Entanglement, Complexity and Quantum Simulations of Nuclear Many-Body Systems

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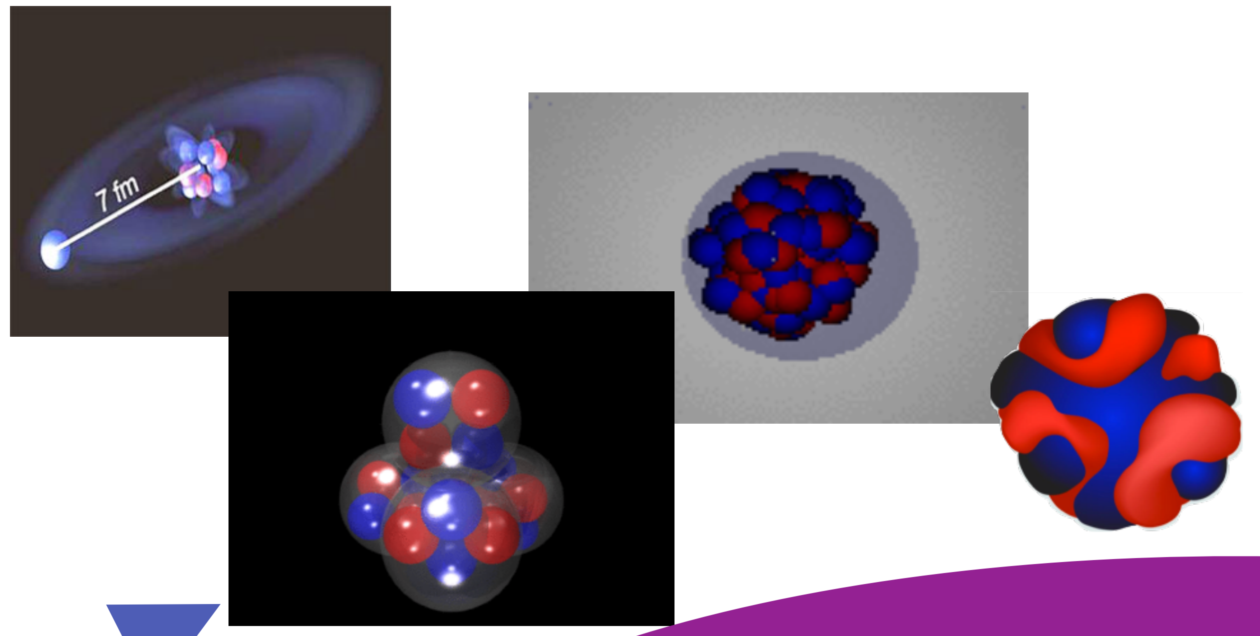
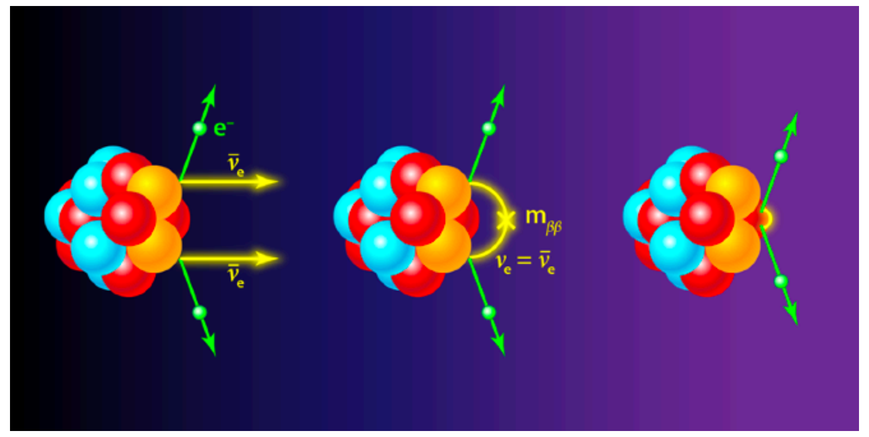
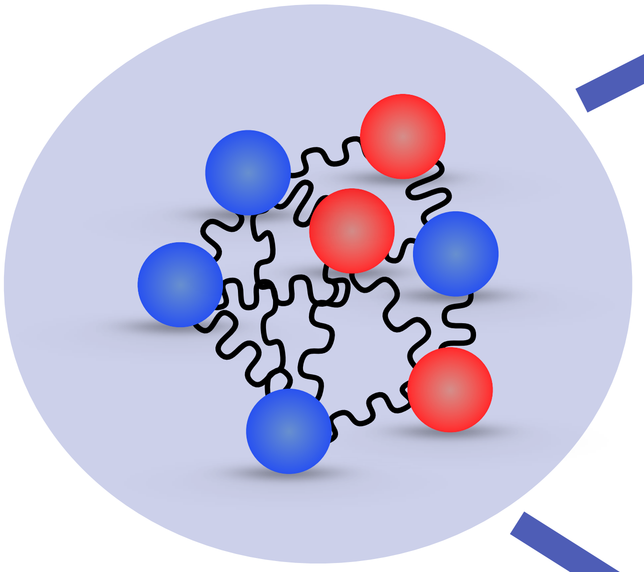
Nuclei to address fundamental questions

Understand how protons and neutrons bind together to form nuclei and predict the structure and dynamics of nuclei to address fundamental science questions





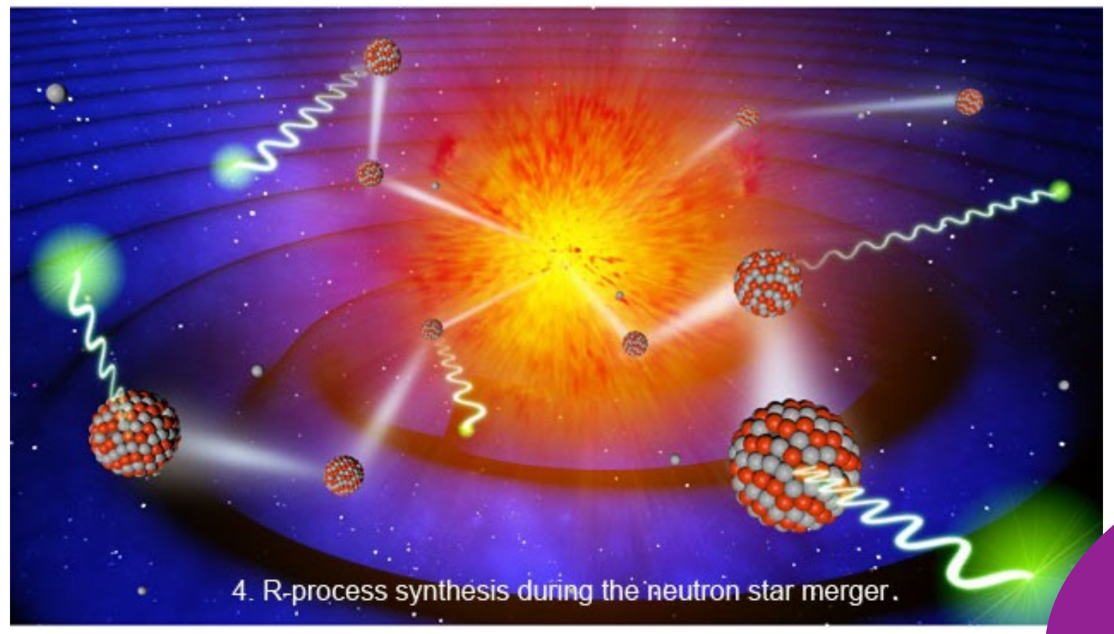


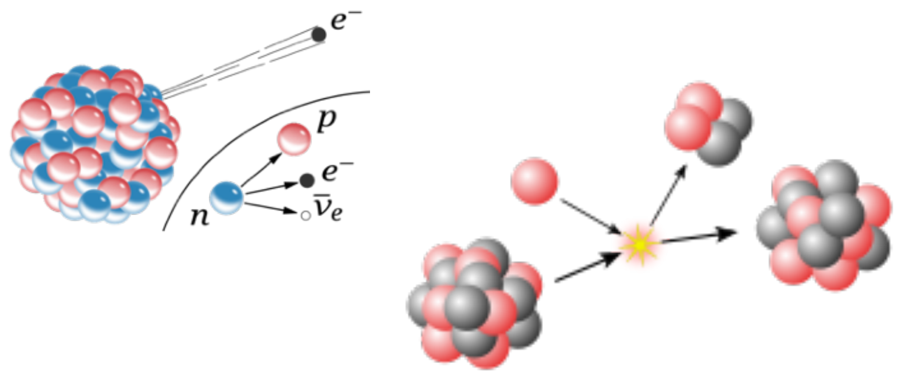
emergent phenomena

"How does subatomic matter organize itself and what phenomena emerge?"

fundamental symmetries

"Are the fundamental interactions that are basic to the structure of matter fully understood?"

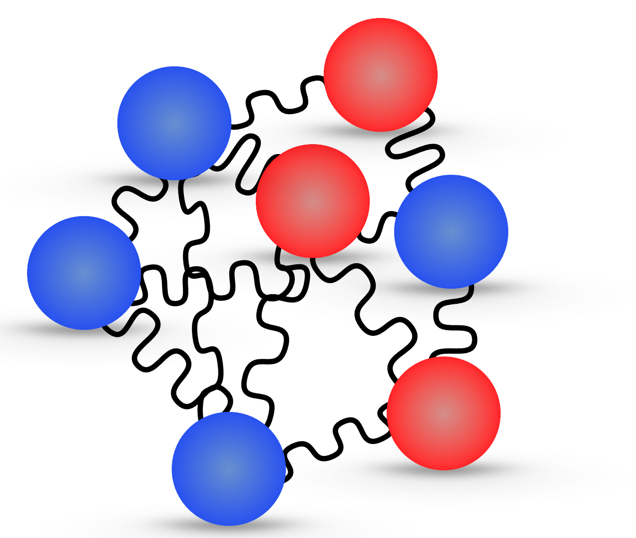




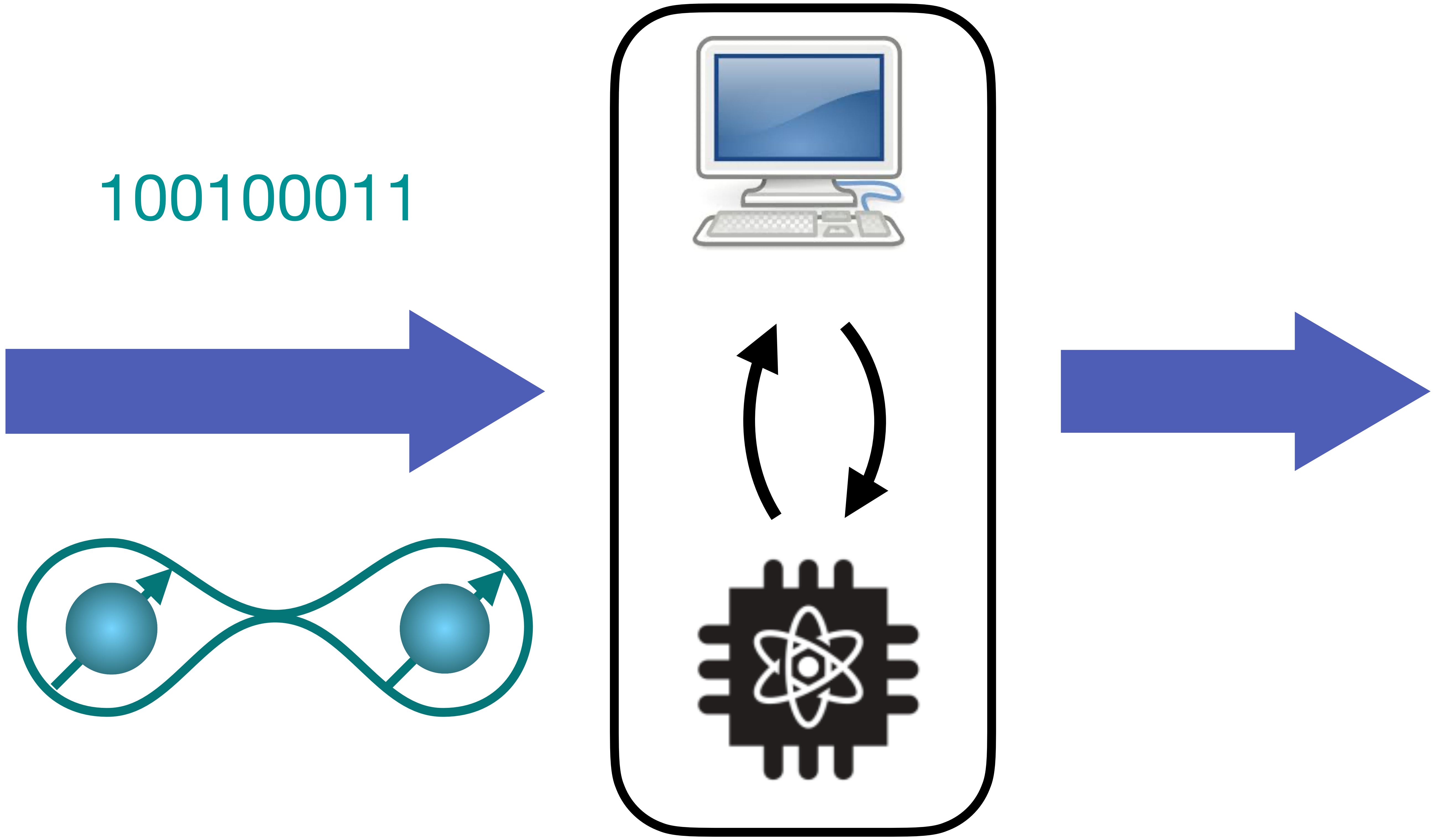
origin of the elements in the cosmos

"How did matter come into being and how does it evolve?"

Motivations

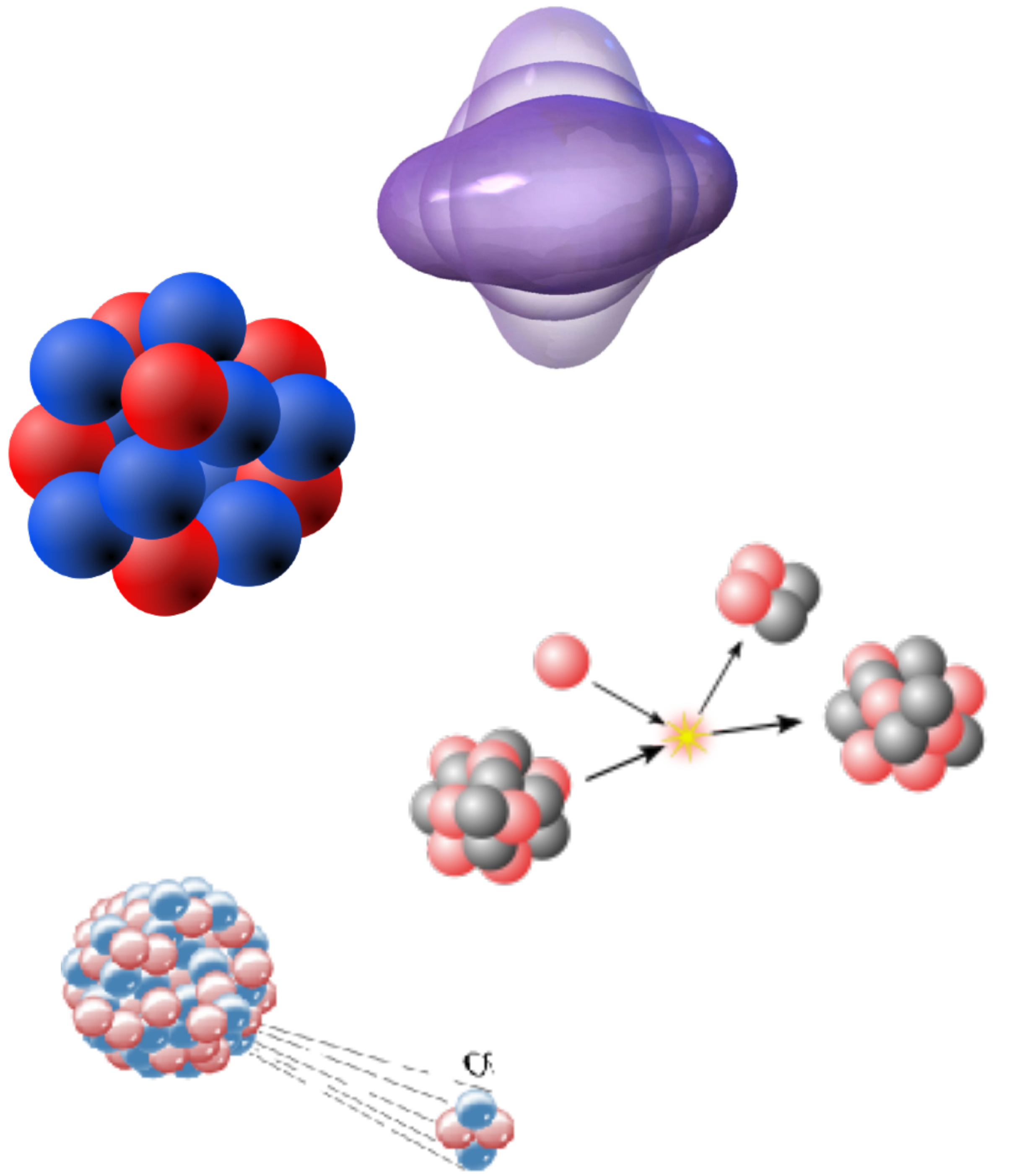


Nucleons and interactions

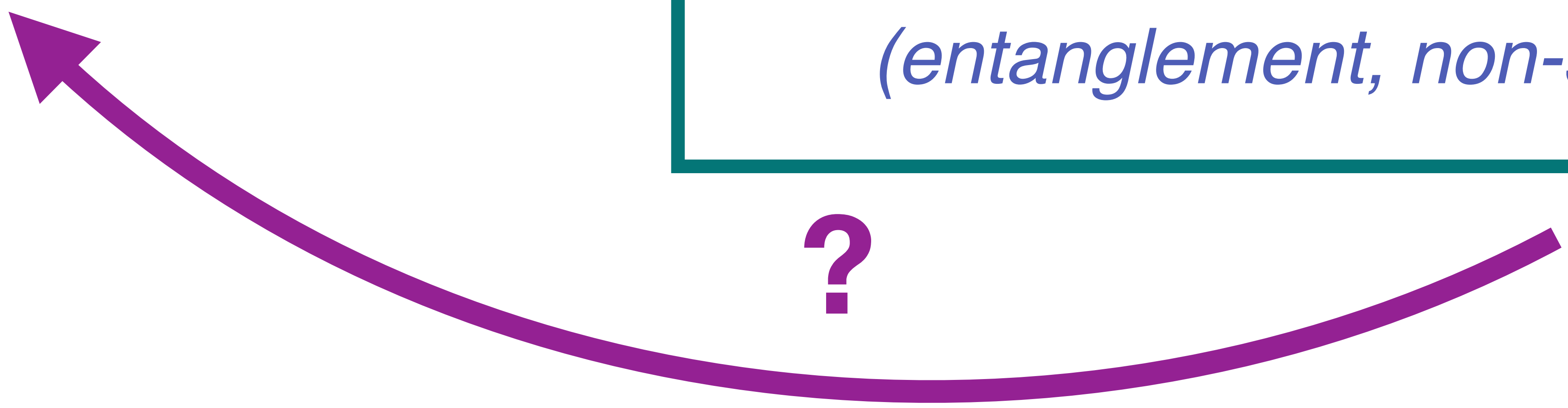


Physics-guided Mappings & Algorithms

(entanglement, non-stabilizerness (magic), symmetries)



Structure, Reactions and Decays of Nuclei

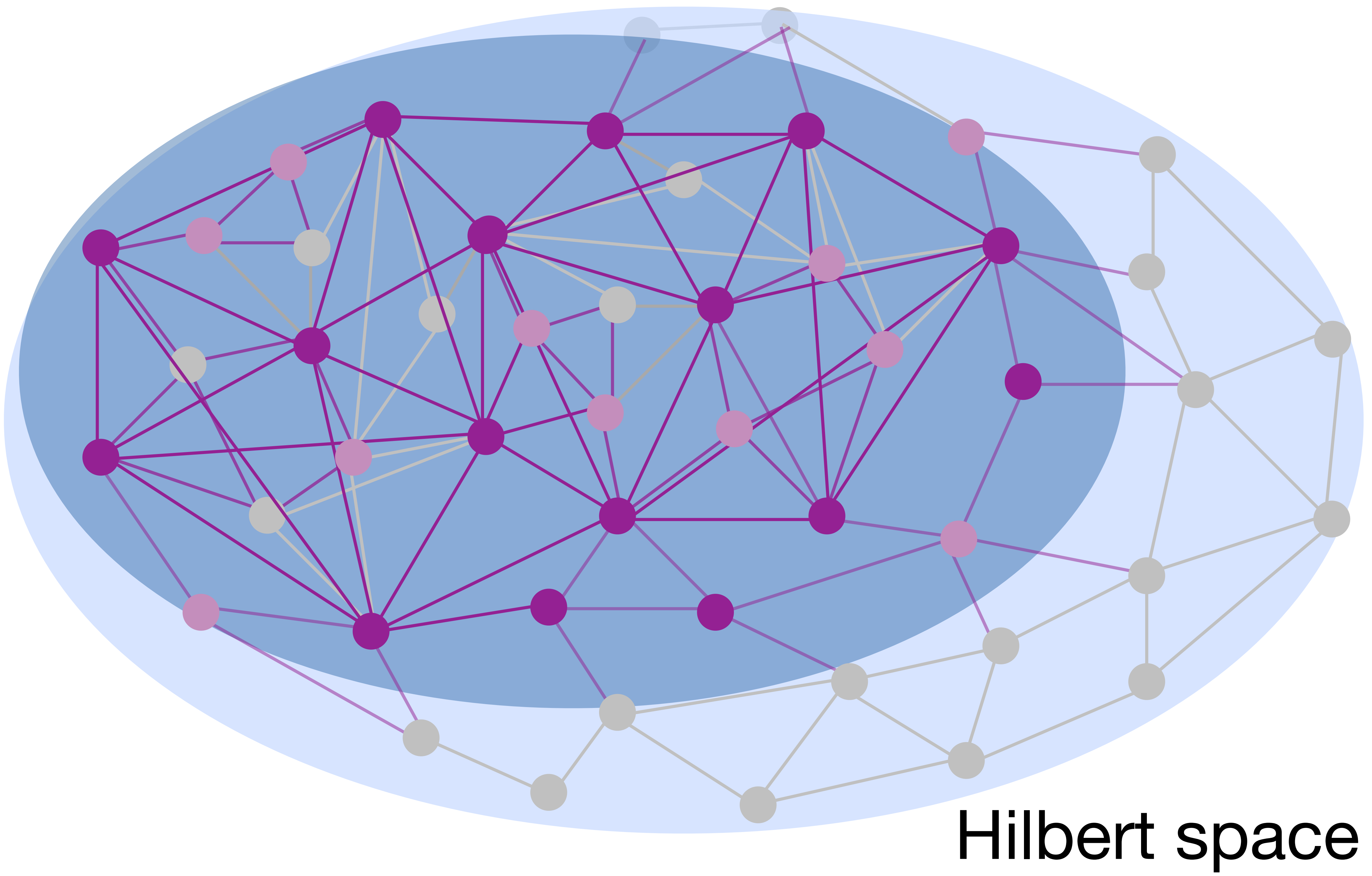


Simulations

Inspired by "Quantum simulation of fundamental particles and forces", Bauer, Davoudi, Klco, Savage, Nature Rev. Phy. 5, 420 (2023)

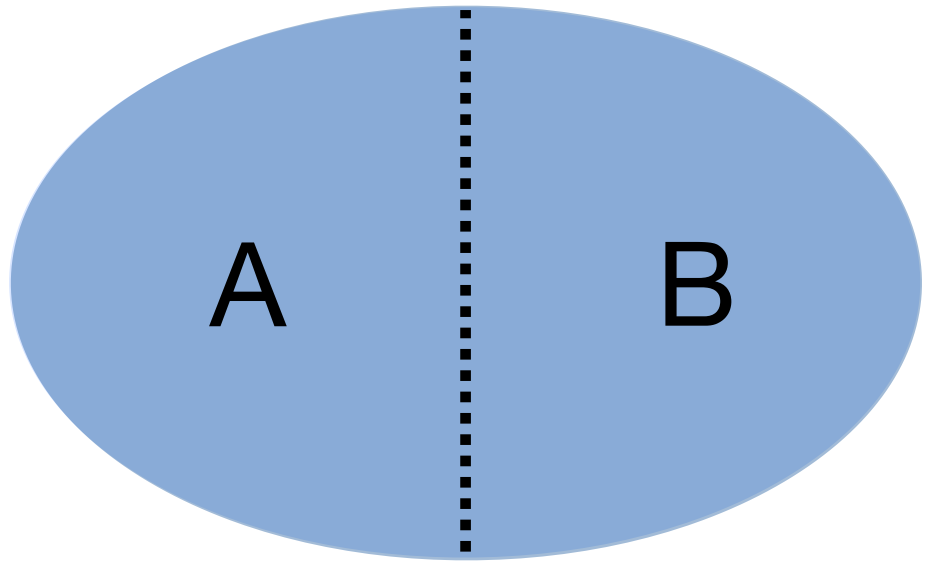
Quantum Complexity of Many-Body Systems

(I) Entanglement



$$|\Psi\rangle = \sum_{n}^{\sim 2^N} C_n |\Phi_n\rangle$$

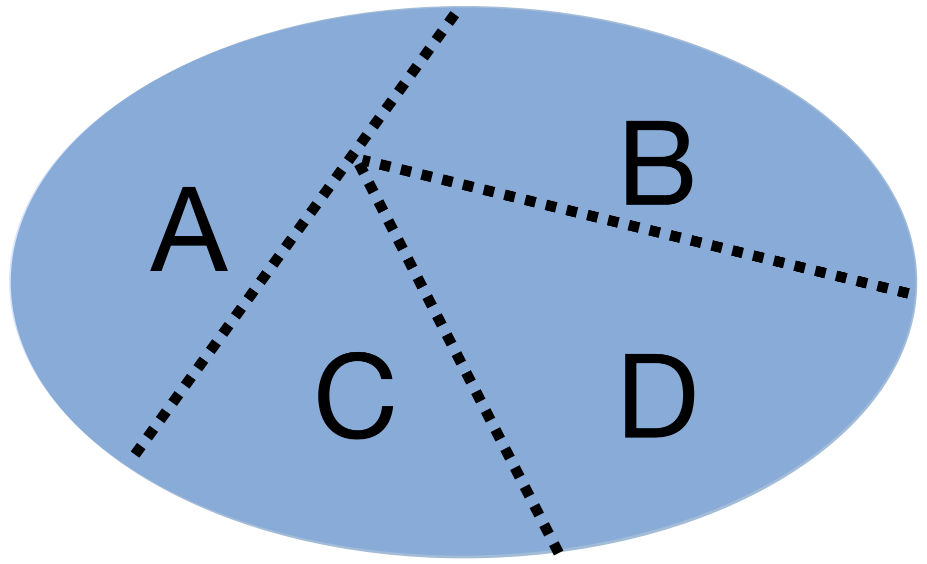
✳ bi-partite pure state:



Von Neumann entanglement entropy

$$S(\rho_A) = -\mathrm{Tr}(\rho_A \ln \rho_A) = S(\rho_B)$$

✳ multi-partite mixed states:



