Strongly Interacting Spinful Particles in 1D

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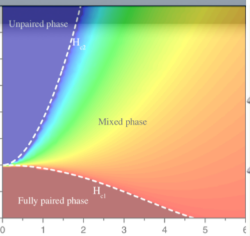
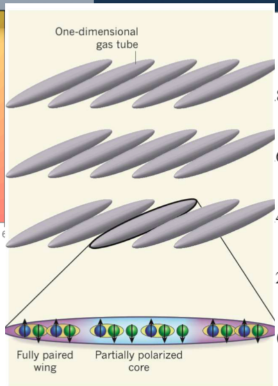
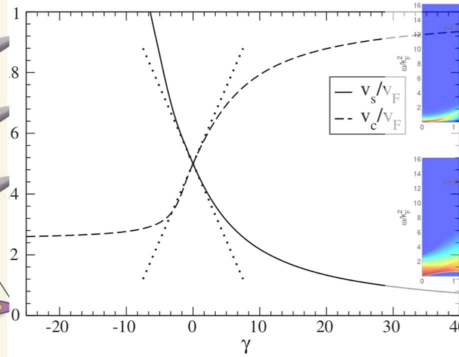
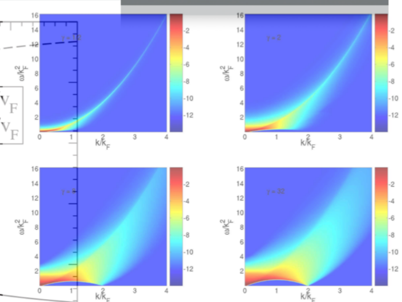
INT, Seattle, WA Oct., 2024

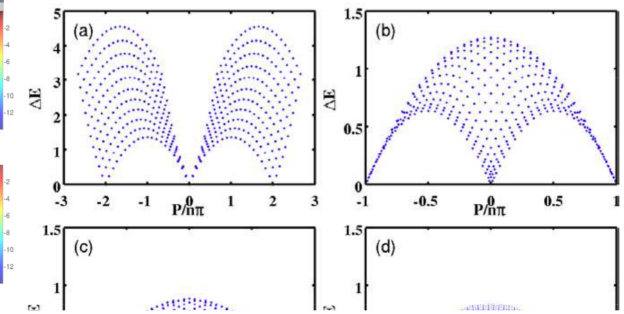
1D quantum gases





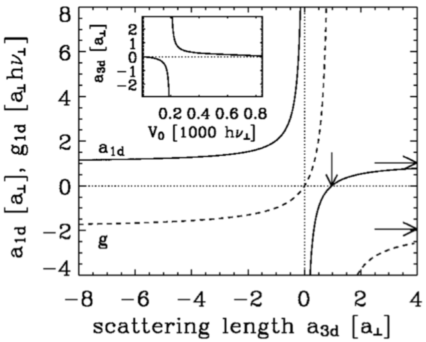




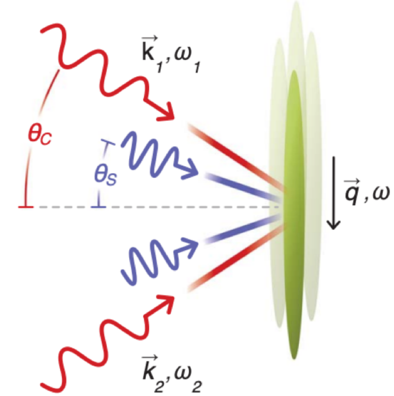


REVIEWS OF MODERN PHYSICS, VOLUME 85, OCTOBER–DECEMBER 2013

Fermi gases in one dimension: From Bethe ansatz to experiments

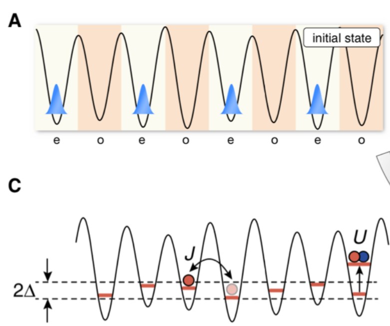


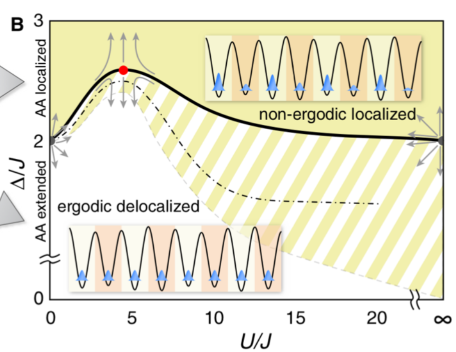
$$H = \underbrace{\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2} + V(x_i)\right]}_{H_f} + \underbrace{\hat{g}\sum_{i<j}\delta(x_i - x_j)}_{H_{\rm int}}$$

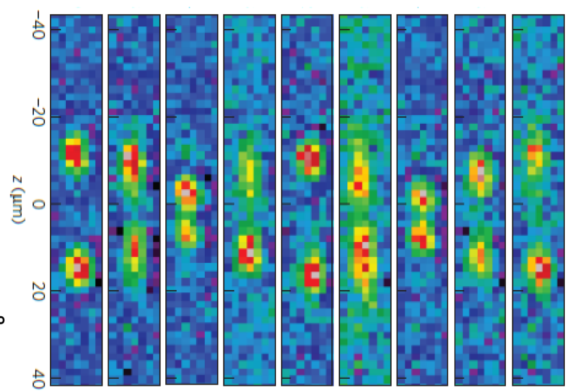


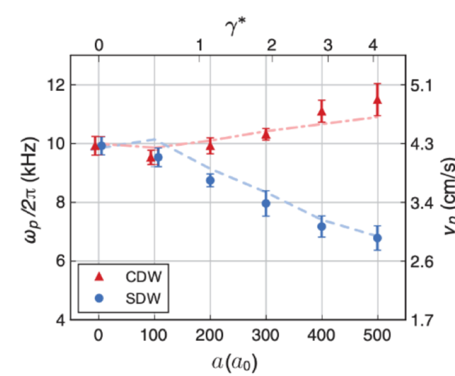
REVIEWS OF MODERN PHYSICS, VOLUME 83, OCTOBER–DECEMBER 2011

One dimensional bosons: From condensed matter systems to ultracold gases









Motivation and Outline

Lower dimension → More quantum

Stronger interaction → More quantum

<u>Outline:</u>

- Strong coupling ansatz w.f. of 1D strongly interacting spinor gas
- Observation of spin-charge separation in 1D Fermi gas

Hard-core spinless bosons: fermionization

$$H = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) \right] + g\sum_{i<j}\delta(x_i - x_j)$$

$g \to \infty$

$$\Psi_B(x_1, x_2, \ldots, x_N)\Big|_{x_i = x_j} = 0$$

Bose-Fermi mapping:

1D hard-core spinless bosons → free fermions

$$\Psi_B(x_1, \ldots, x_N) = \left[\prod_{i>j} \mathrm{sgn}(x_i - x_j) \right] \Psi_F^0(x_1, \ldots, x_N)$$

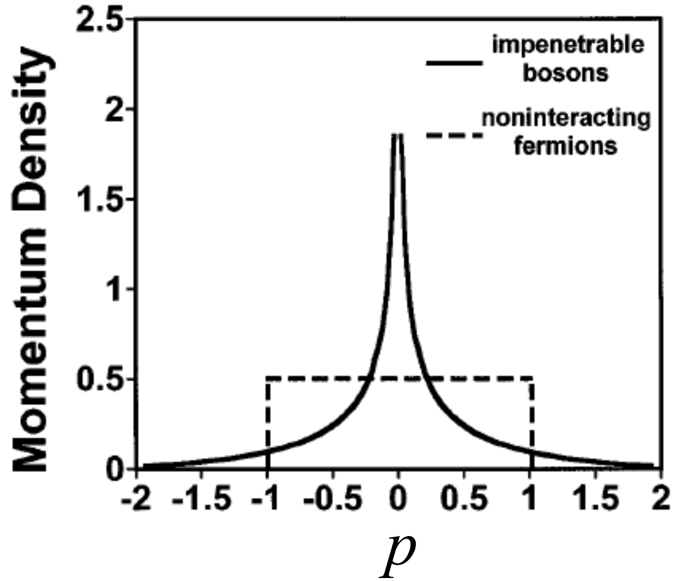
Girardeau, J. Math. Phys. **1**, 516 (1960)

Hard-core spinless bosons: fermionization

Bose-Fermi mapping:

$$\Psi_B\left(x_1,\ldots,x_N\right)=\left[\prod_{i>j}\mathrm{sgn}(x_i-x_j)\right]\Psi_F^0\left(x_1,\ldots,x_N\right)$$

Girardeau, J. Math. Phys. **1**, 516 (1960)



Oshanii, PRL **81**, 938 (1998)

Physical quantities directly related to the w.f. are in general different!

