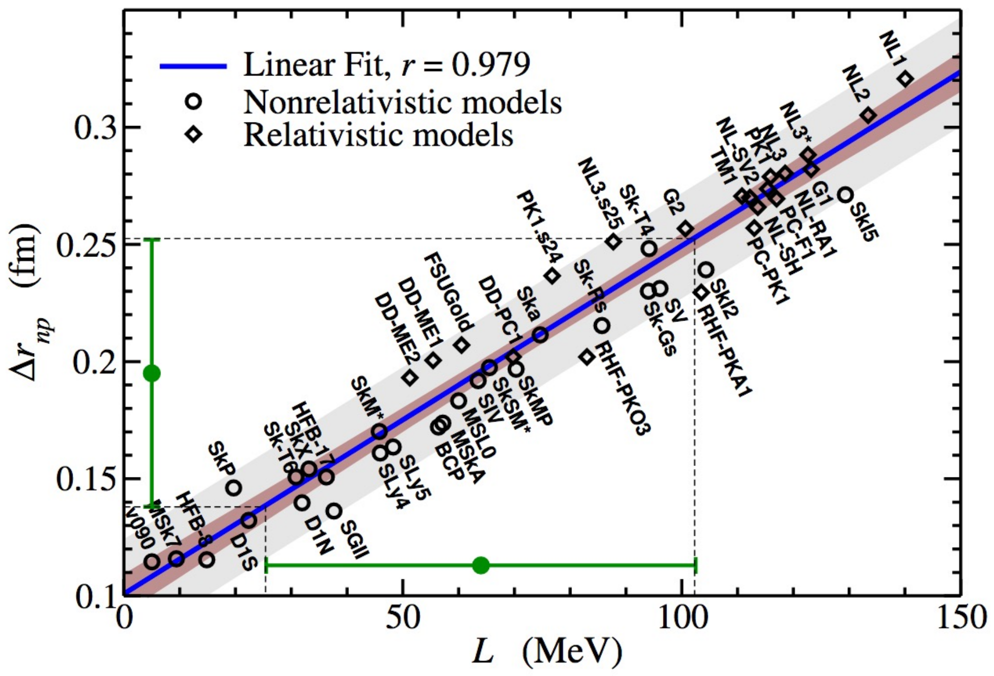
J, L, K_{sym}

(M, R, I, Λ)

Meta-modeling



Existing DFTs predict neutron skin-L relation

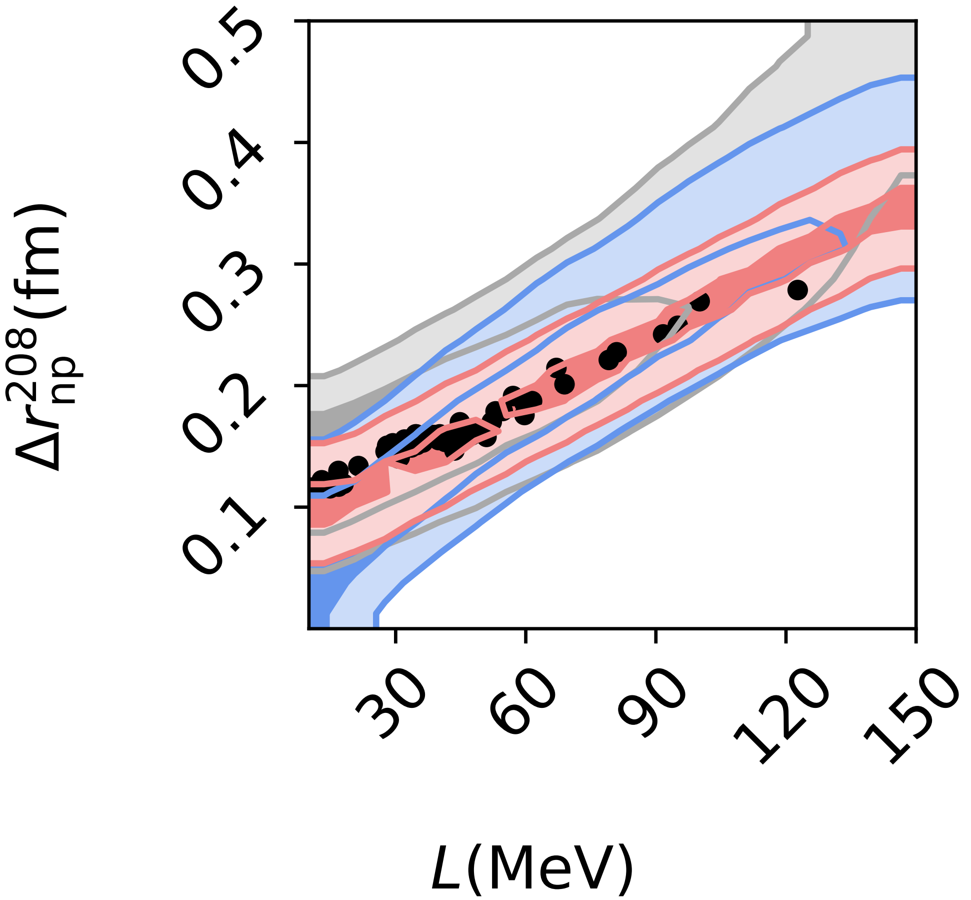


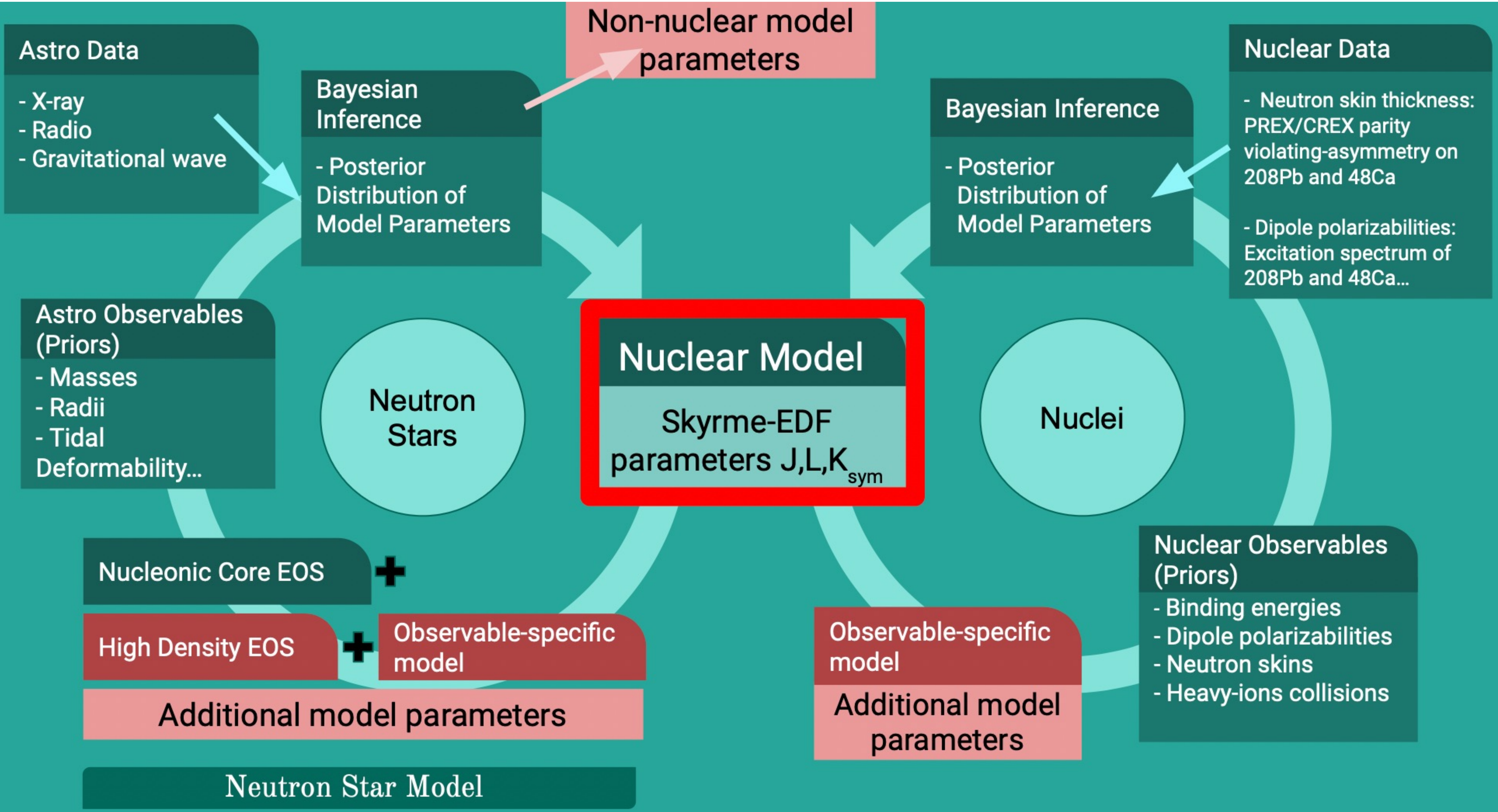
Roca-Maza et al, arxiv:1103.1762

- Models already fit to different datasets which induce additional correlations between symmetry energy parameters
- If, for example, a posterior from Astro observables contains J,L, and K_{sym}. This 3-dimensional parameter space is collapsed to one using the empirical relation

- Selection of EDFs

Linear fit
Liquid Drop Model
Skyrme Hartree-Fock





Non-nuclear model parameters

Astro Data

- X-ray
- Radio
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Bayesian Inference

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Astro Observables (Priors)

- Masses
- Radii
- Tidal Deformability...

Neutron Stars

Nuclear Model

Skyrme-EDF parameters J,L,K_{sym}

Nucleonic Core EOS ✚

High Density EOS ✚ **Observable-specific model**

Additional model parameters

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Observable-specific model

Additional model parameters

Map nuclear matter parameters to model parameters and systematically generate models

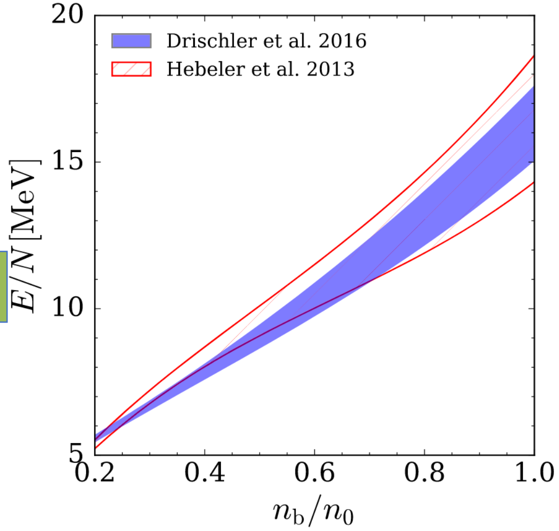
$$\begin{bmatrix} t_0 \\ \vdots \\ t_4 \\ x_0 \\ \vdots \\ x_4 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} \longleftarrow \begin{array}{cl} \left[\begin{matrix} E_0 \\ K_0 \\ n_0 \\ m^*_{\mathrm{p}} \\ m^*_{\mathrm{n}} \\ G_{\mathrm{s}} \\ G_{\mathrm{v}} \\ J \\ L \\ K_{\mathrm{sym}} \end{matrix}\right] & \begin{matrix} \\ \text{SNM} \\ \\ \text{Dynamics of n,p} \\ \\ \text{Surface energy} \\ \\ \\ \text{Symmetry Energy} \\ \\ \end{matrix} \end{array}$$

Map nuclear matter parameters to model parameters and systematically generate models

$$\begin{bmatrix} x_0 \\ x_3 \\ x_4 \end{bmatrix} \longleftarrow \begin{bmatrix} J \\ L \\ K_{\mathrm{sym}} \end{bmatrix}$$

$$\begin{bmatrix} t_0 \\ \vdots \\ t_4 \\ x_1 \\ x_2 \end{bmatrix} \longleftarrow \begin{bmatrix} E_0 \\ K_0 \\ n_0 \\ m_{\mathrm{p}}^* \\ m_{\mathrm{n}}^* \\ G_{\mathrm{s}} \\ G_{\mathrm{v}} \end{bmatrix} \begin{bmatrix} x_0 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_3 \\ \alpha_4 \end{bmatrix} \longleftarrow$$



Haensel, Fortin JPhysG 2017
Lim, Holt arXiv:1702.02898

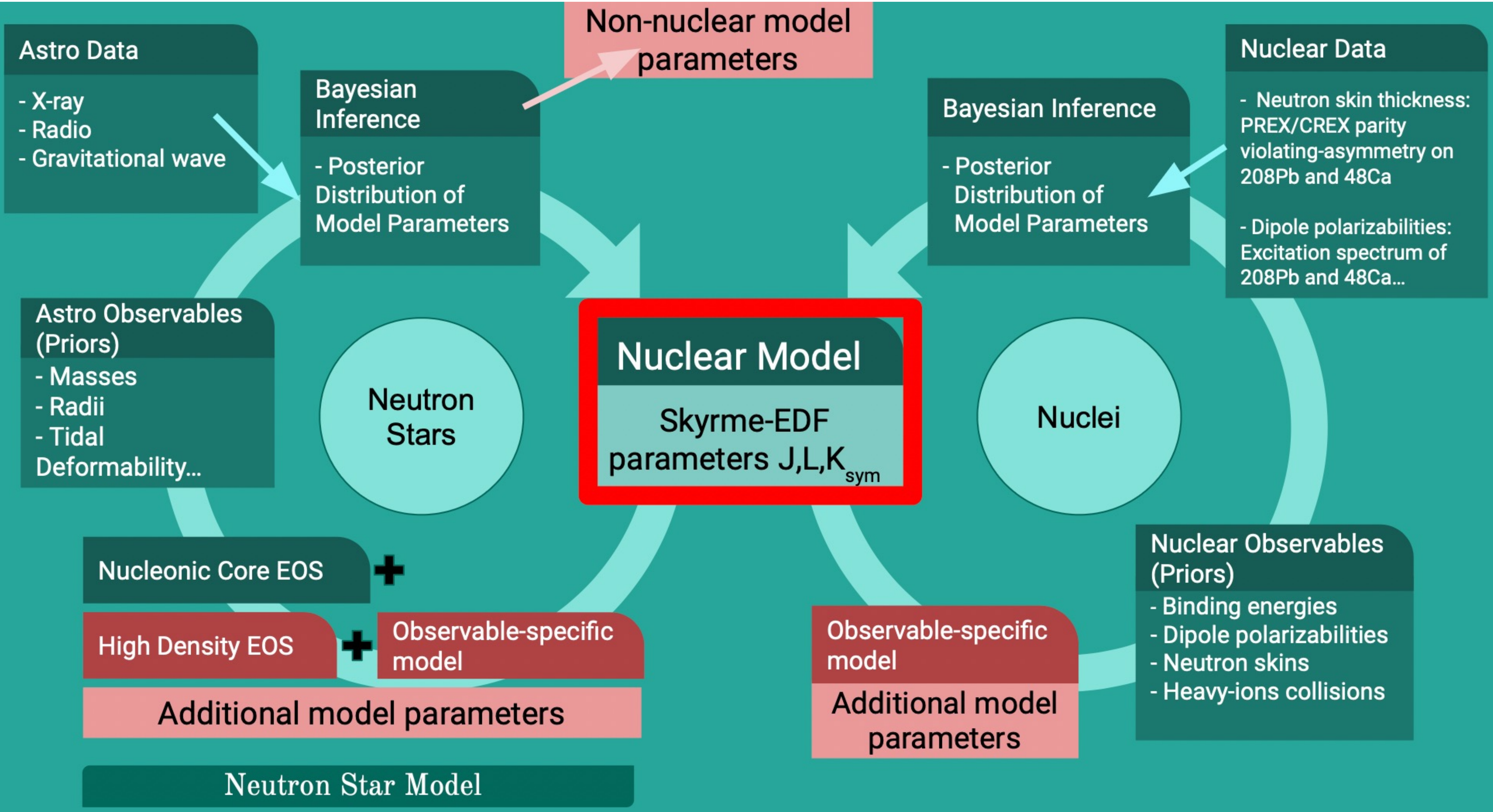
Nuclear masses,
giant resonances

ρ_0 (fm^{-3})	0.160 ± 0.005
B (MeV)	16.0 ± 0.5
K (MeV)	230 ± 30

Lim, Holt arXiv:1702.02898

Fixed: potential source of systematic model error

Neutron skins, dipole polarizability, binding energy: SkyrmeRPA Comp Phys Comms, 184, (2013)