Mutual information, negativity, n-tangles…

Quantum Complexity of Many-Body Systems

(II) Magic (non-stabilizerness)

But … some highly entangled states can be simulated efficiently with classical computers:

Quantum Complexity of Many-Body Systems

(II) Magic (non-stabilizerness)

But … some highly entangled states can be simulated efficiently with classical computers:

Quantum Gate Set:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix} \qquad \mathrm{CNOT} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

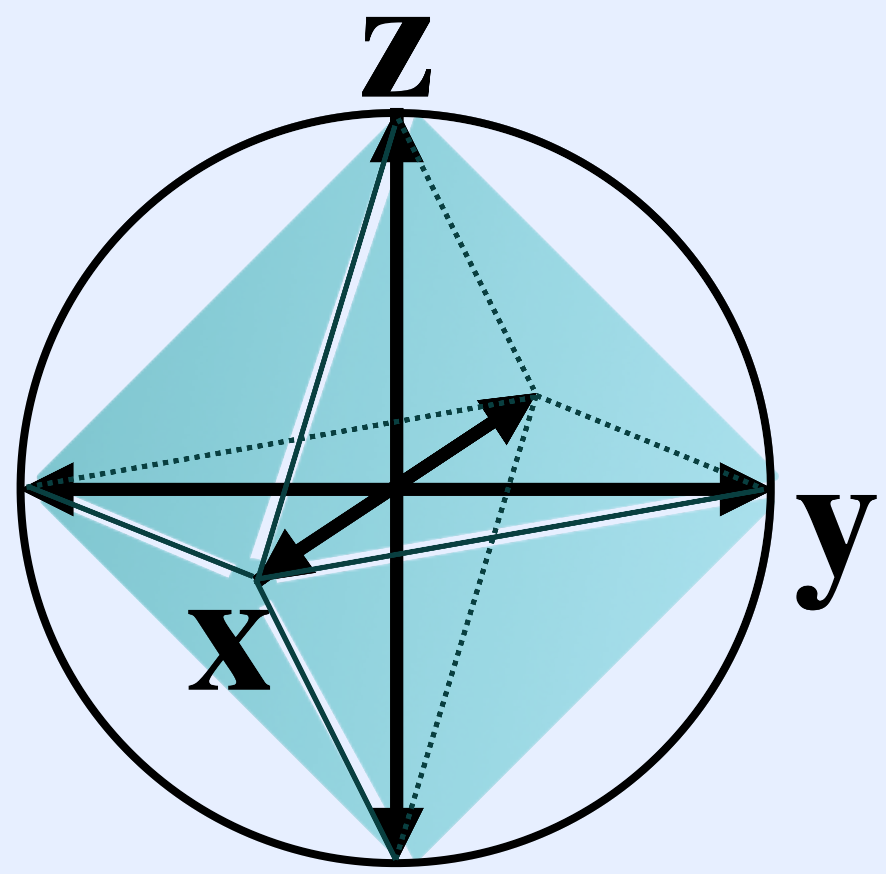
{H,S,CNOT} are generators of the Clifford group

$$\hat{U}_{\mathrm{Clifford}}|00...0\rangle = |\mathrm{stabilizer\ state}\rangle$$

Bravyii & Kitaev (2005)

- one qubit:

  6 stabilizer states



- two qubits: 60 stabilizer states incl. 24 entangled states

  e.g. $\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, $\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$

- Three qubits: 1080,
- Four qubits 36720…

Gottesman-Knill theorem (1998): *Any stabilizer state can be efficiently simulated with a classical computer (incl. highly entangled states)*

Quantum Complexity of Many-Body Systems

(II) Magic (non-stabilizerness)

But … some highly entangled states can be simulated efficiently with classical computers:

Universal Quantum Gate Set:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix} \qquad \mathrm{CNOT} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

$$+\ T = \begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}$$

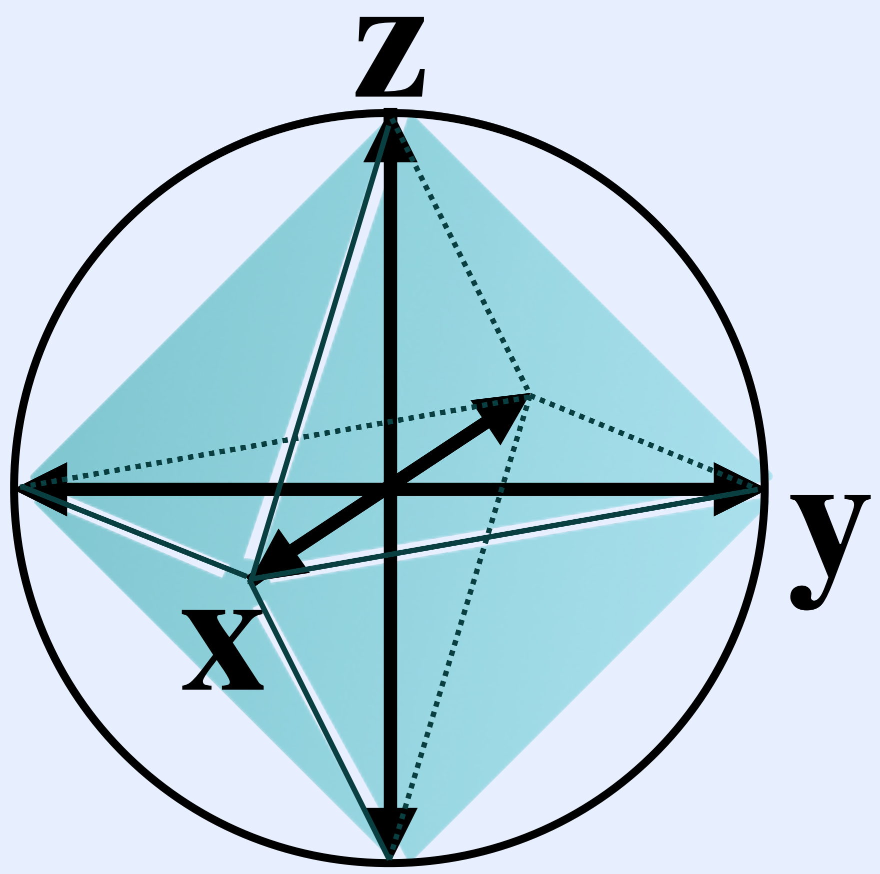
{H,S,CNOT} are generators of the Clifford group

$$\hat{U}_{\mathrm{Clifford}}|00\ldots0\rangle = |\mathrm{stabilizer\ state}\rangle$$

Bravyii & Kitaev (2005)

- one qubit:

  6 stabilizer states



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Quantum Complexity of Many-Body Systems

Magic = measure of non-stabilizerness

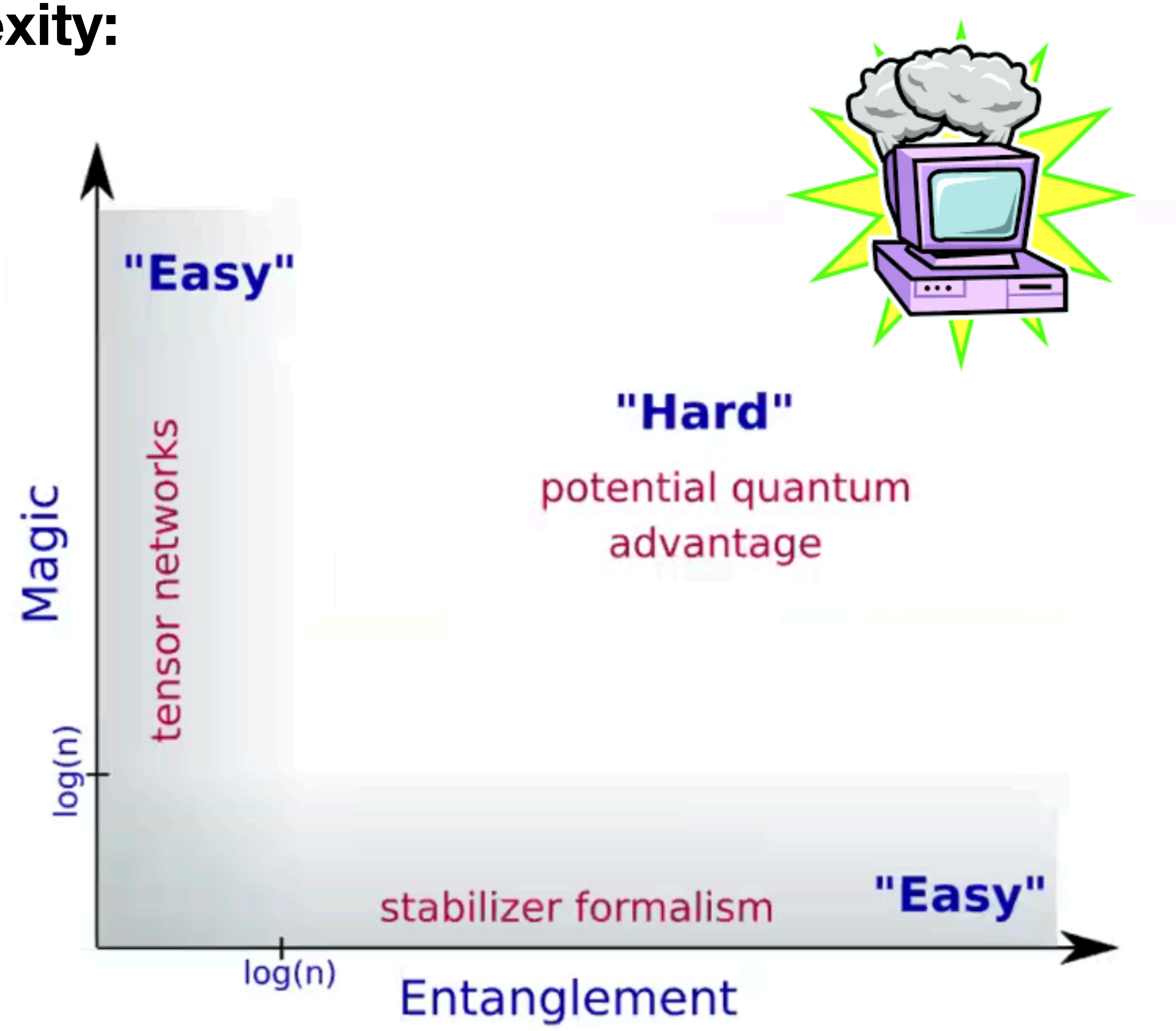
~ how far a state $|\Psi\rangle$ is from a stabilizer state

~ scales with the number of non-Clifford operations (T gates) needed to prepare $|\Psi\rangle$

Aaronson-Gottesman (2004): classical resources to simulate $|\Psi\rangle$ scale exponentially with the number of T gates / with the magic

Quantum Complexity of Many-Body Systems

Quantum Complexity:



(Fig. adapted from Emanuele Tirrito)

Quantum Complexity of Many-Body Systems

How to quantify magic?

Pauli string

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \frac{1}{d}\sum_P \langle\Psi|\hat{P}|\Psi\rangle\hat{P}$$

$$d = 2^{n_{qubits}}$$

Stabilizer states have:

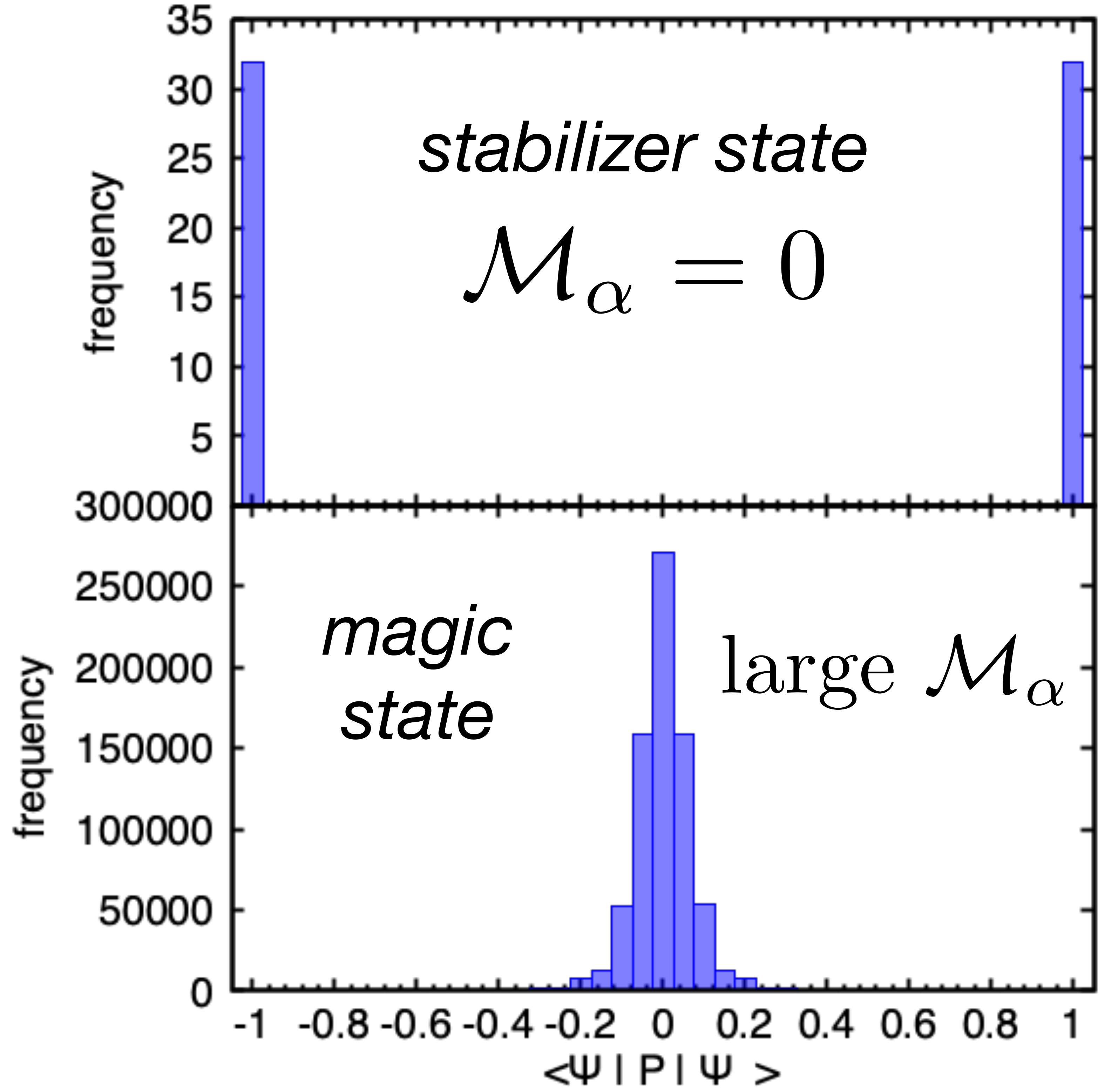
$$\langle\Psi|\hat{P}|\Psi\rangle = \pm 1 \quad \text{for d Pauli strings}$$
$$= 0 \quad \text{for the rest}$$

Stabilizer Rényi Entropy:

Leone, Oliviero, Hamma, PRL 128, 050402 (2022)

$$\mathcal{M}_\alpha(|\Psi\rangle) = -\log(d) + \frac{1}{1-\alpha}\log\left(\sum_P \frac{\langle\Psi|\hat{P}|\Psi\rangle^{2\alpha}}{d^\alpha}\right)$$

Distribution of non-zero expectation values



Entanglement and Magic Phase Transitions

Entanglement–magic separation in hybrid quantum circuits

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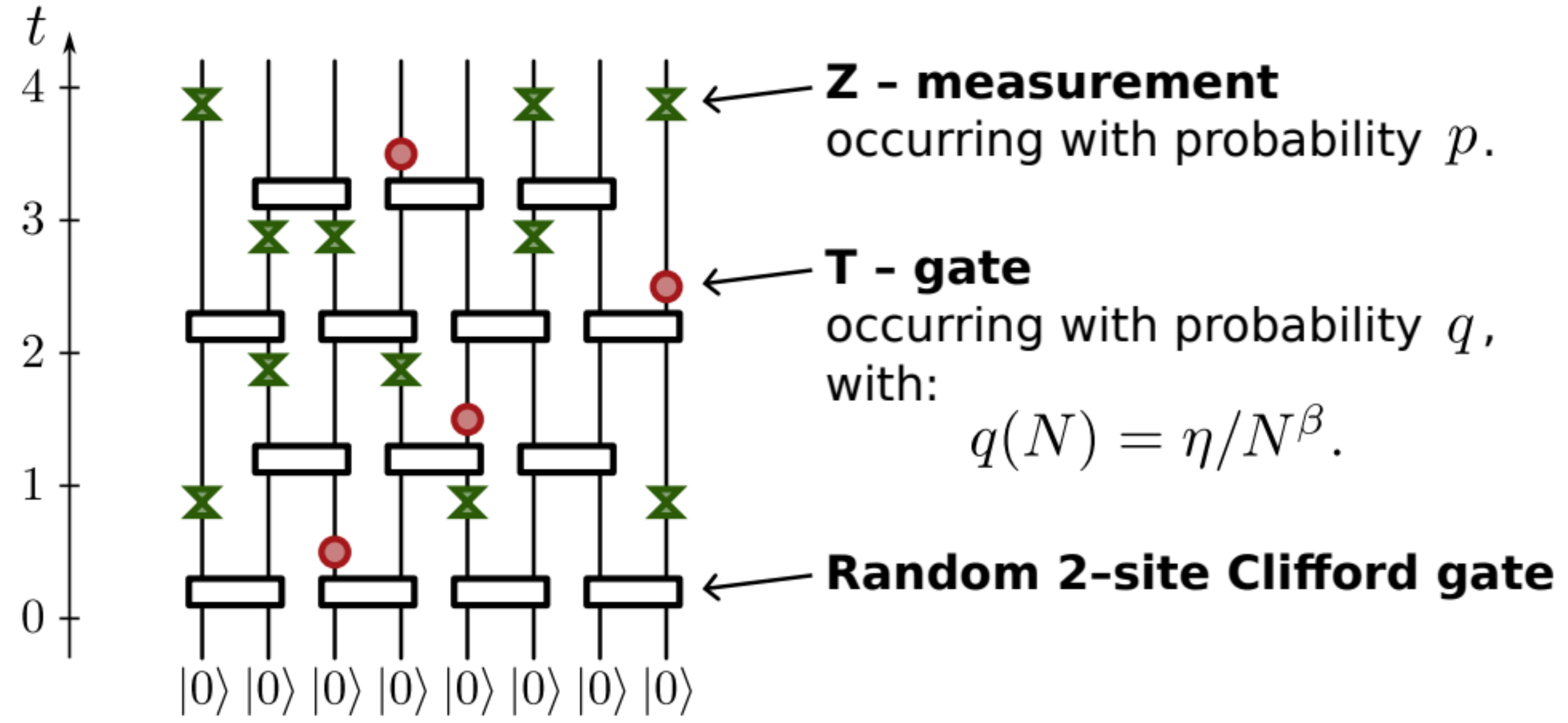
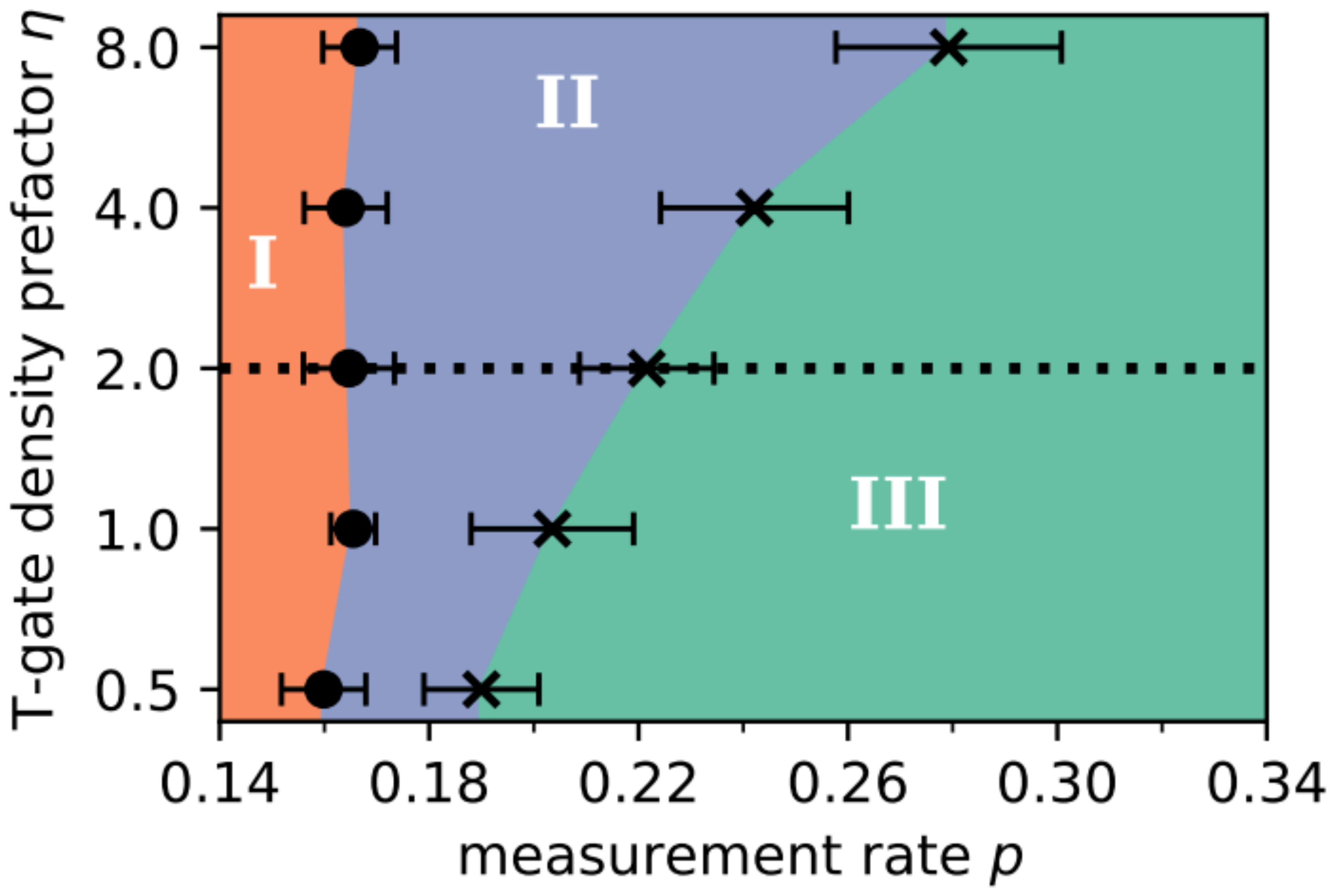
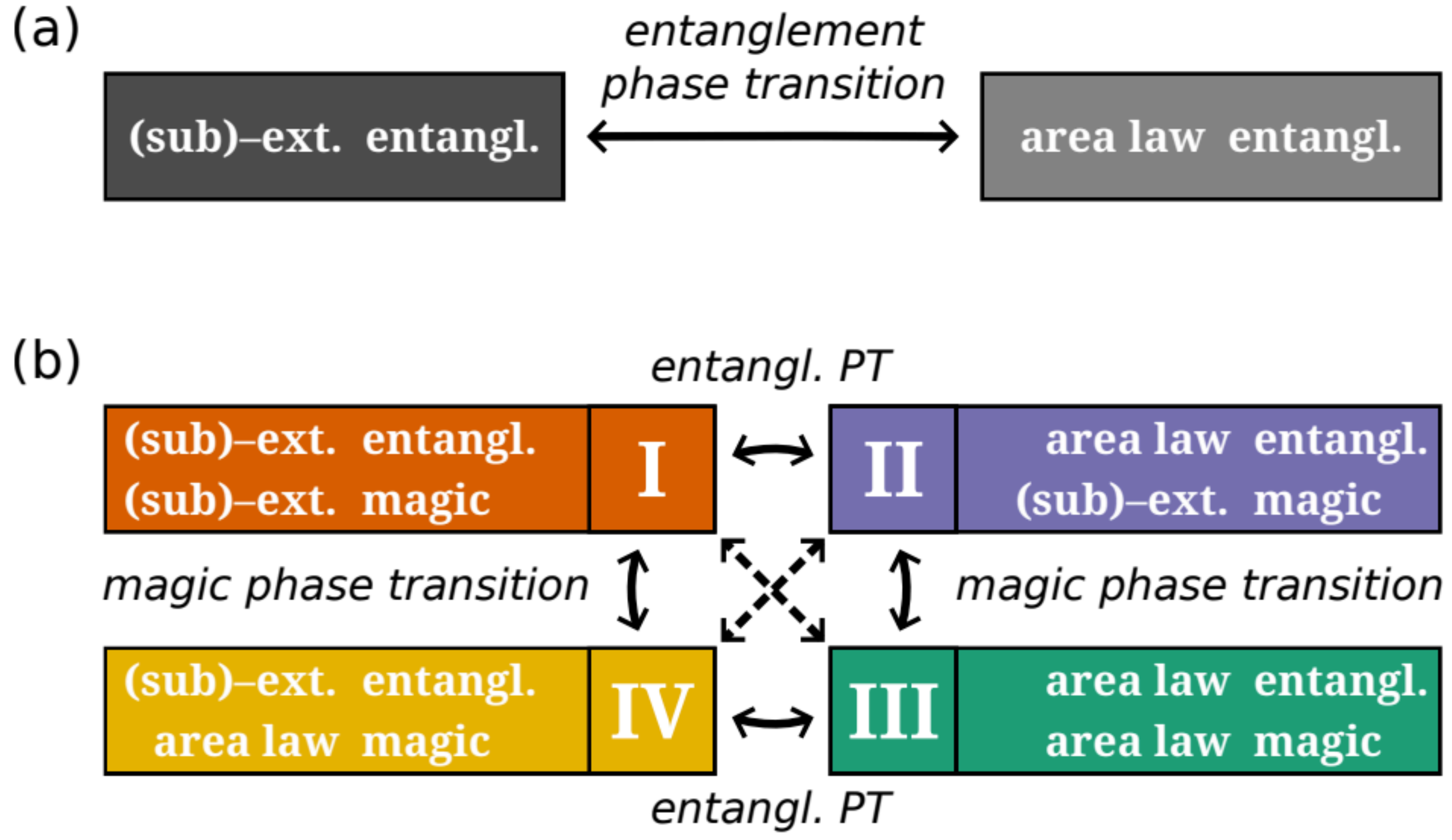
[3] *Scuola Internazionale Superiore di Studi Avanzati (SISSA), Via Bonomea 265, 34136 Trieste, Italy*
[4] *Dipartimento di Fisica "E. Pancini", Università di Napoli "Federico II", Monte S. Angelo, I-80126 Napoli, Italy*

(Dated: December 12, 2023)

See also Bejan+ PRX Quantum 5, 030332 (2024)



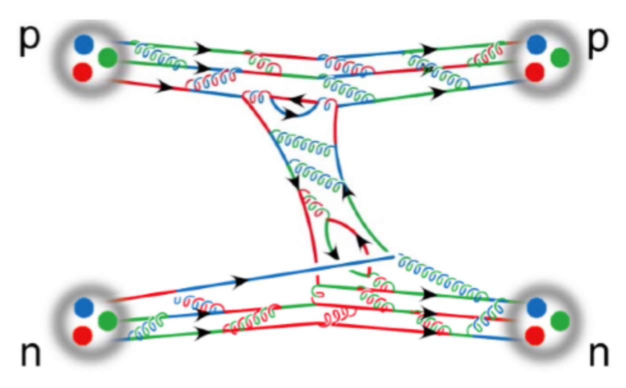




Different measurement rates for Magic and Entanglement PT ->"This suggest that the mechanism that drives the observed magic phase transition is different from the mechanism driving the entanglement phase transition"

Motivational Questions

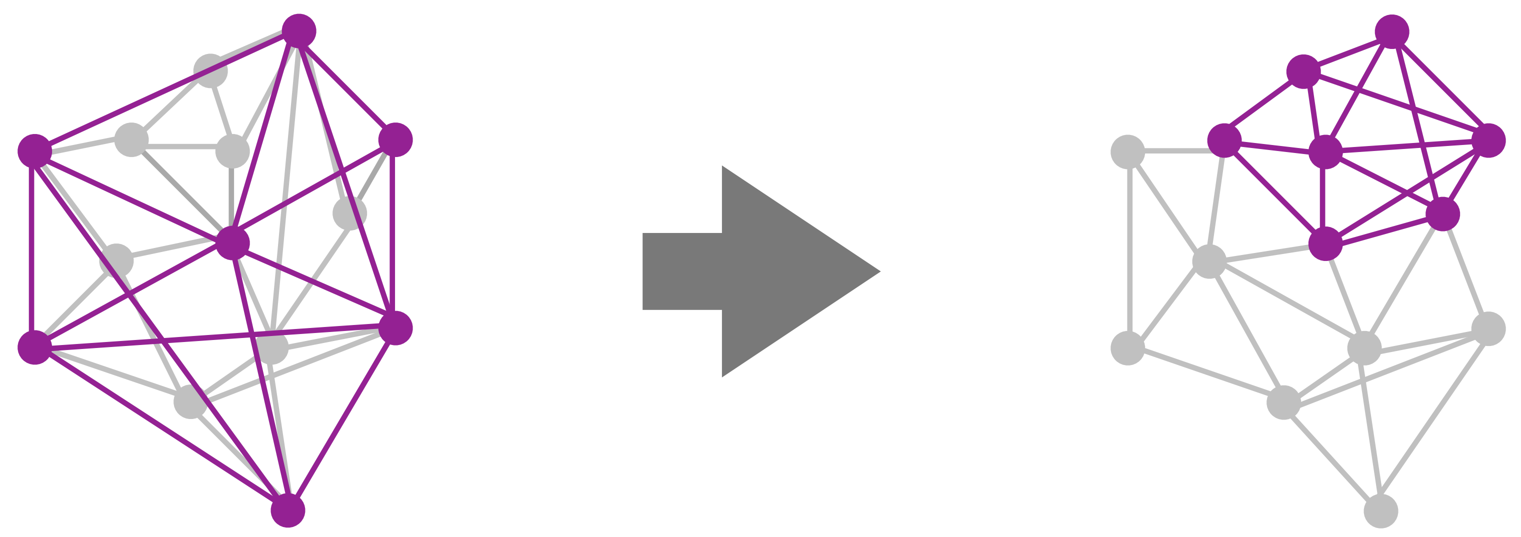
- What is the role played by entanglement and magic in the structure and dynamics of nuclear systems? What are possible connections with underlying forces and symmetries?



e.g. "Entanglement Suppression and Emergent Symmetries of Strong Interactions" Beane, Kaplan, Klco, Savage, PRL122,102001 (2019).