Hard-core spinless anyon: wavefunction

$$\Psi^\kappa(\cdots x_j, x_{j+1}, \cdots) = e^{i\pi\kappa\epsilon(x_{j+1}-x_j)}\, \Psi^\kappa(\cdots x_{j+1}, x_j, \cdots)$$

$$\epsilon(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

$\kappa = 0$: boson

$\kappa = 1$: fermion

Exp. realization (Greiner group) arXiv:2306.01737

Spinless anyons can help us understand strongly interacting spinor gas!

Anyon-fermion mapping: hard-core anyons also fermionize!

$$\Psi^\kappa(x_1,\ldots,x_N) = \left[\prod_{1\le i<j\le N} A^\kappa(x_j - x_i)\right]\Psi_F^0(x_1,\ldots,x_N)$$

$$A^\kappa(x_j - x_i) = e^{i\pi(1-\kappa)\theta(x_j - x_i)}$$

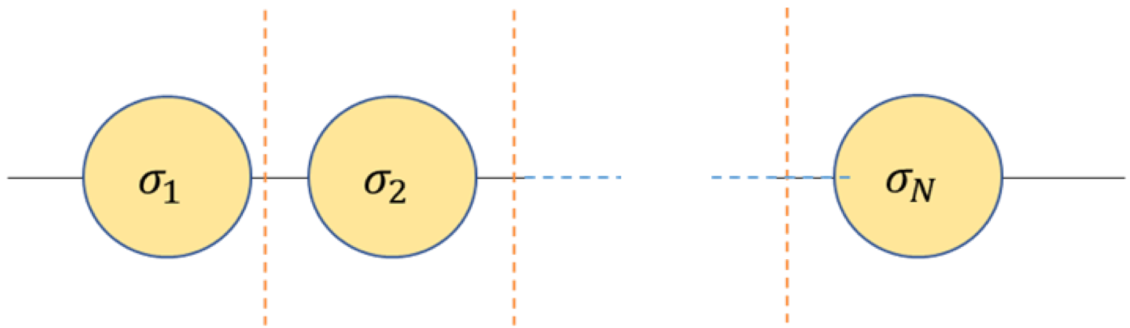
Girardeau, PRL **97**, 100402 (2006)

Hard-core particles with spin

$$H=\sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}+V(x_i)\right]+g\sum_{i<j}\delta(x_i-x_j)$$

$$g\to\infty$$

$$\Psi(x_1,\sigma_1;x_2,\sigma_2;\ldots;x_N,\sigma_N)\Big|_{x_i=x_j}=0$$



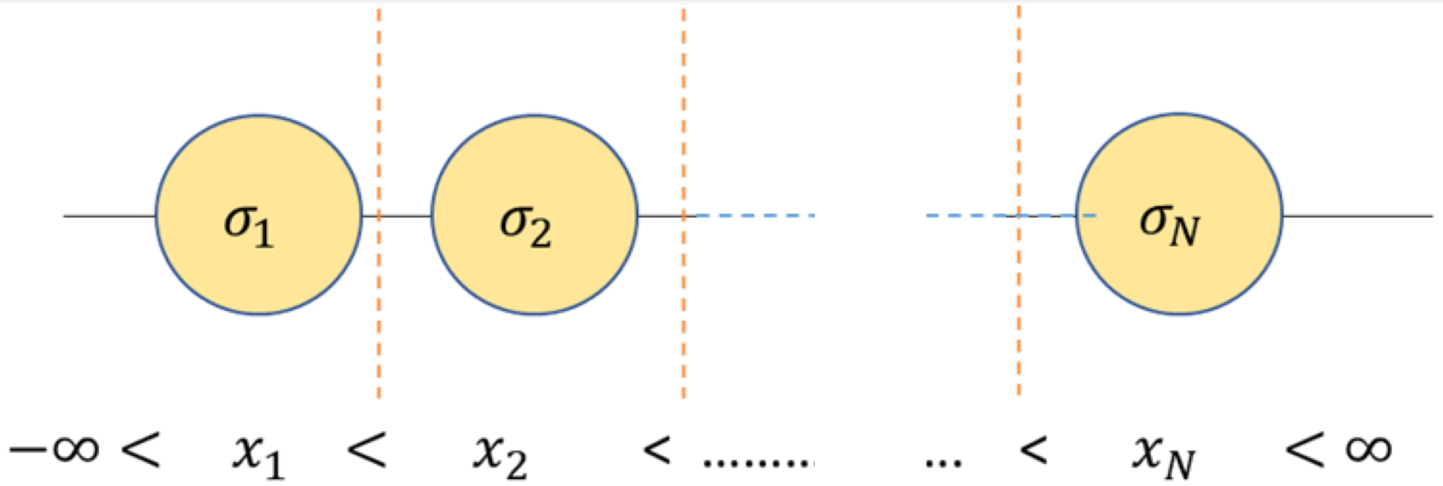
$-\infty < \quad x_1 \quad < \quad x_2 \quad < \ldots\ldots\ldots \quad \ldots \quad < \quad x_N \quad < \infty$

w.f. in spatial sector with $x_1<x_2<\cdots<x_N$

$\Psi_F^0(x_1,x_2,\cdots,x_N)\theta^1\otimes\chi$ $\qquad\chi$: arb. spin w.f.

$\theta^1=1,$ if $x_1<x_2<\cdots<x_N \quad$ (0, otherwise)

Hard-core particles with spin



w.f. in one spatial sector determines the full w.f.

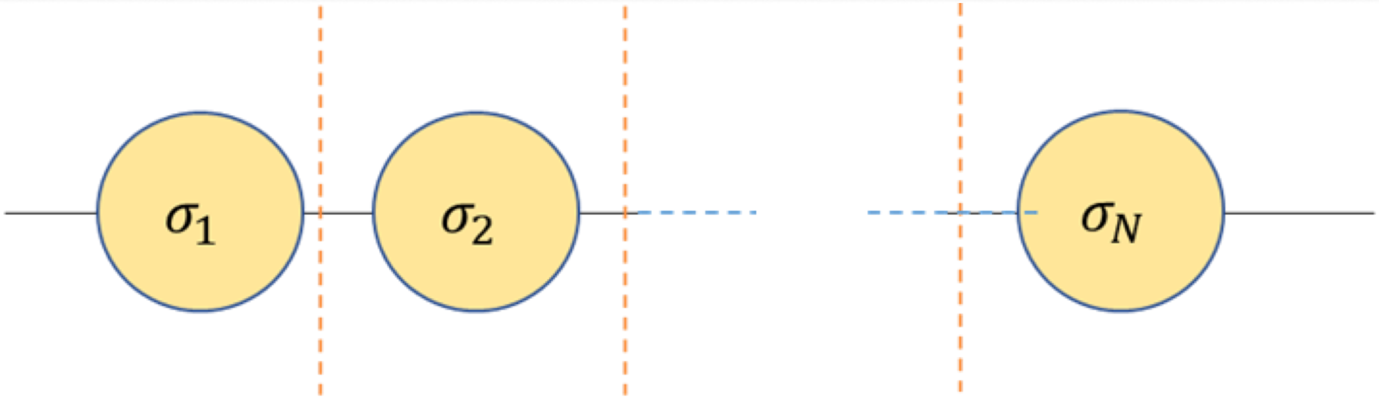
For N spinful bosons or fermions: strong coupling ansatz

$$\Psi\left(x_1,\sigma_1;x_2,\sigma_2;\ldots;x_N,\sigma_N\right)=\sum_P(\pm1)^P P\left[\Psi_F^0(x_1,x_2,\cdots,x_N)\theta^1\otimes\chi\right]$$

spin degeneracy: $(2s+1)^N$

Deuretzbacher *et al.*, PRL **100**, 160405 (2008)
Guan, Chen, Wang, and Ma, PRL **102**, 160402 (2009)