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Nucleonic Core EOS ✚

High Density EOS ✚ Observable-specific model

Additional model parameters

Neutron Star Model

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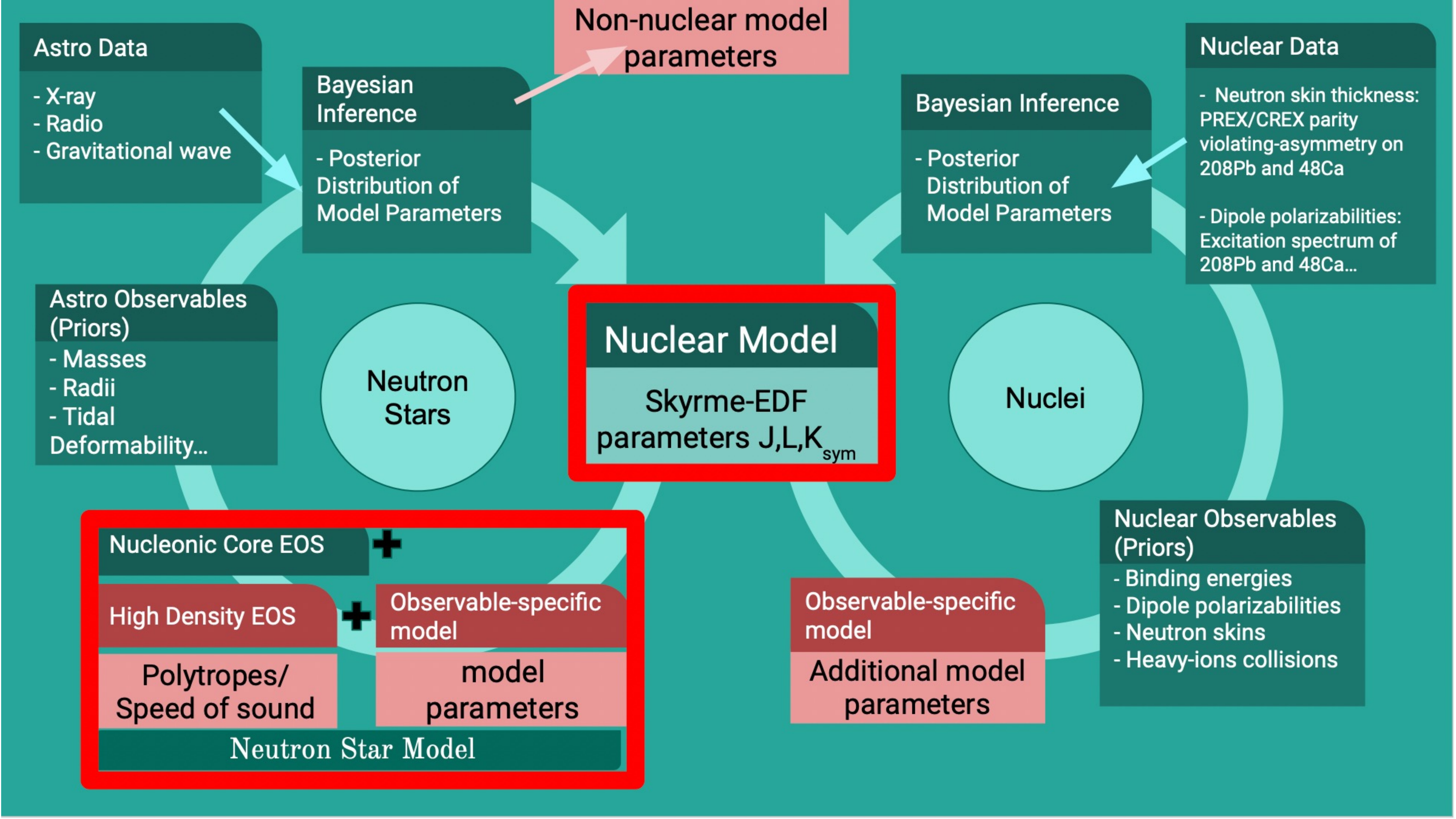

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Nucleonic Core EOS ✚

High Density EOS ✚ **Observable-specific model**

Polytropes/ Speed of sound

model parameters

Neutron Star Model

Observable-specific model

Additional model parameters

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- Dipole polarizabilities
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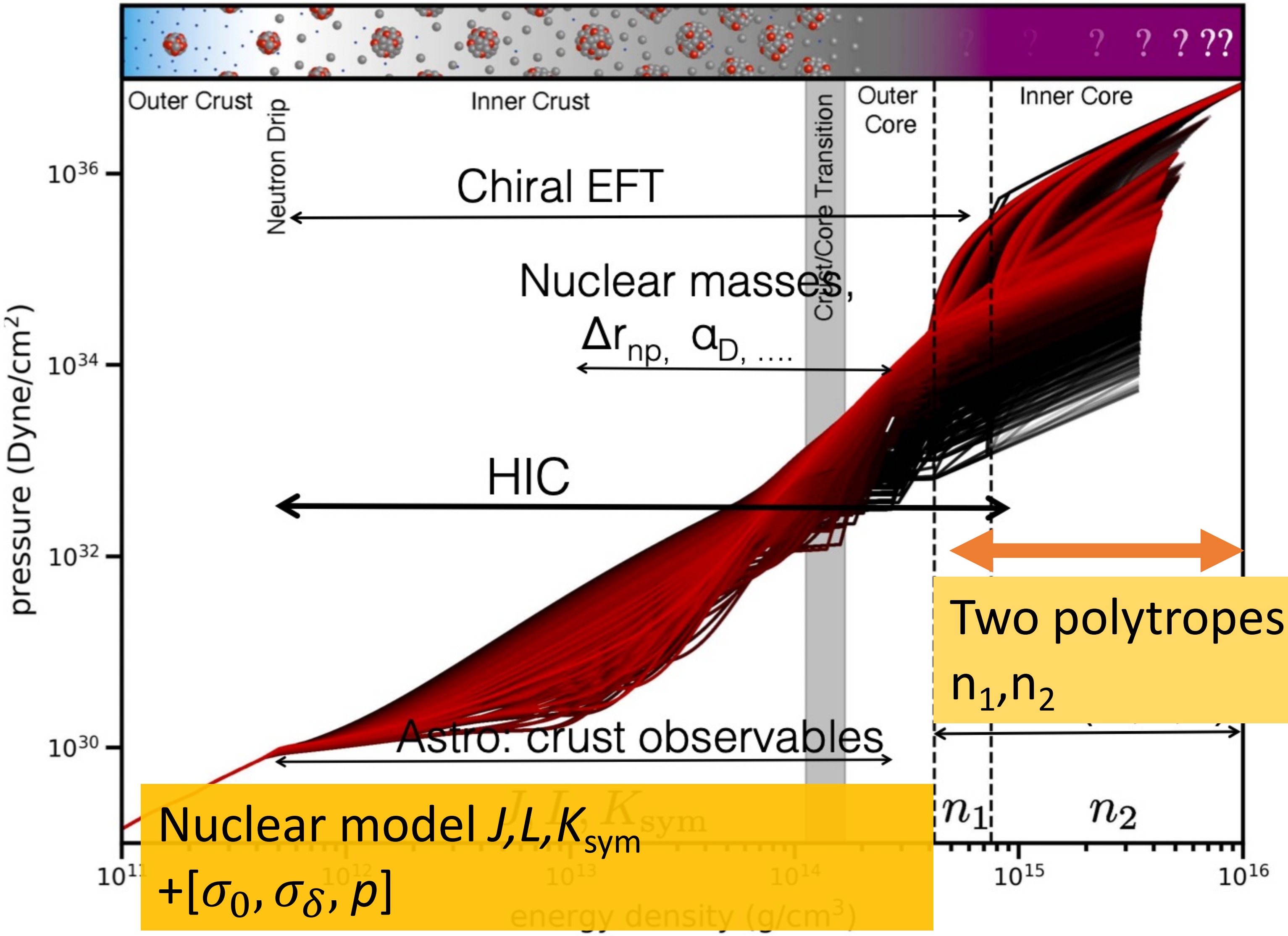
Model parameters



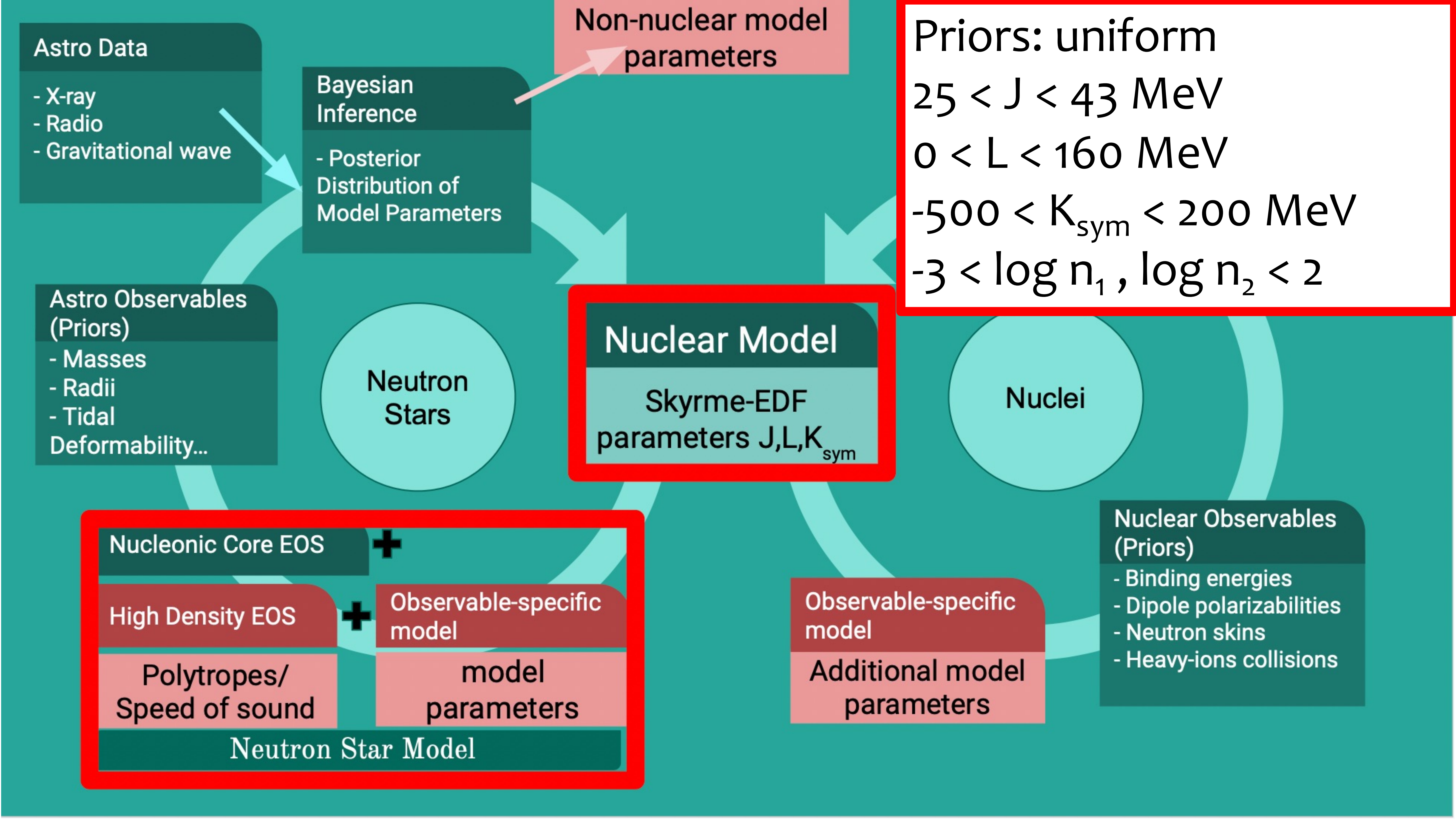
Two polytropes

n_1,n_2

Nuclear model J,L,K_{sym}

+$[\sigma_0, \sigma_\delta, p]$

Neill+ 2208.00994; Sorenson+ 2301.13253



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Nucleonic Core EOS +

High Density EOS + Observable-specific model

Polytropes/ Speed of sound

model parameters

Neutron Star Model

Observable-specific model

Additional model parameters

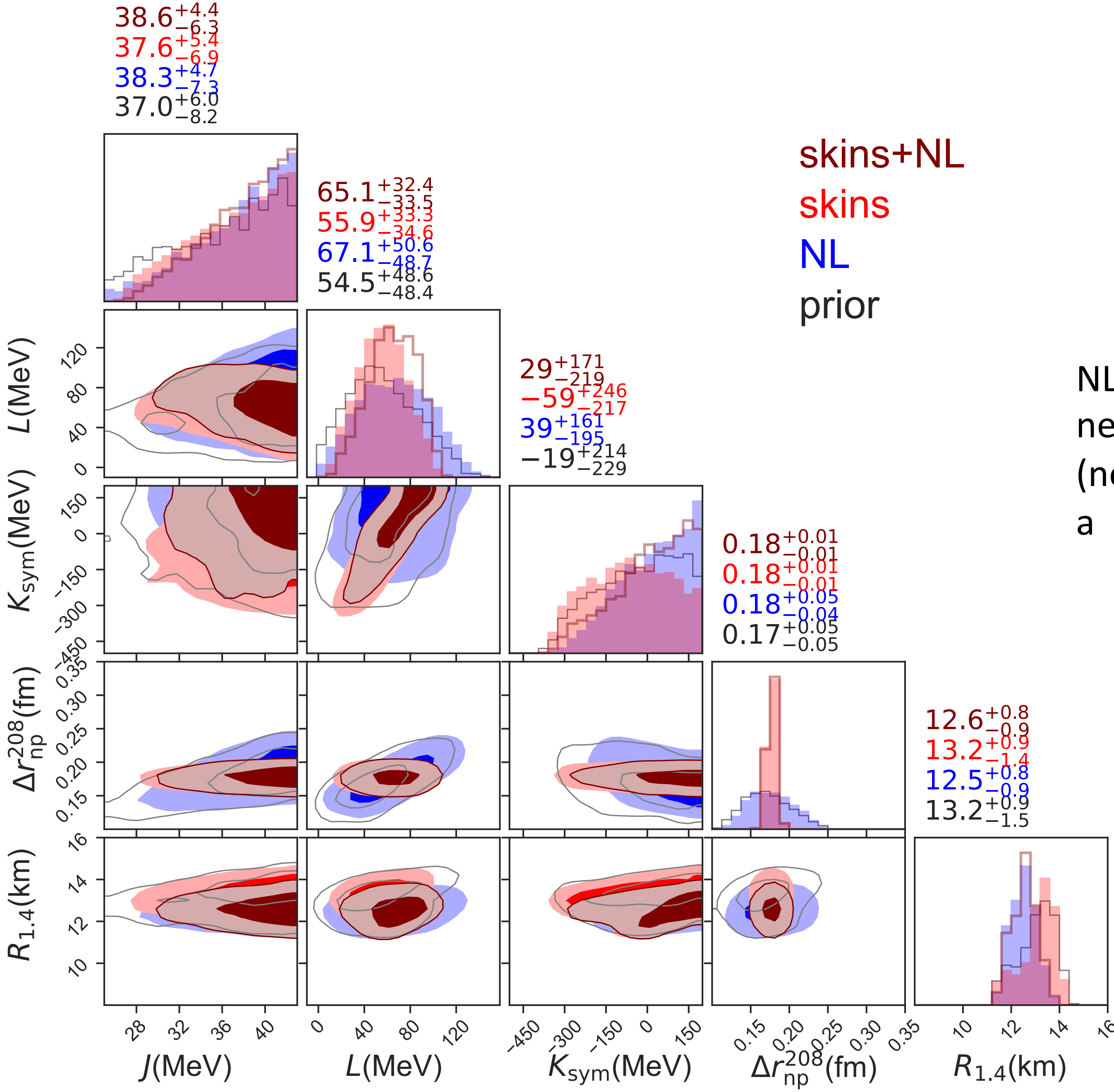
Priors: uniform

$25 < J < 43$ MeV

$0 < L < 160$ MeV

$-500 < K_{sym} < 200$ MeV

$-3 < \log n_1 , \log n_2 < 2$



NL prefers stiffer *L*,K_{sym}
neutron skins prefer softer
(note: PREX alone would come to a different conclusion)

Newton+ in prep

Experiments such as neutron skin and dipole polarizability probes EOS below mostly saturation density, how relevant are those in determine neutron star properties?