"Entanglement minimization in hadronic scattering with pions" Beane, Farrell, Varma. Int. J. Mod. Phys. A 36,2150205 (2021).

- In turn, can these concepts guide the development of new formulations of nuclear QMB problems, and of improved algorithms for hybrid classical/quantum simulations?

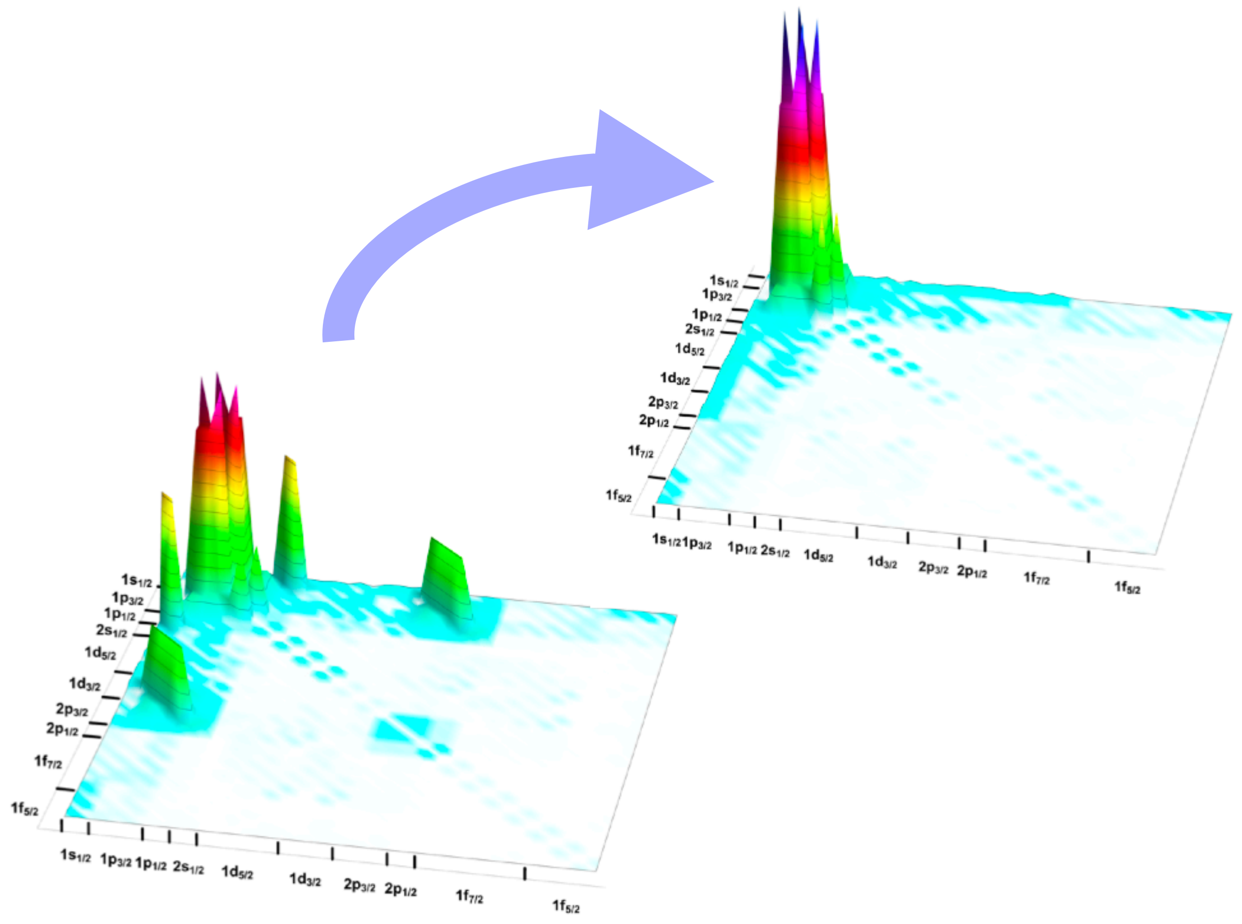


Outline

★ Entanglement and Magic Rearrangement in Nuclear Many-Body Systems

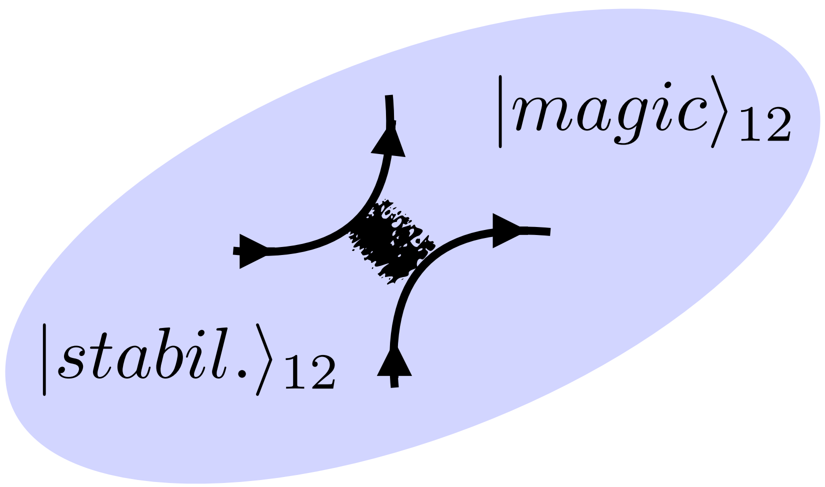
→ From the Lipkin model to nuclei

CR, Savage, Pillet, PRC 103, 034325 (2021);
CR & Savage PRC 108, 024313 (2023);
Hengstenberg, CR, Savage EPJA 59, 231 (2023);
Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064



★ The Magic Power in Nuclear and Hyper-Nuclear Forces

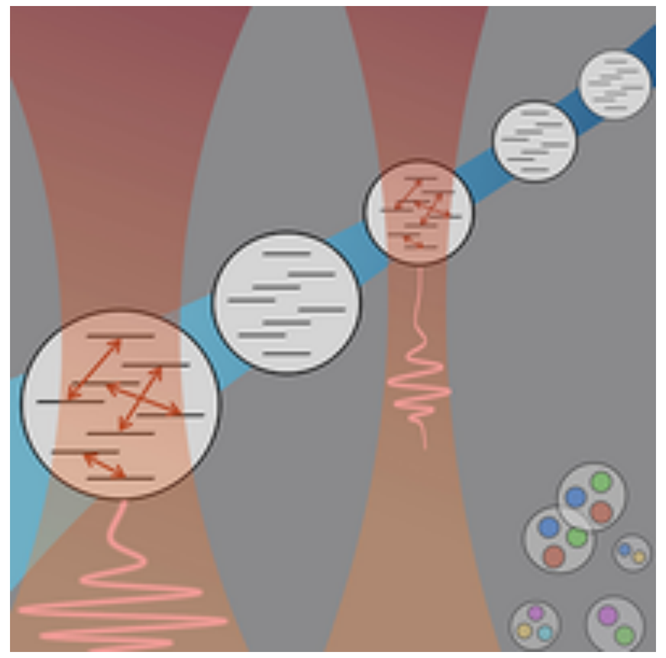
CR & M. J. Savage arXiv:2405.10268



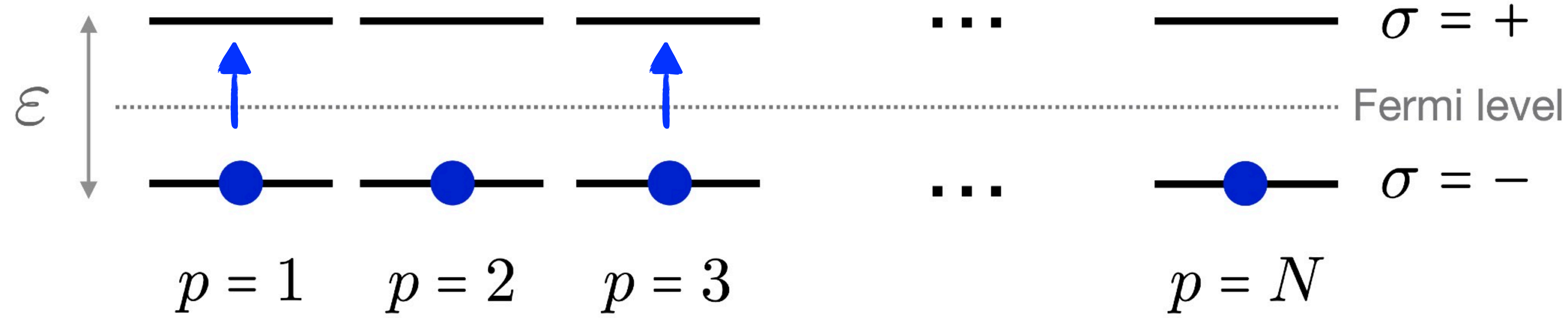
★ Symmetry-guided mapping of quantum systems onto qudits

→ Fermionic model with SO(5) symmetry and 1+1D SU(3) QCD

Illa, CR, Savage PRC 108, 064306 (2023); PRD 110, 014507 (2024)



The Lipkin-Meshkov-Glick Model: a sandbox for new ideas



Lipkin, Meshkov, Glick, Nucl. Phys. 62, 188 (1965)

$$H = \varepsilon J_z - \frac{V}{2}(J_+^2 + J_-^2)$$

$$J_z = \frac{1}{2}\sum_{p\sigma} \sigma c^\dagger_{p\sigma} c_{p\sigma}$$

$$J_+ = \sum_p \sigma c^\dagger_{p+} c_{p-}\ , \qquad J_- = (J_+)^\dagger$$

Relevance for many-body physics, trapped-ion quantum computing, spin squeezing…

▶ *Benchmark for studying relations between entanglement and quantum phase transitions*

See e.g. J. Vidal et al. PRA 69, 022107 & 054101 (2004); Di Tullio et al, PRA 100, 062104 (2019); Faba, Martín, Robledo, PRA, 103, 032426 (2021); PRA 104, 032428 (2021); PRA 105, 062449 (2022); Hengstenberg, CR, Savage EPJA 59, 231 (2023)…

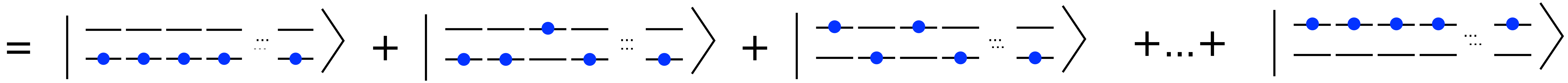
▶ *for testing and comparing new quantum algorithms:*

Cervia et al. PRC 104, 024305 (2021); Chikaoka & Liang, Chin. Phys. C 46 024106 (2022); Romero et al. PRC 105, 064317 (2022); Hlatshwayo et al. PRC 106, 024319 (2022); Robin, Savage PRC 108, 024313 (2023); Beaujeault-Taudiere, Lacroix, arXiv:2312.04703 (2023); Hlatshwayo et al. PRC 109, 014306 (2024)…

The Lipkin-Meshkov-Glick Model in Effective Model Spaces

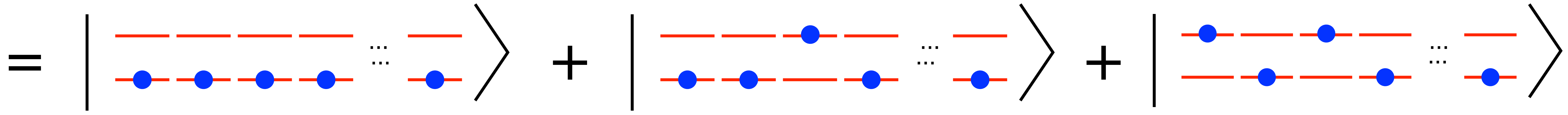
✴ Exact solution:

$$|\Psi\rangle = \sum_{M=-J}^{J} A_{J,M}|J,M\rangle \equiv \sum_{n=0}^{2J} A_n|n\rangle$$

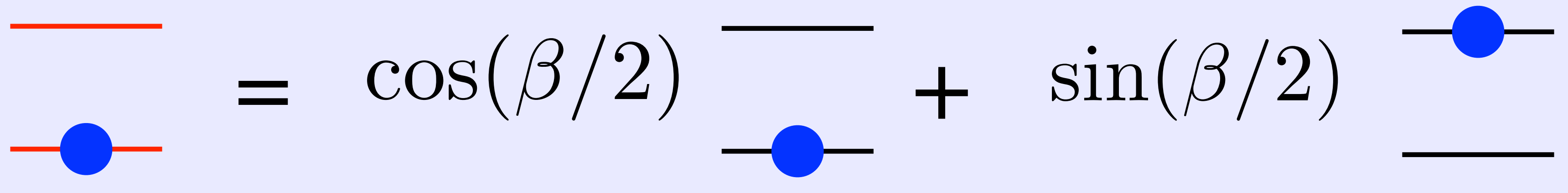


✴ Effective description:

$$|\Psi\rangle^{\Lambda} = \sum_{n=0}^{\Lambda-1} A_n|n\rangle$$



Rotation of the spins as "disentanglers"



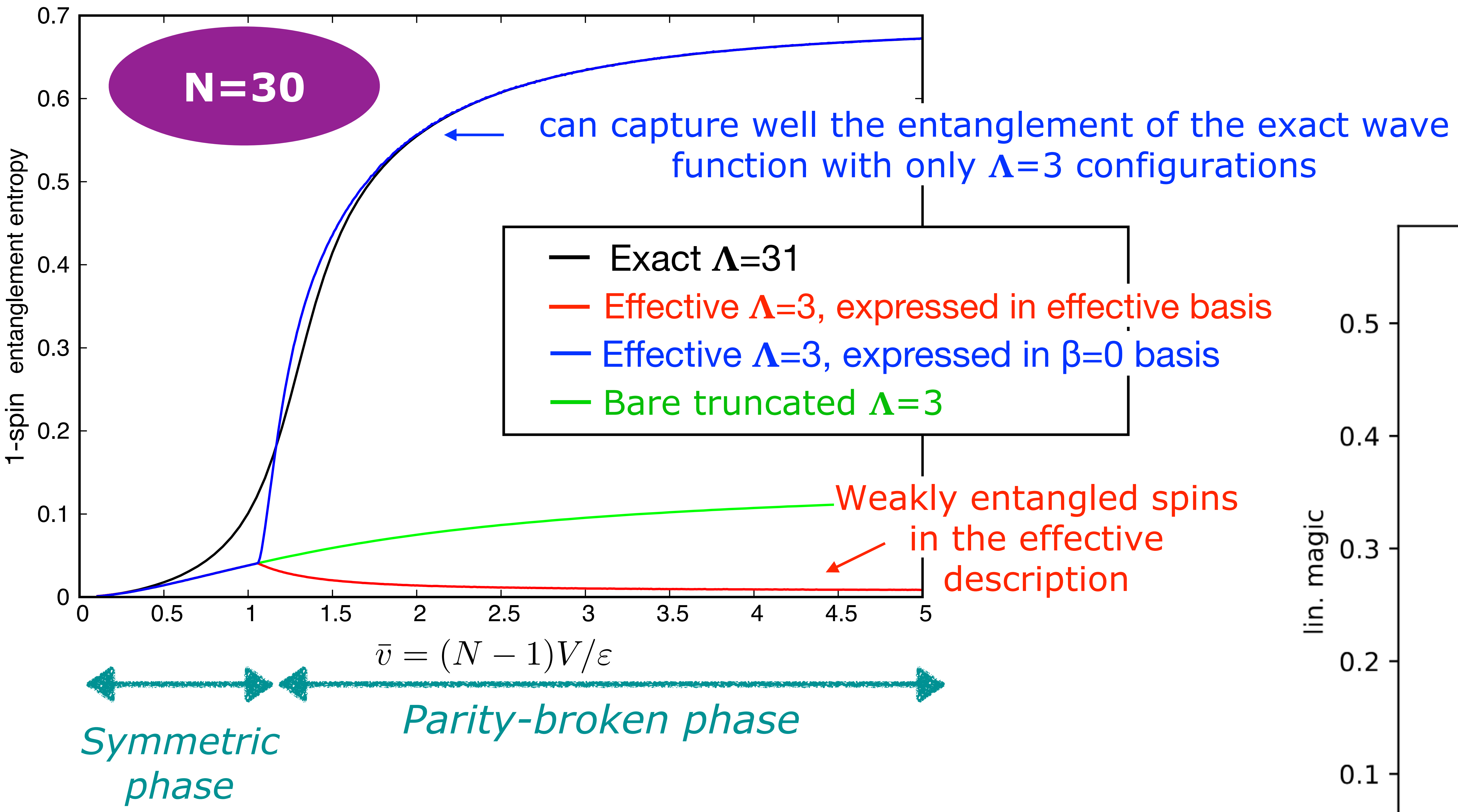
➡ Effective Hamiltonian $H(\beta) = U(\beta)^{\dagger} H U(\beta)$ $U(\beta) = e^{-iJ_y\beta}$

Determined variationally

Similar technique used in tensor networks to disentangle the vertices (e.g. MERA)

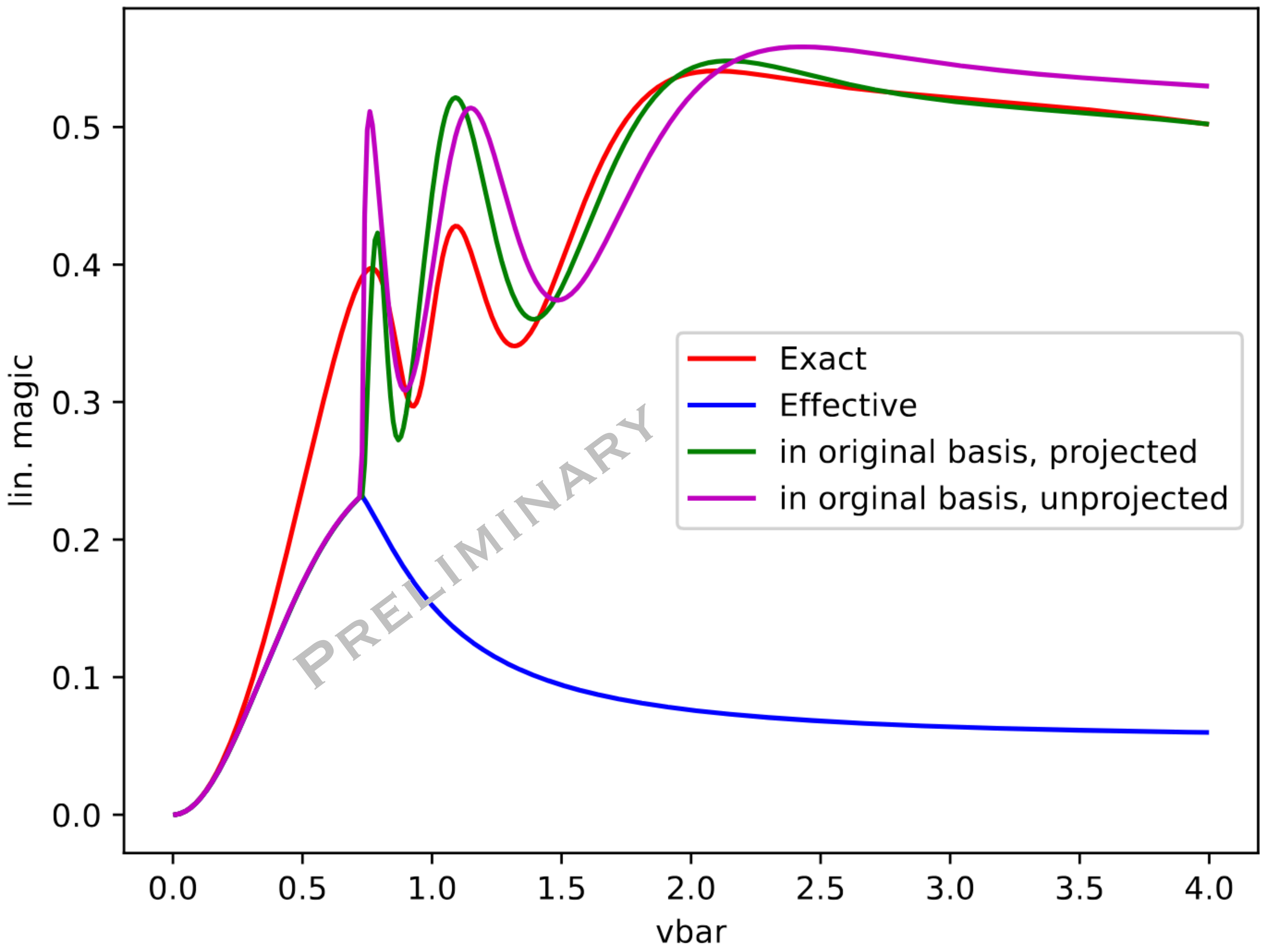
Entanglement Rearrangement and Quantum Simulations

★ 1-spin entanglement entropy



Hengstenberg, CR, Savage EPJA 59, 231 (2023)

★ Linear magic



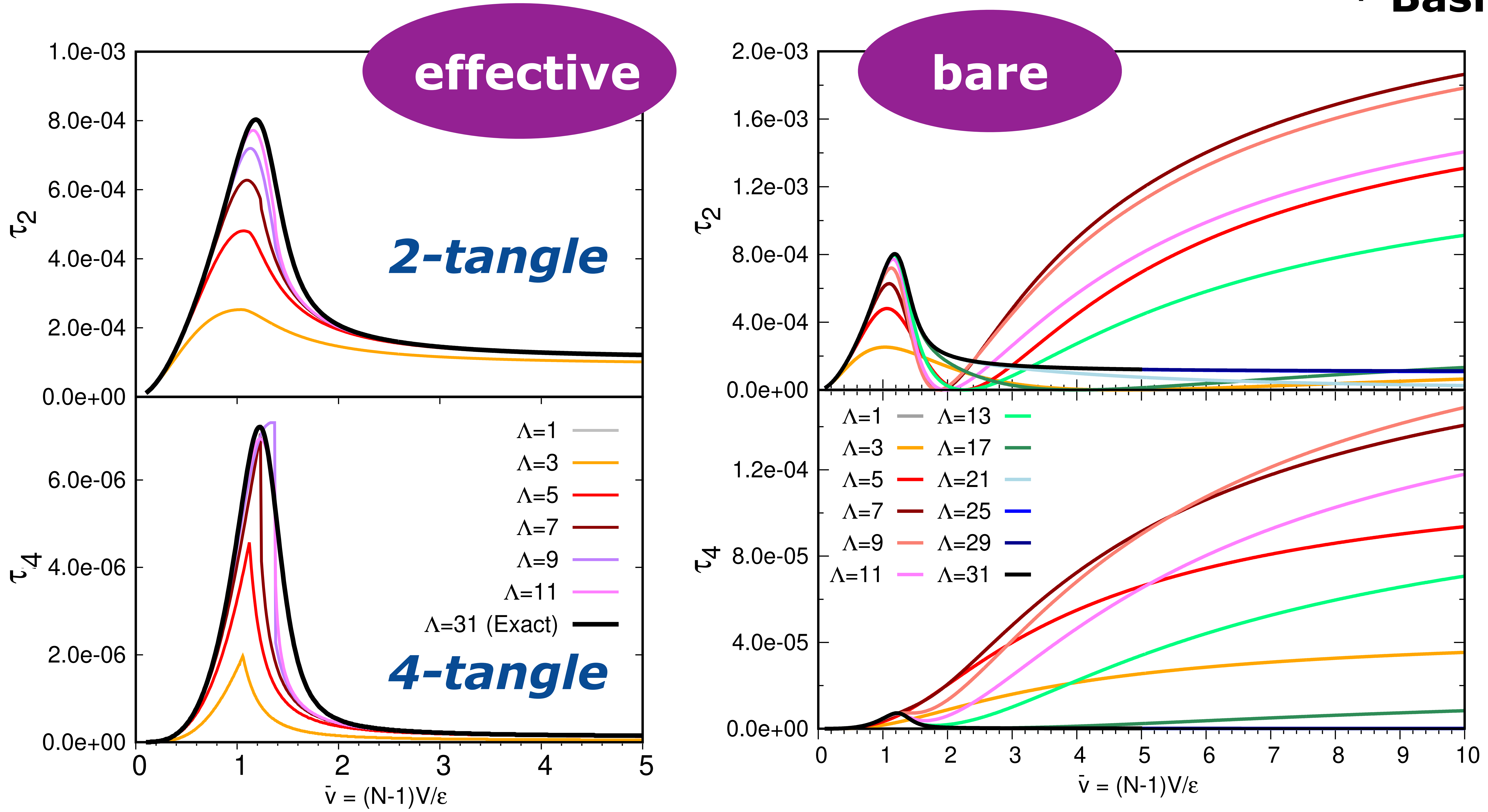
Hengstenberg, CR, Tirrito, in prep.