Away from hard-core limit

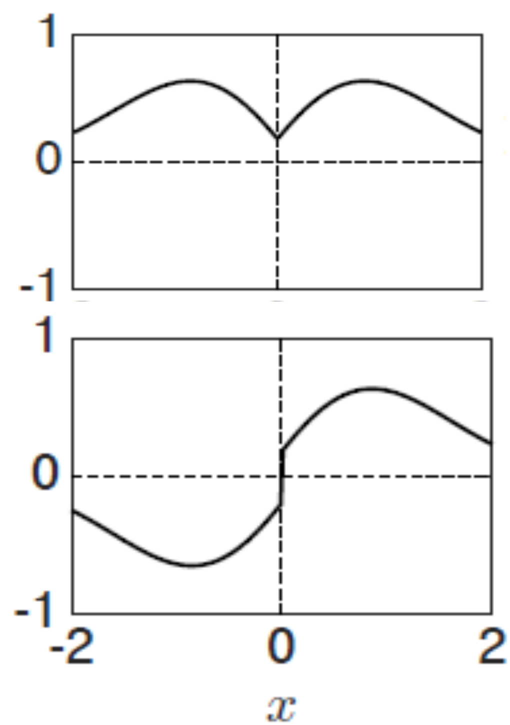


$$-\infty < x_1 < x_2 < \dots\dots\dots \quad \dots < x_N < \infty$$

$$V_B = g\sum_{i<j}\delta\left(x_i - x_j\right)$$

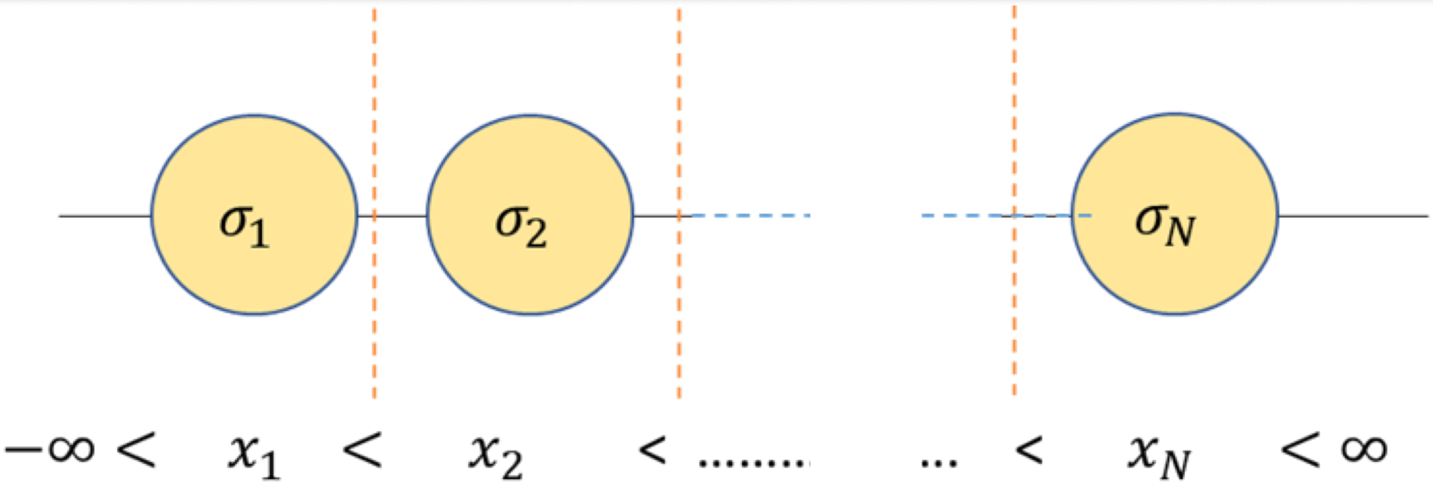
$$\Updownarrow$$

$$V_F = -\frac{4}{g}\sum_{i<j}\overleftarrow{\partial}_{x_{ij}}\delta(x_{ij})\overrightarrow{\partial}_{x_{ij}}, \qquad x_{ij} = x_i - x_j$$



Cheon and Shigehara, PRL **82**, 2536 (1999)

Away from hard-core limit



For N spinful bosons or fermions:

$$\Psi\left(x_1,\sigma_1;x_2,\sigma_2;\ldots;x_N,\sigma_N\right)=\sum_P(\pm 1)^P P\left[\Psi_F^0(x_1,x_2,\cdots,x_N)\theta^1\otimes\chi\right]$$

Spin w.f. χ determined by an effective spin-chain Hamiltonian:

$$H_{\text{eff}}=E_\infty-\frac{1}{g}\sum_{i=1}^{N-1}C_i\left(1\pm\mathcal{E}_{i,i+1}\right)$$

$$C_i=2\cdot S\int\left(\prod_{j=1}^{N}dx_j\right)\left|\partial_i\varphi_0\right|^2\theta^1\delta\left(x_{i+1}-x_i\right)$$

Charge d.o.f. → spinless fermion
Spin d.o.f. → spin-chain

Yang, Guan, HP, PRA **91**, 043634 (2015)

One-Body Density Matrix

Given many-body wavefunction: $\Psi(x_1,\sigma_1;\cdots;x_N,\sigma_N)$

the one-body density matrix (OBDM) is defined as:

$$\rho_{\sigma,\sigma'}(x,x') = \sum_{\sigma_2,\ldots,\sigma_N} \int dx_2 \cdots dx_N \; \Psi^*(x,\sigma;x_2,\sigma_2;\cdots;x_N,\sigma_N)\Psi(x',\sigma';x_2,\sigma_2;\cdots;x_N,\sigma_N)$$

Difficulty in evaluating OBDM:

(N-1)-dim spatial integral

Many-body wavefunction Ψ is very complicated

One-Body Density Matrix

For strong coupling ansatz,

$$\Psi\left(x_1,\sigma_1;x_2,\sigma_2;\ldots;x_N,\sigma_N\right)=\sum_P(\pm1)^P P\left[\Psi_F^0(x_1,x_2,\cdots,x_N)\theta^1\otimes\chi\right]$$

$$\rho_{\sigma,\sigma'}(x,x')=\sum_{m,n}\rho_{m,n}(x,x')S_{m,n}(\sigma,\sigma')$$

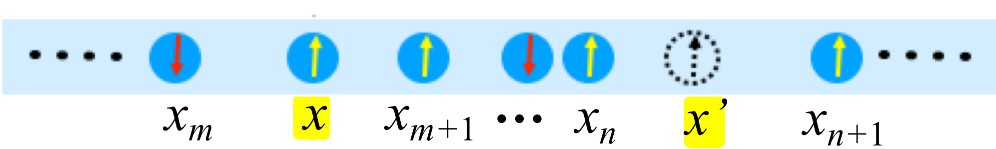
Yang, Guan, and HP, PRA **91**, 043634 (2015)

$$S_{m,n}(\sigma,\sigma')=(\pm1)^{n-m}\left\langle\chi\left|c_{\sigma,m}^{\dagger}c_{\sigma',m}(m\cdots n)\right|\chi\right\rangle$$

loop permutation $(m\cdots n)$: $\quad m\to m+1\to m+2\ldots\to n,\quad n\to m$

$$\rho_{m,n}(x,x')=(-1)^{n-m}(N-1)!\int_{\Gamma_{m,n}}dx_2\cdots dx_N\ \Psi_F^{0\,*}(x,x_2,\cdots,x_N)\Psi_F^0(x',x_2,\cdots,x_N)$$

$$\Gamma_{m,n}:x_2<\cdots<x_m<x<x_{m+1}<\cdots<x_n<x'<x_{n+1}<\cdots<x_N$$



One-Body Density Matrix

$$\rho_{m,n}(x,x') = (-1)^{n-m}(N-1)!\int_{\Gamma_{m,n}} dx_2 \cdots dx_N \ \Psi_F^{0\,*}(x,x_2,\cdots,x_N)\Psi_F^0(x',x_2,\cdots,x_N)$$

$$\Gamma_{m,n} : x_2 < \cdots < x_m < x < x_{m+1} < \cdots < x_n < x' < x_{n+1} < \cdots < x_N$$

