Quantum Magic and Entanglement in Shell Model Nuclei

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Quantum Magic and Multi-Partite Entanglement in the Structure of Nuclei

arXiv:2409.12064

Outlook

- Nuclear Shell Model
- Mapping on a quantum computer
- Multi-partite entanglement in nuclei
- Non-stabilizerness measures in nuclei
- Comparisons
- Conclusions

Spherical Shell Model

- The active valence space is a truncation of the full Hilbert space
- The valence nucleons interact via a phenomenologically-adjusted two-body Hamiltonian

$$\hat{H} = \sum_i \varepsilon_i a_i^\dagger a_i \; + \; \frac{1}{4} \sum_{ijkl} \widetilde{v}_{ijkl} \; a_i^\dagger a_j^\dagger a_l a_k$$

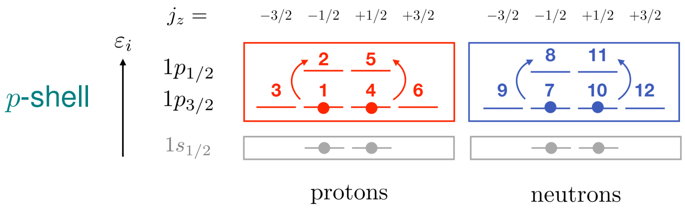
- The nuclear wavefunction is a linear combination of proton and neutron Slater determinants

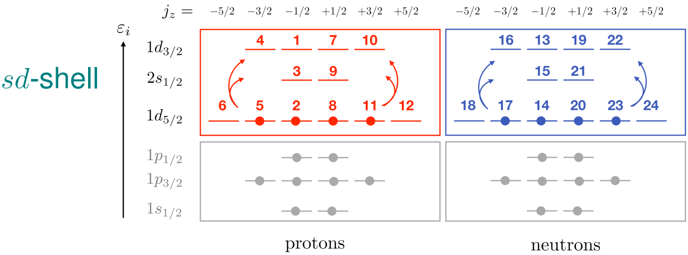
The nuclear wavefunction

$$|\Psi\rangle = \sum_{\alpha_\pi, \alpha_\nu} A_{\alpha_\pi, \alpha_\nu} \, |\Phi\rangle_{\alpha_\pi} \otimes |\Phi\rangle_{\alpha_\nu}$$

$$|\Phi\rangle_{\alpha_\pi} := \Pi_{i \in \alpha_\pi} a_i^\dagger \, |0\rangle$$

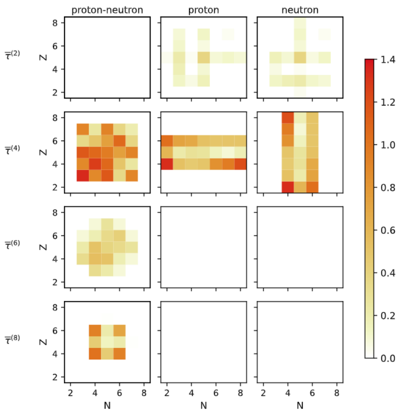
$$|\Phi\rangle_{\alpha_\nu} := \Pi_{i \in \alpha_\nu} a_i^\dagger \, |0\rangle$$

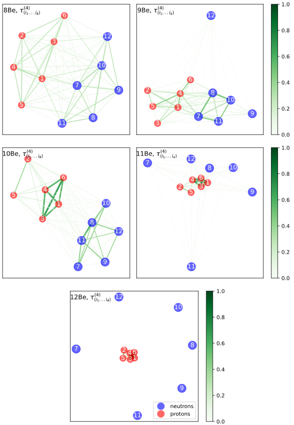




BIGSTICK: Johnson, Ormand, Krastev, Comp. Phys. Comm. 184 (2013)

N-tangles* in the p-shell





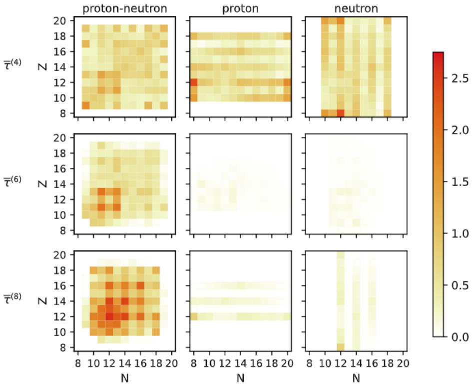
$$\tau^{(n)}_{(i_1...i_n)} = |\langle\Psi|\hat{\sigma}_y^{(i_1)} \otimes ... \otimes \hat{\sigma}_y^{(i_n)}|\Psi^*\rangle|^2$$

