Indirect constraints on the third-generation baryon number violation

Based on:
M. Beneke, G. Finauri, AAP, *JHEP* 09 (2024) 090

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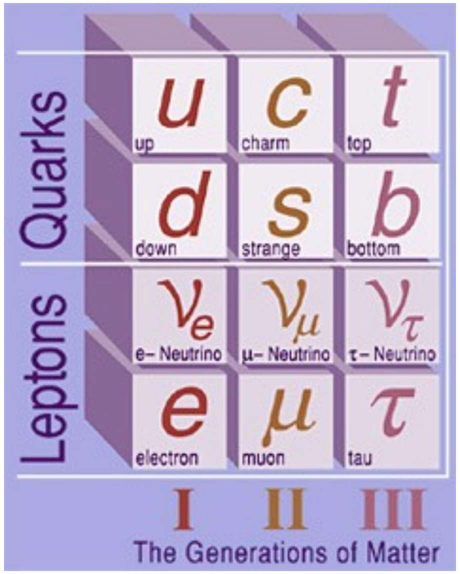
- Baryon and lepton numbers are accidental symmetries of the standard model: no reason to be conserved in general

Sakharov's conditions require BNV

- There are excellent experimental constraints on BNV in proton decays and/or $n - \bar{n}$ oscillations

But: initial and finals states only include quarks of the first two generations



- Can the third generation of fermions be special?

Can new flavor physics with highly specific generation-dependent couplings be baryon-number violating?

I. Dorsner, S. Fajfer, N. Kosnik

- Let's be agnostic (SMEFT):

Warsaw basis

$$\mathcal{Q}_{duu} = \varepsilon^{abc}[\widetilde{D}^a U^b]\,[\widetilde{U}^c E]\,,$$
$$\mathcal{Q}_{duq} = \varepsilon^{abc}\varepsilon_{jk}[\widetilde{D}^a U^b]\,[\widetilde{Q}^c_j L_k]\,,$$
$$\mathcal{Q}_{qqu} = \varepsilon^{abc}\varepsilon_{jk}[\widetilde{Q}^a_j Q^b_k]\,[\widetilde{U}^c E]\,,$$
$$\mathcal{Q}_{qqq} = \varepsilon^{abc}\varepsilon_{jn}\varepsilon_{km}[\widetilde{Q}^a_j Q^b_k]\,[\widetilde{Q}^c_m L_n]\,.$$

with $\widetilde{\psi} \equiv \bar{\psi}^c$
and $\psi^c = C\bar{\psi}^T$

Introduction: experimental constraints

- There are plenty of experimental searches for baryon and lepton number violation with the 2nd and 3rd generations

PHYSICAL REVIEW D **108,** 012021 (2023)

Search for the baryon- and lepton-number violating decays $B^0 \to p\mu^-$ and $B_s^0 \to p\mu^-$

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 26 October 2022;

A search for the baryon- and lepton-num
at the LHCb experiment using data collec
corresponding to integrated luminosities o
$B^0 \to p\mu^-$ and $B_s^0 \to p\mu^-$ decays is found an
be $\mathcal{B}(B^0 \to p\mu^-) < 2.6(3.1) \times 10^{-9}$ and $\mathcal{B}($
confidence level. These are the first limits o

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PHYSICAL REVIEW D **106,** 112009 (2022)

Search for baryon and lepton number violating decay $D^\pm \to n(\bar{n})e^\pm$

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R. A. Briere,[5] A. Brueggemann,[63] H. Cai,[71] X. Cai,[1,53] A. Calcaterra,[26a] G. F. Cao,[1,58] N. Cao,[1,58] S. A. Cetin,[57a]
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[53] Z. J. Chen,[23,h] W. S. Cheng,[69c]
Dai,[73] A. Dbeyssi,[17] R. E. de Boer,[4]
Mori,[69a,69c] Y. Ding,[36] J. Dong,[1,53]
Fang,[1,53] S. S. Fang,[1,58] W. X. Fang,[1]
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RAPID COMMUNICATIONS

PHYSICAL REVIEW D **83,** 091101(R) (2011)

Searches for the baryon- and lepton-number violating decays $B^0 \to \Lambda_c^+ l^-$, $B^- \to \Lambda l^-$, and $B^- \to \bar{\Lambda} l^-$

P. del Amo Sanchez,[1] J. P. Lees,[1] V. Poireau,[1] E. Prencipe,[1] V. Tisserand,[1] J. Garra Tico,[2] E. Grauges,[2] M. Martinelli,[3a,3b]
D. A. Milanes,[3a] A. Palano,[3a,3b] M. Pappagallo,[3a,3b] G. Eigen,[4] B. Stugu,[4] L. Sun,[4] D. N. Brown,[5] L. T. Kerth,[5]
Yu. G. Kolomensky,[5] G. Lynch,[5] I. L. Osipenkov,[5] H. Koch,[6] T. Schroeder,[6] D. J. Asgeirsson,[7] C. Hearty,[7] T. S. Mattison,[7]
J. A. McKenna,[7] A. Khan,[8] V. E. Blinov,[9] A. R. Buzykaev,[9] V. P. Druzhinin,[9] V. B. Golubev,[9] E. A. Kravchenko,[9]
A. P. Onuchin,[9] S. I. Serednyakov,[9] Yu. I. Skovpen,[9] E. P. Solodov,[9] K. Yu. Todyshev,[9] A. N. Yushkov,[9] M. Bondioli,[10]

PHYSICAL REVIEW D **102,** 111101(R) (2020)