Take a discrete Fourier transformation:

$$\rho_{m,n}(x',x) = N^{-2}\sum_{\kappa,\kappa'} \rho^{\kappa,\kappa'}(x',x) e^{i\pi\kappa' m} e^{-i\pi\kappa n}$$

$$\kappa,\ \kappa' = 2j/N; \quad j = 0,\ 1,\ \ldots,\ N-1$$

$$\rho^{\kappa,\kappa'}(x,x') = N\int dx_2 \cdots dx_N \prod_{j=2}^{N} A^{\kappa *}(x_j - x)\Psi_F^{0\,*}(x,x_2,\cdots,x_N)$$

$$\times\ A^{\kappa'}(x_j - x')\Psi_F^0(x',x_2,\cdots,x_N)$$

$$A^{\kappa}(x_j - x_i) = e^{i\pi(1-\kappa)\theta(x_j - x_i)}$$

$\rho^{\kappa,\kappa}(x',x)$ is the OBDM of hardcore spinless anyons with w.f.

$$\Psi^{\kappa}(x_1,\ldots,x_N) = \left[\prod_{1\le i<j\le N} A^{\kappa}(x_j - x_i)\right]\Psi_F^0(x_1,\ldots,x_N)$$

One-Body Density Matrix

Homogeneous system with periodic boundary condition:

$$\rho_{m,n}(x',x) = \rho_{n-m}(x'-x) = N^{-1}\sum_{\kappa}\rho^{\kappa}(x'-x)e^{i\pi\kappa(n-m)}$$

$$\kappa = 2j/N; \quad j = 0,\ 1,\ \ldots,\ N-1$$

$\rho^{\kappa}(x'-x)$: OBDM of homogeneous hard-core spinless anyon

$$\rho_{\sigma',\sigma}(x',x) = \sum_{\kappa}\rho^{\kappa}(x'-x)S^{\kappa}(\sigma',\sigma);$$

$$S^{\kappa}(\sigma',\sigma) = N^{-1}\sum_{n-m}S_{m,n}e^{-i\pi\kappa(n-m)}$$

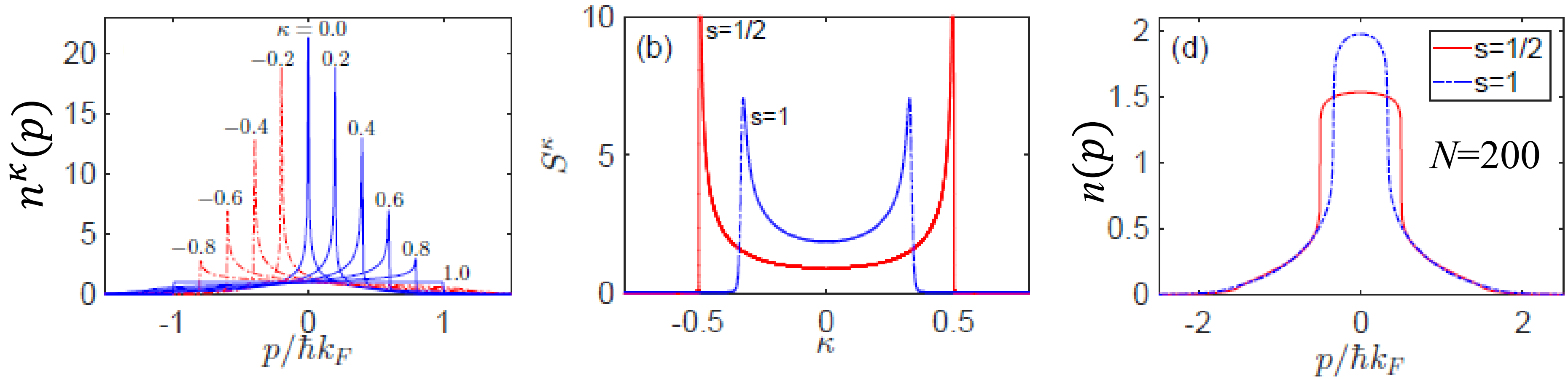
Yang, and HP, PRA **95**, 051602(R) (2017)

Generalization to arb. trapping potential: Patu, arXiv:2408.06060

Momentum Distribution

Momentum distribution: $n_\sigma(p) = \frac{N}{2\pi}\iint dx dx' e^{ip(x-x')}\rho_{\sigma,\sigma}(x,x')$

$$n_\sigma(p) = \sum_\kappa n^\kappa(p) S^\kappa(\sigma,\sigma)$$



Santachiara and Calabrese, J. Stat. Mech. P06005 (2008)

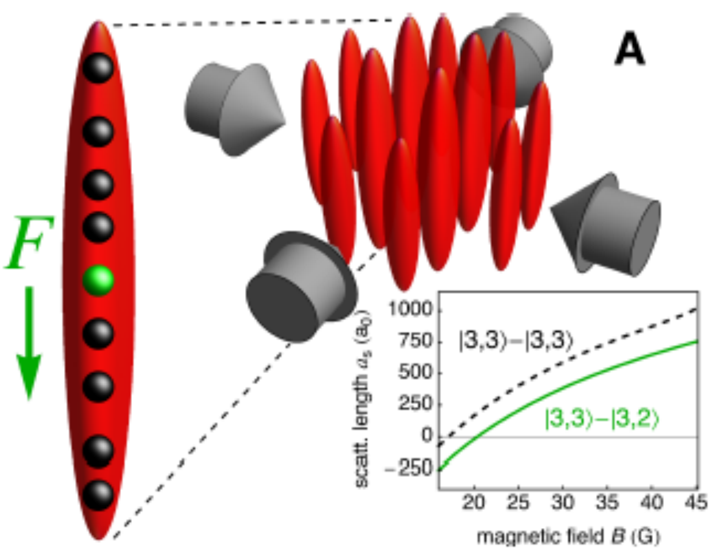
Yang, and HP, PRA **95**, 051602(R) (2017)

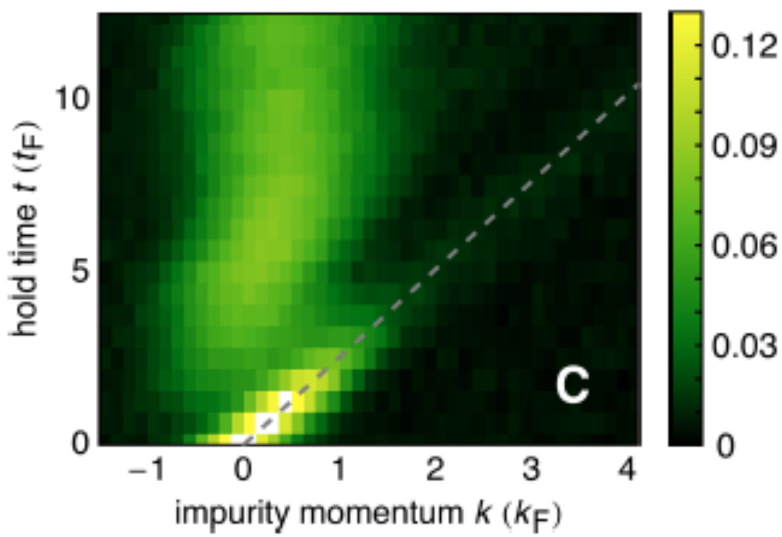
An example: single impurity in a TG gas

Bloch oscillations in the absence of a lattice

Florian Meinert,[1] Michael Knap,[2] Emil Kirilov,[1] Katharina Jag-Lauber,[1] Mikhail B. Zvonarev,[3] Eugene Demler,[4] Hanns-Christoph Nägerl[1*]

Science **356**, 945 (2017)