Experiments such as neutron skin and dipole polarizability probes EOS below mostly saturation density, how relevant are those in determine neutron star properties?

(with apologies to Matt Groening)

Why care about the crust?

Pulsar glitches

Link, Lattimer, Epstein PRL 1999

Magnetic field evolution

Pons, Vigano, Rea, Nature Physics 2013

Crust cooling

Newton, Murphy, Li ApJL 2013

Brown and Cumming, ApJ 2009

Horowitz+, PRL 2015

GWs from mountains

Caplan,Horowitz,Schneider, PRL 2018

Spin evolution, r-modes

Fattoyev, Newton, Li PRC 2014

Crust shattering flares

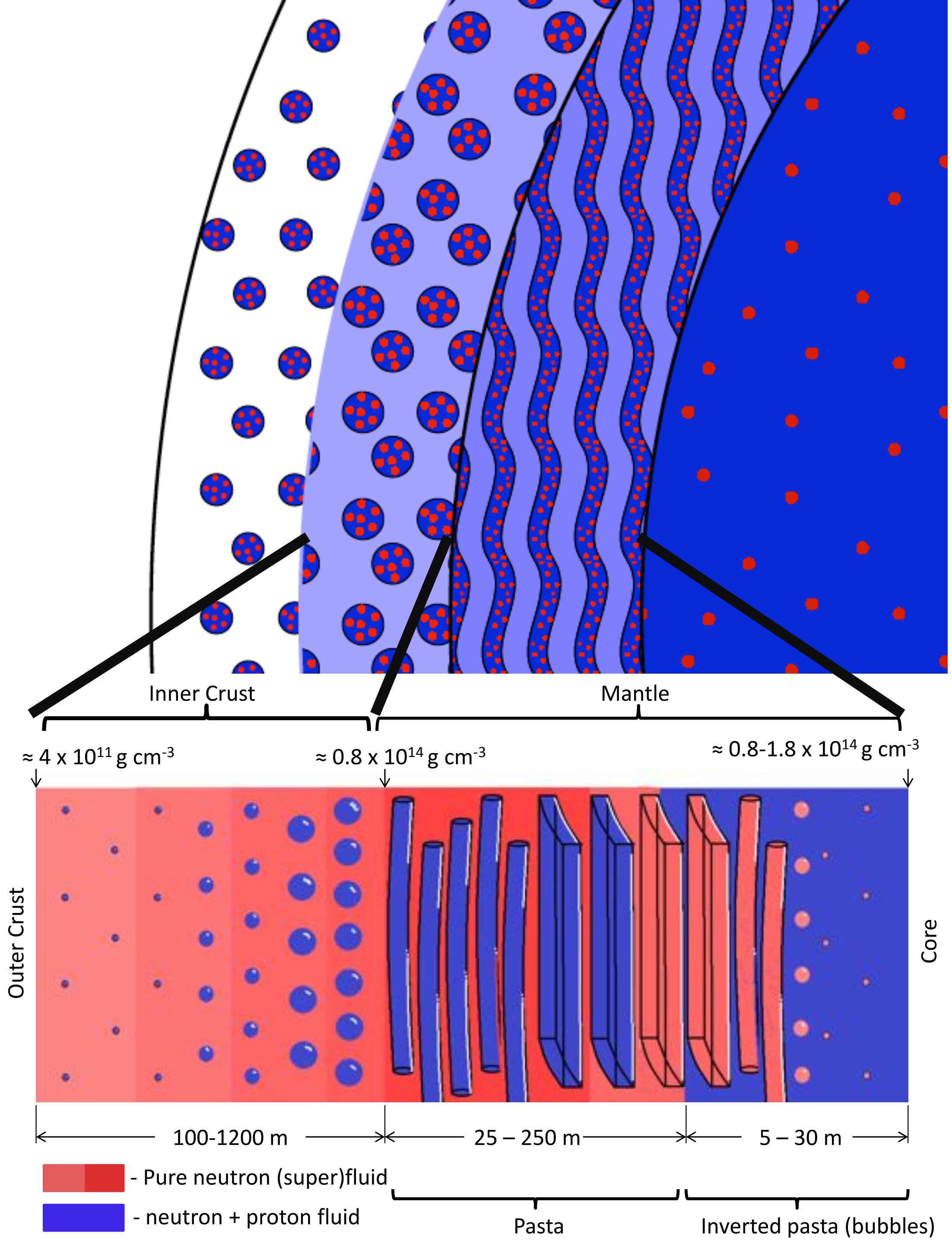
Tsang et al PRL108, 2012

Chamel, Haensel,

Living Reviews in Relativity 2008

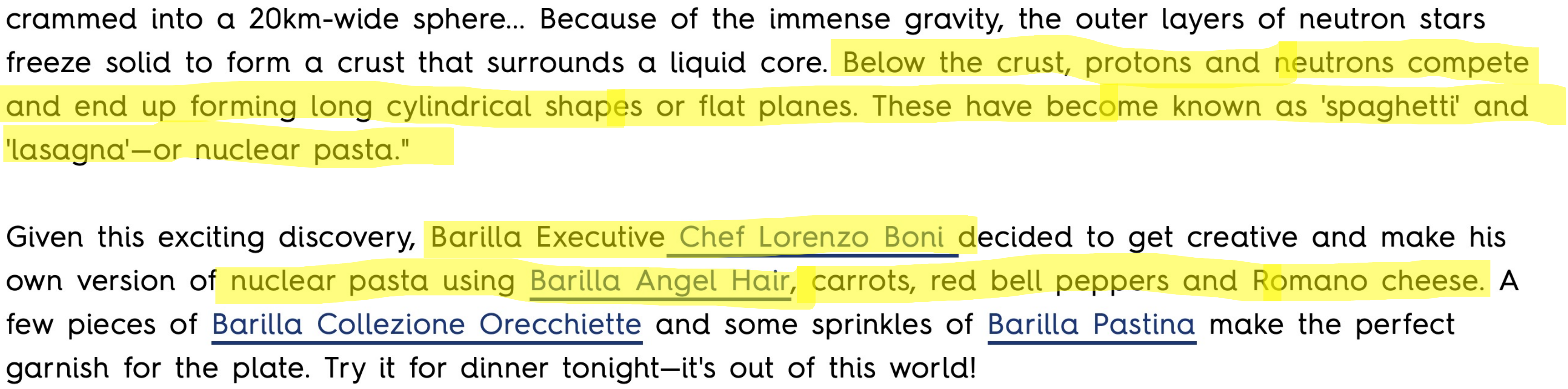
Constraining the symmetry energy:

Newton+ EPJA 2014



NUCLEAR PASTA RECIPE: ANGEL HAIR WITH CARROTS

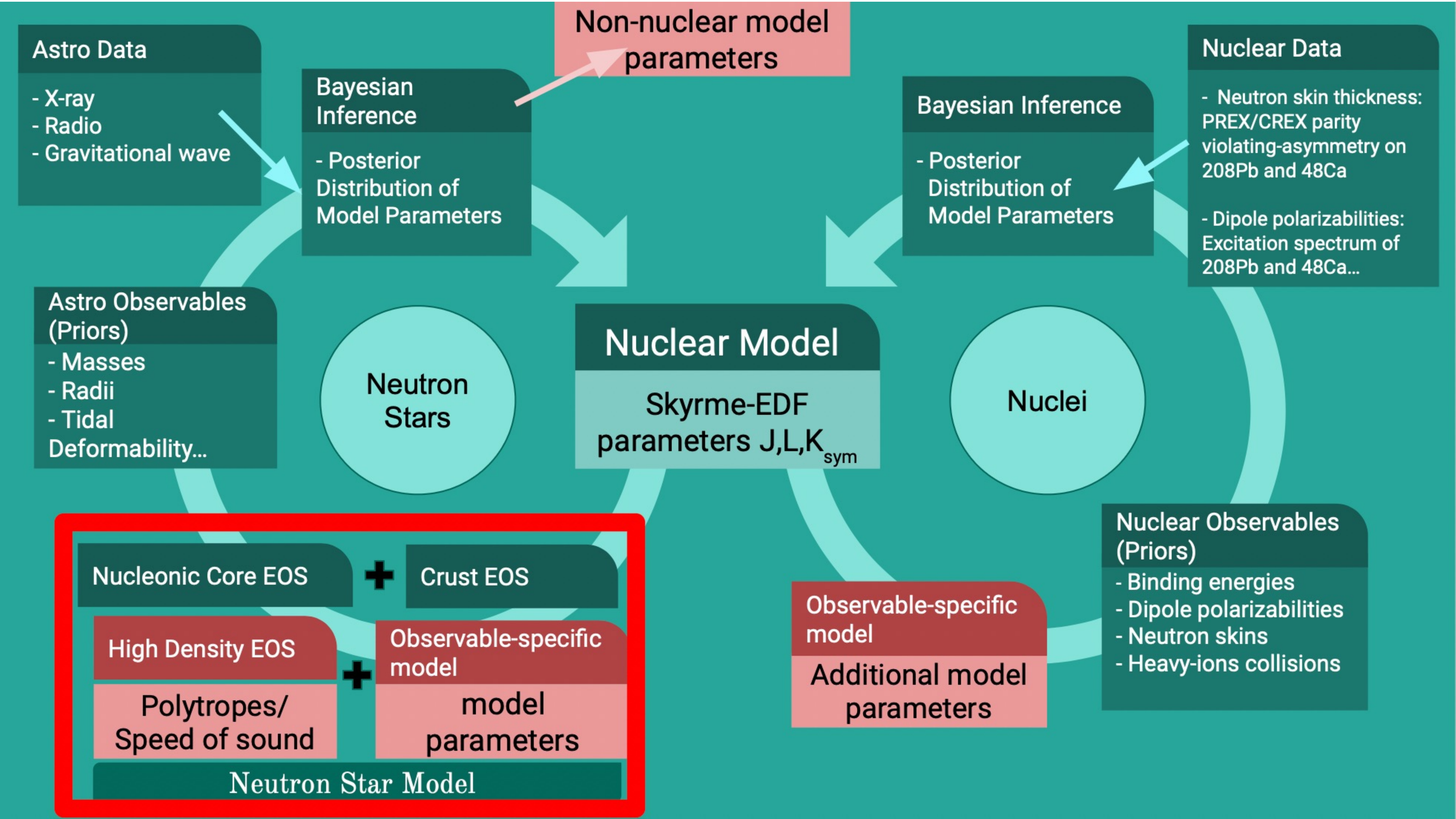
DAL 1877
Barilla®



crammed into a 20km-wide sphere... Because of the immense gravity, the outer layers of neutron stars freeze solid to form a crust that surrounds a liquid core. Below the crust, protons and neutrons compete and end up forming long cylindrical shapes or flat planes. These have become known as 'spaghetti' and 'lasagna'—or nuclear pasta."

Given this exciting discovery, Barilla Executive Chef Lorenzo Boni decided to get creative and make his own version of nuclear pasta using Barilla Angel Hair, carrots, red bell peppers and Romano cheese. A few pieces of Barilla Collezione Orecchiette and some sprinkles of Barilla Pastina make the perfect garnish for the plate. Try it for dinner tonight—it's out of this world!

https://www.barilla.com/en-us/posts/2018/10/22/nuclear-pasta-recipe-angel-hair-with-carrots



Astro Data

- X-ray
- Radio
- Gravitational wave

Bayesian Inference

- Posterior Distribution of Model Parameters

Non-nuclear model parameters

Nuclear Data

- Neutron skin thickness: PREX/CREX parity violating-asymmetry on 208Pb and 48Ca
- Dipole polarizabilities: Excitation spectrum of 208Pb and 48Ca…

Bayesian Inference

- Posterior Distribution of Model Parameters

Astro Observables (Priors)

- Masses
- Radii
- Tidal Deformability…

Neutron Stars

Nuclear Model

Skyrme-EDF parameters J,L,K_{sym}

Nuclei

Nucleonic Core EOS + **Crust EOS**

High Density EOS

Polytropes/ Speed of sound

+

Observable-specific model

model parameters

Neutron Star Model

Observable-specific model

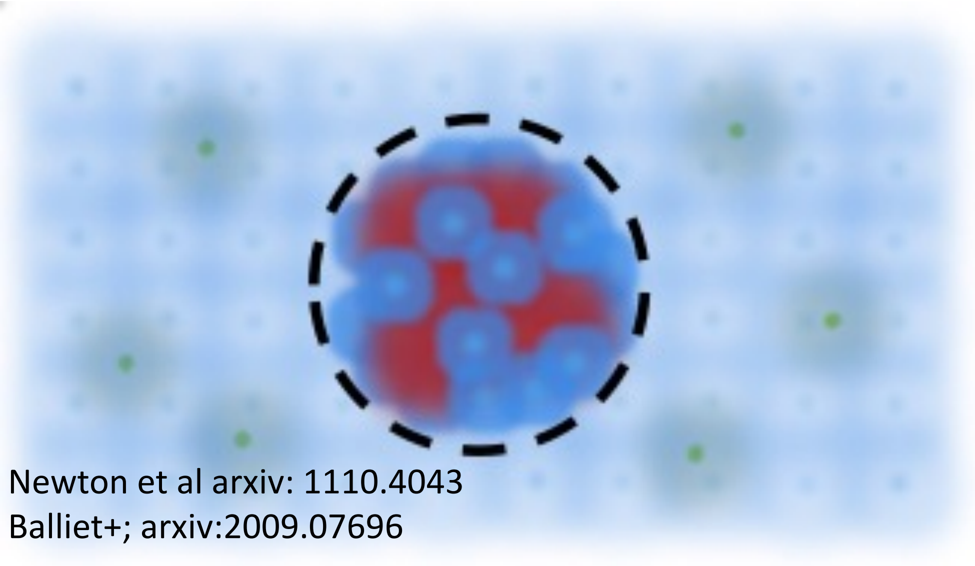
Additional model parameters

Nuclear Observables (Priors)

- Binding energies
- Dipole polarizabilities
- Neutron skins
- Heavy-ions collisions

