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Universal properties of equations of state of dense nuclear matter and mass-radius curves of neutron stars

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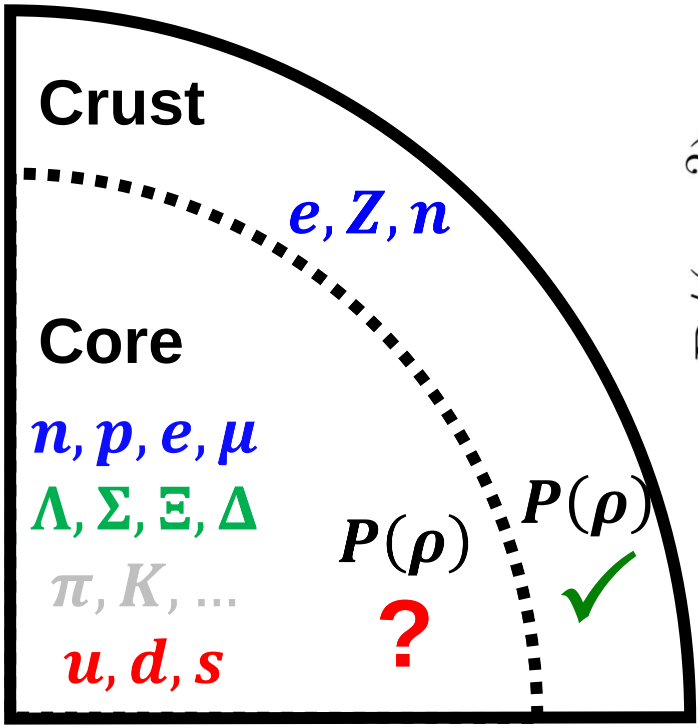
INT-N3AS workshop 24-89W

EOS Measurements in the Era of Next-Generation GW Detectors

2 September 2024

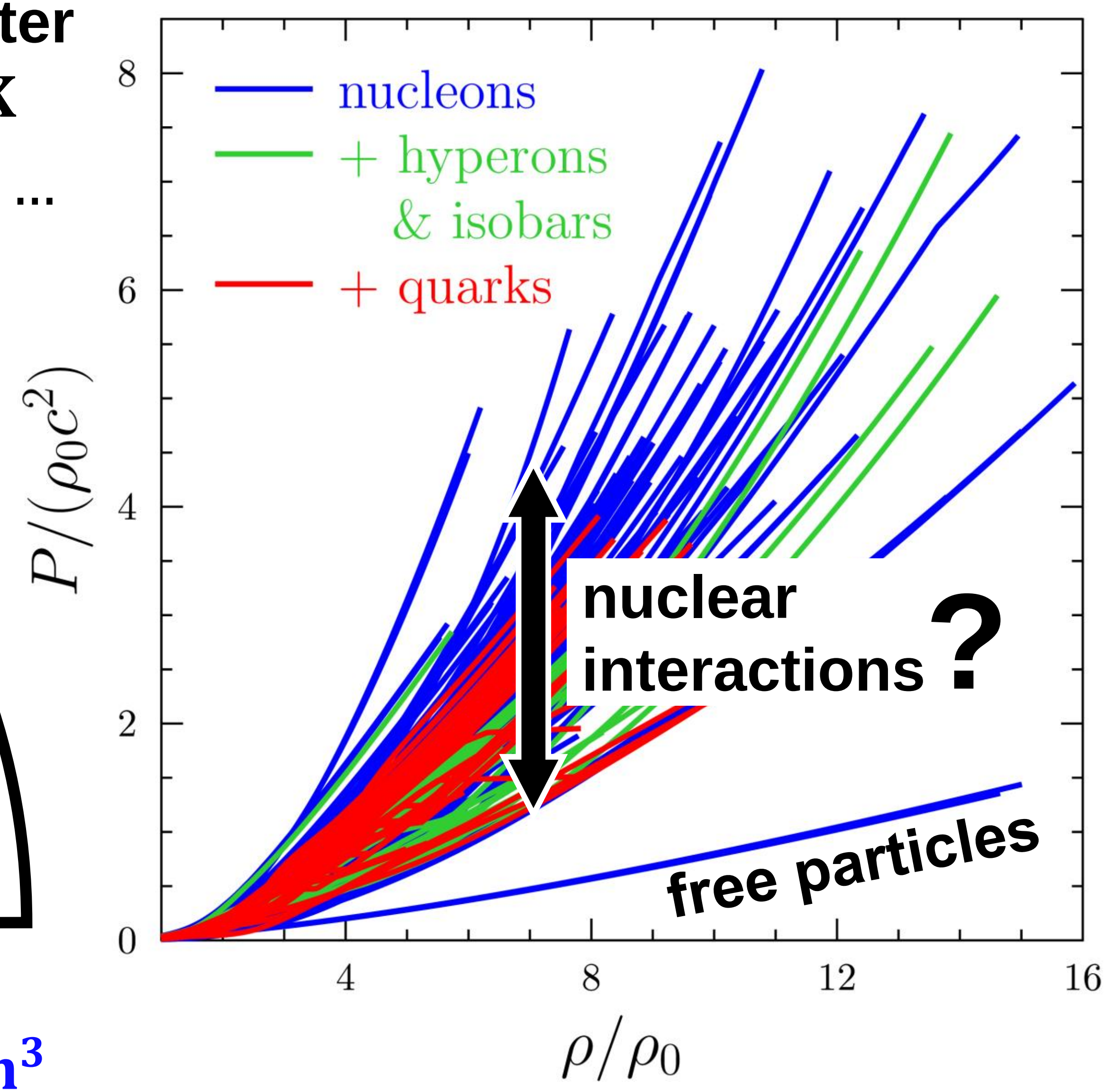
Introduction: NS EoS

- **Cold degenerate matter** $T < 10^{10}\text{K},\ T_F \sim 10^{12}\text{K}$
- $P(\rho), n(\rho), \text{compos}(\rho), \ldots$



$\sim 0.5\rho_0$

$\rho_0 = 2.8 \times 10^{14}\ \text{g/cm}^3$



Relation to Observables

- **Tolman-Oppenheimer-Volkoff (TOV) equations**
  - ***GR hydrostatic equilibrium***
  - ~~***Rotation***~~

Tolman (1939); Oppenheimer & Volkoff (1939)

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -\dfrac{Gm\rho}{r^2}\dfrac{\left(1+\dfrac{P}{\rho c^2}\right)\left(1+\dfrac{4\pi r^3 P}{mc^2}\right)}{1-\dfrac{2Gm}{rc^2}} \\ \dfrac{dm}{dr} = 4\pi r^2 \rho \end{cases}$$

+ $P(\rho)$ as a given function

Initial conditions

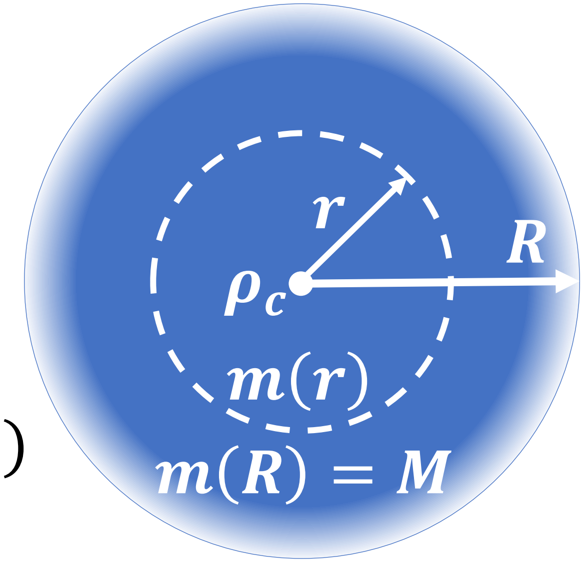
$$\rho(r=0) = \rho_c$$
$$m(r=0) = 0$$

NS surface

$$P \to 0$$

NS mass & radius

$$r\Big|_{P\to 0} = R(\rho_c)$$
$$m(r=R) = M(\rho_c)$$



Oppenheimer-Volkoff Mapping

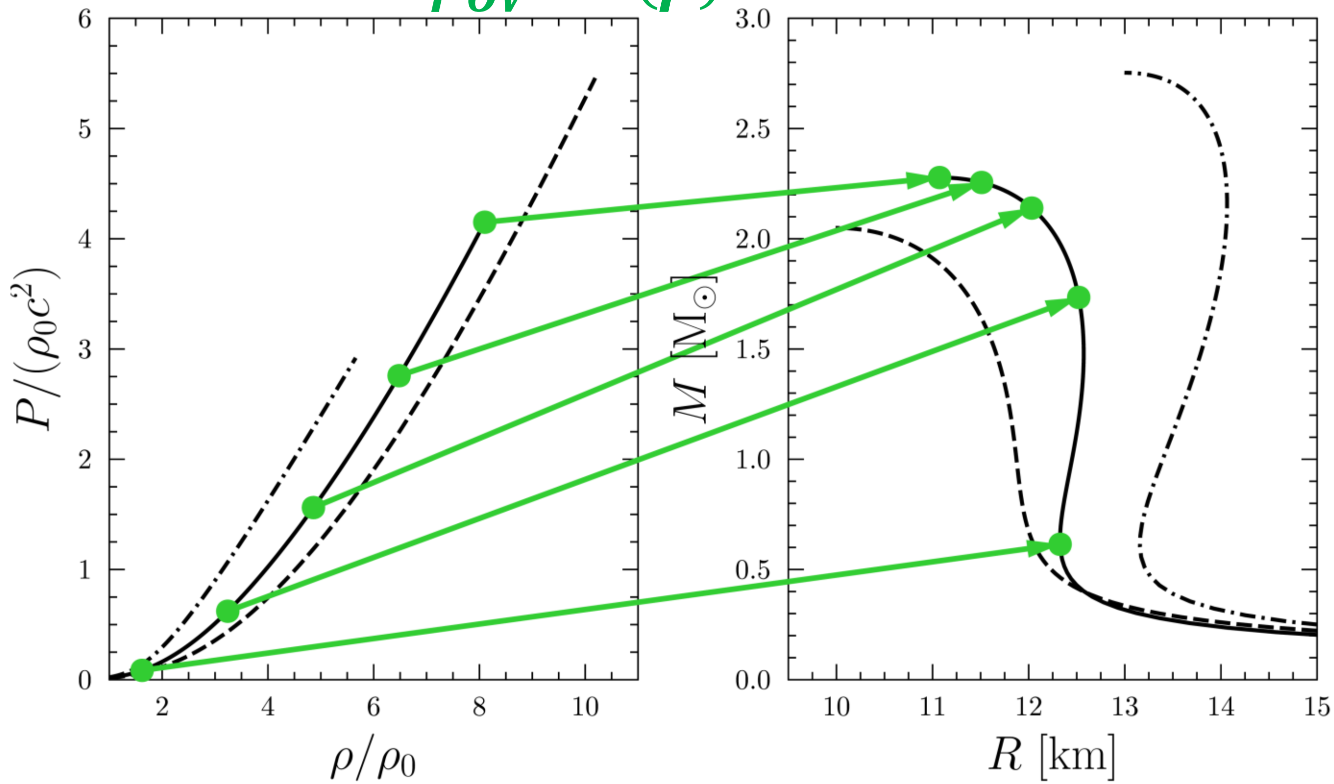
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\frac{\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)}{1-\frac{2Gm}{rc^2}}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

- ***GR hydrostatics***
- ***~~Rotation~~***

Tolman (1939); Oppenheimer & Volkoff (1939)

ψ_{OV}: $P(\rho) \mapsto M - R$



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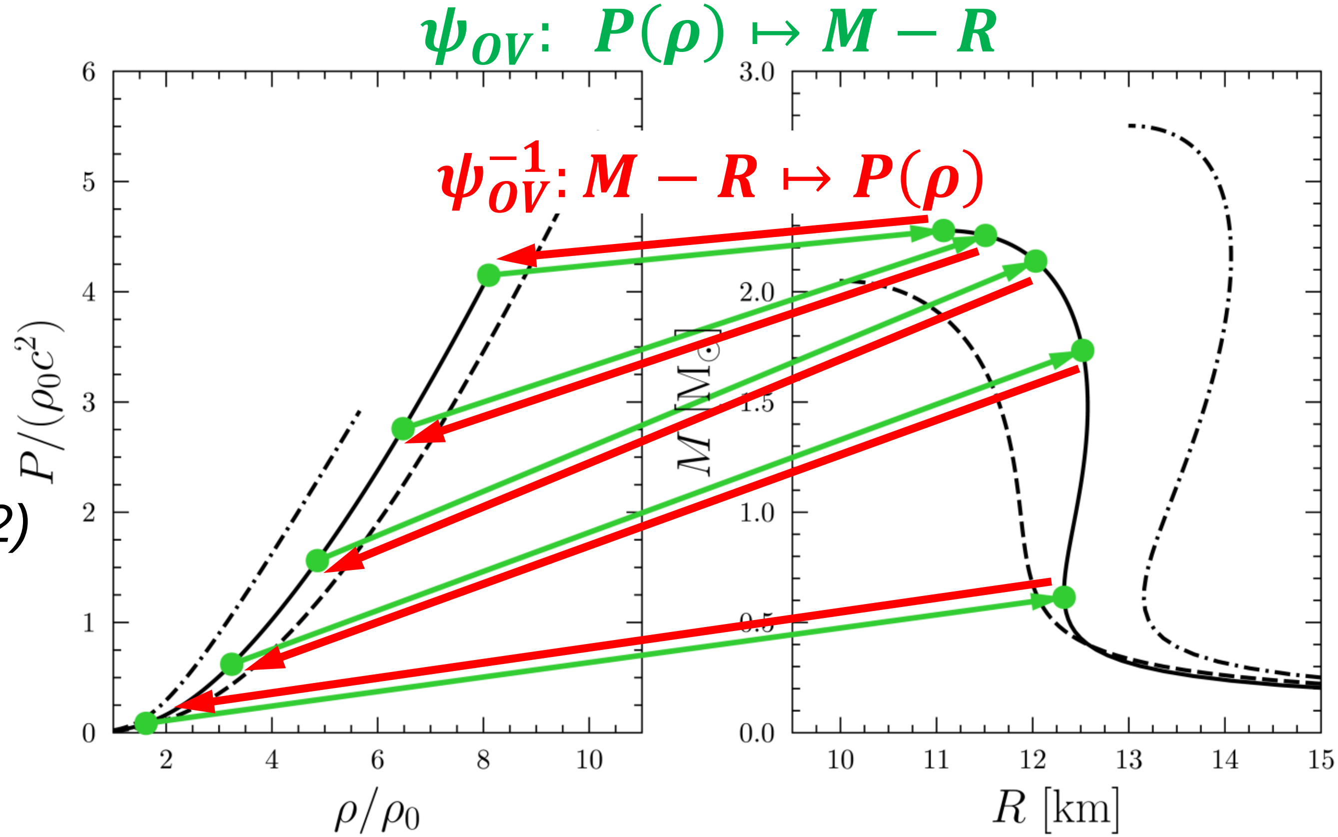
- ***GR hydrostatics***
- ~~***Rotation***~~

Tolman (1939); Oppenheimer & Volkoff (1939)

- **Lindblom (1992):** $\exists \psi_{OV}^{-1}$
  - numerics
  - neural networks *(Soma+ JCAP 2022)*