For a single impurity:

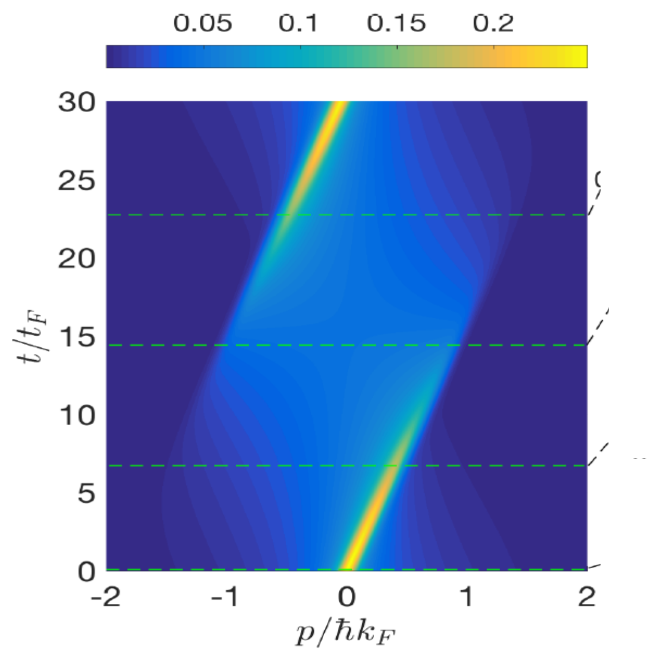
$$H_{\rm eff} = -\frac{1}{g}\sum_{i=1}^{N-1} C_i\left(1+\mathcal{E}_{i,i+1}\right) \Longleftrightarrow H_{\rm sc} = -\frac{\pi}{\sqrt{2N}\gamma_i}\sum_{j=1}^{N-1} C_j[c_j^\dagger c_{j+1} + \text{H.c.}] + \left[\frac{1}{\pi}\sqrt{2N}\right]^3 \mathcal{F}\sum_{j=1}^{N-1} D_j n_j$$

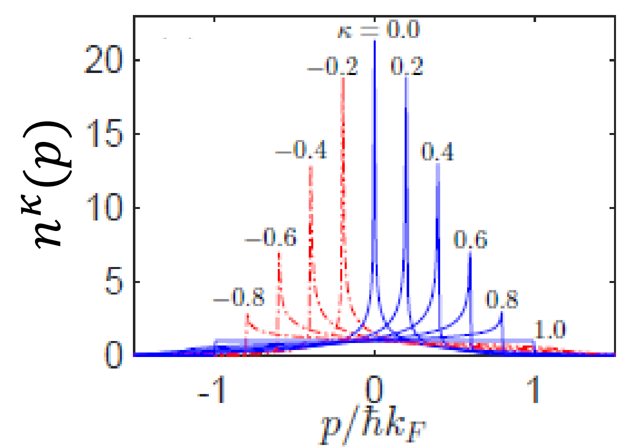
Single impurity in Tonks gas: momentum distribution

$$H_{\rm sc} = -\frac{\pi}{\sqrt{2N}\gamma_i}\sum_{j=1}^{N-1} C_j[c_j^\dagger c_{j+1} + \text{H.c.}] + \left[\frac{1}{\pi}\sqrt{2N}\right]^3 \mathcal{F}\sum_{j=1}^{N-1} D_j n_j$$

$$n_\sigma(p,t) = \sum_\kappa n^\kappa(p,t) S^\kappa(\sigma,t)$$

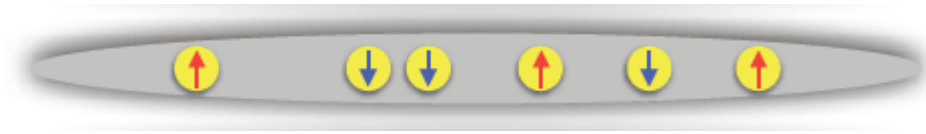
$$S^\kappa(t) = \delta_{\kappa,Ft} \quad \Rightarrow \quad n_{\text{impurity}}(p,t) = n^{\kappa=Ft}(p)$$





Yang, and HP, PRA **95**, 051602(R) (2017)

Motivation and Outline



Lower dimension → More quantum

Stronger interaction → More quantum

Outline:

- Strong coupling ansatz w.f. of 1D strongly interacting spinor gas
- Observation of spin-charge separation in 1D Fermi gas

Yang-Gaudin model: 1D spin-1/2 Fermi gas

$$\mathcal{H} = \sum_{j=\uparrow,\downarrow} \int_0^L \phi_j^+(x) \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right) \phi_j(x) dx$$
$$+ g_{1D} \int_0^L \phi_\downarrow^+(x)\phi_\uparrow^+(x)\phi_\uparrow(x)\phi_\downarrow(x) dx$$
$$- \frac{H}{2} \int_0^L \left(\phi_\uparrow^+(x)\phi_\uparrow(x) - \phi_\downarrow^+(x)\phi_\downarrow(x)\right) dx$$

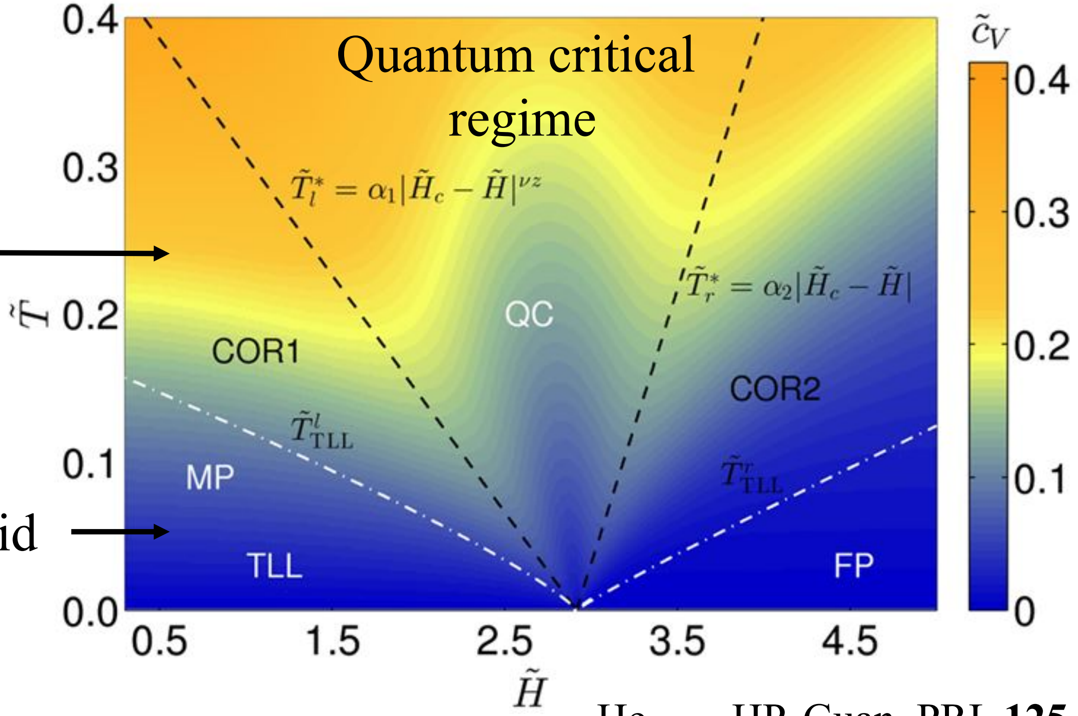
Antiferromagnetic coupling: $g_{1D} > 0$

Yang, PRL 19, 1312 (1967)

Gaudin, Phys. Lett. 24, 55 (1967)