Sensitivity of multi-body entanglement to truncation and optimization

★ **n-tangles** $\tau_n = \left|\langle \Psi | \hat{\sigma}_y^{\otimes n} | \Psi^* \rangle\right|^2$

Multi-"spin" entanglement

*** Basis independent ***



Effective: *Rapid convergence which can be further improved with projection*

Bare: *convergence badly behaved*

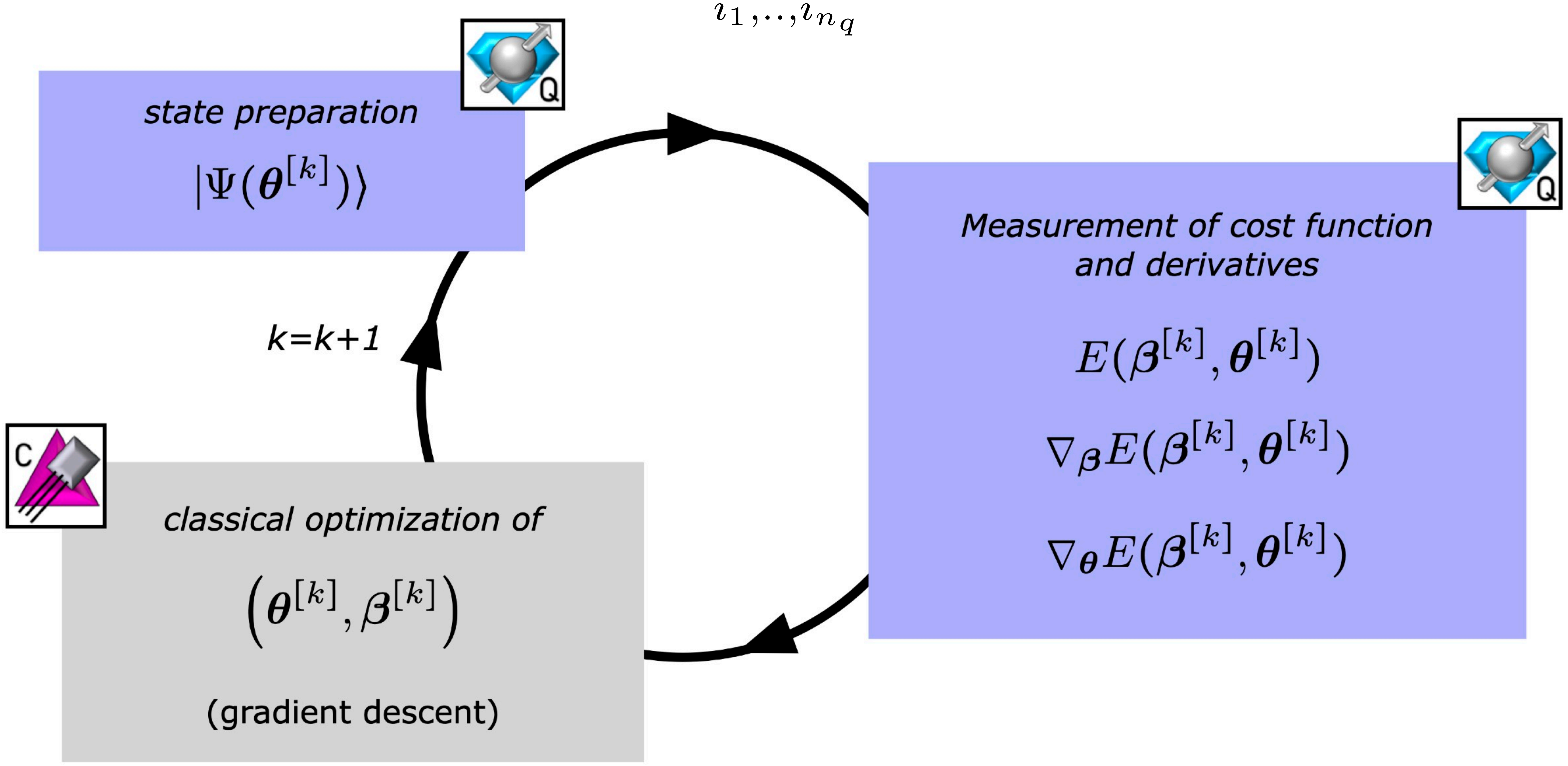
Entanglement Rearrangement and Quantum Simulations

★ *Hamiltonian-Learning-VQE Algorithm:*

CR, Savage PRC 108, 024313 (2023)

Cost function to minimize: $E(\beta, \boldsymbol{\theta}) = \langle \Psi(\boldsymbol{\theta}) | \hat{H}(\beta) | \Psi(\boldsymbol{\theta}) \rangle$

$$= \sum_{i_1,..,i_{n_q}} h_{i_i,..,i_{n_q}}(\beta) \, \langle \Psi(\boldsymbol{\theta}) | \overline{\sigma}_{i_1} \otimes \ \ldots \otimes \overline{\sigma}_{i_{n_q}} | \Psi(\boldsymbol{\theta}) \rangle$$



⇒ learns the effective Hamiltonian and identifies the associated ground state simultaneously

Entanglement Rearrangement and Quantum Simulations

★ *Implementation of HL-VQE for the LMG model on a digital quantum computer:*

$$|\Psi\rangle^{\Lambda} = \sum_{n=0}^{\Lambda-1} A_n |n\rangle$$

Map the many-body (Dicke) states |n> onto qubits:

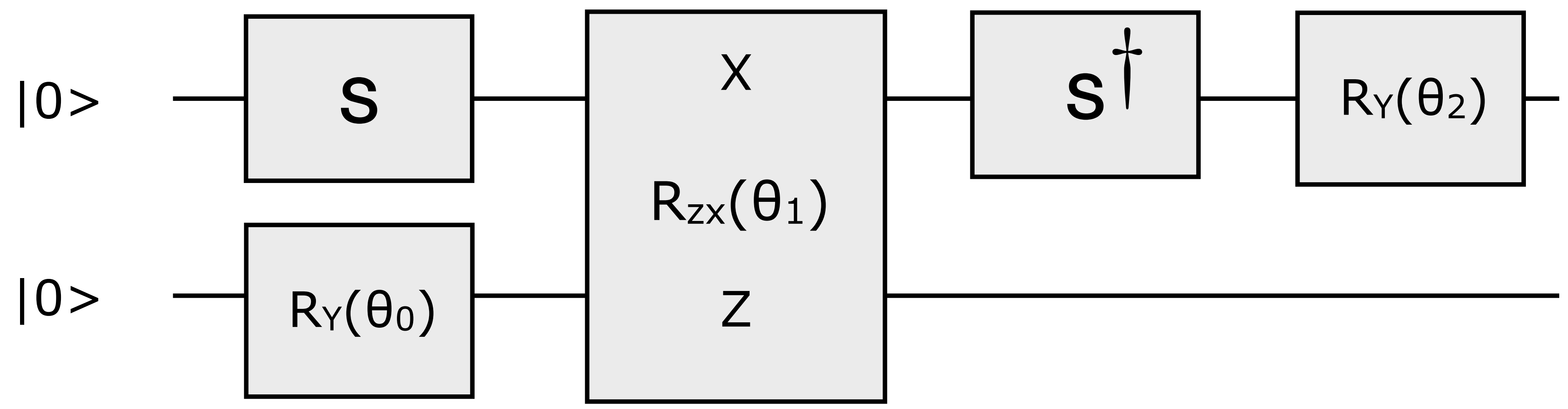
$$\Rightarrow \ \Lambda = 2^{n_{qubits}}$$

Number of qubits only depends on the cut-off $\boldsymbol{\Lambda}$, not the particle number

✳Example: 2 qubits (Λ = 4):

$$|\Psi(\theta_0, \theta_1, \theta_2)\rangle$$

$$= \cos\frac{\theta_0}{2}\cos\frac{\theta_2-\theta_1}{2}|00\rangle + \sin\frac{\theta_0}{2}\cos\frac{\theta_2+\theta_1}{2}|10\rangle + \cos\frac{\theta_0}{2}\sin\frac{\theta_2-\theta_1}{2}|01\rangle + \sin\frac{\theta_0}{2}\sin\frac{\theta_2+\theta_1}{2}|11\rangle$$

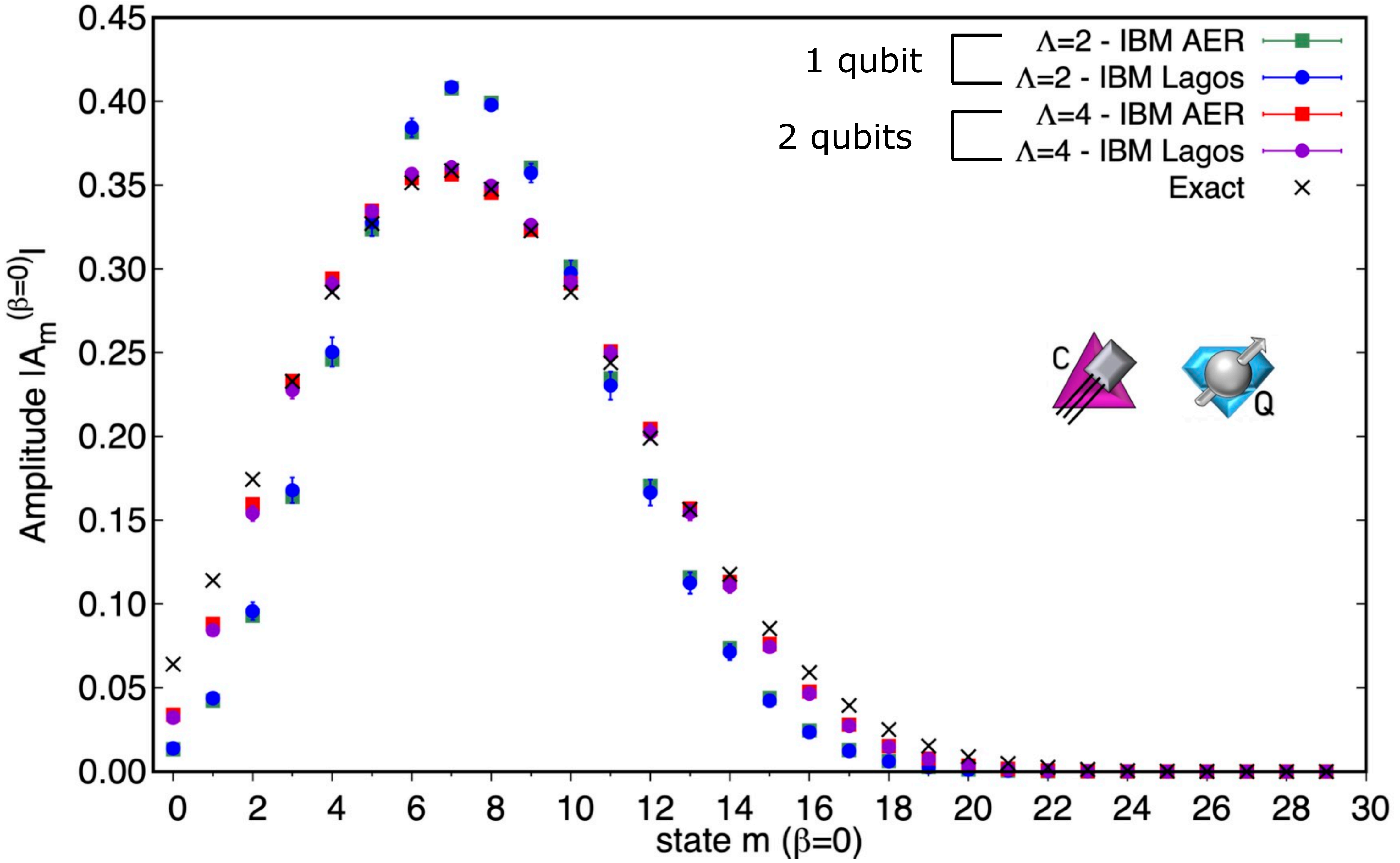


Entanglement Rearrangement and Quantum Simulations

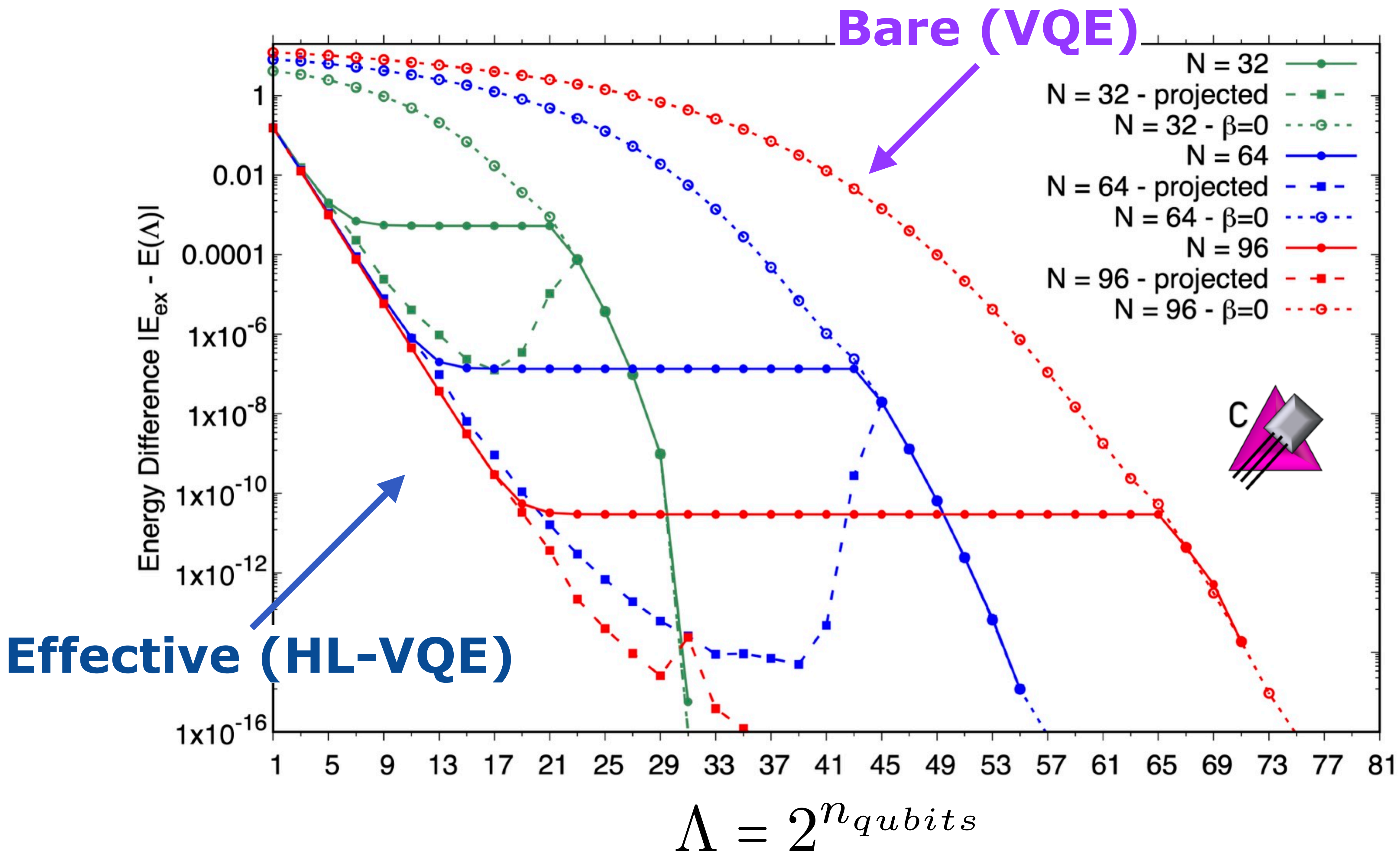
CR, Savage PRC 108, 024313 (2023)

★ New Hamiltonian-Learning-VQE Algorithm:

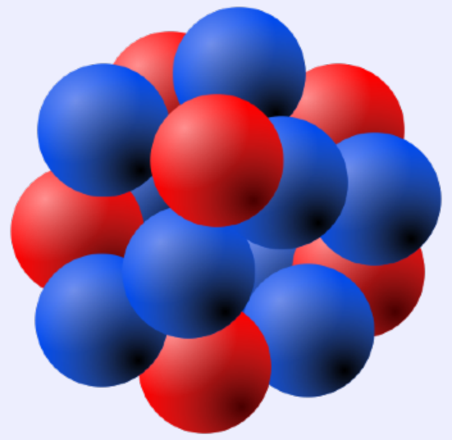
Wave function extracted from IBM quantum computer



Exponential Acceleration in the expected convergence:

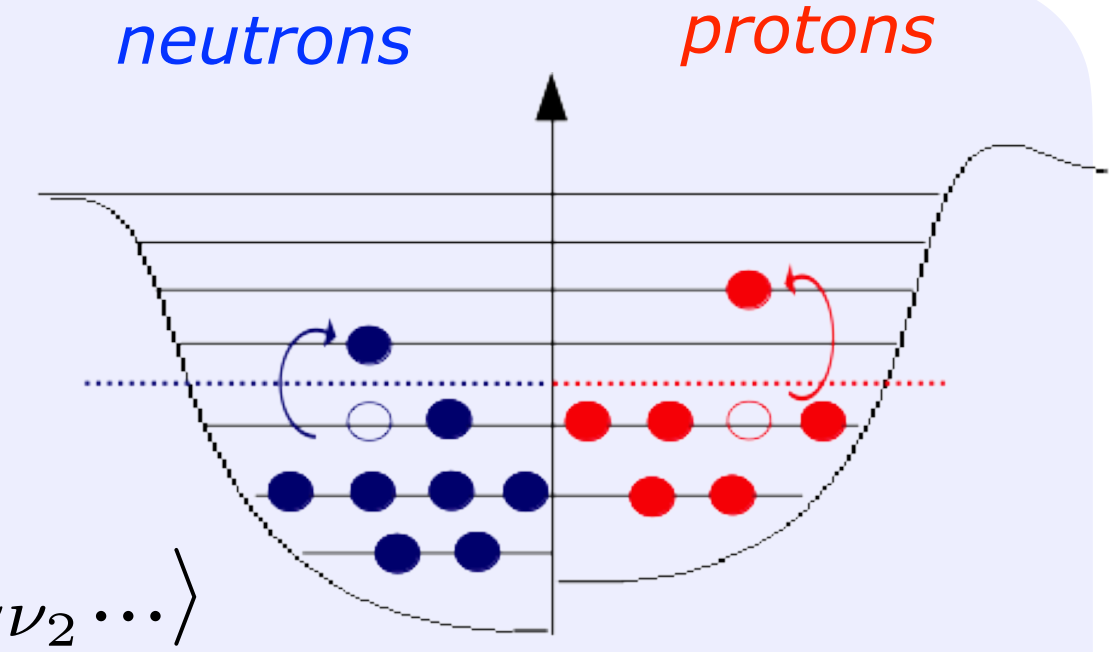


Entanglement in Nuclei



$$|\Psi\rangle = \sum_{\pi\nu} C_{\pi\nu} |\phi_\pi\rangle \otimes |\phi_\nu\rangle$$

$$= \sum_{n_{\pi_1} n_{\pi_2} \ldots n_{\nu_1} n_{\nu_2} \ldots} C_{n_{\pi_1} n_{\pi_2} \ldots n_{\nu_1} n_{\nu_2} \ldots} |n_{\pi_1} n_{\pi_2} \ldots n_{\nu_1} n_{\nu_2} \ldots\rangle$$



occupation numbers $n_i = 0 \text{ or } 1$

✳*Entanglement between proton and neutron subsystems*

See e.g. Papenbrock & Dean (2003), Gorton & Johnson (2023)

→ *Von Neumann Entropy* $S(\rho_\pi) = -\mathrm{Tr}(\rho_\nu \ln \rho_\nu)$

✳*Entanglement of modes (single-particle orbitals)*

See e.g. Legeza+ (2015), CR & Savage (2020), Tichai+ (2022), Pérez-Obiol+ (2023)

→ *One-Orbital Von Neumann Entropy; Two-Orbital Mutual Information, Negativity*

Entanglement Rearrangement In Nuclei

“Entanglement distance”:

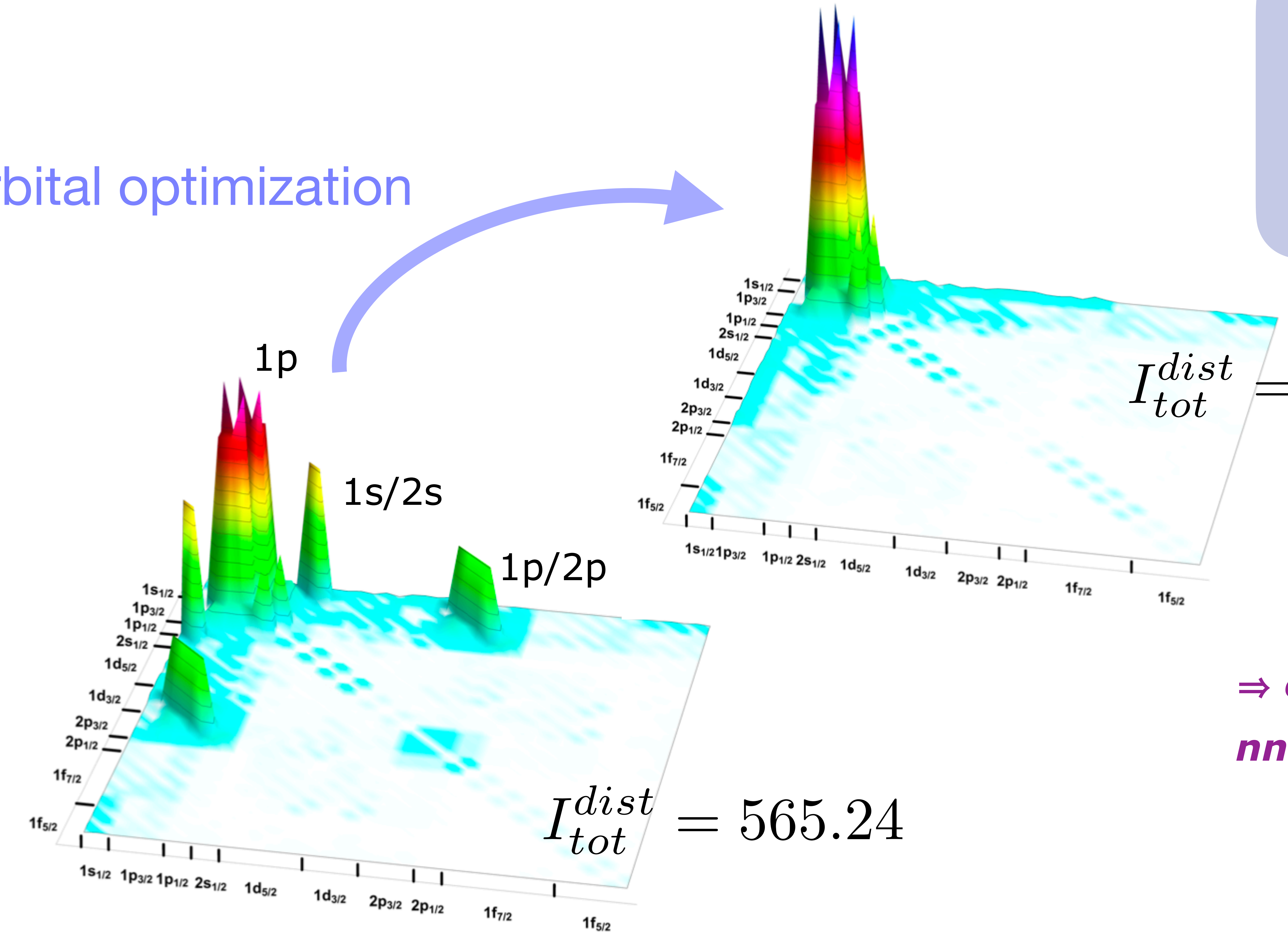
$$I_{ij}^{dist} = I_{ij} \times |i-j|^2$$

$$I_{tot}^{dist} = \sum_{ij} I_{ij}^{dist}$$

^{6}He

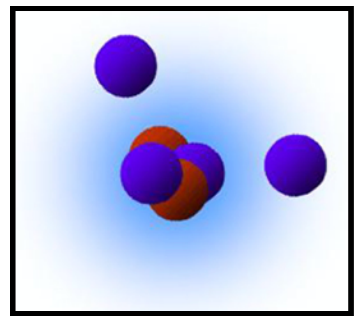
Neutron-neutron mutual information

Orbital optimization



$$I_{tot}^{dist} = 565.24$$

$$I_{tot}^{dist} = 115.90$$



⇒ emergence of ^{4}He-core + nn-valence structure?