Modeling the crust

CLDM:Bulk fluid and surface degrees of freedom

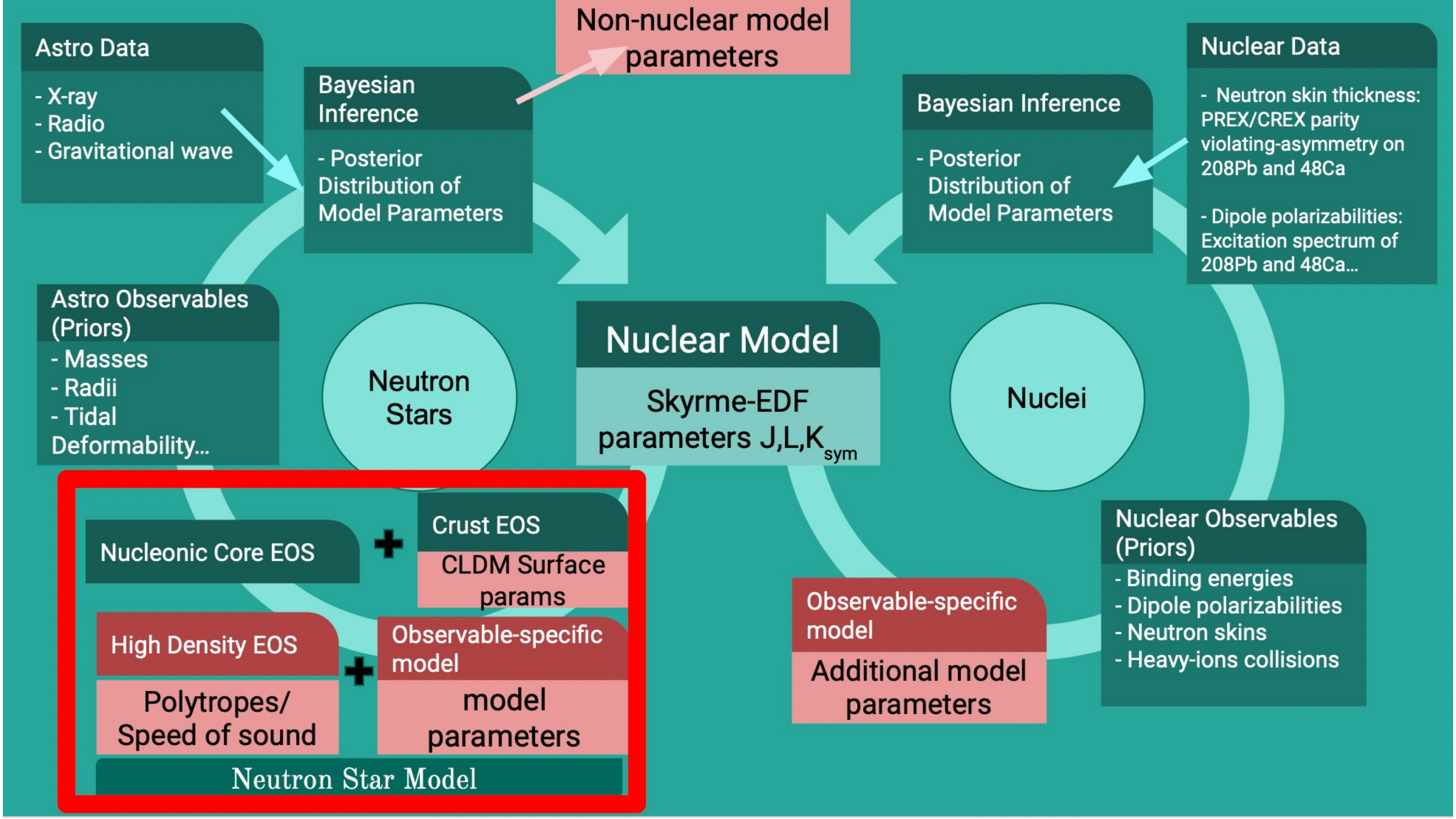


Newton et al arxiv: 1110.4043
Balliet+; arxiv:2009.07696

$$\mathcal{H}_\delta + \mathcal{H}_\rho + \mathcal{H}_{\rm eff} \qquad \sigma_{\rm s}(y_{\rm p}) = \sigma_0 \frac{2^{p+1}+b}{\frac{1}{y_{\rm p}^p}+b+\frac{1}{(1-y_{\rm p})^p}}$$

Skyrme provides uniform nuclear and neutron fluid model+ surface energy function (Lattimer, Lamb, Pethick, Ravenhall 1985)

Nuclear model parameters $J, L, K_{\rm sym}$
Surface parameters $\sigma_0, \sigma_\delta, p$



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Nucleonic Core EOS + **Crust EOS**

CLDM Surface params

High Density EOS

Polytropes/ Speed of sound

+ **Observable-specific model**

model parameters

Neutron Star Model

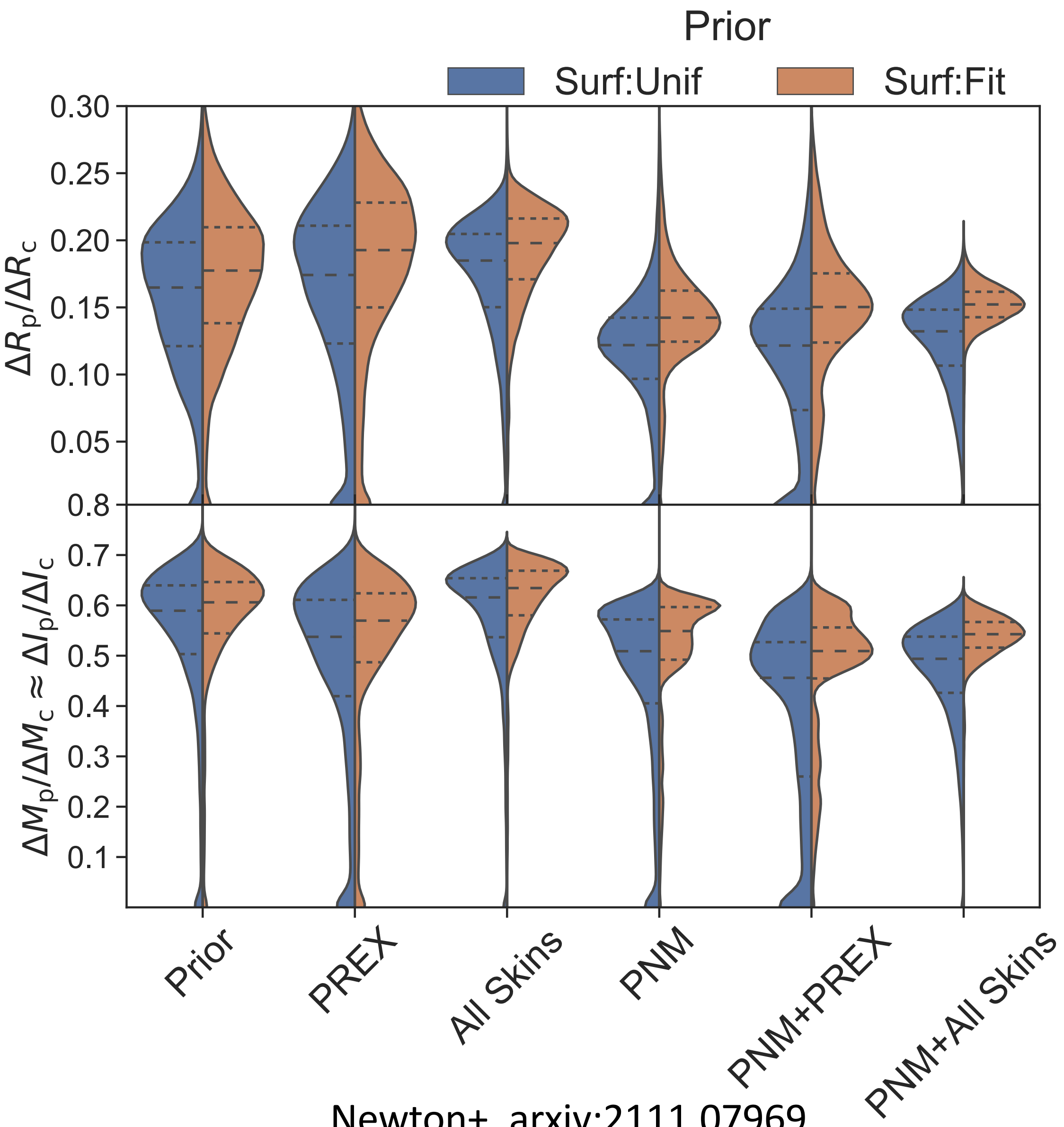
Observable-specific model

Additional model parameters

Nuclear Observables (Priors)

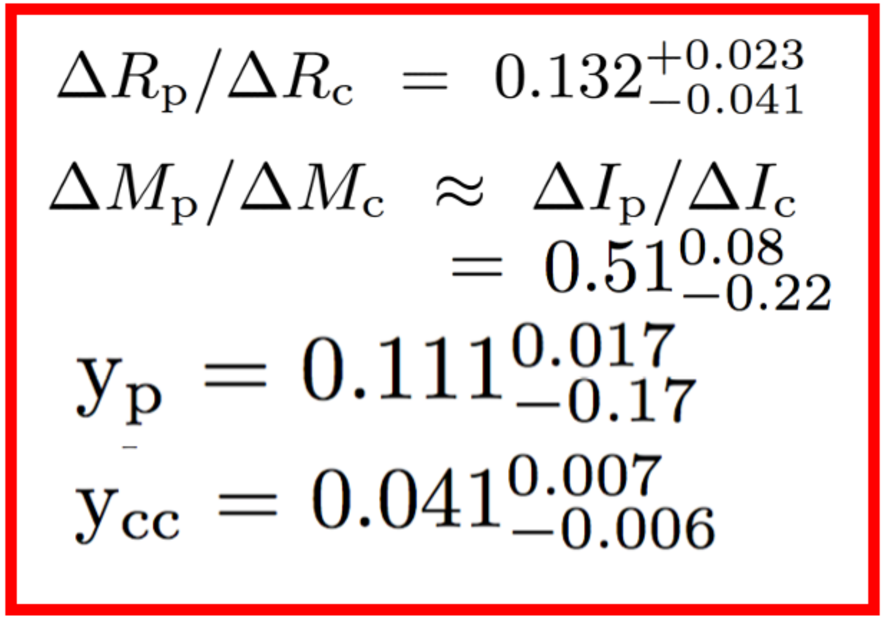
- Binding energies
- Dipole polarizabilities
- Neutron skins
- Heavy-ions collisions

Relative thickness and mass of pasta



Newton+, arxiv:2111.07969
Balliet+, arxiv:2009.07696

Concordance of CLDM models



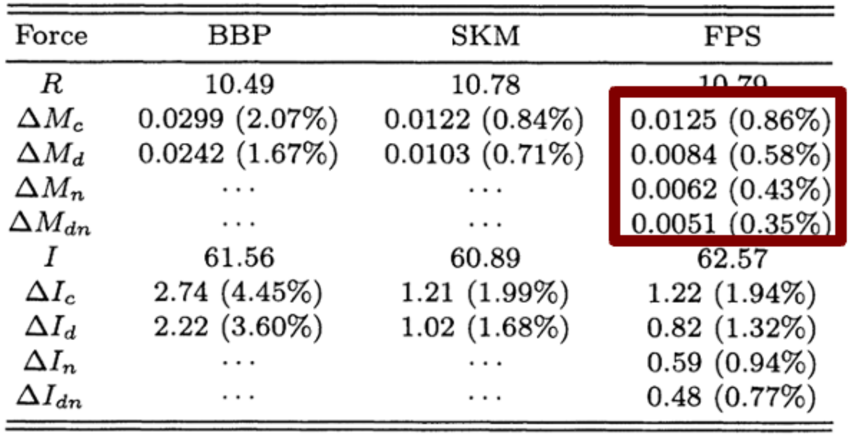
$$\Delta R_{\rm p}/\Delta R_{\rm c} = 0.132^{+0.023}_{-0.041}$$

$$\Delta M_{\rm p}/\Delta M_{\rm c} \approx \Delta I_{\rm p}/\Delta I_{\rm c} = 0.51^{0.08}_{-0.22}$$

$$y_{\rm p} = 0.111^{0.017}_{-0.17}$$

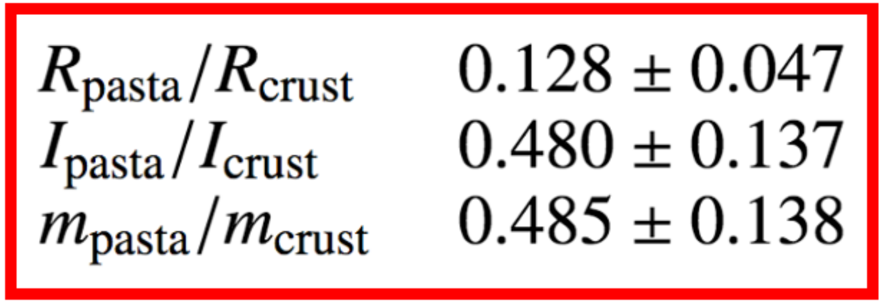
$$y_{\rm cc} = 0.041^{0.007}_{-0.006}$$

Newton+, arxiv:2111.07969



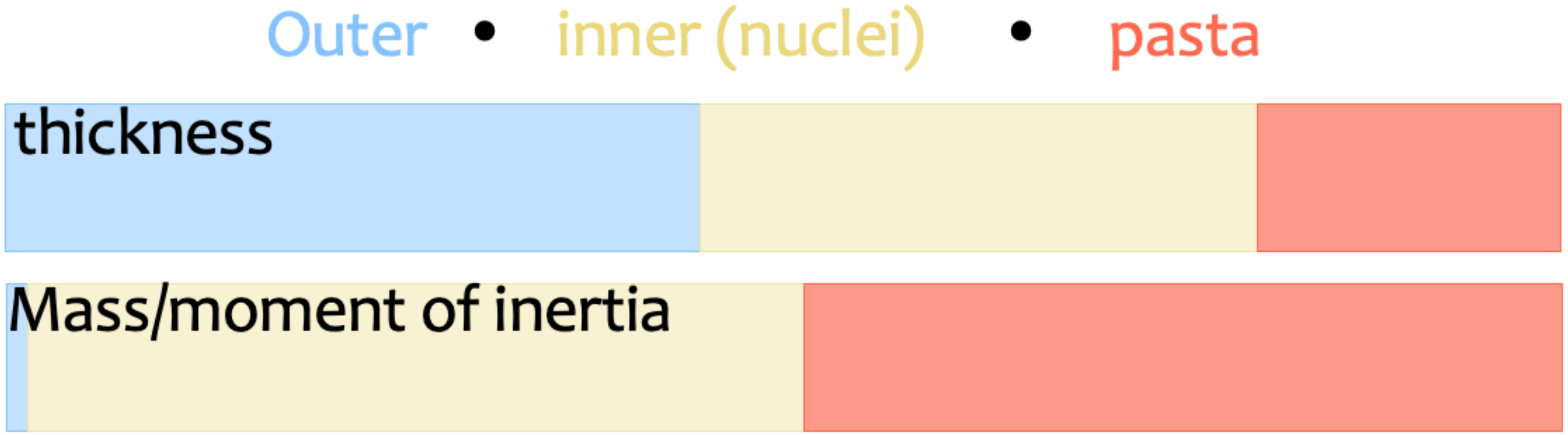
Force	BBP	SKM	FPS
R	10.49	10.78	10.70
ΔM_c	0.0299 (2.07%)	0.0122 (0.84%)	0.0125 (0.86%)
ΔM_d	0.0242 (1.67%)	0.0103 (0.71%)	0.0084 (0.58%)
ΔM_n	...	...	0.0062 (0.43%)
ΔM_{dn}	...	...	0.0051 (0.35%)
I	61.56	60.89	62.57
ΔI_c	2.74 (4.45%)	1.21 (1.99%)	1.22 (1.94%)
ΔI_d	2.22 (3.60%)	1.02 (1.68%)	0.82 (1.32%)
ΔI_n	...	...	0.59 (0.94%)
ΔI_{dn}	...	...	0.48 (0.77%)

Lorenz et al PRL70 (1993)