Rapid Communications

Search for lepton-number- and baryon-number-violating tau decays at Belle

D. Sahoo,[82,91] G. B. Mohanty,[82] K. Trabelsi,[42] I. Adachi,[17,13] K. Adamczyk,[61] H. Aihara,[88] S. Al Said,[81,36] D. M. Asner,[3]
T. Aushev,[19] R. Ayad,[81] T. Aziz,[82] V. Babu,[8] S. Bahinipati,[23] P. Behera,[25] J. Bennett,[51] M. Bessner,[16] V. Bhardwaj,[22]
T. Bilka,[5] J. Biswal,[33] G. Bonvicini,[93] A. Bozek,[61] M. Bračko,[48,33] T. E. Browder,[16] M. Campajola,[30,56] L. Cao,[2]
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S.-K. Choi,[14] Y. Choi,[79] S. Choudhury,[24] D. Cinabro,[93] S. Cunliffe,[8] S. Das,[47] N. Dash,[25] G. De Nardo,[30,56] F. Di Capua,[30,56]
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A. Frey,[12] B. G. Fulsom,[67] R. Garg,[68] V. Gaur,[92] A. Garmash,[4,65] A. Giri,[24] P. Goldenzweig,[34] B. Golob,[45,33] Y. Guan,[7]
K. Gudkova,[4,65] C. Hadjivasiliou,[67] S. Halder,[82] T. Hara,[17,13] O. Hartbrich,[16] K. Hayasaka,[63] H. Hayashii,[57] M. T. Hedges,[16]
M. Hernandez Villanueva,[51] W.-S. Hou,[60] C.-L. Hsu,[80] K. Huang,[60] T. Iijima,[55,54] K. Inami,[54] G. Inguglia,[28]
A. Ishikawa,[17,13] R. Itoh,[17,13] M. Iwasaki,[66] Y. Iwasaki,[17] W. W. Jacobs,[26] H. B. Jeon,[41] S. Jia,[11] Y. Jin,[88] C. W. Joo,[35]
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S. H. Kim,[75] Y.-K. Kim,[95] K. Kinoshita,[7] P. Kodyš,[5] T. Konno,[37] S. Korpar,[48,33] D. Kotchetkov,[16] P. Križan,[45,33]
R. Kroeger,[51] P. Krokovny,[4,65] T. Kuhr,[46] M. Kumar,[47] R. Kumar,[71] K. Kumara,[93] A. Kuzmin,[4,65] Y.-J. Kwon,[95]
K. Lalwani,[47] I. S. Lee,[15] S. C. Lee,[41] C. H. Li,[44] J. Li,[41] L. K. Li,[7] Y. B. Li,[69] J. Libby,[25] K. Lieret,[46] D. Liventsev,[93,17]
T. Luo,[11] J. MacNaughton,[52] M. Masuda,[87,72] T. Matsuda,[52] D. Matvienko,[4,65,43] M. Merola,[30,56] F. Metzner,[34]

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

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Search for baryon number violation in top quark production and decay using proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration[*]

Jun 2024

BNV decays of b-flavored hadrons

- Experimental studies of exclusive decays of B-mesons

difficult matrix elements

Branching ratio constraints (on $B_{(s)} \to p\mu^-$ etc.) are in the ballpark of 10^{-9}

- Possible experimental studies of inclusive decays of B-mesons

easy matrix elements

Bonus: kinematical window of lepton energy E_ℓ of about 250 MeV that can only be reached by baryon-number violating decays

$$\frac{m_B}{2}\left(1+\frac{m_\ell^2-4m_N^2}{m_B^2}\right) \leq E_\ell \leq \frac{m_B}{2}\left(1+\frac{m_\ell^2-m_N^2}{m_B^2}\right)$$

A rough estimate of the branching ratio of BNV inclusive B-decay $B \to \ell X$:

$$\Gamma(\bar{B} \to X\ell) = \frac{1}{2m_B}\int d\Pi_{\mathrm{LIPS}}\, |\langle X\ell|\mathcal{H}_{\mathrm{BNV}}|\bar{B}\rangle|^2 \approx \frac{4\pi}{2m_B(2\pi)^3}\int_0^{E_\ell^{\max}} dE_\ell\, \frac{E_\ell}{2}\, \frac{[L\cdot W]}{\Lambda_{\mathrm{BNV}}^4}\,,$$

with $E_\ell^{\max} \approx m_B/2$, also $L \sim E_\ell$, and $W \sim \pi m_B^3/(16\pi^2)$

Branching ratio $\mathcal{B}(\bar{B} \to X\ell) = \frac{\Gamma(\bar{B} \to X\ell)}{\Gamma_{\mathrm{tot}}^B} \approx \frac{m_b^5}{2^{10}\, 3\pi^3 \Lambda_{BNV}^4 \Gamma_{\mathrm{tot}}^B} \approx (8|V_{cb}|G_F\Lambda_{BNV}^2)^{-2}$

- Can any of those decays of B-mesons be seen? What is Λ_{BNV}?

- Matching SMEFT operators to Weak Effective Theory (WET) at the electroweak scale ($\Lambda_{BNV} \gg m_W$)

$$\mathcal{H}_{\rm BNV} = \frac{1}{\Lambda^2_{\rm BNV}} \sum_{p,r,s,t} \sum_{X=L,R} \sum_{Y=L,R,\nu} C^{prst}_{XY} Q^{prst}_{XY} \,,$$

W. Dekens and P. Stoffer

with

$$\begin{aligned}
Q_{RR} &= \varepsilon^{abc}[\widetilde{d}^a_p P_R u^b_r]\,[\widetilde{u}^c_s P_R \ell_t] \quad \left[= \mathcal{O}^{S,RR}_{duu}\right], \\
Q_{RL} &= \varepsilon^{abc}[\widetilde{d}^a_p P_R u^b_r]\,[\widetilde{u}^c_s P_L \ell_t] \quad \left[= \mathcal{O}^{S,RL}_{duu}\right], \\
Q_{LR} &= \varepsilon^{abc}[\widetilde{d}^a_p P_L u^b_r]\,[\widetilde{u}^c_s P_R \ell_t] \quad \left[= \mathcal{O}^{S,LR}_{duu}\right], \\
Q_{LL} &= \varepsilon^{abc}[\widetilde{d}^a_p P_L u^b_r]\,[\widetilde{u}^c_s P_L \ell_t] \quad \left[= \mathcal{O}^{S,LL}_{duu}\right], \\
Q_{R\nu} &= \varepsilon^{abc}[\widetilde{d}^a_p P_R u^b_r]\,[\widetilde{d}^c_s P_L \nu_t] \quad \left[= \mathcal{O}^{S,RL}_{dud}\right], \\
Q_{L\nu} &= \varepsilon^{abc}[\widetilde{d}^a_p P_L u^b_r]\,[\widetilde{d}^c_s P_L \nu_t] \quad \left[= -\mathcal{O}^{S,LL}_{udd}\right].
\end{aligned}$$

generation indices

- Working assumption: $C^{prst}_{XY} = 0\,, \quad \text{for} \quad p, r, s \neq 3\,,$

- What operators would potentially allow for $\Lambda_{\rm BNV} \ll \Lambda_{\rm GUT}$ without anomalously small Wilson coefficients?