CR, Savage, Pillet, PRC 103, 034325 (2021)

Multi-Partite Entanglement in Shell-Model Nuclei

Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064

*Multi-Partite entanglement via n-tangles**

$$\tau^{(n)}_{(i_1 \ldots i_n)} = \left| \langle \Psi | \hat{\sigma_y}^{(i_1)} \otimes \ldots \otimes \hat{\sigma_y}^{(i_n)} | \Psi^* \rangle \right|^2$$

Jordan Wigner Mapping

$$a_i^\dagger \rightarrow \left(\prod_{j<i} \hat{\sigma}_z^{(j)}\right)\left(\hat{\sigma}_x^{(i)} - i\hat{\sigma}_y^{(i)}\right)/2$$

$$a_i \rightarrow \left(\prod_{j<i} \hat{\sigma}_z^{(j)}\right)\left(\hat{\sigma}_x^{(i)} + i\hat{\sigma}_y^{(i)}\right)/2$$

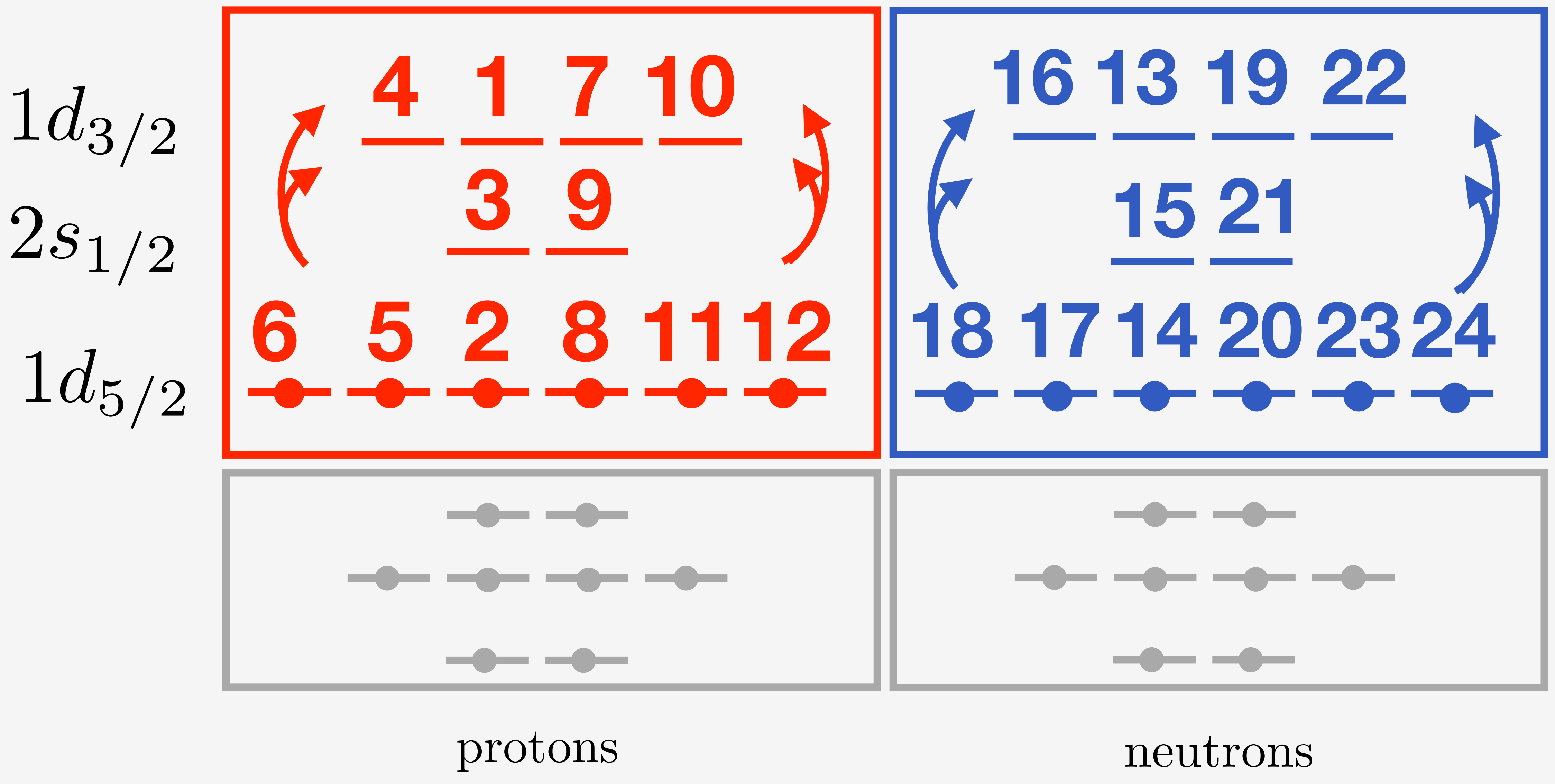
⇒ n-tangles related to n/2-body entanglement

*Wong, Christensen, PRA 63, 044301 (2001)

Nuclear wavefunction computed with the shell-model BIGSTICK code

C. W. Johnson+ Comp. Phys. Comm. 184, 2761 (2013).

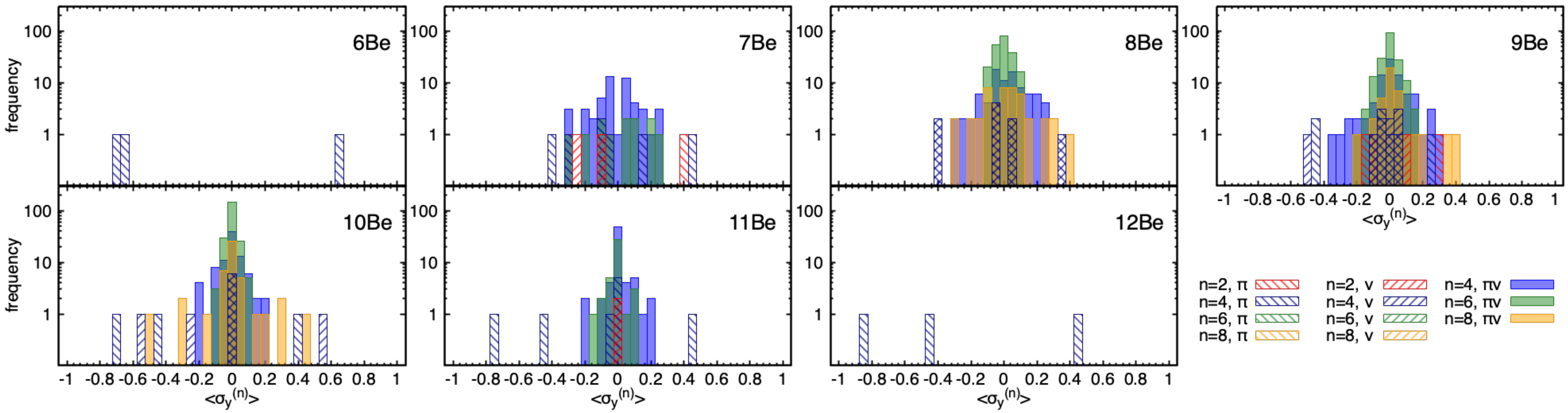
sd-shell nuclei



Multi-Partite Entanglement in Shell-Model Nuclei

Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064

Distribution of the Pauli strings expectation values in the Be chain:

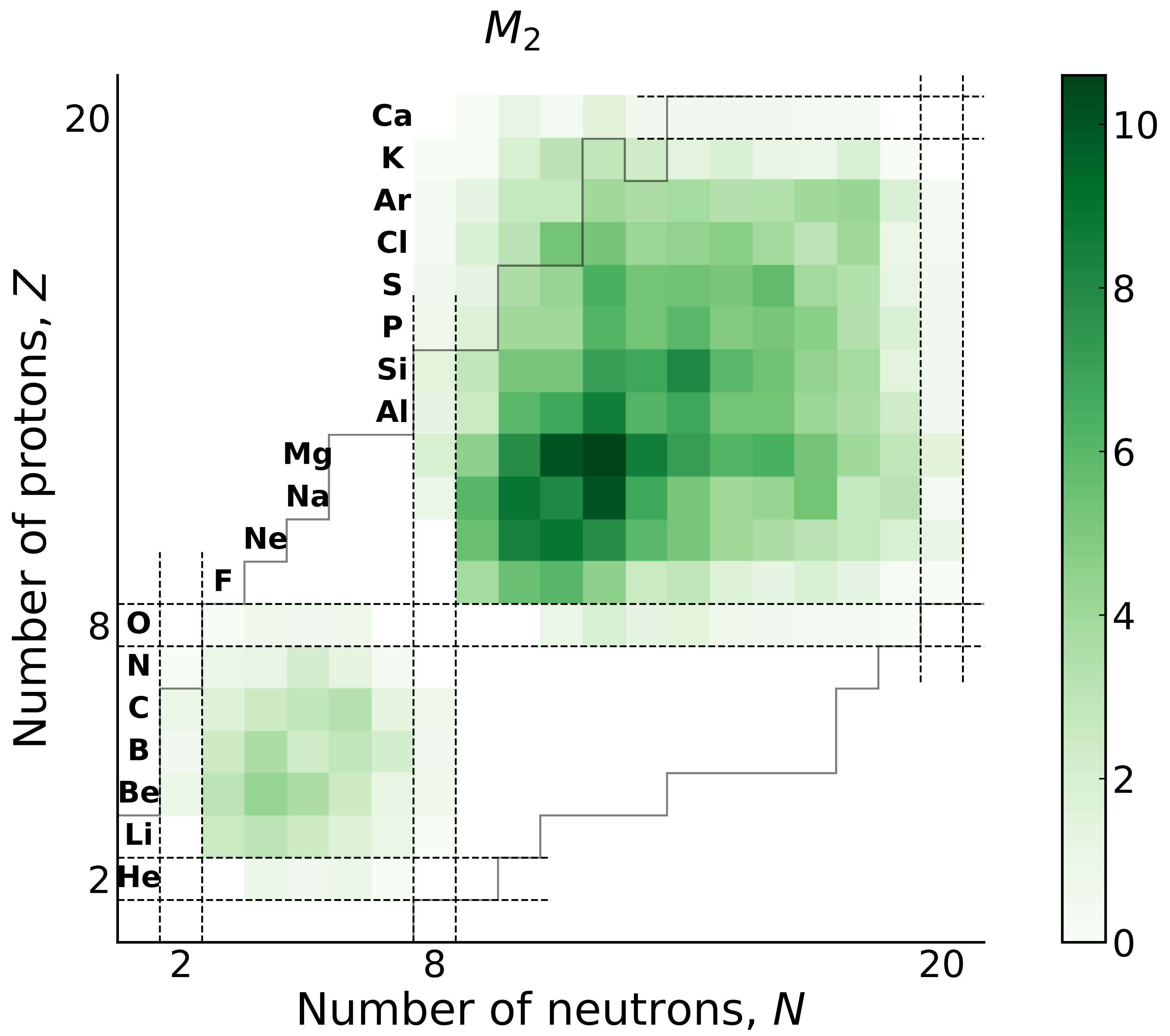


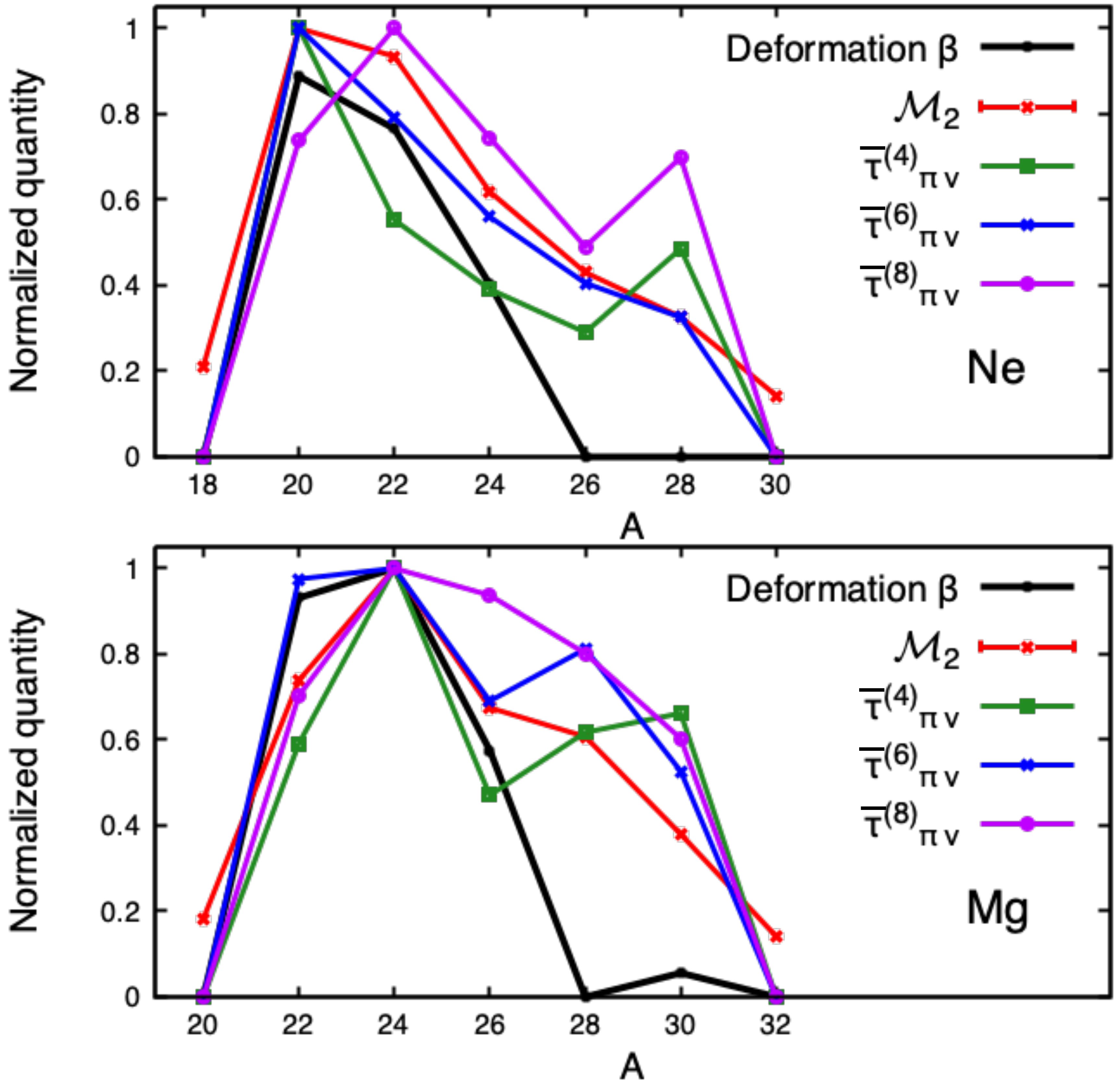
- large many-body entanglement when the model space and symmetries allow it
- proton-neutron entanglement is more collective than pure proton or neutron entanglement
- large proton-neutron 8-tangles → hint of alpha correlations?

Magic in Shell-Model Nuclei

$$\overline{\tau}^{(n)}_{\pi,\nu,\pi\nu} \equiv \sum_{i_1,i_2,\ldots i_n \in \pi,\nu,\pi\nu} \tau^{(n)}_{(i_1\ldots i_n)}$$

Magic calculations with exact and MCMC techniques:





- Maximal magic and proton-neutron tangles coincides with maximal deformation in nuclei
- Magic and tangles also persist in the region where axial deformation vanishes (shape co-existence)

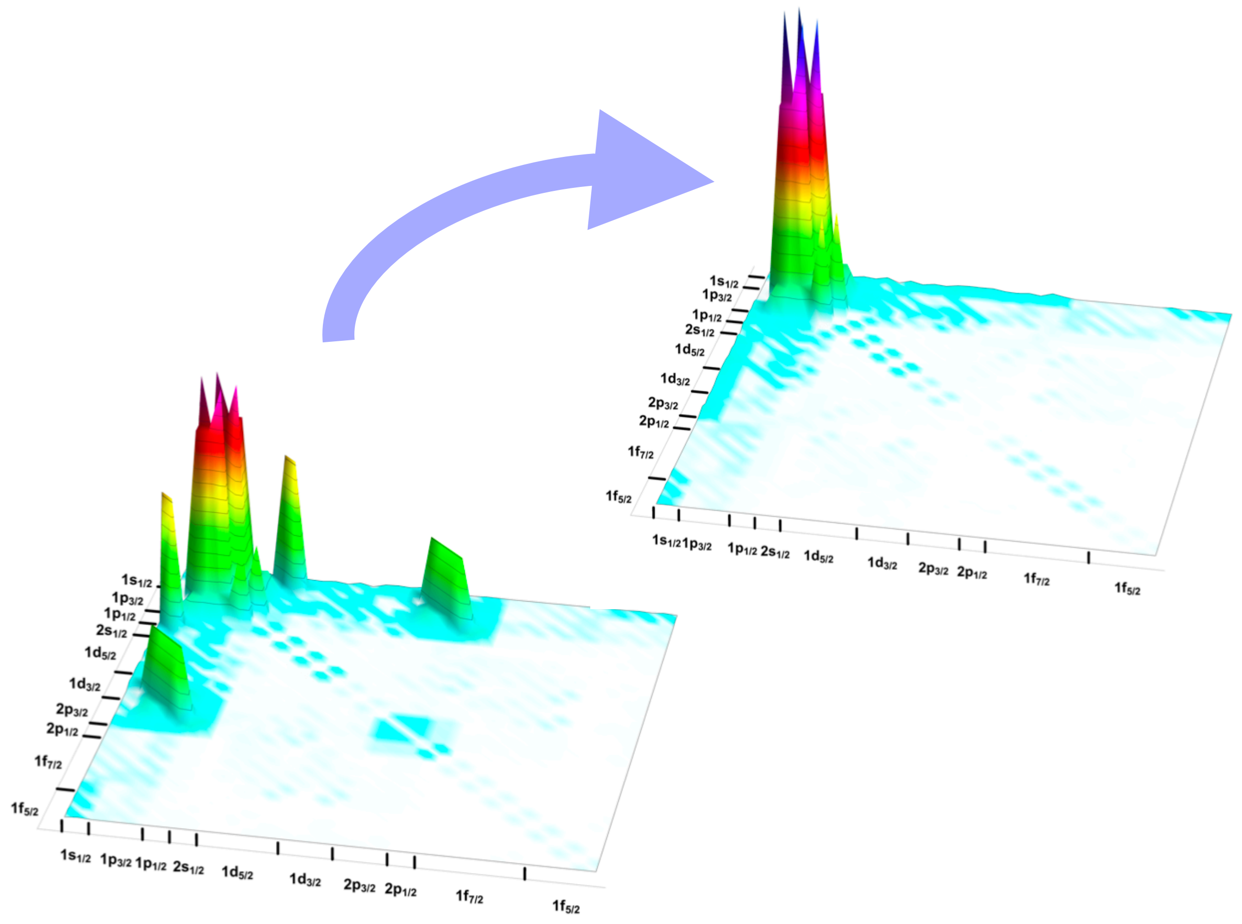
→ See Federico Rocco's talk on Friday

Outline

★ Entanglement and Magic Rearrangement in Nuclear Many-Body Systems

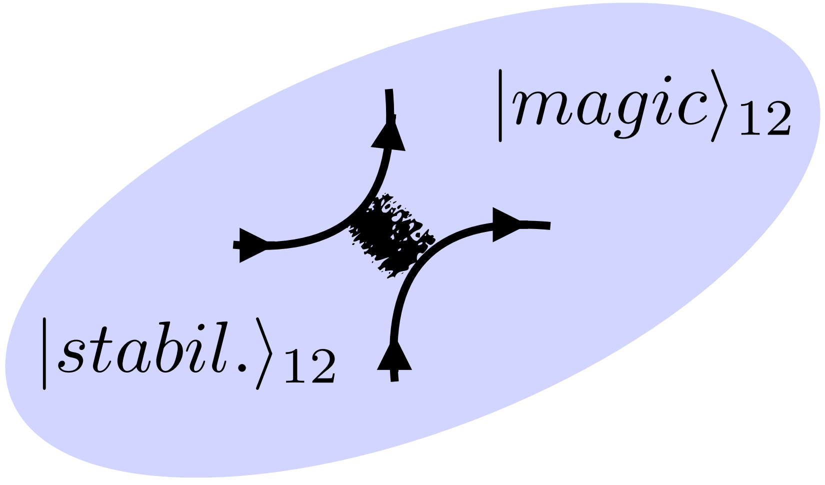
→ From the Lipkin model to nuclei

CR, Savage, Pillet, PRC 103, 034325 (2021);
CR & Savage PRC 108, 024313 (2023);
Hengstenberg, CR, Savage EPJA 59, 231 (2023);
Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064



★ The Magic Power in Nuclear and Hyper-Nuclear Forces

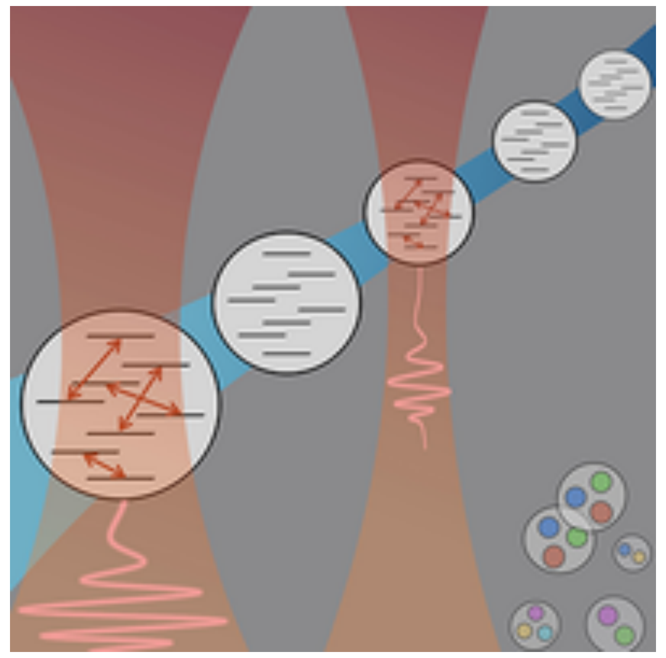
CR & M. J. Savage arXiv:2405.10268



★ Symmetry-guided mapping of quantum systems onto qudits

→ Fermionic model with SO(5) symmetry and 1+1D SU(3) QCD

Illa, CR, Savage PRC 108, 064306 (2023); PRD 110, 014507 (2024)



The Magic Power of Nuclear and Hyper-Nuclear Forces

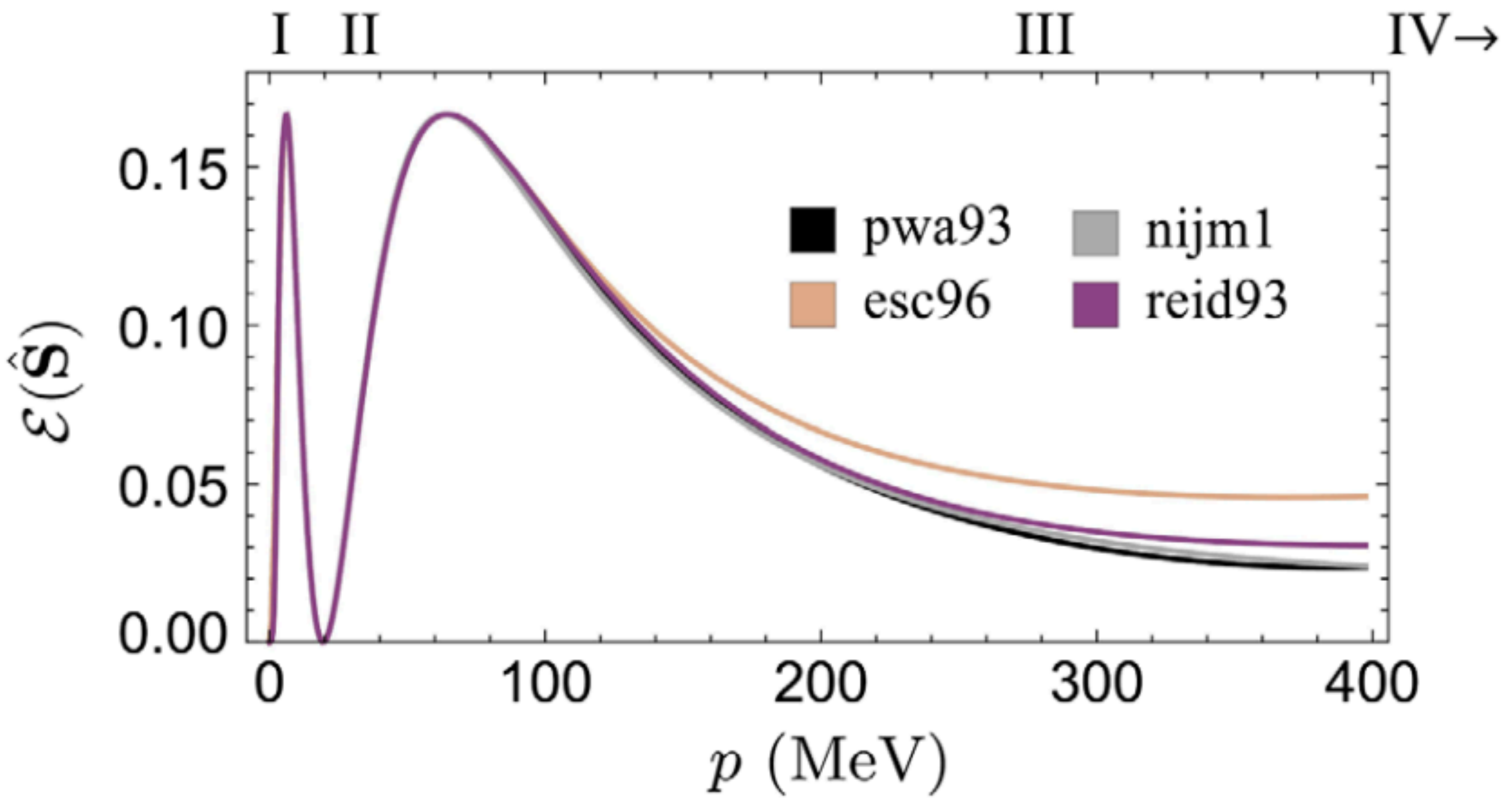
Beane, Kaplan, Klco, Savage PRL 122, 102001 (2019)

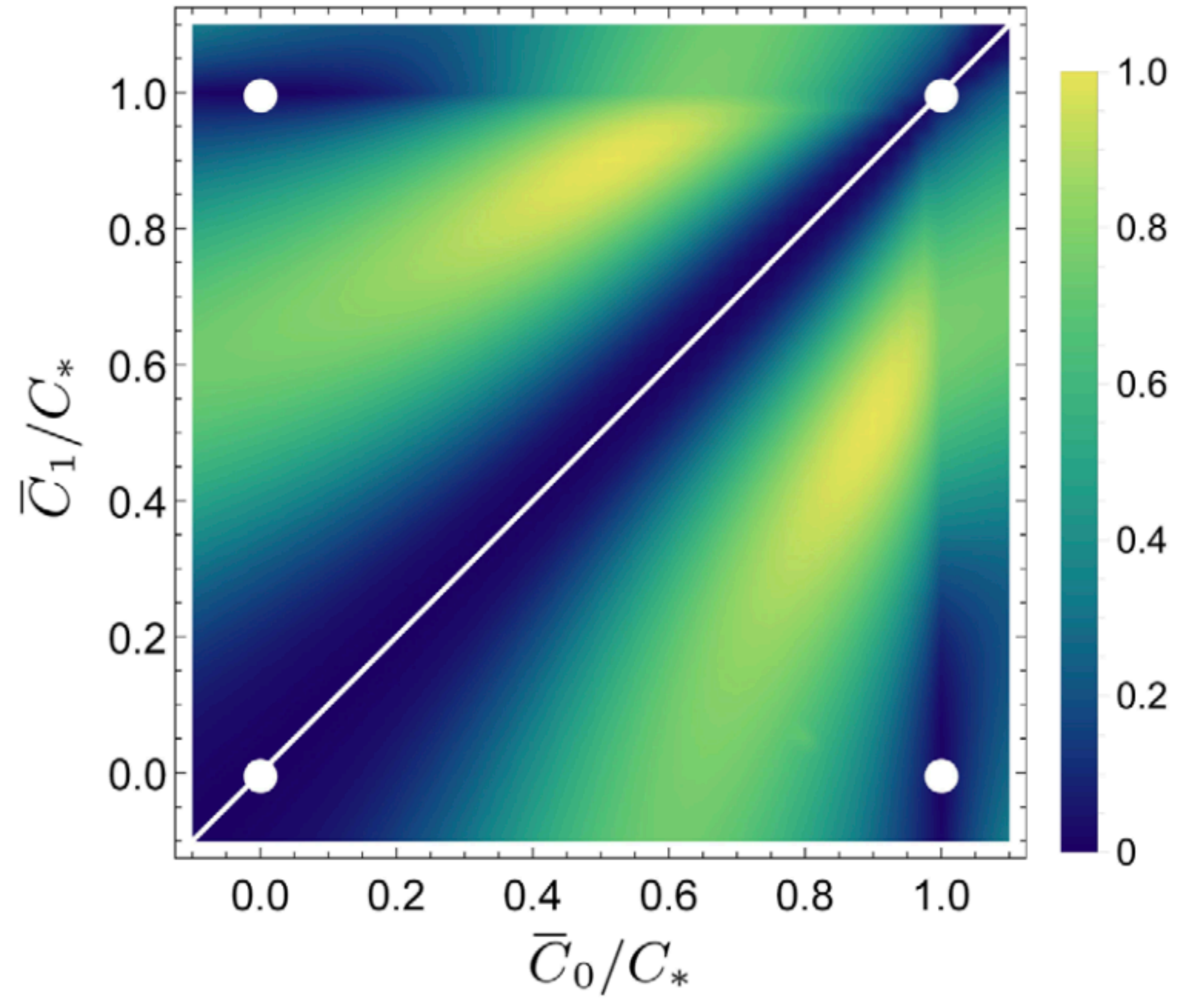
S-wave NN scattering

$$\hat{\mathbf{S}} = \frac{1}{4}\left(3\, e^{2i\delta_1} + e^{2i\delta_0}\right)\hat{\mathbb{1}} + \frac{1}{4}\left(\, e^{2i\delta_1} - e^{2i\delta_0}\right)\hat{\boldsymbol{\sigma}}.\hat{\boldsymbol{\sigma}}$$