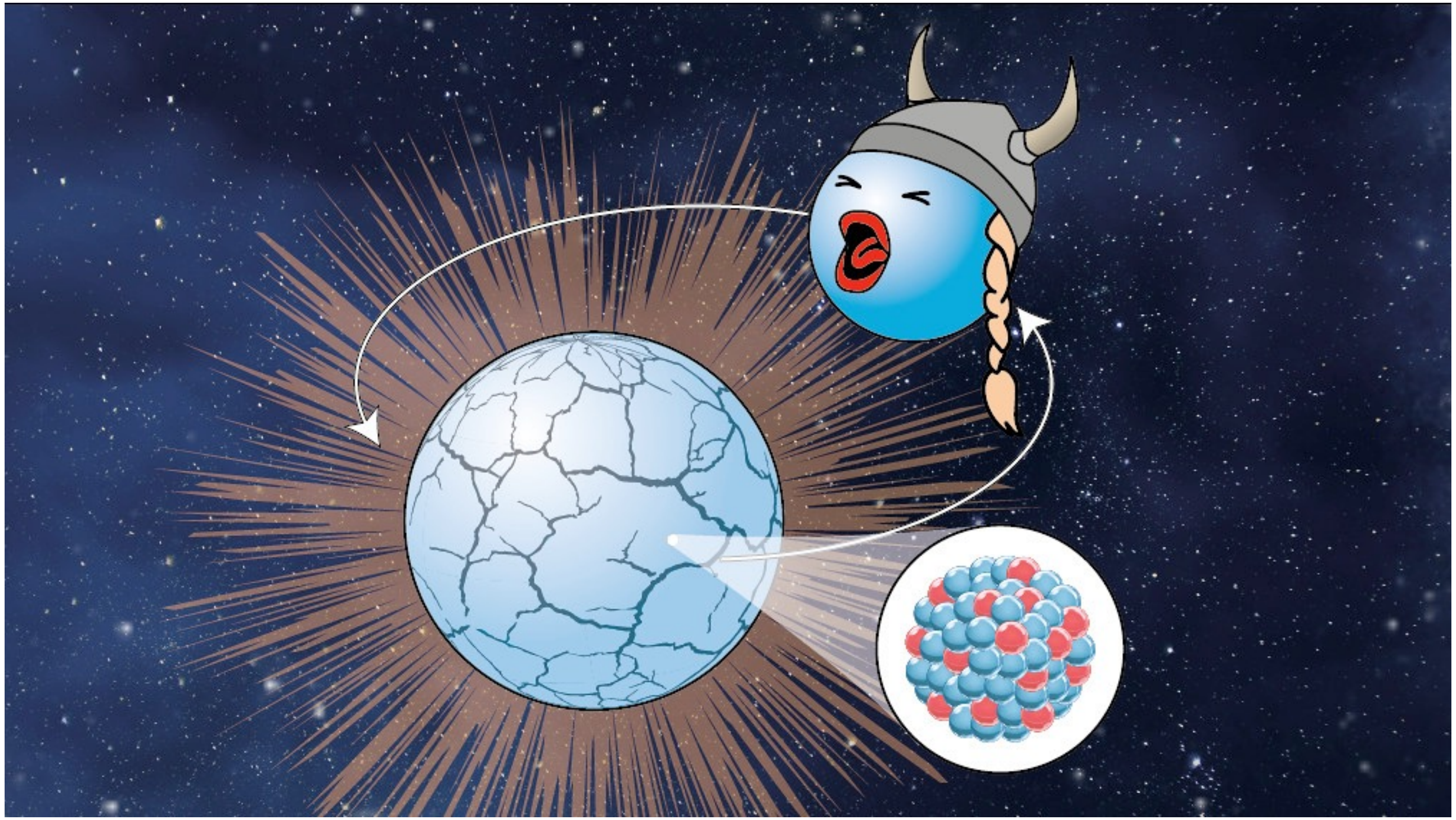
$R_{\rm pasta}/R_{\rm crust}$	0.128 ± 0.047
$I_{\rm pasta}/I_{\rm crust}$	0.480 ± 0.137
$m_{\rm pasta}/m_{\rm crust}$	0.485 ± 0.138

Dinh Thi+ arxiv: 2109.13638



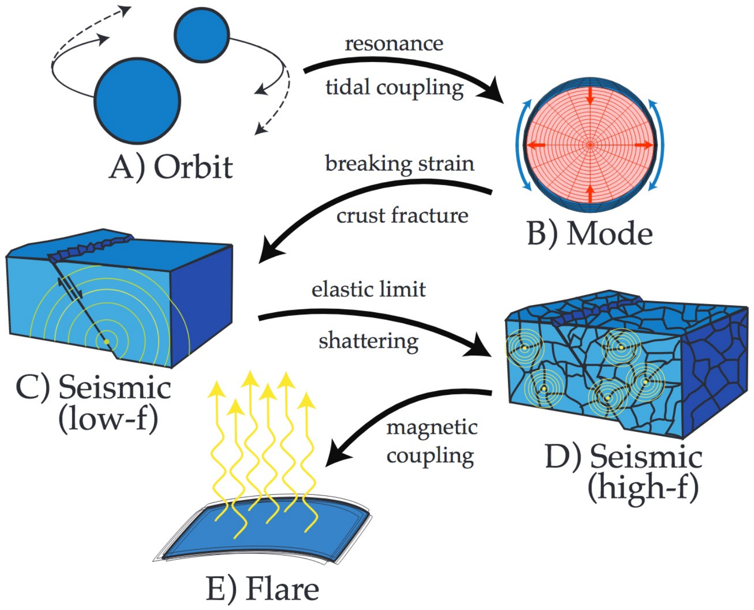
Application: resonant crust-shattering flares

Neill, Newton & Tsang, MNRAS 504, 2021
Neill,Preston,Newton,Tsang, PRL130, 2022
Neill, Tsang, Newton, MNRAS 532, 1, 827



Picture: David Tsang

The elastic crust can be made to resonantly vibrate by the tidal field of its companion – something we can potentially measure!



D.Tsang, Apj 777, 2013
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Neill,Preston,Newton,Tsang, PRL130, 2022

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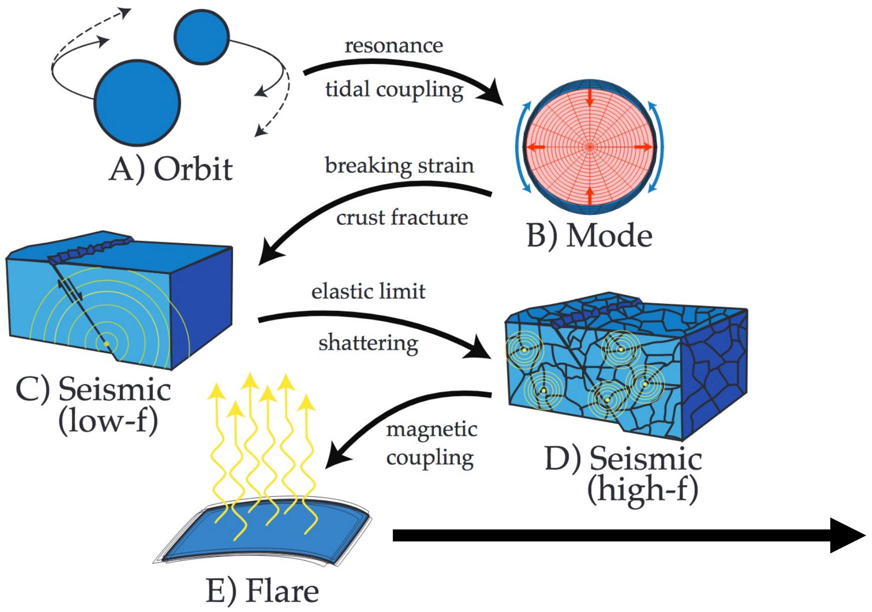

D.Tsang, Apj 777, 2013

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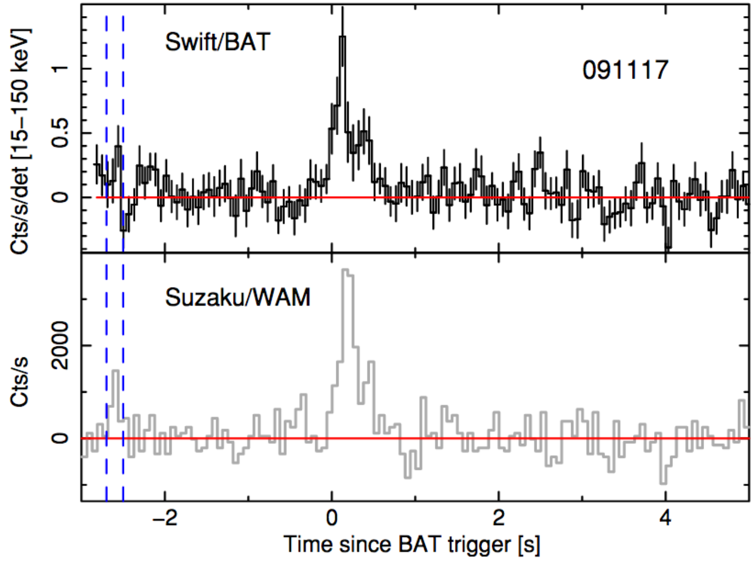
Neill,Preston,Newton,Tsang, PRL130, 2022



Troja, Rosswog, Gehrels, ApJ723, 2010

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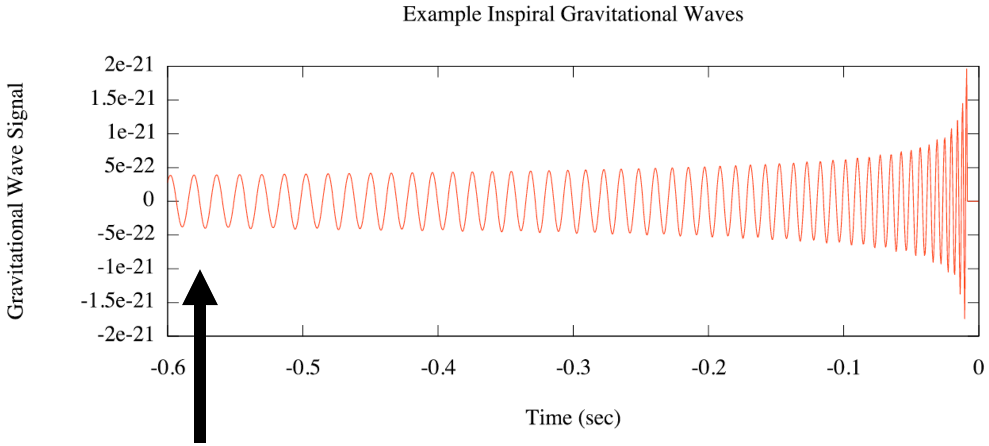

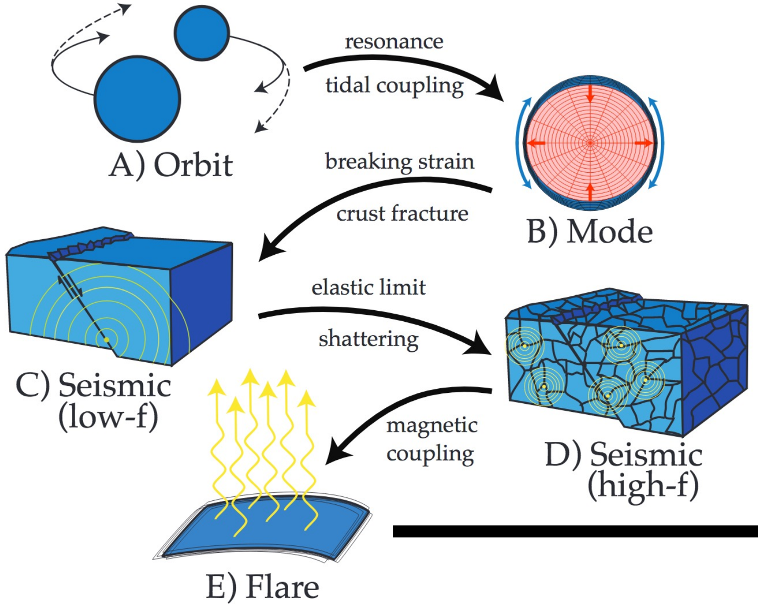

D.Tsang, Apj 777, 2013
Neill, Newton & Tsang, MNRAS 504, 2021
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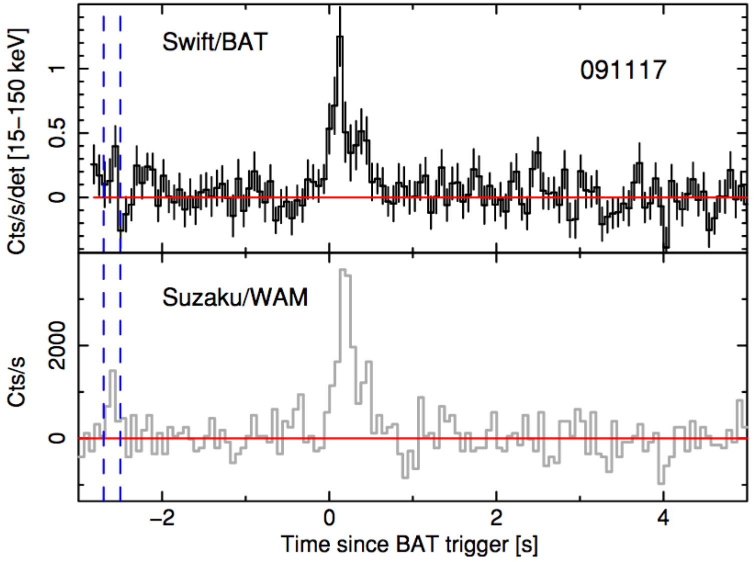

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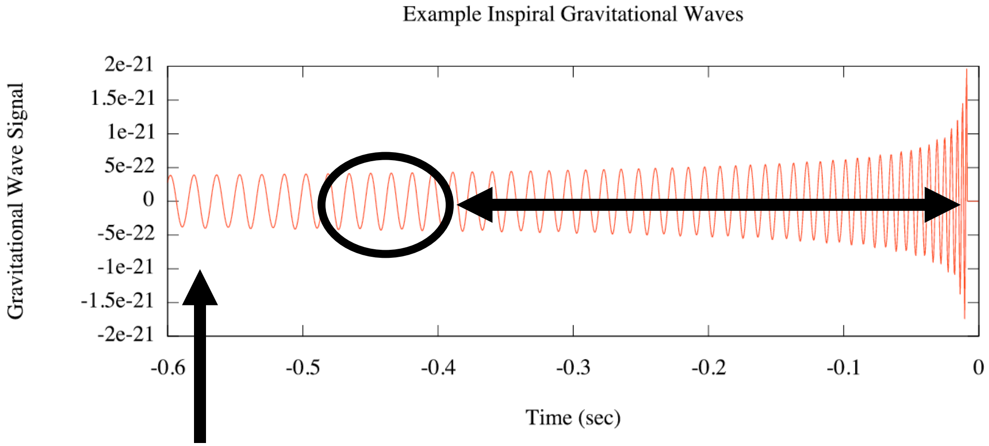