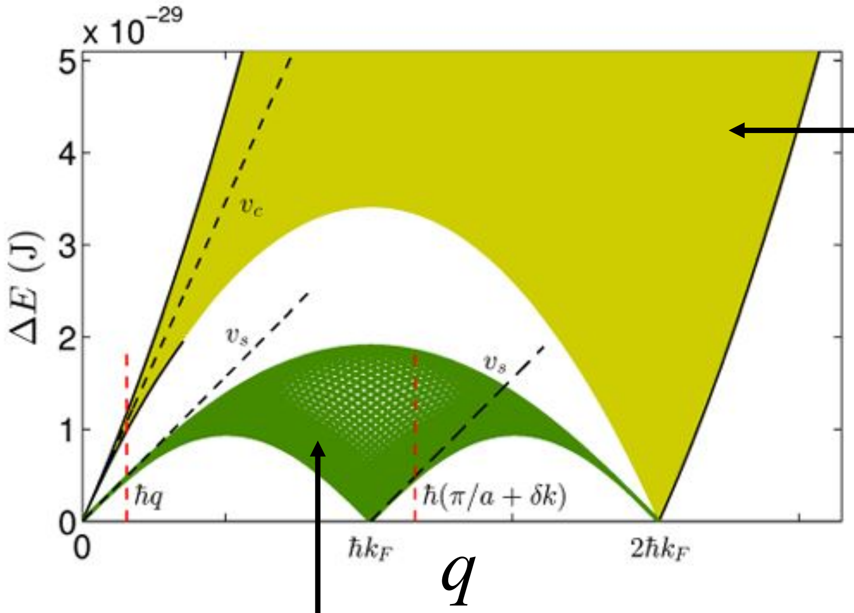
Spin-incoherent TLL

Tomonaga-Luttinger liquid



He, …, HP, Guan, PRL **125**, 100401 (2020)

Collective excitation in TLL



Charge: Particle-hole continuum

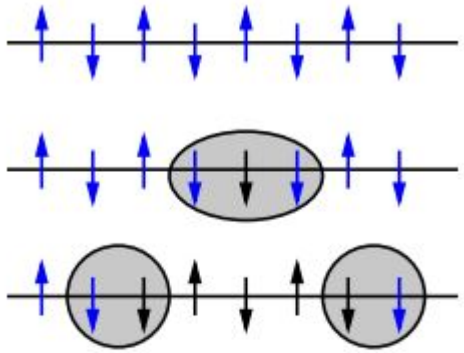
$$\omega_{\pm} = v_c q \pm \frac{1}{2m^*} q^2$$

$v_c \neq v_s$ spin-charge separation

Spin: Two-spinon excitation

$$w_{s+}(q) = v_s|q| - \frac{v_s q^3}{2K_s^2} + \cdots$$

$$w_{s-}(q) = v_s|q| - \frac{2v_s q^3}{K_s^2} + \cdots$$

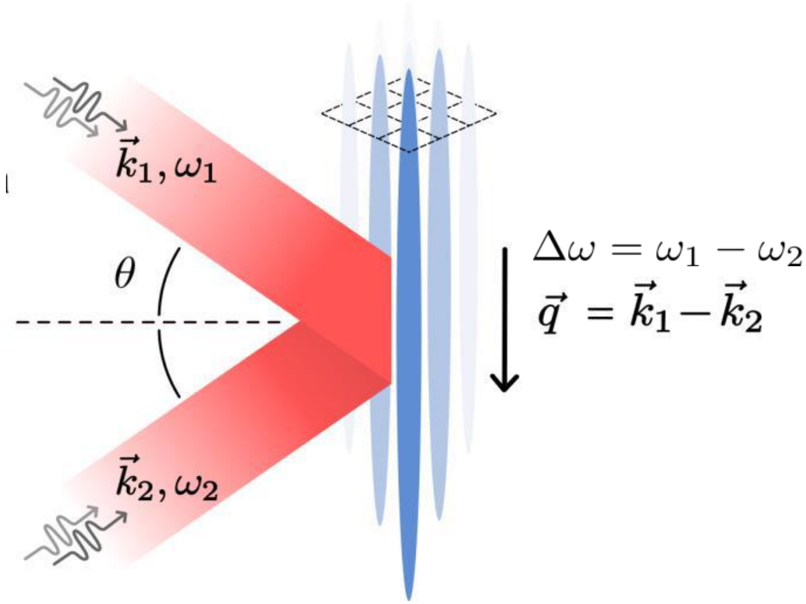


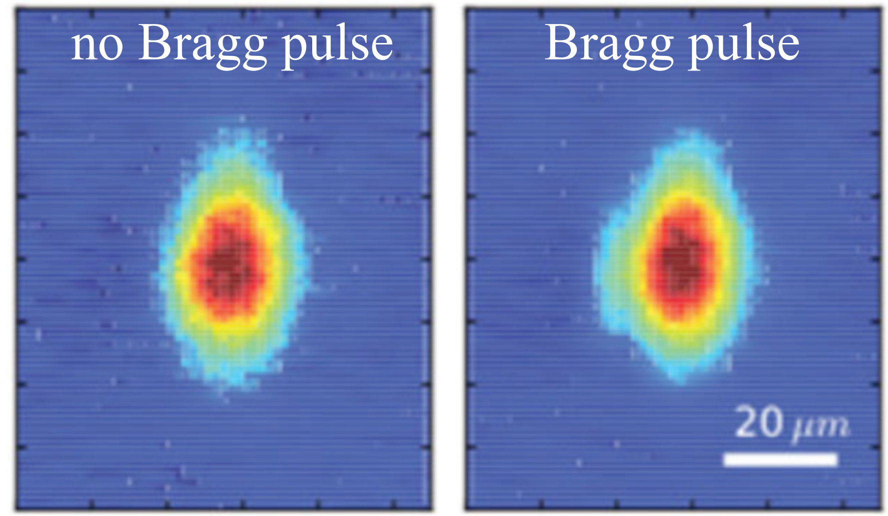
A single spin flip decouples into 2 spinons

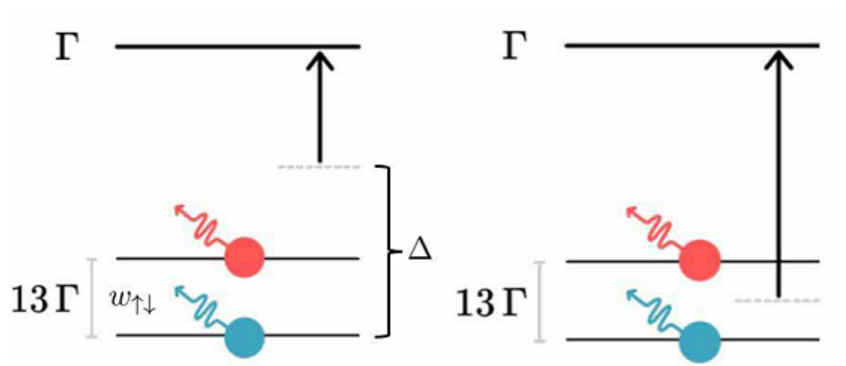
Faddeev and Takhtajan, Phys. Lett. **85**, 49 (1981)

Observing spin-charge separation in cold atoms

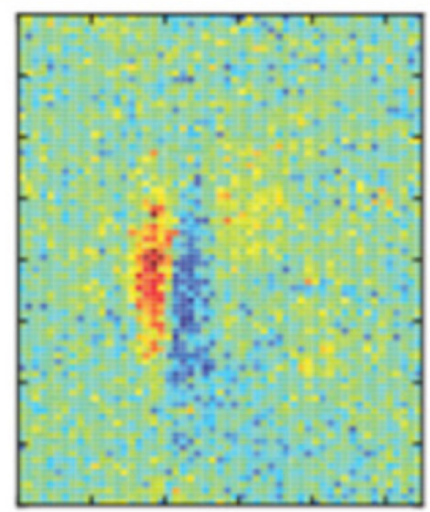
Creating and measuring collective excitation: Bragg spectroscopy