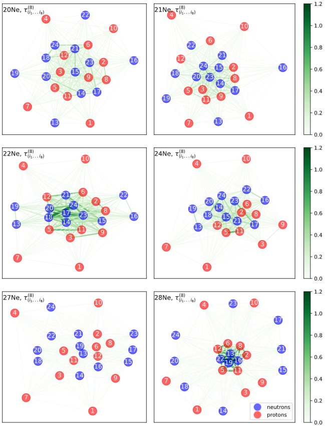
$$\overline{\tau}^{(n)}_{\pi,\nu,\pi\nu} \equiv \sum_{i_1,i_2,..i_n \in \pi,\nu,\pi\nu} \tau^{(n)}_{(i_1 i_2..i_n)}$$

$$e^{(4)}_{i_1 i_2} = \sum_{i_3<i_4} \tau^{(4)}_{(i_1,i_2,i_3,i_4)}$$

*Wong, Christensen, PRA 63, 044301 (2001)

N-tangles in the sd-shell

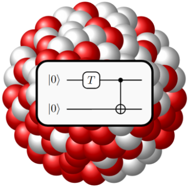




$$e^{(8)}_{i_1 i_2} = \sum_{i_3<i_4<i_5<i_6<i_7<i_8} \tau^{(8)}_{(i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8)}$$

Non-stabilizerness (magic)

- Stabilizer formalism centered around the Pauli group: $\mathcal{G}_{n_Q} = \{\varphi\, \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)} \otimes ... \otimes \hat{\sigma}^{(n_Q)}\}$
- Pauli stabilizer group of $|\Psi\rangle$: $\mathcal{S} = \{\hat{P} \in \mathcal{G}_{n_Q} s.t. \hat{P}\,|\Psi\rangle = |\Psi\rangle\}$
- $|\Psi\rangle$ is a stabilizer state if $\mathcal{S}$ contains $d = 2^{n_Q}$ elements and is fully specified by it
- Stabilizer states can be prepared with Clifford operations only (Gottesman-Knill theorem) $\rightarrow$ efficient classical simulation



Stabilizer Rényi entropies

α-Rényi entropy

$$\mathcal{M}_\alpha(|\Psi\rangle) = -\log_2 d + \frac{1}{1-\alpha}\log_2\left(\sum_{\hat{P}\in\widetilde{\mathcal{G}}_{n_Q}} \Xi_P^\alpha\right), \qquad \Xi_P = \frac{\langle\Psi|\hat{P}|\Psi\rangle^2}{d}$$

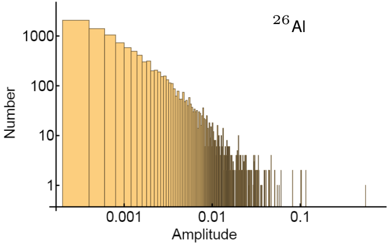
- Vanishing SREs for stabilizer states
- Shannon entropy in the $\alpha = 1$ limit
- $\alpha > 1 \rightarrow$ distance from the nearest stabilizer state
- $\alpha < 1 \rightarrow$ stabilizer rank and complexity of classical simulations

Leone et al Phys. Rev. Lett. 128.5 050402 (2022)

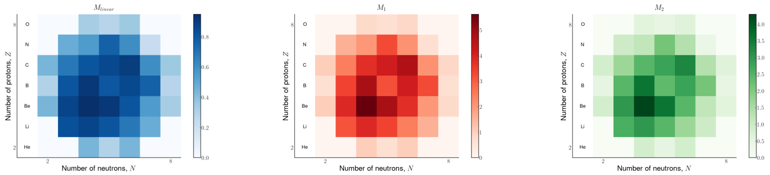
Haug, Aolita, Kim, Probing quantum complexity via universal saturation of stabilizer entropies, (2024), arXiv:2406.04190