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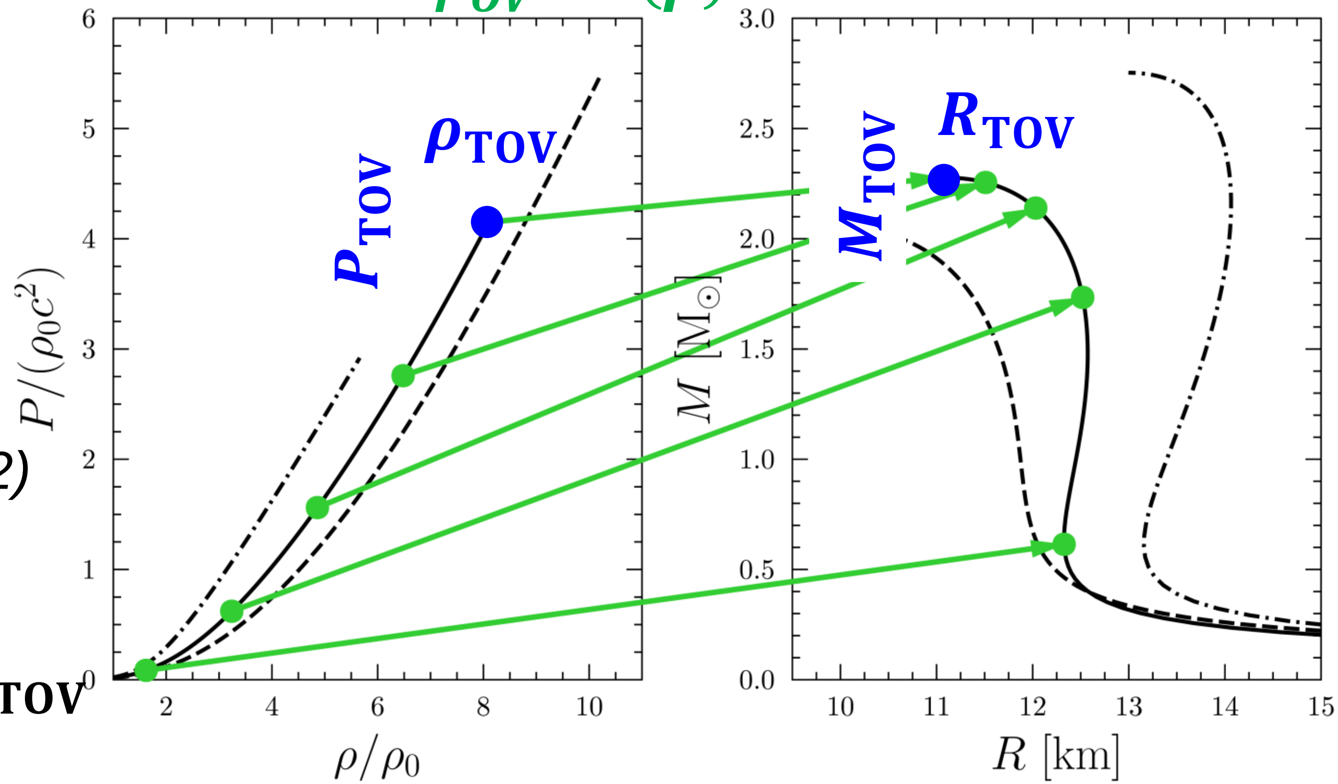
- ***GR hydrostatics***
- ~~***Rotation***~~

Tolman (1939); Oppenheimer & Volkoff (1939)

$\psi_{OV}\colon\ P(\rho) \mapsto M - R$

- **Lindblom (1992):** $\exists \psi_{OV}^{-1}$
  - numerics
  - neural networks *(Soma+ JCAP 2022)*
- **TOV limit** $\exists\, M_{\mathrm{TOV}} \leftrightarrow \rho_{\mathrm{TOV}}, P_{\mathrm{TOV}}$



Universalities of NSs

= EoS-independent relations

- **Binding energy** $\approx f(M, R)$ *Lattimer&Yahil'89; Lattimer&Prakash'01*
- **Oscillation frequences & damping times** $\approx f(M, R)$
  Andersson&Kokkotas'98; Manoharan&Kokkotas'24
- **I-Love-Q, I-Love-C, binary I-Love-Q** *Yagi&Younes'17*
- **Prompt collapse threshold for BNS merger**
  $(M_1 + M_2)_{\rm thres} \approx f(M_{\rm TOV}, R_{\rm TOV})$ *Bauswein+'17*
- $P(2n_0) \leftrightarrow R_{1.4}$ *Lattimer&Prakash'01*
- **Merger ringdown** $\approx f(M_{\rm TOV}, R_{\rm TOV}, P_{\rm TOV}, R_{1.4})$
  Ecker+'24; ***Tyler Gorda's talk***
- **…** ***(Jim Lattimer's talk)***

Common feature: ***based on TOV background***

This work

- **Novel universalities: $M-R$ & $P-\rho$**
  - ***3 key parameters*: *2 of TOV limit + $R(M_{\mathrm{TOV}}/2)$***

(cf. Tyler Gorda's talk)

- **Explicit (semi)analytic Inverse OV mapping**

(cf. Jim Lattimer's talk)

- **Novel method to constrain EoS from observations**

(DO, Shternin, & Piran arXiv:2404.17647)

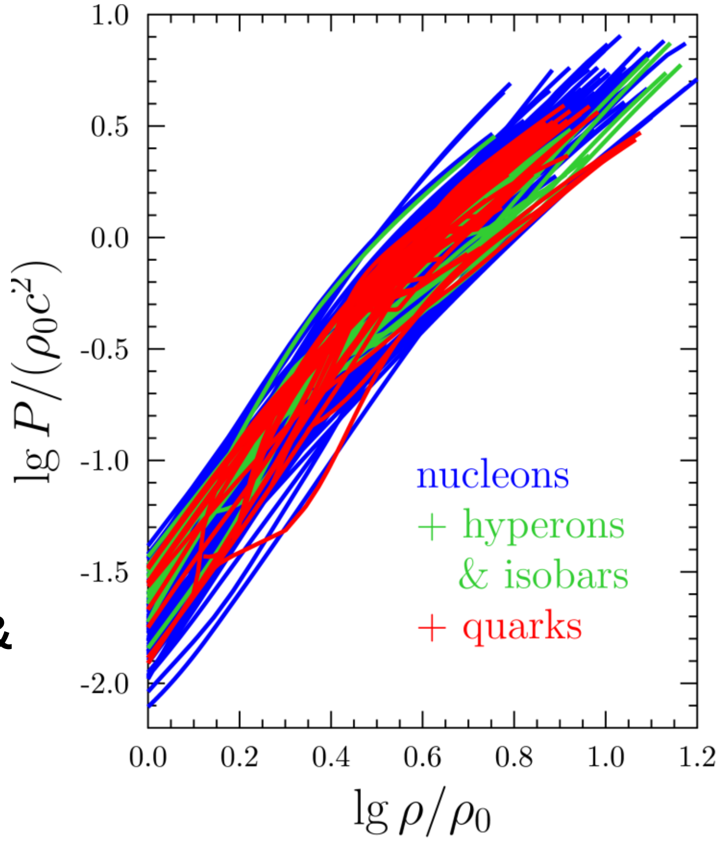
Further talk plan

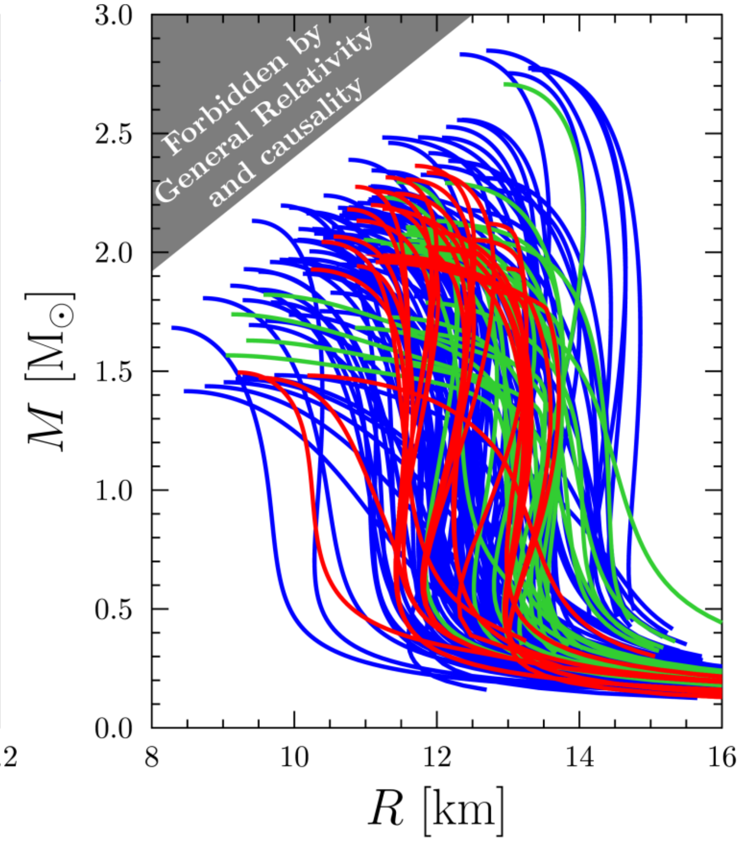
1. **Describe the EoS zoo**
2. **Show the universal fits**
3. **Build inverse OV mapping**
4. **Constrain EoS from observations via Inverse OV**

EoS Zoo (a representative sample)

169 models → **100 $npe\mu$** → **36 $+\Lambda\Sigma\Xi\Delta$** → **33 $+uds$**

- **CompOSE**
  https://compose.obspm.fr/
- **Read+2009**
  widely used for universality tests
- **Ozel & Freira 2016**
- **Gusakov, Kantor & Haensel 2014, Fortin+2017, Ofengeim+2019, Maslov+2016,…**





…phenomenological, variational, Skyrme, RMF, QMC, QHC,…

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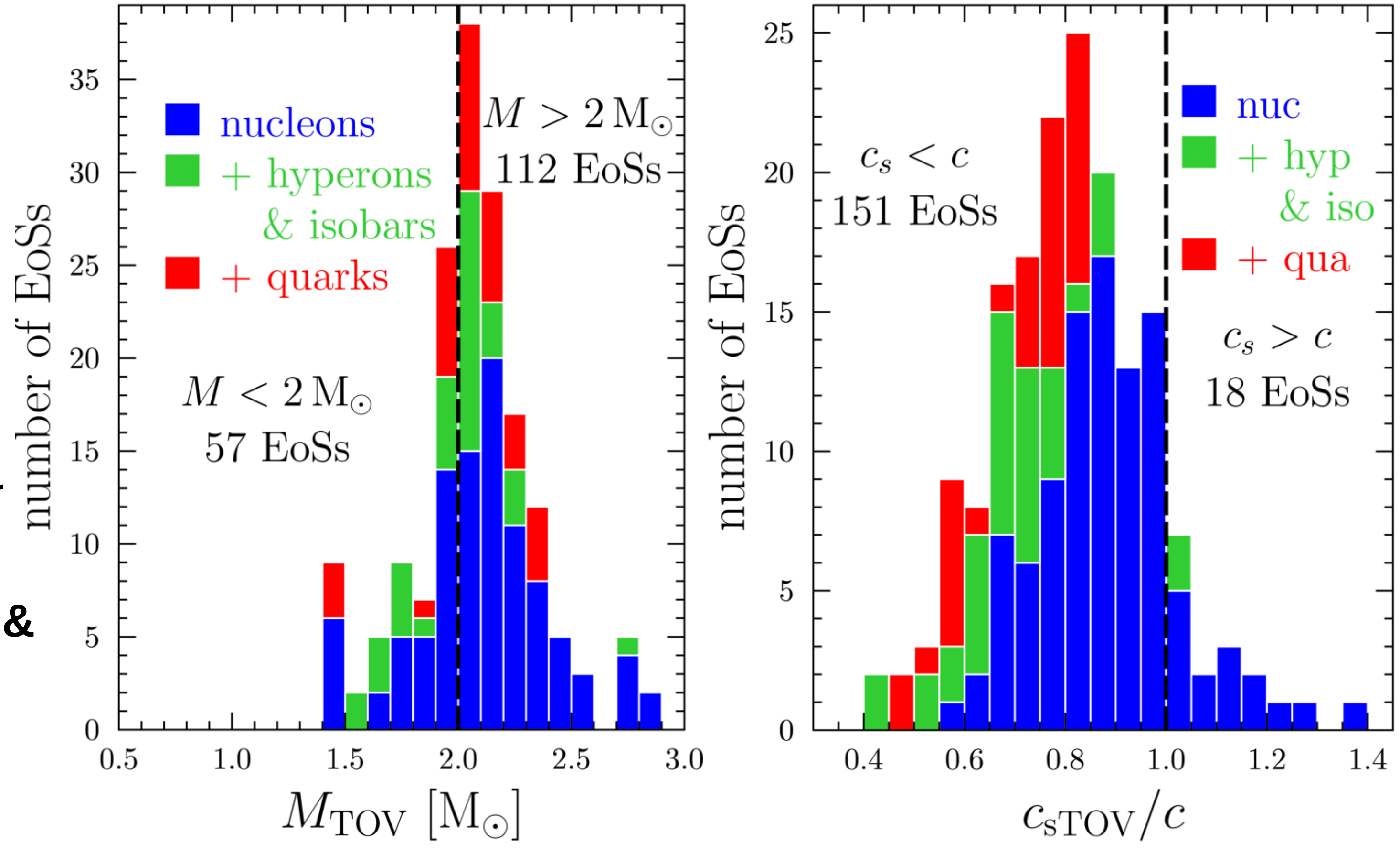


…phenomenological, variational, Skyrme, RMF, QMC, QHC,…

Universal Fit $R(M)$

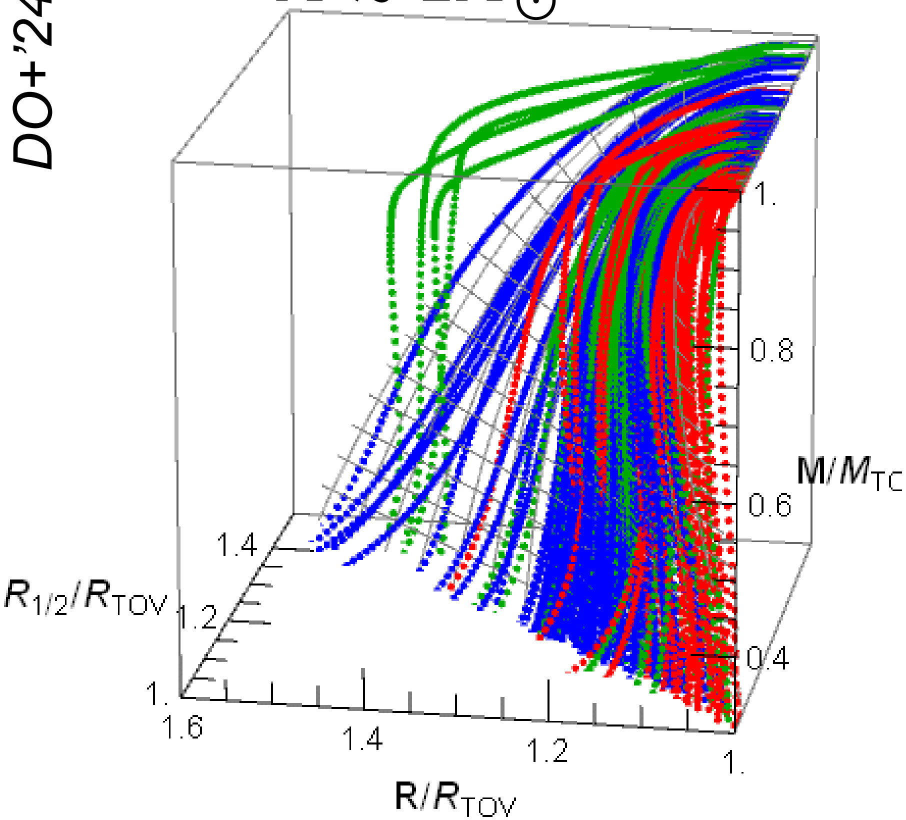
$$\frac{R}{R_{\rm TOV}} = 1 + \left[2\left(\sqrt{2}-1\right)\frac{R_{1/2}}{R_{\rm TOV}} - a\right]\sqrt{1-\frac{M}{M_{\rm TOV}}} + \left[2\left(\sqrt{2}-1\right)\frac{R_{1/2}}{R_{\rm TOV}} - 2 + a\sqrt{2}\right]\left(1-\frac{M}{M_{\rm TOV}}\right)$$