- Operators with left-handed b-quarks and first-family quarks

Tree level matching (with our choice of SMEFT operators): rotation of the down-type quarks by CKM

E.g., for $Q_{R\nu}^{113}$ (in WET) generated by Q_{duq}^{prs} (in SMEFT): $C_{RL}^{prs} = C_{duq}^{prs}\,, \quad C_{R\nu}^{prs} = -C_{duq}^{prv} V_{vs}$

Thus, $Q_{R\nu}^{113}$ (in WET) depends on the same building blocks in SMEFT as the light-quark only BNV operators Q_{RL}^{111}, $Q_{R\nu}^{111}$, and $Q_{R\nu}^{112}$:

$$
\begin{aligned}
C_{RL}^{111} &= C_{duq}^{111}\,,\\
C_{R\nu}^{111} &= -V_{ud} C_{duq}^{111} - V_{cd} C_{duq}^{112} - V_{td} C_{duq}^{113}\,,\\
C_{R\nu}^{112} &= -V_{us} C_{duq}^{111} - V_{cs} C_{duq}^{112} - V_{ts} C_{duq}^{113}\,,\\
C_{R\nu}^{113} &= -V_{ub} C_{duq}^{111} - V_{cb} C_{duq}^{112} - V_{tb} C_{duq}^{113}
\end{aligned}
$$

The only suppression comes from the CKM parameters, e.g., $\sqrt{|V_{ub}|}$, ruling out $\Lambda_{\rm BNV} \ll \Lambda_{\rm GUT}$

- Operators with right-handed b-quarks and first family quarks

$$
\begin{aligned}
Q_{RR}^{311} &= \varepsilon^{abc} [\widetilde{b}^a P_R u^b]\, [\widetilde{\ell} P_R u^c]\,,\\
Q_{RL}^{311} &= \varepsilon^{abc} [\widetilde{b}^a P_R u^b]\, [\widetilde{\ell} P_L u^c]\,,\\
Q_{R\nu}^{311} &= \varepsilon^{abc} [\widetilde{b}^a P_R u^b]\, [\widetilde{\nu} P_L d^c]\,.
\end{aligned}
$$

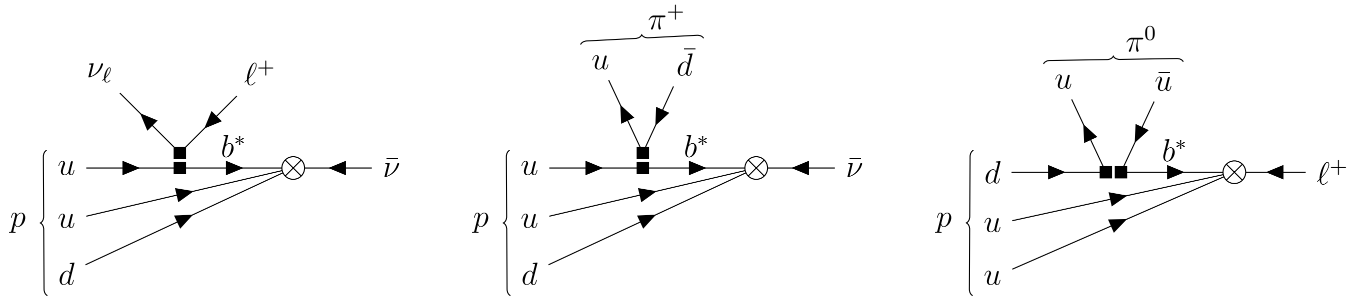
with

$$
\begin{aligned}
C_{RR}^{311} &= C_{duu}^{311}\,,\\
C_{RL}^{311} &= C_{duq}^{311}\,,\\
C_{R\nu}^{311} &= -V_{ud} C_{duq}^{311} - V_{cd} C_{duq}^{312} - V_{td} C_{duq}^{313}
\end{aligned}
$$

The operators with right-handed b-quarks do not mix with light BNV operators at dim-6 level: study those

- The relevant proton decay Hamiltonian contains the baryon-number violating part and the Standard Model Hamiltonian for b-decays

$$\mathcal{H}_{\rm BNV} = \frac{1}{\Lambda^2_{\rm BNV}}\left(C_L Q^{311}_{RL} + C_R Q^{311}_{RR} + C_\nu Q^{311}_{R\nu}\right) + {\rm h.c.}\,.$$



- The SM Hamiltonians for $b \to u\ell^-\bar\nu_\ell$ and $b \to u\bar u d$ decays

$$\mathcal{H}_{\rm sl} = 4\frac{G_F}{\sqrt{2}} V_{ub} Q_{\rm sl} + {\rm h.c.}\,, \qquad \text{with} \quad Q^\dagger_{\rm sl} = [\bar b\gamma^\mu P_L u]\,[\bar\nu_\ell\gamma_\mu P_L \ell]\,.$$

$$\mathcal{H}_W = 4\frac{G_F}{\sqrt{2}} V_{ub}V^*_{ud}\,(C_1 Q_1 + C_2 Q_2) + {\rm h.c.}\,, \quad \text{with} \quad \begin{aligned} Q^\dagger_1 &= [\bar b\gamma^\mu P_L T^A u]\,[\bar u\gamma_\mu P_L T^A d]\,,\\ Q^\dagger_2 &= [\bar b\gamma^\mu P_L u]\,[\bar u\gamma_\mu P_L d]\,.\end{aligned}$$