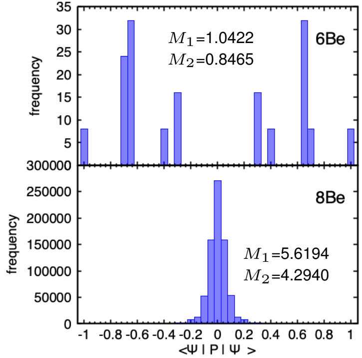
New PSIZe-MCMC algorithm

- First use of MCMC techniques to compute SREs in Tarabunga et al
- Slow thermalization of the chains in non-spherical nuclei, due to the amplitude distribution in the wavefunction
- $d = 2^{n_Q}$ strings have typically a higher probability ($\Xi_P \sim 1$) than the other $d^2 - d$
- **P**auli-**S**tring $\hat{\mathbf{I}}\hat{\mathbf{Z}}$ **e**xact **MCMC**

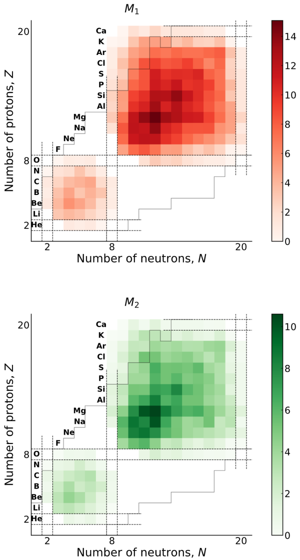


Tarabunga et al PRX Quantum 4, 040317 (2023)



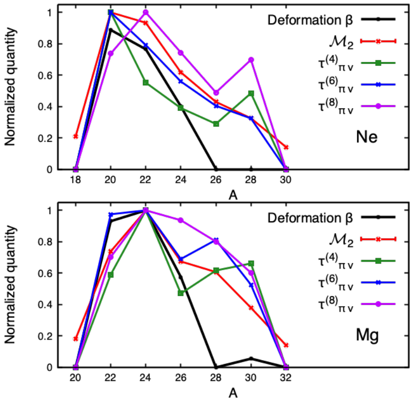


Collective structure phenomena influence quantum complexity



- Maximal magic is found to coincide with the maximal deformation β
- The quantum complexity persists beyond where β becomes small and extends through the region of shape co-existence
- The magic dependence upon J_z is a modest-sized effect

Comparisons for the Ne and Mg isotope chains



- Magic and n-tangles remain significant after β drops to zero
- The classical computing resources scale exponentially with the "shape-complexity" of the nucleus

Conclusions

- We need both entanglement and magic to represent quantum complexity
- We introduced the PSIZe-MCMC algorithm to accelerate the convergence of MCMC evaluations in deformed nuclei
- The complexity of p-shell and sd-shell nuclei is reflected in multi-nucleon entanglement and magic
- Transformations among the basis states and the use of a deformed/collective basis are expected to reduce the quantum complexity
- End goal is to gain insight to develop optimal partition of the workflow between classical and quantum computation in hybrid algorithms

Backup

$$\mathsf{M}_1 = -\sum_{\hat{P}\in\widetilde{\mathcal{G}}_{n_Q}} \Xi_P \log_2 d\, \Xi_P$$

$$\sum_{\hat{P}\in\widetilde{\mathcal{G}}_{n_Q}} \Xi_P \log_2 \Xi_P = \langle \log_2 \Xi_P \rangle_{\Xi_P}$$

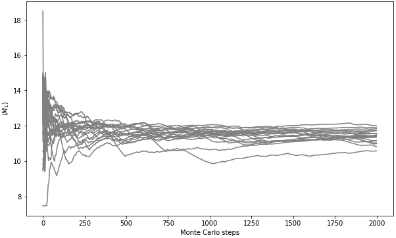
$$\mathsf{M}_1 \approx -\left\langle \log_2 \langle\Psi|P|\Psi\rangle^2 \right\rangle_{\Xi_P}$$

$$\mathsf{M}_2 = -\log_2 d \sum_{\hat{P}\in\widetilde{\mathcal{G}}_{n_Q}} \Xi_P^2, \qquad \mathbb{E}(\alpha) = \left\langle \frac{\langle\Psi|P|\Psi\rangle^{2(\alpha-1)}}{d^{(\alpha-1)}} \right\rangle_{\Xi_P}$$