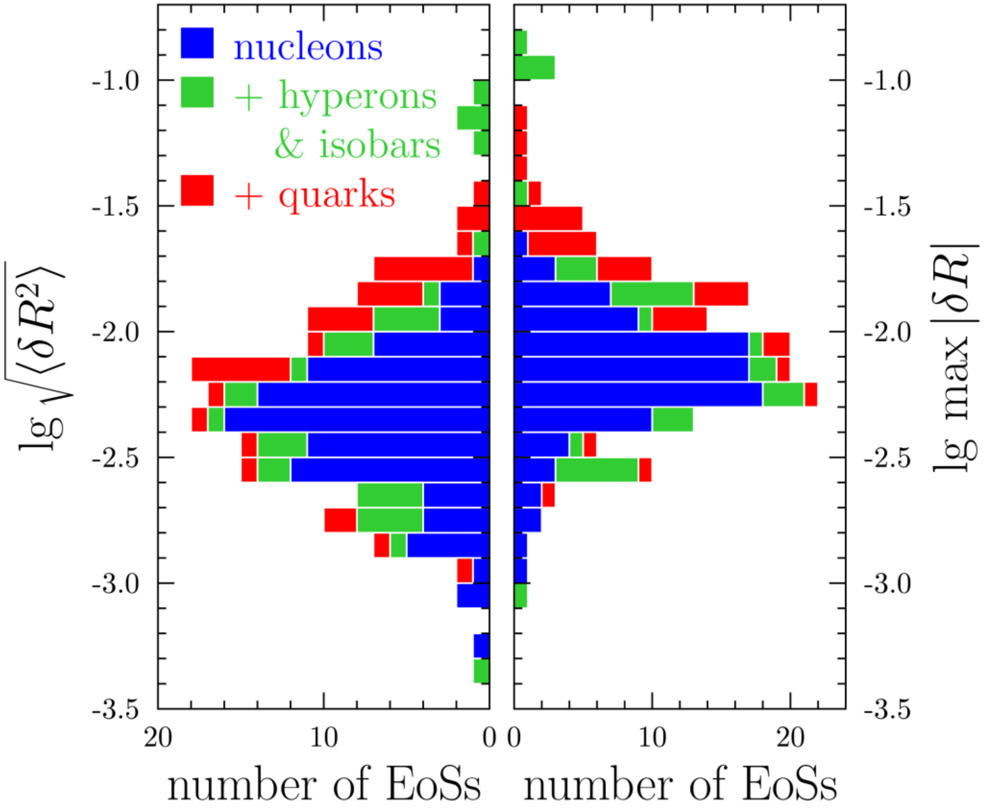
$a = 0.492$

$R_{1/2} = R(M_{\rm TOV}/2)$

$M \gtrsim 1 M_\odot$

DO+'24





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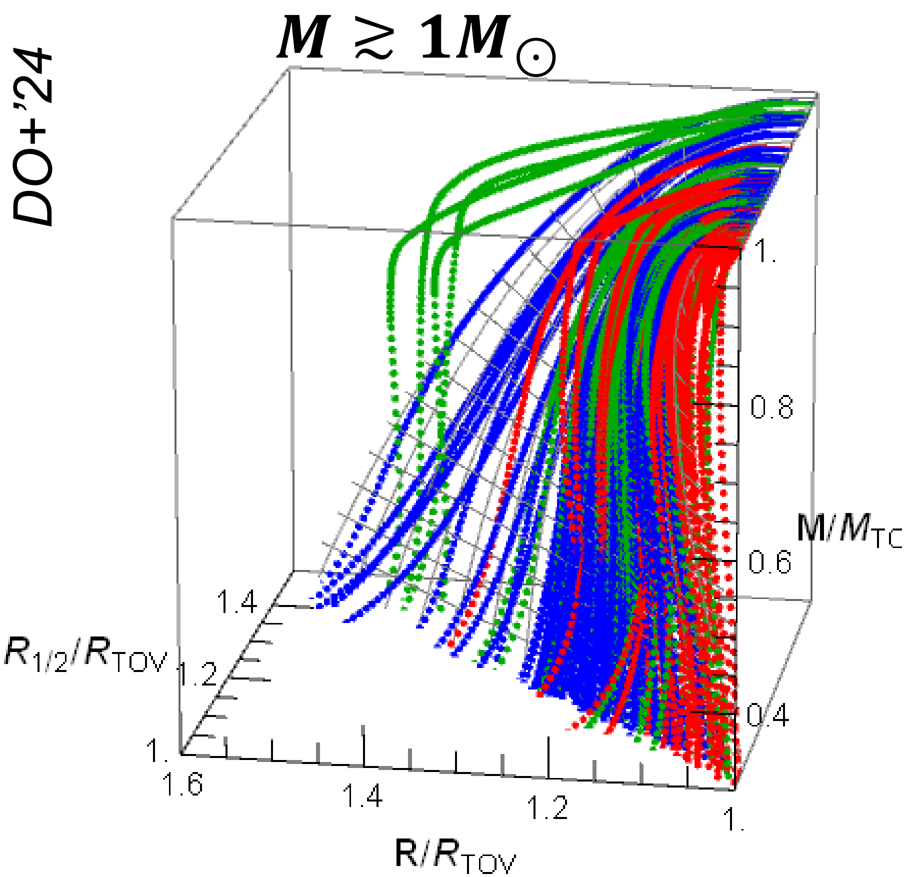
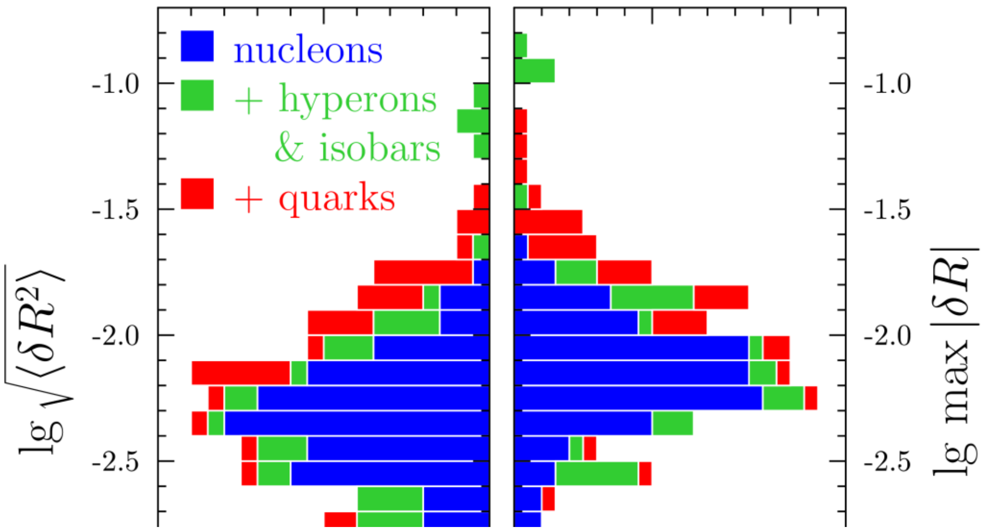




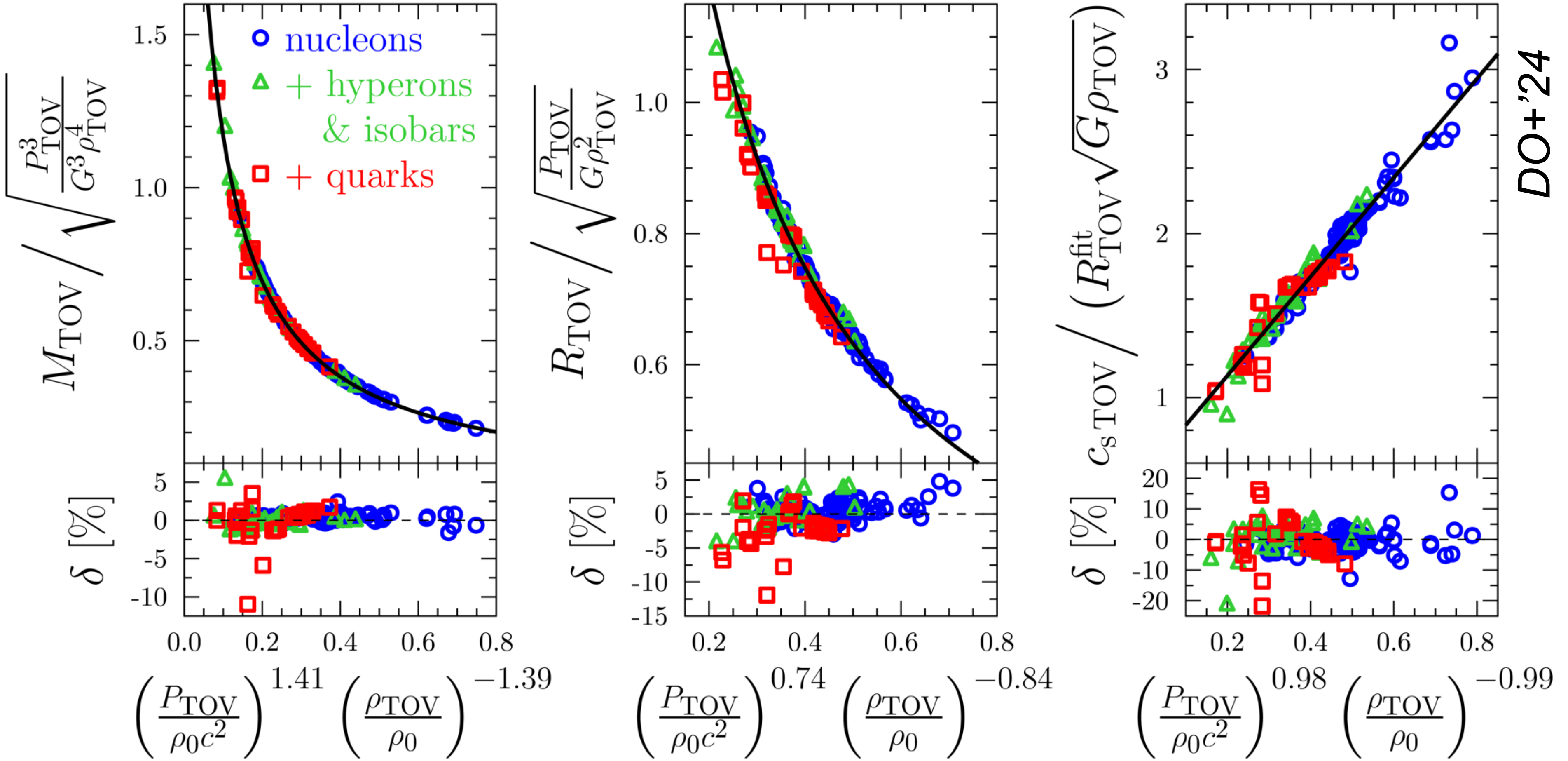
<u>"Effective" EoS manifold dimension = 3</u>

$M_{\rm TOV}, R_{\rm TOV}, \frac{R_{1/2}}{R_{\rm TOV}}$

$M_{\mathrm{TOV}}, R_{\mathrm{TOV}} \leftrightarrow P_{\mathrm{TOV}}, \rho_{\mathrm{TOV}}$

$$M_{\mathrm{TOV}} = \sqrt{\frac{P_{\mathrm{TOV}}^3}{G^3 \rho_{\mathrm{TOV}}^4}} f_M\left(\frac{\rho_{\mathrm{TOV}}}{\rho_0}, \frac{P_{\mathrm{TOV}}}{\rho_0 c^2}\right) \qquad R_{\mathrm{TOV}} = \sqrt{\frac{P_{\mathrm{TOV}}}{G \rho_{\mathrm{TOV}}^2}} f_R\left(\frac{\rho_{\mathrm{TOV}}}{\rho_0}, \frac{P_{\mathrm{TOV}}}{\rho_0 c^2}\right)$$

$$c_{s\mathrm{TOV}} = R_{\mathrm{TOV}}^{\mathrm{fit}} \sqrt{G \rho_{\mathrm{TOV}}} f_c\left(\rho_{\mathrm{TOV}}/\rho_0\,, P_{\mathrm{TOV}}/\rho_0 c^2\right)$$



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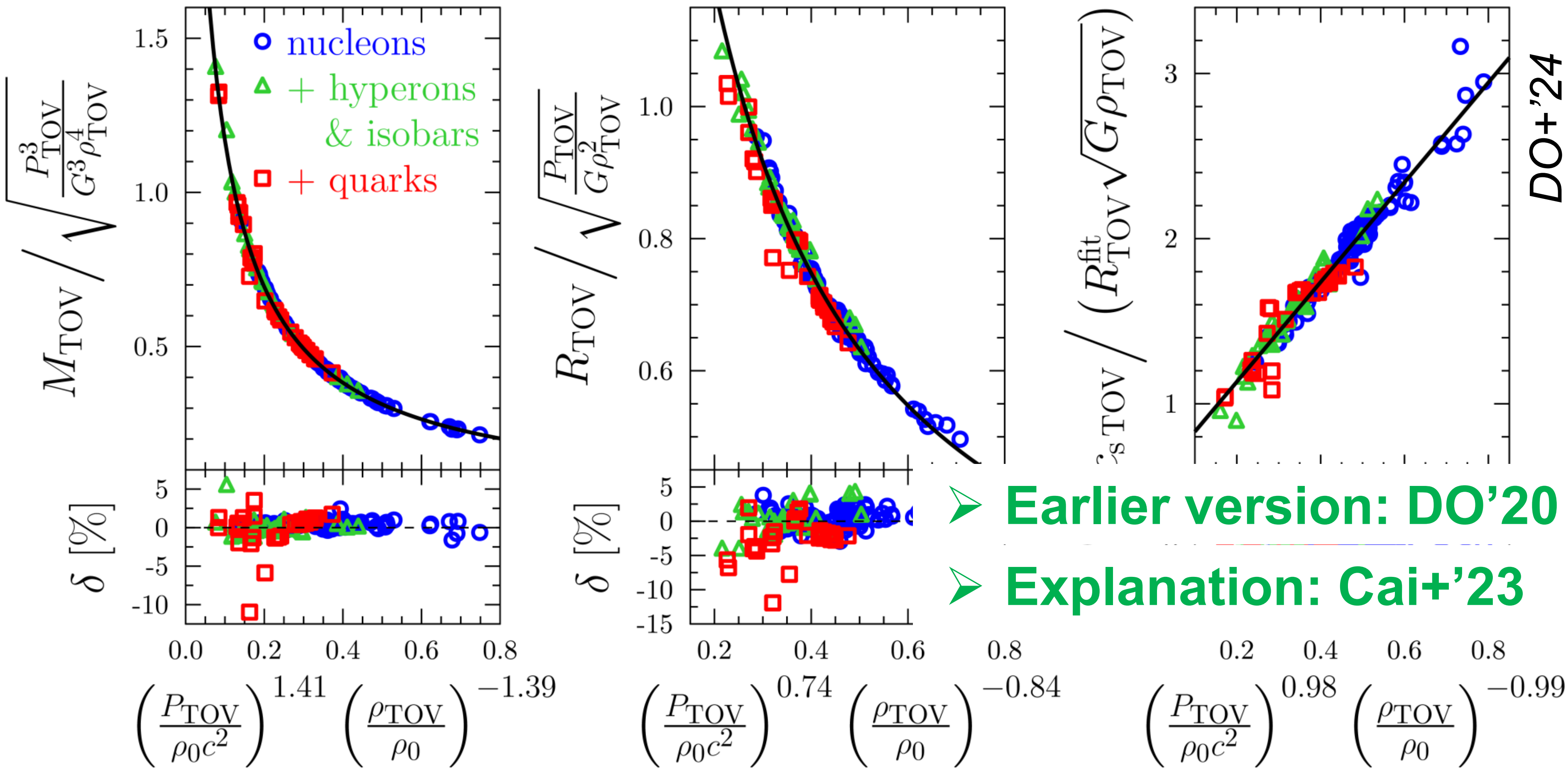