Contact six-fermion interaction

- Evolving the WET operators down to m_b and integrating out b-quark, we get the local effective Hamiltonian $\mathcal{H}_{6\mathrm{f}} = \mathcal{H}_{p\to\ell^+\nu_\ell\bar{\nu}} + \mathcal{H}_{p\to\pi^+\bar{\nu}} + \mathcal{H}_{p\to\pi^0\ell^+}$,

$$\mathcal{H}_{p\to\ell^+\nu_\ell\bar{\nu}} = -2\sqrt{2}\,\frac{G_F C_\nu V_{ub}^*}{m_b\Lambda_{\rm BNV}^2}\,\mathcal{O}_{\nu,\rm sl} + \text{h.c.}\,,$$

$$\mathcal{O}_{\nu,\rm sl} = \varepsilon^{abc}[\widetilde{u}^a\gamma^\mu P_L u^b]\,[\widetilde{\nu}P_L d^c]\,[\bar{\nu}_\ell\gamma_\mu P_L\ell]\,,$$

$$\mathcal{H}_{p\to\pi^+\bar{\nu}} = -2\sqrt{2}\,\frac{G_F C_\nu V_{ub}^* V_{ud}}{m_b\Lambda_{\rm BNV}^2}\Big(C_1\mathcal{O}_{\nu,1} + C_2\mathcal{O}_{\nu,2}\Big) + \text{h.c.}\,,$$

$$\mathcal{O}_{\nu,1} = \varepsilon^{abc}[\widetilde{u}^a\gamma^\mu P_L T^A_{bi} u^i]\,[\widetilde{\nu}\,P_L d^c]\,[\bar{u}^f\gamma_\mu P_L T^A_{fj} d^j]\,,$$
$$\mathcal{O}_{\nu,2} = \varepsilon^{abc}[\widetilde{u}^a\gamma^\mu P_L u^b]\,[\widetilde{\nu}\,P_L d^c]\,[\bar{u}^f\gamma_\mu P_L d^f]$$

$$\mathcal{H}_{p\to\pi^0\ell^+} = -2\sqrt{2}\,\frac{G_F V_{ub}^* V_{ud}}{m_b\Lambda_{\rm BNV}^2}\sum_{X=L,R} C_X\Big(C_1\mathcal{O}_{X,1} + C_2\mathcal{O}_{X,2}\Big) + \text{h.c.}\,,$$

$$\mathcal{O}_{X,1} = \varepsilon^{abc}[\widetilde{u}^a\gamma^\mu P_L T^A_{bi} u^i]\,[\widetilde{\ell}\,P_X u^c]\,[\bar{u}^f\gamma_\mu P_L T^A_{fj} d^j]\,,$$
$$\mathcal{O}_{X,2} = \varepsilon^{abc}[\widetilde{u}^a\gamma^\mu P_L u^b]\,[\widetilde{\ell}\,P_X u^c]\,[\bar{u}^f\gamma_\mu P_L d^f]$$

- We need matrix elements of these operators to compute the proton decay rates

- Parameterization of relevant proton matrix elements

$$G_{\alpha\beta\gamma}(p) \equiv \langle 0|\varepsilon^{abc}\widetilde{u}^a_\alpha\, u^b_\beta\, d^c_\gamma\,|p(p)\rangle = \sum_{i,j} f^{ij} M^i_{\beta\alpha}(p)[\Gamma^j u_p(p)]_\gamma$$

with
$$\Gamma^j = \{1, \gamma^5, \gamma_\mu, i\gamma_\mu\gamma^5, \sigma_{\mu\nu}\}$$
$$M^i(p) = \{1, \gamma^5, \not{p}, \not{p}\gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, p_\nu\sigma^{\mu\nu}, \varepsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}\}$$

Parity: $M^i_{\beta\alpha}(p)[\Gamma^j u(p)]_\gamma = -[\gamma^0 M^i(\tilde{p})\gamma^0]_{\beta\alpha}[\gamma^0\Gamma^j\gamma^0 u_p(p)]_\gamma$ (as $u_p(\tilde{p}) = \gamma^0 u_p(p)$)

A/commuting u-fields: $M^i(p) = \mathcal{C}M^i(p)^T\mathcal{C}$

$$G_{\alpha\beta\gamma}(p) = V_P\ \not{p}_{\beta\alpha}[\gamma^5 u(p)]_\gamma + \left(V_A\gamma_\rho + T_A p^\sigma\sigma_{\rho\sigma}\right)_{\beta\alpha}[\gamma^\rho\gamma^5 u(p)]_\gamma + T_T[\sigma_{\rho\sigma}]_{\beta\alpha}[\sigma^{\rho\sigma}\gamma^5 u(p)]_\gamma$$

- … or relating them to the structures computed on the lattice

RQCD, 1903.12590

$$G_{\alpha\beta\gamma}(p) = -\frac{f_p}{4}\left(\not{p}_{\beta\alpha}[\gamma^5 u_p(p)]_\gamma + ip^\nu[\sigma_{\rho\nu}]_{\beta\alpha}[\gamma^\rho\gamma^5 u_p(p)]_\gamma\right)$$
$$+\frac{m_p}{16}(\lambda_1 - f_p)[\gamma_\rho]_{\beta\alpha}[\gamma^\rho\gamma^5 u_p(p)]_\gamma + \frac{m_p}{96}(\lambda_2 - 6f_p)[\sigma_{\rho\sigma}]_{\beta\alpha}[\sigma^{\rho\sigma}\gamma^5 u_p(p)]_\gamma\,.$$

- Parameterization of relevant proton matrix elements

$$G_{\alpha\beta\gamma}(p) = -\frac{f_p}{4}\left(\not{p}_{\beta\alpha}[\gamma^5 u_p(p)]_\gamma + ip^\nu[\sigma_{\rho\nu}]_{\beta\alpha}[\gamma^\rho\gamma^5 u_p(p)]_\gamma\right) + \frac{m_p}{16}(\lambda_1 - f_p)[\gamma_\rho]_{\beta\alpha}[\gamma^\rho\gamma^5 u_p(p)]_\gamma + \frac{m_p}{96}(\lambda_2 - 6f_p)[\sigma_{\rho\sigma}]_{\beta\alpha}[\sigma^{\rho\sigma}\gamma^5 u_p(p)]_\gamma\,.$$

- … or numerically

Decay constants (at 2 GeV for the proton [29])	
$f_p = 3.54^{+0.06}_{-0.04} \cdot 10^{-3}$ GeV^2	$\lambda_1 = -(44.9^{+4.2}_{-4.1}) \cdot 10^{-3}$ GeV^2
$\lambda_2 = 93.4^{+4.8}_{-4.8} \cdot 10^{-3}$ GeV^2	$f_\pi = 130.2$ MeV [30]