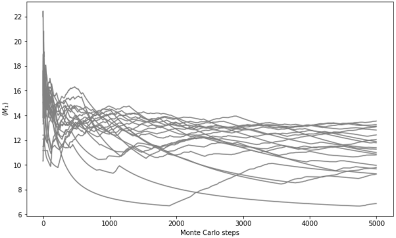
$$\mathsf{M}_2 \approx -\log_2 (d\, \mathbb{E}(2)) = -\log_2 \left\langle \langle\Psi|P|\Psi\rangle^2 \right\rangle_{\Xi_P},$$

$$\mathsf{M}_{lin} = 1 - d \sum_{\hat{P}\in\widetilde{\mathcal{G}}_{n_Q}} \Xi_P^2$$

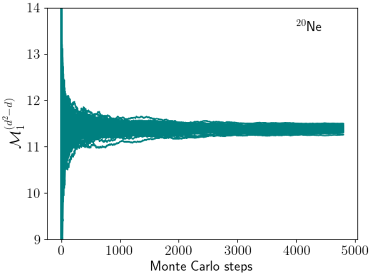
$$\mathsf{M}_{lin} \approx 1 - d\, \mathbb{E}(2) = 1 - \left\langle \langle\Psi|P|\Psi\rangle^2 \right\rangle_{\Xi_P} .$$



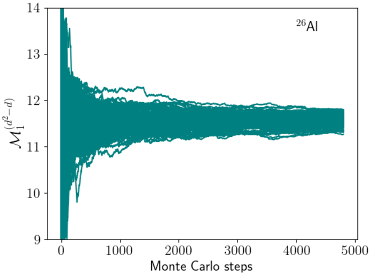
⟨M₁⟩
Monte Carlo steps



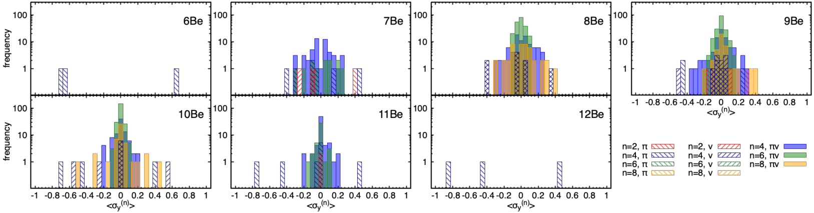
⟨M₁⟩
Monte Carlo steps



$\mathcal{M}_1^{(d^2-d)}$
^{20}Ne
Monte Carlo steps



$\mathcal{M}_1^{(d^2-d)}$
^{26}Al
Monte Carlo steps



6Be
7Be
8Be
9Be
10Be
11Be
12Be
frequency
100
10
1
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
<σy(n)>
n=2, π
n=4, π
n=6, π
n=8, π
n=2, ν
n=4, ν
n=6, ν
n=8, ν
n=4, πν
n=6, πν
n=8, πν

PSIZe-MCMC

$$\Xi_{\overline{P}} = \frac{\langle\Psi|\overline{P}|\Psi\rangle^2}{d} \ ,$$

$$\mathcal{P} = \sum_{\overline{P}} \Xi_{\overline{P}} \ \leq \ 1 \ ,$$

$$\mathcal{S}_{\overline{P}} = \sum_{\overline{P}} \Xi^2_{\overline{P}} \ , \ \mathcal{L}_{\overline{P}} \ = \ -\sum_{\overline{P}} \Xi_{\overline{P}} \log_2 d \ \Xi_{\overline{P}} \ .$$

$$\mathcal{S}_{P\notin\overline{P}} = \sum_{P'\notin\overline{P}} \Xi^2_{P'} \ \approx \ (1-\mathcal{P}) \ \langle\Xi^2_{P'}\rangle_{\mathrm{MCMC}}$$

$$\mathcal{L}_{P\notin\overline{P}} = -\sum_{P'\notin\overline{P}} \Xi_{P'} \log_2 d \ \Xi_{P'} \ \approx \ -(1-\mathcal{P}) \ \langle\Xi_{P'} \log_2 d \ \Xi_{P'}\rangle_{\mathrm{MCMC}} \ .$$

$$\sum_{P} \Xi^2_{P} = \mathcal{S}_{\overline{P}} \ + \ \mathcal{S}_{P\notin\overline{P}} \ , \ \sum_{P} \Xi_{P} \log_2 d \ \Xi_{P} \ = \ \mathcal{L}_{\overline{P}} \ + \ \mathcal{L}_{P\notin\overline{P}} \ .$$