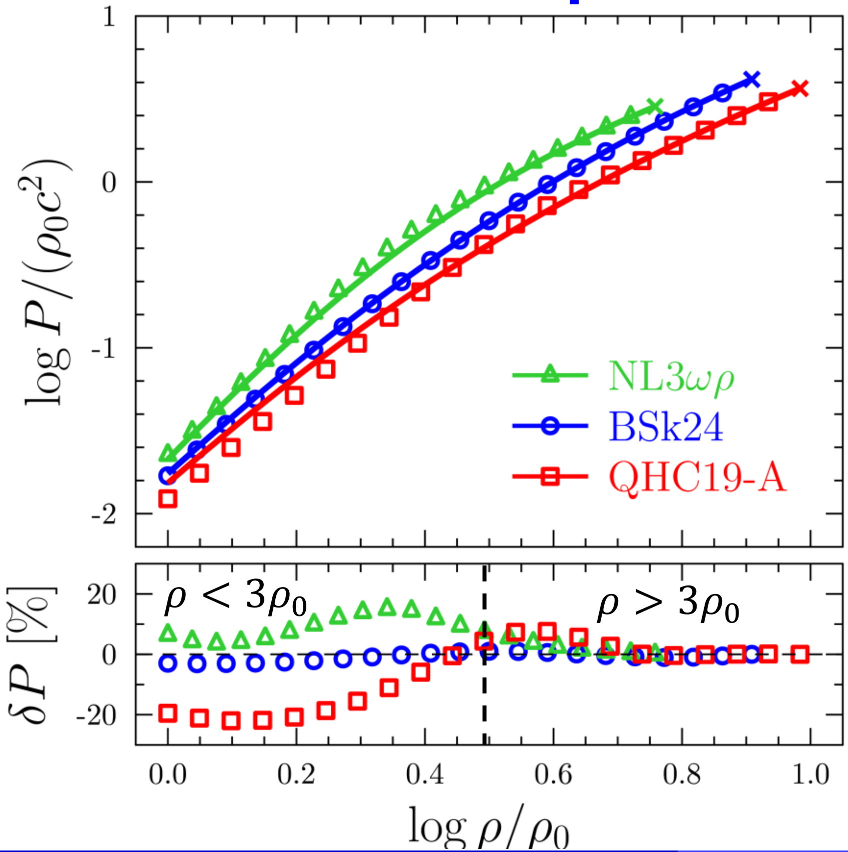
- **Earlier version: DO'20**
- **Explanation: Cai+'23**

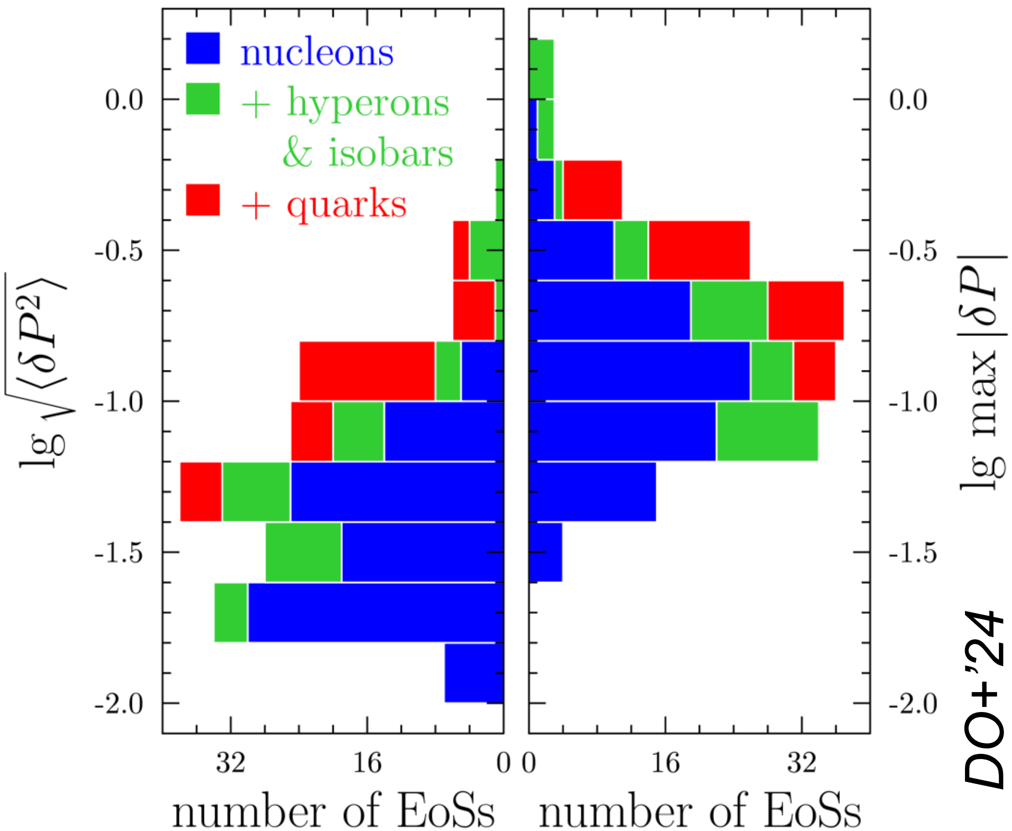
Universal $P(\rho)$

$$\frac{P}{P_{\rm TOV}} = \underbrace{\mathcal{F}\left(\frac{\rho}{\rho_{\rm TOV}}; P_{\rm TOV}, \rho_{\rm TOV}\right)}_{\rho > 3\rho_0} \underbrace{\mathcal{K}\left(\frac{\rho}{\rho_0}; \frac{R_{1/2}}{R_{\rm TOV}}, \frac{\rho_{\rm TOV}}{\rho_0}\right)}_{\rho_0 < \rho < 3\rho_0}$$

2 parameters (for $\rho > 3\rho_0$)

+ 3rd parameter (for $\rho_0 < \rho < 3\rho_0$)



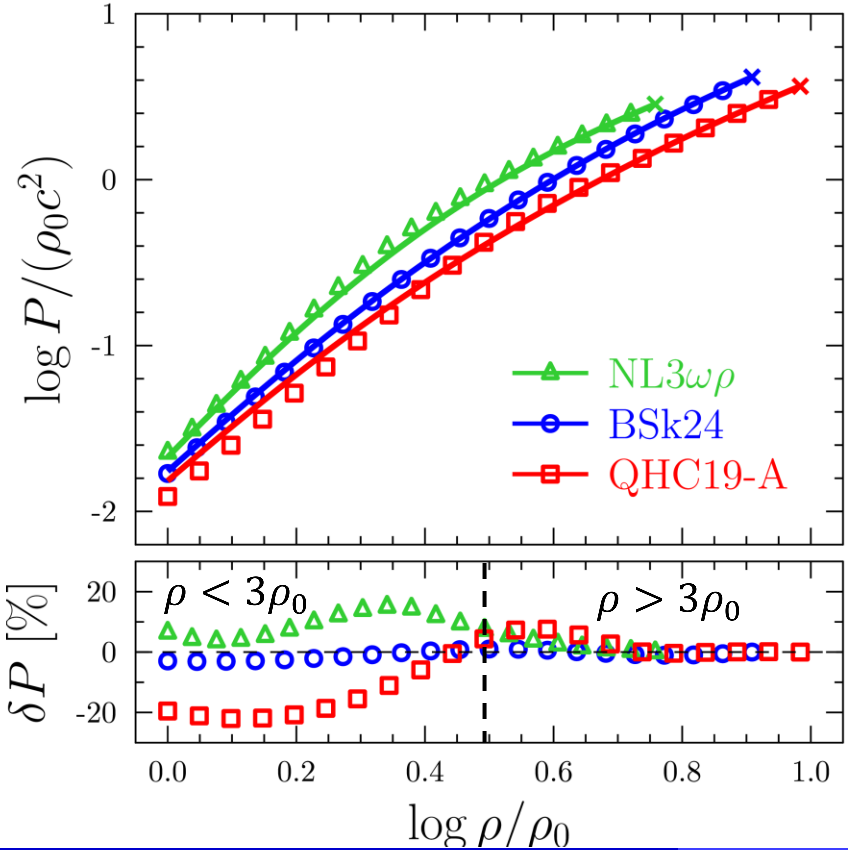


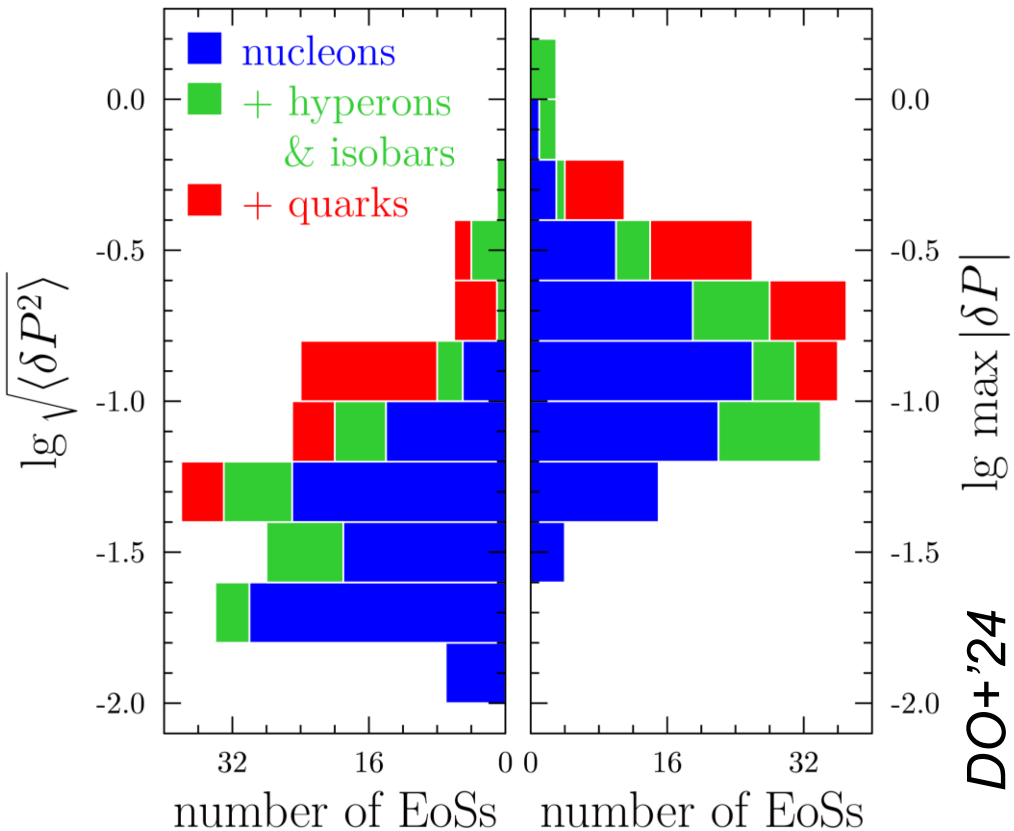
DO+'24

Universal $P(\rho)$

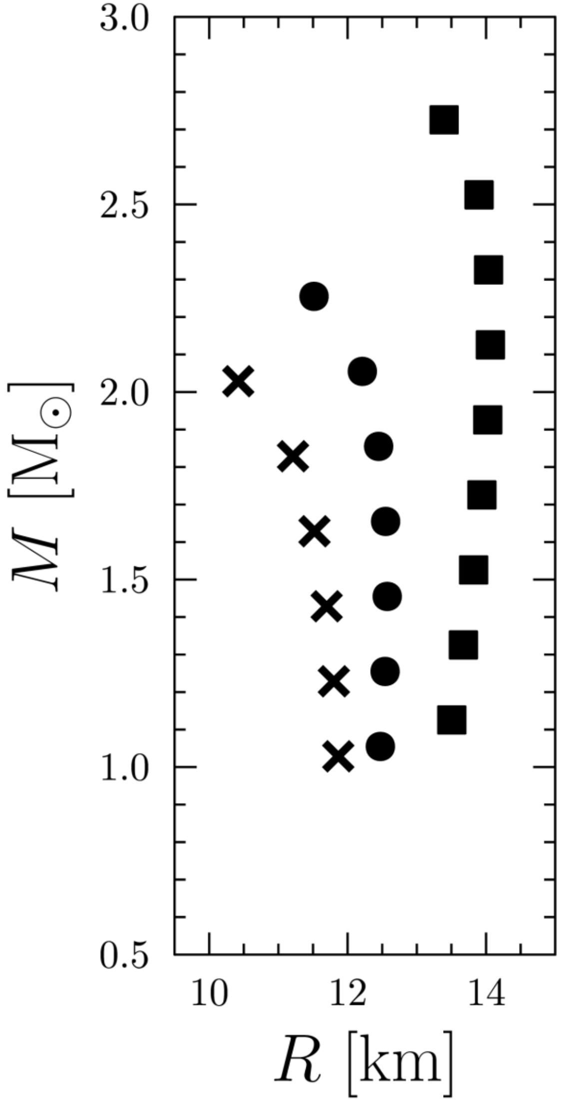
$$\frac{P}{P_{\rm TOV}} = \underbrace{\mathcal{F}\left(\frac{\rho}{\rho_{\rm TOV}}; P_{\rm TOV}, \rho_{\rm TOV}\right)}\ \underbrace{\mathcal{K}\left(\frac{\rho}{\rho_0}; \frac{R_{1/2}}{R_{\rm TOV}}, \frac{\rho_{\rm TOV}}{\rho_0}\right)}$$

Dimension = 3: $\rho_{\rm TOV}, P_{\rm TOV}, \frac{R_{1/2}}{R_{\rm TOV}} \longleftrightarrow M_{\rm TOV}, R_{\rm TOV}, \frac{R_{1/2}}{R_{\rm TOV}}$





Inverse Oppenheimer-Volkoff Mapping



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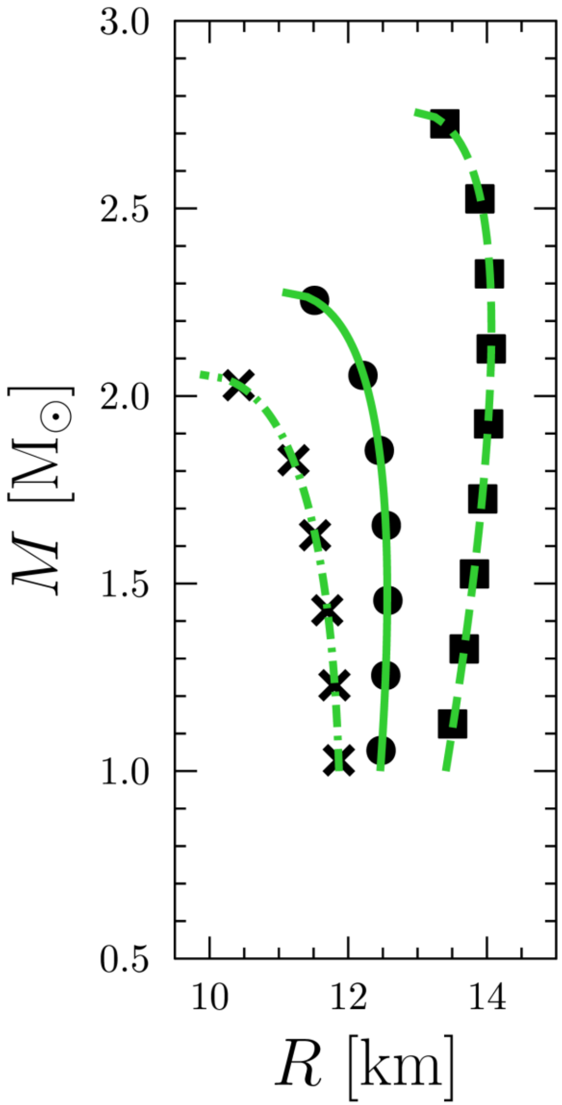
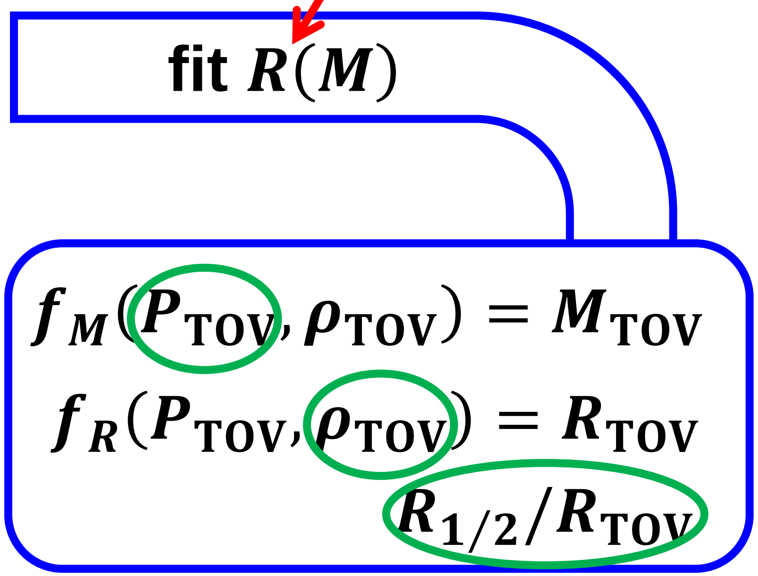