RQCD, 1903.12590

- Parameterization of relevant pion matrix elements

$$\langle\pi^+(p)|\bar{u}^a_\alpha(0)d^b_\beta(0)|0\rangle = \frac{i\delta^{ab}}{4N_c}f_\pi(\not{p}\gamma^5 - \mu_\pi\gamma^5)_{\beta\alpha}$$

Leptonic decays of the proton: $p \to \ell^+ \nu_\ell \bar{\nu}$

- The leptonic BNV decay rate is given by

$$\Gamma(p \to \ell^+ \nu_\ell \bar{\nu}) = \frac{4G_F^2 |V_{ub}|^2 |C_\nu|^2}{m_p m_b^2 \Lambda_{\rm BNV}^4} \int d\Pi_{\rm LIPS} \, \frac{1}{2} \sum_{\rm spins} |\langle \ell^+ \nu_\ell \bar{\nu} | \mathcal{O}_{\nu,{\rm sl}} | p \rangle|^2$$

Operators: $\mathcal{O}_{\nu,{\rm sl}} = \frac{1}{2} [\bar{\nu}_\ell \gamma_\mu P_L \ell] \, [\widetilde{\nu} P_L]_\alpha [\mathcal{O}_q^\mu]_\alpha \,, \quad \mathcal{O}_{\nu,{\rm sl}}^\dagger = \frac{1}{2} [\bar{\ell} \gamma_\nu P_L \nu_\ell] \, [\bar{\mathcal{O}}_q^\nu]_\beta \, [P_R \nu^c]_\beta$

with $\mathcal{O}_q^\mu = \varepsilon^{abg} [\widetilde{u}^a \gamma^\mu u^b] \, d^g \,, \quad \bar{\mathcal{O}}_q^\nu = \varepsilon^{abg} [\bar{u}^b \gamma^\nu (u^c)^a] \, \bar{d}^g$

- Compute the matrix element squared…

$$\frac{1}{2} \sum_{\rm spins} |\langle \ell^+ \nu_\ell \bar{\nu} | \mathcal{O}_{\nu,{\rm sl}} | p \rangle|^2 = \frac{1}{4} L_{\mu\nu} \sum_{p \ {\rm spin}} {\rm Tr} \Big[\langle 0 | \mathcal{O}_q^\mu | p \rangle \langle p | \bar{\mathcal{O}}_q^\nu | 0 \rangle \, \not{q} P_L \Big]$$

q is the momentum of antineutrino

with the leptonic tensor $L^{\mu\nu} = p_\ell^\mu p_n^\nu - p_\ell \cdot p_n g^{\mu\nu} + p_\ell^\nu p_n^\mu + i \varepsilon^{\mu\nu\alpha\beta} p_{\ell\,\alpha} p_{n\,\beta}$

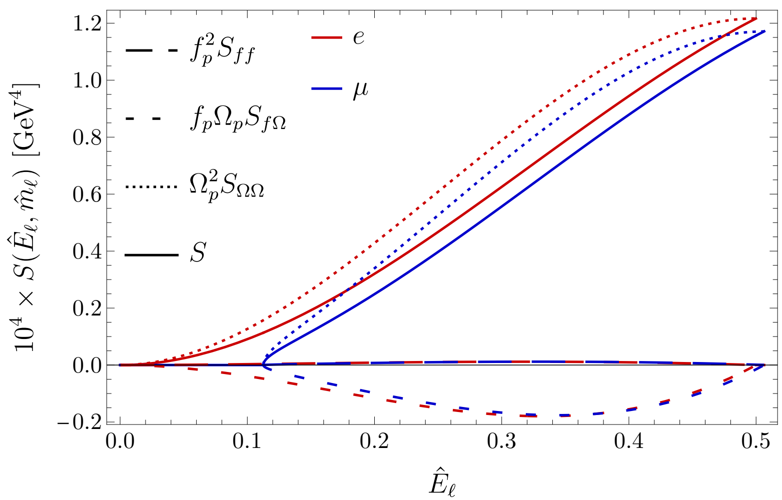
and $\langle 0 | \mathcal{O}_q^\mu | p(p) \rangle = - f_p p^\mu [\gamma^5 u_p(p)] + m_p \Omega_p [\gamma^\mu \gamma^5 u_p(p)]$

phase space:

$$\int d\Pi_{\rm LIPS} = \frac{1}{4(2\pi)^3} \int_0^\infty dE_q \int_{m_\ell}^\infty dE_\ell \, \theta\Big(m_p - E_\ell - E_q - \big| E_q - \sqrt{E_\ell^2 - m_\ell^2} \big| \Big) \times \theta\Big(2E_q + E_\ell + \sqrt{E_\ell^2 - m_\ell^2} - m_p \Big)$$

Leptonic decays of the proton: $p \to \ell^+ \nu_\ell \bar{\nu}$

- Charged lepton spectrum



- … and total decay width (with $\hat{m}_\ell = m_\ell / m_p$)

$$\Gamma(p \to \ell^+ \nu_\ell \bar{\nu}) = |V_{ub}|^2 |C_\nu|^2 \frac{G_F^2 m_p^7}{7680 \pi^3 m_b^2 \Lambda_{\rm BNV}^4} \times \left[(1 - \hat{m}_\ell^2)^5 f_p^2 + \frac{5}{8} (1 - 8\hat{m}_\ell^2 + 8\hat{m}_\ell^6 - \hat{m}_\ell^8 - 24 \hat{m}_\ell^4 \ln \hat{m}_\ell)(\lambda_1^2 - f_p^2) \right]$$

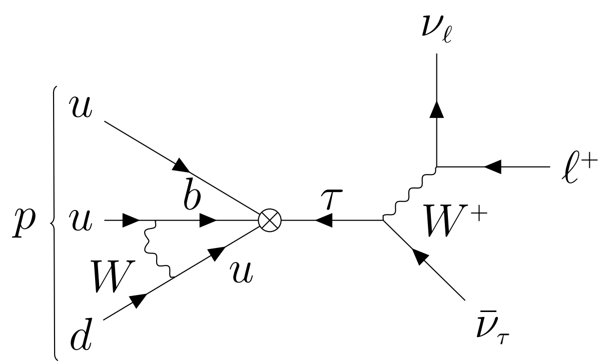
… and constraints on the 3rd generation BNV scale

$$\left. \frac{\Lambda_{\rm BNV}}{\sqrt{|C_\nu|}} \right|_{p \to e^+ \nu_e \bar{\nu}} > 6.59 \cdot 10^9 \text{ GeV} ,$$

$$\left. \frac{\Lambda_{\rm BNV}}{\sqrt{|C_\nu|}} \right|_{p \to \mu^+ \nu_\mu \bar{\nu}} > 6.86 \cdot 10^9 \text{ GeV}$$