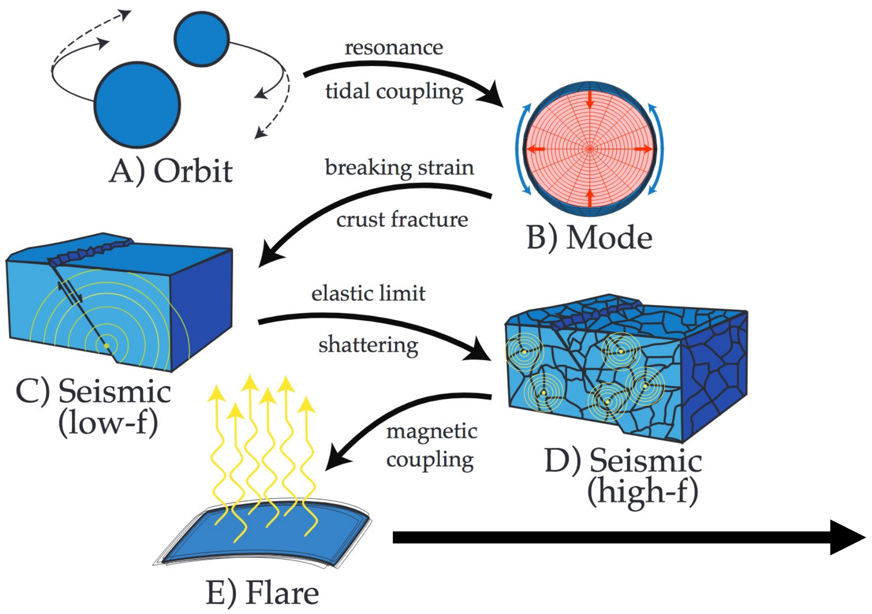

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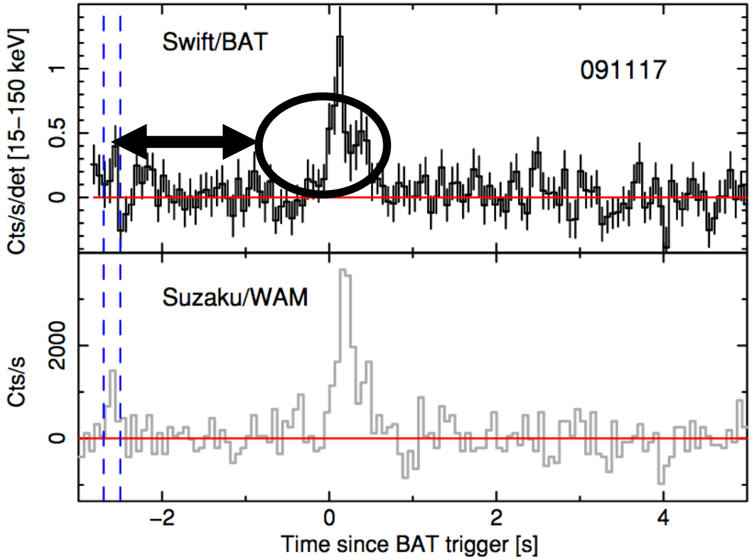
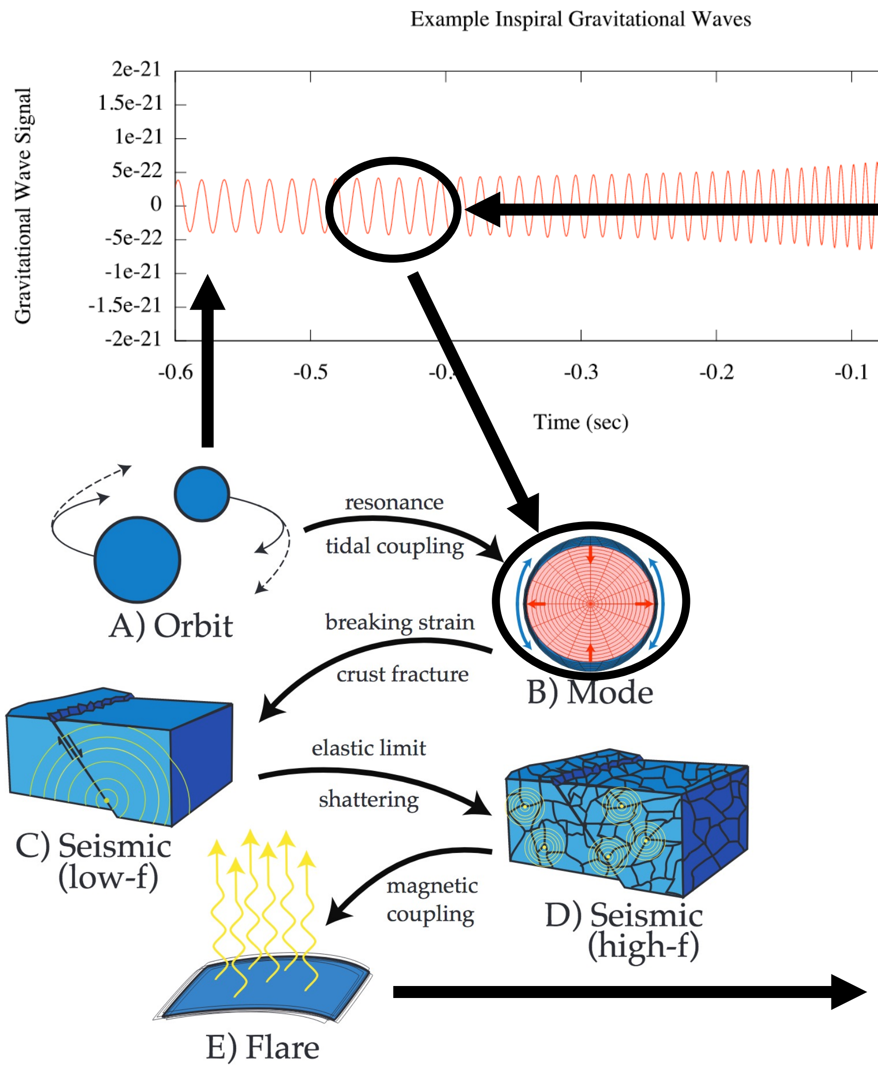


Troja, Rosswog, Gehrels, ApJ723, 2010

The elastic crust can be made to resonantly vibrate by the tidal field of its companion – something we can potentially measure!



D.Tsang, Apj 777, 2013
Neill, Newton & Tsang, MNRAS 504, 2021
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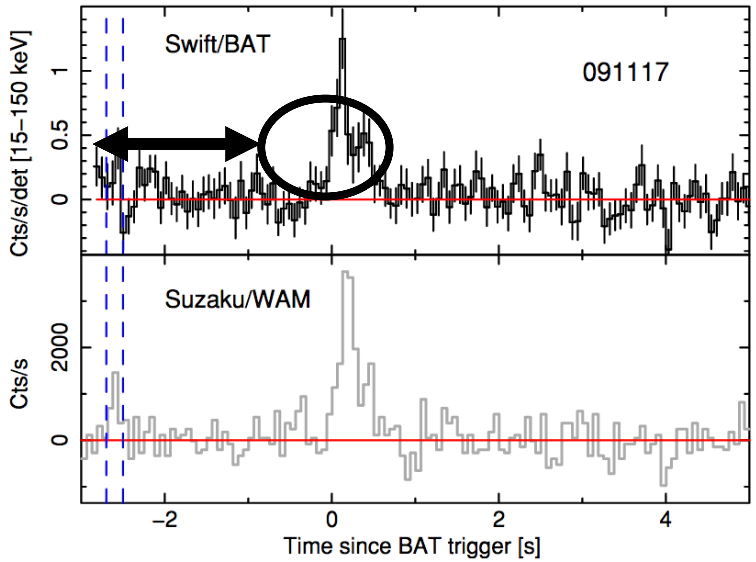
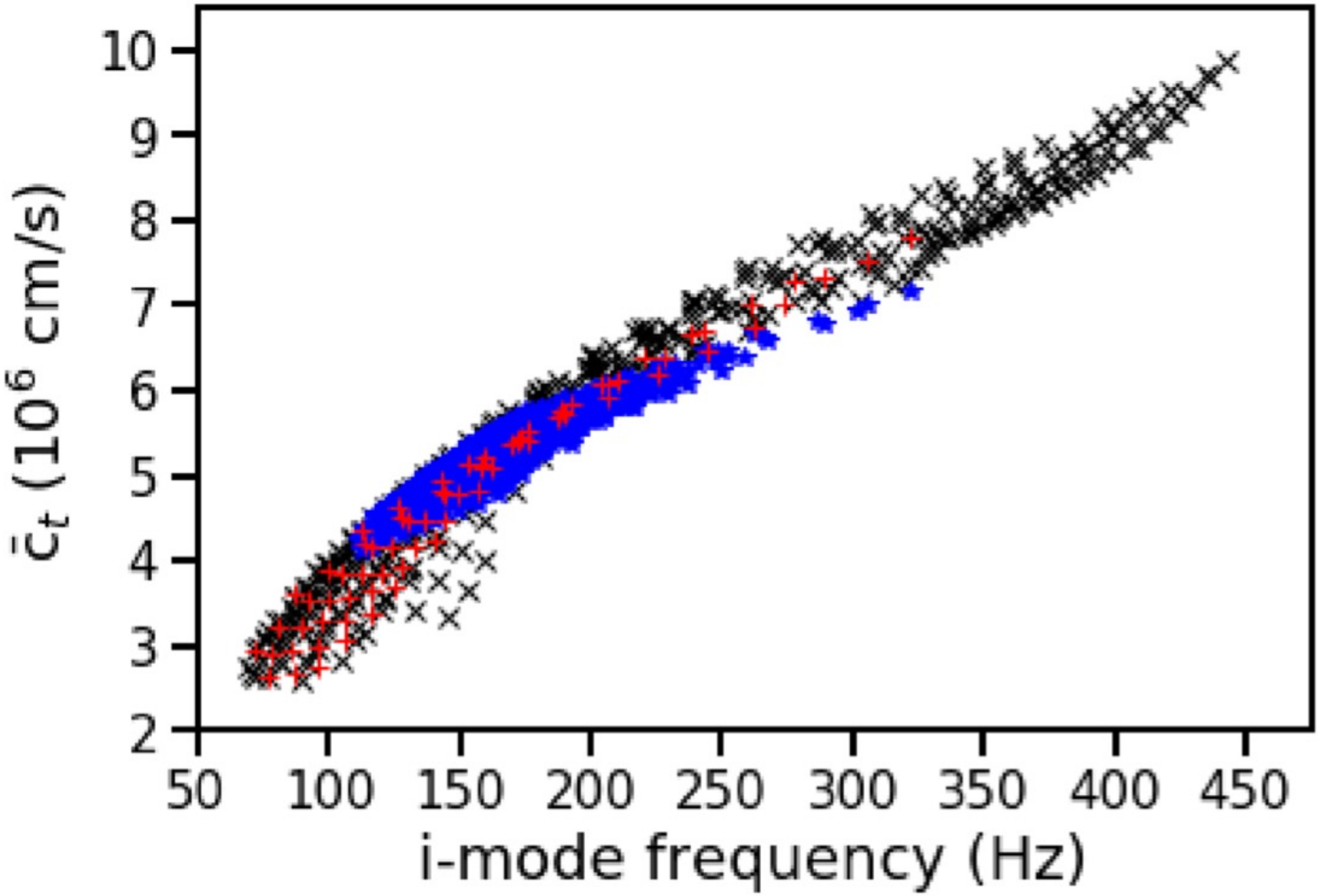
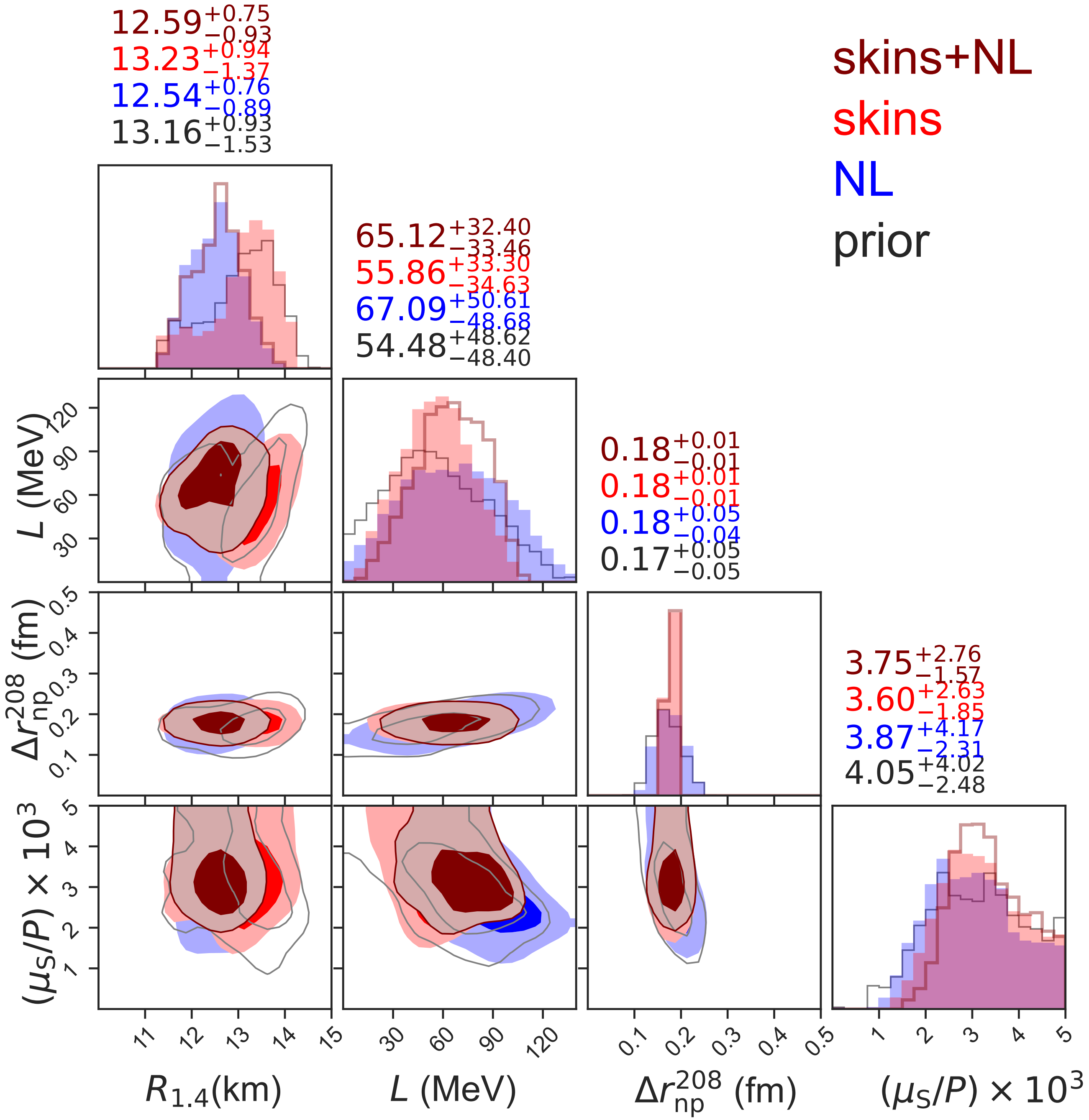


Troja, Rosswog, Gehrels, ApJ723, 2010

Strong correlation between shear speed and *i*-mode frequency



Neill, Newton & Tsang, MNRAS 504, 2021

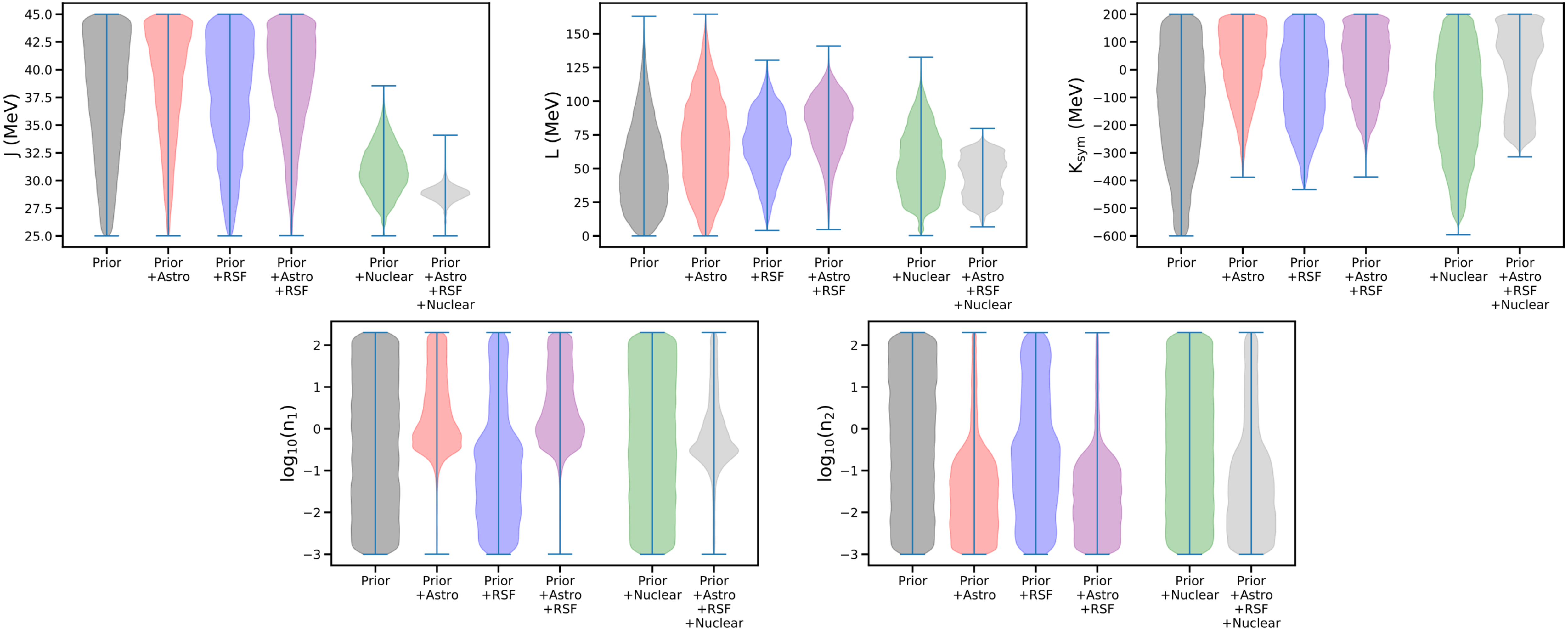


Newton+ in prep

We can also obtain the posteriors on the shear modulus at the base of crust (with and without pasta)

(using form of Strohmayer+ 1991)