parameters: $M_{TOV}, R_{TOV}, \frac{R_{1/2}}{R_{TOV}}$



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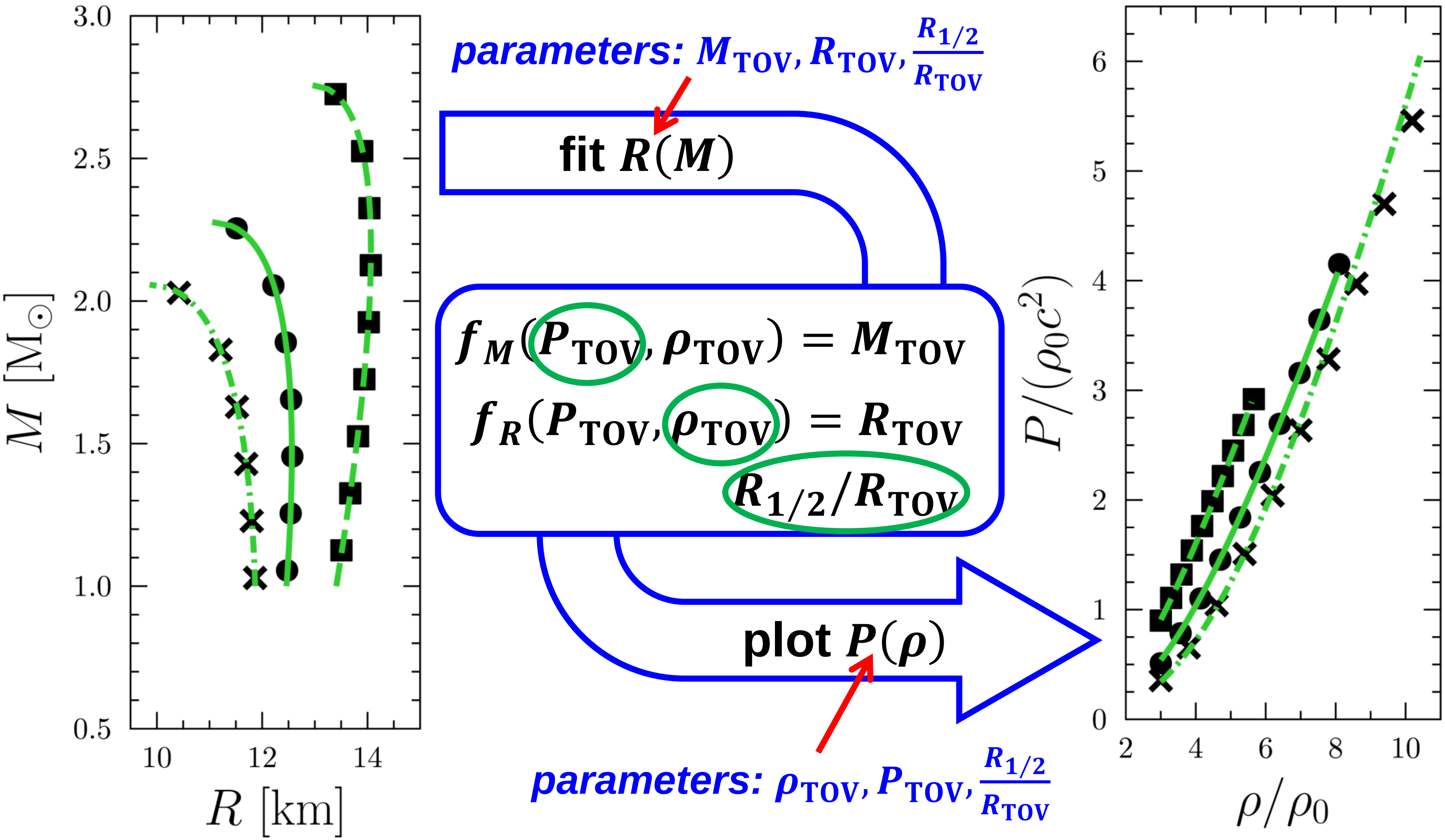
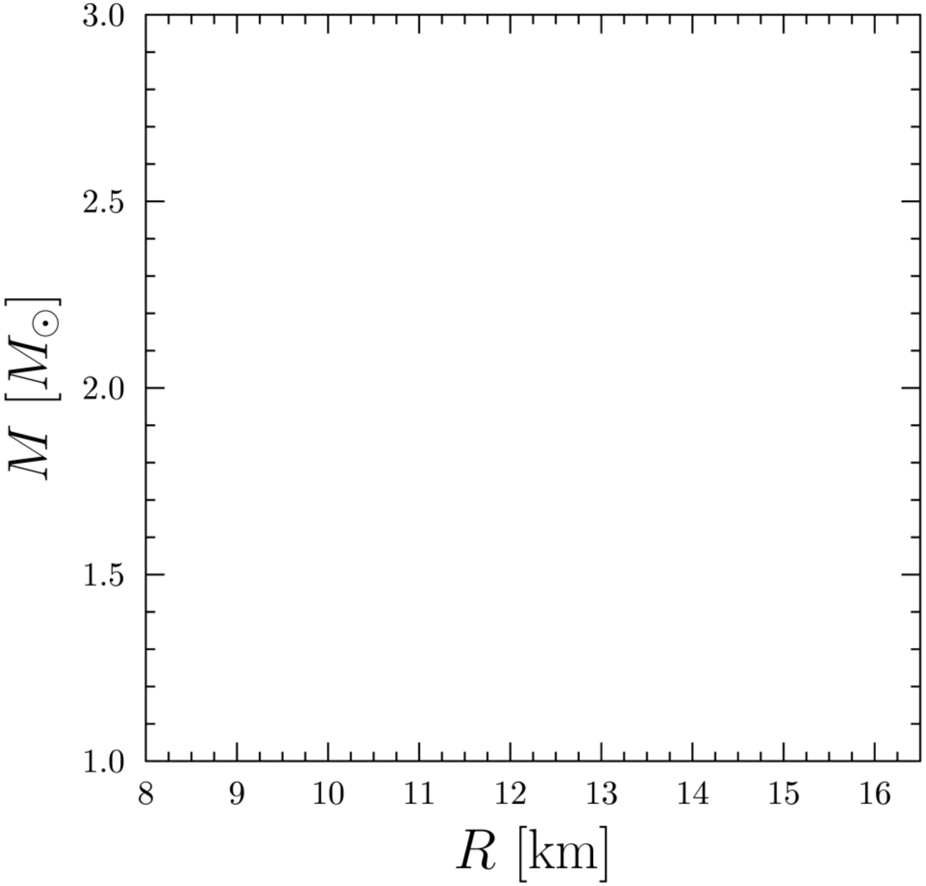
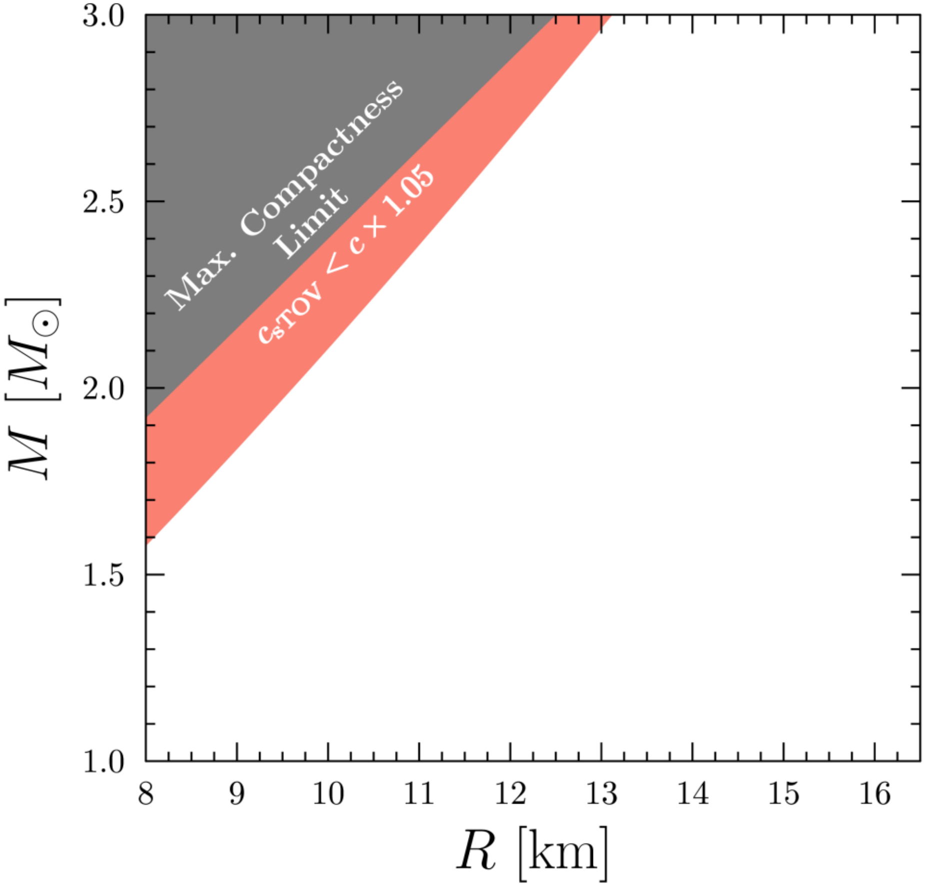


Application to $M - R$ Observations



Theoretical Constraints



- **Max compactness:**

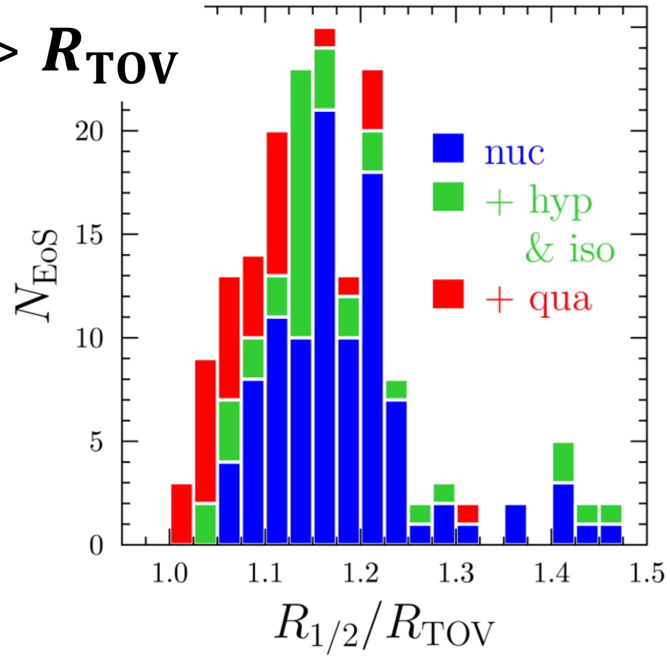
$$P = \begin{cases} (\rho - \rho_*)c^2 & \rho > \rho_* \\ 0 & \rho < \rho_* \end{cases}$$

Rhoades&Ruffini'74

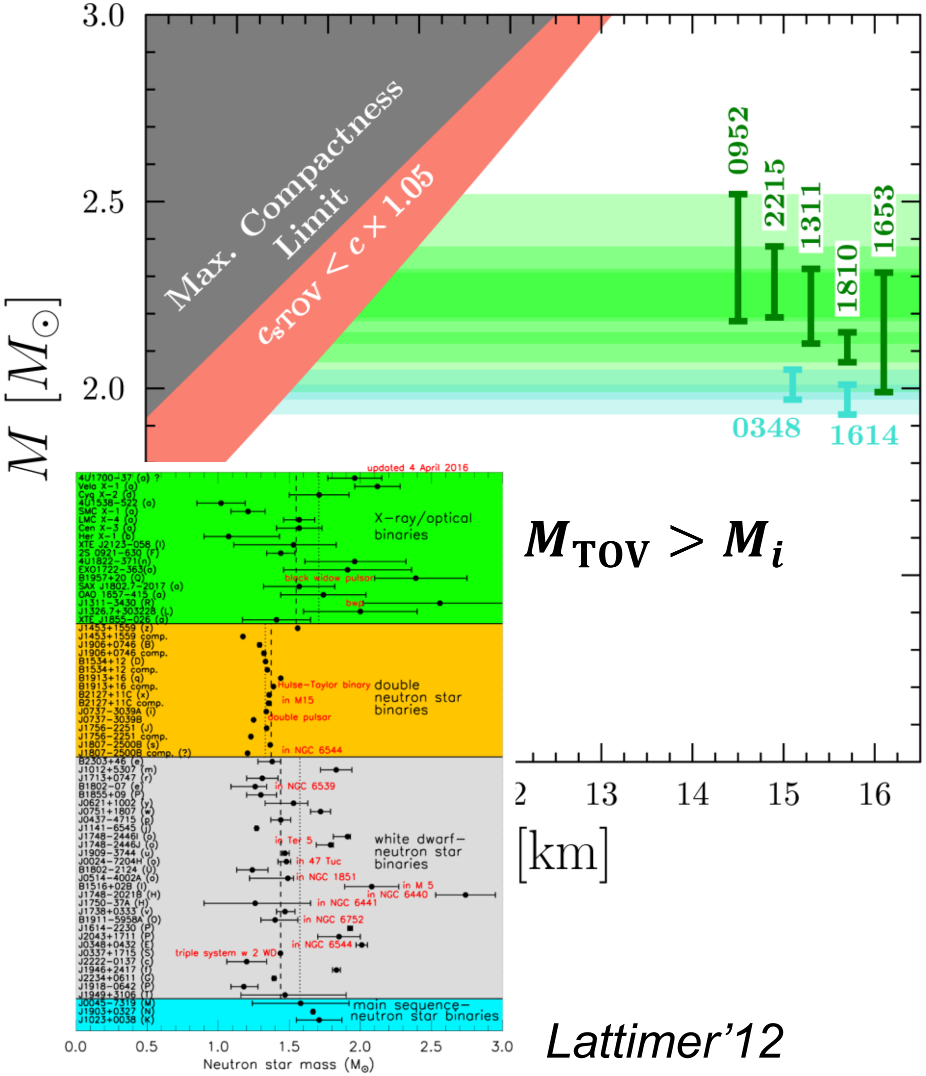
- $$\begin{cases} c_{s\mathrm{TOV}}(P_{\mathrm{TOV}}, \rho_{\mathrm{TOV}}) < c \\ M_{\mathrm{TOV}}, R_{\mathrm{TOV}} = f(P_{\mathrm{TOV}}, \rho_{\mathrm{TOV}}) \end{cases}$$