Leptonic decays of the proton: $p \to \ell^+ \nu_\ell \bar{\nu}_\tau$

- Note: a contribution of an operator containing both b and τ is allowed

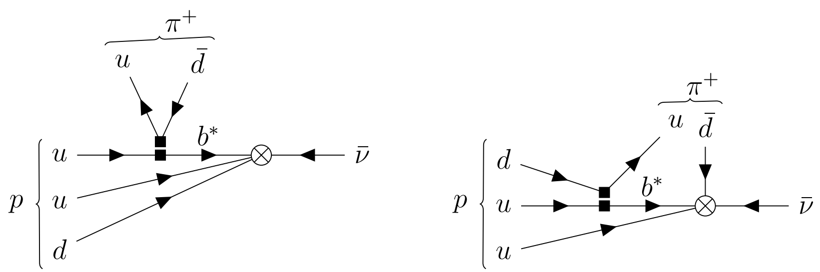


- … can estimate its contribution as

$$\frac{\Lambda_{\rm BNV}}{\sqrt{|C_{RX}^{3113}|}} \approx \left((0.2 \div 4) \frac{|V_{ud}| m_b^2 m_u m_\tau}{4\pi^2 v^2 (m_\tau^2 - m_p^2)} \right)^{1/2} \frac{\Lambda_{\rm BNV}}{\sqrt{|C_\nu|}} \bigg|_{p \to \ell^+ \nu_\ell \bar{\nu}_\tau} \gtrsim (0.4 \div 1.8) \cdot 10^6 \ {\rm GeV}$$

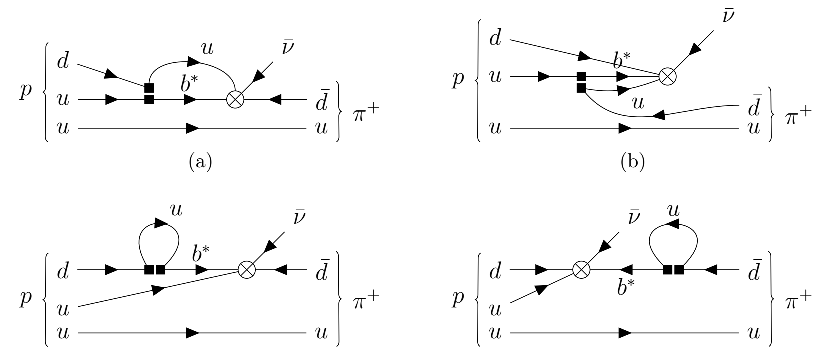
- These effective operators will also mediate two-body proton decays.
- Let's consider $p \to \pi^+\bar{\nu}$

Adopt naive factorization assumption: neglect strong interactions connecting pion to the rest of the process



(a) "colour-allowed" (b) "colour-suppressed"

factorizable



(a) (b) (c) (d)

non-factorizable

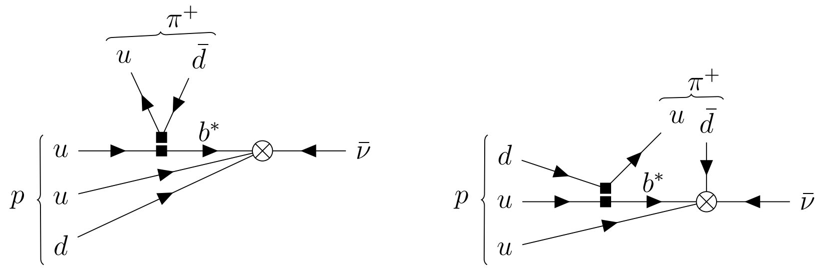
$$\Gamma(p \to \pi^+\bar{\nu}) = \frac{4G_F^2|V_{ub}|^2|V_{ud}|^2|C_\nu|^2}{m_p m_b^2 \Lambda_{\rm BNV}^4} \int d\Pi_{\rm LIPS}\, \frac{1}{2} \sum_{\rm spins} |\langle \pi^+\bar{\nu}| C_1 \mathcal{O}_{\nu,1} + C_2 \mathcal{O}_{\nu,2} |p\rangle|^2$$

Two-body decay: $\int d\Pi_{\rm LIPS} = \frac{1}{8\pi}(1-\hat{m}_\pi^2)$ where for any A: $\hat{A} = A/m_p$

Operators:

$$\mathcal{O}_{\nu,1} = \varepsilon^{abc}[\widetilde{u}^a \gamma^\mu P_L T^A_{bi} u^i]\,[\widetilde{\nu}\, P_L d^c]\,[\bar{u}^f \gamma_\mu P_L T^A_{fj} d^j]\,,$$
$$\mathcal{O}_{\nu,2} = \varepsilon^{abc}[\widetilde{u}^a \gamma^\mu P_L u^b]\,[\widetilde{\nu}\, P_L d^c]\,[\bar{u}^f \gamma_\mu P_L d^f]$$

- Compute the required matrix elements



(a) "colour-allowed" (b) "colour-suppressed"

$$\langle \pi^+(p_\pi)\bar{\nu}(q)|\mathcal{O}_{\nu,1}|p(p)\rangle = \frac{N_c+1}{4N_c^2}\frac{i}{4}m_p^2 f_\pi^2 (f_p + 4\Omega_p)[v^T(q)\mathcal{C}P_L u_p(p)]$$