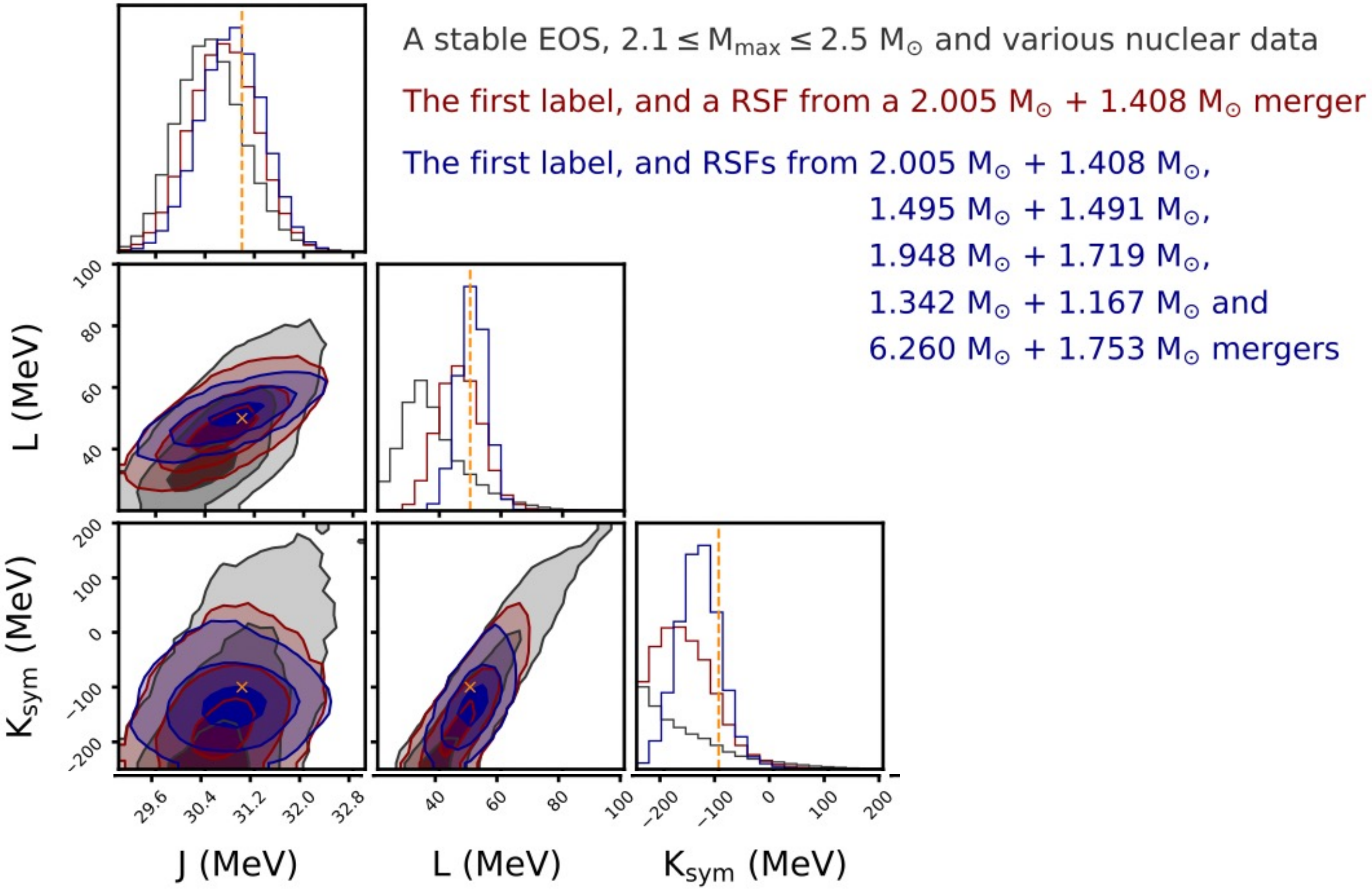
Inference using a synthetic detection of an RSF at a frequency of 250 Hz, comparison with Nicer-Ligo and nuclear binding energy data



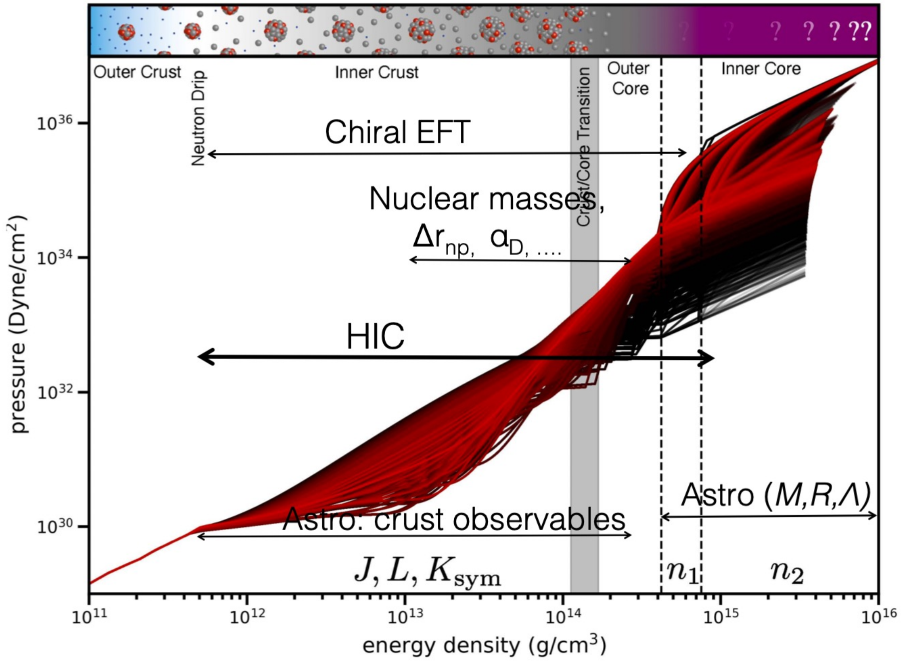
- *J* not constrained by astro
- *L* constrained by nuclear, RSF
- K_{sym} constrained by RSF/NL
- Polytrope parameters constrained by NL

Take-aways

- Many paths to enlightenment, presented here is perhaps one
- Microscopic model, DFT. Uninformative priors on its parameters. Add in empirical information through statistical modeling of data
- Allows us to step closer to the quantities we actually measure
- Don’t forget the crust! Many observables are sensitive to the layers of the neutron star around the crust-core transition, *exactly where much of our experimental measurements probe*
- Crust physics is messy, requiring modeling at several different scales, so quantifying uncertainty in the EOS is important
- But we have a lot of data that probes that physics (glitches, crust cooling,…)
- L, K_{sym} are sensitive to astrophysical observables, even when we decouple higher densities.


A stable EOS, 2.1 ≤ M_{max} ≤ 2.5 $M_\odot$ and various nuclear data
The first label, and a RSF from a 2.005 $M_\odot$ + 1.408 $M_\odot$ merger
The first label, and RSFs from 2.005 $M_\odot$ + 1.408 $M_\odot$,
1.495 $M_\odot$ + 1.491 $M_\odot$,
1.948 $M_\odot$ + 1.719 $M_\odot$,
1.342 $M_\odot$ + 1.167 $M_\odot$ and
6.260 $M_\odot$ + 1.753 $M_\odot$ mergers
L (MeV)
100
80
60
40
K_{sym} (MeV)
200
100
0
−100
−200
29.6
30.4
31.2
32.0
32.8
J (MeV)
40
60
80
100
L (MeV)
−200
−100
0
100
200
K_{sym} (MeV)

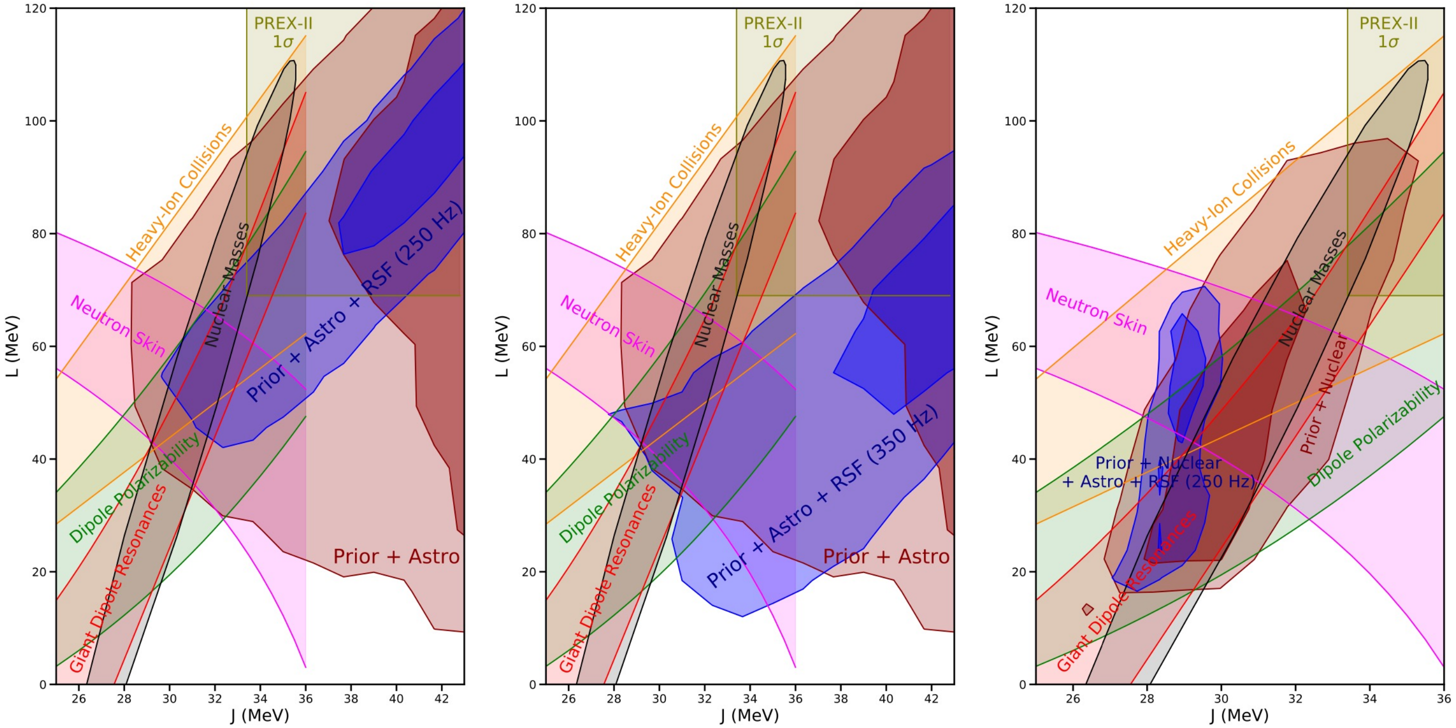
Take-aways



Many observables bear the signature of crust physics

There is already a lot of data related to Crust observables

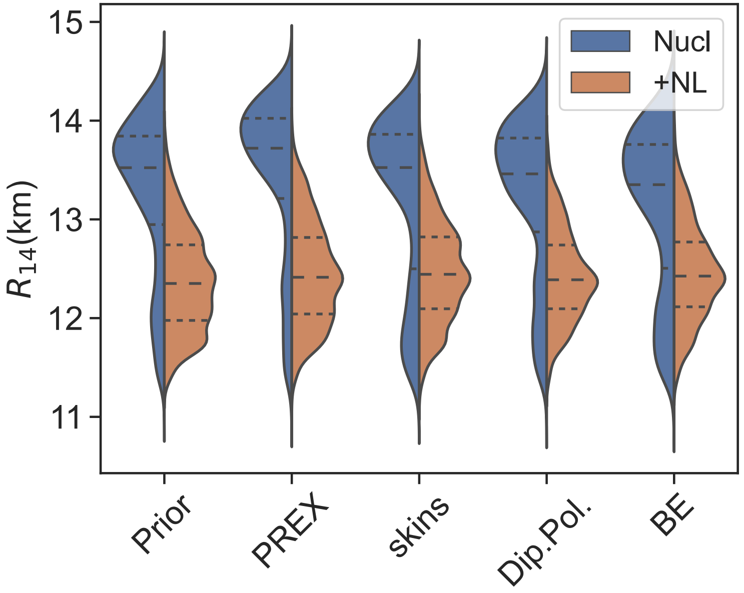
TO use this data to uncover crust physics, we need to include crust models in our statistical inference pipelines



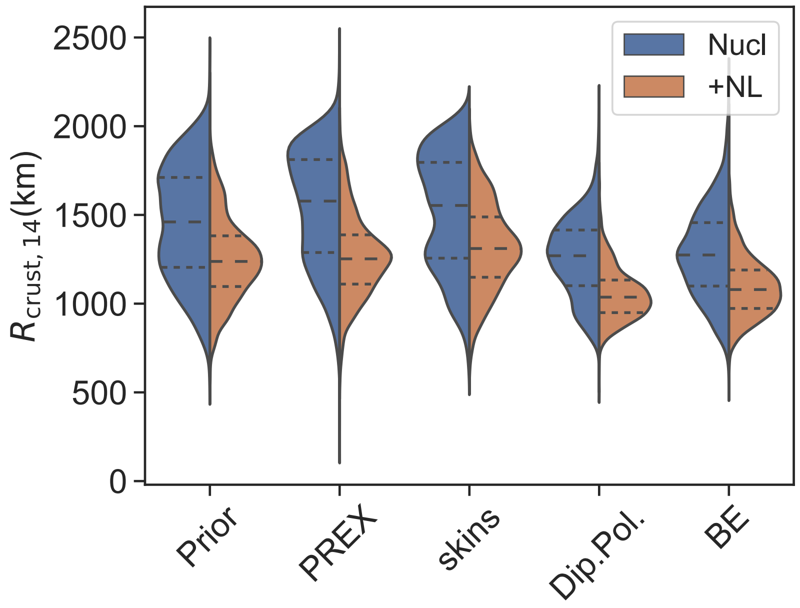
It is possible to construct neutron star and nuclear models with a single central nuclear model (we do

Measuring crust physics constrains Astro (radius, etc) and nuclear (symmetry energy)
- example: RSF

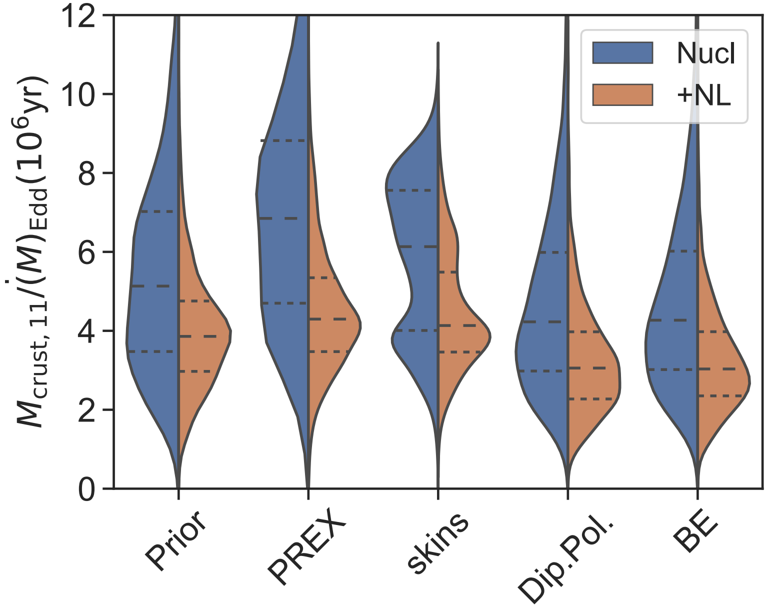
Both nuclear data and astrophysical radius/tidal deformability measurements inform crust