Entanglement Power

$$\mathcal{E}(\hat{\mathbf{S}}) = 1 - \int \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi}\,\mathrm{Tr}_1\left[\hat{\rho}_1^2\right],$$

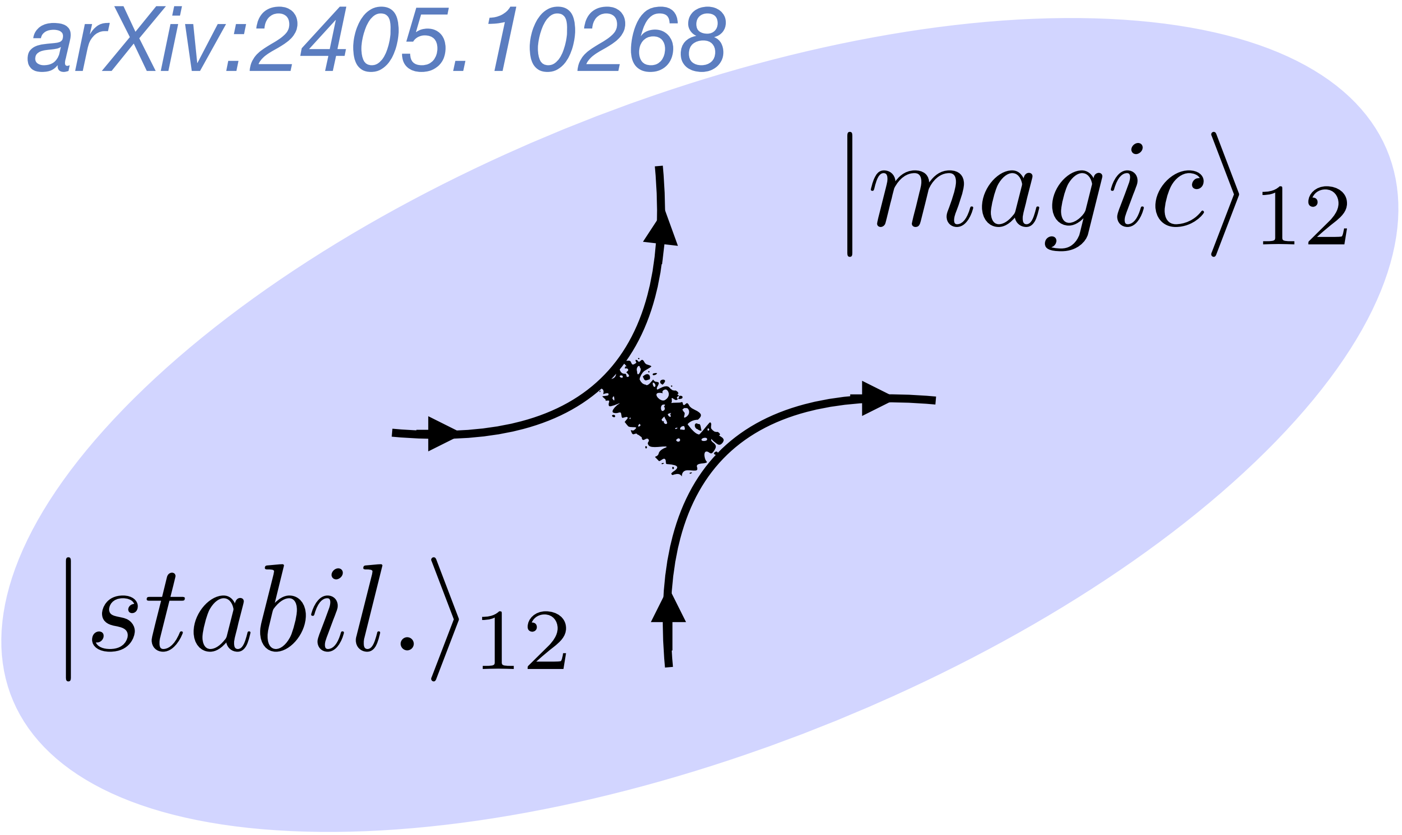




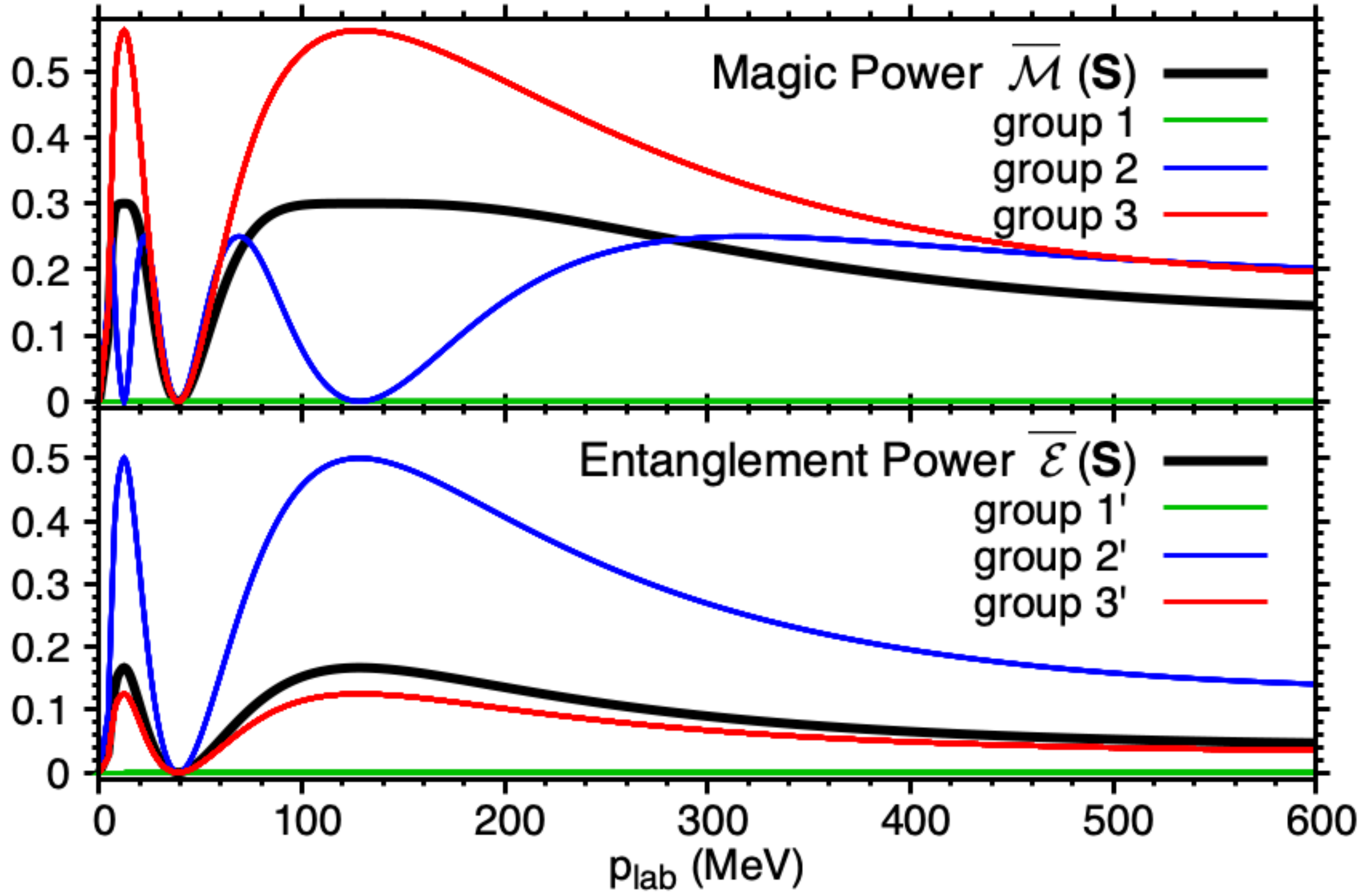
vanishing entanglement power occurs at points of emergent global symmetries

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268



$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) = \frac{3}{20}\Big(3+\cos(4\,\Delta\delta)\Big)\sin^2(2\,\Delta\delta) \qquad \overline{\mathcal{E}}(\hat{\mathbf{S}}) = \frac{1}{6}\sin^2(2\,\Delta\delta)$$



Magic power of the S-matrix:

$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}}\sum_{i=1}^{\mathcal{N}_{ss}}\mathcal{M}\left(\hat{\mathbf{S}}\,|\Psi_i\rangle\right)$$

Average fluctuations in magic induced by the S-matrix

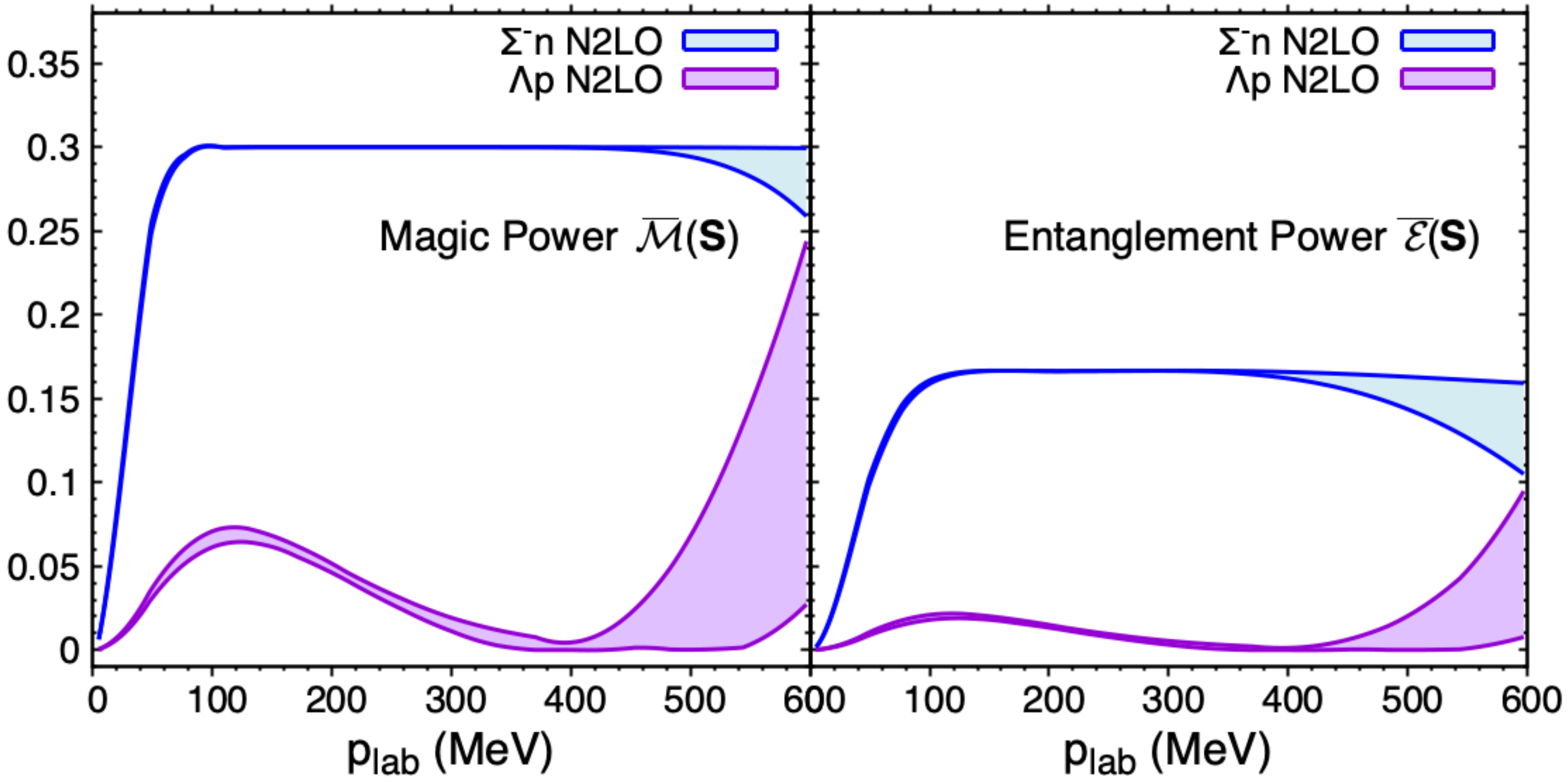
Entanglement power of the S-matrix

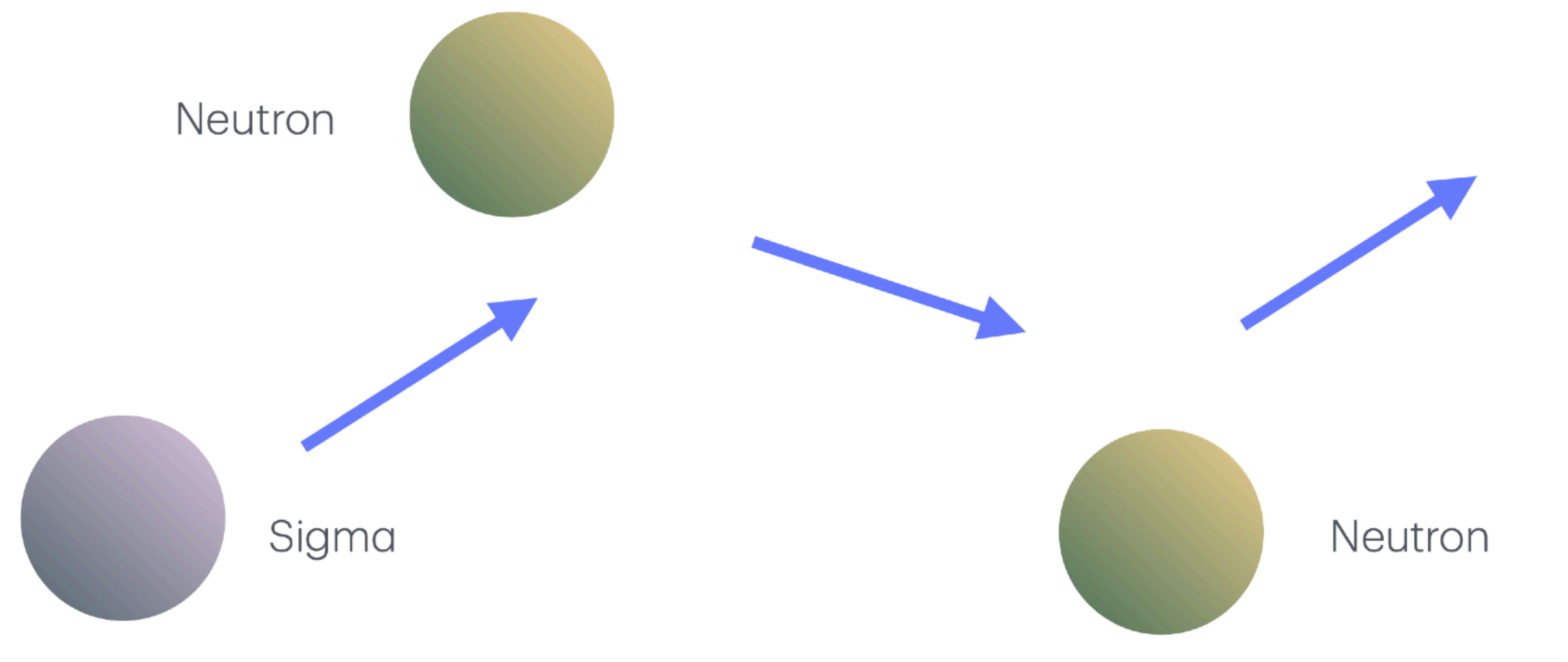
$$\overline{\mathcal{E}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}^{TP}}\sum_{i=1}^{\mathcal{N}_{ss}^{TP}}\mathcal{E}\left(\rho_i^{(1)}(\hat{\mathbf{S}})\right)$$

→ Same results as in Beane+ PRL 122, 102001 (2019) with continuous integration over spin orientations of initial tensor-product states

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268





➡ Σ^--baryon is identified as a potential candidate catalyst for enhanced spreading of magic and entanglement in dense matter

Entanglement power also in Beane+ PRL 122, 102001 (2019); Liu+ PLB 856, 138899 (2024)

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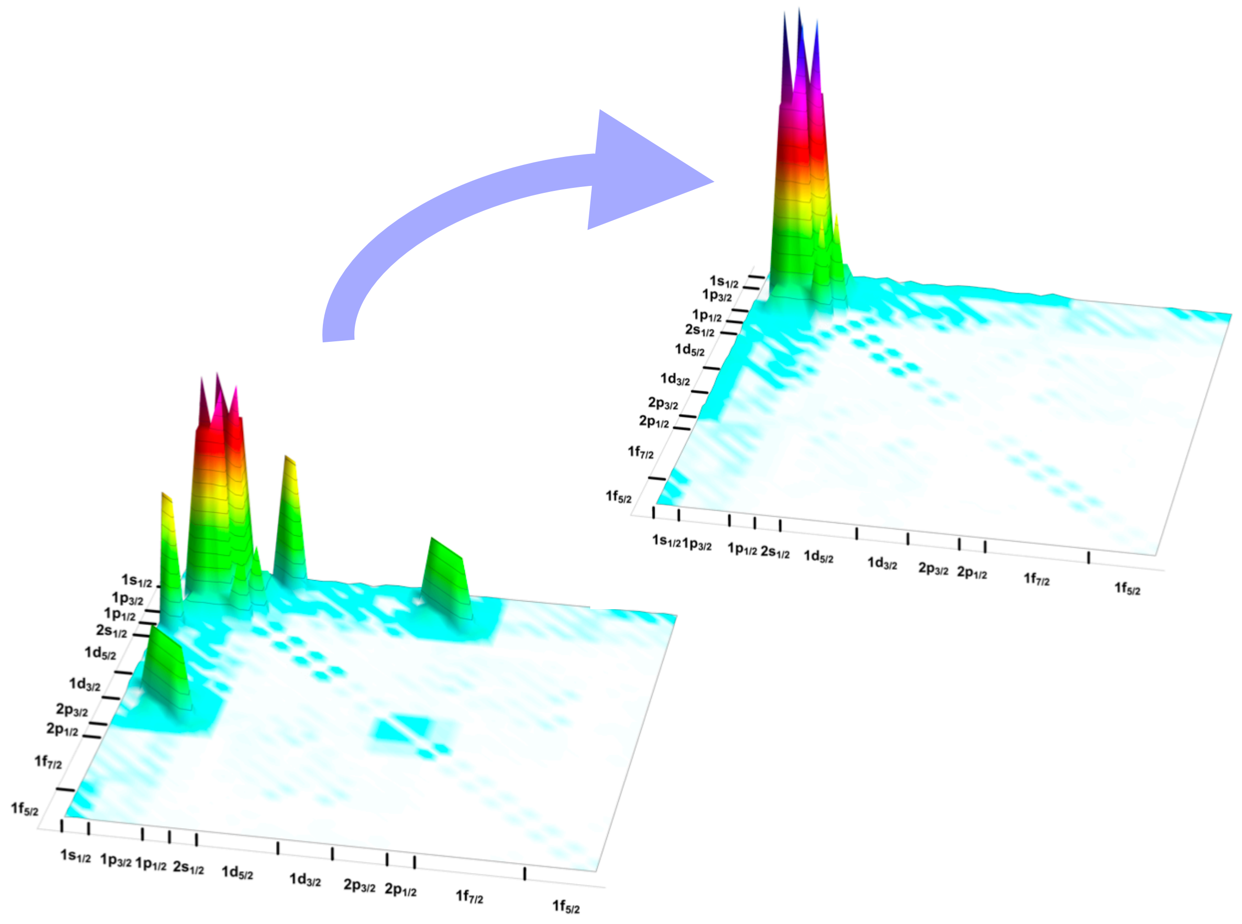

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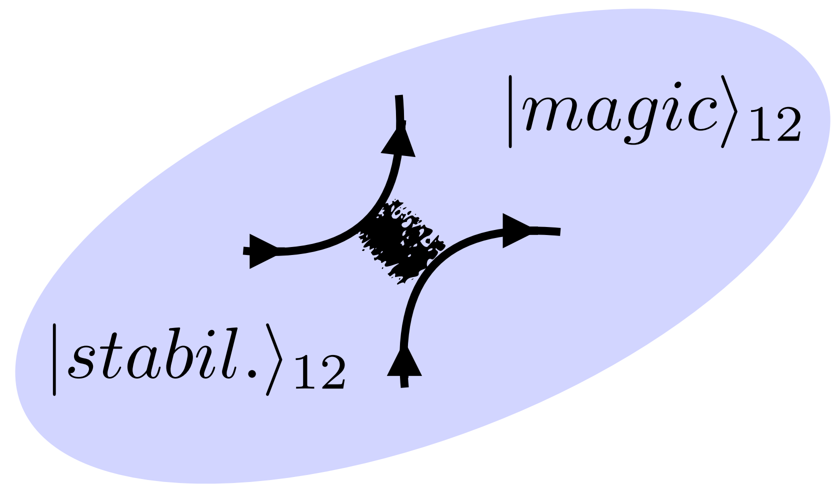

★ Symmetry-guided mapping of quantum systems onto qudits

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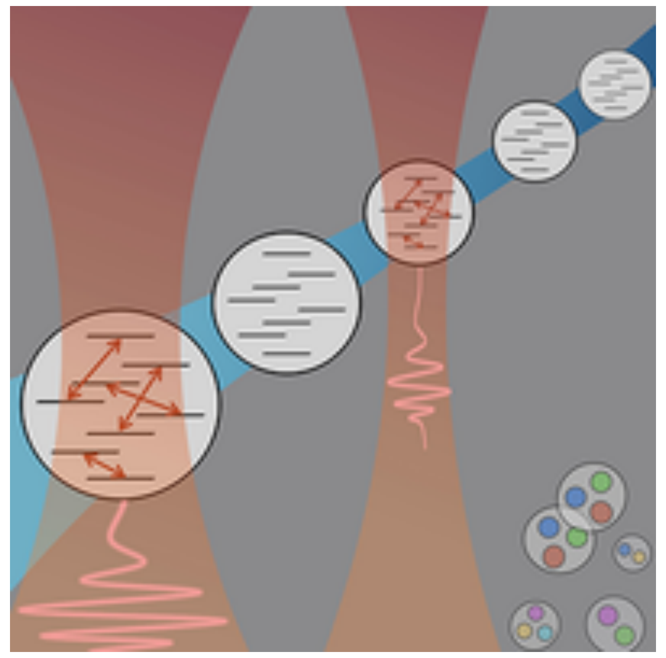
Illa, CR, Savage PRC 108, 064306 (2023); PRD 110, 014507 (2024)



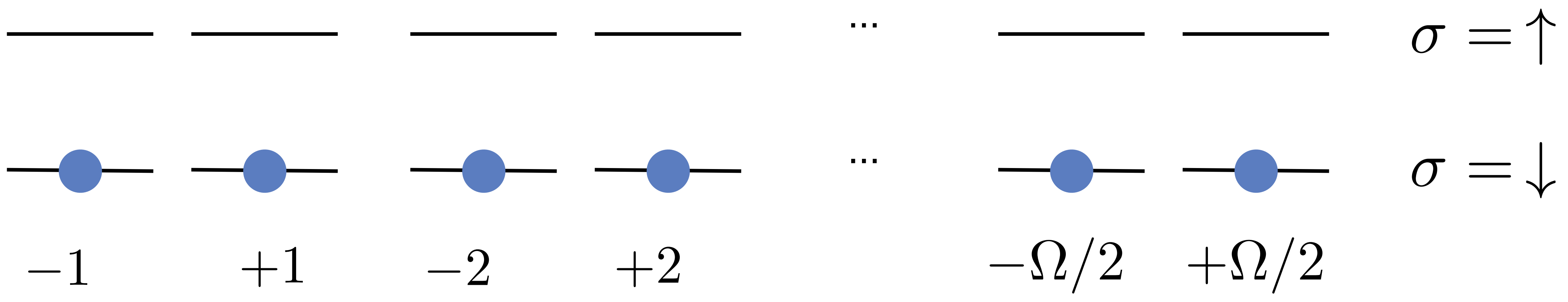
The Agassi model as demonstration of symmetry-guided mapping

*D. Agassi, Nucl. Phys. A 116, 49 (1968)