DO'20

- $R_{1/2} > R_{\mathrm{TOV}}$



Observations: Massive NSs



$M_{\mathrm{TOV}} > M_i$

Lattimer'12

- **Radio timing**

  PSR J0348+0432

  Antoniadis+'13

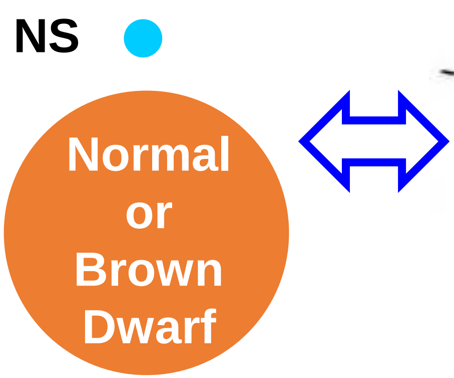
  PSR J1614-2230

  Demorest+'10

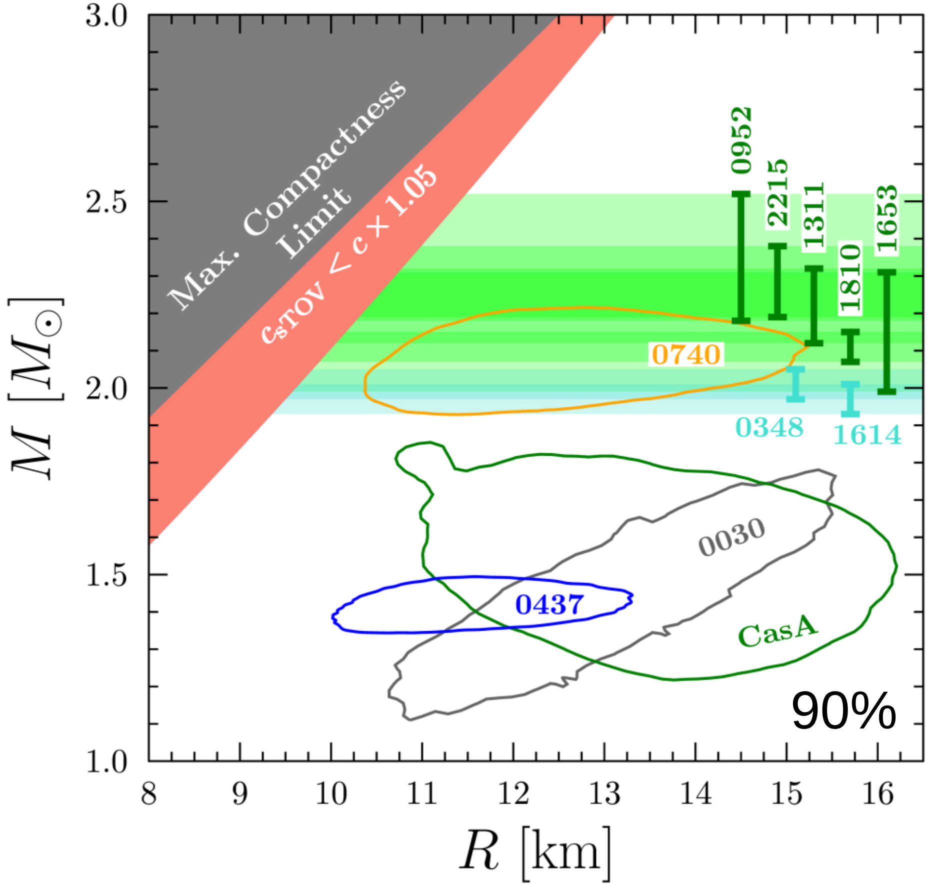
- **"Spider" binaries**

  PSRs J0952−0607, J2215+5135, J1311−3430, J1810+1744, J1653−0158

Kandel&Romani'23



Observations: NS atmosphere fits



- **NICER data**

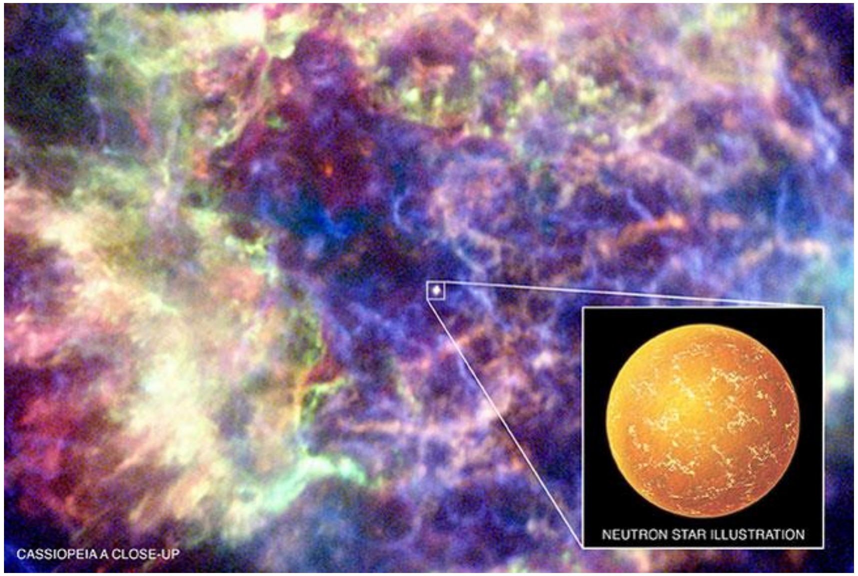
  PSR J0740+6620 *Riley+'21*

  PSR J0030+0451 *Miller+'19*

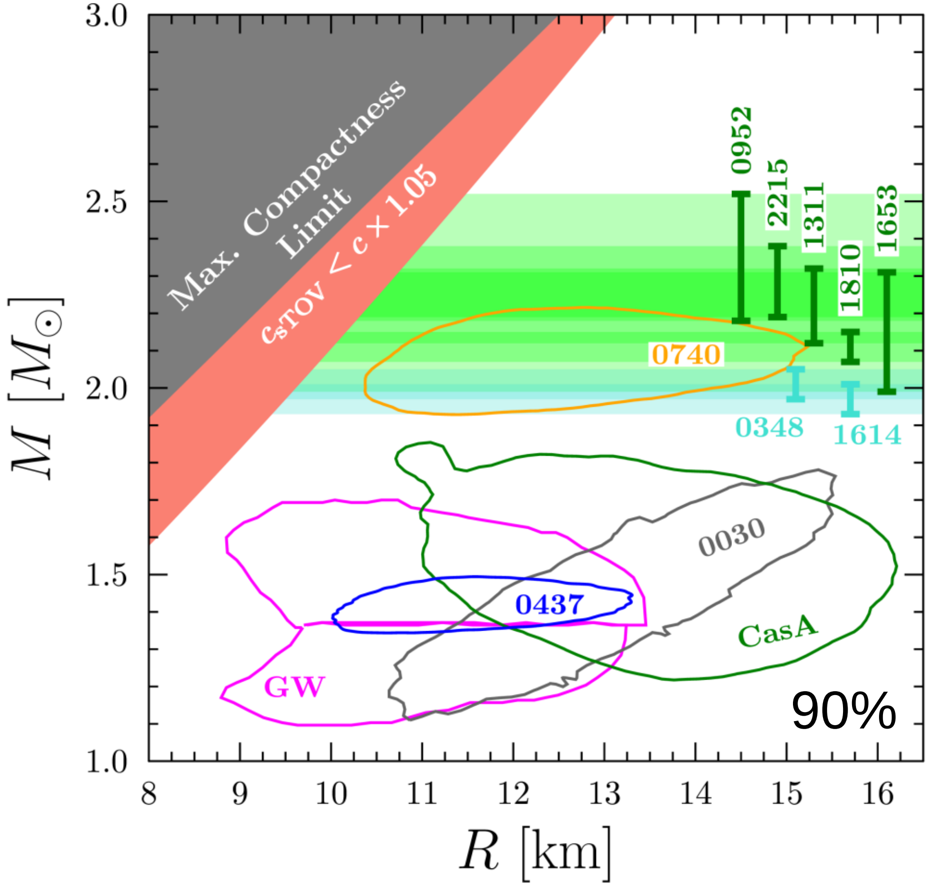
  PSR J0437−4715 *Choudhury+'24*

- **NS in Cassiopeia A SNR**

  Shternin, DO,+'23

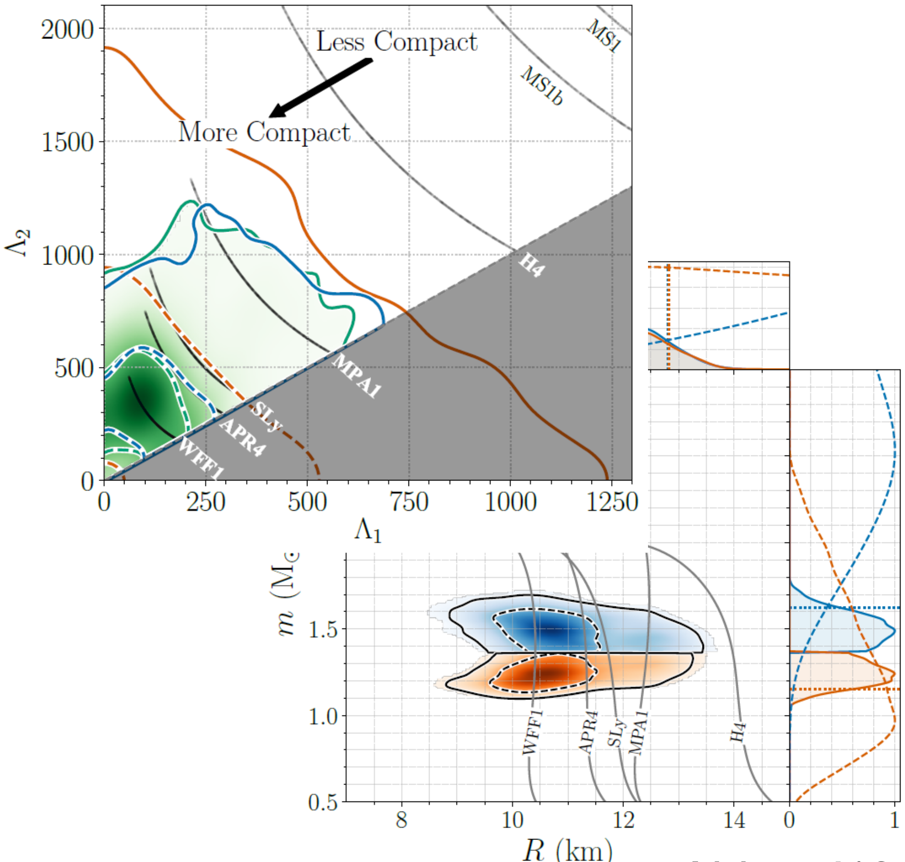


Observations: GWs from mergers



- **GW170817**

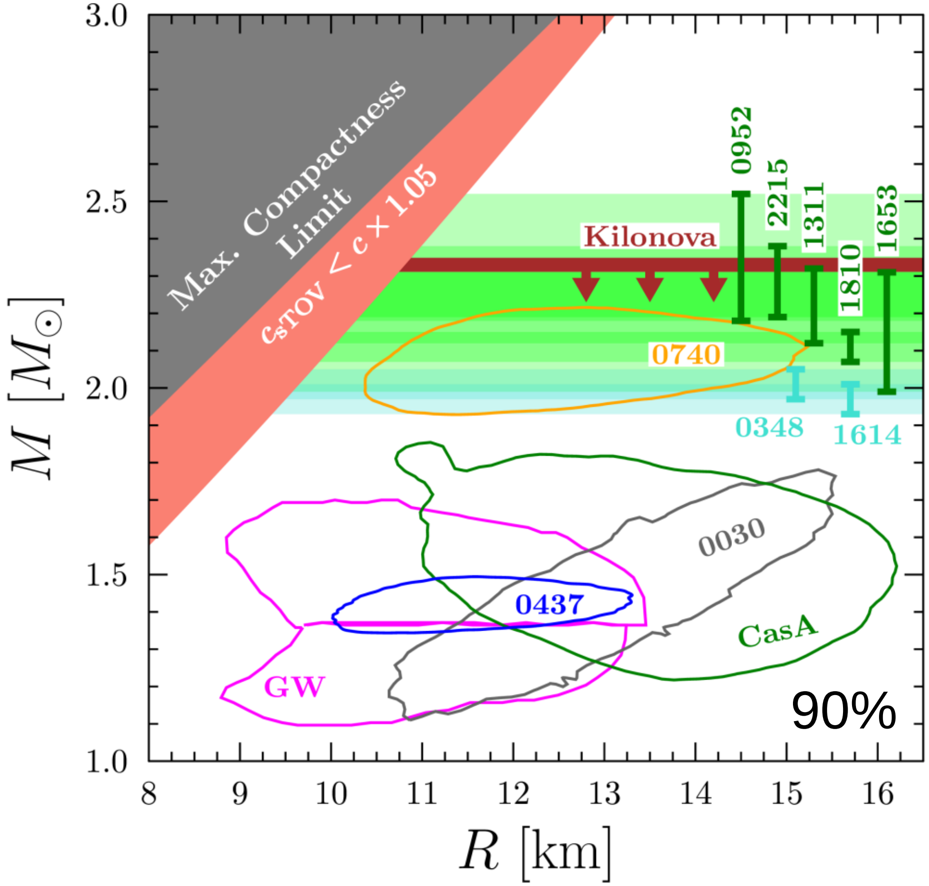
"binary I-Love-Q" / Love-C ⇒ M, R



Abbot+'18

Observations: GW counterparts

- **Rezzolla, Most, Weih, ApJL'18**



blue kilonova ⇒ **no prompt collapse**

GRB 170817A ⇒ **BH formation**

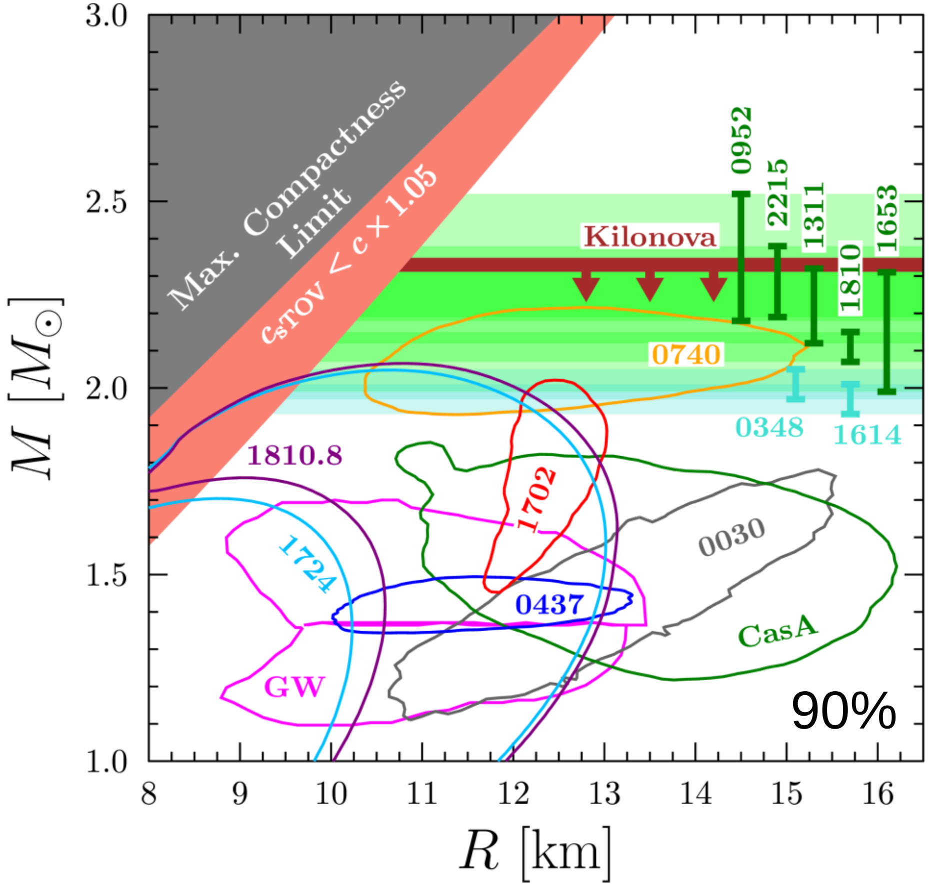
⇓

$$M_{\mathrm{TOV}} < 2.16^{+0.17}_{-0.15} M_\odot\ (90\%)$$

- ***Other KN interpretations?***