Nuclear: Almost nothing to say
Everything to say



Nuclear: A lot to say
Astro: A lot to say

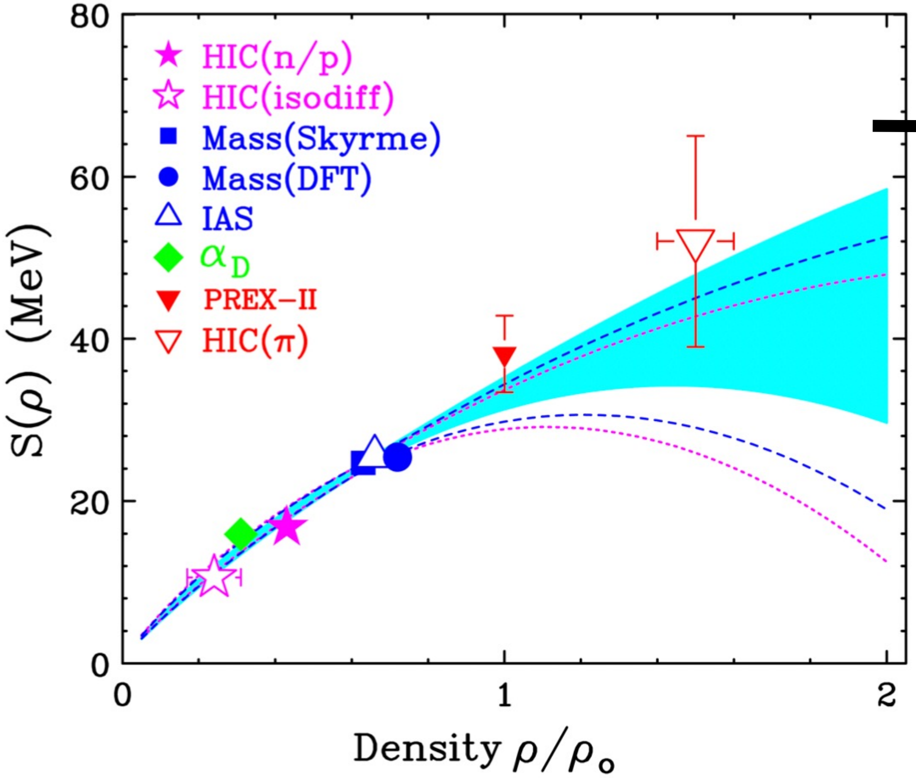


Nuclear: A lot to say
Astro: A lot to say

Neutron skins prefer thicker, more massive crusts than dipole polarizability, BE
Astro data: Prefers significantly thinner, less massive crusts

Different observables constrain at different densities…

… so resulting constraints on nuclear matter parameters at saturation density involve model-dependent extrapolation

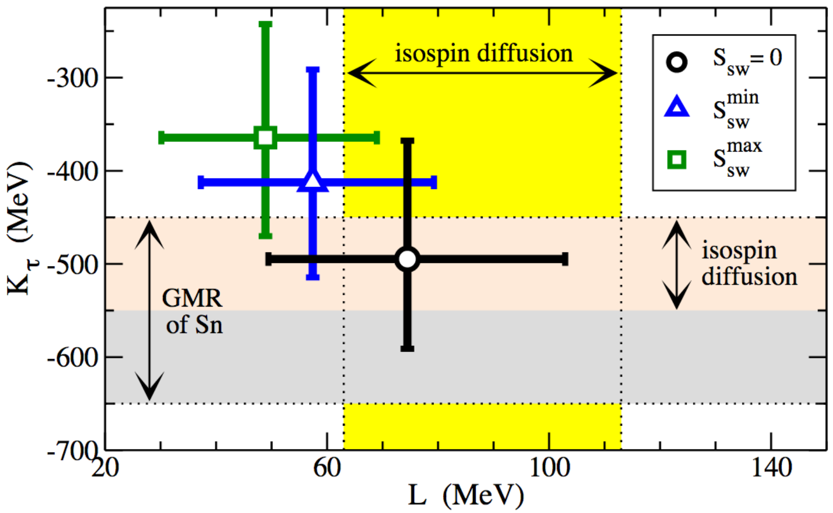


Tsang and Lynch, arxiv:2106.10119

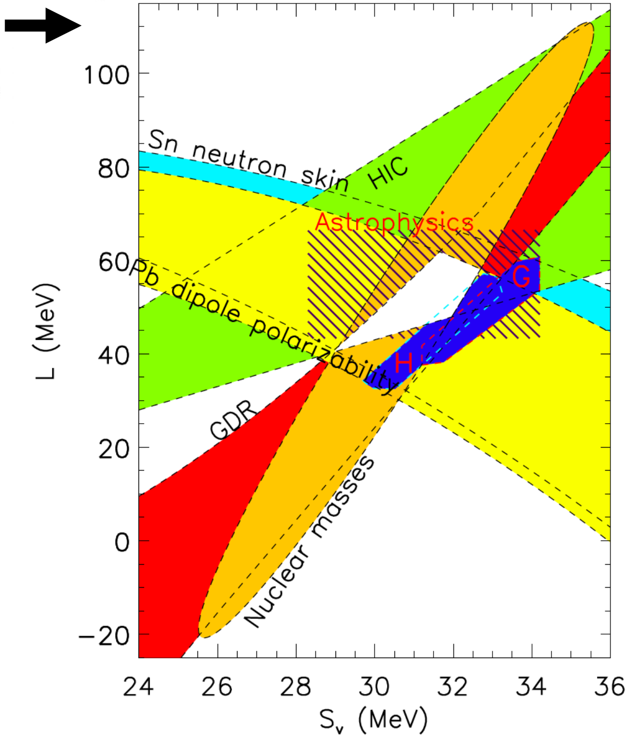
$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3$$

$$S_{\rm RMF}(\rho) = A(\rho)\rho^{2/3} + B(\rho)\rho \ ,$$

$$S_{\rm SHF}(\rho) = a\rho^{2/3} - b\rho - c\rho^{5/3} - d\rho^{\sigma+1}$$



Centelles et al, arxiv:0806.2886



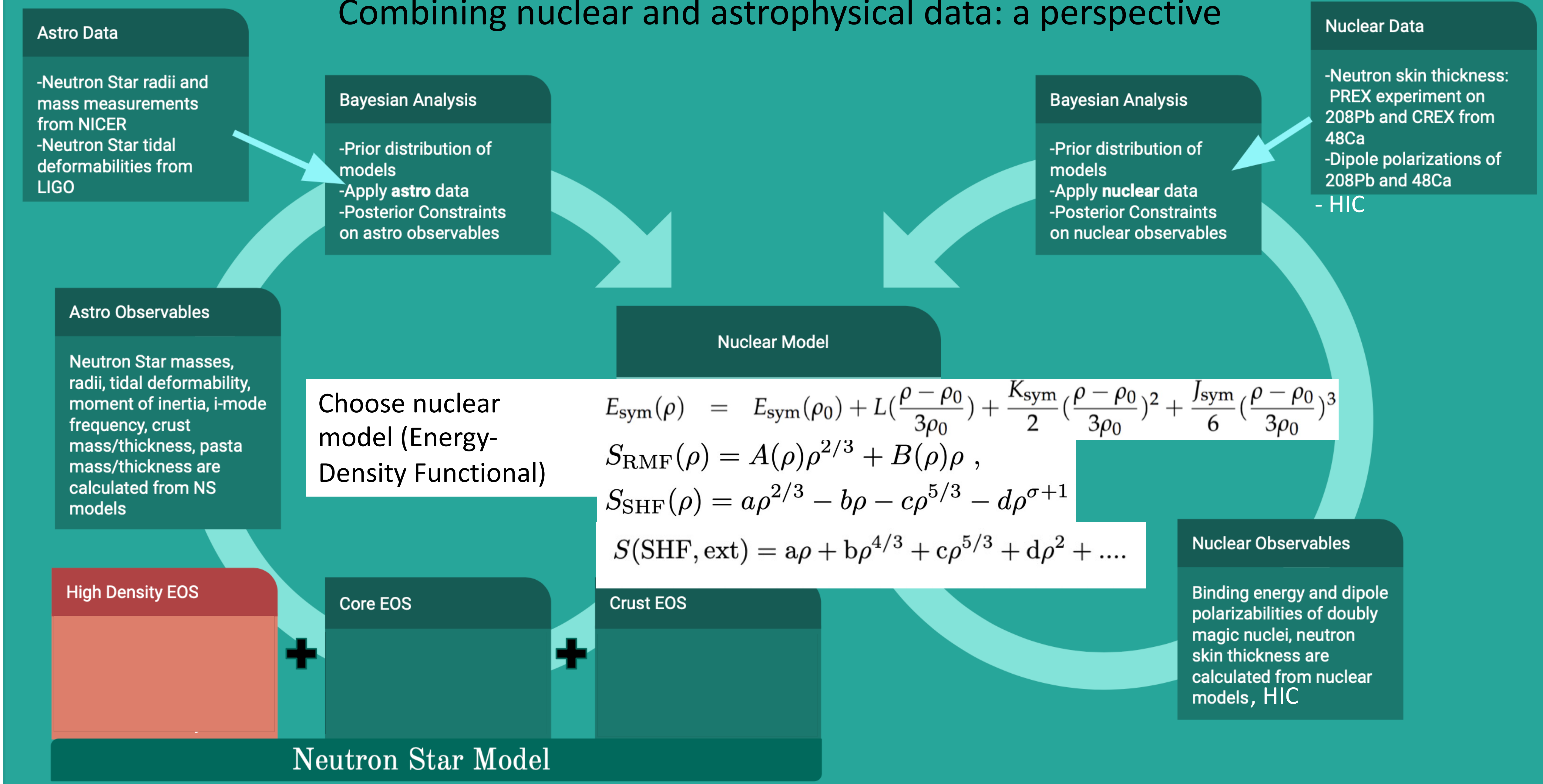
Lattimer, Lim ApJ771(2013)
Lattimer, Steiner EPJA50 (2013)

$$\begin{aligned}
\rho\omega^2 U = {} & \rho\frac{\mathrm{d}\hat{\chi}}{\mathrm{d}r} - A\Gamma_1 p\hat{\alpha} - \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{3}\mu\hat{\alpha}\right) + \frac{\mathrm{d}\mu}{\mathrm{d}r}\left(\hat{\alpha} - 2\frac{\mathrm{d}U}{\mathrm{d}r}\right) \\
& - \mu\left(\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}U}{\mathrm{d}r}\right) - \frac{\ell(\ell+1)}{r^2}U \right. \\
& \left. + \frac{2\ell(\ell+1)}{r^2}V - \frac{2}{r^2}U\right), \\
\rho\omega^2 V = {} & \rho\frac{\hat{\chi}}{r} - \frac{1}{3}\frac{\mu\hat{\alpha}}{r} - \frac{\mathrm{d}\mu}{\mathrm{d}r}\left(\frac{\mathrm{d}V}{\mathrm{d}r} - \frac{V}{r} + \frac{U}{r}\right) \\
& - \mu\left(\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}V}{\mathrm{d}r}\right) - \frac{\ell(\ell+1)}{r^2}V + \frac{2}{r^2}U\right),
\end{aligned}$$

where:[2]

$$\begin{aligned}
\hat{\alpha} &= \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 U) - \frac{\ell(\ell+1)}{r}V, \\
\hat{\chi} &= -\frac{\Gamma_1 p}{\rho}\hat{\alpha} - \frac{1}{\rho}\frac{\partial p}{\partial r}U.
\end{aligned}$$

Combining nuclear and astrophysical data: a perspective



Astro Data

-Neutron Star radii and mass measurements from NICER
-Neutron Star tidal deformabilities from LIGO

Bayesian Analysis

-Prior distribution of models
-Apply **astro** data
-Posterior Constraints on astro observables

Nuclear Data

-Neutron skin thickness: PREX experiment on 208Pb and CREX from 48Ca
-Dipole polarizations of 208Pb and 48Ca
- HIC

Bayesian Analysis

-Prior distribution of models
-Apply **nuclear** data
-Posterior Constraints on nuclear observables

Astro Observables

Neutron Star masses, radii, tidal deformability, moment of inertia, i-mode frequency, crust mass/thickness, pasta mass/thickness are calculated from NS models

Nuclear Model

Choose nuclear model (Energy-Density Functional)

$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\mathrm{sym}}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\mathrm{sym}}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3$$

$$S_{\mathrm{RMF}}(\rho) = A(\rho)\rho^{2/3} + B(\rho)\rho\ ,$$

$$S_{\mathrm{SHF}}(\rho) = a\rho^{2/3} - b\rho - c\rho^{5/3} - d\rho^{\sigma+1}$$

$$S(\mathrm{SHF,ext}) = \mathrm{a}\rho + \mathrm{b}\rho^{4/3} + \mathrm{c}\rho^{5/3} + \mathrm{d}\rho^2 + ...$$

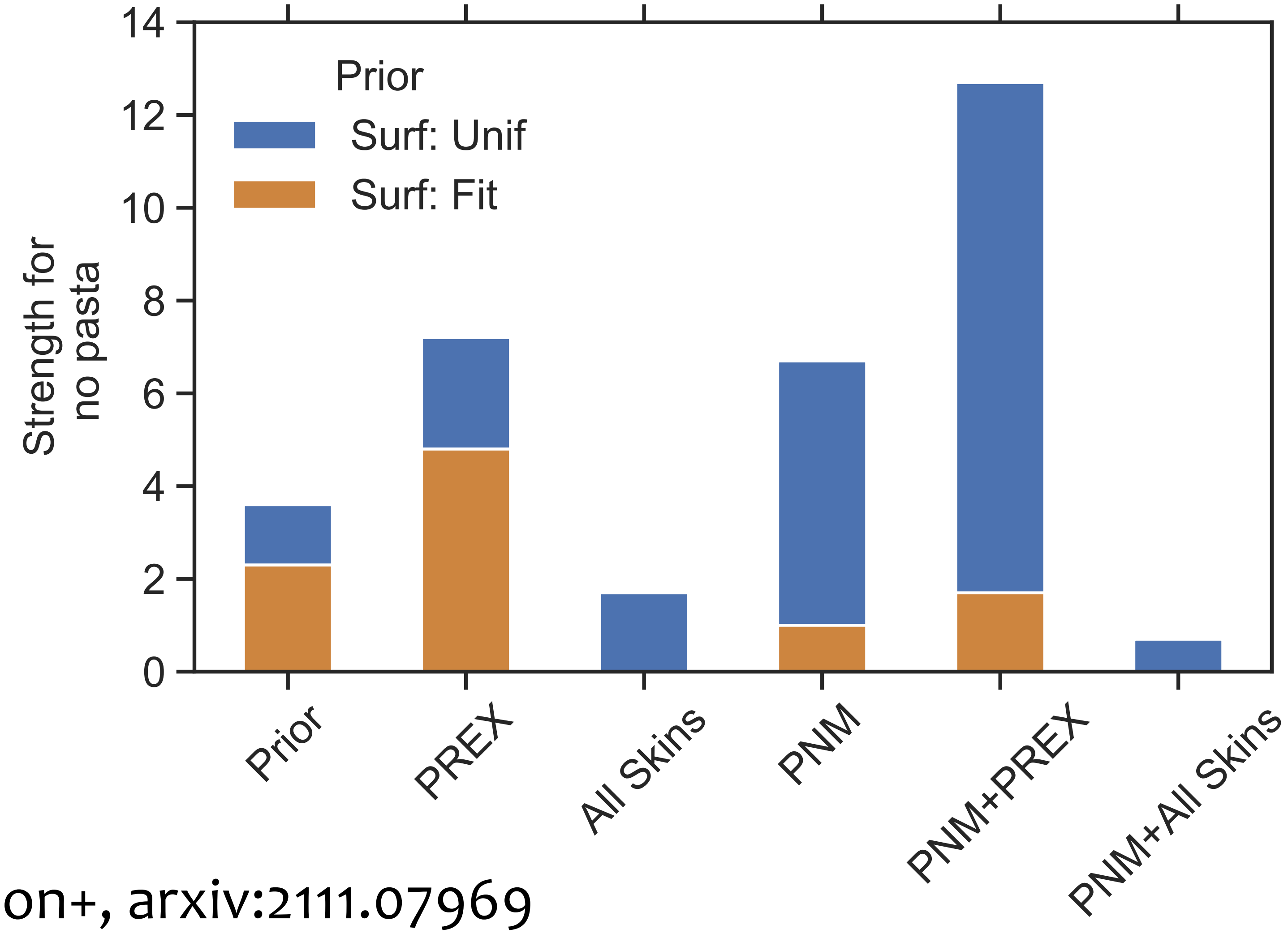
Nuclear Observables

Binding energy and dipole polarizabilities of doubly magic nuclei, neutron skin thickness are calculated from nuclear models, HIC

High Density EOS + **Core EOS** + **Crust EOS**

Neutron Star Model

There’s a non-negligible range of models that predicts no pasta



Newton+, arxiv:2111.07969
Balliet+, arxiv:2009.07696