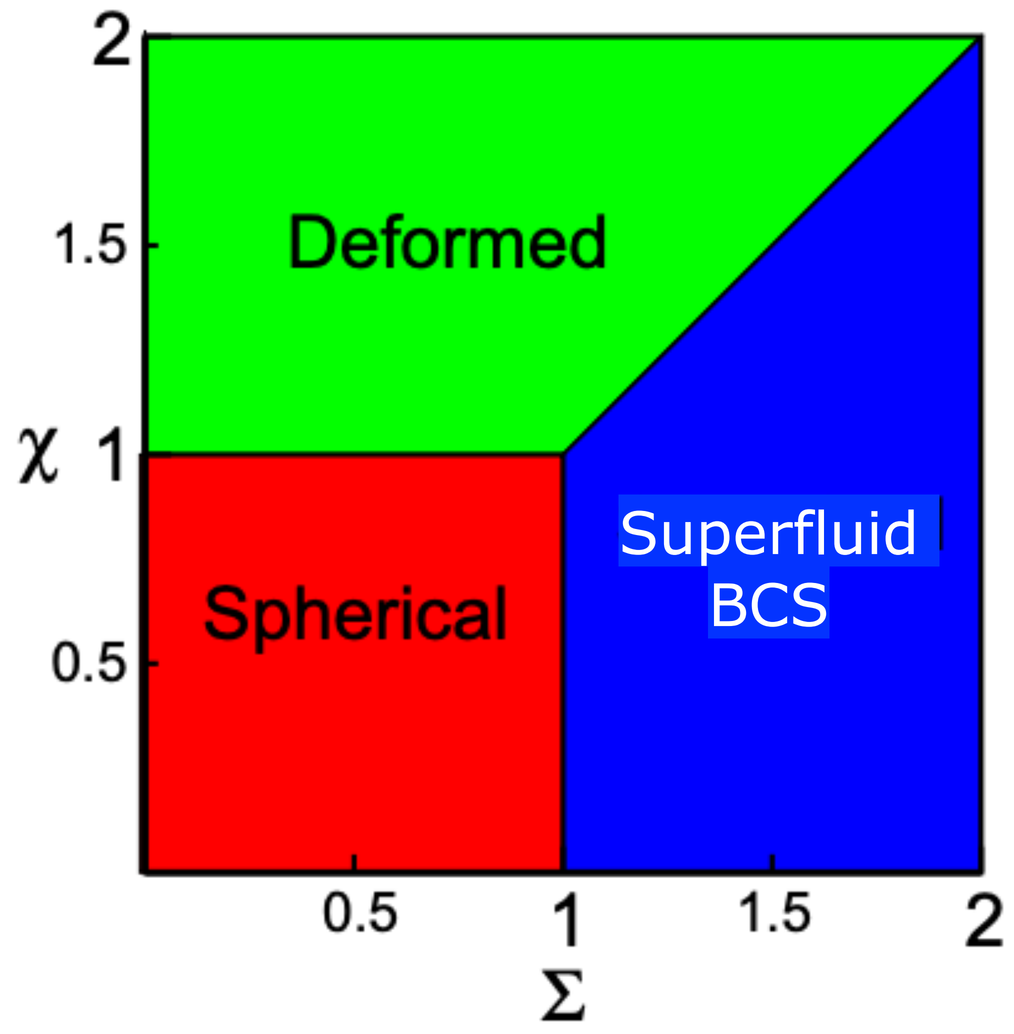
★The Agassi model

= extension of the LMG model with superfluid pairing

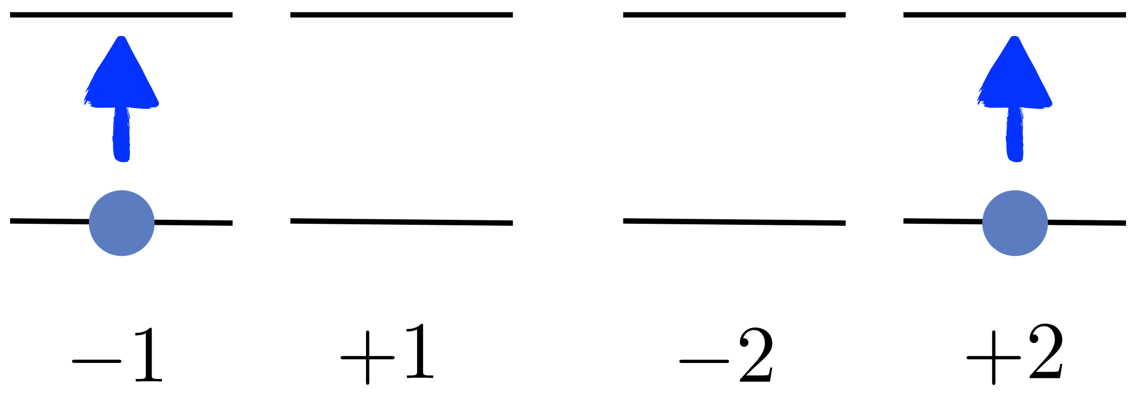
"mode" $k =$



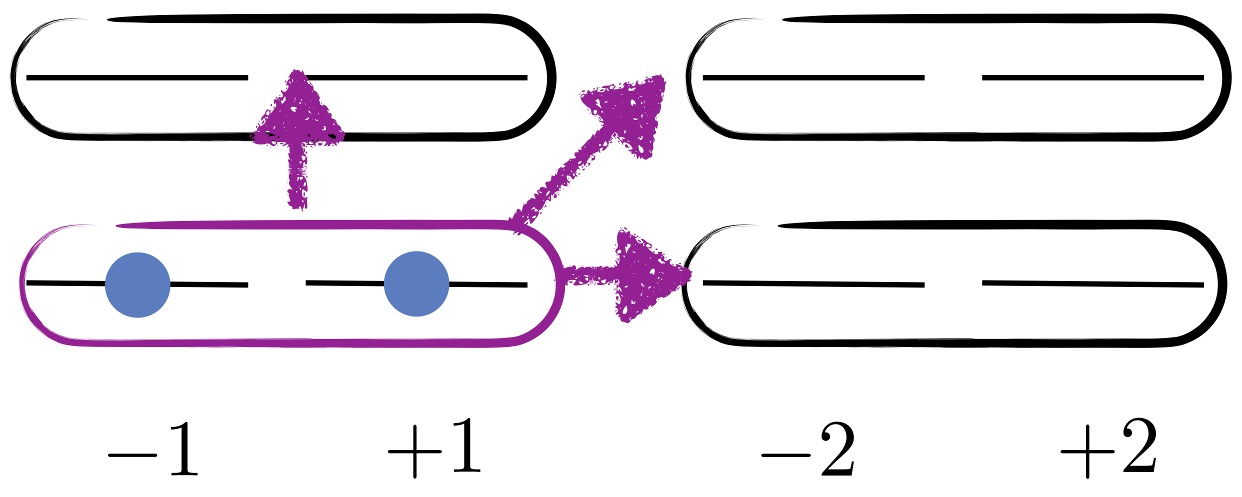
$$\hat{H} = \varepsilon \hat{J}_z - \frac{V}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right) - g \sum_{\sigma\sigma'} \hat{B}_\sigma^\dagger \hat{B}_{\sigma'}$$



[Pérez-Fernández+ PLB 829 137133 (2022)]



particle-hole interaction V



pairing interaction g

Symmetry-guided mapping of the Agassi model onto qudit systems

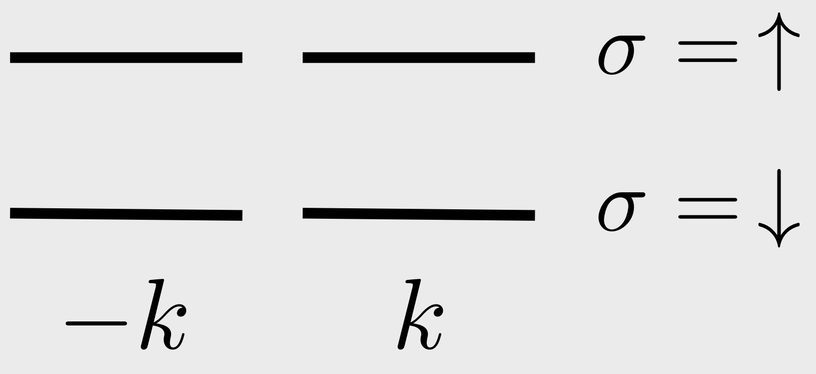
- Previous quantum simulations of the Agassi model:

  Pérez-Fernández,et al. PLB 829, 137133 (2022); Sáiz, García-Ramos,et al. PRC 106, 064322 (2022): Ω=2 & 4 with 4 & 8 qubits

  Jordan-Wigner mapping of the sites (k, σ) onto qubits: —— ≡ $|0\rangle$ —●— ≡ $|1\rangle$

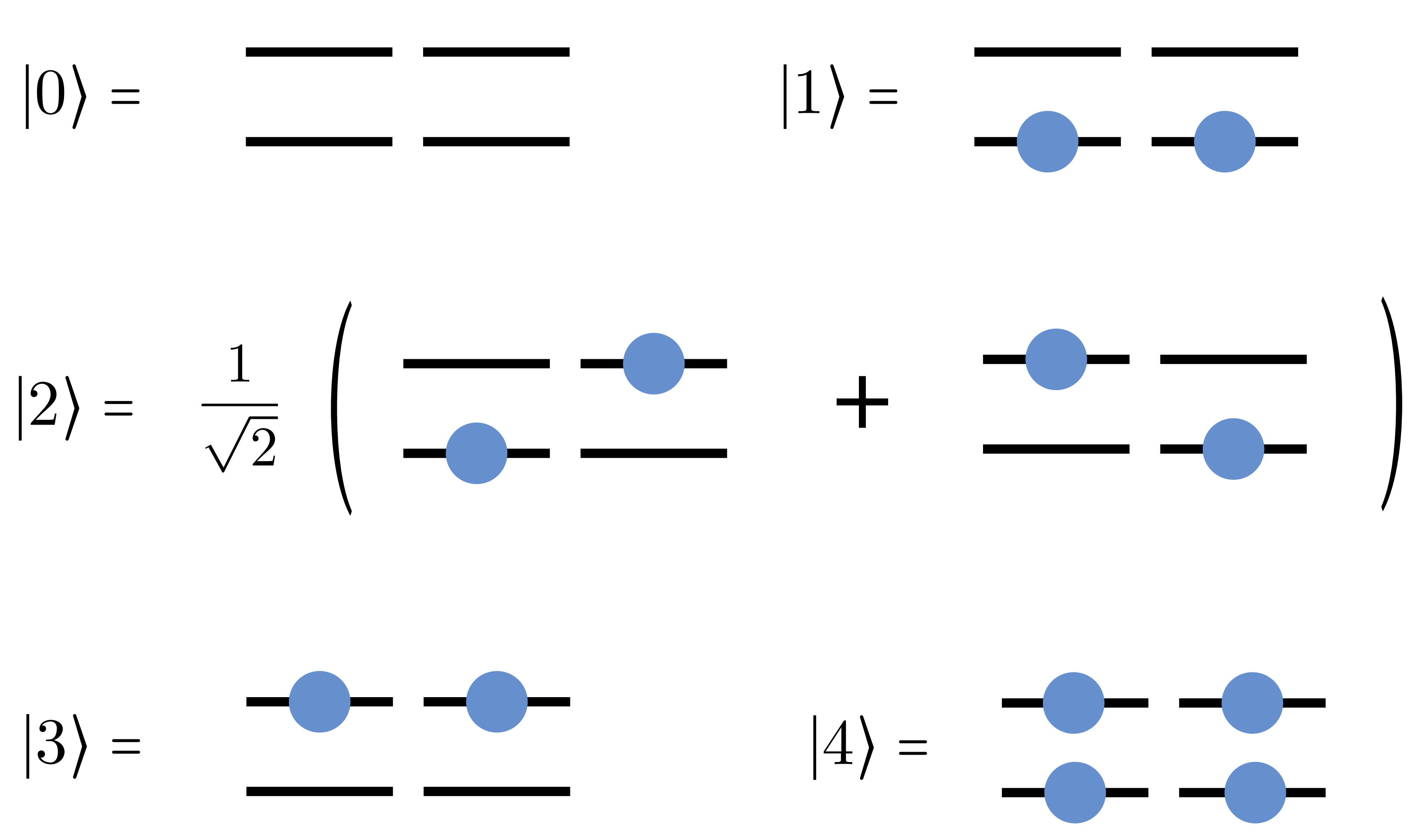
- **Here we make use of the SO(5) symmetry:**

Degrees of freedom = pairs of modes

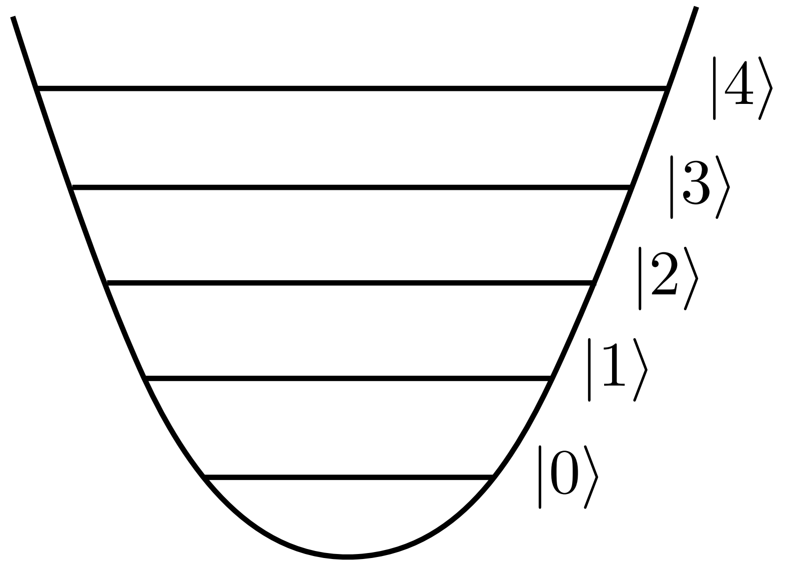


$J_z, J_\pm, B_{\uparrow,\downarrow}, B^\dagger_{\uparrow,\downarrow}$ = generators of SO(5)

⇒ 5 states:



Naturally maps onto "qu5its"
[qudits with d=5]



Symmetry-guided mapping of the Agassi model onto qudit systems

★ ***Time evolution — circuits for simulations using qu5its***

- **Hamiltonian mapping to qu5its:**

$$\hat{H} = \sum_{j=1}^{\Omega/2} \sum_{j' \neq j=1}^{\Omega/2} H^{(2)}_{(jj')}$$

Acts on 2 qu5its j, j'

$$
\begin{aligned}
H^{(2)} &\equiv \sum_a \hat{H}^{(2,a)} \\
&= \left[\varepsilon\, \hat{j}_z - (V+g)\hat{\mathcal{X}}_{13} - g\hat{N}_{\text{pairs}}\right] \otimes \hat{I}_5 \\
&\quad + \hat{I}_5 \otimes \left[\varepsilon\, \hat{j}_z - (V+g)\hat{\mathcal{X}}_{13} - g\hat{N}_{\text{pairs}}\right] \\
&\quad - V \sum_{r,s \in \{(12),(23)\}} \left(\hat{\mathcal{X}}_r \otimes \hat{\mathcal{X}}_s - \hat{\mathcal{Y}}_r \otimes \hat{\mathcal{Y}}_s\right) \\
&\quad - \frac{g}{2} \sum_{\substack{r,s \in \{(01),(03), \\ -(14),-(34)\}}} \left(\hat{\mathcal{X}}_r \otimes \hat{\mathcal{X}}_s + \hat{\mathcal{Y}}_r \otimes \hat{\mathcal{Y}}_s\right)
\end{aligned}
$$

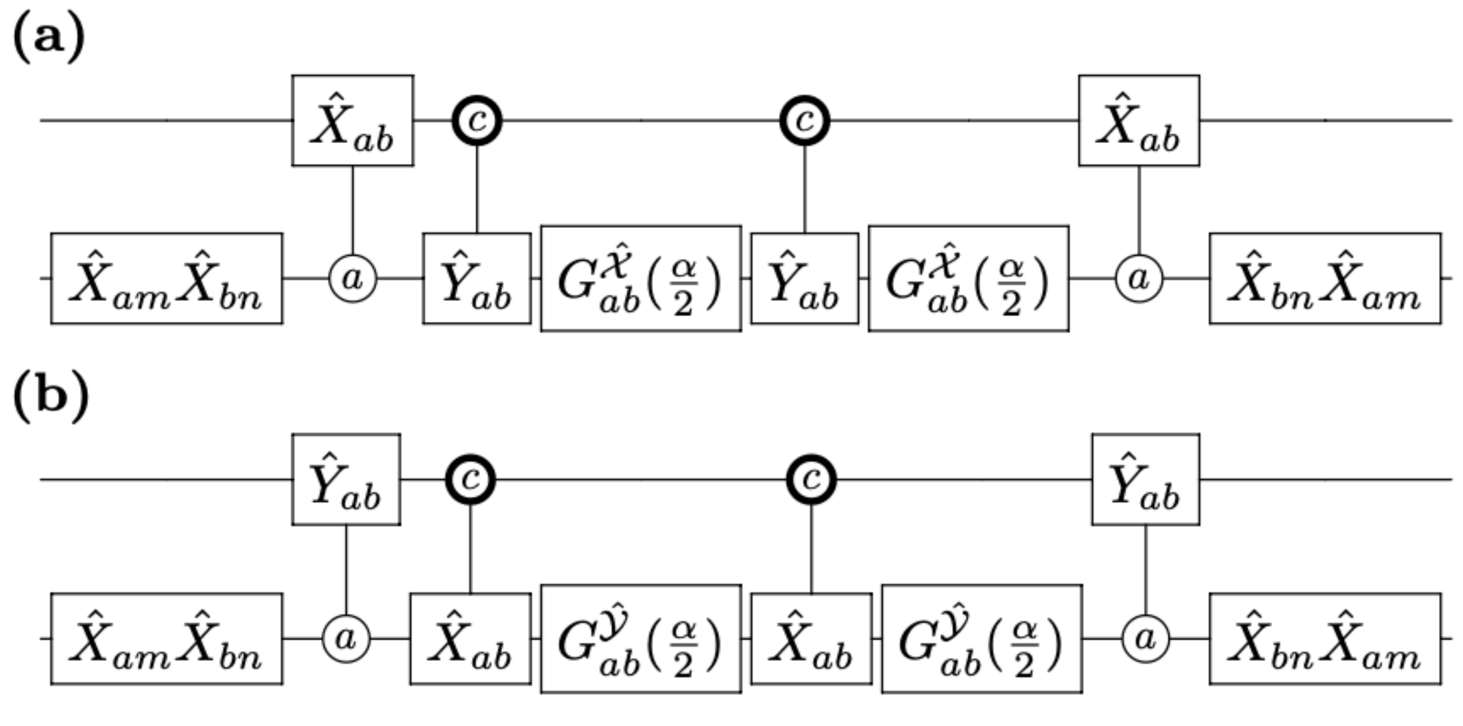
generators of Givens rotations

$G^{\mathcal{X}\mathcal{X}}_{abmn}(\alpha)$ $G^{\mathcal{Y}\mathcal{Y}}_{abmn}(\alpha)$

- **Trotter decomposition at leading order:**

$$\hat{U}(t) = e^{-i\hat{H}t} \simeq \left(e^{-i\hat{H}\Delta t}\right)^{n_{Trot}}$$

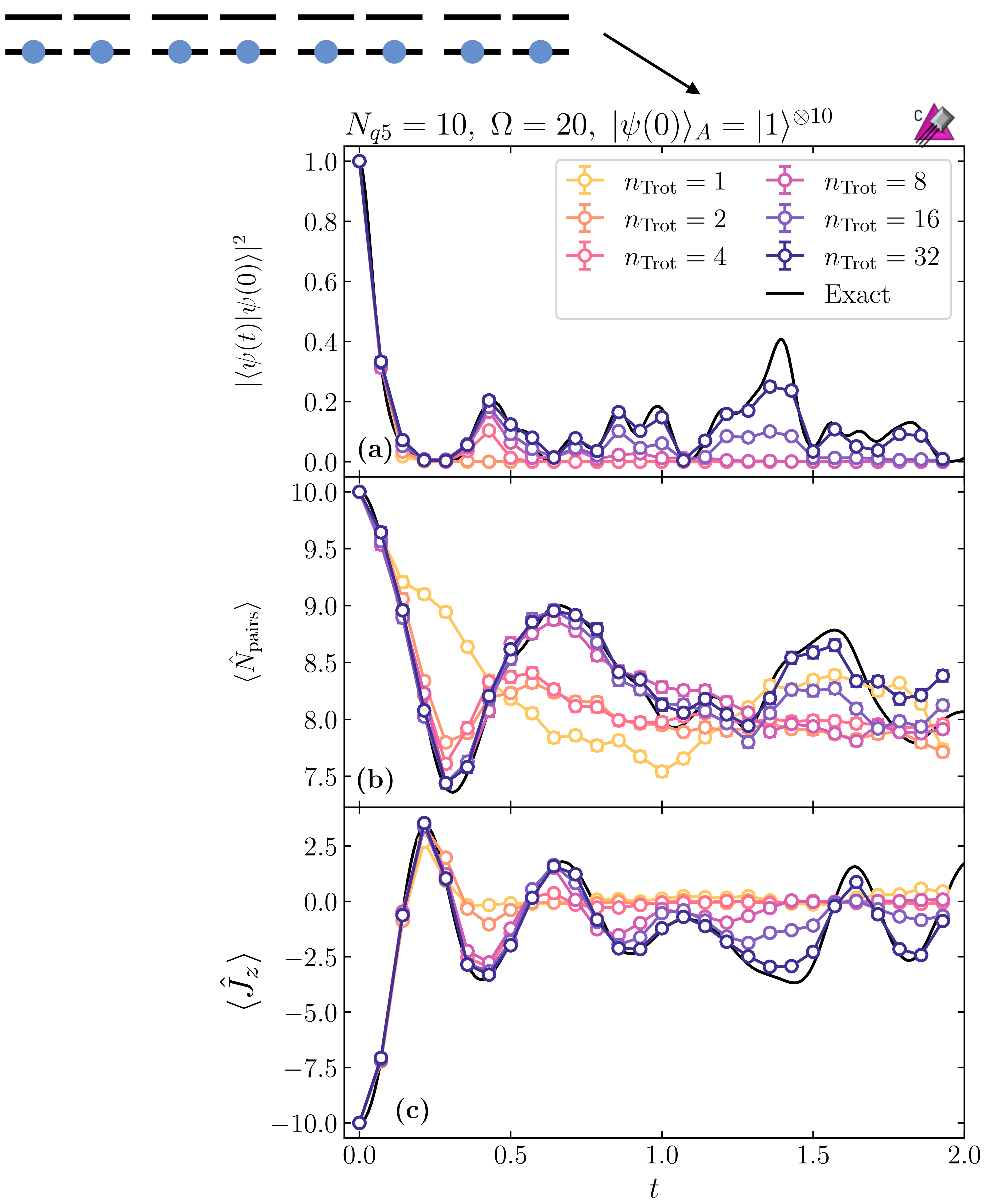
$$e^{-i\hat{H}\Delta t} = e^{-i\sum_{jj'} \hat{H}^{(2)}_{jj'}\Delta t} \simeq \prod_{jj'} \prod_a e^{-i\hat{H}^{(2,a)}_{jj'}\Delta t}$$

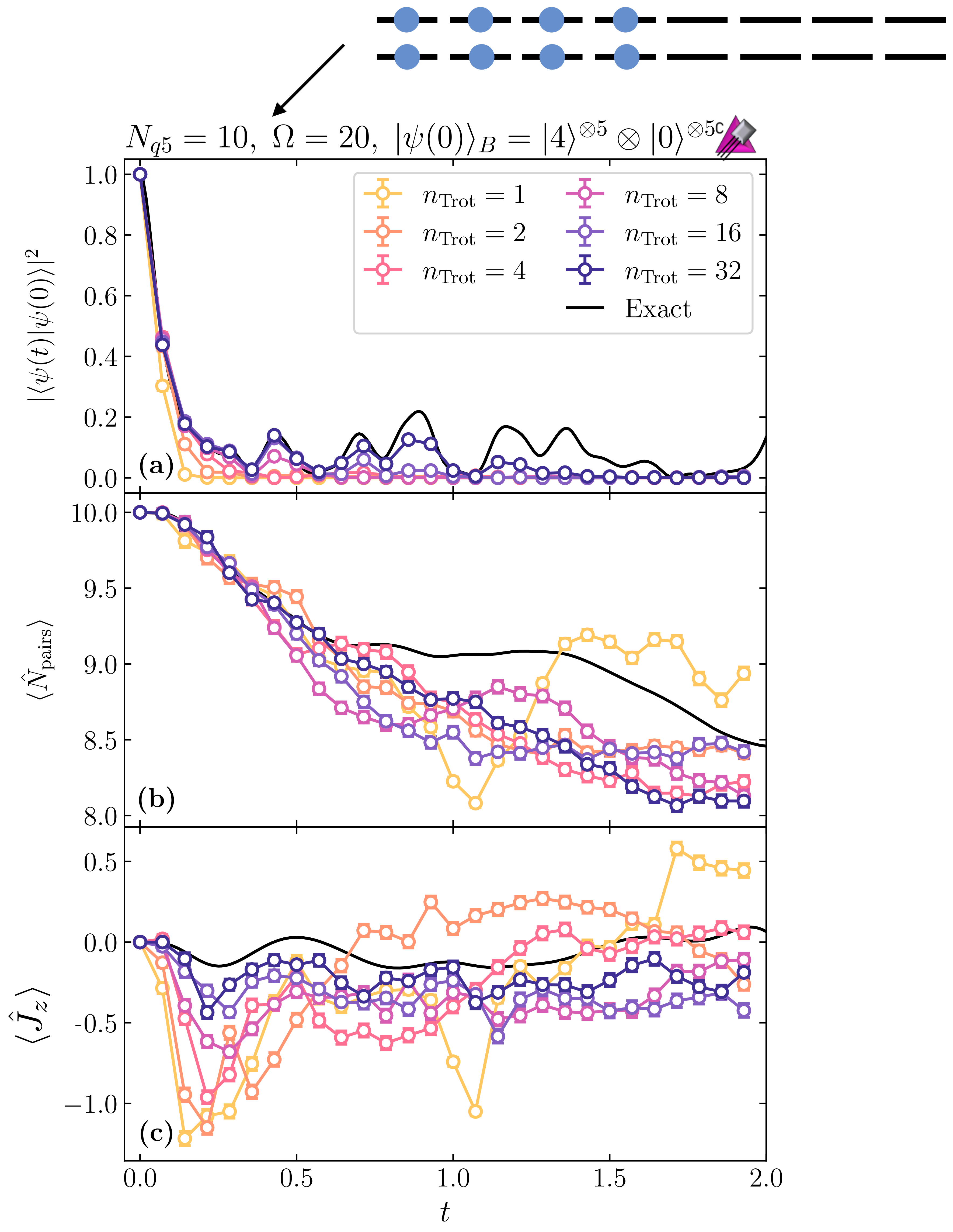


Circuits for $G^{\mathcal{X}\mathcal{X}}_{abmn}(\alpha)$ and $G^{\mathcal{Y}\mathcal{Y}}_{abmn}(\alpha)$

Symmetry-guided mapping of the Agassi model onto qudit systems

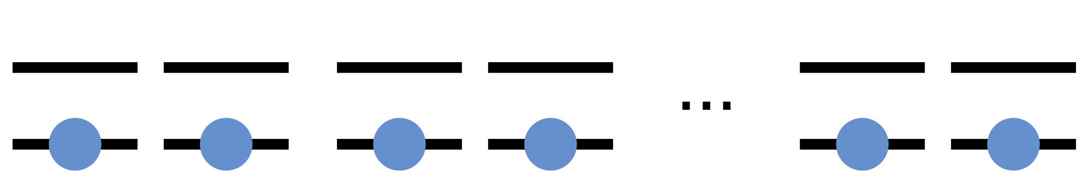
★ ***Developed a qudit-system simulator using Google's `cirq` software:***