(only color-suppressed diagram contributes)

$$\langle \pi^+(p_\pi)\bar{\nu}(q)|\mathcal{O}_{\nu,2}|p(p)\rangle = \frac{i}{8N_c}m_p^2 f_\pi \left\{(2N_c\hat{E}_\pi - 1)f_p + (2N_c-4)\Omega_p\right\}[v^T(q)\mathcal{C}P_L u_p(p)]$$

with $\Omega_p = \frac{1}{4}(\lambda_1 - f_p)$

- Thus, the virtual b-quark induces the decay width for $p \to \pi^+ \bar{\nu}$

$$\Gamma(p \to \pi^+\bar{\nu}) = |V_{ud}|^2|V_{ub}|^2|C_\nu|^2 \frac{G_F^2 m_p^5 f_\pi^2}{1024\pi m_b^2 \Lambda_{\rm BNV}^4}(1-\hat{m}_\pi^2)^2 \left[\left((1+2\hat{m}_\pi^2)f_p + \frac{\lambda_1}{3}\right)C_2 + \frac{4}{9}\lambda_1 C_1\right]^2$$

… and a constraint on the 3rd generation BNV scale

$$\left.\frac{\Lambda_{\rm BNV}}{\sqrt{|C_\nu|}}\right|_{p\to\pi^+\bar{\nu}} > 3.34\cdot 10^9 \text{ GeV}$$

Two-body proton decays: $p \to \pi^0 \ell^+$

- Let's consider $p \to \pi^0 \ell^+$

Adopt naive factorization assumption: neglect strong interactions connecting pion to the rest of the process

$$\Gamma(p \to \pi^0 \ell^+) = \frac{4G_F^2 |V_{ub}|^2 |V_{ud}|^2}{m_p m_b^2 \Lambda_{\rm BNV}^4} \int d\Pi_{\rm LIPS} \frac{1}{2} \sum_{\rm spins} \Big| \sum_{X=L,R} \sum_{i=1,2} C_X C_i \langle \pi^0 \ell^+ | \mathcal{O}_{X,i} | p \rangle \Big|^2$$

Two-body decay: $$\int d\Pi_{\rm LIPS} = \frac{1}{8\pi} \sqrt{(1-\hat{m}_\pi^2)^2 - 2\hat{m}_\ell^2 (1+\hat{m}_\pi^2) + \hat{m}_\ell^4}$$

Operators:
$$\mathcal{O}_{X,1} = \varepsilon^{abc} [\widetilde{u}^a \gamma^\mu P_L T^A_{bi} u^i] \, [\widetilde{\ell} P_X u^c] \, [\bar{u}^f \gamma_\mu P_L T^A_{fj} d^j] \, ,$$
$$\mathcal{O}_{X,2} = \varepsilon^{abc} [\widetilde{u}^a \gamma^\mu P_L u^b] \, [\widetilde{\ell} P_X u^c] \, [\bar{u}^f \gamma_\mu P_L d^f]$$

- Compute the required matrix elements

$$\langle \pi^0(p_\pi) \ell^+(q) | \mathcal{O}_{X,1} | p(p) \rangle = \frac{i}{16\sqrt{2} N_c} m_p^2 f_\pi \left[A_1^{XL} M_L + A_1^{XR} M_R \right]$$
$$\langle \pi^0(p_\pi) \ell^+(q) | \mathcal{O}_{X,2} | p(p) \rangle = \frac{i}{16\sqrt{2} N_c} m_p^2 f_\pi \left[A_2^{XL} M_L + A_2^{XR} M_R \right]$$

with $M_X \equiv [v^T(q) \mathcal{C} P_X u_p(p)]$

- The coefficients of the matrix elements are

$$A_1^{XL} = \frac{4}{3}\left[\left(f_p\left(1-2\hat{m}_\ell^2+2\hat{m}_\pi^2\right)+\lambda_2\hat{\mu}_\pi\right)\delta_{XL}+2f_p\hat{m}_\ell\delta_{XR}\right],$$
$$A_1^{XR} = \frac{4}{3}\left[f_p\hat{m}_\ell\,\delta_{XL}+\left(2f_p\left(1-2\hat{m}_\ell^2+2\hat{m}_\pi^2\right)-3\lambda_1\hat{\mu}_\pi\right)\delta_{XR}\right],$$
$$A_2^{XL} = \left(f_p\left(1-2\hat{m}_\ell^2+2\hat{m}_\pi^2\right)-3\lambda_1-2\lambda_2\hat{\mu}_\pi\right)\,\delta_{XL}+2f_p\hat{m}_\ell\,\delta_{XR},$$
$$A_2^{XR} = \left(2f_p\left(1-2\hat{m}_\ell^2+2\hat{m}_\pi^2\right)+6\lambda_1\hat{\mu}_\pi\right)\,\delta_{XR}+\left(f_p\hat{m}_\ell+3\lambda_1\hat{m}_\ell\right)\,\delta_{XL}$$

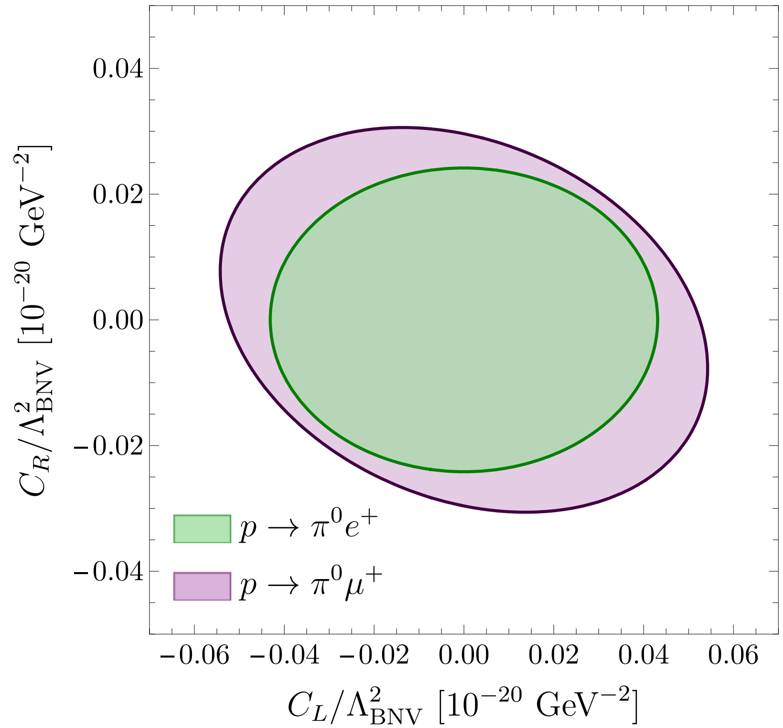
- Thus, the virtual b-quark induces the decay width for $p \to \pi^0 \ell^+$

$$\Gamma(p\to\pi^0\ell^+) = \frac{G_F^2|V_{ub}|^2|V_{ud}|^2 f_\pi^2 m_p^5}{9216\pi m_b^2\Lambda_{\rm BNV}^4}\sqrt{(1-\hat{m}_\pi^2)^2-2\hat{m}_\ell^2(1+\hat{m}_\pi^2)+\hat{m}_\ell^4}\sum_{X,Y=L,R} C_X C_Y^* \times \sum_{i,j=1,2} C_i C_j\left[\left(A_i^{XL}A_j^{YL}+A_i^{XR}A_j^{YR}\right)\hat{E}_\ell+\left(A_i^{XR}A_j^{YL}+A_i^{XL}A_j^{YR}\right)\hat{m}_\ell\right]$$