Blinnikov+22

Observations: X-ray bursters



- **Cooling tail method**

Suleimanov+16,17; Nattila+'16

4U 1702-429

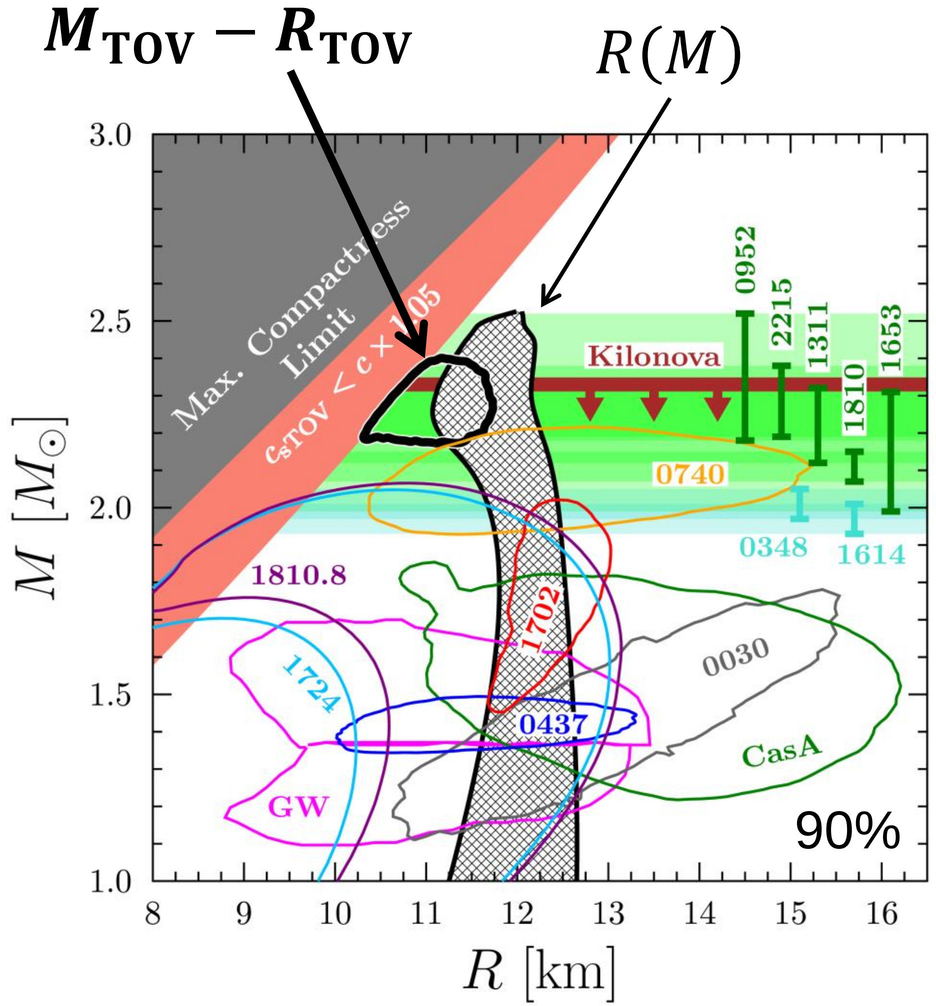
Nattila+'17

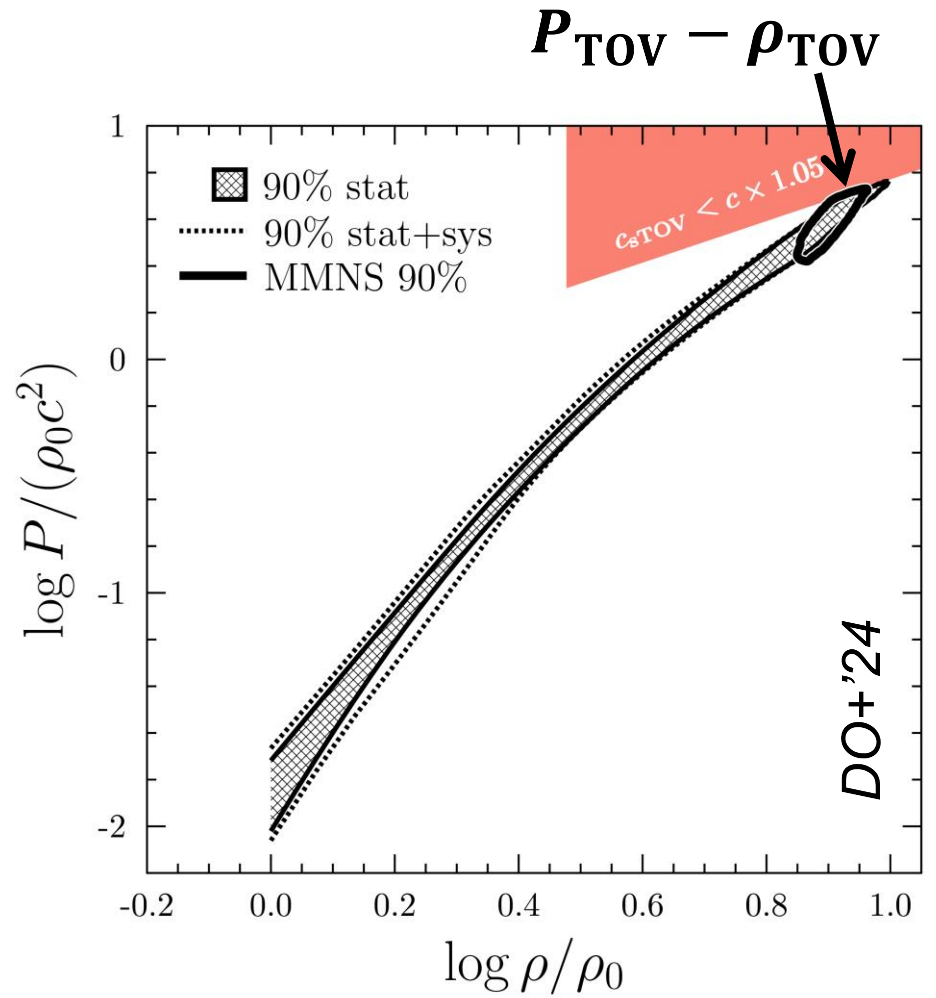
4U 1724-307

SAX J1810.8-260

Nattila+'16

Applying Inverse OV mapping





$M_{TOV} = 2.28^{+0.05}_{-0.06}\, M_\odot$

$R_{TOV} = 11.2^{+0.4}_{-0.3}$ km

$R_{1/2} = 12.0^{+0.4}_{-0.3}$ km

$P_{TOV} = 3.5^{+1.0}_{-0.5}\, \rho_0 c^2$

$\rho_{TOV} = 7.7^{+0.6}_{-0.3}\, \rho_0$

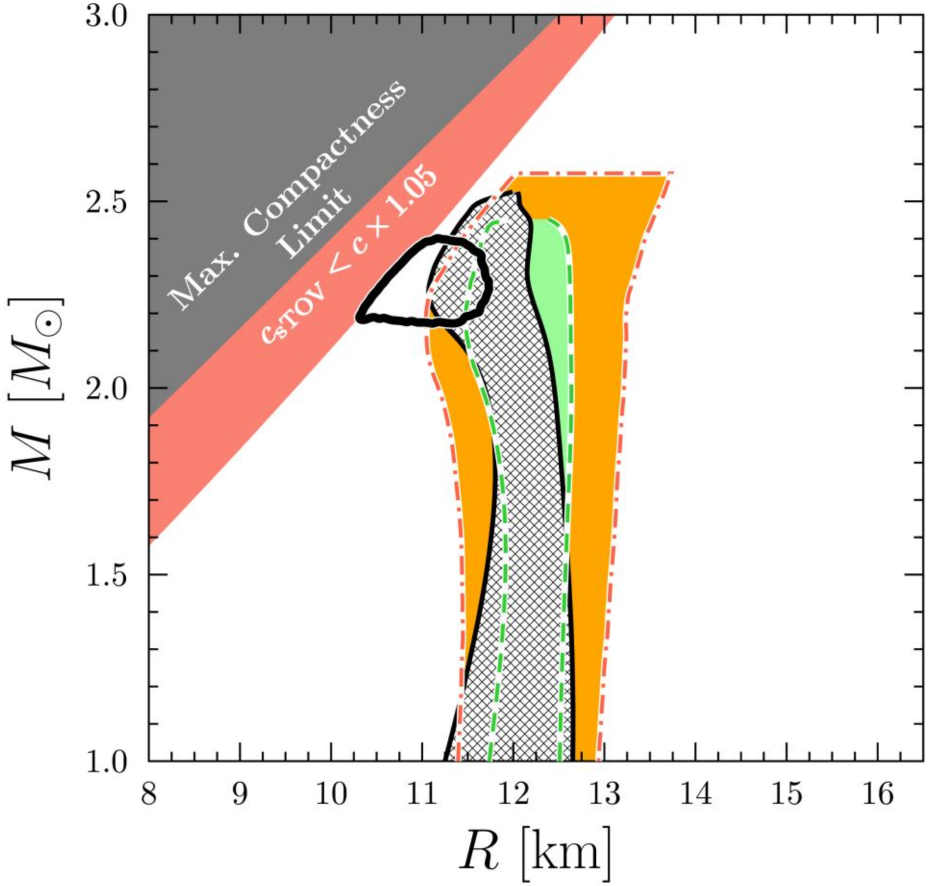
Comparison with Other Works

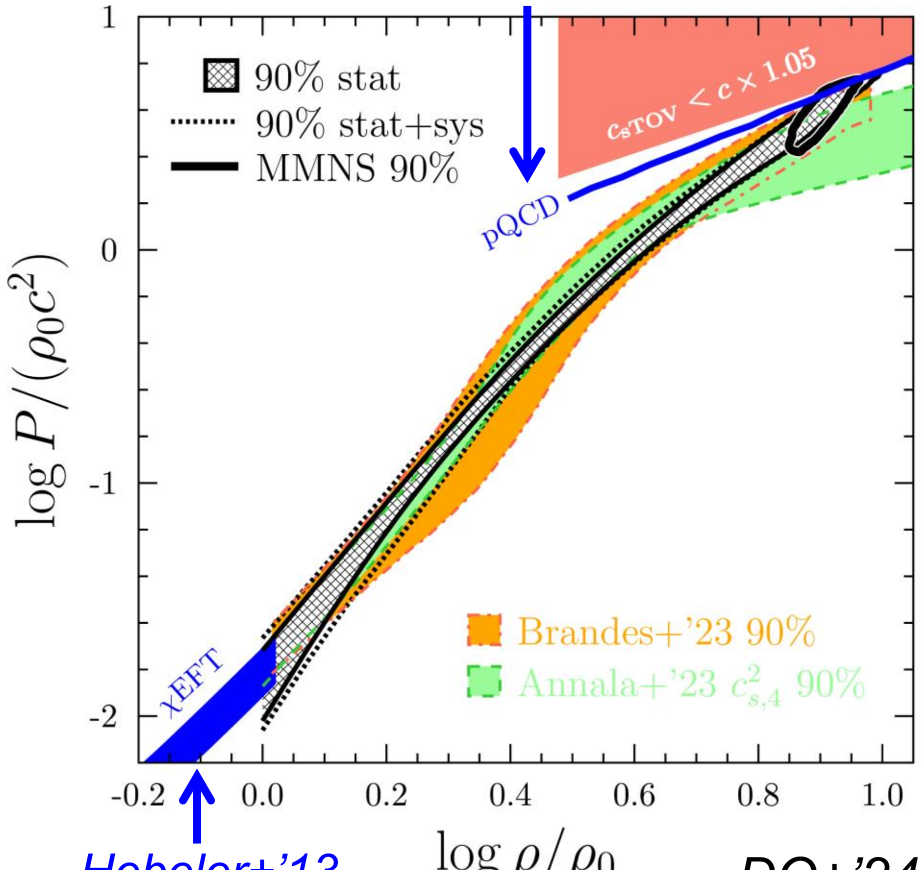
- Annala et al., Nat. Comm. 2023
- Brandes et al. PRD 2023

(Aleksi Vuorinen's talk)

- ➢ piecewise $c_s(\mu)$
- ➢ low ρ: chiral effective field
- ➢ high ρ: perturbative QCD (from $\sim 40\rho_0$)

Komoltsev&Kurkela'22





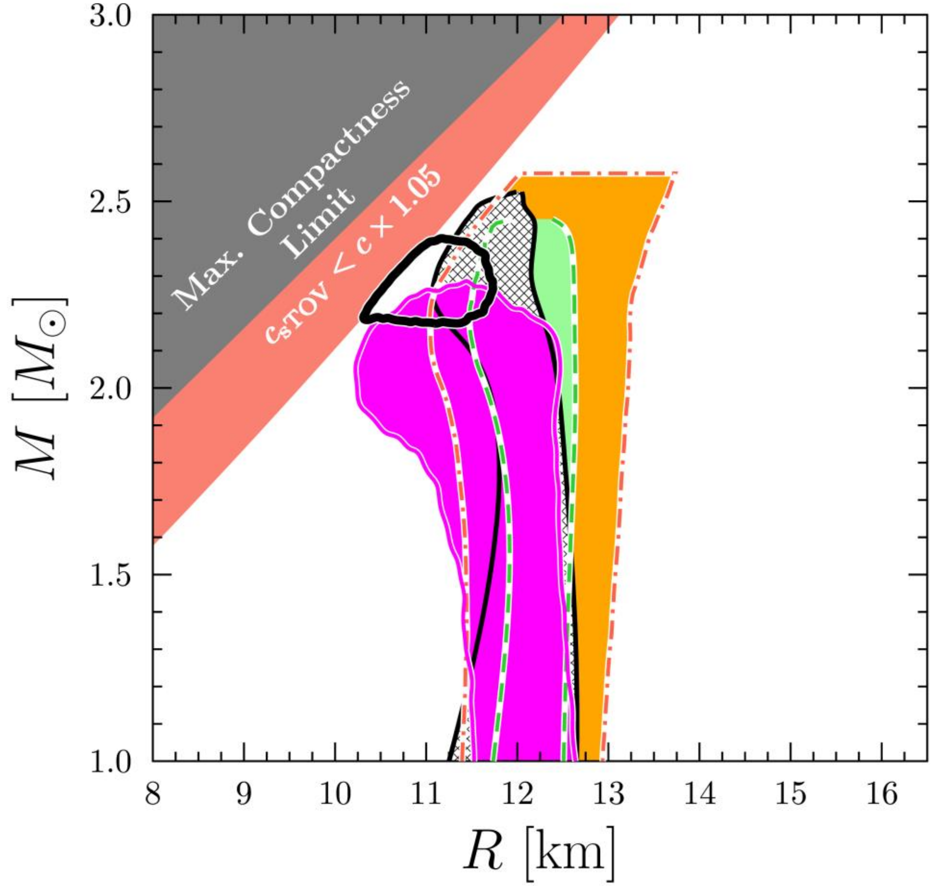
Hebeler+'13

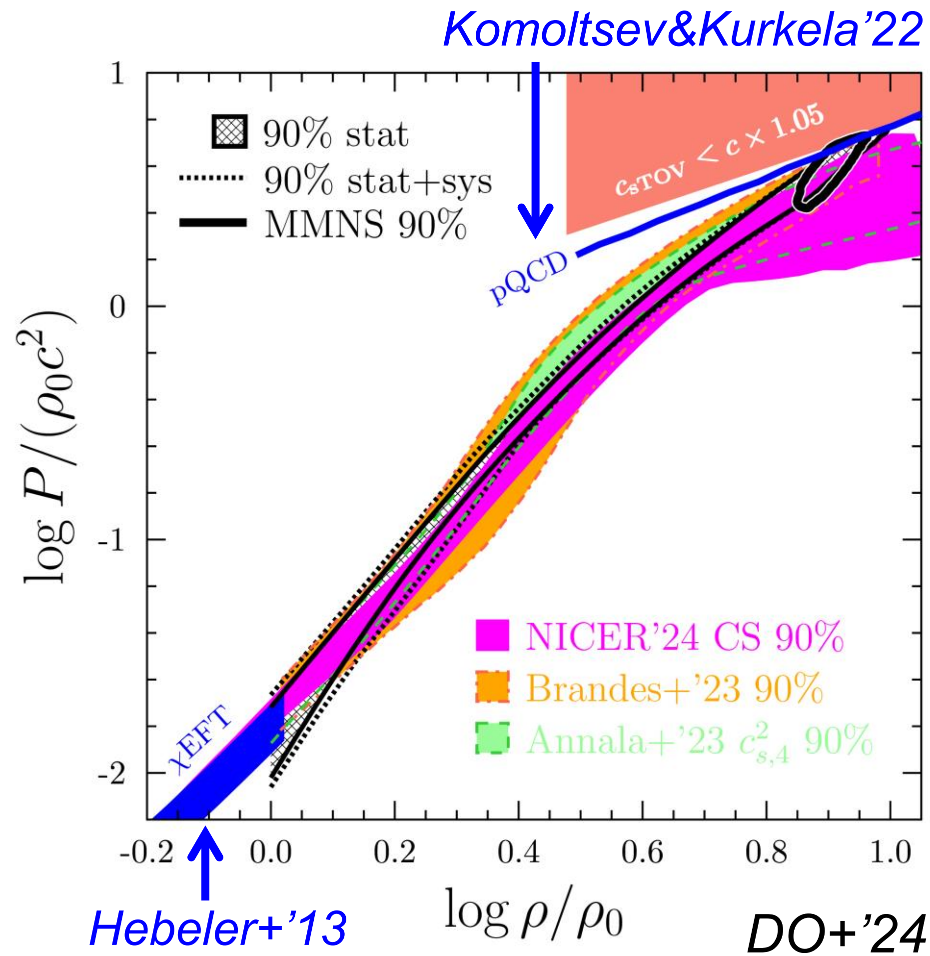
DO+'24

Comparison with Other Works

- **Annala et al., Nat. Comm. 2023** → ~~**PSR J0437−4735**~~
- **Brandes et al. PRD 2023** → ~~**PSR J0437−4735**~~
- **Rutherford et al. ApJL 2024** → **PSR J0437−4735** ✓