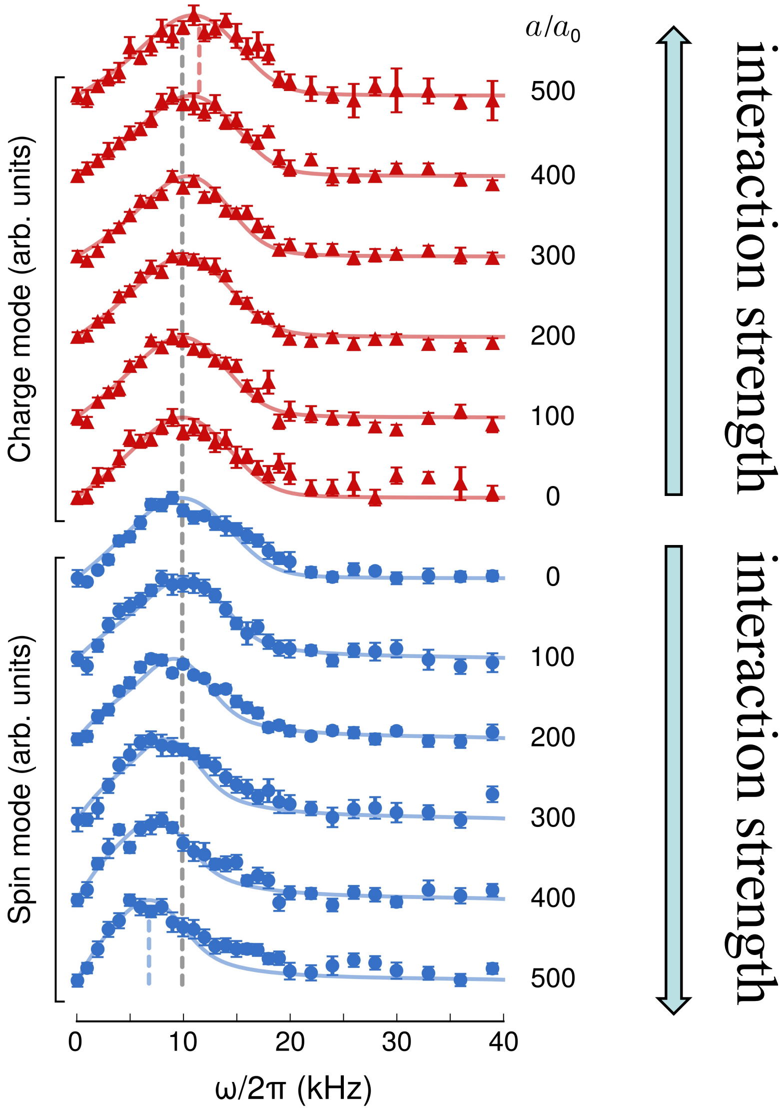


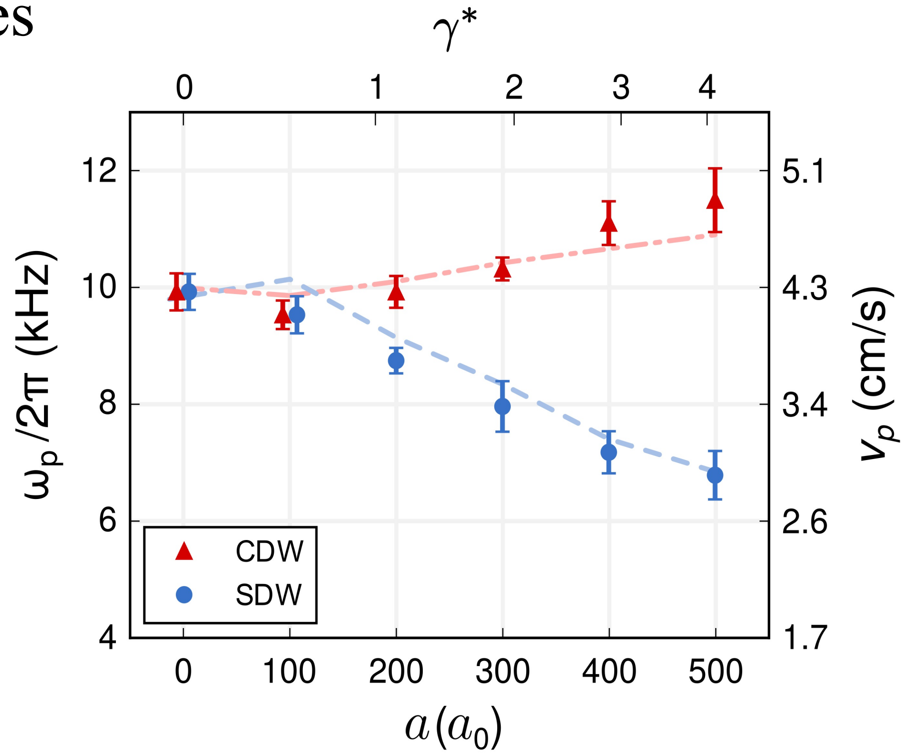
charge excitation spin excitation



Observing spin-charge separation in cold atoms

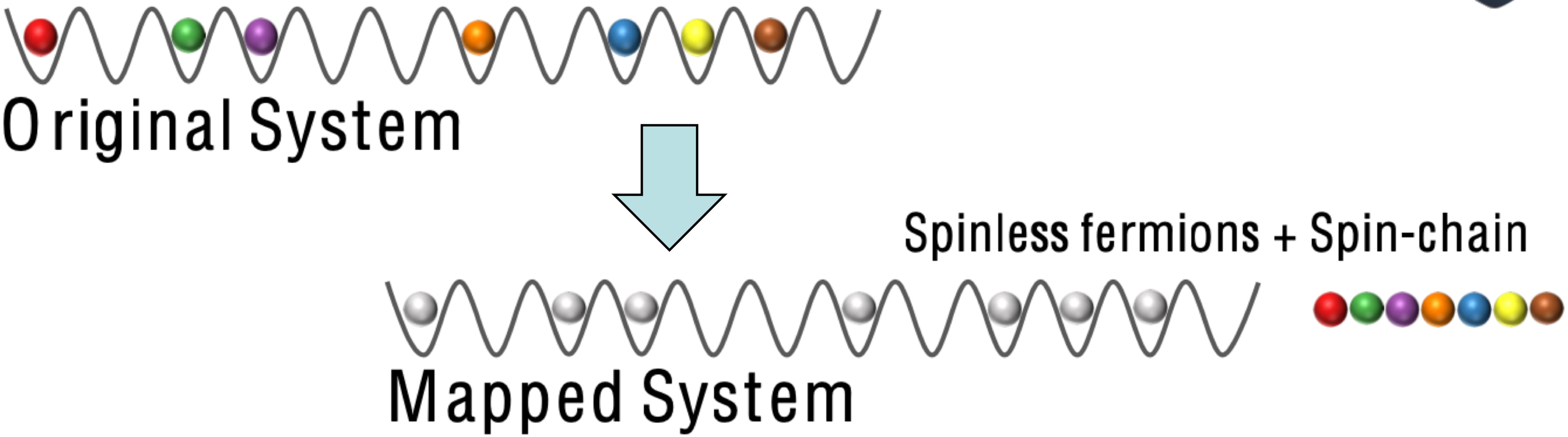
Extracting spin and charge velocities



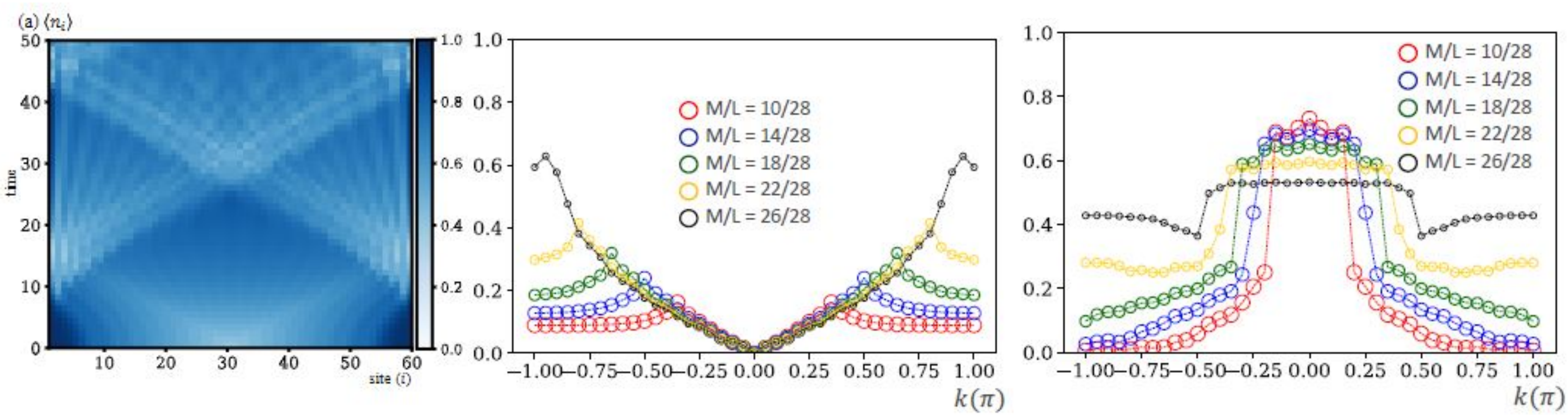


Senaratne *et al.* Science **376**, 1305 (2022)

What about lattice system?