… and constraints on the 3rd generation BNV scale

$$\Lambda_{\rm BNV}\Big|_{p\to\pi^0 e^+} > 6.44\cdot 10^{10}\ \text{GeV}\left(|C_R^e|^2+0.0014\text{Re}[C_L^{e\,*}C_R^e]+0.314|C_L^e|^2\right)^{1/4},$$
$$\Lambda_{\rm BNV}\Big|_{p\to\pi^0 \mu^+} > 5.82\cdot 10^{10}\ \text{GeV}\left(|C_R^\mu|^2+0.285\text{Re}[C_L^{\mu\,*}C_R^\mu]+0.318|C_L^\mu|^2\right)^{1/4}$$

- The allowed region in the $C_L - C_R$ space



Real Wilson coefficients C_L and C_R are assumed; the reference scale $\Lambda_{\rm BNV} = 10^{10}$ GeV

Summary of limits

- All together, the limits on the scale of BNV operators containing right-handed b-quark

Exp. constraint	C	$\Lambda_{\rm BNV}/\sqrt{\lvert C\rvert}$ [10^9 GeV]
dim-8 matching	C_Y	$> \mathcal{O}(1)$
$p \to e^+ \nu_e \bar{\nu}$	C_ν	> 6.59
$p \to \mu^+ \nu_\mu \bar{\nu}$	C_ν	> 6.86
$p \to \pi^+ \bar{\nu}$	C_ν	> 3.34
$p \to \pi^0 e^+$	C_R^e	> 64.4
$p \to \pi^0 e^+$	C_L^e	> 48.2
$p \to \pi^0 \mu^+$	C_R^μ	> 58.2
$p \to \pi^0 \mu^+$	C_L^μ	> 43.7
$p \to \ell^+ \nu_\ell \bar{\nu}_\tau$	C_{RX}^{3113}	$> (0.4 \div 1.8) \cdot 10^{-3}$

- With that, what about those B-decays?

$$\mathcal{B}(\bar{B} \to X\ell) \approx \frac{m_b^5}{2^{10}\, 3\pi^3 \Lambda_{\rm BNV}^4 \Gamma_{\rm tot}^B} \approx (8|V_{cb}|G_F \Lambda_{\rm BNV}^2)^{-2} \lesssim O(5 \cdot 10^{-29})$$

$$\mathcal{B}(\bar{B} \to X\tau) \lesssim \mathcal{O}(10^{-13} \div 10^{-15})$$ (for the operators containing both b and τ)

- Computed BNV proton decay processes $p \to \ell^+ \nu_\ell \bar{\nu}$, $p \to \pi^+ \bar{\nu}$, and $p \to \pi^0 \ell^+$
  - assumed BNV is flavor-specific and occurs at $\Lambda_{\rm BNV} \ll \Lambda_{\rm GUT}$
  - similar studies for the operators involving the tau lepton

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- Derived bounds on $\Lambda_{\rm BNV}$ for the operators containing b-quark
  - experiment: $\Lambda_{\rm BNV} > 10^9 - 10^{10}$ GeV, ruling out BNV at $\Lambda_{\rm flavor}$
  - lower scales are possible ($\Lambda_{\rm BNV} \sim 10^6$ GeV) if tau-lepton is also involved

- Obtained limits rule out any possibility of observing BNV directly in B-decays


EXTRA!

Numerical inputs

Masses [1]	
$m_b = 4.8$ GeV $m_e = 0.511$ MeV $m_\tau = 1.777$ GeV $m_u(2\text{ GeV}) = 2.16$ MeV	$m_p = 0.938$ GeV $m_\mu = 105.66$ MeV $m_\pi = 139.57$ MeV $m_d(2\text{ GeV}) = 4.67$ MeV
Coupling constants [1]	
$G_F = 1.1663788 \cdot 10^{-5}\ \text{GeV}^{-2}$	$\alpha_s^{(5)}(m_Z) = 0.1179$
CKM matrix elements	
$\|V_{ud}\| = 0.9737$ [27]	$\|V_{ub}\| = 3.77 \cdot 10^{-3}$ [28]
Decay constants (at 2 GeV for the proton [29])	
$f_p = 3.54^{+0.06}_{-0.04} \cdot 10^{-3}\ \text{GeV}^2$ $\lambda_2 = 93.4^{+4.8}_{-4.8} \cdot 10^{-3}\ \text{GeV}^2$	$\lambda_1 = -(44.9^{+4.2}_{-4.1}) \cdot 10^{-3}\ \text{GeV}^2$ $f_\pi = 130.2$ MeV [30]
Wilson coefficients [22]	
$C_1(1\text{ GeV}) = -0.829$	$C_2(1\text{ GeV}) = 1.050$
Partial lifetimes (90% CL) & decay rates	
$\tau_{p\to e^+\nu\nu} > 1.7 \cdot 10^{32}$ yr [12] $\tau_{p\to\pi^+\bar{\nu}} > 3.9 \cdot 10^{32}$ yr [11] $\tau_{p\to\pi^0 e^+} > 2.4 \cdot 10^{34}$ yr [13]	$\tau_{p\to\mu^+\nu\nu} > 2.2 \cdot 10^{32}$ yr [12] $\tau_{p\to\pi^0\mu^+} > 1.6 \cdot 10^{34}$ yr [13] $\Gamma^B_{\text{tot}} = 4.4 \cdot 10^{-13}$ GeV [1]