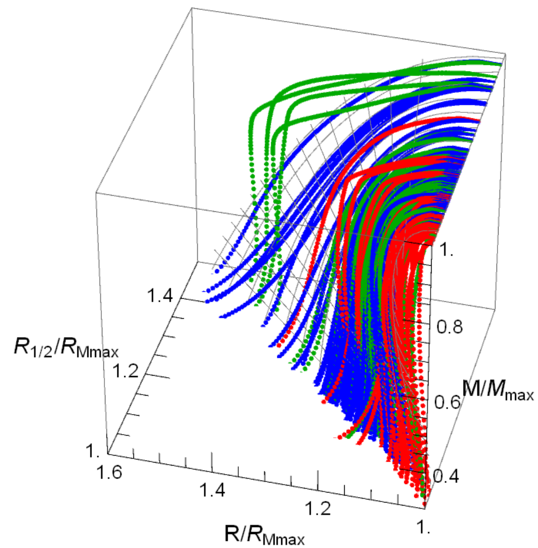
(Melissa Mendes' talk)





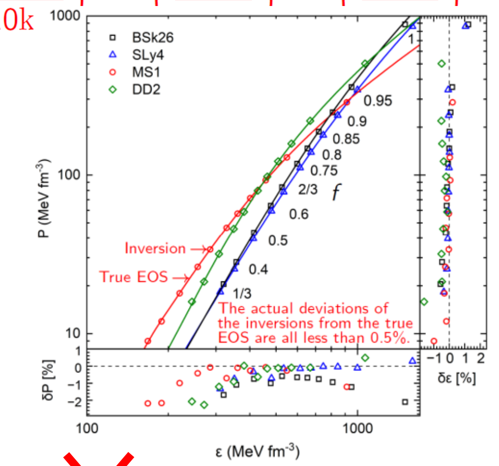
Comparison with J. Lattimer’s talk

This talk | **J. Lattimer’s talk**



$$\mathcal{E}_f = a_{\mathcal{E},f}\left(\frac{R_{f_1}}{10\mathrm{km}}\right)^{b_{\mathcal{E},f_1}}\left(\frac{R_{f_2}}{10\mathrm{km}}\right)^{c_{\mathcal{E},f_2}}\left(\frac{M_{max}}{M_\odot}\right)^{d_{\mathcal{E},f}}$$

$$P_f = a_{P,f}\left(\frac{R_{f_1}}{10\mathrm{k}}\right)^{b_{P,f_1}}\left(\frac{R_{f_2}}{}\right)^{c_{P,f_2}}\left(\frac{M_{max}}{}\right)^{d_{P,f}}$$



	This talk	J. Lattimer’s talk
• **Restore $R(M)$ from observations**	✓	✗
• **Inverse OV mapping** $R(M) \mapsto P(\rho)$	✓	✓*
• **Inverting TOV equation** $M \mapsto P_c, \rho_c$	✗	✓*

***if $R(M)$ is given**

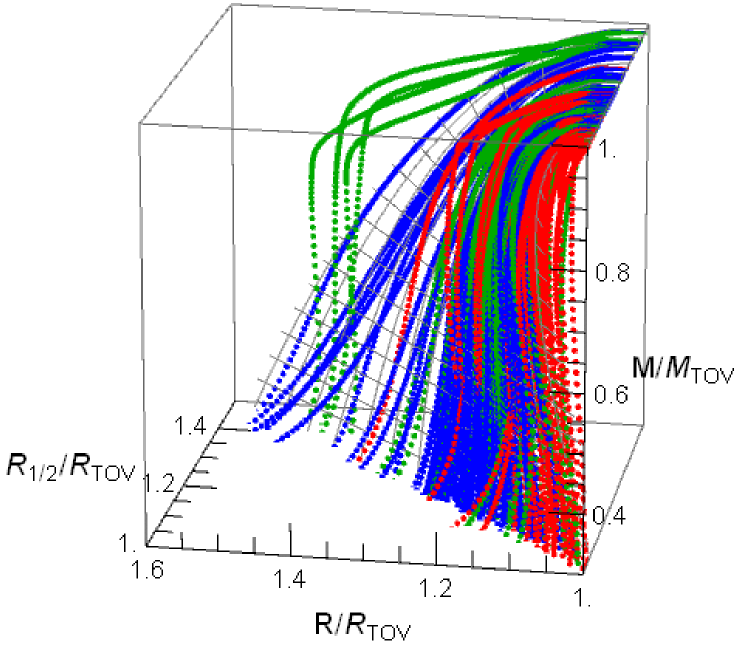
Conclusion

- **Effective dimension of the EoS manifold = 3** (at least, for densities dominating in NSs with $M > 1M_\odot$)
  - **loss of nuclear-physics information?**
- **Handful parametrization:**

$$M_{\mathrm{TOV}}, R_{\mathrm{TOV}}, {}^{R_{1/2}}/_{R_{\mathrm{TOV}}} \quad \text{or} \quad P_{\mathrm{TOV}}, \rho_{\mathrm{TOV}}, {}^{R_{1/2}}/_{R_{\mathrm{TOV}}}$$

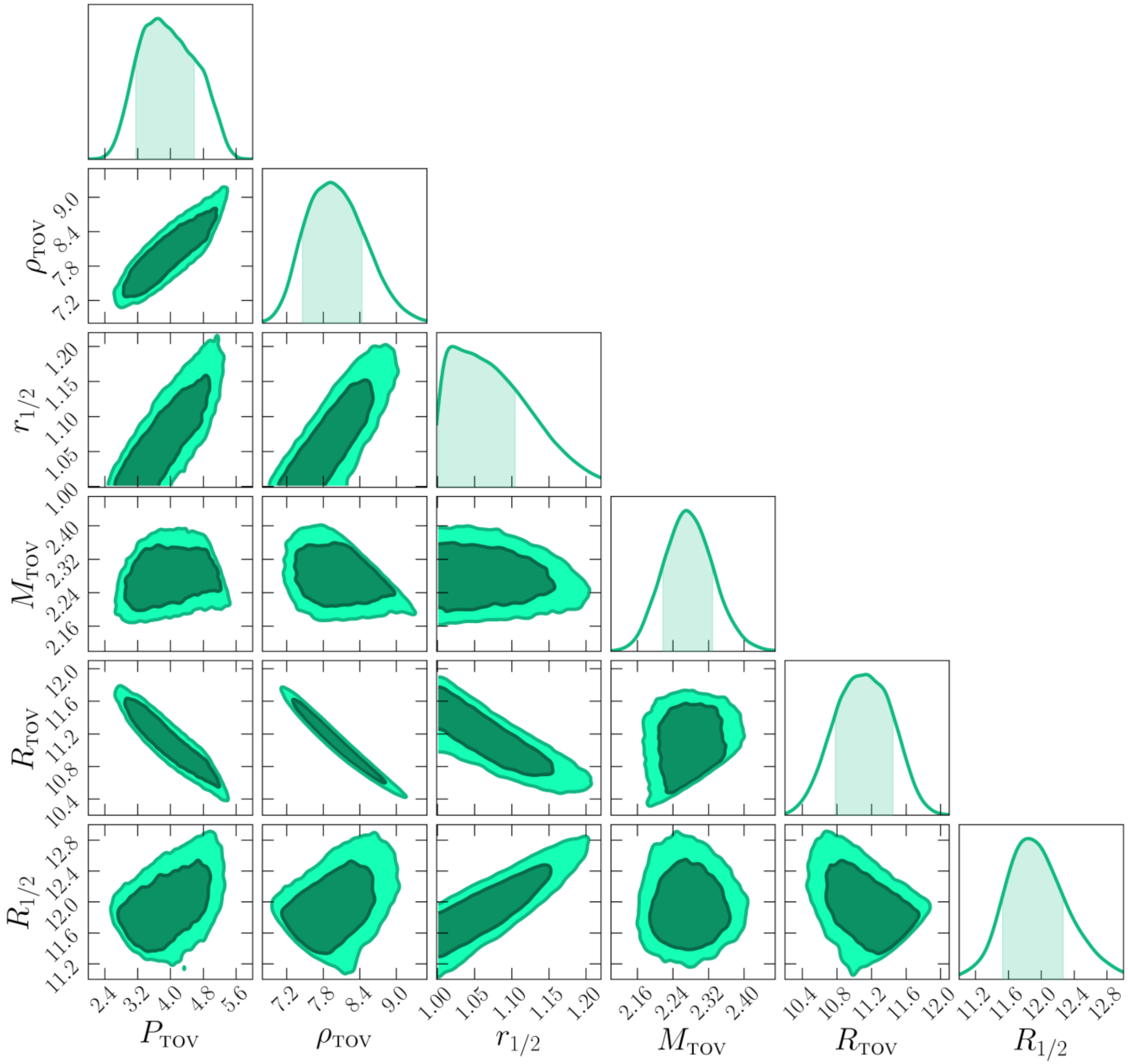
- **Using such the parametrization we built**
  - **universal approximations for $P-\rho$ and $M-R$**
  - (approximate) **explicit analytic inverse Oppenheimer-Volkoff mapping**
- **Applications:**
  - **constraining EoS through observations**
  - **other observables $= f(3 \text{ parameters})$?**

Thank you!



arXiv:2404.17647

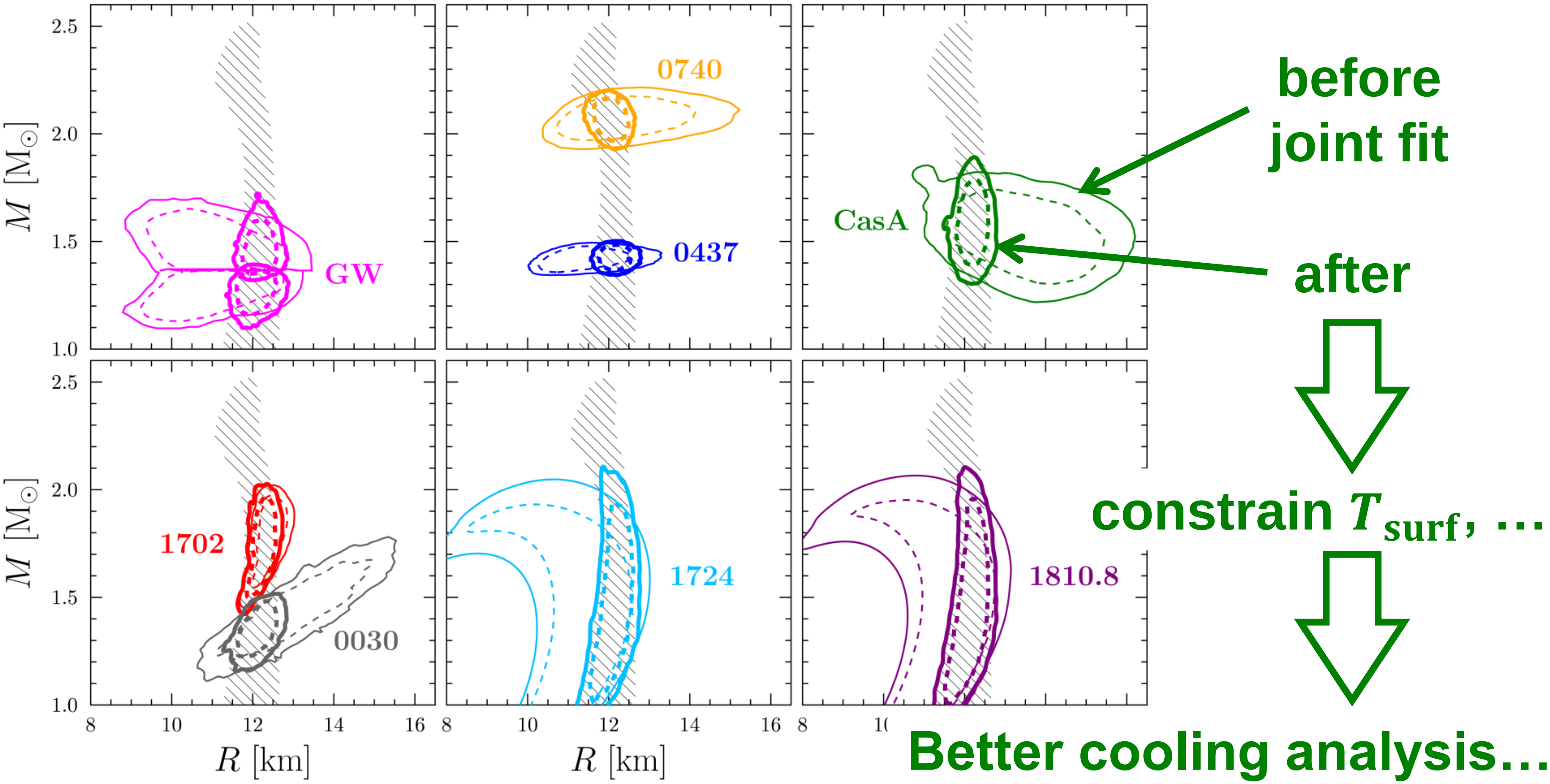
The Triangle Plot



Other Possible Applications

- **joined spectral fits of multiple sources; avoid "the information loss" due to intermediate M and R determination**
- **constraining individual sources**

(Brandes+'24)



Universal $P(\rho)$ at high ρ

$$\frac{P}{P_{\rm TOV}} = g\left(\frac{\rho}{\rho_{\rm TOV}}; c_{s\rm TOV}, \gamma_{\rm TOV}\right)$$

$c_{s\rm TOV}(P_{\rm TOV}, \rho_{\rm TOV})$

$$\gamma_{\rm TOV} = \frac{\rho_{\rm TOV}}{P_{\rm TOV}} c_{s\rm TOV}^2(P_{\rm TOV}, \rho_{\rm TOV})$$

$\rho \gtrsim 3\rho_0$ ↔ center of $1M_\odot$

2 parameters only