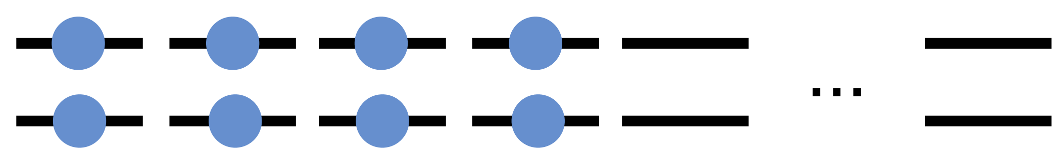




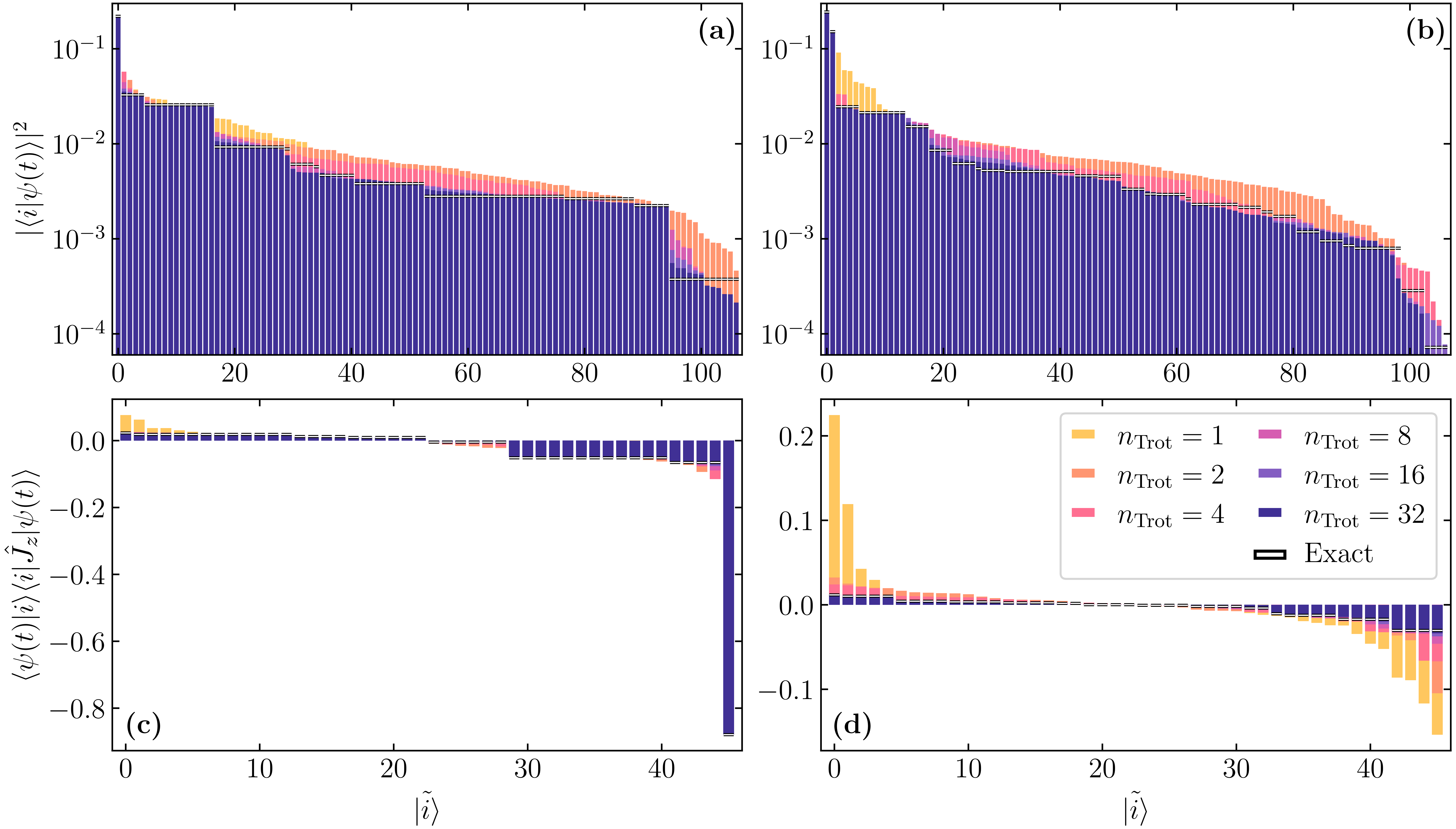
A new sign problem for quantum simulations

★ ***A new sign problem:*** $|\psi(0)\rangle = \sum_i c_i(0)|i\rangle$ ← Computational-basis states





$|\psi(0)\rangle_A = |1\rangle^{\otimes 4}$ (Low-energy state) $|\psi(0)\rangle_B = |4\rangle^{\otimes 2} \otimes |0\rangle^{\otimes 2}$ (high-energy state)

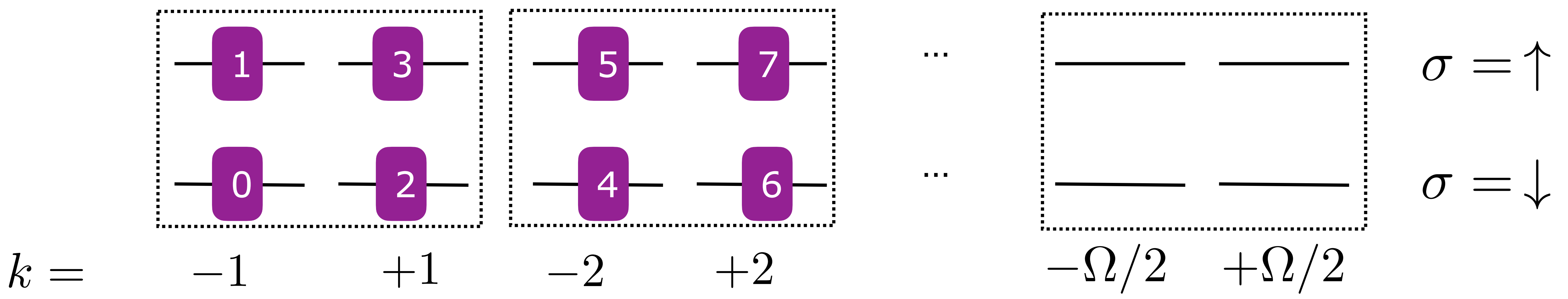


Qu5it resource requirements and comparison with qubit mappings

★ *Resource requirements and comparison with mappings onto qubits*

A) "Physics-Aware" Jordan-Wigner (paJW) mapping

Organization in terms of mode-pairs:



Bonsonization made explicit

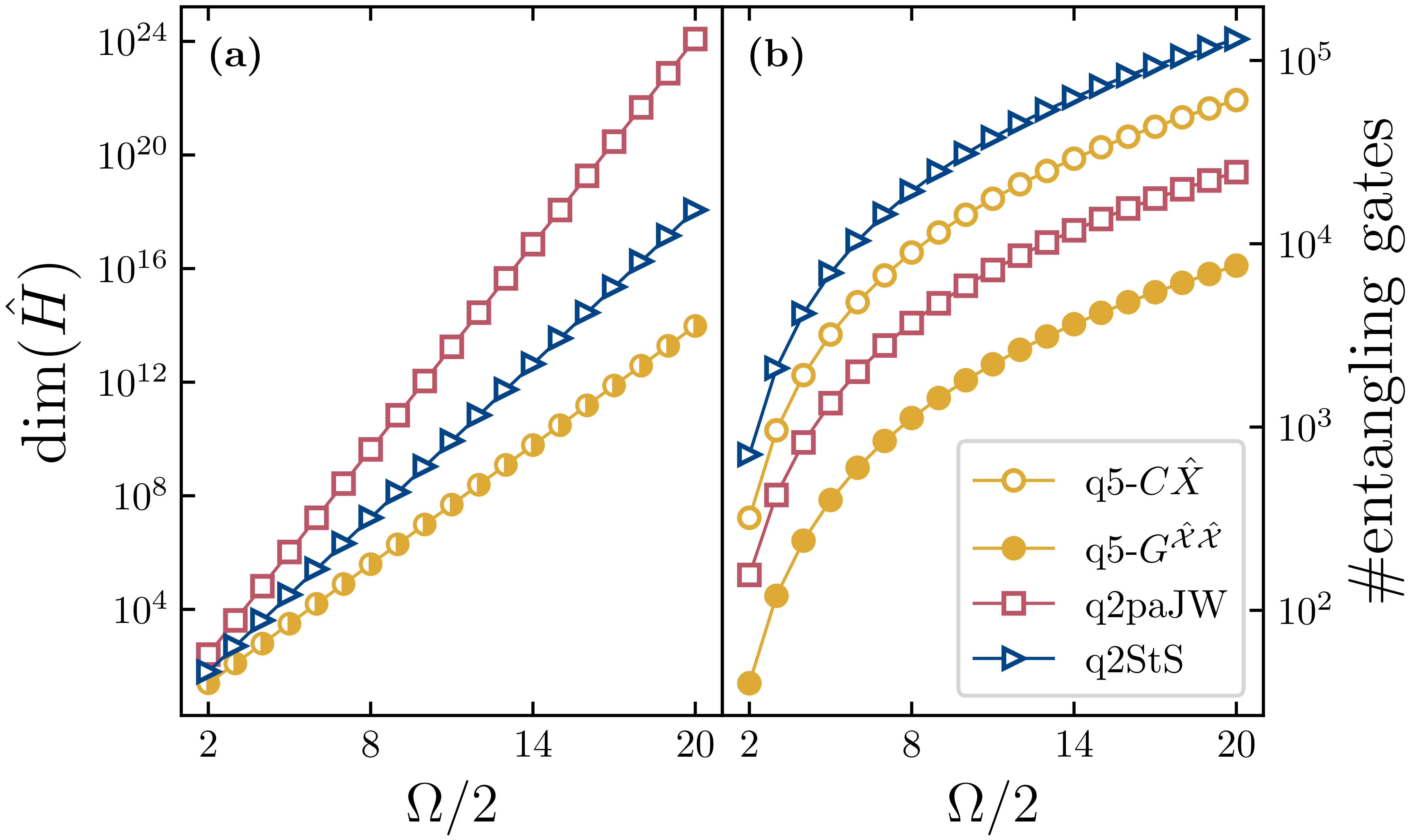
→ minimizes the number of phase operators Z
→ 4 qubits per mode pair

B) State-to-state (StS) qubit-qu5it mapping

→ 3 qubits are used to map the 5 states of one mode pair

$$|0\rangle = |000\rangle \;, \quad |1\rangle = |001\rangle \;, \quad |2\rangle = |010\rangle \;, \quad |3\rangle = |011\rangle \;, \quad |4\rangle = |100\rangle$$

Qu5it resource requirements and comparison with qubit mappings



Mappings to qubits/qudits guided by physics are typically advantageous

"Physics-aware" JW mapping to qubits

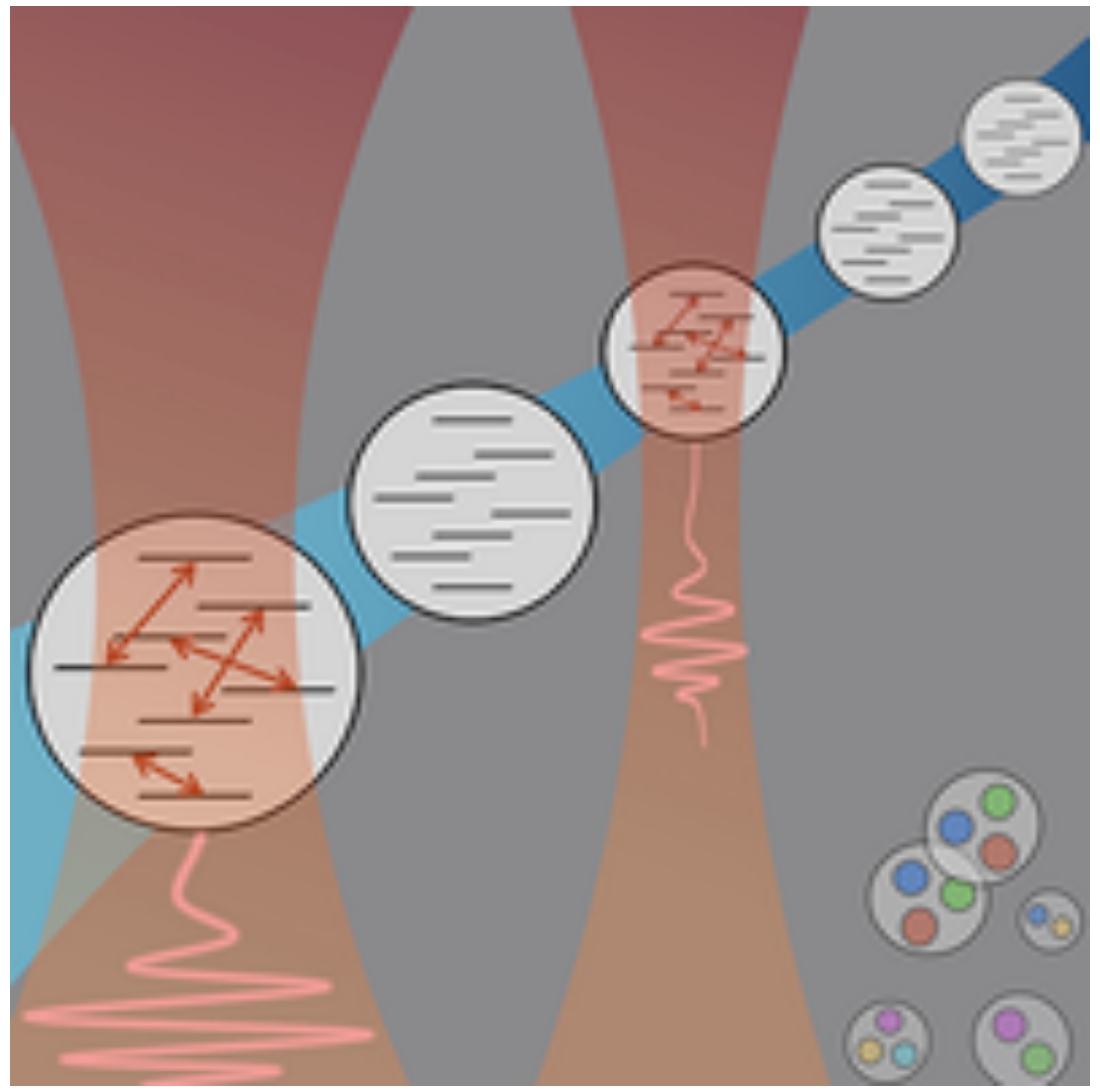
qu5it-state to qubit-states mapping

: Two-qu5it Givens rotations $G^{\mathcal{X}\mathcal{X}}_{pq,rs}(\alpha) = e^{-i\alpha \mathcal{X}_{pq}\otimes\mathcal{X}_{rs}}$ are available on the device

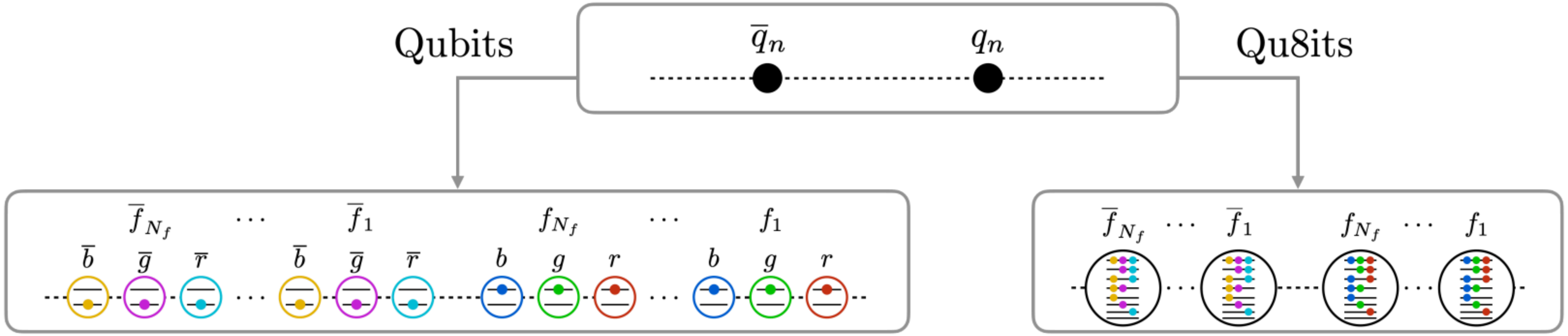
: They are implemented via generalized CX, CY

Qu8its for Quantum Simulations of 1+1D Lattice QCD

Illa, CR, Savage, PRD 110, 014507 (2024)
Editor's suggestion



$$H = \sum_f \left[\frac{1}{2} \sum_{n=0}^{2L-2} \left(\phi_n^{(f)\dagger} \phi_{n+1}^{(f)} + \text{h.c.} \right) + m_f \sum_{n=0}^{2L-1} (-1)^n \phi_n^{(f)\dagger} \phi_n^{(f)} \right] + \frac{g^2}{2} \sum_{n=0}^{2L-2} \sum_{a=1}^{8} \left(\sum_{m \leq n} Q_m^{(a)} \right)^2$$



Resource for time evolution (single Trotter step):

Qudits	Number of qudits	U_{kin} ent. gates	U_{el} ent. gates
Qubit ($d=2$)	$6N_f L$	$6N_f(8L-3)-4$	$N_f(2L-1)[23N_f(2L-1)-17]$
Qu8it ($d=8$)	$2N_f L$	$6N_f(2L-1)$	$4N_f(2L-1)[N_f(2L-1)-1]$
Reduction in resources ($L \to \infty$)	3	4	5.75

Conclusion

★ Quantum Information and Quantum Simulations represent new opportunities that can potentially advance nuclear physics both conceptually and computationally

★ Entanglement, Magic and Symmetries are key ingredients for designing efficient hybrid classical/quantum simulations of nuclear structure and real-time dynamics

★ More questions to address:
→ relations between entanglement and magic? Relations between quantum complexity and physical phenomena? (see works by Hamma, Dalmonte, Tirrito, Gu…)
→ How to probe quantum complexity in (nuclear physics) experiments?

★ Exchanges of ideas and techniques between fields of QMB physics and QIS is essential

Workshop on

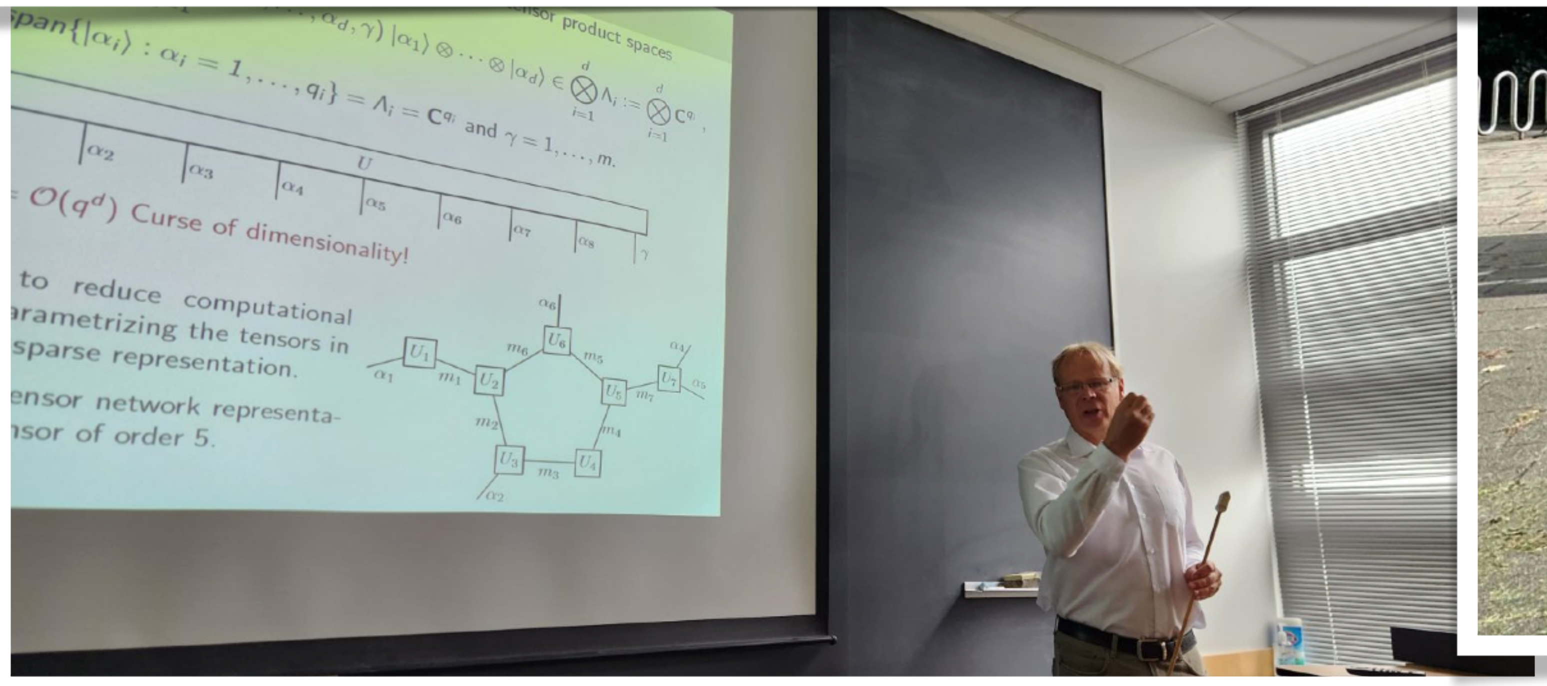
“Entanglement in Many-Body Systems: From Nuclei to Quantum Computers and Back”

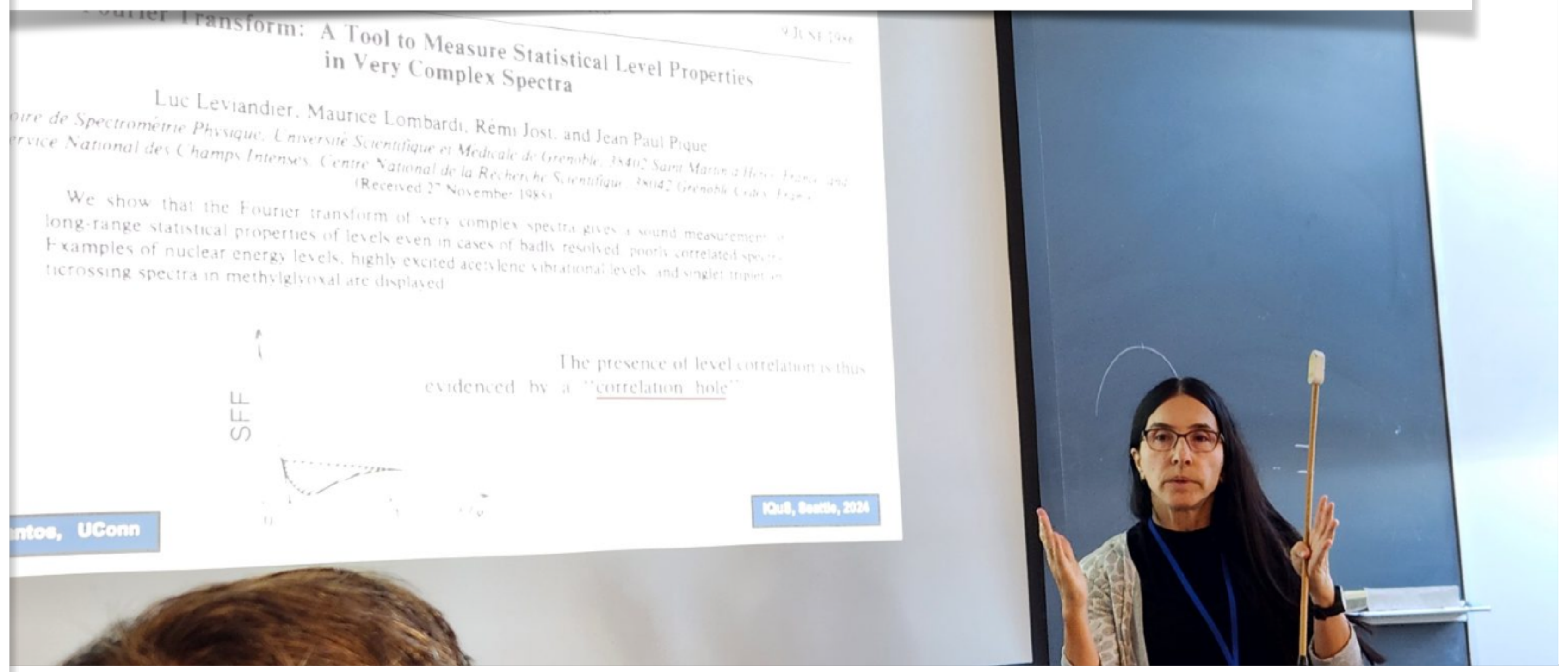
IQuS InQubator for Quantum Simulation

September 09-20, 2024

ORGANIZERS

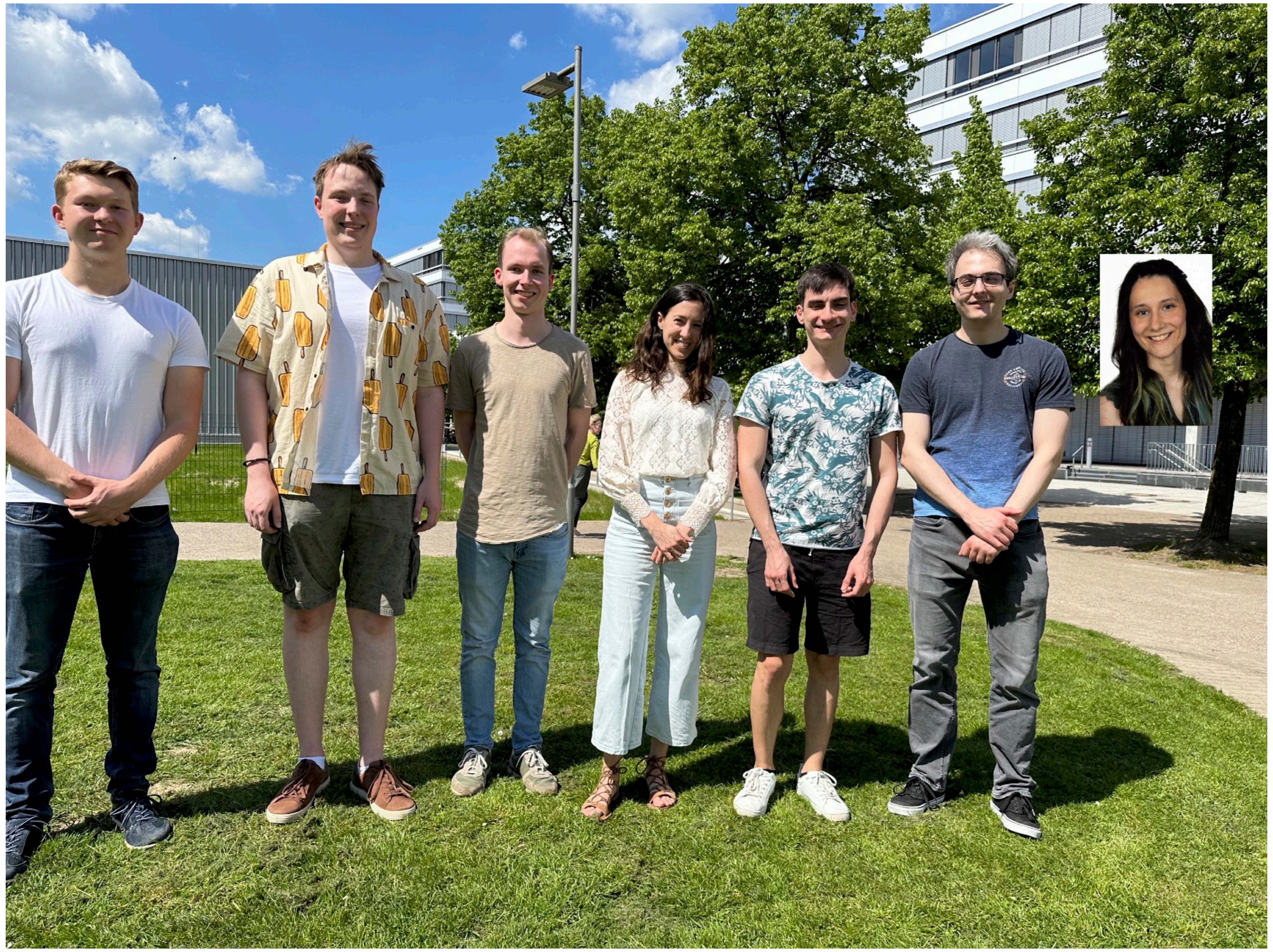
Entanglement Team : Mari Carmen Banuls, Susan Coppersmith, Calvin Johnson and Caroline Robin





Thanks to Collaborators!

Uni Bielefeld group



From left to right:
Erik Müller, Florian Brökemeier, Momme Hengstenberg, CR, Federico Rocco, James Keeble, Elisabeth Hahm

IQuS @ UW Seattle

Martin Savage

Marc Illa

ICTP @ Trieste

Emanuele Tirrito