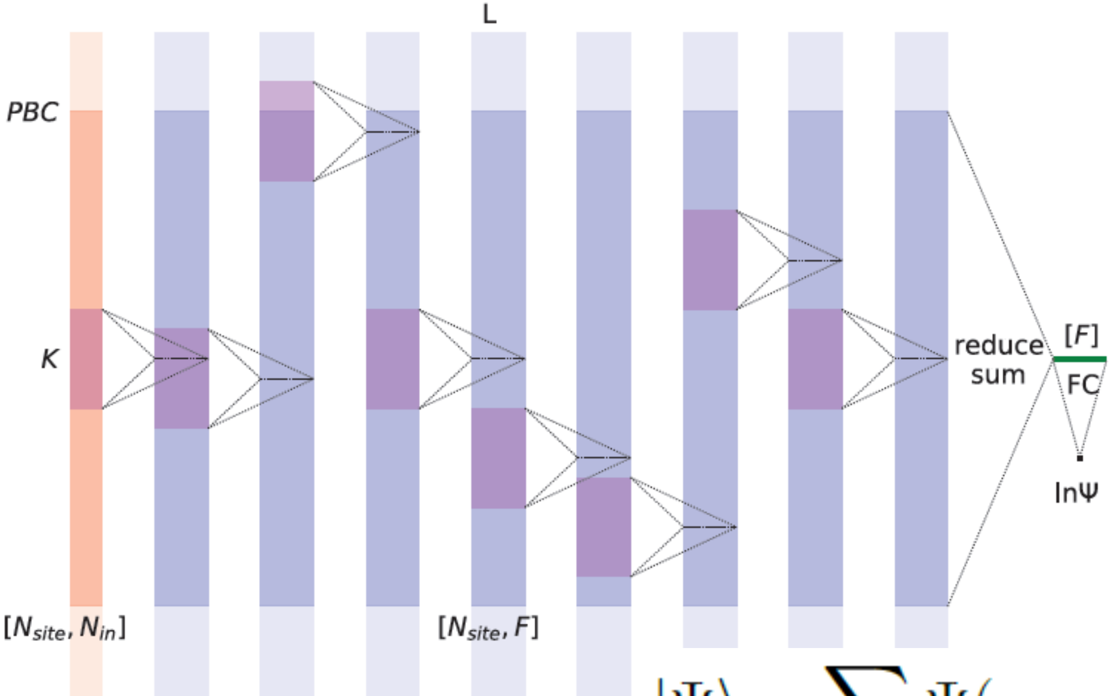
Quantum dynamics | Spin structure factor | Momentum distribution



S. Basak and HP, PRA **108**, 063315 (2023)

Neural Networks for spin chain

Variational QMC + deep CNN



1D SU(N) spin chain:

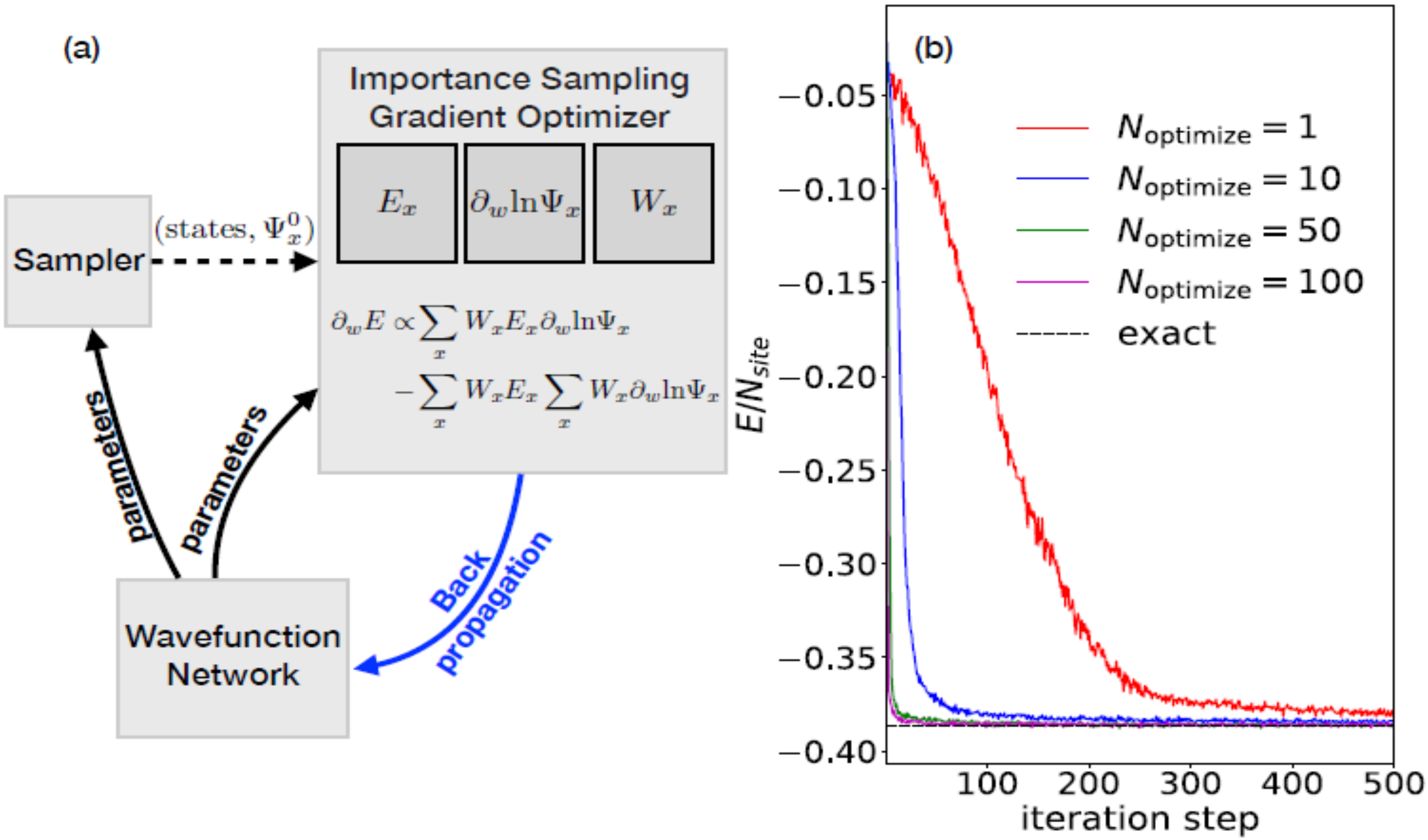
$$H = \sum_{i=1}^{N_{\text{site}}} P_{i,i+1}$$

$$|\Psi\rangle = \sum_{\{s_i\}} \Psi(s_1, s_2, \ldots, s_{N_{\text{site}}}) \, |s_1, s_2, \ldots, s_{N_{\text{site}}}\rangle$$

$$\Psi_s(w) \equiv \Psi(s_1, s_2, \ldots, s_N; w) \qquad \text{minimize:} \quad E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

$$w \leftarrow w - \alpha \frac{\partial E}{\partial w}$$

Neural networks for quantum many-body physics



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Summary and Outlook

- Concepts of fermionization are extended to 1D strongly interacting *spinful* particles, using the **strong coupling ansatz**.
- It provides new insights into the system and serves as an extremely efficient computational tool.
- Spin-charge separation unambiguously observed in cold atoms.

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